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# Financing Higher Education and Labor Mobility 

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# Financing Higher Education and Labor Mobility 


#### Abstract

This paper analyzes how mobility of post-graduate skilled workers and students across different countries affects the quality level of higher education and the way education is financed. We start by examining a closed economy. In the presence of imperfect credit markets the education level with pure fee-financing is lower than the optimal level. If the credit market imperfections are not too large, a mix of tax- and fee-financing is optimal. The reason for this is that with pure fee-financing too few individuals decide to study. With mobility of skilled workers, both countries have an incentive to attract foreign skilled mobile workers as tax-payers while - at least partially - free-riding on the other country's provision of education. Both countries thus increase the tuition fee above the optimum and change the level of education correspondingly. If countries maintain the financing mix foreign skilled workers are attracted by suboptimal levels of educational quality. Allowing also for mobile students may intensify the upward race of fees. The case of free-riding on the education provided by other countries may be strengthened. However, countries may anticipate this race and abstain from engaging in fee competition in the first place.


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Keywords: higher education, labor migration, tuition fees, taxes.

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## 1 Introduction

Individual mobility becomes more and more a driving force that influences national policies. It is especially crucial as higher education and research are concerned as it enlarges the opportunities of students and affects their returns to education. In addition, the mobility of students and skilled workers induces changes in governmental educational policies and introduces competition between educational institutions or countries.

We are here particularly interested in the impact of the financing mix (public or private) of higher education and of the realised quality level on the individual incentives to attain higher education and in how those decisions influence in turn governmental educational policies. Moreover, the financing and the provision of higher education may be affected by labor mobility. The aim is thus to analyze which quality levels of education can be sustained in countries open to migration and which financing (mix) is optimal.

The analysis is conducted in a general equilibrium setup. We analyze a two-period overlapping generations model with two jurisdictions and individuals who differ in their innate abilities. In the first period, individuals decide whether (and where) to study and in the second period educated workers decide where to work. Since students do not earn an income in the first period their costs incurred by higher education can either be subsidized by wage taxes or they can obtain a credit to pay the costs in the form of fees. In a first part, we look at the closed economy case to derive results which can serve as benchmarks for the open economy setting. In a second part, we analyze open economies where migration is restricted to post-graduate skilled workers. We also discuss the case when migration of skilled workers and of students is possible.

The first part on closed economies looks in detail at different financing instruments - namely financing via tuition fees and via wage taxes as well as mixed financing. The education levels are derived for each system and compared to the optimal allocation of a social planner for both countries. In the case of perfectly competitive credit markets we find that for the same education level more individuals choose to become educated when education is tax-financed than when it is fee-financed. Tax-financing thus induces individuals also with lower ability levels to opt for acquiring education. Furthermore, we show that the chosen education level in the tax-financing country is higher than in the fee-financing country with the latter corresponding to the social planner's optimum. This no longer holds in the presence of imperfections on the credit markets. The education level with pure fee-financing is lower than the optimal level. If the credit market imperfections are not too large, we show that a mix-financing system is optimal. The reason for this is that with pure fee-financing too few individuals decide to study and to supply labor as skilled workers. The welfare can thus be increased by partially subsidizing education via taxes. If the credit market imperfections are, however, important, pure tax-financing leads to a second-best level of education.

The second part addresses the mobility issue in an integrated labor market of both countries. In accordance with evidence we assume that only post-graduate skilled workers are mobile in the second period (see, e.g., Demange, Fenge and Uebelmesser, 2008). Thus individuals who decide not to study in the first period and to work as unskilled workers stay in their home country. We choose the optimal mixed-financing system and the optimal educational level as the starting point in both countries. We are interested in whether mobility of skilled workers generates distortions on these optimal levels.

For the scenario where only skilled workers are mobile, the optimal allocation is distorted in two ways. For a given education level, a country can increase the tuition fees which implies that it can lower the wage tax to finance higher education. At the same time, fewer individuals decide to attain higher education since subsidization of their study is reduced. This eases the country's budget constraint because the tax-financed spending per student diminishes and fewer individuals take up higher education. However, there are two countervailing effects. The ratio of the wage of skilled workers to the wage of unskilled workers increases and the wage tax rate decreases so that the higher net wage of skilled workers incites more individuals to study. We show that those general equilibrium effects do not offset the first effect. We find that both countries have an incentive to attract foreign skilled mobile workers as tax-payers while - at least partially - free-riding on the other country's provision of education. Both countries thus increase the fee above the optimal level.

Taken the tuition fee level as given, a country can also decide to decrease the educational level in order to attract foreign skilled workers. Lowering the educational level reduces directly the cost of higher education and, thereby, the tax revenue needed to finance higher education. Under the assumption that the number of individuals who decide to study decreases with lower educational levels (which depends on the production technology and the cost function of higher education; see section 3.3) the indirect effect of fewer students reduces further the educational costs and increases the wage rate of skilled workers in relation to the wage rate of unskilled workers. Hence, the overall impact of lowering the educational level increases the net wage of skilled workers which attracts foreign workers and increases welfare. In a migration equilibrium the educational qualities in both countries will be set at levels below the optimal one.

Finally we discuss the scenario where not only skilled workers in the second period but also students in the first period are mobile. In a Nash-equilibrium, the countries may intensify the race to the top as fees are concerned as they try to attract skilled workers and to deter students in order to save educational costs. The free-riding of countries may escalate although in equilibrium the number of students does not change. Depending on the optimal financing mix to start with, however, it is also possible that countries abstain from increasing fees in the first place as they anticipate this race to the top. Then, the optimal financing mix is maintained.

Our paper is related to the literature of higher education subsidies. One of the earlier contributions is Johnson (1984) who analyzes the distributional effects of subsidies. He argues that even though these subsidies benefit only those who study, there is not necessarily a conflict of interest due to complementarities between skilled and unskilled labour. Creedy and Francois (1990) more directly address majority voting of higher education subsidies when education generates a positive externality leading to growth. Both Johnson and Creedy/Francois abstract, however, from credit market imperfections and uncertainties related to the education investment. The riskiness of this investment is at the core of the analysis by García-Penalosa and Wälde (2000) who compare the efficiency and equity effects of a tax-subsidy scheme to loan schemes and graduate taxes. All these papers have in common that they focus on a closed economy. As we have already pointed out, however, mobility is an important consequence of labor market integration and even more so as high-skilled workers are concerned.

The analysis has therefore been extended to an open economy framework in more recent contributions. Wildasin (2000) analyzes the effects of labor market integration on human capital investment in a general equilibrium model with uncertainty where education may be either
publicly or privately financed. (Industry-specific) skills expose individuals to wage risks while mobility across jurisdictions can help to eliminate these risks. The focus is thus on the decision to acquire education in an open economy setting with uncertainty where two financial regimes are compared and workers are mobile. In Del Rey (2001), on the contrary, students are mobile. The analysis concentrates on the ensuing fiscal competition and how this affects the governmental decision about the public provision of higher education.

A further aspect is central in Kemnitz (2005). He analyzes the impact of tuition fees on the quality of higher education under decentralized and centralized decision making. Special attention is given to the question as to what extent fees crowd out public funds under both regimes. Busch (2007) and Mechtenberg and Strausz (2008) also look at the quality level of education in an open economy. While in Busch the positive correlation between education quality and the mobility of graduates induces governments to lower the quality level to counteract the threat of a brain drain, Mechtenberg and Strausz come to similar conclusions in a setting with mobile students where governments fear free-riding. ${ }^{1}$

Our paper is clearly related to Wildasin (2000) in the sense that we also look at an integrated labor market with mobility of skilled workers in a general equilibrium framework. We abstract, however, from uncertainty and focus instead on the choice of financing of higher education and the educational quality while allowing for heterogeneous individuals with respect to their innate ability.

We proceed as follows: In the next section, we present the model. In section 3, we analyze the closed economy where migration is not possible. In section 4 we analyze the impact of migration of skilled workers on the financing mix of higher education and the educational level. Furthermore, we discuss the modification of results when students are mobile, too. Section 5 concludes.

## 2 The model

To analyze the interaction of higher education and migration between the labor markets, each country is described by the same overlapping generations model. Individuals live for two periods and the population growth rate is nil. There is one consumption good, which cannot be stored. The good is produced from two kinds of labor, skilled and unskilled and there is no capital. The labor supply is determined by the individuals decisions to acquire higher education as follows. Individuals differ with respect to their innate ability. In the first period of their life, they decide whether (and where if they are mobile) to receive higher education. Those who choose to do so will supply skilled labor at the second period of their life and others will supply unskilled labor at both periods their life. Hence the structure of labor supply at a given period is determined by the current and past individuals decisions to acquire higher education. These decisions affect the structure of labor supply in the following period. We shall concentrate on steady state situations.

In each period higher education has to be financed via wage taxes by workers in the same

[^0]period and via tuition fees by the students. To analyze these choices, we consider the following set-up. At the first stage, governments choose the quality level of education and how higher education is financed, i.e. via taxes and/or via fees. At the second stage, individuals make their education and migration decisions given the governmental arrangements for higher education.

We describe in more details a closed economy in which all individuals at each period of their life are immobile. This simplifies the presentation since wages, taxes and education parameters are those of the home country. The modifications under the mobility of skilled workers or of students will be introduced when needed in subsequent sections.

