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# The Political Economy of Post-Compulsory Education Policy with Endogenous Credit Constraints

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# The Political Economy of Post-Compulsory Education Policy with Endogenous Credit Constraints

## Abstract

Altruistic parents, who differ in income, make financial transfers to their children, who differ in ability. The children invest in post-compulsory education, subject to an endogenous credit constraint, and taking policy as given. There are two policy tools: a subsidy to those who participate in education and a proportional income tax. Not all children participate; a larger subsidy encourages participation, and a larger income tax discourages it. The parents, prior to making transfers, vote on policy. A voting equilibrium, if it exists, is such that voters in the two tails of the income distribution support a reduction, while the “middle-class” supports an expansion, of the education subsidy. Public support of education is a policy with regressive elements as it entails, among other things, a redistribution from the poor to the middle-earners. We characterise a local equilibrium analytically, verify its existence numerically, and finally perform a number of comparative statics exercises.

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# I Introduction

In the Western countries, the State has played a dominant role in the financing, regulation and provision of education for the past two centuries and a half, and despite the more recent calls for a "rolling back" of the Welfare State, its presence remains strongly felt. It is also widely recognised that understanding the educational process is key for addressing important issues such as intergenerational mobility, growth and development, and income equality. The choices made by the families for the education of their children must be based, among other things, on the existing education policies, and in turn these policies react to the political orientation of the families. In the present paper, we address this circular relationship with a political economy model that investigates the political support for post-compulsory education.

There is a general perception that post-compulsory education policies are at least partially regressive, redistributing income from the lower income groups to middle- and high income groups (see e.g. Fernandez and Rogerson 1995). The present paper focuses mostly on this redistributive issue, trying to disentangle its components. We argue that a more precise reading is that public financing of non-compulsory education tends to redistribute towards the middle-earners both from the high- and the low-earners. Hence, it has both a progressive and a regressive element. This interpretation is consistent with the analysis offered by most commentators on recent political conflicts over education policy such as those over the tuition fees increase introduced in England by the Labour government in 2006. It was clear at the time that much of the resistance against such an increase came from the middle-class, not the citizens at either tail of the income distribution. In the context of our model, this can be explained as follows. A move in the direction of financing public education through user fees rather than general taxation will of course be unwelcome to the user themselves (among them the middle-earners), but very welcome to the non-users (the low-earners) and to those users who carry a disproportionately large share of the tax burden (the high-earners).

In order to build a model to address these issues, we first have to note a number of features that distinguish post-compulsory education from basic education:

- in most Western countries, by the time an individual reaches post-compulsory education she is old enough to make economic decisions; hence parents and children should reasonably be treated as separate individuals each with their own individual preferences;
- a key decision for a young individual is whether or not to participate in post-compulsory

education; the individual has the option of leaving education at the minimum school leaving age; moreover, in line with the stylized facts, the participation decision should be related to parental income;

- young individuals may be financially constrained with respect to post-compulsory educational choices, leading either to non-participation or to a downward bias in the chosen quality/expenditure; parents can, however, financially assist their children and hence mitigate any potential credit constraints.

We embed the above features in our model: the parents, who are altruistic, make financial transfers to the children who, in turn, decide whether or not to participate in post-compulsory education and, if so, how much to spend. A standard way of incorporating credit constraints is simply to assume that children cannot borrow against future income. We assume instead that children can borrow any amount that they can credibly promise to repay. This has the effect that e.g. parental transfers and/or government subsidies can boost a child's credit limit.

Prior to the children entering post-compulsory education, the parents vote on educational policy. We assume that the government can observe only the participation decision, not how much a child spends on education. An educational policy hence consists of a fixed subsidy that is received by any participating child, financed via a proportional tax on income. Despite the relative complexity of the model, the parents' preferences over policy has a surprisingly simple structure. Indeed, the modeling of parents and children as separate individuals linked via the parents' financial transfers is key to this result: the assumptions imply that the parents are, in the language of Becker, "effective altruists", implying that they have simple preferences defined over total family resources.

In line with Fernandez and Rogerson (1995) we find that existence of a majority voting equilibrium is not guaranteed. However, following the approach of Epple and Romano (1996a,b), we can give necessary conditions that must be satisfied by any majority voting equilibrium. Indeed, just as in the Epple and Romano approach, we find that a political equilibrium, when one exists, will typically be of an "ends-against-the-middle" type, where parents in both tails of the income distribution want to reduce the tax/subsidy policy. However, the intuition for this is quite different from the Epple and Romano result. In the current context the poor will offer less political support to a policy subsidising post-compulsory education since they participate less frequently. The very rich would like to see the policy scaled back since they pay a large tax price. The support for an expansion of the education subsidy comes primarily from the

“middle-class”. So, in this sense, we have the progressive/regressive effect we mentioned above: redistribution flows not only from the high-income but also from the low-income to the middle-income individuals.

As is well-known, "ends-against-the-middle" equilibria pose existence problems. In order to check the existence of the equilibrium and to investigate its main properties we use a numerical example, adapting and simplifying a computational model calibrated to match some key parameters of the UK economy that is developed in full by Anderberg (2008). We round off the paper by performing a comparative statics exercise.

The paper is structured as follows. Section 1 depicts the model; Section 2 discusses the political equilibrium; Section 4 summarises.

## II The Model

Consider an economy populated by a large number of households, each made of a parent and a child. The parents are distinguished by their earnings  $y$ ; children have different abilities  $\theta$ . Each household is identified by a  $(y, \theta)$  pair; the parent observes her own earnings as well as her child’s ability. Both variables are continuously distributed on the supports  $[\underline{y}, \bar{y}]$  (with  $\underline{y} > 0$ ) and  $[\underline{\theta}, \bar{\theta}]$  (with  $\underline{\theta} > 0$ ) and the number of households is normalised to unity,  $\int \int f(y, \theta) dy d\theta = 1$ .

In line with the empirical literature (e.g. Mayer 1997 and Blau 1999), we allow  $y$  and  $\theta$  to be positively, but not perfectly, correlated. Parents are altruistic towards their offspring: each adult makes a transfer  $b$  to her child. The child decides whether or not to participate in post-compulsory education. If she does participate she makes a decision about how much to spend,  $z$ , on that education. Hence a child that attends education and spends  $z$  obtains the final income  $w(z, \theta)$ . The earnings function satisfies<sup>1</sup>

$$w_z > 0; w_\theta > 0; w_{zz} < 0; w_{\theta\theta} < 0; w_{z\theta} > 0. \quad (1)$$

We assumed that the two inputs in the earnings function are complements,  $w_{z\theta} > 0$ ; this reflects an idea that people with higher innate ability can profit more from their own human capital investments. For simplicity, we take it that a child that does not participate in education obtains fixed earnings  $w_0 > 0$ .<sup>2</sup>

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<sup>1</sup>Subscripts denote partial derivatives.

