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## Specialists or All-Rounders: How Best to Select University Students?

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DISCUSSION PAPER SERIES

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MAY 2022

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## ABSTRACT

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# Specialists or All-Rounders: How Best to Select University Students?\*

This paper studies whether universities should select their students only using specialised subject-specific tests or based on a broader set of skills and knowledge. I show that even if broader skills are not improving graduates' outcomes in the labour market, the university chooses to use them as a criterion for selection alongside the mastery of more subject-specific tools. Empirically, I exploit the variation between subject-specific and non-specific entrance exam sets on Portuguese students' large administrative dataset. My central finding is that, on average, universities with less specialised admission policies admit a pool of students who obtain a higher final GPA.

**JEL Classification:** I23, I24, I28, J24

**Keywords:** university choice, admission tests, job market, general skills

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# 1 Introduction

Which university entrance criteria matter for the academic performance of the students admitted? Most higher education institutions (HEIs) rely on standardized tests as the primary mechanism to select their students. Additionally, they can activate a second source of information. In some cases, the institution considers student essays, interviews, as well as information on students' extracurricular activities and experience. Other HEIs design their own entrance exams as a complement to standardized tests. The purpose is to obtain a more comprehensive assessment of higher education (HE) applicants (for reviews see, e.g., [Hoxby, 2009](#), and [Zwick, 2017](#)). However, how broad the admission criteria should be is a question that has been given little attention in the literature.

I study the role of specialisation of the admission criteria as a choice variable used by HEIs to maximise their objective function of admitting the best pool of students possible. These are expected to have better academic performance, better placement after degree completion, and better future labour market achievement. As a result, HEIs benefit from improvements in their own reputation.

The central finding of this paper is that universities with less specialised admission policies admit a pool of students who perform better at university by the end of the degree (namely, they obtain a higher final GPA on average). This is a surprising and relevant result. Many universities operate under the assumption that candidates who perform better at a subject-specific test, will go on to perform better in that subject at university. For instance, in Physics Sciences, maths is often perceived as a relevant *field-specific skill*, while languages convey information on broader cognitive skills. I define *general skills* as those that are not directly rewarded by the field of study. They might be informative about the student's ability, but they are not specific to the field. Ability refers to the potential or innate ability that an individual is born with, while skills are learned and acquired with time. Strong emphasis on field-specific skills rewards students that specialise early in high school rather than cultivate a more versatile portfolio of skills.

In my theoretical framework, I consider a single university and a continuum of students. The university's objective is to admit the best pool of students possible. In turn, students aim to maximise an utility function that depends on their own future performance and the effort level exerted on the university admission tests. I assume that the

university can select students based on a wider and broader set of tests. Of the tests available, some tests measure skills that will be considered specialised, while others are tests of skills that have no direct influence on future student performance (the general skills) but serve solely as a signal of ability. Thus, the university must decide how to set its admission criteria, either i) using only a field-specific test; or ii) combine it with a test of a more general nature. In the latter case, the university gains additional information about the student's ability. However, in return, it must contend with less effort being put by students into the specialised test, because students optimally re-allocate the effort. In the case of a single field-specific test, the students' effort would be directed solely towards acquiring knowledge and preparing for the exam that contributes to future academic performance and is directly valued in the labour market (see, e.g., [Card et al., 2018](#)).

From a theoretical point of view, I find that the university chooses to use a general skill test as a criterion for selection alongside a test to evaluate subject-specific skills. Although, in equilibrium, students deviate effort from the field-specific test (the productive test) to the general skill test, a different set of students (more able on average) is admitted and ultimately performs better than when only a field-specific requirement is in place.

In my empirical analysis, I use Portugal as a case study. The admission process in Portugal to public universities is comparable to several other countries (e.g. Spain, Brazil, Colombia, Hungary and Denmark). Admission to public HEIs in Portugal is centralized and managed by the government. HEIs choose their admission requirements. Students, in turn, can apply to up to six HE programmes (pairs institution-degree), ranked by order of preference. Although the Portuguese system has specificities, the lessons one learns from it provide generalizable insights to any HE system that considers at least one of two different metrics in the admission process: one that matches closely to what the student actually learns in the subject of study (a field-specific test for instance); and another one that conveys information of a more generic nature, not directly rewarded by the field of study.

I rely on an extremely rich administrative dataset of the population of HE applicants, from 2008 to 2018. Over that period, I observe approximately 800 thousand students' applications to the first year of a HE programme. For each applicant, there is informa-

tion regarding personal characteristics, socio-economic background, previous academic achievement, the application process (including all programme preferences stated with the corresponding overall application score and each exam score), and the HE placement. Moreover, for five years, I have information on students' performance at HE for each year of their degree, and subsequent graduation information (namely, the graduation date and the final GPA).

I consider the Portuguese exam as a general skill test. The Portuguese exam is compulsory and the only common exam in all academic tracks at high school. The performance in this exam is available for all applicants to HE. With some exceptions, most programmes have multiple tracks of admission. They either require a subject-specific set of exams as admission criteria or a broader set of exams. The student will be ranked according to whichever set of exams yields the highest overall application score. For instance, suppose that a student applies to an Economics degree at a particular university. The score that determines the ranking in the application process is either the Mathematics exam score or the average of Mathematics and Portuguese exam scores, depending on which generates the highest application score and hence the highest feasible position in the ranking. The Mathematics exam is considered to be the field-specific requirement, while the Portuguese exam is the general skill test. However, not all HE programmes consider a non-specific set of exams as an admission criterion. Thus, I perform my empirical analysis on HE programmes that include the general skill test (the Portuguese exam) in the broader set of exams.<sup>1</sup> Therefore, my results do not apply to highly specialised subjects, such as Medicine.

Due to this admission criteria design, I observe students being admitted to the same programme based on performance in different exam sets. In my empirical analysis, the treatment is the inclusion of the Portuguese exam as an admission criterion at university. Nevertheless, students might self-select into programmes on the basis of unobserved attributes. I tackle the problem of selection into treatment by first determining an alternative application score for each student. The application score in the case that the student was admitted with a different exam set. For each student, I compute: i) her application score in the case the Portuguese exam was considered (my measure of gen-

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<sup>1</sup>Note that although the definition of a general skill test may be context-dependent, I exclude from my analysis all programmes for which the Portuguese exam is a mandatory admission criterion (such as linguistic degrees). Later in this paper, I discuss differences across fields of study.

eral skills); ii) her application score in the case that the Portuguese exam score was not considered (my field-specific skills measure).

I define exposure to treatment in the following way. I focus my analysis around the programme admission threshold (the admission score of the last admitted student). Within each programme, I define three categories of admitted students: the *specialists* (those admitted with a field-specific test and whose general skills would not have been enough to gain entry in HE), the *generalists* (those admitted purely based on their performance on the general skill test who would not have been admitted only based on the field-specific test result), and the *all-rounders* (those that could have been admitted with each one of the two types of tests). I observe differences in academic student performance across the different groups at distinct landmarks of the academic programme. In particular, I find that: i) the *generalists* perform no worse than their *specialist* peers, by the end of the first academic year; and ii) by the end of the programme the *specialists* are outperformed by the *all-rounders* and the *generalists*.

My results have substantial implications for university admission practices. Universities that include a general skill test in their entrance set of exams benefit at the margin of admission, as this has a positive effect on the average student performance. Although this effect is not observable by the end of the first year, students admitted based on a broader set of skills and knowledge perform better on average towards the end of the programme.

An additional policy implication surrounds the role of the field-specific exam, as it may or not be fit for purpose. Differences in student performance across groups suggest that the field-specific exam is not an effective test of the specialist skills associated with high achievement at university, which is the main policy message of my paper. Although I find evidence that the field-specific exam is a predictor of student performance by the end of the first academic year, the exam is not informative of overall performance. Thus, the field-specific exam does not guarantee that those who perform well on it will perform better at university.

Alternatively, another possible interpretation is that the general and field-specific exams measure the same characteristics, but the general exam is just a more precise signal. In that case, the result would be driven by the exams' precision rather than the differences between general and specific skills. Thus, the generalization of the previous



policy implication will depend on how exams are designed in other settings.

I contribute to different strands of literature. First, my paper contributes to the body of work that seeks to evaluate the role of standardized tests in the admission process to HE (e.g., Rothstein, 2004; Zwick and Green, 2007). Examples of standardized tests include A-levels in the UK and SAT/ACT in the US. The widespread use and reliance of these tests has been criticised in the literature for favouring ethnicity (e.g., Bridgeman et al., 2000; Freedle, 2003), gender (e.g., Leonard and Jiang, 1999) or socio-economic background (e.g., Zwick, 2019; Wyness et al., 2021). Despite a lack of consensus in the literature on whether such tests predict performance at university (e.g., Burton and Ramist, 2001; Kuncel and Hezlett, 2007; Radunzel and Noble, 2012), they are the most common instrument used to admission. In this paper, I conclude that admission tests indeed help to predict student performance at university. Namely, the non-specific tests are informative of overall performance while the specific tests only predict performance in the first year of the HE programme.

Secondly, my paper contributes to the literature on the effects of broadening admission criteria in HE (e.g., Sternberg, 2010; Sternberg et al., 2012; Stemler, 2012; Schmitt, 2012; Niessen and Meijer, 2017) and the impact of collaborative and other skills in the labour market analysis (e.g., Deming, 2017). Some HEIs rely on a second source of information. For instance, in the US, some Ivy League universities rely on interviews. In the UK, Oxford and Cambridge establish their own entrance exams for some degrees. Often, this extra step in admission broadens the type of information gathered about the student by the institution. My work suggests that broadening the nature of cognitive admission tests leads to increased performance at university because abler students are admitted. Nevertheless, the general admission test should be considered as a complement to field-specific requirements.

Thirdly, this paper contributes to the literature on the predictive performance of tests scores on the implications for admission policies. For instance, although high-stakes assessments influence students' decision to apply to HE (Papay et al., 2016), Bettinger et al. (2013) proposes to reduce the number of ACT components to improve college admissions. At the same time, other papers show the predictive power of past performance, such as high school transcripts (e.g., Belfield and Crosta, 2012; Cyrenne and Chan, 2012; Dooley et al., 2012). Moreover, prior work shows that combining teacher scores and high stake

assessments is a better practice for selecting students (Westrick et al., 2015; Zwick, 2019; Silva et al., 2020). My results corroborate that high-stakes assessments are a predictive tool of students' future academic performance.

Finally, my work relates to literature in the field of economics of HE. While previous work has focused on different aspects of admission to HE, such as the role of student portfolios, admission tuition, or college choice and selectivity (e.g., Epple et al., 2006; Avery and Levin, 2010; Avery et al., 2012; Hoxby et al., 2013), my paper focuses on one novel aspect of the admission process to HE — the choice of the examination structure, namely how specific should be the admission exams. Even under the assumption that some skills have no direct influence on student performance, and thus evaluating them may distract from learning more productive skills, I show that the best way for the university to resolve the resulting trade-off is to have a non-zero weight on the general skill test. My empirical evidence also shows that universities with broader admission exams select a better pool of incumbents.

The remainder of the paper proceeds as follows. Section 2 sets up the theoretical framework and its assumptions. Section 3 defines the equilibrium in the two stages: the students' effort and the university's choice of admission criteria. In that section, I also consider an extension of the model where the social planner plays a role. Section 4 describes the institutional setting and section 5 presents the datasets. Section 6 presents the empirical set-up and section 7 presents the estimation strategy. Section 8 presents the results and section 9 discusses possible mechanisms. Section 10 concludes.

## 2 Theoretical Framework

The design of admission policies aims at overcoming problems of imperfect information (Stiglitz, 2000), namely information asymmetries (Teixeira et al., 2006) that characterize HE markets, given that it is not possible to directly observe a candidate's ability.<sup>2</sup> College admission assessments provide incentives for signalling via both productive and non-productive activities. The assumption is that it is worthwhile to invest in skills for the specialized assessment but not for the general-interest assessment. The key finding highlighted before is that HEIs should use a combination of field-specific and general ad-

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<sup>2</sup>For a review of the student portfolio problem see, e.g., Araujo et al. (2007); Chade et al. (2014); Che and Koh (2016).

mission requirements to overcome the problems of imperfect information. In this section, I build a simple model to illustrate that point.

In my theoretical framework, I focus on the choice of the examination structure in HE during the process of admission.<sup>3</sup> I study a single university (or college) offering a single degree. A continuum of individuals of measure 1 want to apply to university. However, there are more applicants than university places. I assume that only half of the applicants are able to be admitted to the university. The time of the sequential game that I model below is as follows: the university sets the admission exams required to get admission, and then students decide the amount of effort to exert on each exam. The text below describes the two types of admission exams considered, the goals of each one of the agents, and the key assumptions of the model.

## 2.1 Ability and Admission Tests

Students are identical in every respect except their innate ability, and there are two levels of ability, the low type  $\alpha_L$  and the high type  $\alpha_H$ . I follow MacLeod and Urquiola (2015) and Gary-Bobo and Trannoy (2008) by assuming that students do not observe their own ability. Both types of students have the same expectation of their own ability,<sup>4</sup>  $\bar{\alpha}$ . In order to be admitted to university, each student needs to perform on two admission tests. I assume that the observed score  $T_i$  on the admission test  $i \in \{1, 2\}$  provides a noisy measure of ability and effort:

$$(1) \quad T_i = \alpha + e_i + \epsilon_i,$$

where  $e_i$  is the effort the student put into studying for test  $i$ , and  $\alpha$  denotes the innate ability of each student. Additionally, I assume that the two tests are independent,  $\epsilon_1$  follows the uniform distribution  $[0, a]$ , where  $a < 1$ , and  $\epsilon_2$  follows the uniform distribution  $[0, 1]$ . Thus,  $T_1$  is more precise than  $T_2$ . Think on  $T_1$  as a field-specific exam and  $T_2$  as a

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<sup>3</sup>To simplify the analysis, I ignore the student choice of subjects as well as the existence of competition effects between universities. For a survey of the literature on returns to curriculum and college major choices see, e.g., Bound and Turner (2011), Altonji et al. (2012), and Patnaik et al. (2020).

<sup>4</sup>Albeit her expectation might not be the correct one, I assume that the expectation is constant for simplicity purposes.

general admission test.<sup>56</sup>

## 2.2 Labour Market and Wages

Similarly to Gary-Bobo and Trannoy (2008), I assume that there are two categories of workers only, the graduates from the university (the skilled ones) and the ones that did not study at university (unskilled). I assume that an individual's wage,  $w$ , as a possible measure of future student's performance that depends on individual's innate ability,<sup>7</sup> and on the effort exerted for  $T_1$ . The individual's wage is given by the following relation:

$$(2) \quad w = \begin{cases} \gamma\alpha + e_1, & \text{if the individual attends university} \\ \beta\alpha + e_1, & \text{if the individual does not attend university} \end{cases}.$$

The presence of the term  $e_1$  in both equations indicates that  $T_1$  (the field-specific exam) has a long term benefit to the individual.  $T_1$  is assumed to be strongly related to potential future performance. Nevertheless, both admission tests produce a signal of ability which affect wages.<sup>8</sup> There is an increment in wages from attending university,  $v\alpha = (\gamma - \beta)\alpha$ , which is proportional to the individual's ability. I assume that  $v = \gamma - \beta$  is positive. Students with higher ability are expected to earn higher wages and to benefit more from attending university.

For clarity, labour market potential performance will be a function of student's innate ability,  $\alpha$ , and of  $e_1$ . As opposed to ability  $\alpha$ , which is time-invariant,  $e_1$  is acquired with maturity. Therefore, even if the student does not attend university, those skills have an impact on students' future performance.<sup>9</sup>

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<sup>5</sup>An alternative way of dichotomizing admission tests is to think about student's talent as multidimensional. The student has a portfolio of skills, cognitive and non-cognitive. In that setting, you can consider  $T_1$  as a test that measures cognitive skills, and the student can prepare for it, and on  $T_2$  as a non-cognitive test, for instance, an IQ test.

<sup>6</sup>The assumption that students do not know their ability will allow me to present a "representative agent" setting from the students' perspective. Nevertheless, students who perceive themselves to be specialists, for instance, might decide to invest solely in  $T_1$ , while other might take the opposite approach. Specialized strategy is a phenomenon that might play a role and it is studied in the counter-signalling literature (e.g., Feltovich et al., 2002).

<sup>7</sup>Following the neoclassical framework proposed by Becker (2009) and Mincer (1974).

<sup>8</sup>I consider the extreme situation where  $T_2$  is not informatively, directly, of labour market performance. Nevertheless, although  $e_2$  is not considered in the equation, a high score on  $T_2$  may be indicative of ability  $\alpha$ , which is reflected in the wage equation.

<sup>9</sup>For a review regarding the impact of tests scores on labour market outcomes, and national income see Chetty et al. (2014) and Hanushek and Woessmann (2010).

## 2.3 University

The university needs to determine the weight which it allocates to each admission test. Let  $\lambda$  be the normalized weight of  $T_2$ . The goal of the university is to maximise the total wage of its own students by using  $\lambda$  as an instrument, such that all students with  $T_1 + \lambda T_2 \geq \tau^*$  are admitted, where  $\tau^*$  is an endogenous threshold.<sup>10</sup>  $\tau^*$  it is the minimum admission score of the weighted average of the two tests scores among the admitted students. Considering that the university admits half of the population,  $\tau^*$  is the median of the weighted average of the two admission tests. I assume that there are as many low ability students as high ability students in the population and, the university is not able to distinguish between high and low ability students. Depending on  $\lambda$ , measures  $n_H$  of high ability and  $n_L$  of low ability, students will be admitted to the university.

## 2.4 Students

In the model, the track of a student is as follows: a student leaves the High School, and she applies to the university. Conditional on passing the admission tests she is accepted at university. Otherwise, she goes directly to the labour market. A student's objective function is the maximisation of the difference between her future wage and effort cost. When applying to university she knows  $\lambda$  and she exerts effort  $\mathbf{e} = (e_1, e_2) \subseteq R^2$  on the two admission tests. The effort has a utility cost measured by  $C(\mathbf{e})$ , increasing and convex. In the model, I assume the cost function to be<sup>11</sup>  $C(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2) + \delta e_1 e_2$  with  $0 < \delta < 1$ .

By exerting effort the student knows that with probability  $\Pi$  she would pass the admission threshold (where  $\Pi = \text{Prob}(T_1 + \lambda T_2 \geq \tau^*)$ ). The benefit to the student of attending university is that her wage will be boosted. The potential increase in earnings will be proportional to the student's ability. The student's ability is revealed when she leaves university.