### 2.1 The production sector

The production sector in each country uses two kinds of input: ${ }^{2}$ labor supplied by individuals with and without higher education, $L_{s}$ (skilled labor) and $L_{u}$ (unskilled labor) respectively, where it is assumed that only skilled labor is mobile in open economy. ${ }^{3}$ Production takes place according to a neoclassical production function with constant returns to scale so that:

$$
\begin{equation*}
F\left(L_{u}, L_{s}\right)=L_{u} f\left(\frac{L_{s}}{L_{u}}\right)=L_{u} f(l) \tag{1}
\end{equation*}
$$

where $l=\frac{L_{s}}{L_{u}}$ denotes the ratio of skilled to unskilled labor. We assume competitive labor markets in each country. The optimal demand for labor implies that productivities of skilled and unskilled workers are equal to their respective wage rates $w_{s}$ and $w_{u}$ :

$$
\begin{align*}
w_{s} & =f_{l}  \tag{2}\\
w_{u} & =f-l f_{l} \tag{3}
\end{align*}
$$

It follows that

$$
\begin{equation*}
\frac{\partial w_{u}}{\partial L_{s}}>0 ; \quad \frac{\partial w_{s}}{\partial L_{s}}<0 ; \quad \frac{\partial w_{u}}{\partial L_{u}}<0 ; \quad \frac{\partial w_{s}}{\partial L_{u}}>0 . \tag{4}
\end{equation*}
$$

Throughout the paper, to avoid corner solutions, we shall assume Inada conditions, according to which marginal productivities with respect to a factor increase indefinitely as the factor becomes scarce and that marginal cost to education increases indefinitely with the level:

Assumption 1: $\lim _{L_{u} \rightarrow 0} F_{L_{u}}\left(L_{u}, L_{s}\right)=\infty$ and $\lim _{L_{s} \rightarrow 0} F_{L_{s}}\left(L_{u}, L_{s}\right)=\infty ;$

$$
\lim _{e \rightarrow \infty} c^{\prime}(e)=\infty
$$

### 2.2 The demand for higher education

Individuals are distinguished by an ability parameter, $y$, which reflects individually different benefits from higher education. The distribution of abilities is identical in each country and

[^1]assumed for simplicity to be uniform in the range $[0, \bar{y}]$. Only some individuals of a generation with high enough abilities decide to study. The rest starts working as unskilled workers.

To be skilled, an individual must receive higher education. Education quality or level is denoted by $e$. The quantity of skilled labor provided by an educated worker with education level $e$ depends on her ability $y$ : it is given by $y e$. For simplicity, we assume that the amount of money spent for higher education per individual only depends on the education level, given by $c(e)$. Put differently, costs in education are proportional to the number of students, given the quality. ${ }^{4}$ The cost function $c$ is assumed to be increasing and convex.

If an individual decides to study, she pays a fraction $0 \leq f \leq 1$ of her education costs as fees during the first period, $f c(e)$, and earns no wage income. In the second period, the educated worker receives a gross wage income which depends on her ability $y: w_{s} y e$. The wage income net of tax is $w_{s} y e(1-\tau)$ where $\tau$ is the tax rate levied to finance the remaining costs of higher education. Thus her lifetime income is

$$
\begin{equation*}
(1-\tau) w_{s} \frac{y e}{1+r}-f \cdot c(e) \tag{5}
\end{equation*}
$$

If the individual decides not to study she receives a wage income net of tax of $(1-\tau) w_{u}$ in both periods. Hence, her lifetime income is

$$
\begin{equation*}
(1-\tau) w_{u} \frac{2+r}{1+r} \tag{6}
\end{equation*}
$$

The individual compares the lifetime incomes and chooses the option that maximizes her income. The decision whether to study or not depends on the ability of the individual. The marginal ability type who is indifferent between both options is given by

$$
\begin{equation*}
y^{F T}=\frac{w_{u}(2+r)}{w_{s} e}+\frac{(1+r) f c(e)}{(1-\tau) w_{s} e} \tag{7}
\end{equation*}
$$

Of course, the pure fee-financed and the pure tax-financed systems are obtained as particular cases. With obvious notation, taking respectively $(\tau=0 ; f=1)$ and ( $f=0 ; 1>\tau>0$ ), we have

$$
\begin{gather*}
y^{F}=\frac{w_{u}(2+r)+(1+r) c(e)}{w_{s} e}  \tag{8}\\
y^{T}=\frac{w_{u}(2+r)}{w_{s} e} \tag{9}
\end{gather*}
$$

### 2.3 Imperfections on the credit market

We interpret a positive interest rate as the result of imperfections on the credit markets. In an overlapping generation model without a storable good, borrowing takes place between the individuals of one generation (not all decide to study) and possibly between generations. All the involved interest payments accrue from the borrowers to the lenders. Financial institutions,

[^2]if any, are pure intermediaries. As shown by Gale (1973), feasible states are characterized by the condition that interest payments are zero. There are two possible steady states: either the equilibrium interest rate is equal to the population growth rate (the golden rule equilibrium) or there is no borrowing between different generations (the autarky or no-trade equilibrium). We rule out autarky. Thus, without frictions, the interest rate is the golden rule interest rate, which is here is nil because population is constant. When the credit market is not perfectly competitive, the net interest payments are positive and accrue to financial intermediaries. When on aggregate a young generation borrows, net interest payments are positive if the interest rate is positive. This is the more plausible situation because young individuals who decide to study have no income in the first period and need to borrow in order to finance their studies and to consume. It is unlikely that the unskilled young workers will save enough to compensate these needs. In that case, the positive interest rate characterizes a deadweight loss on the imperfect credit markets. Another interpretation of the positive interest rate is that there are moral hazard problems (see von Weizsäcker and Wigger, 2001). The positive interest rate can be interpreted as a risk premium charged by credit markets due to the risky investment in human capital. In the following we will show how this imperfection affects the financing of higher education.

## 3 Financing higher education without migration

As a benchmark, this section disregards migration effects and analyzes the individual and governmental decisions within a closed country. Higher education may be financed partly by fees paid by students and partly subsidized by taxes levied on labor income. We start by considering the second stage of the game. Hence, we analyze the individual choice of studying and the resulting equilibrium on the labor market.

Then we turn to the first stage of the game. First we derive the optimal policy when abilities can be observed by a social planner and both the education level and the number and the abilities of students can be chosen. We then study the decision problem faced by a government which chooses the education level without observing abilities.

### 3.1 Equilibrium employment

We are interested in the impact of the education level on individual decisions and the resulting impact on the labor markets. We describe here how an education level $e$ and the financing parameters, $f, \tau$, and $r$, determine a (steady state) equilibrium of the labor markets.

Basically we look at an equilibrium under rational expectations as follows. The individuals' decision to be skilled or unskilled just derived are based on 'expected' wages. These decisions, more precisely the ability threshold level, determine the supply of skilled and unskilled labor, which in turn determine the wages that clear the markets. At an equilibrium, these realized wages must be equal to the initial expected wages.

Let us describe more precisely the labor market. As already mentioned, the population growth rate is assumed to be nil. In each period, employment consists of young and old unskilled workers and old skilled workers. Let an education level $e$ and a threshold ability level of skilled workers $y$ be given. The number of unskilled workers per generation, denoted by $N_{u}$, is equal
to $y$ and the number of skilled workers, denoted by $N_{s}$, is equal to $\bar{y}-y$. The employment of unskilled labor is given by

$$
\begin{equation*}
L_{u}=2 \int_{0}^{y} 1 d z=2 y=2 N_{u} \tag{10}
\end{equation*}
$$

and the effective skilled labor by

$$
\begin{align*}
L_{s} & =\int_{y}^{\bar{y}} z e d z=e\left(\frac{\bar{y}^{2}-(y)^{2}}{2}\right)=(\bar{y}-y) e\left(\frac{\bar{y}+y}{2}\right) \\
& =N_{s} e\left(\frac{\bar{y}+y}{2}\right) \tag{11}
\end{align*}
$$

which is equal to the number of skilled workers multiplied by their average ability and the educational level.

The above expressions determine the labor forces and hence the wages of skilled and unskilled labor thanks to (2) and (3) as a function of the threshold $y$ and the educational level $e$. We denote these wages by $w_{s}(y, e)$ and $w_{u}(y, e)$.

These wages in turn determine the incentives to be skilled, i.e. they determine $y^{F T}$ as given by (7). At an equilibrium of the labor markets, the obtained value $y^{F T}$ must be equal to the initial value $y$, that is $y^{F T}(e)$ solves

$$
\begin{equation*}
y-\frac{w_{u}(y, e)(2+r)}{e w_{s}(y, e)}-\frac{(1+r) f c(e)}{(1-\tau) e w_{s}(y, e)}=0 \tag{12}
\end{equation*}
$$

Equilibrium is unique (see Section 6.1). The intuition is that as there are fewer skilled individuals, the incentives to become skilled are enhanced through the impact on wages, which gives an equilibrating force. In other words, increasing the threshold ability means that fewer workers become skilled which raises the wage rate for skilled and decreases the wage rate for unskilled.

### 3.2 The incentives to acquire education: comparative statics

How the educational level affects the ability threshold turns out to be important for the sequel. As we have just seen, the threshold is given by the implicit equation (12). It is convenient to rewrite this equation by defining the net benefit for a $y$-agent to acquire education,

$$
\begin{equation*}
B(y, e)=(1-\tau)\left[y e . w_{s}(y, e)-w_{u}(y, e)(2+r)\right]-(1+r) f c(e) \tag{13}
\end{equation*}
$$

With this notation, equation (12) writes as $B(y, e)=0$, which says that the net benefit of education is null for the marginal student. The benefit increases with $y .{ }^{5}$ Hence the monotony property of $y^{F T}$ with respect to the educational level $e$ depends on how the net benefit to the

[^3]marginal student varies with $e$. More precisely $\frac{\partial y^{F T}}{\partial e}=-\frac{B_{e}}{B_{y}}\left(y^{F T}, e\right)$ : if the net benefit increases, $y^{F T}$ decreases, raising the number of students, and similarly if the net benefit decreases, $y^{F T}$ increases.

The educational level has two impacts on the net benefit: a direct one because the total wage of a skilled worker is proportional to $e$ and the cost varies with $e$, and an indirect one through the equilibrium wages. The indirect impact always lowers the net benefit: increasing the education level is akin to increasing skilled labor, hence $w_{u}$ increases and $w_{s}$ decreases. The direct impact is ambiguous and depends in particular on how steep the marginal cost is. The sign of the overall effects is determined by differentiating the net benefit (13) with respect to $e$ :

$$
\begin{equation*}
B_{e}\left(y^{F T}, e\right)=(1-\tau)\left[y^{F T}\left(w_{S}+e \frac{\partial w_{S}}{\partial e}\right)-(2+r) \frac{\partial w_{U}}{\partial e}\right]-(1+r) f c^{\prime}(e) \tag{14}
\end{equation*}
$$

The skilled wage decreases with a higher education level since the effective labor supply of skilled increases ${ }^{6}$ with $e$. Thus the ability threshold increases with the educational level (fewer individuals decide to study) if the net wage income of the skilled decreases or if it increases to a smaller extent than the share of marginal educational costs increases which has to be paid by fees. It is convenient to reformulate $B_{e}$ in terms of the elasticities of wages with respect to the educational level, and the elasticity of substitution.