<sup>2</sup>In fact, this simplification is extreme. The model would work as long as ability has a larger impact on the earnings of those who acquire an education than on the earnings of those who don’t. In the numerical model

The timing of the model is as follows. First the parents vote over policy, consisting of a fixed subsidy  $\sigma$ , conditioned on participation only, and a proportional income tax  $\tau$  levied on the parents' income (we disregard any taxes on income that the children may face). Then each parent makes a transfer  $b$  to her children. Finally, the child makes her decision about education. The model is solved by backward induction.

## The Investment in Education

Consider a child who has received a transfer  $b$  from her parent. She faces the decision of whether or not to participate in post-compulsory education, and, if she decides to participate, how much to invest. We start by exploring the latter decision and its consequences in terms of the child's final resources.

If a child participates in education and invests the amount  $z$ , her final resources will be  $x = w(z, \theta) - z + \sigma + b$ . In order to finance the investment  $z$  she will need to borrow  $z - \sigma - b$ ; assuming a zero interest rate, she will have to repay exactly this amount. A particular loan will be available if and only if it is in the child's interest to subsequently repay that loan. This in turn hinges on the consequences of defaulting. Following Lochner and Monge-Naranjo (2002), we assume that if a child defaults all her assets will be seized by the lender, and she will also have to pay a penalty equal to a fraction  $\gamma \in (0, 1)$  of her future earnings: final resources for a defaulter are thus  $(1 - \gamma)w(z, y)$ . Hence a child will be able to borrow the amount  $z - \sigma - b$  if and only if she is better off by not defaulting. i.e.  $w(z, \theta) - z + \sigma + b \geq (1 - \gamma)w(z, \theta)$  or simply

$$\gamma w(z, \theta) - z + \sigma + b \geq 0. \quad (2)$$

The objective of the child is to maximize her final resources  $x$ . The child's maximal final resources, conditional on participation, can then be written as

$$x^p(\sigma + b, \theta) = \max_z (w(z, \theta) - z + \sigma + b) \quad \text{s.t.} \quad (2) \quad (3)$$

Note that  $\sigma$  and  $b$  enter as argument in  $x(\cdot)$  in the form of the sum  $\sigma + b$ . Hence we can define  $a \equiv \sigma + b$  as the child assets and write  $x^p = x^p(a, \theta)$  and  $z(a, \theta)$  as the solution to the above problem. Letting  $\mu$  denote the Lagrange multiplier (to be interpreted as the marginal value of credit), the first order condition can then be written as

$$(w_z - 1) + \mu(\gamma w_z - 1) = 0 \quad (4)$$

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below, we will relax the assumption that uneducated workers earn a fixed income.

If the credit constraint does not bind,  $\mu = 0$  and (4) reduces to  $w_z = 1$ . From (1), we have:

$$z_\theta = -\frac{w_z\theta}{w_{zz}} > 0; \quad z_a = 0. \quad (5)$$

Thus, the unconstrained child's educational investment is increasing in her own ability; also, neither the government grant nor the domestic transfer affect the child's optimal level of investment (it may, of course, affect the child's decision to participate).

If the constraint binds, the optimal investment satisfies the credit constraint (2) with equality, and can be obtained as an implicit solution to  $\gamma w(z, \theta) - z + a = 0$ . Note that we can determine bounds on the marginal return to  $z$  as (i) it must be that  $w_z > 1$  since the unconstrained investment is not available, and, (ii) it must be that  $\gamma w_z < 1$  since a marginal increase in the investment necessarily leads the credit constraint to be violated.<sup>3</sup> Combining the two inequalities yields that

$$1 < w_z < \frac{1}{\gamma} \quad (6)$$

Treating  $\gamma w(z, \theta) - z + a = 0$  as an identity and using (6) and (1) we can sign the following comparative statics in the constrained case

$$z_a = (1 - \gamma w_z)^{-1} > 0 \quad (7)$$

$$z_\theta = \gamma w_\theta (1 - \gamma w_z)^{-1} > 0 \quad (8)$$

that is, if the child is credit constrained, then her investment is increasing in her initial assets and in her ability (and thus tendentially in her parent's income). Note that the constrained investment is increasing in the child's ability *for any given transfer*  $b$ . This implies that a high ability child (tendentially, then, a child to a rich parent) can borrow more than a low ability child given the same transfer; the reason is that the child's innate ability will boost the child's earnings, which allows the credit to the child to be extended without violating the constraint.

For future use, we need a few results concerning the marginal value to the child of her initial assets and of her innate ability:

**LEMMA 1** *For unconstrained children, the marginal value of the assets is constant ( $x_a^p = 1$  and  $x_{aa}^p = x_{a\theta}^p = 0$ ), and that of ability is positive. For constrained children, the marginal value of assets exceeds unity, since it also has the effect of relaxing the credit constraint, and*

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<sup>3</sup>The constraint's derivative w.r.t.  $z$  is  $\gamma w_z - 1$ ; the constraint has a positive intercept ( $a$ ) and to be binding it must cross the abscissa from above, hence  $\gamma w_z < 1$ .



is decreasing, since the marginal value of credit  $\mu$  decreases as initial assets increase; that of ability is positive. Formally:

$$x_a^p = 1 + \mu \geq 1; \quad x_\theta^p = w_\theta (1 + \gamma\mu) \geq w_\theta > 0. \quad (9)$$

and, for constrained children,

$$x_{aa}^p = \mu_a < 0; \quad x_{a\theta}^p = \mu_\theta. \quad (10)$$

**Proof.** See the Appendix. ■

This completes the description of the child's educational investment given that she has decided to enroll in further education. Before characterizing the child's participation choice we need to consider the parent's decision on the transfer  $b$ .

### The Parent's Transfer Decision

We assume that all parents make "interior" (strictly positive) transfers. This will imply that the parents are, in Beckerian parlance, "effective altruists", a fact that has strong implications for the structure of their indirect utilities. The parent cares about her own consumption and the child's final resources, and makes a transfer to the child, taking policy as given. For simplicity, we let the parents' utility be additively separable.

For a participating family, we can write  $U(b, y, \theta; \tau, \sigma) = u((1 - \tau)y - b) + v(x^p(\sigma + b, \theta))$ , with  $u(\cdot)$  and  $v(\cdot)$  strictly concave. The optimal transfer  $b^p(y, \theta; \tau, \sigma)$  will satisfy

$$-u' + v'x_a^p = 0; \quad (11)$$

(it is easy to check that the second order condition is satisfied). We are interested first in determining how the transfer varies with the parent's earnings. We expect that richer parents make larger transfers (everything else, and in particular child's ability, being equal), and in fact:

$$b_y^p = -\frac{-u''(1 - \tau)}{u'' + v''(x_a^p)^2 + v'x_{aa}^p} > 0. \quad (12)$$

Second, we want to investigate the role of the child's ability; a standard result is that altruistic parents compensate for the children's failures, hence transfers should be decreasing in the child's ability. In our case, the outcome is complicated by the presence of the credit constraint:

$$b_\theta^p = -\frac{v''x_\theta^px_a^p + v'x_{a\theta}^p}{u'' + v''(x_a^p)^2 + v'x_{aa}^p}. \quad (13)$$