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<sup>10</sup>Universities may care about outcomes other than academic performance in college or wages. That would be directly related with the stated mission of each university, and using a single proxy to measure college quality may induce measurement error (see, [Black and Smith, 2006](#)). However, for simplification purposes, I will assume that universities care about academic performance and wages.

<sup>11</sup>The parameter  $\delta$  measures the interaction between the two efforts. If I assume that they are complements,  $\delta$  would be negative. However, that would drive me to the solution where the student should only exert effort to one test. A more interesting case is when  $\delta$  is positive, which means that the marginal cost of one type of effort is not independent of the other type.

### 3 Equilibrium

In this section, I find the equilibrium by first determining the student's optimal level of effort exerted in the two admission tests and then incorporating that into the university's admission problem.

#### 3.1 Effort of Students

In equilibrium, each student knows the effort distribution of the entire cohort. As so, each student takes as given the effort choice of all her fellow students  $(e_1^*, e_2^*)$ , thus  $\tau^*$  is a function of  $(e_1^*, e_2^*)$ .

The assumptions that all students have the same expectation about their ability, and ability is revealed in the job market implies that the student expected wage is as follows:<sup>12</sup>

$$(3) \quad w = (\gamma\bar{\alpha} + e_1)\Pi + (\beta\bar{\alpha} + e_1)(1 - \Pi) = v\bar{\alpha}\Pi + e_1 + \beta\bar{\alpha}.$$

where  $\Pi = Prob(T_1 + \lambda T_2 \geq \tau^*)$ . The student's objective function is the maximisation of the difference between the expected wage and cost of effort. The maximization problem of a student is given by:

$$(4) \quad \text{Max}_{e_1, e_2} \quad v\bar{\alpha} \times Prob(T_1 + \lambda T_2 \geq \tau^*) + e_1 + \beta\bar{\alpha} - C(e_1, e_2).$$

The solution of the student's problem is stated in the following result.<sup>13</sup>

**Proposition 1.** *The optimal effort  $\mathbf{e}^*(\lambda) = (e_1^*, e_2^*)$  is:*

$$(5) \quad \begin{aligned} e_1^*(\lambda) &= \begin{cases} \frac{v\bar{\alpha}}{a(1-\delta^2)}(1 - \lambda\delta) + \frac{1}{1-\delta^2}, & \text{if } \delta\hat{\lambda} \leq \lambda < \frac{\hat{\lambda}}{\delta} \\ \frac{v}{a}\bar{\alpha} + 1, & \text{if } \lambda < \delta\hat{\lambda} \end{cases} \\ e_2^*(\lambda) &= \begin{cases} \frac{v\bar{\alpha}}{a(1-\delta^2)}(\lambda - \delta) - \frac{\delta}{1-\delta^2}, & \text{if } \delta\hat{\lambda} \leq \lambda < \frac{\hat{\lambda}}{\delta} \\ 0, & \text{if } \lambda < \delta\hat{\lambda} \end{cases}. \end{aligned}$$

<sup>12</sup>When determining the student expected wage, I relied on the fact that  $\alpha$  and  $\Pi$  are independent, given the assumption of everyone having the same expectation for  $\alpha$ .

<sup>13</sup>In a Nash Equilibrium every student tries to manipulate her scores and therefore all students exert the same level of effort,  $(e_1, e_2) = (e_1^*, e_2^*)$ , and no student is able to manipulate their test scores (see De Fraja and Landeras, 2006).

where  $\hat{\lambda} = 1 + \frac{a}{v\alpha}$ .

*Proof.* Available in appendix A.1. □

According to Proposition 1, for small values of  $\lambda$  I obtain a partial corner solution (for  $\lambda < \delta\hat{\lambda}$ ) and thereafter I have an interior solution (for  $\lambda > \delta\hat{\lambda}$ ). In other words, when the weight on  $T_2$  is too small the student would not exert effort for that test and  $e_1$  is constant on  $\lambda$ .<sup>14</sup>

As a result, the optimal amount effort varies with  $\lambda$  as follows:<sup>15</sup>

[Insert Figure 1]

Figure 1 represents the optimal student's strategy in a Nash Equilibrium. For low values of  $\lambda$ , the student would not change her decision when compared to the case where  $\lambda = 0$ . The same occurs if the cost of switching from one test to another is too high (which means high value for  $\delta$ ). Alternatively, when the university increases the weight on  $T_2$ , for values of  $\lambda > \delta\hat{\lambda}$ , the student reallocates effort from  $T_1$  to  $T_2$ . Thus, under some conditions, allowing for a second exam deviates effort from the productive test ( $T_1$ ). From a student's perspective, this would be the optimal strategy to maximise the difference between expected wage and costs.

## 3.2 University Choice

In this subsection, I show that not considering the second test ( $\lambda = 0$ ) is generally not optimal for the university. I normalize the high ability level to  $\alpha_H = 1$  and, consequently,  $\alpha_L < 1$ .

The university cares about the wage of their own students. Let  $\lambda_U^*$  be the solution of the university problem and  $(n_H(\lambda), n_L(\lambda))$  the measure of high and low ability students admitted at university. The maximization problem of the university is as follows:

$$(6) \quad \begin{aligned} \text{Max}_{\lambda} \quad & \Pi_U = (\gamma\alpha_H + e_1^*) \times n_H(\lambda) + (\gamma\alpha_L + e_1^*) \times n_L(\lambda) \\ \text{s.to} \quad & n_H(\lambda) + n_L(\lambda) = \frac{1}{2} \end{aligned}.$$

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<sup>14</sup>Similarly, for  $\frac{\lambda}{\delta} > \frac{\hat{\lambda}}{\delta^2}$  the converse situation occurs. The student will exert effort only in the second test, ( $e_2 > 0$ ) and  $e_1 = 0$ . For simplicity I neglect that case. I am only interested on showing that  $\lambda = 0$  is not necessarily optimal for the university.

<sup>15</sup>From proposition 1 I infer that  $\mathbf{e}^*(\lambda)$  is a continuous function on its domain,  $D = [0, \frac{1}{\delta}\hat{\lambda}]$ . Additionally, it is differentiable on its domain except at  $\frac{\lambda}{\delta} = \hat{\lambda}$ .  $\mathbf{e}^*(\lambda)$  is a smooth function on its domain except at  $\delta\hat{\lambda}$ .

In equilibrium, the university takes the student's optimal effort in consideration to maximise students' wage for those they admit.<sup>16</sup> As a result, the payoff function is  $(\gamma\alpha_H + e_1^*)n_H + (\gamma\alpha_L + e_1^*)n_L$ . The university payoff function is a function of  $e_1^*$  (defined on proposition 1) and  $(n_H(\lambda), n_L(\lambda))$ . The measure of admitted students is determined in the following lemma.

**Lemma 1.** *The measure  $n_H$  of high ability students admitted at university is:*<sup>17</sup>

$$(7) \quad n_H(\lambda) = \begin{cases} \left[ \frac{1+a-\alpha_L}{2} + \frac{\lambda}{2}(1-\alpha_L) \right] \times \frac{1}{2a}, & \text{if } \lambda \leq \tilde{\lambda} \\ \left[ a - \lambda \frac{\alpha_L^2}{8} - \frac{\alpha_L}{4}(a + \alpha_L - 1) - \frac{(a+\alpha_L-1)^2}{8\lambda} \right] \times \frac{1}{2a}, & \text{if } \tilde{\lambda} < \lambda \leq 1 \end{cases}$$

with  $\tilde{\lambda} = \frac{a+\alpha_L-1}{2-\alpha_L}$ .

*Proof.* Available in appendix A.3. □

Instead of determining  $\lambda_U^*$ , I show that the optimal solution is not zero. The main result is presented in the next theorem.

**Theorem 1.** *If  $a > 1 - \alpha_L$ , the solution of the university's problem is  $\lambda_U^* > 0$ .*

*Proof.* The university chooses  $\lambda$  to maximise its payoff. According to the result presented on proposition 1,  $e_1^*$  is a continuous function on  $\lambda$ . Following lemma 1, I can also verify that the measure of admitted students is a continuous function on  $\lambda$ .<sup>18</sup> Additionally,  $e_1^*$  is not differentiable at  $\delta\hat{\lambda}$  and  $n_H(\lambda)$  is not differentiable at  $\tilde{\lambda}$  (for  $\lambda < 1$ ). Thus,  $\Pi_U$  is continuous on  $\lambda$  and differentiable at its domain except at  $\delta\hat{\lambda}$  and  $\tilde{\lambda}$ .

Given that I know the payoff function ( $\pi_U$ ), the number of admitted students ( $n_H(\lambda), n_L(\lambda)$ ), and the student's optimal effort ( $e_1^*(\lambda)$ ), I can determine the first derivative of the payoff function. Namely,

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<sup>16</sup>According to Hoxby (2009) and MacLeod et al. (2017), for instance, universities care about reputation issues which translates on student's lifetime wage. However, for simplifications purposes, I focus on the wage immediately after the job market. For research on labour market returns to school identity see, for example, Dale and Krueger (2002) on college selectivity, Author et al. (2014) on skills formation, and MacLeod and Urquiola (2019) on school choice.

<sup>17</sup>The increase of high ability students admitted to university when  $\lambda$  changes is equal to the change on the number of low ability students (in absolute value).

<sup>18</sup>According to Table A1 in appendix A.3, the limit of the number of admitted students when  $\lambda \rightarrow \tilde{\lambda}^+$  is equal to the limit when  $\lambda \rightarrow \tilde{\lambda}^-$ .



$$(8) \quad \frac{\partial \pi_U}{\partial \lambda} = \gamma(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda} + \frac{N}{2} \frac{\partial e_1^*}{\partial \lambda}$$

For  $\lambda \in ]0, \min\{\delta\hat{\lambda}, \tilde{\lambda}\}[$  I have  $\frac{\partial e_1^*}{\partial \lambda} = 0$  (according to proposition 1) and  $\frac{\partial n_H}{\partial \lambda} > 0$  (according to lemma 1). That implies that  $\frac{\partial \pi_U}{\partial \lambda} > 0$  and  $\lambda = 0$  is not a solution of the university problem.  $\square$

### 3.3 Interpreting the Results

I model the university admission problem with the focus on two aspects: the noise and the nature of the admission tests. Figure 2 presents the two test scores distribution for students of both abilities. On the horizontal axis, I have the test scores distribution of  $T_1$  and the test scores distribution of  $T_2$  are represented on the vertical axis. Ability is fixed for each student type, and  $(e_1, e_2)$  is also fixed in the equilibrium. Given that the two idiosyncratic terms of both tests follow a uniform distribution, the shape of the distribution is rectangular. Besides the gap on ability, the noise of the two tests is a key factor in both distributions.<sup>19</sup>

[Insert Figure 2]

According to Figure 2, when  $\lambda$  increases, the university can select more high ability students.<sup>20</sup> The increment in  $\lambda$  is illustrative of the following: if a university is faced with two students with the same score on  $T_1$ , the university should select the one with the highest score on  $T_2$ . Indeed, as argued by [Hölmstrom \(1979\)](#), additional information would allow to a more accurate judgement of student's performance. For students with the same score on  $T_1$ , the university would like to pick the ones that are also good on  $T_2$  (even if  $T_2$  conveys less relevant skills).<sup>21</sup>

<sup>19</sup>According to Figure 2, if  $a < 1 - \alpha_L$  the two test score distributions will not overlap, and that would mean that university would use a very precise test and, with that, they can admit only high ability students. I preclude that case by imposing  $a > 1 - \alpha_L$ . The case of low noise is not very interesting because one test is enough to completely separate students by ability.

<sup>20</sup>Notice that when  $\lambda$  increases to values higher than  $\delta\hat{\lambda}$  the two rectangles move on the northwest direction, but the two areas do not change.

<sup>21</sup>According to my model, it is also true that two field-specific exams provide a more efficient solution rather than only one. That is because, in the model, I assume that the errors are uncorrelated. If you assume the errors of field-specific exams to be correlated, then it is no longer true that two field-specific exams are better than one. Take the extreme case where  $\epsilon$  is the same in all the field-specific tests. Then, having a second specialist test is not necessarily optimal.

The other important feature of my model is the nature of the second test. In my model, I assume that there are no administrative costs of introducing a second test. However, there is an inefficiency associated to that test: it reduces the productive effort,  $e_1$  (see Figure 1).<sup>22</sup> The cost of running a less relevant test (the general admission exam) is a decrease in student's future productivity, which reduces the university payoff.

Hence, when the university is deciding about introducing a second admission test, they face a trade-off between gaining new information about the student's ability and losing productive effort. On the one hand, geometrical intuition shows that by increasing  $\lambda$  the university is able to increase the number of high ability students. On the other hand, from Figure 1, I conclude that an increase of  $\lambda$  has a detrimental effect on  $e_1$ , which has a negative impact on student's future wage. The two effects move in opposite directions.

Nevertheless, in theorem 1, I have shown that the overall effect of introducing a second admission test is positive. Even if students deviate effort from the relevant test, the university still benefits from including a general admission exam on its admission criteria.<sup>23</sup> A separated question is whether that is socially desirable.

In appendix A.4, I take an utilitarian approach to government intervention. The government's goal is to maximise the unweighted sum of wages of all individuals, internalising the cost of effort exerted in the two admission tests.<sup>24</sup> I find that the university and the government do not always agree about  $\lambda$ . Nevertheless, both the university and the government agree that the weight on  $T_2$  should be positive.

## 4 Admission Policies in Portugal

The theoretical framework can be applied to any HE system where different admission conditions are combined, such as field-specific tests and additional requirements. This combination is common across countries, and Portugal is one such case. In Portugal, universities use national central exams and the high school score as mandatory requirements. As explained before, I aim to test whether the inclusion of a general skill test as

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<sup>22</sup>That occurs when  $\lambda > \delta\hat{\lambda}$ . I believe that small values of  $\delta$  are more realistic than high values.

<sup>23</sup>This idea traces back to the debate in Italy about the survival of the *liceo classico* and the role of skills. It has been suggested that learning classic languages (e.g., Latin and Ancient Greek) is useful because those who studied at *liceo classico* do better in life (e.g., obtain higher earnings on average).

<sup>24</sup>This approach is common in the literature since Arrow (1971).

admission criteria generates a better pool of students. However, it is crucial to clarify HE's admission procedure in Portugal before presenting my estimation strategy.

## 4.1 Centralized allocation of candidates

The process of admission to the public HE system in Portugal is centralized. Each year, the government sets the number of vacancies for each institution and degree, the *Numerus Clausus*. There are approximately 50 thousand vacancies each year, over 170 public HE institutions, offering a total of 1,180 programmes.

Table B3 of the appendix provides further information. The Portuguese HE system is binary and composed of Polytechnics and Universities, which can be either public or private. I focus on the public HE system. An institution offering a degree can be a department/faculty within a polytechnic or university or can be the university or polytechnic itself, depending on its organizational structure. I only consider first degree cycles in my analysis. Throughout the text I will refer to the pair institution-degree as an *HE programme* or simply a *programme*.<sup>25</sup>

The government also sets boundaries on admission requirements that universities must respect. The weight to attach to the high school score (which is an average of all courses taken by the student at high school) must be between 50% and 65%. The remaining weight is allocated to the standardized test(s), chosen by the university out of the national exams.

Finally, the government manages the allocation of candidates via a deferred acceptance (DA) mechanism (Gale and Shapley, 1962).<sup>26,27</sup> Having set the number of vacancies at each programme, it then ranks all the candidates to that programme based on the admission criteria set by the university. Note that the admission score of each student is specific to each programme, as different exams and different weights are enforced across

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<sup>25</sup>For more details on the distinction between choosing a combination of a college/major instead of only a college see Bordon and Fu (2015). The authors develop a sorting equilibrium model where they exploit variation in the college-major-specific admissions regime.

<sup>26</sup>For a review see, e.g., Roth and Sotomayor (1992) and Kara and Sönmez (1997).

<sup>27</sup>A DA mechanism is an algorithm that finds a stable matching between agents on both sides of the problem, taking into account their preferences. In this case, the government considers the capacity constraint for each programme and matches the students with explicitly stated preferences to the institutions; the institutions, in turn, have stated their preferences through the choice of admission criteria. This is an efficient and fair mechanism, which ensures that students with the same preferences and admission scores have the same opportunity of being admitted to a programme (equal treatment of equals property, ETE). When vacancies are binding, the assignment procedure described above generates quasi-experimental variation in institution assignment. For a review see Abdulkadiroğlu et al. (2017).

programmes. Therefore, the government computes each year approximately one thousand rankings. Each candidate will be listed in as many rankings as the number of programmes she applied to. She will be offered admission into her highest feasible stated preference, given the number of vacancies and the quality of the pool of competing applicants. Each student is allocated to a single programme, and she cannot change the result of the allocation.

## 4.2 The university problem in practice

Each HEI defines, for each programme, the number and nature of national exams required as its admission criteria, as well as their weights. The institution has a set of 19 entrance exams available to choose from (see Table 1).

[Insert Table 1]

All students at high school must take the core field-specific exam at the end of the 12th grade, defined according to the academic track they have followed: Arts, Science and Technology, Socio-Economics or Languages and Humanities. They must additionally have taken at the end of grade 11 two other field-specific exams, chosen out of a set of three to eight possible exams (see Table 1). Independently of the track chosen, all students must take a general exam of Portuguese at the end of 12th grade.<sup>28</sup>

The university must respect one single constraint when defining the number of exams required, their nature and weights. The number of exams required is either one or two, but there may be different exam combinations considered by the university.<sup>29</sup> If requiring one exam, the institution can specify it or allow candidates a choice among a defined set. In either case, the university is free to require a field-specific exam or a general one (see Table 1). If requiring a second exam, the same procedure applies. If considering two exams, they must have equal weight (thus, the exams' weight is divided equally by the two exams). Whether the institution allows for different access options to a programme (exam combinations), and the student fulfils different access options (given the exams she took), the combination that is considered for ranking her among the candidates to that programme is the one that yields her the highest admission score. As a result,

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<sup>28</sup>Since 2011, they can as well swap one field-specific optional exam with a general exam in Philosophy at the end of grade 11.

<sup>29</sup>Except for the Medicine degree, which requires three exams.

within the same programme, I can observe students being admitted with different exam combinations.

### 4.3 The student problem in practice

The process of application to HE requires students to rank their preferred programmes. After the announcement of the admission criteria rules, and after observing her own scores in the national exams, each student can order up to six programmes to which she applies.

Students have an incentive to report a set of six preferences that they judge feasible. Since they observe their scores and they know past programme thresholds of admission, they will try to exclude options of programmes that are beyond reasonable choice. They will not waste preference slots with scenarios that are way out of their possibilities. Hence, the student has an incentive to report her truthful rank of preferences. She knows that she will be allocated to her highest feasible stated preference.