These elasticities are given respectively by

$$
\eta_{w_{s}, e}=\frac{e}{w_{S}} \frac{\partial w_{S}}{\partial e}, \eta_{w_{u}, e}=\frac{e}{w_{u}} \frac{\partial w_{u}}{\partial e}, \sigma=\frac{\partial \ln \left(\frac{L_{u}}{L_{s}}\right)}{\partial \ln \left(\frac{w_{s}}{w_{u}}\right)}
$$

Note that $\eta_{w_{s}, e}<0, \eta_{w_{u}, e}>0$, and $0<\sigma<\infty$, and they are related by

$$
\eta_{w_{s}, e}-\eta_{w_{u}, e}=\frac{\partial \ln \left(\frac{w_{s}}{w_{u}}\right)}{\partial \ln e}=-\frac{1}{\sigma}
$$

because $L_{s}=e \frac{\left(\bar{y}^{2}-y^{F T 2}\right)}{2}$ and $L_{u}=2 y$ gives $\frac{\partial \ln \left(L_{u} / L_{s}\right)}{\partial \ln e}=-1$. This means the larger the substitutability between the two labor inputs is (the larger $\sigma$ is) the smaller is the decrease in the wage ratio due to a higher educational level.

Using this notation, we have

$$
\begin{equation*}
B_{e}=(1-\tau)\left[y w_{S}\left(1+\eta_{w_{s}, e}\right)-(2+r) \frac{w_{u} \eta_{w_{u}, e}}{e}\right]-(1+r) f c^{\prime}(e) \tag{15}
\end{equation*}
$$

At the threshold level, equation (12) is satisfied. Plugging the value of $w_{u}$ given by this equation and using the elasticity $\sigma$ yields

$$
\begin{equation*}
B_{e}\left(y^{F T}, e\right)=(1-\tau)\left[y^{F T} w_{S}\left(1-\frac{1}{\sigma}\right)\right]-(1+r) f\left[c^{\prime}(e)-c(e) \frac{\eta_{w_{u}, e}}{e}\right) \tag{16}
\end{equation*}
$$

[^4]Observe that it must hold that $\frac{(1-\tau) y^{F T} w_{S} e}{(1+r) f c(e)}>1$. By short-writing $R \equiv \frac{(1-\tau) y^{F T} w_{S} e}{(1+r) f c(e)}$, the following lemma holds:

Lemma 1 Let $R=\frac{(1-\tau) y^{F T} w_{S} e}{(1+r) f c(e)} . R$ is strictly larger than 1. We have

$$
\begin{equation*}
\frac{\partial y^{F T}}{\partial e} \leq 0 \Leftrightarrow \frac{\left[R-\frac{c^{\prime}(e) e}{c(e)}+\eta_{w_{s}, e}\right]}{R-1} \geq \frac{1}{\sigma} \tag{17}
\end{equation*}
$$

and similarly replacing large inequalities by strict ones.
The conditions in the cases of pure tax or fee-financing can be easily derived by setting $f=0$ or $f=1, \tau=0$. By taking the limit in the first case, the condition in lemma 1 reduces to $1 \geq 1 / \sigma$ so that financing education only by taxes implies that the ability threshold decreases (increases) with the educational level if and only if $\sigma$ is larger (smaller) than unity.

Lemma 1 gives a characterization of the impact of education on the threshold. Basically the condition says that increasing educational level incites more individuals to acquire education if the wages do not react too much compared to a measure composed with the cost elasticity and skilled wage elasticity. A contrario a higher educational level leads to a lower number of students if and only if

$$
\begin{equation*}
\frac{\left[R-\frac{c^{\prime}(e) e}{c(e)}+\eta_{w_{s}, e}\right]}{R-1}<\frac{1}{\sigma} \tag{18}
\end{equation*}
$$

This inequality holds under two alternatives: either (a) the return to education is small enough relative to the sum of the cost elasticity and skilled wage elasticity (i.e., $R-\frac{c^{\prime}(e) e}{c(e)}+\eta_{w_{s}, e}<$ 0 ) or (b) wages react strongly to educational choices, so that $1 / \sigma$ is large enough. Observe that this is surely the case for $\sigma \leq 1$ : since $\frac{c^{\prime}(e) e}{c(e)}>1$ because we assume a convex educational cost function and $\eta_{w_{s}, e}<0$, the left hand side of (18) is smaller than 1 .

As an illustration, with a linearly homogeneous Cobb-Douglas production function $F\left(L_{s}, L_{u}\right)=$ $L_{s}^{\alpha} L_{u}^{1-\alpha}, \alpha \in(0,1)$, the substitution elasticity is unity and $\frac{\partial y^{F T}}{\partial e}>0$. If education is purely tax-financed $(f=0)$ we find that $\frac{\partial y^{T}}{\partial e}=0$ in this case.

In case of a CES production function $F\left(L_{s}, L_{u}\right)=A\left[\theta\left(L_{S}\right)^{-\rho}+(1-\theta)\left(L_{U}\right)^{-\rho}\right]^{-1 / \rho}$ with $A>0, \theta \in(0,1)$ and $-1<\rho \neq 0$, we have $1 / \sigma=\rho+1$. This shows that both cases of an increasing or decreasing impact of education level on the number of students are possible.

### 3.3 Government decisions

Now we analyze the first stage of the game. We first derive the optimal allocation in the absence of any informational constraints. Then, we compare it to the optimal decisions of governments for cases where higher education is financed either via fees or via taxes or where there is mixedfinancing and individuals freely choose to study.

### 3.3.1 Optimal allocation

Under complete information on individuals' abilities, a social planner can decide on the level of education and on the ability of those who study. The criterion is aggregate production net of education cost at a steady state, given by $F\left(L_{s}, L_{u}\right)-N_{s} c(e)$. This is the criterion that obtains in a fully fledged overlapping generations economy in which the planner treats all generations equally. In other words, we are at the golden rule with an implicit interest rate equal to the population growth rate, which is here equal to zero (see Gale, 1973).

The choice of the level of education and of the minimum ability of those who study, $e$ and $y$ respectively, fully determines skilled and unskilled labor from (10) and (11). Hence defining

$$
\begin{equation*}
W(y, e)=F\left(L_{s}, L_{u}\right)-N_{s} c(e) \tag{19}
\end{equation*}
$$

where $L_{s}, L_{u}$ are functions of $e$ and $y$ and $N_{s}$ is a function of $y$ alone, the objective is to maximize $\underset{e, y}{\operatorname{Max}} W(y, e)$.

The objective is concave, and thanks to Assumption 1, there must be both skilled and unskilled workers at an optimum. Besides, the education level has to be bounded. ${ }^{7}$ Hence the optimum is interior. The impact of a marginal increase in $e$ keeping the set of students fixed is given by

$$
\begin{align*}
\frac{\partial W}{\partial e} & =F_{L_{s}} \frac{\partial L_{s}}{\partial e}+F_{L_{u}} \frac{\partial L_{u}}{\partial e}-N_{s} c^{\prime}(e) \\
& =(\bar{y}-y)\left[w_{s} \frac{\bar{y}+y}{2}-c^{\prime}(e)\right] \tag{20}
\end{align*}
$$

It is equal to the effect on the production of the skilled minus the increase in cost.
The impact of a marginal increase in the minimum ability level $y$, keeping the education level fixed is given by

$$
\begin{align*}
\frac{\partial W}{\partial y} & =F_{L_{s}} \frac{\partial L_{s}}{\partial y}+F_{L_{u}} \frac{\partial L_{u}}{\partial y}-c(e) \frac{\partial N_{s}}{\partial y} \\
& =-w_{s} e y+2 w_{u}+c(e) \tag{21}
\end{align*}
$$

It is equal to the net impact on the productivity of a student of ability just equal to $y$ from becoming skilled compared to remaining unskilled where the impact is measured at the steady state situation.

At the optimum, the level of education and the threshold ability level are given by the following first-order conditions

$$
\begin{align*}
(\bar{y}-y)\left[w_{s} \frac{\bar{y}+y}{2}-c^{\prime}(e)\right] & =0  \tag{22}\\
-w_{s} e y+2 w_{u}+c(e) & =0 \tag{23}
\end{align*}
$$

[^5]that is, the marginal gain from a change in education on the average student, $w_{s} \frac{\bar{y}+y}{2}$ is equal to the marginal cost, and the net gain of education for the marginal student is null.

In the sequel, we put a superscript * to indicate the values at the optimum solution for wages, education levels, etc.. In the following analysis, two features may hamper this optimum solution to be reached. First, individuals' abilities are no longer assumed to be observable (or contractible). Due to these informational asymmetries, the set of students cannot be chosen as an omniscient social planner does. The government chooses the optimal level of education taking account of the individual decisions which are determined by the threshold level of ability. Second, the interest rate faced by the individuals is not at the golden rule level (equal to zero) but it is positive.

### 3.3.2 Education level with pure fee-financing

The welfare criterion of the government is still the aggregate production net of education cost at a steady state. Given an education level, the equilibrium ability threshold which determines who decides to study is denoted by $y^{F}(e)$. Thus, the government's objective is

$$
\begin{equation*}
M_{e} \operatorname{Max} W\left(y^{F}(e), e\right)=F\left(L_{s}, L_{u}\right)-N_{s} c(e) \tag{24}
\end{equation*}
$$

in which skilled and unskilled labor levels are those determined by the equilibrium threshold ability level:

$$
\begin{equation*}
N_{s}=\bar{y}-y^{F}(e), L_{u}=2 y^{F}(e), L_{s}=N_{s} e\left(\frac{\bar{y}+y^{F}(e)}{2}\right) \tag{25}
\end{equation*}
$$

The impact on welfare due to a marginal change of education is

$$
\begin{equation*}
\frac{\partial W}{\partial y} \frac{d y^{F}}{d e}+\frac{\partial W}{\partial e} \tag{26}
\end{equation*}
$$

where $\frac{d y^{F}}{d e}$ denotes the equilibrium change in the threshold ability level - and thus in the selection of abilities - that results from an increase in the education level. The impact on welfare due to a marginal change of education is thus composed of two terms: an indirect one through the selection of abilities and a direct one.