The second term at the numerator,  $v'x_{a\theta}^p$ , equals zero for unconstrained children, see the discussion of (10) above; then,  $b_\theta^p < 0$  by concavity of  $v(\cdot)$  and (9). Thus, the standard result is confirmed for unconstrained children. For constrained ones, the effect remains ambiguous as the term  $v'x_{a\theta}^p$  cannot be signed; on the one hand, the parent do try and compensate for reduced ability, but they have also to account for the fact that changes in  $\theta$  affect the child's capability to obtain a loan. Hence, we have:

LEMMA 2 *The parental transfer is increasing in the parent's earnings for all participating families; it is decreasing in the child's ability for unconstrained families, but is ambiguously related to the child's ability for constrained families.*

As for the effects of policy on the transfer, we have

$$b_\tau^p = -\frac{u''}{u'' + v''(x_a^p)^2 + v'x_{aa}^p}y < 0; |b_\tau^p| \leq y; \quad (14)$$

$$b_\sigma^p = -\frac{v''(x_a^p)^2 + v'x_{aa}^p}{u'' + v''(x_a^p)^2 + v'x_{aa}^p} < 0; |b_\sigma^p| < 1 \quad (15)$$

Intuitively, an increase in the tax rate affects negatively the transfer due to an income effect, although an increase in the tax by 1% leads to a reduction in the transfer of less than 1%, as  $\partial(b^p/y)/\partial\tau = b_\tau^p/y \in -(0, 1)$ . Also, increasing the education grant crowds out the domestic transfer, although by less than one-for-one.

Finally, note that the parent's problem can be rewritten using  $c \equiv (1 - \tau)y - b$  and  $a = b + \sigma$  to eliminate  $b$ . We thus have:

$$V^p((1 - \tau)y + \sigma; y, \theta) = \max_{c, a} \{u(c) + v(x^p(a, \theta)) \mid c + a = (1 - \tau)y + \sigma\} \quad (16)$$

This is how the assumption of effective altruism comes into play: conditional on the child participating in education, the parent evaluates alternative policies only by how they affect net family resources,  $m^p \equiv (1 - \tau)y + \sigma$ .

In a non-participating family, the final resources for the child will be  $x^{np}(b) \equiv w_0 + b$  since she is not eligible for the subsidy  $\sigma$ . We thus have  $U(b; y, \tau) = u((1 - \tau)y - b) + v(w_0 + b)$ , with first order condition (both necessary and sufficient),

$$-u' + v' = 0 \quad (17)$$

We can now compute:

$$b_y^{np} = \frac{u''(1 - \tau)}{u'' + v''} > 0; b_\tau^{np} = -\frac{u''y}{u'' + v''} < 0, \quad (18)$$

i.e. the transfer is increasing in parental income and decreasing in the income tax rate for non-participating families. Note that the child's earnings do not depend on her ability; hence, the parental transfer is also independent from the child's ability. Finally, writing the indirect utility as

$$V^{np}((1 - \tau)y) = \max_{c,b} \{u(c) + v(w_0 + b) \mid c + b = (1 - \tau)y\} \quad (19)$$

shows that it can be written simply as a function of the parent's net-of-tax income.

## The Participation Choice

The participation choice formally rests with the child and the child can take this decision after the parent has made the financial transfer. Hence, we assume that *the parent cannot condition the transfer on the child's education choice*. The reason for this is that, if the child needs the transfer to finance her investment, it must occur upfront, i.e. before the child implements her education choice. It is however easy to see that there is no conflict between the parent and the child, in general and specifically where the participation choice is concerned. Whatever maximises the child's resources also maximises the parent's utility, as the latter cares for the child's objective. This has the useful implication that we can study the participation decision from the point of view of the parent, because in equilibrium, the child will always make the educational choice most preferred by the parent. This approach implies that we can analyse a family's participation choice using the parent's indirect utility rather than focusing on the child's final resources.

We establish first a preliminary result. The result states that, if a family is indifferent between participating and not participating and if they *are not* credit constrained, then the outcome for both the child and the parent is the same under each choice. In contrast, if a family is indifferent, but *are* credit constrained, then the parent enjoys higher own consumption under non-participation while the child enjoys higher a higher net income under participation.

**LEMMA 3** *Consider a family indifferent between participating and not participating in education: if the child is not credit constrained, both  $x$  and  $c$  are the same no matter whether the child participates or not; if she is constrained,  $c$  is larger and  $x$  is smaller when non-participating than when not participating.*

**Proof.** See the Appendix. ■

We can now show a number of results that characterise the participation choice. First, we show that this choice is strictly monotonic in the child's ability level within each parental income class:

**PROPOSITION 4** *Within any group of parents with the same income  $y$ , there exists a cut-off child's ability level  $\hat{\theta}(y)$  such that all parents whose children are of ability  $\theta \geq \hat{\theta}(y)$  prefer them to enter education, while all parents whose children are of ability  $\theta < \hat{\theta}(y)$  prefer them not to enter education.*

**Proof.** See the Appendix. ■

Strict monotonicity in the participation choice follows the fact that high-ability children obtain a larger income from education than low-ability children at the same level of investment.

The cut-off level  $\hat{\theta}(y)$  is in general also a function of policy. We can show that, predictably, participation within each  $y$ -class is boosted by an increase in  $\sigma$ , everything else being equal; instead, an increase in  $\tau$ , everything else being equal, will reduce participation within the  $y$ -class, but *only if* the marginal family is credit constrained:

**PROPOSITION 5** *Within any group of parents with the same income  $y$ , the cut-off child's ability level  $\hat{\theta}$  is increasing in the subsidy  $\sigma$ , decreasing in the tax rate  $\tau$  if the family is credit constraint and unaffected by changes in  $\tau$  otherwise.*

**Proof.** See the Appendix. ■

The first part follows because only participating families enjoy the education grant; the second because participation becomes less attractive only if a family is credit constrained: if the child attains the efficient investment level, then there will be no income effect of taxation.

It is also true that participation choice is *weakly* monotonic in parental income keeping  $\theta$  constant. However, note that this obtains if and only if there are binding credit constraints: if there are no binding credit constraints, then the investment undertaken by a family (both in terms of participation and in terms of the level of investment in case of participation) is solely a function of the child's ability  $\theta$ . When there are binding credit constraints, there will generally be a set of ability types for which the participation decision (and investment level) depends positively on family income.

### III On the Political Equilibrium

A parent's indirect utility, taking into account the endogenous participation choice, is

$$V(\sigma, \tau; y, \theta) = \max \{V^p((1 - \tau)y + \sigma, y, \theta), V^{np}((1 - \tau)y)\} \quad (20)$$

For participating families, the marginal rate of substitution between policy tools is

$$-\frac{u_\tau}{u_\sigma} = y; \quad (21)$$

for a non-participating family, there is no way in which an increase in  $\sigma$  can compensate an increase in  $\tau$ , since they are not entitled to the grant. Note that for each family we can identify, in the  $(\tau, \sigma)$ -space, a locus along which the family is indifferent between participating and not participating; this is the locus of all  $(\tau, \sigma)$  pairs such that  $V^p((1 - \tau)y + \sigma, y, \theta) = V^{np}((1 - \tau)y, \theta)$ , as we know that the child's choice agrees with that of the parent. Note that an increase in  $\sigma$ , everything else being the same, will trivially break the indifference in favour of participation; hence, all policies "above" the indifference locus will determine participation, and all policies "below" the locus will determine non-participation.<sup>4</sup>

The above formulation makes it easy to see that a typical indifference curve is vertical below the family's participation locus and has slope  $y$  above the locus (importantly, children's ability does not affect policy preferences; the marginal rate of substitution between policy instruments only depends on  $y$ ).