## 5 Datasets

In my study I link three primary data sets: i) Applications to public HEIs (DGES, 2019b); ii) Students' Performance and iii) Graduation at HE (DGEEC, 2019).<sup>30</sup>

The application dataset provides information on the population of applicants to the HE system. For each student, I have demographic characteristics, socio-economic background, previous academic achievement, the application process, including all programme preferences stated, and the placement. I have microdata for eleven years, from 2008/2009 to 2018/2019.

The students' performance and graduation at HE datasets are comprehensive data sources that cover all the HE institutions in Portugal and have information on all the students enrolled. They report the student's performance each year she has been enrolled and her graduation, whenever applicable. The performance dataset also reports information regarding the mobility status, such as the placement at an exchange programme. Tables B4 to B6 of the appendix present descriptive statistics on the variables of interest

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<sup>30</sup>The performance and graduation dataset are supplied linked by DEGADI (*Divisão de Estudos e Gestão do Acesso a Dados para Investigação*) from DGEEC and the link between the application and performance dataset was made by the author at the premises of the Ministry of Science, Technology and Higher Education in Portugal.

for the full datasets; the description of all variables is presented in Tables B8 to B10 of the appendix.

Over the period 2008-2018, I observe approximately 800 thousand students' applications to the first year of a HE programme under the General Access Regime (GAR) (the centralized process of application to public HEIs). Additionally, I track enrolled students over the five years of 2013 to 2018 (see Table B5). This panel comprises 1,5 million observations on individual-year on 760 thousand individuals.<sup>31</sup> I also observe approximately 330 thousand individuals graduating from 2012 to 2017, irrespectively of their application year and years of enrolment.<sup>32</sup>

I link the three datasets (application, performance and graduation) restricting the link to those individuals that applied to public HEIs through the centralized national system, who were offered a place and who enrolled.<sup>33</sup> From the 214,165 individuals observed in the performance dataset, I was able to match 96% to the application dataset.

Table 3 of the appendix presents descriptive statistics for the linked individuals. The majority of the individuals studied at a public high school, and 57% are female. One-third of the individuals are non-local students, which means that those students live in a geographical area different from their household.<sup>34</sup> Moreover, only 30% of the individuals have a mother/father with a HE degree.

Once I define the outcomes of interest, the variables of interest, and the methodology to be used, I will impose further constraints that should be applied to the linked dataset in order to determine the analysis population.

[Insert Table 3]

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<sup>31</sup>In this paper, I define an individual as a student enrolled in a specific programme. If a student is enrolled at two distinct programmes at the same time, according to this definition, that will count as two individuals. The reader should bear in mind that a student can only be admitted to one programme in each contest.

<sup>32</sup>These students may have obtained more than one diploma over this period, for instance, a bachelor and master degrees.

<sup>33</sup>In my analysis I only consider individuals that were admitted, for the first time, into a Bachelor or Integrated Master programme in a public university through the GAR (in order to match those to the application dataset). A detailed list of the diploma types included in each degree is provided in Table B13 of the appendix.

<sup>34</sup>This variable is self-reported. However, I also have information regarding the council of residence for each individual.

## 6 Empirical Set-up: Tests and Performance

### 6.1 Outcomes

In this study I consider three measures of student performance at university ( $y_i$ ), for an individual  $i$ : i) the number of credits obtained through the *European Credit Transfer System* (ECTS) by the end of the first academic year; ii) whether the individual completed their HE programme on time<sup>35</sup> (Completion on Time); and, iii) Final GPA.<sup>36</sup> These outcomes are different in their nature. They measure student performance at different stages of the degree. For each measure, the sub-population of analysis is different. In Table 4, I report descriptive statistics for each subset that will be considered in my analysis (for descriptive statistics of the total sample population see Table 3).

I only observe the number of ECTS for students that completed the first academic year of their degree. I do not have information on the ECTS for the 2017/2018 entry cohort and individuals that dropped out during their first year (on average, dropouts represent 17% of enrolments per year). Overall, there are no differences between the sub-sample of individuals with ECTS credits (see Table 4) and the total sample (see Table 3). I define “Completion on Time” as a dummy variable equal one if a student graduated on time and zero otherwise. I only consider students enrolled either in a three-year bachelor programme or in an integrated master (that awards a bachelor certificate after successfully completing the first three academic years). I exclude from the analysis the cohorts 2015/2016 to 2017/2018 and those individuals enrolled in four or six-year bachelor programmes on the 2013/2014 and 2014/2015 cohorts. By comparison to the number of ECTS, in this sub-sample of analysis students have, on average, a lower admission score, but better performance at high school. Additionally, I observe a slightly higher share of students admitted with the Portuguese exam (28%).

Finally, the Final GPA<sup>37</sup> is only available for students that have finished their degree during the period of analysis, and it is an average score of all courses taken at university (including those taken in the first year). For that reason in this subset I only consider students enrolled in a three-year or four-year programmes for the 2013/2014 cohort and

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<sup>35</sup>In order to finish the degree each student needs to complete a minimum of 180 ECTS.

<sup>36</sup>Although, for instance, on-the-job human capital accumulation might play a role on wage determination (Stinebrickner et al., 2019), I am using Final GPA as a proxy for income. Students with better academic achievement are expected to obtain better earnings (see, e.g., Jones and Jackson, 1990).

<sup>37</sup>For a review on the determinants of Final GPA (see, e.g., Betts and Morell, 1999).

those enrolled in three-year programmes of the 2014/2015 cohort (the ones that could potentially have graduated). In this sub-sample of analysis, I observe a higher share of female graduates and graduates have, on average, a higher high school score.

Comparing the three different sub-samples of analysis, the majority of the descriptive statistics remain the same. The major difference is associated with the gender and the high school score variables. In particular, both the share of females and the average high school score tend to increase when I look at those that graduate. It is important to control for both characteristics in the empirical analysis. Although the descriptive statistics are similar, I believe that it is relevant to look at the three different outcomes.

The number of ECTS and Final GPA can vary across programmes. Namely, some programmes might be more restrictive in their marking standards compared to others. Thus, those outcomes can be very different across programmes. For that reason, I consider deviations to the mean instead of total scores in my analysis. I standardize the number of ECTS and Final GPA within the programme by subtracting the mean and dividing by the standard deviation. I consider for each student the deviation to the mean outcome for the programme that she is enrolled in.

[Insert Table 4]

## 6.2 $T_2$ in practice: the Portuguese exam

From my model, there are two relevant attributes of the admission requirements that should be considered: the number of exams and their nature (generic or field-specific). In fact, the existence of a second exam always adds more information to the selection process (e.g., [Holmström, 1999](#)). The addition of a general exam diversifies the nature of the information gathered by the institution. Furthermore, allowing the inclusion of a general exam favours candidates who are relatively better at general skills than at field-specific skills. However, the general exam is only taken into account when it improves the student's admission score.

Consider the following example in Table 5, where two students (Pedro and Alexandre) apply to the same degree at universities  $A$  and  $B$ . Both students prefer university  $A$  over  $B$ . However, university  $A$  only considers a field-specific exam ( $T_1$ ) as an admission criterion, while university  $B$  allows students to apply with a general exam ( $T_2$ ) combined with  $T_1$ . Both Pedro and Alexandre have taken  $T_1$  and  $T_2$  at high school. However,



when applying to university  $B$ ,  $T_2$  is only considered in Alexandre's case. Only in his case, the score on  $T_2$  increases the application score. On the other hand, in Pedro's case, only the score on  $T_1$  is considered given that he performed better in  $T_1$  relatively to  $T_2$ . That means, holding constant preferences, Pedro will be allocated to university  $A$  and Alexandre to  $B$ .

[Insert Table 5]

In this example, I observe that Pedro performed much better on  $T_1$  relatively to  $T_2$ . Instead, Alexandre had a similar score on  $T_1$  compared to Pedro but performed even better on  $T_2$ . Pedro represents a student profile strong on field-specific skills (specialist). Alexandre represents students that performed sufficiently well across disciplines, both field-specific and general skills (all-rounder profile). University  $A$  places a higher value on specialists students while university  $B$  allows all-rounders to compete with specialists for the same place. Thus, university  $B$  captures a different pool of students compared to  $A$ .

Note that if both universities had given the students the option to add the exam  $T_2$ , Alexandre would have displaced Pedro at university  $A$ . Indeed, for the student, the option of inclusion of a general exam favours a student profile relatively more robust on general skills, all else equal. I aim to understand whether allowing all-rounders to compete with specialists would have increased student performance at university.

The example also illustrates another implication for the university. If both universities had considered  $T_1$  as the single admission criteria, the allocation of students would have remained unchanged. However, the admission threshold of university  $B$  would have decreased from 180 to 160 (the admission score of the last admitted student). Therefore, one might imagine that the introduction of  $T_2$  can be seen as an incentive that universities use to boost the admission threshold score and signal themselves as more selective universities.

### **Alternative Entry Requirements**

For each programme, there may be different alternative entry requirements (exam combinations). On average, each programme allows for three alternative entry exam combinations. Some combinations include the Portuguese exam, and others do not. Within and across programmes, it is up to the discretion of each institution, whether to include the Portuguese exam in one or more of the exam combinations allowed.

[Insert Table 6]

Table 6 presents the total number of exam combinations within programmes. In total, there are 11,951 programmes over the eleven years of analysis. I refer to each programme per year as programme-year. For instance, there are 2,316 programme-year that allow for a single entry exam combination, of which only 204 included the Portuguese exam.<sup>38</sup> For each of the individual's stated preferences (up to six), her application score is computed as the highest score out of the exam combinations set by the programme.<sup>39</sup>

In my analysis, I only consider programmes that include Portuguese as an alternative requirement (and not mandatory). That is, Portuguese is included in at least one of the allowed exam combinations, but not in all. Additionally I define  $\tau^*$  as the programme admission threshold.

## 7 Estimation Strategy: Partial equilibrium analysis

### 7.1 Selection into Treatment

The treatment is the inclusion of the Portuguese exam as an admission criterion at the individual level. I define  $y(1)$  as the observed outcome of student performance in case of treatment and  $y(0)$  otherwise. The problem I must tackle is that selection into treatment might not be random. However, I can accommodate selection in my model by understanding how individuals are assigned to treatment.

In my analysis, I define three different groups of students: i) the *specialists*; ii) the *generalists*; and, iii) the *all-rounders* (see Figure 3). Firstly, I observe students admitted without the Portuguese exam, and so their field-specific skills are better than their general skills ( $T_1 > T_2$  and  $T_1 > \tau^*$ ). Among these students, some of them could not have been admitted with a general admission exam (specialists). Some could have been admitted with such an exam (all-rounders). Secondly, I also observe students admitted with

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<sup>38</sup>Moreover, there are 136 programme-years that select students based on their best result among five different exam combinations. Among those 136, only two programmes did not allow for the Portuguese Exam while 117 programmes allowed for it in one out of the five exam combinations. Additionally, 15 programmes allowed for five different exam combinations where all of them included Portuguese.

<sup>39</sup>The computation is conditional on the result of the national exams the student takes at high school. For instance, imagine that Pedro only took the Mathematics and the Portuguese exams and he is applying to digital marketing at a particular university, as one of his stated preferences. The digital marketing programme specifies that students can apply with the Mathematics exam or the combination of the IT and the Portuguese exams. As Pedro did not take the IT exam, his application score is determined by the Mathematics exam only.

Portuguese but could have been admitted even without it (another type of all-rounders). These are students that are relatively better at general skills but whose field-specific skills would have been enough to get them admitted ( $T_2 > T_1 > \tau^*$ ). Finally, I observe students who were only admitted because their Portuguese exam score was sufficiently high (generalists). These are students whose fields specific skills are relatively weak compared to their peers, who would not have been admitted without the Portuguese exam. ( $\frac{T_1+T_2}{2} > \tau^* > T_1$ ).

[Insert Figure 3]

There is a potential trade-off in allowing generalists to compete with specialists. The university can increase the average quality of the pool of students by admitting all-rounders. At the same time, the university might get pure generalists in their pool. In the example, all students gained entry to the programme. A consequence of including Portuguese as an alternative requirement ( $T_2$ ) is that generalists may take places that would have been otherwise allocated to specialists. Thus, I frame my research question in the following way: Does the *generalists* perform better at university relatively to their peers?

To answer the question, I look at performance in  $T_1$  and  $T_2$ . For individuals that were admitted with Portuguese, I dichotomize them according to whether their performance in  $T_1$  was sufficient by itself to meet the entry requirement ( $\tau^*$ ).<sup>40</sup> For individuals that were admitted without Portuguese, I look at performance in  $T_2$ . In my estimation strategy, the specialists will be the comparison group. I hypothesise that the effect of allowing the inclusion of the Portuguese exam on individual performance will be different across groups. Namely, I expect generalists to perform worst at university compared to their peers, given that the generalists are weaker in terms of field-specific skills.

In the data, I attempt to determine the application score for each student in the case that she was admitted with a different exam combination. For each student I compute: i) her application score in the case the Portuguese exam was considered ( $\tau_{PT}$ ); ii) her application score in the case that the Portuguese exam was not considered ( $\tau_{\sim PT}$ ). Based on four blocks of information (admitted with Portuguese,  $\tau^*$ , ( $\tau_{PT}$ ), and ( $\tau_{\sim PT}$ )) I can

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<sup>40</sup>Remember that for each programme and each student I observe the actual application score and the actual placement threshold ( $\tau^*$ ).

distinguish between the different comparison groups. Table 7 reports the number of students for which I can identify each one of the two types of application scores.<sup>41</sup>

In the case that  $\tau_{PT}^i < \tau^*$ , student  $i$  is classified as being a generalist. Otherwise, she is classified as an all-rounder. As a result, according to Table 2, I observe that eight per cent of the enrolled students are all-rounders (that were admitted with Portuguese), and seven per cent are generalists. Additionally, for 38 % I cannot distinguish between the two groups. This occurs because I cannot observe their scores on the exam combination required in the case that Portuguese was not considered.

[Insert Table 2]

To distinguish between specialists and all-rounders (that were admitted without Portuguese), I need to look at performance in  $T_2$ . Individual  $i$  is classified as being a specialist when I observe that  $\tau_{PT}^i < \tau^*$ . Otherwise, she is classified as an all-rounder. In fact, I observe 21 % of the enrolled students are specialists, and 15 % are all-rounders, while 13 % I am not able to distinguish between the two groups.

In both distinctions, I assume that the pool of allocated students and the threshold ( $\tau^*$ ) would remain the same in the case of changing the admission criteria.<sup>42</sup> Due to data constraints, the division between specialists and all-rounders is more accurate than the division between generalists and all-rounders. In the dataset, I observe the Portuguese exam score for the majority of students. As a result, when looking at those admitted without Portuguese there is 13% of students for which I cannot identify the group, while for those admitted with Portuguese the percentage increase to 39%. The unbalance on the assigning the sample into groups can potentially bias the results.<sup>43</sup>

In my analysis I consider two different ways of addressing the problem of not being able to identify the corresponding group for each student: i) I run my analysis without making a distinction between generalists and all-rounders admitted with the Portuguese exam. In that case, I only have 13% of missing observations;<sup>44</sup> and ii) I use regression analysis to predict to which group the student should be allocated. I regress the observed application

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<sup>41</sup>In Table B12 of the appendix, I compare both types of programmes for fields that consider the Portuguese exam as an optional requirement.

<sup>42</sup>This is not necessarily true because a change on admission criteria rules would change the pool of applicants, their preferences and also their performance at the high school exams, for instance.

<sup>43</sup>In Table C16 of the appendix I analyse whether the individuals for which I do not know their group are missing at random.

<sup>44</sup>Alternatively, I also consider only individuals for which I can identify the group, but that approach has a drawback of having 52% of missing observations.

score with and without Portuguese on individual and programme characteristics.<sup>45</sup> I obtain the predicted application score for each group of peers, and I replace the missing values for each individual within those groups. As a result, according to Table 2, when I consider a distance of ten or five points from the threshold, I observe a higher percentage of specialists, following those admitted with Portuguese, and finally a small percentage of all-rounders that were admitted without Portuguese.

Figures 10 and 11 of the appendix presents the distribution of the scores for the identified observed groups. As expected generalists perform very well in Portuguese and worse in terms of field-specific skills. For specialists, the opposite occurs. For all-rounders, the densities are similar according to the two different application scores distributions.

Additionally, I run my analysis around the programme admission threshold. I am interested in comparing student performance among those whose Portuguese exam was, at the margin, the only reason why they were admitted (generalists). In this set-up, I consider the reduced form regression:

$$(9) \quad y^i = \alpha_0 + \alpha_1 \text{Generalists} + \alpha_2 \text{All-Rounders with Portuguese} + \\ + \alpha_3 \text{All-Rounders without Portuguese} + \alpha_i' \mathbf{X}^i + \Theta + u^i$$

where  $y^i$  is the outcome,  $\mathbf{X}^i$  is the same vector of controls as in the previous section,  $\Theta$  represents the fixed effects, and *Generalists* is an indicator equals one if the student's application score without Portuguese is smaller or equal than the threshold. The treatment toggle on whether the student needed her result on the Portuguese exam to gain entry in the HE programme.  $\alpha_1$  is the coefficient of interest.  $\alpha_1$  tell the reader whether the *generalists* performed better at university in comparison to the *specialists*, at the margin of gaining entry in HE.  $\alpha_2$  and  $\alpha_3$  measures differences on students performance when comparing *all-rounders* with *generalists*.

The advantage of comparing students close to the threshold in the distribution of the admission score is that students were admitted to the same programme, and I can observe their outcome. I assume that individuals around the assignment rule are comparable (comparing students that at the margin were admitted only because of the Portuguese exam with the ones for which was not necessary). I run my analysis within 10 and 5 points distance from the threshold.<sup>46</sup>

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<sup>45</sup>For details see appendix C17.

<sup>46</sup>Table B14 of the appendix provides descriptive statistics for all variables considered in the estimations

## 8 Results

I perform my analysis at the margin of gaining entry in HE. All results in this section should be interpreted in that context. I consider three different approaches. Firstly, I look at the groups of students admitted with Portuguese jointly. Secondly, I consider them separately. Thirdly, I run the analysis considering the division of groups when I use the imputation technique described on section 7.

Tables 9 presents a summary of the different estimations.<sup>47</sup> Similarly to the previous section, I start my analysis by considering the reduced form proposed in equation (9) (columns (1) and (4)). Additionally, in columns (2) and (4) I control for individual characteristics, and in columns (3) and (6) I include programme fixed effects as well. For that reason, the complete estimation is reported in columns (3) and (6). Coefficients in Panel A and C should be interpreted in standard deviation changes like as before.