For $r=0$, the optimal ability associated with a given education level coincides with that chosen by individuals, that is $\frac{\partial W}{\partial y}\left(y^{F}(e), e\right)$ is identically null as can be seen from (8) and (21). An immediate consequence is that the optimal level of education coincides with the optimum level as given by (23).

Consider now the more plausible situation in which, due to distortions on the credit market, $r$ is positive.

The direct impact on welfare of education level is given by expression (20) computed at the ability threshold:

$$
\begin{equation*}
\frac{\partial W}{\partial e}\left(y^{F}(e), e\right)=\left(\bar{y}-y^{F}(e)\right)\left[w_{s} \frac{\bar{y}+y^{F}(e)}{2}-c^{\prime}(e)\right] \tag{27}
\end{equation*}
$$

At the optimal level $e^{*}$, this expression is positive because a positive interest rate reduces the incentives to study, that is, increases $y^{F}(e)$ and the skilled wage: the average ability of students is larger than at the optimum, enhancing the benefits to improve education level.

As for the indirect impact of changing education level we have with (8) and (21):

$$
\begin{equation*}
\frac{\partial W}{\partial y}\left(y^{F}(e), e\right)=-r\left(w_{u}+c(e)\right) \tag{28}
\end{equation*}
$$

This reflects the interest on the effective cost of education to an individual, i.e. the fee plus the forgone wage in the first period. Since this is negative, welfare would be improved by lowering the ability threshold, this means by educating some unskilled individuals.

The government however can choose only the education level. Consider the optimum level of education $e^{*}$ and the chosen level $y^{F}\left(e^{*}\right)$. On the one hand, welfare is increased by increasing $e$ and on the other hand by decreasing $y$ below $y^{F}\left(e^{*}\right)$. Thus if $y^{F}$ decreases with $e$, there is a double benefit in increasing the educational level above the optimal one. Otherwise, if $y^{F}$ increases with $e$, there is a trade off between improving quality of education but decreasing even further the number of students.

Now, we can use the results obtained in section 3.2 where the behavior of $y^{F}$ is analyzed. (Since the equilibrium on the labor markets changes with the threshold ability level, the behavior of $y^{F}$ cannot be directly seen from equation (8) due to the impact on wages.) We can therefore state

Proposition 2 Consider an economy with purely fee-financed education.
With a perfect credit market, $r=0$, the optimal level of education leads to the optimal allocation.
With an imperfect credit market, $r>0$, the level of education is higher than the optimum if more individuals will study with a higher e, that is if the condition in lemma 1 holds (elasticity of substitution $\sigma$ is sufficiently large and average cost of education does not increase too much).

### 3.3.3 Education level with pure tax-financing

We consider the same setting except that now the government levies a tax on all workers in a given period to finance education. We assume that there is no distortionary impact of taxes on the labor-leisure choice nor any redistributive considerations outside the educational system. The decision problem for the optimal level of education is then given by

$$
\begin{equation*}
M_{e}^{\operatorname{ax}} W\left(y^{T}(e), e\right)=F\left(L_{s}, L_{u}\right)-N_{s} c(e) \tag{29}
\end{equation*}
$$

subject to the budget constraint

$$
\tau\left(w_{s} L_{s}+2 w_{u} N_{u}\right)=N_{s} c(e)
$$

in which skilled and unskilled labor levels are those determined by the threshold ability level $y^{T}(e)$, as in (25) replacing $y^{F}(e)$ by $y^{T}(e)$. Since the level $y^{T}(e)$ is independent of $\tau$, the government can choose the education level and then set the tax rate so as to finance the costs, provided this gives a value for $\tau$ smaller than 1 .

Maximization with respect to the education level $e$ yields

$$
\begin{equation*}
\left[\frac{\partial W}{\partial y} \frac{d y^{T}}{d e}+\frac{\partial W}{\partial e}\right]\left(y^{T}(e), e\right)=0 \tag{30}
\end{equation*}
$$

For the direct impact of the education level, we have with (20)

$$
\begin{equation*}
\frac{\partial W}{\partial e}\left(y^{T}(e), e\right)=\left(\bar{y}-y^{T}(e)\right)\left[w_{s} \frac{\bar{y}+y^{T}(e)}{2}-c^{\prime}(e)\right] \tag{31}
\end{equation*}
$$

and the indirect impact, with (9) and (21) is

$$
\begin{align*}
\frac{\partial W}{\partial y}\left(y^{T}(e), e\right) & =-w_{s} e y^{T}(e)+2 w_{u}+c(e) \\
& =-r w_{u}+c(e) \tag{32}
\end{align*}
$$

Let us start with the direct impact (cf. (31)). We discuss here the case of a financial market in which the interest rate $r$ is not too large, more precisely where $-r w_{u}+c\left(e^{*}\right) \geq 0$ at the wage resulting from the threshold $y^{T}\left(e^{*}\right) .{ }^{8}$ In that case, the absence of fees for becoming skilled gives to some individuals with low ability too much incentive to study, that is $y^{T}\left(e^{*}\right)$ is smaller than $y^{*}$. As a result, $w_{s}$ is smaller than $w_{s}^{*}$. Since at the optimal level $e^{*}$, the right hand side of equation (31) is zero when $y^{T}\left(e^{*}\right)$ is replaced by the optimal level $y^{*}$ and $w_{s}$ by $w_{s}^{*}$, the direct marginal impact of education on welfare is negative at $e^{*}$ : in the absence of an indirect impact through the selection of students, the education level in the tax country must be lower than the optimum. Under the assumption that $r$ is not too large (see (32)), the indirect impact is positive: increasing the ability threshold above that chosen by individuals is welfare improving. Again, the absence of costs for becoming skilled gives to some individuals too much incentive to study, contrary to the fee setting.

To sum up, if $y^{T}$ decreases with $e$, there is a double benefit in decreasing the educational level below the optimum level: on the one hand, welfare is increased by decreasing $e$ and on the other hand by increasing $y$ above $y^{T}\left(e^{*}\right)$. From section 3.2 , this occurs when the elasticity of substitution between skilled and unskilled labor $\sigma$ is larger than 1 in $e$. For a Cobb Douglas function for instance ( $\sigma$ is equal to 1 ), the threshold level is constant and only the direct effect of education affects welfare: the chosen level is lower than the optimum. We can therefore state

Proposition 3 Consider a country with tax-financing.
With a perfect credit market, $r=0$, the level of education may be higher, equal to or lower than the optimum. It is smaller than the optimum (or that in a country with fee-financing) if the elasticity of substitution between skilled and unskilled labor $\sigma$ is larger than 1.
With an imperfect credit market, $r>0$, assuming the elasticity of substitution $\sigma$ to be larger than 1 , the level of education is smaller than the optimum (or that in a country with fee-financing) if $r<\frac{c\left(e^{*}\right)}{w_{u}}$.

### 3.3.4 Education level and financing if both taxes and fees are available

Now we assume that the costs of higher education are partly financed by tuition fees paid by the students and the remainder by taxes levied on wage income. The budget of the government

[^6]is given by:
\[

$$
\begin{equation*}
\tau\left(w_{s} L_{s}+2 w_{u} N_{u}\right)=(1-f) c(e) N_{s}, \quad f \in[0,1] \tag{33}
\end{equation*}
$$

\]

The government maximizes aggregate production net of education costs by choosing simultaneously the education level $e$ and the share of costs financed by fees, $f$ :

$$
\begin{equation*}
\underset{e, f}{\left.\operatorname{Max}_{f} W\left(y^{F T}(e), e\right)=F\left(L_{s}, L_{u}\right)-N_{s} c(e), ~\right)} \tag{34}
\end{equation*}
$$

where the tax rate is endogenously determined by the budget constraint (33). Since $F\left(L_{s}, L_{u}\right)=$ $w_{s} L_{s}+2 w_{u} N_{u}$, welfare is also equal to $\left.[1-\tau /(1-f)] F\left(L_{s}, L_{u}\right)\right)$ and we may restrict our analysis to $\tau<(1-f)$. The threshold ability for studying is now given by (7).

Since the planner can use two instruments, $f$ and $\tau$, and there are two parameters of interest, the education level $e$ and the ability of students as determined by the threshold, there is some hope that the optimum can be achieved. To check whether this is possible, let us consider the optimum levels $e^{*}$ and $y^{*}=y^{F T}\left(e^{*}\right)$. To be implemented, one must find $f$ and $\tau$ for which individuals have incentives such that the threshold equilibrium value $y^{F T}$ is given by $y^{*}$ and the budget constraint (33) is satisfied.

Given $e^{*}$ and $y^{*}$ the budget constraint determines the ratio $\rho=\tau /(1-f)$ (which is smaller than 1). Now consider the expression of $y^{F T}$ as given by (7) where the right hand side is computed at the optimum levels (including the wages) and $\tau=\rho(1-f)$. Using $y^{*}=y^{F T}\left(e^{*}\right)=$ $\frac{1}{w_{s}^{*} e^{*}}\left[2 w_{u}^{*}+c\left(e^{*}\right)\right]$, we have

$$
y^{F T}=y^{*}+\frac{1}{w_{s}^{*} e^{*}}\left[r w_{u}^{*}-c\left(e^{*}\right)+c\left(e^{*}\right) \frac{(1+r) f}{1-\rho(1-f)}\right]
$$

The optimum is implemented for $f$ such that $y^{F T}=y^{*}$, or equivalently for $f$ for which the term in square brackets is null. As expected, for $r=0$, the optimum is reached with pure fee-financing, i.e. $f=1$. Let us assume $r>0$. The term in square brackets increases with $f$ (since $\rho$ is smaller than 1 ), and is positive at $f=1$. Thus the optimum can be reached if $r w_{u}^{*}-c\left(e^{*}\right)<0$, i.e. if the distortion on the credit market is not too high.