#### Preferences over Policy

The revenue constraint can be written, in per-capita terms, as

$$\tau y^a - \sigma Q(\tau, \sigma) = 0, \quad (22)$$

where  $y^a$  is average income and  $Q(\cdot)$  is the total share of the population participating in post-compulsory education. This can be seen as implicitly defining e.g.  $\sigma$  as a function of  $\tau$ . For the ideal policy problem (conditional on participation) to be well-behaved, we need the revenue curve in the  $(\tau, \sigma)$ -space to be strictly concave. Intuitively, we expect this to be the case: as the tax rate grows, so does the budget-balancing grant, but at a decreasing pace, because more and more people are entitled to it – aggregate participation increases with  $\sigma$ , as can be inferred by (A13). At the present level of generality, we can however only prove the following:

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<sup>4</sup>It is also possible to show that the locus is flat as long as the family is unconstrained, and positively sloped when it becomes constrained.

PROPOSITION 6 *The revenue curve in the  $(\tau, \sigma)$ -space is increasing,*

$$\sigma'(\tau) > 0. \quad (23)$$

**Proof.** See the Appendix. ■

Strict concavity will have to be confirmed numerically; for now, we simply assume it.

Using the revenue constraint to eliminate one policy tool, we may rewrite the indirect utility functions of participating and non-participating families as follows:

$$V^P(\tau; y) = u((1 - \tau)y - b) + v(x^p(\sigma(\tau) + b, y)); \quad (24)$$

$$V^{np}(y; \tau) = u((1 - \tau)y - b) + v(x_0 + b), \quad (25)$$

where in both cases it is understood that  $b$  is chosen optimally. The derivatives w.r.t.  $\tau$  are:

$$V_\tau^P = -yu' + v'x_a^p\sigma'; \quad V_\tau^{np} = -yu'. \quad (26)$$

For families outside education, it is immediate to see that the preferred policy is no policy at all: they only lose from an income tax liability that gives them nothing in return. For these families, the highest attainable indifference curve is the one through the origin – the revenue curve is all below such indifference curve. For participating families, the ideal tax rate satisfies, using that  $u' = v'x_a^p$  by (11):

$$y = \sigma'. \quad (27)$$

The indifference curve is tangent to the revenue curve somewhere along the former's positively sloped tract. Note that it does not make a difference for the policy preferences whether a family is credit-constrained or not as long as it is participating in education: credit constraints only affect a family's policy preferences in so far as it affects the participation decision. The ideal tax rate has then the standard interpretation of being the tax rate that equates the MRS with the slope of the revenue curve.

### The Ideal Policy as a Function of Income

If concavity of  $\sigma(\cdot)$  holds the following “algorithm” can be used to obtain the ideal policy for each family  $(\theta, y)$  given that the budget set is concave (over the relevant region): (i) find the policy that maximizes the participation utility  $V^P((1 - \tau)y + \sigma, y, \theta)$  over the budget set  $\sigma(\tau)$  using the first order condition  $\sigma'(\tau) \leq y$  with complementary slackness (i.e.  $[\sigma'(\tau) - y]\tau = 0$ ); this ideal policy trivially only depends on  $y$  (not on  $\theta$ ). Then, (ii) compare the “participation

utility” at this policy to the “non-participation” utility at *laissez-faire*  $V^{np}(y)$  (since the ideal policy under non-participation is always  $\tau = \sigma = 0$ ).

Note that there will be a group of families for whom  $y \geq \sigma'(0)$ ; these families will favour the *laissez-faire* policy both under participation and under non-participation. This is because, even when they participate, the subsidy they receive is simply not enough to compensate the taxes they pay. Hence, any family with income  $y \geq y^\circ$ , where  $y^\circ \equiv \sigma'(0)$ , will have the ideal policy  $\tau = \sigma = 0$ . Note that all these families have incomes above the average: evaluating (23) at the origin yields

$$y^\circ = \frac{y^a}{Q(0,0)}, \quad (28)$$

where  $Q(0,0)$  is the participation in *laissez-faire*. Since  $Q(0,0)$  is in the interval  $(0,1)$ , then  $y^\circ > y^a$ .

For families with income below  $y^\circ$ , refer to the above algorithm, and think about part (i) as generating a “participating” ideal tax  $\tau^p(y)$  (which is a well-behaved decreasing function in  $y$ ) and part (ii) as generating a “non-participating” ideal tax  $\tau^{np}(y)$  (which is trivially zero). Then the family’s ideal tax is

$$\tau(\theta, y) = \begin{cases} \tau^p(y) \geq 0 & \text{if } V^p(\tau^p(y), y, \theta) \geq V^{np}(\tau^{np}(y), y, \theta) \\ \tau^{np}(y) = 0 & \text{if } V^p(\tau^p(y), y, \theta) < V^{np}(\tau^{np}(y), y, \theta) \end{cases} \quad (29)$$

Again, it should be stressed that ability only affects the family’s ideal policy through its effect on the participation decision.

## Necessary Conditions for Local Equilibria

We now discuss political equilibria under majority voting. For the purpose of such a discussion, we postulate that all parents attend elections; in the numerical example below, we relax this assumption.

Voting is over  $\tau$ , with  $\sigma$  implicitly defined through the budget constraint; we refer to  $\tau^*$  as the equilibrium tax rate, and to  $\sigma^* = \sigma(\tau^*)$  as the equilibrium, budget-balancing, subsidy. Given the nature of the policy preferences, and the way they vary with income, it is difficult to make general statements. We cannot invoke neither single-peakedness nor single-crossing, and hence the median voter theorem does not apply. Another difficulty is that “large” policy changes (i.e. non-marginal changes) induce a correspondingly “large” change in participation. This implies possibly complex voting behaviour: for example, a typical reaction for a parent would be that if a given tax-subsidy pair that induces non-participation is taken as a starting point, she will

certainly oppose a marginal expansion of policy but may well favour a non-marginal one that makes her jump out of the non-participation area.

With this in mind, we proceed in our discussion focusing on "small" policy changes and local equilibria. We distinguish between a trivial equilibrium, in which the implemented policy is in fact the *laissez-faire*,  $\tau^* = \sigma^* = 0$ , and a non-trivial one in which  $\tau^* > 0$  and  $\sigma^* > 0$ . There are two possibilities:

1. If *more* than half the population have a *laissez-faire* ( $\tau = \sigma = 0$ ) as their ideal policy, this is trivially a majority voting equilibrium. The group of families who have *laissez-faire* as their ideal policy may be diverse, consisting both of families with children with low ability and families with high income ( $y \geq y^o$ ).
2. A sufficient condition for *laissez-faire* not to be a majority voting equilibrium is that the fraction of families that (i) participate in education at *laissez-faire* and (ii) have  $y < y^o$  exceeds 1/2. All families in that satisfy (i) and (ii) would favour the introduction of a subsidy policy, thus ruling *laissez-faire* out as an equilibrium.