According to Table 9, I observe differences in student's performance regarding all-rounders admitted without Portuguese relatively to specialists. I find evidence that *all-rounders* with better field-specific skills perform better than their *specialists'* peers at university in all outcomes and specifications. For instance, according to column (3), an all-rounder admitted without Portuguese would have, on average, an increment of 0.138 standard deviations on her Final GPA when compared to a specialist student of the same ability (within the same programme).

Moreover, I find no evidence that students admitted with the Portuguese exam perform worse than the *specialists*. There is a positive effect of being admitted with Portuguese in terms of Final GPA. The result is pronounced even when I control for individual and programme characteristics. Within programmes, I observe that a student admitted with Portuguese obtains 0.077 standard deviations more on her Final GPA on average than a specialist' student of the same ability. Although the overall effect of those admitted with Portuguese is statistically significant, it is relevant to distinguish between generalists and all-rounders to fully understand whether such effect is driven by one of the two groups jointly considered. The aggregation of the two groups might lead to misleading conclusions.<sup>48</sup>

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(according to all constraints imposed in the analysis population).

<sup>47</sup>Tables D18 and D19 of the appendix provide a more detailed version, including different distance bands around the threshold.

<sup>48</sup>In Table D20 of the appendix I test the statistical difference between all-rounders and generalists.

Thus, regarding the number of ECTS accumulated by the end of the first year, I observe no difference in performance between generalists and specialists. Instead, I observe a positive effect associated with all-rounders admitted without Portuguese, which becomes non-significant when introducing programme fixed effects. When I consider completion on time as an outcome, generalists instead of all-rounders drive the positive effect. Finally, in terms of Final GPA seems that all groups outperform specialists in all specifications considered.<sup>49</sup>

At the margin of the cutoff, I expect students to have a similar ability, and I would not expect many differences across groups. My results show that differences in performance across groups exist. Firstly, I find no evidence that students admitted purely based on their performance on the Portuguese exam perform worse at university than the others. I do not observe *generalists* performing any worse than the *specialists*, which is a surprising result.

In turn, I observe consistent differences on performance regarding all-rounders relatively to specialists. All-rounders tend to perform better at the end of the first academic year, to have a higher Final GPA, and they have a higher probability of completion on time. In other words, I find evidence that *all-rounders* perform better than their *specialists* peers at university. This implies that students that performed sufficiently well in both admission tests perform better at university. Overall, these results suggest that general skills matter at the margin of gaining entry in HE.

[Insert Table 9]

## 9 Discussion and mechanisms

Universities that allow for Portuguese as an alternative requirement benefit at the margin of admission. Students that perform better in general skill exams perform no worse at university than their specialist peers. Alternatively, there is some evidence suggesting that students with a sufficiently high score in the Portuguese exam perform better at university, even when they did not need the general skills exam to gain entry in HE. As

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<sup>49</sup>In Table D21 of the appendix, I consider the Final GPA only for those that completed the degree on time, and in Table D22 of the appendix, I consider take into account the possibility of dropout during the programme. My main results remain the same.

a result, the inclusion of a general skill test as an admission criterion can have a positive effect on student performance at the margin of admission.

The evidence suggests that all groups performed better relative to *specialists* in terms of Final GPA. However, that effect is less pronounced when I look at completion on time and at the number of ECTS by the end of the first year. Differences across groups vary according to the type of measure I consider. Namely, at the beginning of the programme differences are less pronounced rather than towards the end.<sup>50</sup> I interpret that result from two perspectives.

Final GPA includes all course scores while ECTS only measure the number of credits accumulated, independently of the scores. *Specialists* might have a head start, but they do not capitalize on that advantage. Students with better general skills do not substantially outperform *specialists* by the end of the first year. On average, both groups have the same quantity of ECTS accumulated within the same programme. Even so, *generalists* outperform *specialists* when I consider the final GPA as outcome. To a certain extent, this signals that *generalists* are able to adapt to changes. Students with different skills need time to adapt.

Additionally, I can also infer that the specialist exam may not be well designed. This result assumes particular relevance in a centralized admission system that emphasizes the importance of field-specific skills. Throughout the paper, I made the assumption that the field-specific exam provides a good measure of specialists skills. However, in the light of my results, I can hypothesise that the specialist exam is not very accurate at checking the field-specific skills required. In fact, by the end of the programme *specialists* obtain a worse Final GPA on average. The field-specific exam seems not to match the skills needed to obtain a good performance in the degree.

Nevertheless, my empirical analysis only considers programmes that already allow for Portuguese as an alternative requirement. A sceptic reader might argue that such a policy has different impacts according to the goal and type of institution. For instance, in less well-regarded programmes, often the goal is to admit as many students as possible. They have an incentive to allow for as many exam combinations as possible in order to fill the number of vacancies. Opening the application process to all type of students

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<sup>50</sup>Even when I consider the final GPA only for those that finished on time (see Table D21 of the appendix) I observe a positive difference on performance regarding generalists and all-rounders relatively to specialists.



might be perceived in different ways. Allowing for different alternative entry requirements can be perceived as a sign of being a less rewarded programme. Students that perceive themselves as high ability students might not apply to those programmes.<sup>51</sup>

I acknowledge that the type of institution can potentially drive the results.<sup>52</sup> My results might not be verified for elite institutions. Nevertheless, the reader should keep in mind that those institutions are able to select the top distribution of students, even with only one admission criterion. Regardless of the nature of the admission exams, there is always a high demand for programmes in elite institutions. I still believe that my results are relevant for the average institution, which represents the majority of the HE system.

Additionally, the reader might ask herself whether my results would depend on the field of study. Even controlling for differences across fields, my main results remain the same. The access to some fields might be more restricted than others and may affect students' choices (see, [Hastings et al., 2013](#); [Kirkeboen et al., 2016](#)). My results might be driven by the type of field that allows more often for Portuguese as an alternative admission requirement. Therefore, the distribution of students across different fields of study is relevant to understand my results.<sup>53</sup> In the sub-population of analysis, I observe very few Natural Sciences programmes. My results might be misinterpreted that universities should be asking all Physics, Maths, or Engineering students to do Portuguese, while my empirical analysis does not allow me to conclude that.<sup>54</sup>

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<sup>51</sup>Figure 12 of the appendix, shows which type of programmes allow for Portuguese and how students are distributed between them. Additionally, Figure 13 presents students' distribution by programme in terms of ability.

<sup>52</sup>An alternative approach would have been to perform a general equilibrium analysis where I contemplate the scenario of all institutions allowing for the Portuguese exam as an alternative entry requirement. However, in that scenario, students could have performed differently in the Portuguese exam. Students might have allocated more effort to the Portuguese exam if they knew that such an exam would have been a valid admission criterion for all programmes.

<sup>53</sup>Figure 14 of the appendix show how students are distributed according to fields.

<sup>54</sup>Nevertheless, my model is applicable to all fields, and the optimal choice of  $\lambda$  is not zero. How far  $\lambda$  is from zero depends on other factors, such as the precision of the field-specific exam. Thus, if one believes that for natural sciences the field-specific test is relatively more precise than in fields such as sociology or economics for instance, then my model predicts that for some fields  $\lambda$  is closer to zero in comparison to other fields. For differences across fields see Table D23 of the appendix. I do not find substantial differences across fields.

## 10 Conclusion

Intended and unintended consequences of university admission practices have been under the spotlight of the academic, political, and judicial agendas.<sup>55</sup> I contribute to that debate by analysing the nature of the admission requirements. I test whether the inclusion of a general admission exam generates a better pool of admitted students.

In this paper, my first contribution is theoretical. I set up a simple model in which the university faces a particular trade-off when deciding to combine a general admission requirement with a field-specific exam. The university gains information about the student's ability at the cost of reducing the weight allocated to the field-specific requirement. As a result, in equilibrium, the student would decrease the effort level in the productive test (the field-specific exam). Nevertheless, I find that the university benefits from including a general admission test on its requirements.

My second contribution is the provision of an empirical application of the model using Portugal as a case study. I find evidence that including a general exam as an entry requirement increases the average student performance at university, at the margin of admission to HE. This is a surprising result that has not been a consistent finding in the literature.

I find that in the first academic year the *generalists* tend to fail more when compared to the *specialists*, while towards the end of the programme they have a better performance on average. In other words, *generalists* outperform *specialists*. This result has two possible interpretations.

First, universities that allow for general admission requirements should not put too much weight on first year assessment. Albeit the fact that performance in the first-year is often considered as a good proxy to evaluate who the good students are, *generalists* need time to adapt in order to obtain a better performance.

Secondly, my results show that the field-specific exam may not be well designed in some fields. Students admitted purely based on their field-specific skills perform worse at university. That means that the field-specific exam is not accurate at checking the specialists' skills needed to complete the degree.

My findings have substantial implications for the determination of university selec-

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<sup>55</sup>For instance, in the US there is an ongoing discussion about this topic with centre on the affirmative action exerted by some Ivy League universities (e.g., Card and Krueger, 2005; Arcidiacono et al., 2011; Calsamiglia et al., 2013; Arcidiacono and Lovenheim, 2016).

tion policies. Universities should rethink their admission practices by either designing admission tests that evaluate field-specific skills more accurately or by introducing general admission requirements in order to effectively distinguish their candidates. In short, diversity in admission criteria has a positive effect on student performance at university.

My results offer an optimistic view of the possibility to affect the pool of admitted students at university. Even though I focus on the Portuguese system, this paper provides generalizable insights about the importance of the choice of admission criteria.

Finally, this paper poses some interesting questions that are worthwhile to explore in future research. I have shown that the general admission exam has an important role to play on university admission practices. However, this paper does not provide an answer to what should be the optimal weight to allocate to that exam. Additionally, there might be some unintended consequences of increasing the weight associated with the general exam. For instance, by introducing a general admission exam, the university may change the gender composition of the admitted pool of students and/or it can also change the way students rank their preferences when applying to HE. In sum, these possibilities open scope to future research.

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# 11 Figures

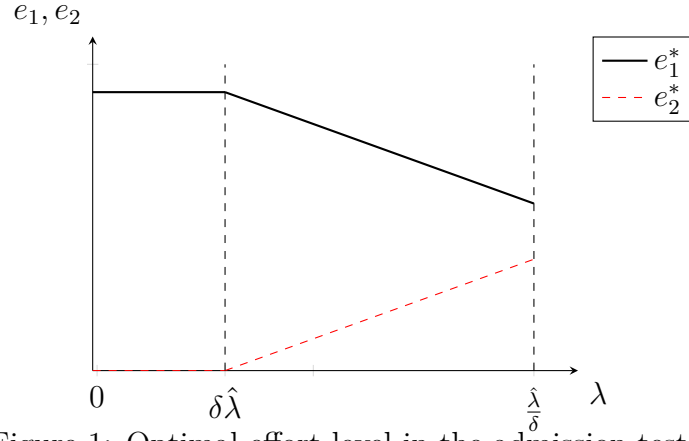


Figure 1: Optimal effort level in the admission tests.

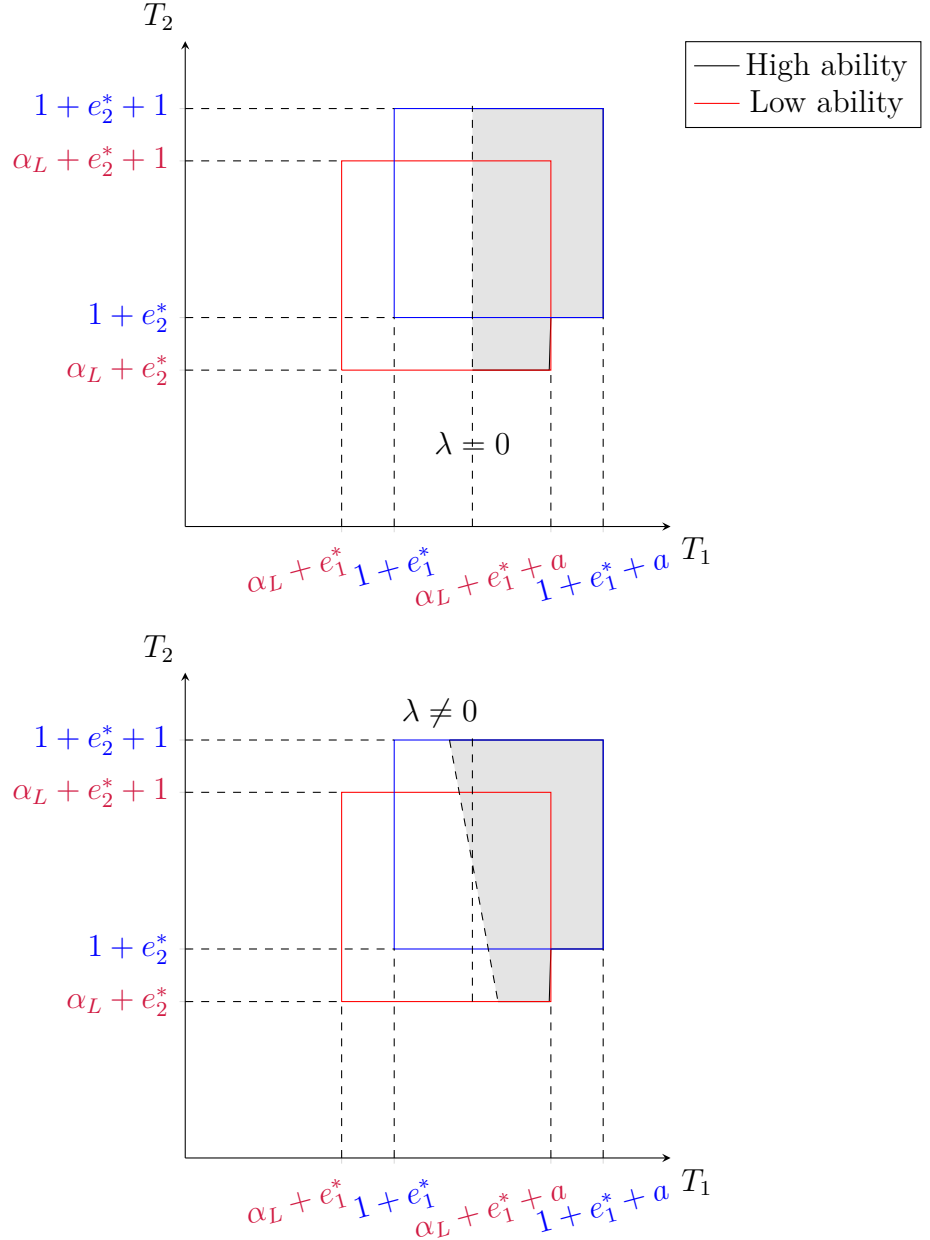


Figure 2: Student's test scores distribution. Shaded area represents half of the population, which is admitted to university.

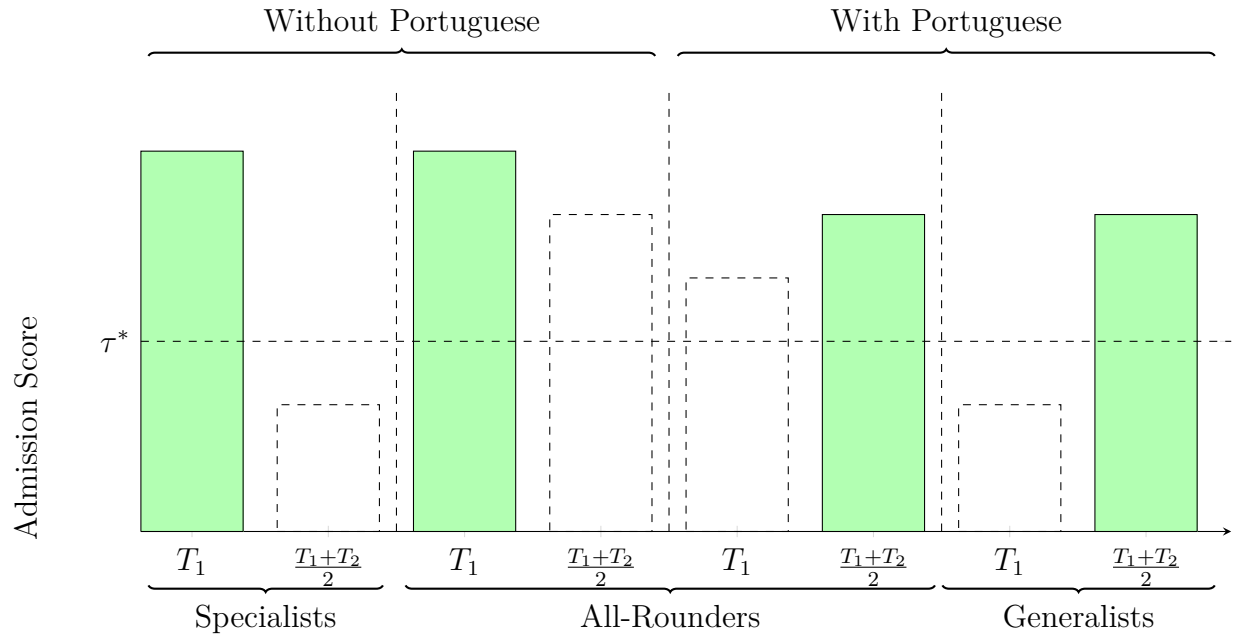


Figure 3: Different groups of admitted students within the same programme

## 12 Tables

Table 1: National Exams at High School (according to the academic track)

Track	Field-Specific Exams		General Exams	
	Core Exam (12th grade)	Additional exams (11th grade) (choice of two)	(12th grade)	(11th grade)
Arts	Drawing	Descriptive Geometry Mathematics History of Culture and Arts	Portuguese	Philosophy <sup>(a)</sup>
Science and Technology	Mathematics	Biology and Geology Physics and Chemistry Descriptive Geometry	Portuguese	Philosophy <sup>(a)</sup>
Socio Economics	Mathematics	Economics Geography History B	Portuguese	Philosophy <sup>(a)</sup>
Languages and Humanities	History A	Geography Latin German French English Spanish Portuguese Literature Applied Mathematics	Portuguese	Philosophy <sup>(a)</sup>

Source: DGE (2019). Notes: This table includes only the general tracks (scientific) and not professional or vocational tracks at high school. For more details see Table B7 of the appendix.; (a) Since 2011 students are allowed to swap one of the two additional field-specific exams with the Philosophy exam.