If this is not the case, presumably, a pure tax system is the second-best solution with an education level chosen as in the previous section. We can therefore state

Proposition 4 Consider an economy where taxes and tuition fees are available to finance higher education.
With a perfect credit market, $r=0$, pure fee-financing is optimal and the education level is at the optimal level.
With an imperfect credit market, $r>0$, it is optimal to have some share of educational costs financed by taxes. If $r<\frac{c\left(e^{*}\right)}{w_{u}^{*}}$, the optimum can be reached.

The reason for the optimality of mixed-financing with credit constraints is that with pure feefinancing too few individuals decide to study. The welfare can thus be increased by subsidizing higher education via taxes as this encourages more students to study.

Having shown under which conditions the government of a country opts for a system which is closer to fee-financing or tax-financing, we are able to give a rationale for the simultaneous
existence of different ways of financing higher education as long as borders are closed. This will serve as a starting point for the following analysis where we allow for mobility of skilled workers. The question will then be how the policy of one country changes taking the policy (financing system and education level) of the other country into account.

## 4 Opening up economies

We consider two identical countries $A$ and $B .{ }^{9}$ There are $\bar{y}$ individuals in each country. The number of students depends on the decisions of the different ability types to take up a university education in one of the two countries. Thus there are $N_{s}^{A}+N_{s}^{B}=2 \bar{y}-y^{A}-y^{B}$ students in both countries. At given education levels, the labor force of skilled workers in the whole economy is $\bar{L}=L_{s}^{A}+L_{s}^{B}=\left(\bar{y}-y^{A}\right) \frac{\bar{y}+y^{A}}{2} e^{A}+\left(\bar{y}-y^{B}\right) \frac{\bar{y}+y^{B}}{2} e^{B}$ and the labor force of unskilled workers is $L_{u}^{A}+L_{u}^{B}=2\left(y^{A}+y^{B}\right)$.

In both countries the credit market is imperfect and we make the plausible assumption that the distortion is not too high so that $\frac{c(e)}{w_{u}}>r>0$. Higher education can be financed via a mix of tuition fees and wage taxes. From the previous analysis we know that as long as the countries are closed a mix-financing is chosen that achieves the optimal levels of ability thresholds and education. Since countries are identical we have: $y^{A}=y^{B}=y^{*}$ and $e^{A}=e^{B}=e^{*}$. In the next step the countries open up and migration takes place. The question is how the financing of education, the education level and the number of students and skilled workers change if the governments take mobility into account.

### 4.1 Only skilled workers are mobile

We assume that skilled workers are mobile while students and unskilled workers are immobile. Mobile skilled workers will migrate between both countries as long as the net-of-tax wage income is different. Thus the migration equilibrium requires that skilled workers receive the same net wage income in both countries yielding the arbitrage condition

$$
\begin{equation*}
w_{s}^{A}\left(1-\tau^{A}\right)=w_{s}^{B}\left(1-\tau^{B}\right) \tag{35}
\end{equation*}
$$

because not all skilled individuals move to the same country (since we have assumed an Inada condition on the production function).

Let us denote by a superscript the variables in country $x=A, B . y^{x}$ then denotes the threshold level above which young agents study in country $x$. This threshold determines the unskilled labor, $L_{u}^{x}$, in country $x$ since unskilled do not move, the number of students, $N_{s}^{x}$, and the skilled labor force of the native students, $L_{s}^{x}$, which are respectively given by :

$$
\begin{equation*}
L_{u}^{x}=2 y^{x}, N_{s}^{x}=\bar{y}-y^{x}, L_{s}^{x}=e^{x}\left(\frac{\bar{y}^{2}-\left(y^{A}\right)^{2}}{2}\right), x=A, B \tag{36}
\end{equation*}
$$

[^7]The skilled labor force in a country comprises the native graduates and skilled migrants, which can be positive or negative. The migrating skilled labor force is by convention counted from country $B$ to country $A$, and is denoted by $m$. Thus the skilled labor force in country $A$ amounts to $L_{s}^{A}+m$ while the skilled labor force in country $B$ is given by $L_{s}^{B}-m$.

### 4.1.1 Changing the tuition fee

Let us consider the mixed system that implements the optimum. By symmetry, the arbitrage condition (35) for the skilled workers is satisfied. Starting from this situation, we want to show how the welfare in one country, say country $A$, changes, when this country modifies its feefinancing, $f^{A}$ in order to determine whether the optimum is a Nash equilibrium, and if not, in which direction a country is incited to change the fee. To be more precise, A's welfare is given by $F\left(L_{s}^{A}+m, L_{u}^{A}\right)-N_{s}^{A} c\left(e^{*}\right)$, which is evaluated at the new migration equilibrium induced by the new value for the fee: keeping the education level fixed, the equilibrium is determined by values for the thresholds, the taxes and the migration level that satisfy equations (7) and budget balances (33) in the two countries as well as the arbitrage condition (35). In particular a change in a country may induce a change in the tax rate of the other country. To avoid unreasonable levels, we shall assume that there is a maximal tax level $\tau^{\max }$, smaller than 1. Beyond that level, a country changes its policy tools.

A modification of the fee has an impact on welfare through two channels: it modifies the incentives to acquire education within the country and it affects the incentives to migrate. Assume that these variables move smoothly. A marginal change in welfare can be decomposed into the change induced by a marginal change in $y^{A}, d y^{A}$, as if the economy was closed and a change due to a marginal change in migration $d m$. We are thus ultimately interested in

$$
\begin{align*}
& \frac{\partial W^{A}}{\partial f^{A}}=\frac{\partial\left(F\left(L_{s}^{A}+m, L_{u}^{A}\right)-N_{s}^{A} c\left(e^{A}\right)\right)}{\partial f^{A}}=  \tag{37}\\
= & {\left[-w_{s}^{A} e^{A} y^{A}+2 w_{u}^{A}+c\left(e^{A}\right)\right] \frac{\partial y^{A}}{\partial f^{A}}+w_{s}^{A} \frac{\partial m}{\partial f^{A}} }
\end{align*}
$$

At the optimum, the term in bracket is zero: the impact of a policy change within a country is null at the margin. Hence welfare is raised if $d m>0$, i.e. if migration from $B$ to $A$ increases, and is decreased otherwise.

Proposition 5 If only skilled workers are mobile both countries have an incentive to increase fee-financing above the level necessary to achieve the optimum.

Proof. See the proof in Section 6.2.
Here is a sketch of the proof. We show that by increasing its fee, country $A$ spends less on education and decreases its tax, thereby attracting skilled workers from $B$ : country $A$ free rides on country $B$. To understand why, consider first what happens if $A$ increases its fee in a closed economy without migration. The result, as shown in the proof, is that $A$ spends less on education, decreases its tax and has fewer students. (Without equilibrium effect this is obvious: an increase in the fee eases the budget because the spending per student is diminished and fewer individuals want to study. There are however two countervailing effects: the ratio of the skilled
wage to the unskilled one increases and the tax decreases, both of which incite more people to study. These effects however do not offset the first one.) Thus the net skilled wage increases in $A$, and becomes larger than in $B$.

Now open the economy. Since the net skilled wage of country $A$ is larger than that of $B$, skilled workers start to migrate from $B$ to $A$. Although the gap between the net skilled wages may not be decreasing with all levels of $m$, eventually a level is reached that equalizes the net skilled wage, or the maximum tax level is reached in $B$.

Remark. Starting from a net skilled wage larger in closed country $A$ than in $B$ and opening the economies, net skilled wages might not be monotone in $m$. Accounting for equilibrium effects, the impact on the net skilled wages in both countries of an increase of skilled labor in $A$ due to migration is threefold: through the exogenous increase in skilled labor in $A$ and decrease in $B$, through the impact on the ability thresholds due the change in equilibrium wages, and through the impact on wages and taxes. Consider $A$ for instance. An exogenous increase in skilled labor induces a decrease in the skilled wage and a decrease in the tax level because there are more taxpayers without additional cost. It can be shown that this creates a disincentive to acquire education: fewer people become educated, thereby allowing to decrease the tax even further. ${ }^{10}$ Because of these two opposite effects, the net skilled wage in $A$ may decrease or increase, and this may depend on $m$ (of course it also depends on the elasticity of substitution). The fact that net skilled wages are not monotone suggests that, given some level of the fees in $A$ and in $B$, there may be multiple equilibria associated with distinct tax rates and migration levels in opened economies. Migration creates some spillover effects due to the tax decrease. Multiplicity does not occur under a 'stability' condition. This condition bears on the impact of migration of skilled labor on the net skilled wage in a country. To emphasize the various dependencies, let us write the net skilled wage in country $x, x=A, B$, as

$$
w_{s}^{x}\left(L_{s}^{x}+m, L_{U}^{x}\right)\left(1-\tau^{x}\left(L_{s}^{x}+m, L_{u}^{x}\right)\right) .
$$

where $w_{s}^{x}, L_{s}^{x}$ and $L_{u}^{x}$ are the equilibrium levels associated with the threshold level when the country experiences the extra amount $m$ of skilled labor (which can be positive or negative). Stability requires the net skilled wage to decrease with migration. Under this condition, starting from a net skilled wage larger in closed country $A$ than in $B$, skilled workers from $B$ migrate towards $A$ and the gap between the skilled net wage in the two countries diminishes, and equilibrium is unique.

### 4.1.2 Changing the educational level

We consider now how the educational level is influenced by the mobility of skilled workers. The educational level in a country, say $A$ will be chosen so as to maximize welfare measured by $F\left(L_{s}^{A}+m, L_{u}^{A}\right)-N_{s}^{A} c\left(e^{A}\right)$ in which $m$ is the migration of skilled labor. Starting from the optimum, arguing as before, the impact of a marginal policy change within a country is null. Hence the sign of a welfare change depends on the sign of migration: a country has incentives to increase the quality level if this attracts skilled workers from the other country, that is if $\frac{\partial m}{\partial e^{A}}$ is positive (assuming differentiability), and to decrease the level if it is negative.