Let us imagine that voting is over marginal reforms, and that a reform only wins if it collects more than half the votes. There is a status quo, an arbitrary active policy  $\tau^k > 0$  (and  $\sigma^k = \sigma(\tau^k) > 0$ ); let  $y^k = \sigma'(\tau^k)$ .

At the policy  $(\tau^k, \sigma^k)$ , there will be support for a marginal policy expansion for all families who are participating at  $(\tau^k, \sigma^k)$  and who have incomes below  $y^k$ ; in contrast, support for a marginal policy reduction will come from those who do not participate at  $(\tau^k, \sigma^k)$  and/or have incomes above  $y^k$ . Hence for  $(\tau^k, \sigma^k)$  to be a majority voting equilibrium, neither group must be a majority – i.e. the two groups must be of equal size. If  $y^k$  is such that the population is partitioned in the way described above, then  $(\tau^k, \sigma^k)$  is a local equilibrium in the sense that it will beat all marginal reforms.

## A Numerical Model

In order to check the existence (and other properties) of the equilibrium we employ a numerical example. Rather than assigning parameter values arbitrarily, we use those identified by Anderberg (2008), who has calibrated the theoretical model of the present paper to match key features of the UK economy. Details on the procedures for obtaining values of the more immediately



salient features of the model are available in Anderberg (2008). Here, we simply state the most relevant pieces of information, and the functional forms used.

The aggregate rate of staying on in education past the compulsory age of sixteen in the UK is slightly above 71 percent.<sup>5</sup> However, this aggregate value hides significant difference across income groups, as participation rates (as well as educational attainment) is known to vary positively with income (see e.g. Blanden et al., 2003). The calibrated model uses four participation rates, corresponding to the quartiles of an estimated long-run family income distribution: the rates are 60, 65, 76, and 85 percent respectively. In the computation, the aggregate participation rate (72) and the participation gap between the top and the bottom income quartile (25 percentage points) are replicated.

The relationship between parental earnings and children’s abilities is represented using a simple regression-to-the-mean formulation,

$$\ln(a) = C + k \ln(y) + \varepsilon, \quad (30)$$

where  $\varepsilon$  is i.i.d.  $N(0, \sigma_\varepsilon^2)$  and independent of  $y$ . Since the scale of ability is not observed,  $C$  is set so as to normalize log ability to have unit mean; this leaves  $k$  and  $\sigma_\varepsilon^2$  to be determined. To simplify this step, the correlation between parental income and child ability, is estimated, and is found to be in the range of 0.24 to 0.28. Hence, in the model, a correlation between (log) ability and (log) parental income in this range is imposed. Given the variance of (log) parental income  $\sigma_y^2$  this imposes a relationship between  $k$  and  $\sigma_\varepsilon^2$  and leaves one of these variable to be assigned a value (see below).

One of the basic tenets of the present paper is that children might be discouraged from participating in post-compulsory education due to financial constraints. Using an approach originally developed by Carneiro and Heckman (2003), Anderberg (2008) and Dearden et al. (2004) use UK data<sup>6</sup> in order to provide an estimate of the number of children who are discouraged from participating due to financial constraints. The resulting estimates go from of 1-1.2% to 5-7%; the model is calibrated to match the low range of estimates.

As mentioned, in the numerical model we relax the assumption that everybody votes. Using data from the National Child Development Study (a survey similar to the BCS, but following

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<sup>5</sup>Department for Education and Skills, SRF 03/2005.

<sup>6</sup>The data are from the British Cohort Survey (BCS). The BCS originally collected information about all children born in the UK in one specific week in 1970. The study has subsequently carried out follow-up surveys on health, education, family and social influences at various ages (5, 10, 16, 26, and 30).

a cohort born 12 years earlier) on the 1997 UK general election, Anderberg (2008) finds that the average voting frequency was 78 percent and that someone at the top of the distribution is about 30 percentage points more likely to vote than someone at the bottom.

The relevant income concept is the parents' lifetime income (there being no subperiods in the model). Dearden et al. (2006) provide estimates of lifetime income distributions for graduates and non-graduates by gender in the UK. Aggregating their results (at one percent discount rate) yields that the mean is about £700' with a standard deviation of log lifetime income of 0.43. The model is calibrated to match these moments.

For parental preferences, a standard separable iso-elastic formulation is adopted,

$$U = \frac{c^{1-\rho} + \lambda x^{1-\rho}}{1-\rho}. \quad (31)$$

In order to assign a value to the altruism parameter  $\lambda$ , information on parental transfers from the BCS data is used. Among those who left education by age 16, close to 70% of kids reported having received financial support from their parents, including help with accommodation (55% without accomodation). For the risk aversion parameter,  $\rho$  is set to 1.5, which is within the standard range frequently used in the literature.

The model is calibrated to match the average spending on education in terms of its two main components: public expenditure and foregone earnings. Using information on public spending per student per year in further- and higher education respectively and information about the distribution of the years of further study among those staying on past 16 the estimated average total public spending per student staying on is around £14'.<sup>7</sup> BCS data allow to estimate average foregone earnings per student staying on at about £30'. Adding direct costs of participation (e.g. tuition fees), average total private investments is estimated to be £35'.

Human capital accumulation technology is given by a standard log-linear specification:

$$w(z, a) = \eta z^\alpha a^\beta, \quad (32)$$

where  $\alpha$  is the elasticity of earnings with respect to the investment  $z$  and  $\beta$  is the elasticity with respect to ability. As mentioned, in the calibrated model ability influences also the unskilled earnings:

$$w_0(a) = \eta_0 a^{\beta_0}, \quad (33)$$

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<sup>7</sup>Information on spending per student is obtained from the National Statistics Bulletin "Statistics of Education: Education and Training Expenditure Since 1992-93" Issue No 06/02, September 2002, Table 8. The distribution of the years of further study among those staying on past 16 is obtained from the BCS.

Parameter	Value
$\alpha$	0.062
$\beta$	0.550
$\beta_0$	0.475
$\eta$	312.3
$\eta_0$	383.9
$\gamma$	0.233
$\lambda$	0.748
$k$	0.550
$\sigma_\varepsilon^2$	0.902

Table 1: Parameter values in the baseline calibration

where  $\beta_0$  is the elasticity of unskilled earnings with respect to ability.