Table 2: Comparison groups in the overall distribution and around the threshold (shares)

	Overall Sample	Observed $0 \leq \tau - \tau^* \leq x$			Overall Sample	Imputation $0 \leq \tau - \tau^* \leq x$		
		$x = 10$	$x = 5$	$x = 2$		$x = 10$	$x = 5$	$x = 2$
No. Individuals	78,233	42,350	26,358	13,592	78,233	42,350	26,358	13,592
Admitted without PT Exam								
Specialists	0.21	0.29	0.31	0.33	0.28	0.37	0.41	0.43
All-rounders	0.15	0.07	0.04	0.02	0.21	0.11	0.07	0.04
Non identified	0.13	0.11	0.11	0.11	-	-	-	-
Admitted with PT Exam								
All-rounders	0.08	0.06	0.04	0.03	0.38	0.34	0.30	0.27
Generalists	0.04	0.06	0.08	0.09	0.14	0.18	0.23	0.26
Non identified	0.39	0.40	0.41	0.42	-	-	-	-

Source: Author's calculations

Table 3: Descriptive Statistics, Analysis Population, **Linked Dataset**

	Mean	Std. Dev.
A. Data Structure		
Initial year	2013/2014	
Final year	2017/2018	
No. years	5	
No. Individuals	205,297	
B. Individuals		
Age	18.42	1.71
Female (share)	.57	
High school GPA	149.46	20.05
Public high school (share)	.84	
Non-local student (share)	.30	
Mother has HE (share)	.33	
Father has HE (share)	.26	
Applied to a maintenance grant (share)	.32	
Received a maintenance grant (share)	.25	
C. Placement		
Degree of placement (no. individuals)		
Bachelor	166,741	
Integrated Master	38,556	
Preferences of placement (share)		
1st	.56	
2nd	.21	
3rd	.11	
4th	.06	
5th	.04	
6th	.02	
Application score (0-200)	144.72	20.13
No. of admission exams	1.37	0.55
Portuguese admission exam (share)	.22	
Portuguese exam score (0-200)	121.66	28.21
No. of programmes ranked	4.79	1.58
No. of Institutions ranked	2.85	1.46
No. of broad fields ranked	2.02	0.99

Source: Author's calculations. Notes: Scores are measured in a scale between 0 and 200.

Table 4: Descriptive Statistics, Analysis Population according to the **Outcomes**

	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Outcomes	ECTS accumulated by the end of the first year		Completion on Time (for three-year degrees)		Final GPA	
A. Data Structure						
Initial cohort	2013/2014		2013/2014		2013/2014	
Final cohort	2016/2017		2014/2015		2015/2016	
No. years	4		2		2	
No. Individuals	134,143		52,966		29,263	
B. Individuals						
Age	18.40	1.67	18.63	1.99	18.39	1.73
Female (share)	.58		.56		.67	
High school score (0-200)	134,14	19.91	144.69	17.32	149.06	17.19
Public high school (share)	.85		.86		.87	
Non-local student (share)	.31		.29		.31	
Mother has HE (share)	.32		.28		.26	
Father has HE (share)	.26		.21		.19	
Applied to a maintenance grant (share)	.32		.32		.36	
Received a maintenance grant (share)	.24		.26		.28	
C. Placement						
Degree of placement (no. individuals)						
Bachelor	108,373		52,966		29,257	
Integrated Master	25,770		-		6	
Preferences of placement (share)						
1st	0.59		0.58		0.64	
2nd	0.21		0.21		0.20	
3rd	0.10		0.10		0.09	
4th	0.05		0.05		0.04	
5th	0.03		0.03		0.02	
6th	0.02		0.01		0.01	
Application score (0-200)	144.96	20.05	138.95	17.41	143.01	17.48
No. of admission exams	1.37	0.56	1.15	0.36	1.19	0.39
Portuguese admission exam (share)	.22		.28		.29	
Portuguese exam score (0-200)	121.78	28.36	117.00	27.08	120.99	26.94

Table 5: Example with Two Students, Two Universities

Student	Exams		Admission Score		
	Exam	Score	Required	University A	University B
Pedro	$T_1$	170	$T_1$	<b>170</b>	170
	$T_2$	110	$T_1 + T_2$	n.a.	140
Alexandre	$T_1$	160	$T_1$	160	160
	$T_2$	200	$T_1 + T_2$	n.a.	<b>180</b>

Note: Assume that each university has capacity for just one student. University A allows the students to apply with the  $T_1$  and university B allows students to apply either with  $T_1$  only or  $T_1$  and  $T_2$  combined. Scores are in a scale between 0 and 200.

Table 6: No. of exam combinations by programme-year

No. of exam combinations	No. of exam combinations with Portuguese								Total
	0	1	2	3	4	5	6	8	
1	2,112	204							2,316
2	1,305	322	25						1,652
3	3,535	3,657	24	35					7,251
4	63	351	2	0	23				439
5	2	117	2	0	0	15			136
6	9	117	2	2	0	0	19		149
7	0	0	7	0	0	0	0		7
8	0	0	0	0	0	0	0	1	1
Total	7,026	4,768	62	37	23	15	19	1	11,951

Source: Author's calculations based on the application dataset. Notes: This table includes all the different programme/year for the 11 years, from 2008 until 2018.

Table 7: Descriptive statistics on the Portuguese exam

	All programmes			Programmes that allow for PT		
	Mean	Std. Dev.	Freq.	Mean	Std. Dev.	Freq.
No. Individuals			205,297			78,233
Actual application score ( $\tau$ )	144.72	20.13	205,297	139.02	17.66	78,233
Admission score included PT (share)	0.22		45,154	0.51		40,016
Portuguese exam score	127.90	23.08	139,241	120.10	25.90	67,896
Application score with PT ( $\tau_{PT}$ )	134.55	18.05	73,034	133.71	17.79	67,896
Application score without PT ( $\tau_{\sim PT}$ )	146.56	20.31	172,500	141.65	18.06	46,837
( $\tau_{PT} - \tau_{\sim PT}$ )				-9.59	12.40	33,896

Notes: Programmes for which Portuguese is a mandatory requirement were not considered in the analysis. Thus, there are 5,138 students that were admitted to programmes for which Portuguese is mandatory and, for that reason, they were excluded from the analysis. All scores are in a scale between 0 and 200.

Table 8: Individuals around the assignment rule

	Population of Analysis	Overall Sample	$ \tau_{PT} - \tau_{\sim PT}  \leq x$		
			$x = 10$	$x = 5$	$x = 2$
No. Individuals	78,233	33,896	17,405	9,412	3,853
Admitted with the PT exam (share)	0.51	0.18	0.27	0.31	0.30

Source: Author's calculation.

Table 9: At the margin of gaining entry in HE

	(1)	(2)	$0 \leq \tau - \tau^* \leq 10$		(5)	(6)
			(3)	(4)		
<i>Panel A: No. of ECTS by the end of the 1st year</i> (Cohorts 2013/2014 to 2016/2017)						
Admitted with Portuguese	0.009 [0.014]	0.016 [0.014]	-0.026 [0.016]			
Generalists				0.056*** [0.018]	-0.016 [0.018]	-0.009 [0.018]
All-Rounders (with Portuguese)				0.060*** [0.015]	0.060*** [0.015]	-0.001 [0.017]
All-Rounders (without Portuguese)	0.172*** [0.024]	0.153*** [0.023]	0.138*** [0.024]	0.137*** [0.021]	0.099*** [0.021]	0.036 [0.022]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
$R^2$	0.008	0.035	0.062	0.009	0.035	0.058
$N$	23,750	23,750	23,750	26,594	26,594	26,594
<i>Panel B: Completion on time</i> (Cohorts 2013/2014 to 2014/2015) (Probit - Average Marginal Effects)						
Admitted with Portuguese	0.019* [0.010]	0.006 [0.010]	-0.011 [0.011]			
Generalists				0.094*** [0.012]	0.034*** [0.012]	0.006 [0.012]
All-Rounders (with Portuguese)				0.020** [0.010]	0.014 [0.010]	0.005 [0.011]
All-Rounders (without Portuguese)	0.104*** [0.018]	0.069*** [0.018]	0.067*** [0.018]	0.064*** [0.015]	0.024 [0.015]	0.037** [0.015]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
Pseudo $R^2$	0.018	0.064	0.169	0.019	0.064	0.166
$N$	11,431	11,431	11,055	12,976	12,976	12,621
<i>Panel C: Final GPA</i> (Cohorts 2013/2014 to 2014/2015)						
Admitted with Portuguese	0.146*** [0.026]	0.138*** [0.026]	0.077** [0.030]			
Generalists				0.171*** [0.032]	0.127*** [0.032]	0.126*** [0.034]
All-Rounders (with Portuguese)				0.154*** [0.027]	0.152*** [0.027]	0.063* [0.034]
All-Rounders (without Portuguese)	0.223*** [0.042]	0.194*** [0.042]	0.138*** [0.045]	0.192*** [0.037]	0.157*** [0.037]	0.089** [0.041]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
$R^2$	0.023	0.039	0.097	0.027	0.043	0.093
$N$	6,174	6,174	6,174	6,837	6,837	6,837

Notes: Robust standard errors are in parentheses. \*, \*\*, and \*\*\* represents statistical significance from 10%, 5% and 1% respectively. Control variables: Female, High School Score, and Non Local Student. All regressions include cohort and preferences FE. In Tables D18 and D19 of the appendix, I consider a distance of five points from the threshold.



# For Online Appendix

## A Appendix: Model Derivations

### A.1 Proof of Proposition 1

In a pure strategy Nash Equilibrium all students exert the same level of effort, which means that  $\mathbf{e} = (e_1, e_2) = (e_1^*, e_2^*) = \mathbf{e}^*$ . In order to derive the optimal expressions of  $(e_1^*, e_2^*)$  I divide this proof into four parts: i) derivation of  $Prob(T_1 + \lambda T_2 \geq \tau^*)$ ; ii) define  $\tau^*$  as a function of  $\mathbf{e}^*$ ; iii) FOC; iv) SOC v) Corner Solution.

#### 1. Convolution of probability distributions

In order to solve the student maximization problem I need to derive the probability of getting into university,  $\Pi = Prob(T_1 + \lambda T_2 \geq \tau^*) = Prob(\epsilon_1 + \lambda \epsilon_2 \geq \tau^* - (1 + \lambda)\bar{\alpha} - e_1 - \lambda e_2)$ . Let  $f$  denote the density function of  $z = \epsilon_1 + \lambda \epsilon_2$ . Knowing that  $\epsilon_1$  and  $\epsilon_2$  follow uniform distributions  $[0, a]$  and  $[0, 1]$ , respectively,  $f(\epsilon_1 + \lambda \epsilon_2)$  is represented as follows in Figure 4.<sup>56</sup> (see appendix A.2 for the full derivation of the joint distribution of the two errors and its median point).

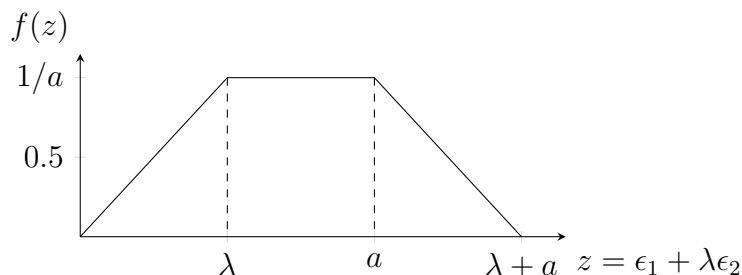


Figure 4: Density function of combined scores from two tests

#### 2. Threshold

The solution to the student's problem depends on  $\tau^*$ .  $\tau^*$  is the admission score of the last student being admitted to the university, and in my model is the median of the student's test scores distribution.  $\tau^*$  depends on the admission test effort of the other students,  $\mathbf{e}^*$ , which implies that

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<sup>56</sup>In Figure 4 I assume  $\lambda > a$  but if I consider that  $\lambda < a$  the density representation would be the same.  $\lambda$  and  $a$  would be in swapped positions on the horizontal axis

$$(10) \quad \tau^*(\mathbf{e}^*) = (1 + \lambda)\bar{\alpha} + e_1^* + \lambda e_2^* + \frac{a + \lambda}{2}.$$

### 3. FOC

The FOC of the student's problem w.r.t  $(e_1, e_2)$  are

$$\begin{aligned} v\bar{\alpha}f(\tau^* - (1 + \lambda)\bar{\alpha} - e_1 - \lambda e_2) &= \frac{\partial C}{\partial e_1} - 1 \\ \lambda v\bar{\alpha}f(\tau^* - (1 + \lambda)\bar{\alpha} - e_1 - \lambda e_2) &= \frac{\partial C}{\partial e_2}. \end{aligned}$$

I can replace  $\tau^*$  by the expression derived in part (2) of this proof, and in equilibrium

I have that  $e_1 = e_1^*$  and  $e_2 = e_2^*$ . Hence, the FOC simplify as follow

$$(11) \quad \begin{aligned} v\bar{\alpha}f\left(\frac{a + \lambda}{2}\right) &= \frac{\partial C}{\partial e_1} - 1 \\ \lambda v\bar{\alpha}f\left(\frac{a + \lambda}{2}\right) &= \frac{\partial C}{\partial e_2}. \end{aligned}$$

In order to find an explicit expression of  $(e_1, e_2)$  I need to replace the derivatives of the cost function and substitute  $f(\frac{a+\lambda}{2})$  by  $\frac{1}{a}$ . As a result,

$$(12) \quad \begin{aligned} e_1 = e_1^* &= \frac{v\bar{\alpha}}{a(1 - \delta^2)}(1 - \lambda\delta) + \frac{1}{1 - \delta^2} \\ e_2 = e_2^* &= \frac{v\bar{\alpha}}{a(1 - \delta^2)}(\lambda - \delta) - \frac{\delta}{1 - \delta^2}. \end{aligned}$$

### 4. SOC

By considering the Hessian matrix,  $H_{(e_1, e_2)}^f = \begin{pmatrix} -\frac{\partial^2 C}{\partial e_1^2} & -\frac{\partial^2 C}{\partial e_1 \partial e_2} \\ -\frac{\partial^2 C}{\partial e_2 \partial e_1} & -\frac{\partial^2 C}{\partial e_2^2} \end{pmatrix} = \begin{pmatrix} -1 & -\delta \\ -\delta & -1 \end{pmatrix}$ ,

I conclude that  $\det(H_1) = -1 < 0$  and  $\det(H_2) = 1 - \delta^2 > 0$  which means that  $H$  is negative definite and  $(e_1, e_2)$  is a local maximum.

### 5. Corner Solution

Finally, the explicit expression of  $(e_1, e_2)$  stated on the previous point is feasible when  $e_1, e_2 > 0$ . Which is true for  $\frac{\lambda}{\delta} > \hat{\lambda}$ , with  $\hat{\lambda} = 1 + \frac{a}{v\bar{\alpha}}$ . For small  $\lambda$ , optimal  $e_2^* = 0$  and optimal  $e_1$  is found from the FOC for  $e_1$ . Thus, for  $\frac{\lambda}{\delta} < \hat{\lambda}$  the solution is

$\mathbf{e}^* = (\frac{v}{a}\bar{\alpha} + 1, 0)$  with  $\lambda = 0$ , provided that  $a > 1 - \alpha_L$  (for  $a < 1 - \alpha_L$  the previous reasoning does not apply).<sup>57</sup>

## A.2 Convolution of the errors

Consider  $Z = \epsilon_1 + \lambda\epsilon_2$  where  $\epsilon_1 \sim U[0, a]$ , with  $0 < a < 1$  and  $\epsilon_2 \sim U[0, 1]$ . This means that the corresponding density functions are  $h(\epsilon_1) = \begin{cases} 1/a, & \text{if } 0 < \epsilon_1 \leq a \\ 0, & \text{otherwise} \end{cases}$  and

$$g(\lambda\epsilon_2) = \begin{cases} 1/\lambda, & \text{if } 0 < \epsilon_2 \leq \lambda \\ 0, & \text{otherwise} \end{cases}$$

Given that  $0 < Z < a + \lambda$  I have that  $f(z) = 0$  for  $z < 0$  and  $z > a + \lambda$

Outside of the previous range of  $z$  I can take the convolution of the two random independent variables:

$$f(z) = \int_{-\infty}^{+\infty} h(z - \lambda\epsilon_2)g(\lambda\epsilon_2)d\epsilon_2 = \int_0^1 h(z - \lambda\epsilon_2)d\epsilon_2$$

I need to consider 3 cases: (i)  $0 \leq z \leq \lambda$ ; (ii)  $\lambda < z \leq a$  and (iii)  $a < z \leq \lambda + a$

For  $0 \leq z \leq \lambda$

$$f(z) = \int_0^z 1 \cdot \frac{1}{a\lambda} d\epsilon_2 = \frac{z}{a\lambda}$$

For  $\lambda < z \leq a$

$$f(z) = \int_0^\lambda 1 \cdot \frac{1}{a\lambda} d\epsilon_2 = \frac{\lambda}{a\lambda} = \frac{1}{a}$$

For  $a < z \leq \lambda + a$

$$f(z) = \int_{z-\lambda}^a 1 \cdot \frac{1}{a\lambda} d\epsilon_2 = \frac{1}{\lambda} + \frac{1}{a} - \frac{z}{a\lambda}$$

Then

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<sup>57</sup>When  $a \rightarrow 0$  I have that  $e_{1j} \rightarrow \infty$  and the student should not exert effort in the second test,  $e_2 = 0$ . With  $a = 0$  the test becomes an all pay auction (Lazear and Rosen, 1981) and there is no equilibrium in pure strategies.

$$(13) \quad f(z) = \begin{cases} \frac{z}{a\lambda}, & \text{if } 0 \leq z \leq \lambda \\ 1/a, & \text{if } \lambda < z \leq a \\ \frac{1}{\lambda} + \frac{1}{a} - \frac{z}{a\lambda}, & \text{if } a < z \leq \lambda + a \end{cases}$$

Equation (13) is represented in Figure 4. Given that  $E[\epsilon_1] = a/2$  and  $E[\epsilon_2] = 1/2$ , the median of the distribution is  $\frac{a+\lambda}{2}$ . By choosing the middle point the university is able to capture half of the student population.

### A.3 Derivation of Lemma 1

In equilibrium, I can think on each admission test as a monotonic transformation of the noise term (effort and ability levels are fixed in equilibrium).<sup>58</sup> To determine the measure of admitted students I focus on the uniform distribution of the error terms  $(\epsilon_1, \epsilon_2)$  associated to the admission tests, as presented in the figure below.