[^8]Let us concentrate on a symmetric equilibrium (in which case no migration takes place). Assuming variables to be differentiable, the first order condition of maximization of welfare, $F\left(L_{s}^{A}+m, L_{u}^{A}\right)-N_{s}^{A} c\left(e^{A}\right)$ for $A$ writes

$$
\begin{equation*}
\left[-w_{s}^{A} e^{A} y^{A}+2 w_{u}^{A}+c\left(e^{A}\right)\right] \frac{\partial y^{A}}{\partial e^{A}}+w_{s}^{A} \frac{\partial m}{\partial e^{A}}=0 \tag{38}
\end{equation*}
$$

The term in bracket is the impact of a policy change within a country on welfare. At the optimum, it is null and the welfare change depends on the sign of $\frac{\partial m}{\partial e^{A}}$ as we have seen.) At a Nash equilibrium, the first order condition gives that the educational choice is suboptimal: it is too low if an increase would incite skilled workers to migrate to the other country (i.e., if $\frac{\partial m}{\partial e^{A}}<0$ ) and too high in the other case.

Arguing as in the previous section, it suffices to consider what happens to the net wage rate of the skilled when a closed economy modifies its educational level. If decreasing the educational level increases the net wage rate of the skilled, this attracts skilled workers from the other country when economies are open. Whether this is true depends on complex effects. We have seen how the quality level of education affects the ability threshold when the financing policy $(f, \tau)$ is kept fixed. When the system is not a pure fee system, a change in quality calls for a change in the financing policy so as to balance the budget, which in turn will have an impact on the ability threshold and the wages.

In most situations however, the net skilled wage is likely to decrease when $e$ is increased. Consider first a fee-financing system. The budget is automatically balanced so that there is no impact on the tax rate. Lemma 1 gives how the level of education quality affects the ability threshold. When the elasticity of substitution between unskilled and skilled labor is large and the marginal cost of education does not increase too much, then increasing $e$ increases the number of students: the skilled wage surely decreases. At the opposite, when the elasticity of substitution is small enough (for example smaller than 1) then increasing $e$ always decreases the number of students whatever the cost function or the financing policy. How the skilled wage reacts then depends on whether this induces an overall decrease in skilled labor, that is whether the increase in the ability threshold is steep enough, which in turn is affected by the cost elasticity.

Next consider the case when the system is not a pure fee-financed system. Budget balance then implies that the tax moves in the same direction as the total costs of education if we assume that fees remain unchanged. In particular, the tax rate increases with $e$ if the number of students increases or does not decrease too much. As for the skilled wage, since we are assuming that $e$ is increasing, it can increase only if the number of those who decide to study decreases fast, that is if the ability threshold is sufficiently steep. Hence overall, the question of whether the net skilled wage decreases with $e$ is largely determined by the steepness of the ability threshold. When this ability is decreasing with $e$, we are sure that the net skilled wage decreases with $e$. When the system is a pure tax-financed system, the threshold behavior only depends on wages, more precisely it is completely determined by the size of $\sigma$ relative to 1 . We are therefore sure that net skilled labor decreases when $e$ increases if $\sigma$ is larger than 1 .

When we have a mixed system, a difficulty is that the tax rate interacts with the ability threshold: an increase for example in the tax rate due to an increase in educational costs has a countervailing effect on the ability threshold since it tends to make studies less attractive. For
the following proposition we make an assumption which is slightly stronger than the condition in Lemma 1. It ensures that the overall impact of an increase in quality on the ability threshold is still negative. Observe however that this is far from being necessary for the net skilled wage to decrease with $e$.

## Assumption 2

$$
\frac{R-\frac{c^{\prime}(e) e}{c(e)}\left[\frac{\tau\left(1-f^{*}-\tau^{*}\right)}{(1-\tau)(1-f)}+1\right]+\eta_{w_{S}, e}}{R-1}>\frac{1}{\sigma}
$$

Thus we can state
Proposition 6 Assume Assumption 2 holds. If only skilled workers are mobile both countries will decrease the educational level below the optimum.

Proof. See the proof in Section 6.
Hence, under the assumption stated in the proposition, if the educational level is decreased fewer individuals decide to study which increases the net wage rate of the skilled. This attracts skilled workers from the other country who increase the net production. Due to symmetry, both countries thus have an incentive to attract foreign skilled mobile workers as tax-payers while at least partially - free-riding on the other country's provision of education.

### 4.2 Skilled workers and students are mobile

Individuals who decide to study and skilled workers can migrate between countries without costs at both periods of their life. Furthermore, in line with EU rules, students have access to the education system of a foreign country at the same conditions as natives. Graduates are assumed to be (partially) mobile as well. We thus allow for some non-perfect link between student and graduate mobility following, e.g., Parey and Waldinger (2007) who provide evidence that mobile students are more likely to work abroad after graduation.

### 4.2.1 Welfare criterion

We first need to make precise the criterion used by a country. We choose here a criterion associated to the welfare of individuals who work in a country. In particular, a student who goes abroad and does not come back is counted in the resident country. Without imperfection on the credit markets, the sum of the lifetime income of all workers in $A$ is

$$
\begin{equation*}
W=F^{A}-N_{A A} c\left(e^{A}\right)-\left(1-f^{A}\right) N_{A B} c\left(e^{A}\right)-f^{B} N_{B A} c\left(e^{B}\right) \tag{39}
\end{equation*}
$$

where $F^{A}$ is the value of production in $A$ and $N_{I J}$ is the number of skilled individuals who study in $I$ and work in $J$. Using the government budget equation

$$
\left(1-f^{A}\right)\left[N_{A A}+N_{A B}\right] c\left(e^{A}\right)=\tau^{A} F^{A}
$$

welfare can also be written as

$$
\begin{equation*}
W=F^{A}-\left[\tau^{A} F_{A}+f^{A} N_{A A} c\left(e^{A}\right)+f^{B} N_{B A} c\left(e^{B}\right)\right] \tag{40}
\end{equation*}
$$

The objective is the value of production in the country less the cost of education supported directly or indirectly by the workers in $A: N_{A A} c\left(e^{A}\right)$ does not deserve comment, $\left(1-f^{A}\right) N_{A B} c\left(e^{A}\right)$ represents the part supported by the workers of $A$ (financed through taxes) of the cost of education for the native who will migrate to $B$, and $f^{B} N_{B A} c\left(e^{B}\right)$ is the cost paid by the workers who have studied abroad through their fees (the remaining part being paid by the tax payers of country $B$ ).

### 4.2.2 Individuals decision

In both countries, in the first period of their life, young individuals not only have to decide whether to study but also where to study and, after having acquired education, they have to decide where to work. They compare their net life-time incomes for all possible education and migration choices. The migration equilibrium for skilled workers still implies the arbitrage condition (35) according to which net wages are equalized: ${ }^{11}$

$$
w_{s}^{A}\left(1-\tau^{A}\right)=w_{s}^{B}\left(1-\tau^{B}\right)
$$

so that skilled workers are indifferent between the two countries.
Now, individuals who decide to study compare their net lifetime income if studying in country $A$ or in country $B$, that is they consider the difference

$$
\begin{equation*}
\left[\left(1-\tau^{A}\right) e^{A} w_{s}^{A} y-(1+r) f^{A} c\left(e^{A}\right)\right]-\left[\left(1-\tau^{B}\right) e^{B} w_{s}^{B} y-(1+r) f^{B} c\left(e^{B}\right)\right] \tag{41}
\end{equation*}
$$

A $y$-student for which this difference is positive (resp. negative) studies in $A$ (resp. in $B$ ). In addition, she decides to study by considering her net lifetime income as an unskilled worker. Since unskilled workers remain immobile, this income depends on the home country, equal to $\left(1-\tau^{A}\right) w_{u}^{A}(2+r)$ or $\left(1-\tau^{B}\right) w_{u}^{B}(2+r)$.

Expression (41) is linear in $y$, giving rise to three possibilities: either (a) individuals are all indifferent between studying in $A$ or in $B$ or (b) they split according to a threshold ability, or (c) they all prefer to study in the same country.

Case (a) holds when the slopes and the intercepts are equal. By the arbitrage condition (35), the net skilled wages are equalized, hence it holds if (and only if)

$$
\begin{equation*}
e^{A}=e^{B}, f^{A}=f^{B} \tag{42}
\end{equation*}
$$

### 4.2.3 Changing the fees

Let us start again from the symmetric mixed-financed system which is optimal for a closed economy. We first consider the incentives to change the fees. Let the educational quality level be kept unchanged in both countries. Under imperfect credit markets, the system is mixed. Let a country, say $A$, contemplate increasing its fee. Only fees matter as by the arbitrage condition the net skilled wages are equalized. It follows that we are in case (c): all individuals will study in country $B$ if they decide to study, and the cost of education in country $A$ falls to zero, allowing for a null tax rate: there is free riding. These changes are far from being marginal, so that we

[^9]cannot argue as previously by looking at small deviations. Furthermore, $B$ may be unable to educate all these students without increasing its fee. We investigate these points.

Consider welfare. At the initial situation, welfare is equal to

$$
\begin{equation*}
F^{*}-N_{s} c\left(e^{*}\right)=\left(1-\tau^{*}\right) F^{*}-f^{*} N_{s} c\left(e^{*}\right) . \tag{43}
\end{equation*}
$$

After the change in fees, since there are no students in $A$, and $f^{B}=f^{*}$, welfare in country $A$ is given by

$$
\begin{equation*}
W^{A}=F^{A}-f^{*} N_{B A} c\left(e^{*}\right) \tag{44}
\end{equation*}
$$

where $F^{A}$ is the production level associated to the new allocation of workers in the two economies. All skilled workers in $A$ study abroad, hence their number is $N_{B A}$. Comparing with the initial expression (43), it is as if the cost of education in $A$ per student was reduced to $f^{*} c\left(e^{*}\right)$ or alternatively as if the tax could be set to zero and still satisfy budget balance.