Nine parameters were obtained by calibrating the model to nine of the stylized facts described above. The “earnings monitoring” parameter  $\gamma$  was set so that the model matches the fraction of families estimated to be credit constrained; the altruism parameter  $\lambda$  was set to match the frequency of parental transfers; the ability transmission parameter  $k$  was set so as to match the observed correlation between parental income and child ability; the remaining calibrated parameters,  $\alpha$ ,  $\beta$ ,  $\beta_0$ ,  $\eta$ ,  $\eta_0$  and  $\sigma_\varepsilon^2$ , were chosen to match the public spending per participating child, average private investments, mean income, the variance of log income, the aggregate participation rate, the participation gap between the top and the bottom income quartile. The calibrated parameter values are summarized in Table 1.<sup>8</sup>

## Existence of a Political Equilibrium

Our first concern is whether we can find a majority voting equilibrium that fits the data. Analytically, we provided conditions for local equilibria; numerically, we can check whether a local equilibrium is also a global one. The calculation proceeds as follows. Starting from an initial income distribution for parents, one has to:

1. characterize individual family behavior at all policies and determine and the government

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<sup>8</sup>Formally, the income distribution is also endogenized; in the equilibrium presented, the economy is in steady state in the sense that the income distribution for the children is the same as that for the parents (i.e. it is stationary). See Anderberg (2008) for further details.

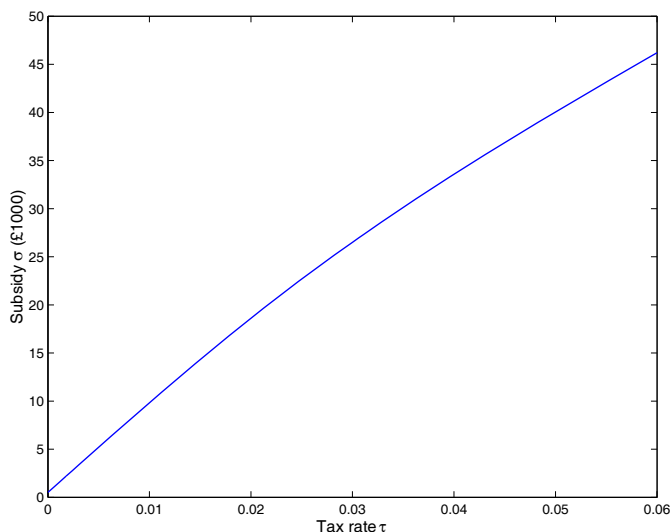


Figure 1: The government budget constraint

budget constraint,  $\sigma(\tau)$ , that is consistent with optimal family behavior;

2. compute the political support for a marginal policy expansion at each policy on the government budget constraint and locate a policy  $(\tau^*, \sigma^*)$  where this support is equal to  $1/2$ ;
3. put  $(\tau^*, \sigma^*)$  to a global test, checking if it is preferred by a majority against all other policies along the government budget constraint;

It will be recalled that we could not show analytically that the budget constraint is concave; we however provided an intuitive argument supporting the view that it should be so. The numerical example confirms the argument: since a more generous education subsidy encourages participation, the constraint is indeed concave (Figure 1 illustrates). The support for a marginal policy expansion comes from those families who participate *and* have  $y \leq \sigma'(\tau)$ : as it happens, there is one policy where the support for a marginal policy expansion is equal to  $1/2$ , namely  $\tau^* = 0.015$  and  $\sigma^* = 14.2$ . This policy then qualifies as a local equilibrium. Figure 2 further shows that the support for this candidate policy against all other feasible alternatives always stays above  $0.5$ ; then, we conclude that the policy is also a global equilibrium.

As a check on the meaningfulness of the whole exercise, it bears verifying the extent to which the model is capable of replicating the stylised fact on the basis of which it was calibrated.

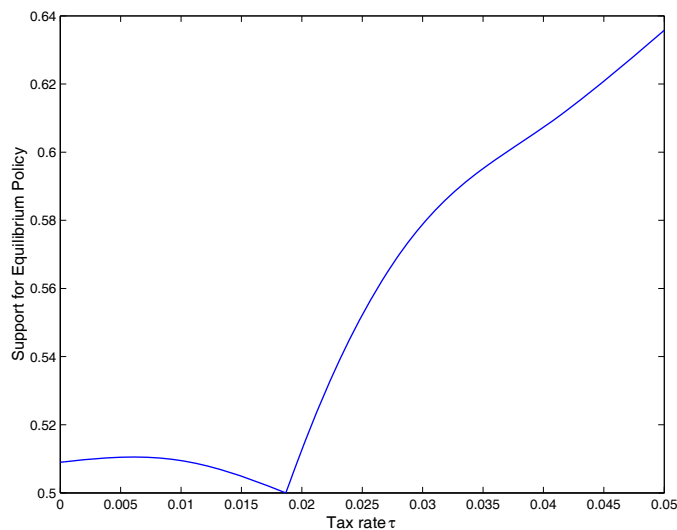


Figure 2: Political support for the winning policy

Table 2 reports the main outcomes of the simulation, and shows that it has worked satisfactorily in this respect. Finally, it is worth remarking that a key simulation outcome (not reported here) was not calibrated: this is the degree of intergenerational social mobility, commonly measured using regressions of log parental earnings or income on the corresponding outcome for the child. Computing this statistic on the simulated data shows that model outcome is well in line with the estimates of 0.4-0.6 for the UK provided in the literature (Dearden et al 1997, Blanden et al 2004) – see Anderberg (2008) for details.

### Comparative Statics

Finally, we present the comparative statics of the political equilibrium with respect to some of the parameters of the model. For ease of comparison results are reported in elasticity form by considering a one percent increase in each parameter on a selected number of outcomes. The results are presented in Table 3.

Consider first  $\alpha$  – the elasticity of skilled earnings with respect to investment. An increase in  $\alpha$  makes participating more attractive. This in turn increases the political support for policy. It also makes investing more attractive. The positive impact on the average investment is fairly small in part due to the fact that new participants make relatively small investments. There is a large negative impact on the fraction of children who are credit constrained. This large effect

Investment levels	Average total $z$	49
	Average private $\hat{z}$	35
Policy	Tax rate $\tau$ (%)	1.5
	Public subsidy $\sigma$	14
Participation rates	1st income quartile (%)	58
	2nd income quartile (%)	71
	3rd income quartile (%)	77
	4th income quartile (%)	83
Lifetime income	Mean	700
	St. dev. of log income	0.43
Transfers	Positive transf.: participants (%)	20
	Positive transf.: non-participants (%)	68
“Credit tightness”	Fraction families constrained (%)	1.1
Intergen. links	Correlation: parental income/ability	0.24

Table 2: Participation rates and investments at the policy equilibrium

reflect the small initial base.

Consider then  $\beta$  – the elasticity of skilled earnings with respect to ability. An increase in this parameter also makes participating more attractive. The positive impact on participation also leads to more political support for policy. However, a higher return to ability does not directly make investing more attractive; indeed due to the new participants making low investments, the overall impact on average investment is a small negative effect. As in the case of  $\alpha$  there is also a large effect on the fraction credit constrained. Hence the main effects are on participation, the level of policy and on the fraction of credit constrained agents.

An increase in  $\beta_0$  makes participation less attractive. This in turn erodes the support for policy. The positive effect on the average investment obtains since those now not participating in education were making relatively small investments.