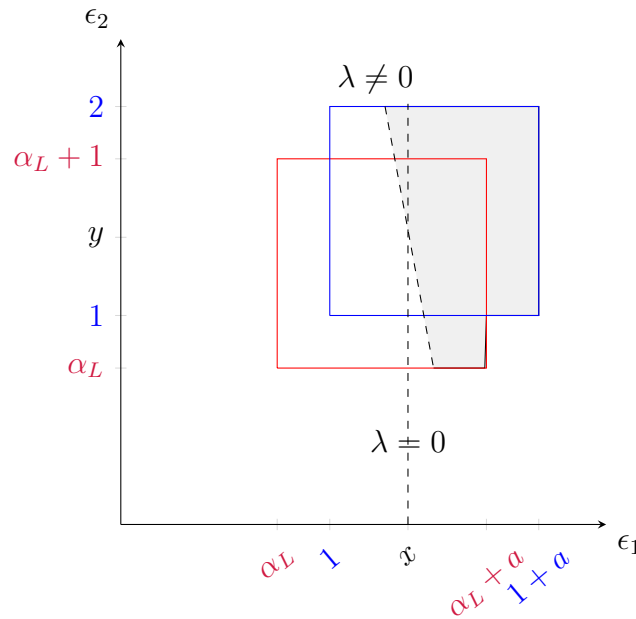


Figure 5: Error terms distribution

<sup>58</sup>In Figure 2 I plotted the distribution of the two tests according to the two levels of ability. When  $\lambda$  increases slightly, there is a rotation of the vertical line. However, the two student's distribution rectangles also change positions. Namely, they move in the Northwest direction (for  $\lambda > \delta\hat{\lambda}$ ). Firstly they move to the west direction (given that  $\frac{\partial \epsilon_1^*}{\partial \lambda} < 0$ ) and thereafter ( $\lambda > \delta\hat{\lambda}$ ) to the north direction (given that  $\frac{\partial \epsilon_2^*}{\partial \lambda} > 0$ ). In both cases what matters to determine the number of admitted students is the slope of the line ( $\lambda$ ) of Figure 2 and not the movement of the rectangles.

where the dotted line labelled  $\lambda \neq 0$  is given by the following expression:

$$(14) \quad \epsilon_2 = k - \frac{\epsilon_1}{\lambda}$$

The line represents the test scores,  $T_1$  and  $T_2$ , such that the combined test score  $T_1 + \lambda T_2$  is constant. I define  $M = (x, y)$  as the center through which the vertical line rotates when  $\lambda$  increases slightly. With  $a = 1$  I have that  $(x, y) = (\frac{\alpha_L}{2} + 1, \frac{\alpha_L}{2} + 1)$  (two exact squares). With  $a \neq 1$  I need to impose that  $(1 + a - x) \times 1 + (\alpha_L + a - x) \times 1 = a$  which implies that  $(x, y) = (\frac{\alpha_L}{2} + \frac{a+1}{2}, \frac{\alpha_L}{2} + 1)$ .

### A.3.1 Graphical definition of $\tilde{\lambda}$

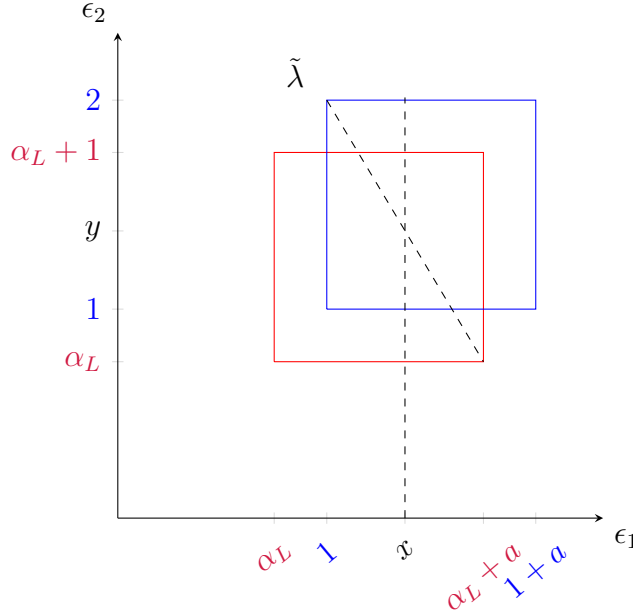


Figure 6: Definition of  $\tilde{\lambda}$

where  $\tilde{\lambda} = \frac{a + \alpha_L - 1}{2 - \alpha_L}$

### A.3.2 For $\lambda \leq \tilde{\lambda}$

In order to find  $(n_H, n_L)$  I determine  $k$ , I impose that  $\frac{\partial n_H}{\partial \lambda} = -\frac{\partial n_L}{\partial \lambda}$  and finally I replace  $k$  as a function of the parameters.

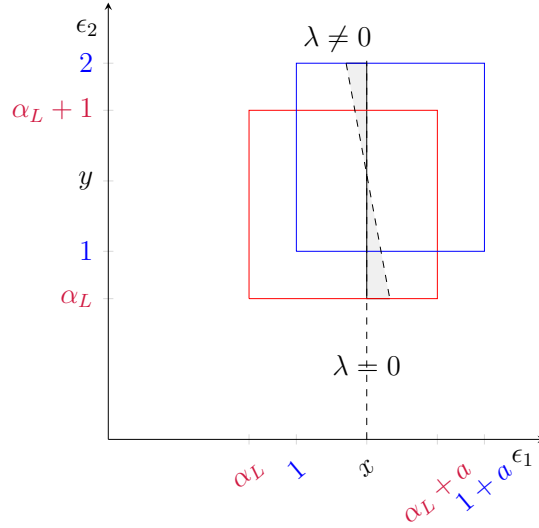


Figure 7: Variation on the number of admitted students for  $0 < \lambda < \tilde{\lambda}$ .

**1st step: Determining  $k$  such as  $n_H + n_L = a$ .**

When  $\lambda$  increases slightly  $n_H = 1 + a - \lambda(k - 1) + \frac{\lambda}{2}$  and  $n_L = \alpha_L + a - \lambda(k - \alpha_L) + \frac{\lambda}{2}$ .

Thus, by imposing that  $n_H + n_L = a$  I obtain that  $k^* = \frac{1+a+\alpha_L}{2\lambda} + 1 + \frac{\alpha_L}{2}$ .

**2nd step: Verifying that the two triangles of Figures 7 are equal.**

Namely,  $[x - \lambda(k - 2)][2 - y]/2 = [y - \alpha_L][\lambda(k - \alpha_L) - x]/2 \Leftrightarrow k^* = \frac{1+a+\alpha_L}{2\lambda} + 1 + \frac{\alpha_L}{2}$

**3rd step: Replace  $k$  by  $k^*$  on  $(n_H, n_L)$ .** As a result, I obtain that  $n_H = \frac{1+a-\alpha_L}{2} + \frac{\lambda}{2}(1 - \alpha_L)$  and  $n_L = \frac{a+\alpha_L-1}{2} + \frac{\lambda}{2}(\alpha_L - 1)$ .

### A.3.3 For $\tilde{\lambda} < \lambda < 1$

The procedure is very similar to the previous one. The difference is that now I am dealing with trapezes instead of triangles (see Figure 8).

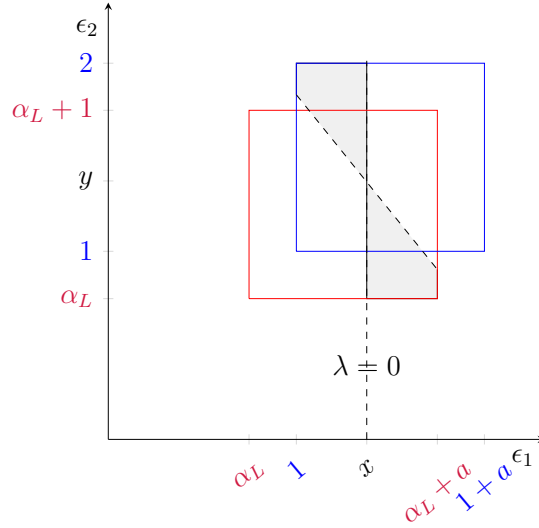


Figure 8: Variation on the admitted students for  $\tilde{\lambda} < \lambda < 1$ .

Again, by imposing that the two shadow areas in Figure 8 must be equal I would obtain  $k^*$ , namely,

$$\begin{aligned} (2 - 1 - \frac{\alpha_L}{2} + 2 - k + \frac{1}{\lambda}) \times (\frac{a + \alpha_L - 1}{2}) / 2 &= (1 + \frac{\alpha_L}{2} - \alpha_L + k - \frac{\alpha_L + a}{\lambda} - \alpha_L) \times (\frac{a + \alpha_L - 1}{2}) / 2 \Leftrightarrow \\ \Leftrightarrow k^* &= 1 + \frac{\alpha_L}{2} + \frac{a + 1 + \alpha_L}{2\lambda} \end{aligned}$$

I replace  $k$  by  $k^*$  on  $(n_H, n_L)$  and I obtain that:

$$\begin{aligned} n_H &= a - \lambda \frac{\alpha_L^2}{8} - \frac{\alpha_L}{4}(a + \alpha_L - 1) - \frac{(a + \alpha_L - 1)^2}{8\lambda} \\ n_L &= \lambda \frac{\alpha_L^2}{8} + \frac{\alpha_L}{4}(a + \alpha_L - 1) + \frac{(a + \alpha_L - 1)^2}{8\lambda} \end{aligned}$$

### A.3.4 Summary

Finally, I need to adjust  $n_H$  and  $n_L$  by  $\frac{1}{a}$ . As a result, I have that:

Table A1: Measure of admitted students

$\lambda$	$\frac{\partial n_H}{\partial \lambda}$	High Ability Students	Low Ability Students
$0 < \lambda \leq \tilde{\lambda}$	$> 0$	$[\frac{1+a-\alpha_L}{2} + \frac{\lambda}{2}(1-\alpha_L)] \frac{1}{2a}$	$[\frac{a+\alpha_L-1}{2} + \frac{\lambda}{2}(\alpha_L-1)] \frac{1}{2a}$
$\tilde{\lambda} < \lambda \leq 1$	$< 0$	$[a - \lambda \frac{\alpha_L^2}{8} - \frac{\alpha_L}{4}(a + \alpha_L - 1) - \frac{(a+\alpha_L-1)^2}{8\lambda}] \frac{1}{2a}$	$[\lambda \frac{\alpha_L^2}{8} + \frac{\alpha_L}{4}(a + \alpha_L - 1) + \frac{(a+\alpha_L-1)^2}{8\lambda}] \frac{1}{2a}$

Notes:  $\tilde{\lambda} = \frac{a+\alpha_L-1}{2-\alpha_L}$ .

## A.4 Government Policy

The maximization problem of the social planner is as follows:

$$(15) \quad \begin{aligned} \text{Max}_{\lambda} \quad & \Pi_S = v(\alpha_H n_H(\lambda) + \alpha_L n_L(\lambda)) + e_1^* + \left(\frac{1}{2}\right)\beta(\alpha_H + \alpha_L) - C(e_1, e_2) \\ \text{s.to} \quad & n_H(\lambda) + n_L(\lambda) = \frac{1}{2} \end{aligned}.$$

The government is interested in maximizing the total wage of the society, taking into consideration individuals that do not attend university and the effort that they exert to the admission tests. All individuals, skilled and unskilled, matter. The approach I follow is very similar to the one presented in the previous section when I considered the university problem. Let  $\lambda_S^*$  be the solution to the government problem. Similar to what I concluded before, I have that:

**Theorem 2.** *If  $a > 1 - \alpha_L$ , the solution of social planner's problem is  $\lambda_S^* > 0$ .*

*Proof.* The proof is similar to the one of theorem 1. The social planner payoff function is a continuous function on  $\lambda$ , and a differentiable function on its domain except at  $\delta\hat{\lambda}$  and  $\tilde{\lambda}$ . For that interval, the first order derivative of the social planner payoff function w.r.t.  $\lambda$  is:

$$(16) \quad \frac{\partial \pi_S}{\partial \lambda} = v(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda} + \frac{\partial e_1^*}{\partial \lambda} - \frac{\partial C}{\partial e_1} \frac{\partial e_1^*}{\partial \lambda} - \frac{\partial C}{\partial e_2} \frac{\partial e_2^*}{\partial \lambda}$$

For  $\lambda \in ]0, \min\{\delta\hat{\lambda}, \tilde{\lambda}\}[$  I have  $\frac{\partial e_1^*}{\partial \lambda} = 0$  and  $\frac{\partial n_H}{\partial \lambda} > 0$  which implies  $\frac{\partial \pi_S}{\partial \lambda} > 0$ .  $\square$

The university and the social planner agree on using the second test as a screening device. However, that does not mean that they would necessarily agree on the weight for



$\lambda$ .<sup>59</sup> I set that result in the theorem below.

**Theorem 3.** *The optimal  $\lambda_U^*$  of university is greater or equal than the optimal  $\lambda_S^*$  of the government,  $\lambda_S^* \leq \lambda_U^*$ .*

*Proof.* The solutions of the university and social planner problems can be either an interior solutions(which satisfies the FOCs) or a corner solution (namely,  $\delta\hat{\lambda}$  or  $\tilde{\lambda}$ ). In this proof I do not determine the exact solution. I want to verify whether the solution of both problems exists and obtain that  $\lambda_S^* \leq \lambda_U^*$ .

I start the proof by computing the first derivatives of both maximization problems. Namely,

$$(17) \quad \frac{\partial \pi_U}{\partial \lambda} = \gamma(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda} + \frac{N}{2} \frac{\partial e_1^*}{\partial \lambda}$$

$$(18) \quad \frac{\partial \pi_S}{\partial \lambda} = v(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda} + \frac{\partial e_1^*}{\partial \lambda} - \frac{\partial C}{\partial e_{1j}} \frac{\partial e_1^*}{\partial \lambda} - \frac{\partial C}{\partial e_{2j}} \frac{\partial e_2^*}{\partial \lambda}$$

If  $\lambda < \delta\hat{\lambda}$  I have that  $\frac{\partial \pi_U}{\partial \lambda} = \gamma(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda}$  and  $\frac{\partial \pi_S}{\partial \lambda} = v(\alpha_H - \alpha_L) \frac{\partial n_H}{\partial \lambda}$ . The solutions cannot be on  $]0, \delta\hat{\lambda}[$ , since  $\frac{\partial \pi_U}{\partial \lambda} > 0$  on this interval. The solutions can only be at  $\lambda_S^* = \lambda_U^*$ . Alternatively, if  $\lambda > \delta\hat{\lambda}$  I have that:<sup>60</sup>

$$(19) \quad \frac{\partial \pi_S}{\partial \lambda} = \frac{\partial \pi_U}{\partial \lambda} - \underbrace{[\beta(\alpha_H - \alpha_L) + \frac{dC}{d\lambda} - \frac{1}{2} \frac{\partial e_{1j}^*}{\partial \lambda}]}_{>0}$$

Consider that  $\lambda_U^*$  exists and satisfies the FOC  $\frac{\partial \pi_U}{\partial \lambda}(\lambda_U^*) = 0$ . Thus,  $\frac{\partial \pi_S}{\partial \lambda}(\lambda_U^*) < 0$  which implies that  $\lambda_S^* < \lambda_U^*$ .

If  $\lambda_U^* = \delta\hat{\lambda}$  or  $\lambda_U^* = \tilde{\lambda}$ , then the solution of the social planner problem would be exactly the same and  $\lambda_S^* = \lambda_U^*$ .

Thus, I always will have that  $\lambda_S^* \leq \lambda_U^*$ . □

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<sup>59</sup>See example in appendix A.4.1.

<sup>60</sup>For  $\lambda > \delta\hat{\lambda}$  I have that  $\frac{dC}{d\lambda} = (e_1 + \delta e_2) \frac{\partial e_1^*}{\partial \lambda} + (e_2 + \delta e_1) \frac{\partial e_2^*}{\partial \lambda} = \frac{v\bar{\alpha}}{a(1-\delta^2)} e_2(1-\delta^2) > 0$

### A.4.1 Disagreement on $\lambda$

Consider the following example where the university would like to consider a weight for  $\lambda$  higher than the one desired by the government. Parameter values assumed in this example are shown in Table A2 and the corresponding payoff functions in the figures below.

Table A2: Parameter Values of Example 1

Parameter	Value
$v$	40
$\gamma$	40.5
$a$	0.85
$\alpha_L$	0.5
$\alpha_H$	1
$\delta$	0.1
$N$	1000
Kink points of the payoff function	Value
$\delta\hat{\lambda}$	0.1028
$\tilde{\lambda}$	0.2333

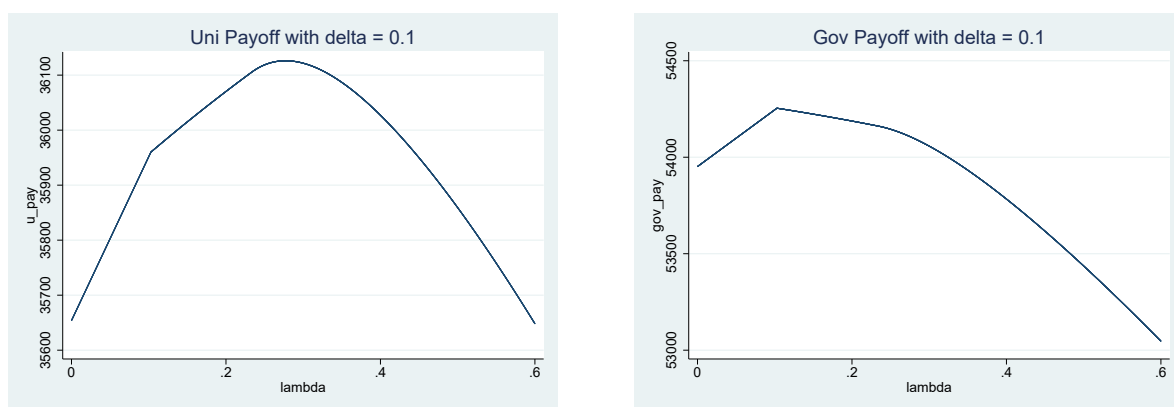


Figure 9: University and Government Payoffs

In this example, university optimal  $\lambda$  is approximately 0.3, and the government's optimal  $\lambda$  is 0.1. The social planner and the university disagree on  $\lambda$ .