As a result of the large inflow of students, country $B$ would then have to change drastically its financing policy. It would have to increase its tax rate, or to increase its fees if the maximum admissible tax level is reached. We discuss both cases.

When the system is mostly financed by taxes, it is likely it cannot absorb the additional costs of education by a sole increase in taxes. Thus, country $B$ would have to increase fees up to the same level as in country $A$. This would lead back to the same financing policy with the same number of students in both countries. Higher education, however, would now be financed by a sub-optimal mixture of fees and taxes. Anticipating the reaction of country $B$, country $A$ abstains from increasing the level of fees in the first place. A symmetric equilibrium then results where the optimal finance mix of the closed economy can be sustained.

In the opposite case where the system is mostly financed by fees, we cannot exclude that country $B$ is able to educate the inflow of students because the taxes are small anyway (To understand better this point, consider the extreme case of a pure fee system. Where students study has no impact at all on government budgets nor on welfare.) Country $A$ is likely to benefit from its deviation: the production would possibly decrease a little bit but the cost of education would drop. We shall argue however that it is unlikely that $B$ does not react by increasing its fee as well.

First observe that by the arbitrage condition and the fact that the tax rate in $A$ is null, the skilled wage in $A$ is smaller than in $B$. Hence, by the homogeneity of the production funtion, the ratio of skilled labor to unskilled labor and the unskilled wage are larger in $A$ than in $B$. This gives the following inequalities:

$$
w_{s}^{A}<w_{s}^{B}, w_{u}^{A}>w_{u}^{B}, \text { and } \frac{L_{s}^{A}}{L_{u}^{A}}>\frac{L_{s}^{B}}{L_{u}^{B}} .
$$

It follows that the net unskilled wage income in $A$ is surely larger than in $B: w_{u}^{A}>\left(1-\tau^{B}\right) w_{u}^{B}$. As a result, natives in $A$ have less incentive to study, and since unskilled do not move, we have $L_{u}^{A}>L_{u}^{B}$. But then the ratio of skilled to unskilled labor can be larger in $A$ than in $B$ only if there is a migration of skilled workers born in $B$ that is sufficiently massive. We think that it is likely that $B$ will react by increasing its fee as well.

### 4.2.4 Changing education

What happens when the education levels can differ across the countries? From the previous argument, a migration equilibrium with different levels of fees and taxes in the two countries can only realise if the quality level of education in the country, say $A$, which increases its fees exceeds the one in country $B$. Then country $A$ specialises in attracting high-ability students while country $B$ focuses on low-ability ones. Whether this constitutes an equilibrium when general equilibrium effects are taken into account, depends on the specific functional forms. The relative importance of student and graduate mobility will be essential for the financial regime and the quality level of education.

## 5 Conclusion

Countries increase their output by employing highly productive skilled workers. And they may benefit from letting those workers graduate in other countries, thus saving on the educational costs. In integrated labor markets where skilled workers are mobile the participating countries can pursue such a policy. The problem of such a policy is that in equilibrium all countries may end up in subsidizing higher education too little by relying too much on tuition fees and providing a suboptimal educational quality. This is the central message of our paper.

Starting with the optimal higher education policy in a closed country without mobility we show that the efficient way of financing higher education is to let students pay tuition fees refraining from state subsidies financed by taxes. If higher education is understood as a more or less private good with negligible external effects and if credit markets work perfectly competitive pure fee-financing is the optimal policy. However, we have reservations with respect to the second assumption: are credit marktes for investments in human capital really competitive? There are several reasons why not which we have discussed in detail in the paper. Hence, we analyze the case of imperfect credit markets and find that a mix of tuition fees and tax-financed subsidies can indeed restore the optimal allocation, i.e. a governmental policy can achieve the optimal number of students and the optimal educational level even in the presence of imperfect credit markets. This mix-financing of higher education is indeed what we observe in all countries.

We take this optimal mix-financing scheme in two identical countries with imperfect credit markets as the outset for analyzing the impact of the mobility of skilled workers on the higher education systems. Against this benchmark we show that if countries open up their borders governments have an incentive to decrease education spending, either by decreasing quality or by increasing fees, so as to increase net skilled wages and attract skilled labor. Hence, mobility generates distortions of the optimal governmental policy. The reason is that countries free-ride on the higher education provided by other countries and therefore reduce the subsidies and the educational quality to inefficiently low levels.

This analysis focuses on the mobility of post-graduate skilled workers. Allowing also for mobility of students may intensify the fee competition and enforce the incentives to increase the fees. Now in addition to attracting skilled worker, the policy also deters students to study within the country which helps to save educational costs. The case of free-riding on the education provided by other countries may be strengthened. However, countries may anticipate this race
which results in equilibrium in the same number of students in each country. Hence, countries may abstain from engaging in fee competition in the first place.

We have restricted our analysis to symmetric countries. If we drop this assumption and allow for different policies in both countries an asymmetric equilibrium could result with differentiated quality levels. Countries may specialize their educational systems by either providing high educational quality at high costs and attracting high ability students or providing low educational quality at low costs and being satisfied with educating low ability students. From an overall welfare point of view, differentiation might be beneficial as it relaxes the constraint of a single quality level of education per country. This extension is left for future research.

## References

[1] Busch, O. (2007): "A Bird in the Land is Worth Two Abroad", mimeo, University of Frankfurt.
[2] Creedy, J. and P. Francois (1990): "Financing Higher Education and Majority Voting", Journal of Public Economics 24, 181-200.
[3] Del Rey, E. (2001): "Economic Integration and Public Provision of Education", Empirica 28, 203-218.
[4] Demange, G., R. Fenge and S. Uebelmesser (2008): "The Provision of Higher Education in a Global World - Analysis and Policy Implications", CESifo Economic Studies 54, 248-276.
[5] Ehrenberg, R.G. and R.S. Smith (1993): Modern Labor Economics: Theory and Public Policy, Harper Collins: New York.
[6] Gale, D. (1973): "Pure Exchange Equilibrium of Dynamic Economic Models", Journal of Economic Theory 6, 12-36.
[7] García-Penalosa, C. and K. Wälde (2000): "Efficiency and Equity Effects of Subsidies to Higher Education", Oxford Economic Papers 52, 702-722.
[8] Johnson, G.E. (1984): "Subsidies for Higher Education", Journal of Labor Economics 2, 303-318.
[9] Kemnitz, A. (2005): "Educational Federalism and the Quality Effects of Tuition Fees", mimeo.
[10] Mechtenberg, L. and R. Strausz (2008): "The Bologna Process: How student mobility affects multi-cultural skills and educational quality ", International Tax and Public Finance 15, 109-130.
[11] Parey, M. and F. Waldinger (2007): "Studying Abroad and the Effect on International Labor Market Mobility", presentation at the CESifo Workshop on "Innovation and Higher Education Reform", CESifo Venice Summer Institute.
[12] Poutvaara, P. (2004): "Educating Europe: Should Public Education be Financed with Graduate Taxes or Income-Contingent Loans?", CESifo Economic Studies 50, 663-684.
[13] Thum, C. and S. Uebelmesser (2003): "Mobility and the Role of Education as a Commitment Device", International Tax and Public Finance 10, 549-564.
[14] Wigger, B.U. and R.K. von Weizsäcker (2001): "Risk, Resources and Education", IMF Staff Papers 48, 547-560.
[15] Wildasin, D.E. (2000): "Labor-Market integration, investment in risky human capital and fiscal competition", American Economic Review, 90, 1, pp 73-95.

## 6 Proofs

### 6.1 Threshold behaviour

We gather some useful properties on the behavior of the ability threshold at an equilibrium of the labor market, given $(f, \tau, e)$. Denote by $\Delta(y, e)$ the function defined for $y$ in $] 0, \bar{y}[$.

$$
\begin{equation*}
\Delta(y, e)=\frac{w_{u}(y, e)(2+r)+(1+r) f /(1-\tau) c(e)}{w_{s}(y, e) e} \tag{45}
\end{equation*}
$$

where $w_{u}(y, e)$ and $w_{s}(y, e)$ are the equilibrium wages if labor quantities are given by (10) and (11). According to (12), an interior equilibrium solves $y-\Delta(y, e)=0$. It is useful to observe that function $\Delta$ decreases with $y$ : As $y$ increases, there are less skilled workers, hence the unskilled wage $w_{u}(y, e)$ decreases and the skilled one $w_{s}(y, e)$ increases.

We first show in point (a) that there is a unique solution, and then derive some comparative statics result about the threshold behavior $y^{F T}(e)$.
(a) There is a unique equilibrium

First note that $\Delta(., e)$ changes sign when $y$ runs over $] 0, \bar{y}[$ : thanks to the Inada conditions on productivities we have $\lim _{y \rightarrow 0} \Delta(y, e)>0$ and $\lim _{y \rightarrow \bar{y}} \Delta(y, e)<0$. Since the function $y \rightarrow y-\Delta(y, e)$ increases in $y$, it follows that there is a unique solution $y^{F T}$ to $y-\Delta(y, e)=0$.

Comparative statics results now are obtained by noticing that the unique solution to $y-$ $\Delta(y, e)=0$ increases (resp. decreaes) with a parameter if the function $\Delta$ is increasing (resp. decreasing) ) with that parameter.
(b) $y^{F T}(e)$ increases with $e$ if $\frac{w_{u}(y, e)}{e \cdot w_{s}(y, e)}$ is non decreasing in $e$.

It suffices to show that $\Delta$ increases with $e$. The term $\frac{c(e)}{e . w_{s}(y, e)}$ is surely increasing because average cost increases and $w_{s}(y, e)$ decreases with $e$. The first term $\frac{w_{u}(y, e)(2+r)}{w_{s}(y, e) e}$ is increasing by assumption. Note that for a tax country, $f=0$, the result can be strengthened as:
(c) $y^{T}(e)$ varies as the ratio $\frac{w_{u}(e)}{w_{s}(e) \cdot e}$.
$y^{T}(e)$ increases if the ratio increases with $e$, that is under Assumption 2, and decreases (resp. is constant) if the ratio decreases (resp. is constant) with $e$. With a Cobb Douglas function, the ratio does not depend on $e$ : this explains why $y^{T}$ is constant in that case.
(d) The threshold $y^{F T}(e)$ increases with $f /(1-\tau)$ and with $r$.