An increase in the transmission of ability  $k$  has a negligible effect on policy and the aggregate participation rate; its largest effects are on the participation gap which increases due to average ability having a larger income gradient. It slightly reduces the fraction of children who are credit constrained which may seem unintuitive; this happens however because it reduces the participation rate in the bottom income quartile. An increase in the variance of ability shocks

Comparative Static		$\alpha$	$\beta$	$\beta_0$	$k$	$\sigma_\varepsilon^2$	$\lambda$	$\rho$
Policy	Tax rate $\tau$ (%)	3.4	9.8	-9.7	-0.5	-0.4	1.7	0.1
	Spending $\sigma$	2.1	8.1	-7.3	-0.4	-0.0	1.6	0.1
Participation	Agg. participation rate	1.7	2.9	-2.4	-0.1	-0.2	0.6	0.0
	Participation gap (%)	-1.7	-3.5	6.5	0.8	-0.1	-2.6	0.0
Investment/ Income	Average total $z$	0.6	-0.3	0.9	0.1	0.2	0.2	0.0
	Average income	0.3	1.1	0.0	0.0	0.2	0.3	0.1
	St. dev. of log income	0.1	1.0	0.1	0.2	0.5	-0.5	-0.1
Credit	Constrained (%)	-17.7	-20.3	13.3	-3.6	-2.6	-16.8	0.9
Transfers	Transfer Freq.	1.0	2.0	-1.8	-0.1	0	1.6	0.1

Table 3: Comparative statics on the policy equilibrium in elasticity form.

$\sigma_\varepsilon^2$  also has a negligible effect on policy and the aggregate participation rate. By increasing the ability spread it also increases the income variance.

An increase in altruism  $\lambda$  increases parental transfers and reduces the fraction of kids who are credit constrained. This also increases the participation rate which in turn generates support for policy expansion. Finally, an increase risk aversion  $\rho$  also implies that parents are more inequality averse between themselves and their children. This generates a small increase in transfers. However, in general, the outcome is not very sensitive to this parameter.

## IV Summary

The present analysis has been motivated by the need to understand political conflicts around education policy like the one raging in England prior to the tuition fees increase in 2006. It has been observed by commentators at the time that while an increase in tuition fees was vehemently opposed by the representatives of the middle-class, it seemed not to rise any comparable concern among the less well-off, as well as in the rich segment of the population.

In fact, the behaviour of the latter was to be expected as they will benefit the most from any corresponding reduction in taxation. A moment of reflection will however reveal that also the behaviour of the low-earners makes perfect economic sense: due to a positive income gradient in participation, low earning families less frequently benefit from subsidies to post-compulsory education and would hence also favour the introduction of use fees and a corresponding tax

reduction.

In order to understand theoretically situations like the one presented above, we developed a model in which altruistic parents differing by their income level make financial transfers to their children, differing in cognitive abilities. The children are sufficiently autonomous to take decisions regarding their investments in post-compulsory education; however, the altruistic link between generation ensures that whatever is optimal for the child is optimal for the parents as well. These investment choices might be endogenously credit-constrained: lenders are supposed to be able to monitor the child's incentives to repay the loan and therefore will only lend amounts that the child can credibly promise to repay. The children's decisions are, among other things, affected by policy. The latter consists of a subsidy to those who enter higher education, financed by a proportional income tax on the parents. We note that not all the children decide to participate in the education policy, that an increase in the tax rate, everything else being equal, will reduce participation within the credit constrained groups, but not for the unconstrained, and that an increase in the subsidy, everything else being equal, will trivially boost participation.

The parents, prior to making transfers, vote over subsidies to those who participate in education, financed by a proportional tax on income. A voting equilibrium, if it exists, will be such that voters in the two tails of the income distribution support a reduction in the education subsidy: the reason is that the "poor" have a low participation rate, while the "rich" pay a particularly high tax price. The support for an expansion of the education subsidy comes primarily from the "middle-class". An education policy has therefore a regressive element in that low-income agents finance a form of public expenditure whose fruits are enjoyed mostly by the middle-earners. This conclusion is perfectly consistent with the intuitive analysis of the tuition fees conflict that we reported above.

Of course, "ends-against-the-middle" equilibria do not always exist. We first provide a necessary condition that has to be satisfied by an interior local equilibrium. We then verify the existence of such an equilibrium in a numerical version of the model, and check successfully that the equilibrium is also global. Finally, we perform a number of comparative statics exercises tracing the effect on the winning policy as well as other outcomes such as participation and intergenerational social mobility.



## References

- Anderberg, D. (2008), ‘Post-compulsory education: Participation and politics’, unpublished, Royal Holloway University of London.
- Blanden, J., Gregg, P. & Machin, S. (2003), ‘Changes in educational inequality’. CMPO WP no. 03/079.
- Blanden, J. & Machin, S. (2004), ‘Educational inequality and the expansion of UK higher education’, *Scottish Journal of Political Economy* 51(2), 230—249.
- Blau, F. (1999), ‘The effect of income on child development’, *Review of Economics and Statistics* 81, 261-276.
- Carneiro, P. & Heckman, J. J. (2003), ‘Human capital policy’, NBER WP no. 9495.
- Dearden, L., Machin, S. & Reed, H. (1997), ‘Intergenerational mobility in Britain’, *Economic Journal* 107, 47—64.
- Epple, D. & Romano, R. E. (1996a), ‘Ends against the middle: Determining public service provision when there are private alternatives’, *Journal of Public Economics* 62, 297-325.
- Epple, D. & Romano, R. E. (1996b), ‘Public provision of private goods’, *Journal of Political Economy* 104, 57-84.
- Fernandez, R. & Rogerson, R. (1995), ‘On the political economy of education subsidies’, *Review of Economic Studies* 62, 249-262.
- Lochner, L. & Monge-Naranjo, A. (2002), ‘Human capital formation with endogenous credit constraints’. NBER WP no. 8815.
- Mayer, S. (1997), *What Money Can’t Buy: Family Income and Children’s Life Chances*, Harvard University Press.

## Appendix

**Proof of Lemma 1.** We first consider the behaviour of the return to educational spending as the child’s own ability changes. In equilibrium, we write  $w_z(z(a, \theta), \theta)$  to emphasize that  $\theta$  affects  $w_z$  through the optimal educational investment as well as directly. The total derivative of  $w_z$  w.r.t.  $\theta$  is

$$\frac{dw_z}{d\theta} = w_{zz}z\theta + w_z\theta. \tag{A1}$$

Using (5), we find that (A1) equals zero for unconstrained families. (This is natural since the unconstrained solution satisfies  $w_z = 1$  as identity). For constrained families instead, it is

positive only if there is enough complementarity between  $z$  and  $\theta$  to overcome the effect of the decreasing returns of  $z$  ( $|w_{zz}z\theta| < w_{z\theta}$ ), and negative (or zero) otherwise. We proceed now by solving the first order condition (4) for the multiplier  $\mu$ , that can be viewed as the marginal value of credit. This yields

$$\mu(a, \theta) = \frac{1 - w_z(z(a, \theta), \theta)}{\gamma w_z(z(a, \theta), \theta) - 1} \geq 0, \quad (\text{A2})$$

where it is emphasized that the first order condition is evaluated at the optimal (unconstrained or constrained) value of  $z$ . Trivially  $\mu = 0$  if the choice is unconstrained, i.e.  $w_z(\cdot) = 1$ . On the other hand,  $\mu > 0$  by (6) if the choice is constrained. Focusing on this case and differentiating  $\mu$  with respect to  $a$  and to  $y$  yields

$$\mu_a = \frac{(1 + \gamma\mu) w_{zz}z_a}{(1 - \gamma w_z)} < 0; \quad (\text{A3})$$

$$\mu_\theta = \frac{(1 + \gamma\mu) (w_{zz}z_\theta + w_{z\theta})}{(1 - \gamma w_\theta)}. \quad (\text{A4})$$