Table B3: First Cycle Degrees in Public HEIs, 2018

	Total Number
<b>Institutions</b>	170
At University	75
At Polytechnic	95
<b>Degrees</b>	529
Bachelors	485
Integrated Masters	38
Prep Bachelors/Master	6
<b>Institutions x Degrees</b>	1,180
Bachelors	1,066
Integrated Masters	112
Prep Bachelors/Masters	2
<b>Vacancies</b>	50,852

Source: DGES (2019a)

## B Appendix: Datasets

Table B4: Descriptive Statistics, Full Population at Public HEIs, **Application Dataset**<sup>(1)</sup>

	Mean	Std. Dev.
A. Data Structure		
Initial year	2008/2009	
Final year	2018/2019	
No. years	11	
No. individuals	790,723	
B. Applicants		
Age	18.74	2.46
Female (share)	.57	
High school GPA (0-200)	147.64	19.41
No. of stated preferences per applicant	4.47	
C. Placement		
Degree of placement (no. individuals)		
Not placed	186,558	
Bachelor	504,474	
Integrated master	97,960	
Other - 1st cycle <sup>(2)</sup>	1,701	
Preferences of placement (share)		
1st	.51	
2nd	.21	
3rd	.12	
4th	.07	
5th	.05	
6th	.03	
Application score	144.22	19.53
No. of admission exams	1.31	.52
Portuguese admission exam (share)	.22	

Source: (DGES, 2019b). Notes: (1) Applications under the General Access Regime (GAR); (2) The majority of applications to “Other - 1st cycle” programmes are not centralized and, instead, made directly to the institution.

Table B5: Descriptive Statistics, Full Population at Public HEIs, **Performance Dataset**

	Mean	Std. Dev.
A. Data Structure		
Initial year	2013/2014	
Final year	2017/2018	
No. years	5	
No. obs. individual-year	1,545,059	
No. individuals <sup>(1)</sup>	756,435	
Bachelor	404,394	
Integrated master	104,613	
Academic year one	360,748	
1st time enrolment	291,042	
Entry through GAR <sup>(2)</sup>	214,165	
Entry through other regime <sup>(3)</sup>	59,957	
Entry regime not specified <sup>(4)</sup>	16,920	
Repeaters	69,706	
Other programmes <sup>(5)</sup>	251,428	
B. Individuals (Academic year one, through GAR)		
No. Individuals	214,165	
Age	18.49	2.11
Female (share)	.57	
Application score (0-200)	144.49	20.24
Public high school (share)	.84	
Non Local Student (share)	.30	
Mother has HE (share)	.33	
Father has HE (share)	.26	
Applied to maintenance grant (share)	.31	
Received a maintenance grant (share)	.17	

Source: (DGEEC, 2019). Notes: (1) Def. individual: unique person ID per programme; (2) GAR regards the national contest of application (General Access Regime); (3) Application through special regime: transference, diplomat son, immigrant status, among other examples; (4) Application through local access contest for instance; (5) Other programmes include Master, PhD, Other - 1st cycle and 2nd cycle programmes.

Table B6: Descriptive Statistics, Full Population at Public HEIs, **Graduation Dataset**

	Mean	Std. Dev.
A. Data Structure		
Initial year	2012/2013	
Final year	2016/2017	
No. years	5	
No. individuals	327,221	
Bachelor	159,283	
Bachelor (part 1 of the integrated master)	32,719	
Other programmes <sup>(1)</sup>	135,219	
B. Graduates		
Age		
Bachelor	26.03	6.50
Bachelor (part 1 of the integrated master)	24.27	3.69
Female (share)		
Bachelor	0.60	
Bachelor (part 1 of the integrated master)	0.50	
Final GPA (0-200)		
Bachelor	136.39	14.78
Bachelor (part 1 of the integrated master)	135.30	14.96
No. years enrolled in the programme		
Bachelor	3.87	1.37
Bachelor (part 1 of the integrated master)	4.09	1.79

Source: (DGEEC, 2019). Notes: (1) Other programmes include Master, PhD, Other - 1st cycle and 2nd cycle programmes.

Table B7: Transition between High School and Higher Education in 2013/2014

Main Tracks at High School	High School		Higher Education	
	National Exams Mandatory	Graduates	Did not continue studying	Enrolled in HE
General Track (Scientific)	Yes	38,382	16%	79%
Professional Track	No	22,842	82%	6%
Vocational Track	No	1,097	39%	53%

Source: DGEEC (2016) Note: This only includes students enrolled at high school in Portugal Mainland. The main tracks at high school in Portugal are the General (Scientific) and Professional tracks. However there are other tracks, with fewer enrolments, such as Vocational, Learning courses (provided by the Institute for Employment and Vocational Training), and Artistic courses, whose goal is to keep everyone in education until certain age (18 years old).

Table B8: Summary of original variables in the Application dataset

Unit	Variable
<b>Programme (Degree x University)</b>	
	Academic year
	Vacancies
	Number of allocated students
	Number of enrolled students
	Admission score threshold
	Scientific area (CNAEF)
<b>Candidate</b>	
Personal Characteristics	Gender
	Date of Birth
Previous Achievement	High school track
	High school course
	High school score
Application Process	Academic year
	Preferences
	Admission exams and scores (for each preference)
Placement	HEI (faculty or university)
	Institution type
	Degree
	Admission score
	Entrance exams and scores
	Preference of placement
	Geographic office of application (GAES)
	Status
	Excluded (reason)
	Type of application
	Enrolment (yes or no)
	Remained enrolled after one year (yes or no)

Source: (DGES, 2019b)

Table B9: Summary of original variables in the Performance dataset

Unit	Variables
<b>Programme (Degree x University)</b>	
	Academic year
	Scientific area (international, CISCED97)
	Scientific area (national, CNAEF)
	Institution type
	Geographic location
<b>Student</b>	
Personal Characteristics	Gender
	Age
Placement	Academic year
	Programme (Institution and degree)
	Admission score
	Preference of placement
	1st enrolment (yes or no)
	Programme transference (yes or nor)
Previous Achievement	Previous university (if applicable)
	High school track
	High school course
	High school score
Socie-conomic Background	Parents education
	Parents job occupation
	Parents job status
	Student job status
	Maintenance grant (applicants and admitted)
Mobility	Home location during high school
	Non-local student (yes or no)
	Exchange programme (type, duration, degree and country)
Performance	ECTS enrolled
	ECTS accumulated

Source: (DGEEC, 2019)

Table B10: Summary of original variables in the Graduation dataset

Unit	Variables
<b>Programme (Degree x University)</b>	
	Scientific area (international, CISCED97)
	Scientific area (national, CNAEF)
	Institution type
	Geographic location
<b>Student</b>	
Personal Characteristics	Gender
	Age
Placement	Programme (Institution and degree)
Graduation	Final GPA
	Degree type
	Diploma type
	Date of graduation

Source: (DGEEC, 2019)



Table B11: Broad Fields of Study

Code	Field	REF (UK)
1	Science: biology; chemistry; computer science; mathematics; physics	B
2	Business: administration; accounting; business studies	C
3	Social science: sociology; political science; anthropology; economics; psychology	C
4	Teaching: kindergarten teacher; school teacher	D
5	Humanities: history; philosophy; languages; media	D
6	Health: nursing; social work; physical therapy	A
7	Engineering (BSc): electrical; construction; mechanical; computer	B
8	Technology: MSc engineering; biotechnology; information technology	C
9	Law: law	A
10	Medicine: medicine; dentistry; pharmacology	A
11	Agriculture: agriculture veterinary science; forestry and fishing	A
12	Services: tourism; hotels and restaurants; sports	C
13	Unspecified	-

Note: Adapted from [Kirkeboen et al. \(2016\)](#). REF Categories: A) Medicine, Health and Life Sciences; B) Physical Sciences, Engineering and Maths; C) Social Sciences; D) Arts and Humanities

Table B12: Descriptive statistics for fields that consider Portuguese

	Programmes that not consider PT		Programmes that consider PT	
	Mean	Std. Dev.	Mean	Std. Dev.
No. Individuals	71,647		78,233	
High School Score	142.42	17.50	150.96	18.62
Non-local student (share)	0.28		0.30	
Applied to a maint.grant (share)	0.31		0.36	
Received a maint. grant (share)	0.22		0.31	
Female (share)	0.50		0.65	
Mother has HE (share)	0.35		0.23	
Father has HE (share)	0.27		0.16	

Notes: I only include fields that consider the Portuguese exam. For instance, if in the field of civil engineering, there is no programme which considers Portuguese as an optional requirement, then that field is not included in the left panel.

Table B13: Diploma types

Degree classification	(code)	Diploma type (in PT)	Translation
Bachelor	(PL)	Preparatórios de licenciatura 1.º ciclo	Bachelor preparatory programmes 1st cycle
	(PM)	Preparatórios de mestrado integrado	Integrated master preparatory programmes
	(L1)	Licenciatura 1.º ciclo	Bachelor 1st cycle
	(L)	Licenciatura	Bachelor
Integrated master	(MI)	Mestrado integrado	Integrated master
	(MT)	Mestrado integrado terminal	Terminal integrated master
Master	(M2)	Mestrado 2.º ciclo	Master 2nd cycle
	(M)	Mestrado	Master
PhD	(D3)	Doutoramento 3.º ciclo	PhD 3rd cycle
	(D)	Doutoramento	PhD
Other - 1st Cycle	(T)	Curso técnico superior profissional	Professional higher technical courses
	(CF)	Complemento de formação	Complementary course
Others - 2nd Cycle	(C0)	Curso de especialização tecnológica	Technological specialization course
	(E)	Especialização pós-licenciatura	Pos-bachelor specialization
	(GB)	Especialização pós-bacharelato	Pos-master specialization

Source: (DGEEC, 2019). Notes: Aggregation made by the author.

Table B14: Analysis dataset according to all constraints<sup>(a)</sup>

Interval Band	Analysis by groups $0 \leq \tau - \tau^* \leq 10$					
	ECTS		Completion on Time		Final GPA	
Outcomes	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	
Mean						
No. Individuals	26,594		12,976		6,837	
Female (share)	.66		.62		.71	
High school score (0-200)	141.30	16.61	139.39	15.49	142.90	15.34
Non-local student (share)	.32		.31		.32	
Preferences of placement (share)						
1st	0.50		0.49		0.53	
2nd	0.23		0.24		0.24	
3rd	0.13		0.13		0.12	
4th	0.07		0.07		0.06	
5th	0.04		0.04		0.03	
6th	0.02		0.02		0.02	
Application score (0-200)	137.37	16.31	134.26	14.99	137.12	14.76
Portuguese admission exam (share)	0.54		0.55		0.56	
Portuguese exam score (0-200)	119.27	24.96	115.67	23.91	118.20	23.74

Notes: (a) I only consider: i) programmes that allow for PT as an optional requirement; and ii) individuals for which I observe the outcome.

## C Appendix: Groups' Identification

### C.1 Example with observed partial scores

Consider again the Pedro and Alexandre's example. Imagine that both brothers were admitted to the same programme. The university allows the candidates to apply with the Mathematics exam score only or with the combination of Economics and Portuguese exam scores (see Table C15).

Table C15: Observed partial scores

	Exams' scores			Actual admission score	Application score without PT	Application score with PT
	Mathematics	Economics	Portuguese			
Pedro	170	150	110	170		130
Alexandre	n.a.	170	200	185	n.a.	

Note: Assume Pedro and Alexandre were admitted to the same programme. The university allows the students to apply with the Mathematics exam score only or with the combination of the Economics and Portuguese exam scores. Alexandre was admitted with the Portuguese exam and Pedro without it.

In the case of Alexandre, the application score without Portuguese should be the Mathematics exam score. Since I do not observe the Mathematics score, I consider the score of Economics as the combination itself,  $\tau_{\sim PT}^{Alexandre} = 170$ .

## C.2 Missing at random

Table C16: Group classification missing at random

	Total Sample		$0 \leq \tau - \tau^* \leq 10$		$0 \leq \tau - \tau^* \leq 5$	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Outcome: Non Identified Group</i>						
Probit - Average Marginal Effects						
Female	-0.002 [0.004]	0.002 [0.004]	0.006 [0.005]	0.007 [0.005]	0.012* [0.006]	0.013** [0.006]
Non Local Student	-0.007* [0.004]	0.004 [0.005]	-0.002 [0.005]	0.006 [0.006]	-0.003 [0.006]	0.007 [0.007]
High School Score	-0.004*** [0.000]	0.001*** [0.000]	-0.005*** [0.000]	0.002*** [0.000]	-0.005*** [0.000]	0.001*** [0.000]
Mother has HE degree	-0.017*** [0.005]	0.004 [0.005]	-0.016** [0.006]	0.006 [0.006]	-0.019** [0.008]	0.005 [0.007]
Father has HE degree	-0.032*** [0.006]	0.009* [0.005]	-0.026*** [0.007]	0.014** [0.007]	-0.021** [0.009]	0.017** [0.008]
Programme FE		✓		✓		✓
Control and Preference FE	✓	✓	✓	✓	✓	✓
Pseudo $R^2$	0.021	0.167	0.023	0.173	0.022	0.216
$N$	70,833	70,746	42,866	42,712	28,443	28,190

Notes: Robust standard errors are in parentheses. \*, \*\* and \*\*\* represents statistical significance from 10%, 5% and 1% respectively. Out of the 78,233 students there are 7,400 students for which I do not have information regarding either the Mother or Father level of education. The outcome variable is a dummy equal one if I cannot allocated the student to one of the four groups and 0 otherwise.

## C.3 Imputation of application score

Consider the following example in Table C17, where three female students were admitted, without the Portuguese exam, to the same programme whose admission threshold was 110. I have the following information for each one of the six students:

There is one student, Joana, for which I was not able to identify the group. However, I know that she is either a specialist or an all-rounder. To determine the corresponding group I estimate the alternative application score using regression analysis. In order to determine Joana's application score with Portuguese I run the following regression:

Table C17: Imputation of the alternative application score

Student	Admitted with the PT exam $1\{\tau_{PT} - \tau_{\sim PT}\}$	Actual admission score $\tau$	Application score with PT $\tau_{PT}$	Group
Ana	0	150	130	All-Rounder
Joana	0	120	n.a.	n.a.
Marta	0	140	100	Specialist

Notes: n.a. - non available. Assume that there are three female students admitted to the same programme. Three were admitted without the Portuguese exam while the other students were admitted with it.

$$(20) \quad \tau_{PT}^i = \beta_0 + \beta_i' \mathbf{X}^i + \Theta + \epsilon^i \quad \text{if} \quad 1\{\tau_{PT} - \tau_{\sim PT}\} = 0$$

where  $\mathbf{X}^i$  represents the vector of controls (gender, individual ability and home location) and  $\Theta$  the programme and year fixed effects. Thus,

$$(21) \quad \tau_{PT}^{Joana} = \widehat{\beta}_0 + \widehat{\beta}_i' \mathbf{X}^i + \Theta$$

As a result if  $\tau_{PT}^{Joana} \geq 110$  then Joana is considered to be an all-rounder, otherwise she is consider to be a specialist. In order words, I estimate Joana's predicted application score with Portuguese based on the information of her peers that match the same individual characteristics.

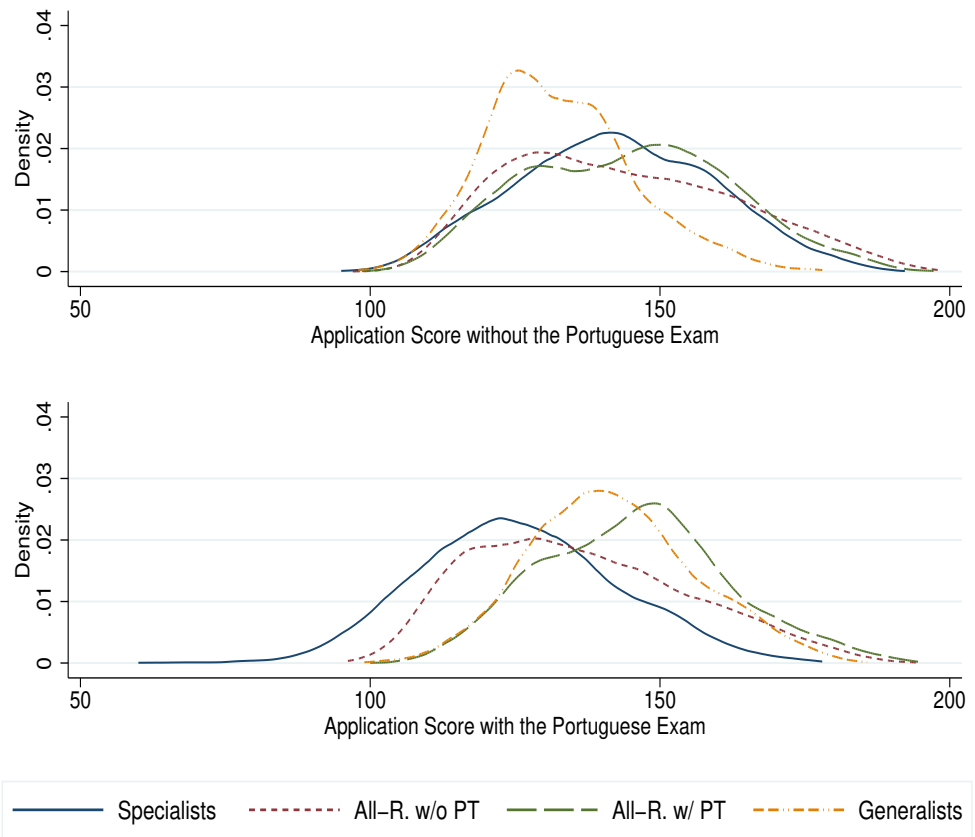


Figure 10: Distribution of application scores by groups - w/o imputation

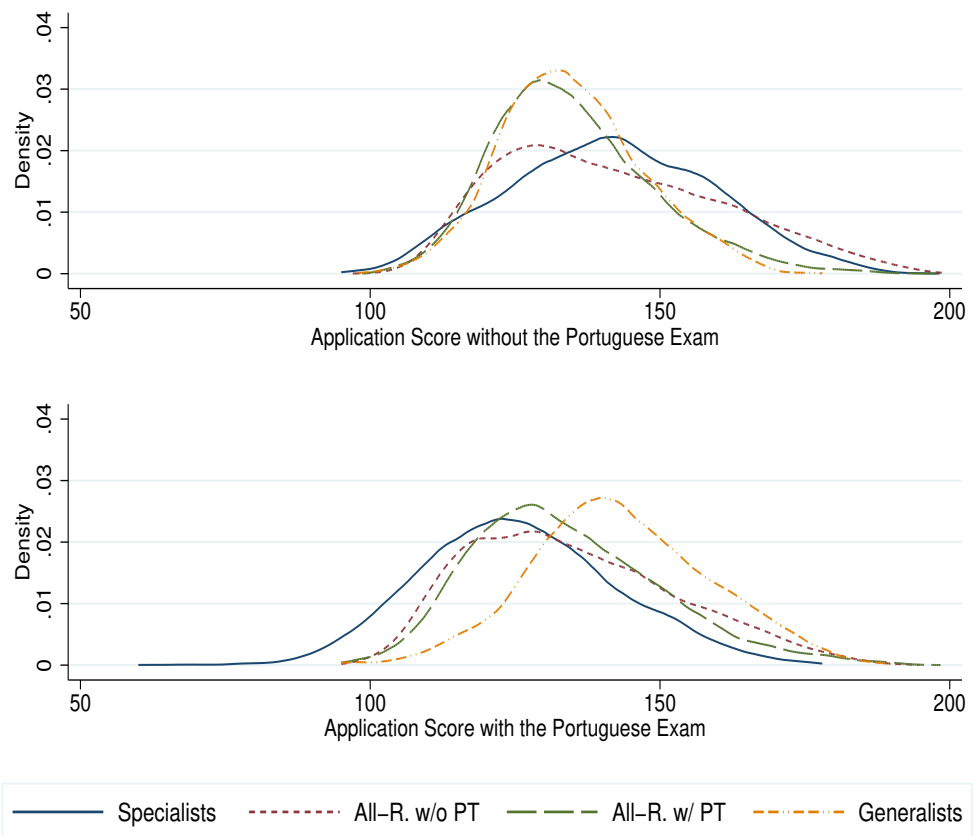


Figure 11: Distribution of application scores by groups - w/ imputation

## D Appendix: Results

Table D18: At the margin of gaining entry in HE  
(considering those admitted with Portuguese jointly)

	$0 \leq \tau - \tau^* \leq 5$		
	(1)	(2)	(3)
<i>Panel A: No. ECTs accumulated by the end of the 1st year</i> <i>Cohorts 2013/2014 to 2016/2017</i>			
Admitted with Portuguese	0.025 [0.018]	0.009 [0.018]	-0.029 [0.020]
All-Rounders (without PT)	0.162*** [0.038]	0.131*** [0.037]	0.091** [0.038]
Controls		✓	✓
Programme FE			✓
$R^2$	0.009	0.034	0.075
$N$	14,652	14,652	14,652
<i>Panel B: Completion on time</i> <i>Cohorts 2013/2014 to 2014/2015</i> <i>(Probit - Average Marginal Effects)</i>			
Admitted with Portuguese	0.022* [0.013]	0.009 [0.013]	-0.011 [0.013]
All-Rounders (without PT)	0.132*** [0.030]	0.097*** [0.029]	0.115*** [0.029]
Controls		✓	✓
Programme FE			✓
Pseudo $R^2$	0.016	0.069	0.176
$N$	6,990	6,990	6,599
<i>Panel C: Final GPA</i> <i>Cohorts 2013/2014 to 2014/2015</i>			
Admitted with Portuguese	0.142*** [0.033]	0.139*** [0.033]	0.072* [0.039]
All-Rounders (without PT)	0.175*** [0.068]	0.155** [0.067]	0.124* [0.074]
Controls		✓	✓
Programme FE			✓
$R^2$	0.025	0.033	0.123
$N$	3,628	3,628	3,628

Notes: Robust standard errors are in parentheses. \*, \*\* and \*\*\* represents statistical significance from 10%, 5%, and 1% respectively. All regression include cohort and preferences FE. The number of ECTS by the end of first year and Final GPA were standardized within programmes. All coefficients in panels A and C should be interpreted in terms of standard deviation changes in the outcome. Control variables: Female, High School Score, Non-Local Student.

Table D19: At the margin of gaining entry in HE

	$0 \leq \tau - \tau^* \leq 5$					
	Without Imputation			With Imputation		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel A: No. of ECTS by the end of the 1st year - Cohorts 2013/2014 to 2016/2017</i>						
Generalists	0.058* [0.031]	-0.011 [0.031]	-0.039 [0.034]	0.056*** [0.021]	-0.012 [0.021]	-0.014 [0.022]
All-Rounders (with PT)	-0.042 [0.039]	-0.096** [0.039]	-0.006 [0.049]	0.038* [0.019]	0.043** [0.019]	-0.015 [0.023]
All-Rounders (without PT)	0.163*** [0.038]	0.129*** [0.037]	0.064 [0.040]	0.099*** [0.033]	0.059* [0.033]	-0.003 [0.035]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
$R^2$	0.016	0.043	0.114	0.009	0.034	0.070
$N$	8,003	8,003	8,003	16,397	16,397	16,397
<i>Panel B: Completion on time - Cohorts 2013/2014 to 2014/2015 (Probit - Average Marginal Effects)</i>						
Generalists	0.076*** [0.025]	0.020 [0.025]	-0.008 [0.026]	0.090*** [0.015]	0.034** [0.014]	0.001 [0.015]
All-Rounders (with PT)	0.119*** [0.044]	0.080* [0.044]	0.052 [0.048]	0.018 [0.013]	0.016 [0.013]	0.010 [0.015]
All-Rounders (without PT)	0.131*** [0.029]	0.096*** [0.029]	0.118*** [0.031]	0.077*** [0.024]	0.032 [0.023]	0.062*** [0.023]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
Pseudo $R^2$	0.021	0.066	0.166	0.019	0.064	0.162
$N$	3,186	3,186	2,875	7,923	7,923	7,523
<i>Panel C: Final GPA - Cohorts 2013/2014 to 2014/2015</i>						
Generalists	0.174*** [0.060]	0.126** [0.061]	0.067 [0.074]	0.148*** [0.038]	0.121*** [0.038]	0.104** [0.042]
All-Rounders (with PT)	0.186** [0.092]	0.170* [0.093]	0.143 [0.124]	0.134*** [0.035]	0.139*** [0.035]	0.044 [0.045]
All-Rounders (without PT)	0.174** [0.068]	0.145** [0.068]	0.098 [0.080]	0.131** [0.058]	0.103* [0.058]	0.046 [0.065]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
$R^2$	0.032	0.046	0.220	0.027	0.036	0.114
$N$	1,762	1,762	1,762	4,019	4,019	4,019

Notes: Robust standard errors are in parentheses. \*, \*\* and \*\*\* represents statistical significance from 10%, 5%, and 1% respectively. All regression include cohort and preferences FE. The number of ECTS by the end of first year and Final GPA were standardized within programmes. All coefficients in panels A and C should be interpreted in terms of standard deviation changes in the outcome. Control variables: Female, High School Score, Non-Local Student.



Table D20: T test between Generalists and All-rounders (with PT)

$0 \leq \tau - \tau^* \leq 10$			
	(1)	(2)	(3)
$H_0 : \alpha_1 - \alpha_2 = 0$			
<i>No. ECTS by the end of the 1st year</i>			
F statistic	0.07	18.19	0.18
Prob > F	0.79	0.00	0.67
<i>Final GPA</i>			
F statistic	0.30	0.63	2.93
Prob > F	0.58	0.43	0.09
Controls		✓	✓
Programme FE			✓

Notes: The test regards the regressions presented in Table 9. The table reports the F statistics. Robust standard errors are in parentheses. All regression include cohort and preferences FE. Control variables: Female, High School Score, Non-Local Student.

Table D21: Final GPA for those that completed the degree on time

$0 \leq \tau - \tau^* \leq 10$						
	(1)	(2)	(3)	(4)	(5)	(6)
Admitted with Portuguese	0.141*** [0.027]	0.137*** [0.027]	0.065** [0.031]			
Generalists				0.160*** [0.033]	0.120*** [0.032]	0.118*** [0.035]
All-Rounders (with PT)				0.148*** [0.028]	0.150*** [0.028]	0.046 [0.034]
All-Rounders (without PT)	0.220*** [0.044]	0.192*** [0.043]	0.136*** [0.047]	0.185*** [0.039]	0.152*** [0.038]	0.079 * [0.043]
Controls		✓	✓		✓	✓
Programme FE			✓			✓
$R^2$	0.025	0.041	0.093	0.027	0.043	0.092
$N$	5,813	5,813	5,813	6,441	6,441	6,441

Notes: Robust standard errors are in parentheses. All regression include cohort and preferences FE. \*, \*\* and \*\*\* represents statistical significance from 10%, 5%, and 1% respectively. The number of ECTS by the end of first year and Final GPA were standardized within programmes. All coefficients in panels A and C should be interpreted in terms of standard deviation changes in the outcome. Control variables: Female, High School Score, Non-Local Student.

Table D22: Heckman Selection model, by MLE (with imputation)

	$0 \leq \tau - \tau^* \leq 10$		
	(1)	(2)	(3)
<b>Outcome Eq.</b>			
<b>(Final GPA)</b>			
Generalists	0.170*** [0.032]	0.127*** [0.036]	0.121*** [0.036]
All-Rounders (with PT)	0.155*** [0.027]	0.094*** [0.031]	0.047 [0.036]
All-Rounders (without PT)	0.191*** [0.037]	0.163*** [0.043]	0.124*** [0.044]
Controls		✓	✓
Programme FE			✓
<b>Selection Eq.</b>			
<b>(Prob. of students not dropping out by the end of the first academic year)</b>			
Generalists	0.036*** [0.010]	0.009 [0.009]	0.003 [0.009]
All-Rounders (with PT)	-0.017** [0.008]	-0.018** [0.008]	-0.004 [0.008]
All-Rounders (without PT)	0.027** [0.012]	0.008 [0.011]	0.026** [0.011]
Controls		✓	✓
Programme FE			✓
Number of Obs.	12,707	12,707	12,707
Uncensored of Obs.	6,837	6,837	6,837
$\rho$	-0.024	-0.030	0.834
$\sigma$	0.915	0.908	1.022
$\lambda$	-0.022	-0.027	0.853
Wald test for $\rho = 0$ (Prob > $\chi^2$ )	0.536	0.315	0.000

Notes: Robust standard errors are in parentheses. \*, \*\* and \*\*\* represents statistical significance from 10%, 5%, and 1% respectively. For the selection equation, I report the average marginal effects. All regression include cohort and preferences FE. The number of ECTS by the end of first year and Final GPA were standardized within programmes. All coefficients in panels A and C should be interpreted in terms of standard deviation changes in the outcome. Control variables: Female, High School Score, Non-Local Student.

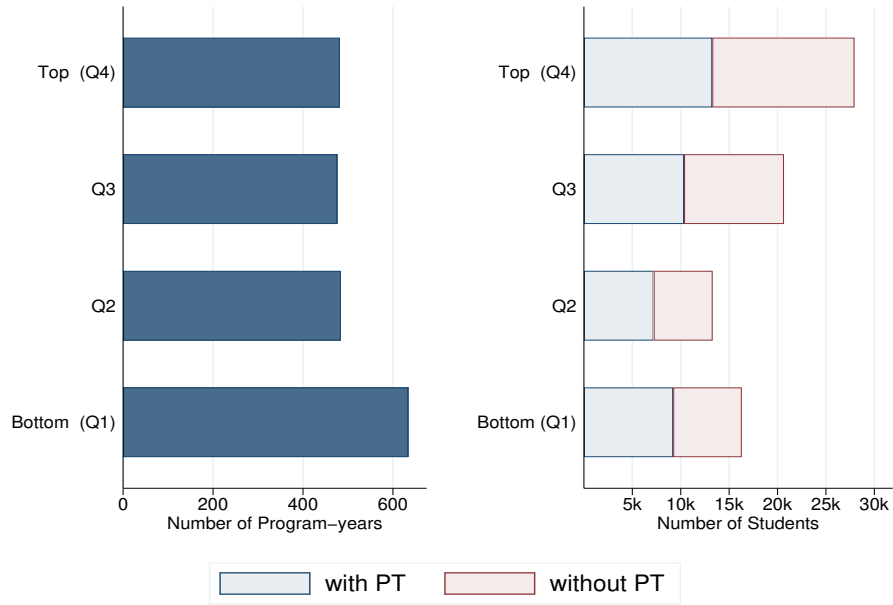


Figure 12: Programmes that allow for PT ranked by threshold in the overall distribution of programmes (quartiles)

Source: Author's calculation. I divide all programmes in the dataset into quartiles according to the admission threshold.

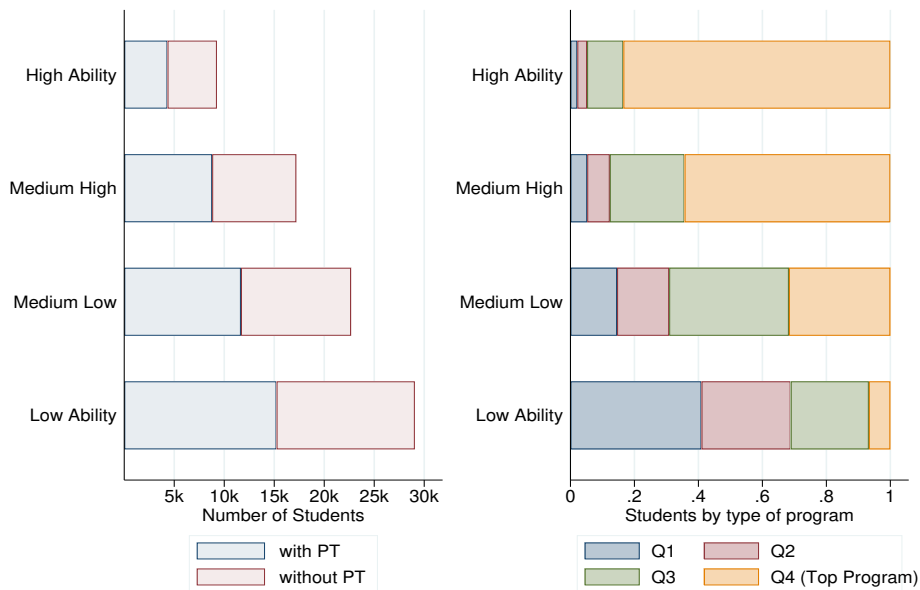


Figure 13: Enrolled students in programmes allow for PT ranked by ability in the overall distribution of student's ability (quartiles)

Source: Author's calculation. I divide all students in the dataset into quartiles according to their ability as measured by their high school score.

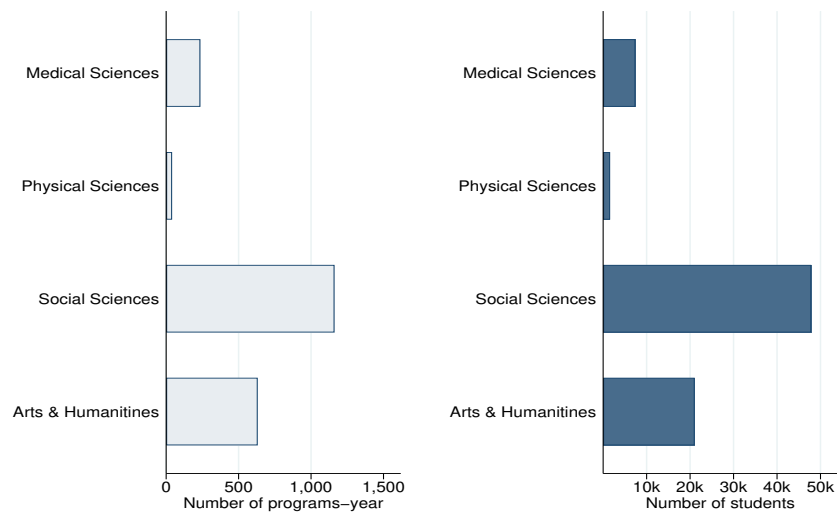


Figure 14: Number of students and programmes by field of study

Source: Author's calculation. Notes: The graphic includes all students enrolled in programmes that allow for Portuguese as an alternative requirement for the entry cohorts 2013/2014 to 2016/2017. For that reason Medical Sciences does not include the course of Medicine, where Portuguese is not allowed. A description of each field is available in Table B11 of the appendix. I group the different fields according to REF (UK) classification.

Table D23: Analysis by Field (with Imputation)

	$0 \leq \tau - \tau^* \leq 10$					
	No. of ECTS 1st year			Final GPA		
	(1)	(2)	(3)	(4)	(5)	(6)
Generalists		-0.021 [0.018]	-0.028 [0.021]		0.121*** [0.032]	0.079** [0.039]
All-Rounders (with PT)		0.059*** [0.015]	0.049*** [0.018]		0.147*** [0.027]	0.132*** [0.034]
All-Rounders (without PT)		0.097*** [0.021]	0.117*** [0.026]		0.151*** [0.037]	0.094** [0.048]
Medical Sciences	0.014 [0.022]	-0.025 [0.022]	-0.011 [0.044]	-0.004 [0.040]	-0.008 [0.040]	-0.073 [0.074]
Physical Sciences	-0.088* [0.045]	-0.090** [0.046]	-0.008 [0.059]	-0.115 [0.138]	-0.127 [0.138]	-0.279** [0.142]
Arts & Humanities	-0.045*** [0.015]	-0.049*** [0.015]	-0.072*** [0.025]	-0.070*** [0.025]	-0.067*** [0.025]	-0.116*** [0.041]
Generalists $\times$ Medical Sciences			0.018 [0.063]			0.175 [0.119]
Generalists $\times$ Physical Sciences			-0.230 [0.154]			-0.710*** [0.147]
Generalists $\times$ Arts & Humanities			0.035 [0.045]			0.092 [0.070]
All-Rounders (with PT) $\times$ Medical Sciences			-0.008 [0.057]			0.031 [0.096]
All-Rounders (with PT) $\times$ Physical Sciences			-0.204 [0.126]			0.171 [0.389]
All-Rounders (with PT) $\times$ Arts & Humanities			0.055 [0.035]			0.043 [0.061]
All-Rounders (without PT) $\times$ Medical Sciences			-0.094 [0.070]			0.131 [0.122]
All-Rounders (without PT) $\times$ Physical Sciences			-0.123 [0.123]			0.681* [0.379]
All-Rounders (without PT) $\times$ Arts & Humanities			-0.024 [0.051]			0.147* [0.085]
Controls		✓	✓		✓	✓
$R^2$	0.008	0.035	0.036	0.021	0.044	0.045
$N$	26,524	26,524	26,524	6,829	6,829	6,829

Notes: Robust standard errors are in parentheses. The omitted field is Social Sciences. \*, \*\* and \*\*\* represents statistical significance from 10%, 5%, and 1% respectively. All regression include cohort and preferences FE. The number of ECTS by the end of first year and Final GPA were standardized within programmes. All coefficients in panels A and C should be interpreted in terms of standard deviation changes in the outcome. Control variables: Female, High School Score, Non-Local Student.