For constrained children, we have that  $\mu_a < 0$  by (6), (A2), (7) and (1); intuitively, the value of credit will decrease as the child's assets go up. As for  $\mu_\theta$ , the sign depends on the sign of (A1) for constrained families – see above. For example, the value of credit for constrained children will increase in parental income ( $\mu_\theta > 0$ ) if complementarity in the earnings function is strong enough, as it induces the child to want to match her own higher ability by spending more on her own education.

We can now recover an expression for the marginal value of the child's assets. Applying the envelope theorem yields that:

$$x_a^p = 1 + \mu \geq 1 > 0; \quad x_\theta^p = w_\theta (1 + \gamma\mu) \geq w_\theta > 0. \quad (\text{A5})$$

Furthermore, (9), (A3) and (A4) allow us to write<sup>9</sup>

$$x_{aa}^p = \mu_a < 0; \quad x_{a\theta}^p = \mu_\theta. \quad (\text{A6})$$

**Proof of Lemma 3.** Let  $(\widehat{c}^p, \widehat{x}^p)$  and  $(\widehat{c}^{np}, \widehat{x}^{np})$  denote an indifferent family's allocation under participation and non-participation respectively. We then write  $\widehat{V}^p = u(\widehat{c}^p) + v(\widehat{x}^p)$  and  $\widehat{V}^{np} = u(\widehat{c}^{np}) + v(\widehat{x}^{np})$  for the utility functions when participating and non-participating respectively;

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<sup>9</sup>Some tedious computations will show that Young's theorem holds, i.e.  $x_{ay}^p = x_{ya}^p = \mu_y$ .

we know that  $\widehat{V}^p = \widehat{V}^{np}$ . Consider the locus of  $(c, x)$  generating the common value  $\widehat{V}^p$ ; formally define

$$\pi \equiv \left\{ (c, x) \mid u(c) + v(x) = \widehat{V}^p \right\}. \quad (\text{A7})$$

By construction  $(\widehat{c}^p, \widehat{x}^p)$  and  $(\widehat{c}^{np}, \widehat{x}^{np})$  both belong to the locus  $\pi$  which is downward sloping. We can now show that  $(\widehat{c}^p, \widehat{x}^p)$  lies to the “north-east” of  $(\widehat{c}^{np}, \widehat{x}^{np})$  if the credit constraint binds on the indifferent family and they coincide if the credit constraint is slack. To see this note from (11) and (17) that

$$\frac{u'(\widehat{c}^p)}{v'(\widehat{x}^p)} = x_a^p \geq 1 \quad \text{and} \quad \frac{u'(\widehat{c}^{np})}{v'(\widehat{x}^{np})} = 1. \quad (\text{A8})$$

If the credit constraint binds,  $x_a^p > 1$  by (9) and then it follows from (A8) and from  $\widehat{V}^p = \widehat{V}^{np}$  that  $\widehat{c}^p < \widehat{c}^{np}$  while  $\widehat{x}^p > \widehat{x}^{np}$ . If the credit constraint is slack,  $x_a = 1$  and  $(\widehat{c}^p, \widehat{x}^p)$  and  $(\widehat{c}^{np}, \widehat{x}^{np})$  coincide.

**Proof of Proposition 4.** A parent will prefer the child to attend education if

$$V^p((1 - \tau)y + \sigma; y, \theta) \geq V^{np}((1 - \tau)y) \quad (\text{A9})$$

If  $V^p$  increases faster in  $\theta$  than  $V^{np}$  at  $\widehat{\theta}$  (i.e. when  $V^p = V^{np}$ ) it follows that  $V^p$  will only ever cut  $V^{np}$  from below in the  $(\theta, V)$  space and the result follows.

Applying the envelope theorem on (16) and on (19) yields (note that this refers to the total derivative of the indirect utility w.r.t. child’s ability)

$$V_\theta^p = v'(x^p) x_\theta^p > 0; \quad V_\theta^{np} = 0, \quad (\text{A10})$$

where the sign of the first expression follows from (9). It is then easy to see that

$$V_\theta^p - V_\theta^{np} \Big|_{\theta=\widehat{\theta}} > 0 \quad (\text{A11})$$

for a parent who is indifferent within each income class.

**Proof of proposition 5.** We can implicitly differentiate  $V^p((1 - \tau)\widehat{y} + \sigma; y, \widehat{\theta}) - V^{np}((1 - \tau)y) = 0$ ; then since for the critical family,  $V_\tau^p = -u'(\widehat{c}^p)y$  and  $V_\tau^{np} = -u'(\widehat{c}^{np})y$ , we have that

$$\widehat{\theta}_\tau = -\frac{\widehat{y}[u'(\widehat{c}^{np}) - u'(\widehat{c}^p)]}{V_\theta^p - V_\theta^{np}} \geq 0; \quad (\text{A12})$$

$$\widehat{\theta}_\sigma = -\frac{v'x_a^p}{V_\theta^p - V_\theta^{np}} < 0. \quad (\text{A13})$$

where the signs follow from Lemma 3 and from (9).

**Proof of Proposition 6.** Let

$$q(y; \tau, \sigma) = \int_{\hat{\theta}(\tau, \sigma)}^{\bar{\theta}} f(y, \theta) d\theta. \quad (\text{A14})$$

be the fraction of participating families within a given  $y$ -class. Note that, by the Leibniz rule,

$$q_\tau = -\hat{\theta}_\tau f(y, \hat{\theta}) \leq 0; \quad q_\sigma = -\hat{\theta}_\sigma f(y, \hat{\theta}) > 0, \quad (\text{A15})$$

where the signs follow from (A12) and (A13) and the interpretation is obvious. The total share of participating families is simply

$$Q(\tau, \sigma) = \int_{\underline{y}}^{\bar{y}} q(y; \tau, \sigma) dy, \quad (\text{A16})$$

with derivatives

$$Q_\tau = \int_{\underline{y}}^{\bar{y}} q_\tau dy \leq 0; \quad Q_\sigma = \int_{\underline{y}}^{\bar{y}} q_\sigma dy > 0, \quad (\text{A17})$$

where the signs follow from (A15). Using the government budget constraint (22) and applying the implicit theorem function we get

$$\sigma'(\tau) = \frac{y^a - \sigma Q_\tau}{Q + \sigma Q_\sigma} > 0, \quad (\text{A18})$$

where we used (A17) to arrive at the sign.

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