It suffices to check that $\Delta(y, e)$ increases with $f /(1-\tau)$ and with $r$.

### 6.2 Proof of proposition 5

Start with the mixed system that implements the optimum. The gap in net skilled wage is

$$
G a p=w_{S}^{A}\left(L_{S}^{A}+m, L_{U}^{A}\right)\left(1-\tau^{A}\left(L_{S}^{A}+m, L_{U}^{A}\right)\right)-w_{S}^{B}\left(L_{S}^{B}-m, L_{U}^{B}\right)\left(1-\tau^{B}\left(L_{S}^{B}-m, L_{U}^{B}\right)\right)
$$

The arbitrage condition writes as $G a p=0$. It is satisfied at the initial situation. The proof is divided into two steps.

Step 1. We first consider a closed economy and show that an increase in the fee induces an increase in the net skilled wage. Consider $A$ for instance. Following a change in the fee $d f^{A}$, the tax rate varies so as to balance the budget, accounting for the impact to acquire education, i.e. the impact on the ability threshold. Denoting the variation in the government budget, the tax rate, and the ability threshold respectively by $d B, d \tau^{A}$, and $d y^{A}$ we have

$$
d B=d \tau^{A} F^{A}+\left[\tau^{*}\left(2 w_{u}^{A}-w_{s}^{A} e y^{A}\right)+\left(1-f^{*}\right) c(e)\right] d y^{A}+d f^{A} N_{s}^{A} c(e)
$$

The tax varies so as to balance the budget, that is so as to satisfy $d B=0$. The term multiplying $d y^{A}$ is the net effect on the budget of a marginal increase in the ability threshold: it is equal to the change in the amount of taxes collected on the marginal skilled worker plus the saving on educational cost. At the optimum, this net effect is positive: ${ }^{12}$ using that $\left[w_{s}^{A} e y^{A}-2 w_{u}^{A}-c(e)\right]=$ 0 , we have $\left[\tau^{*}\left(2 w_{u}^{A}-w_{s}^{A} e y^{A}\right)+\left(1-f^{*}\right) c(e)\right]=\left(1-f^{*}-\tau^{*}\right) c(e)$. Thus the budget variation is

$$
\begin{equation*}
d B=d \tau^{A} F+\left(1-f^{*}-\tau^{*}\right) c(e) d y^{A}+d f^{A} N_{s}^{*} c(e) . \tag{46}
\end{equation*}
$$

We show that an increase in the fee, $d f^{A}>0$ induces $d y^{A}$ to be positive (less students) and $d \tau^{A}$ be negative (lower tax rate). By contradiction, let us assume $d y^{A}$ to be negative. From property (d) of proved in Section 6, the threshold decreases only if the ratio $f /(1-\tau)$ decreases. Using the budget equation $d B=0, d y^{A}<0$ implies $d \tau^{A} F+d f^{A} N_{s}^{*} c(e)>0$. Since $F>N_{s}^{*} c(e)$ and $d f^{A}>0$ we therefore have $d \tau^{A}+d f^{A}>0$ : the possible decrease in the tax rate is smaller than the increase in the fee. This implies that $f /(1-\tau)$ increases :

$$
\frac{d f}{f}+\frac{d \tau}{1-\tau} \geq d f\left[\frac{1}{f}-\frac{1}{1-\tau}\right]
$$

is positive since $1-\tau^{*}-f^{*}>0$. This gives the contradiction: $d y^{A}$ is positive. Then $d B=0$ implies that the tax rate decreases.

Since there are less skilled workers and the tax rate diminishes, the net skilled wage in $A$ increases. In other words, taking $m=0, f^{A}>f^{*}$ implies Gap $>0$.

Step 2. Now open the economy. All skilled workers in $B$ would like to migrate to $A$. Start to increase $m$. As long as the net skilled wage in $A$ is larger than that in $B$, let the threshold $y^{B}$ be determined by taking the net skilled wage to be that in $A$. In particular, at the opening of the economies, there is a discontinuous increase in the number of students in $B$ (i.e. a jump downwards in $y^{B}$ ). The tax rate is adjusted so as to satisfy the budget constraint in country $B: \tau^{B} F^{B} \geq\left(1-f^{*}\right) N_{s}^{B} c(e)=\left(1-f^{*}\right) c(e)\left(\bar{y}-y^{B}\right)$. Two restrictions have to be taken into account: migration $m$ must be smaller than the labor force that is educated in $B$, i.e., $m \leq L_{s}^{B}$ and the tax rate must be smaller than some level $\tau^{\max }<1$ (if this maximum is 1 , the whole production in $B$ is used to cover the education costs).

Thus $m$ is increased until either (a) the arbitrage condition is fulfilled or (b) $\tau^{B}$ reaches $\tau^{\max }=1$ or (c) every skilled migrates to $A$, that is $m$ is equal to $L_{s}^{B}$. In case (a), we are done: country $A$ attracts some skilled workers for free. In case (b) the tax rate reaches its maximum:

[^10]country $B$ stops educating all the students who want to become skilled so as to migrate to country $A$. Country $A$ still attracts some skilled workers for free. Case (c) never occurs before either (a) or (b) occurs: by contradiction, if it would, the skilled wage in country $B$ becomes arbitrarily large so that the net skilled wage in $B$ is surely larger than in $A$ (because $\tau^{B}$ is strictly smaller than 1 ) so that (a) has been fulfilled before.

### 6.3 Proof of proposition 6

Start with the mixed system that implements the optimum. Define the gap in net skilled wage Gap by

$$
G a p=w_{S}^{A}\left(L_{S}^{A}+m, L_{U}^{A}\right)\left(1-\tau^{A}\left(L_{S}^{A}+m, L_{U}^{A}\right)\right)-w_{S}^{B}\left(L_{S}^{B}-m, L_{U}^{B}\right)\left(1-\tau^{B}\left(L_{S}^{B}-m, L_{U}^{B}\right)\right)
$$

Note that the arbitrage condition, $G a p=0$, is satisfied at the initial situation. We consider that country $A$ decreases its educational level in a closed economy and show that the net skilled wage increases.

At the optimum, we have $d F=d\left(c\left(e^{*}\right) N_{s}^{*}\right)$, whatever marginal variation in $e$ or $y$. Since the budget writes $\left.B=\tau F-\left(1-f^{*}\right) c(e) N_{s}\right)$, we obtain that $d B=d \tau^{A} F-\left(1-f^{*}-\tau^{*}\right) d\left(c\left(e^{*}\right) N_{s}^{*}\right)$ and budget balance requires

$$
d B=d \tau^{A} F+\left(1-f^{*}-\tau^{*}\right)\left[-c^{\prime}\left(e^{*}\right) N_{s}^{*} d e^{A}+c(e *) d y^{A}\right]=0
$$

Hence budget balance implies that the tax moves in the same direction as the total costs of education. In particular, it increases with $e$ if either the number of students increase ( $\frac{\partial y^{A}}{\partial e}$ negative) or does not decrease too much. Since $d y^{A}$ is affected both by the change in the tax and the educational level, $d B=0$ finally writes

$$
\begin{equation*}
d \tau^{A}\left[F+\left(1-f^{*}-\tau^{*}\right) c(e *) \frac{\partial y^{A}}{\partial \tau}\right]-d e^{A}\left(1-f^{*}-\tau^{*}\right)\left[c^{\prime}\left(e^{*}\right) N_{s}^{*}-c(e *) \frac{\partial y^{A}}{\partial e}\right]=0 \tag{47}
\end{equation*}
$$

The total effect on the threshold $y^{A}$ of decreasing the educational level is :

$$
\frac{d y^{A}}{d e}=\frac{\partial y^{A}}{\partial e}+\frac{\partial y^{A}}{\partial \tau} \frac{d \tau^{A}}{d e^{A}}
$$

Using (47) this total derivative can be written as

$$
\begin{aligned}
\frac{d y^{A}}{d e} & =\frac{\partial y^{A}}{\partial e}+\frac{\partial y^{A}}{\partial \tau}\left[\frac{\left(1-f^{*}-\tau^{*}\right)\left(c^{\prime}\left(e^{A}\right) N_{s}^{A}-c(e) \frac{\partial y^{A}}{\partial e}\right)}{F+\left(1-f^{*}-\tau^{*}\right) c(e) \frac{\partial y^{A}}{\partial \tau}}\right] \\
& =\frac{\frac{\partial y^{A}}{\partial e} F+\frac{\partial y^{A}}{\partial \tau}\left(1-f^{*}-\tau^{*}\right) c^{\prime}\left(e^{A}\right) N_{s}^{A}}{F+\left(1-f^{*}-\tau^{*}\right) c(e) \frac{\partial y^{A}}{\partial \tau}}
\end{aligned}
$$

Since the denominator is positive, the total derivative is negative if and only if the numerator is negative:

$$
\begin{equation*}
-\frac{\frac{\partial y^{A}}{\partial \tau}}{\frac{\partial y^{A}}{\partial e}}<\frac{F}{\left(1-f^{*}-\tau^{*}\right) c^{\prime}\left(e^{A}\right) N_{s}^{A}} \tag{48}
\end{equation*}
$$

Using the budget equation $F=\frac{1-f}{\tau} N_{s} c(e)$ and the terms of the partial derivatives:

$$
\begin{aligned}
\frac{\partial y^{A}}{\partial e} & =-\frac{w_{S} y+y e \frac{\partial w_{S}}{\partial e}-(1+r) \frac{f}{1-\tau} c^{\prime}(e)-(2+r) \frac{\partial w_{U}}{\partial e}}{w_{S} e+y e \frac{\partial w_{S}}{\partial y}-(2+r) \frac{\partial w_{U}}{\partial y}} \\
\frac{\partial y^{A}}{\partial \tau} & =\frac{(1+r) \frac{f}{(1-\tau)^{2}} c(e)}{w_{S} e+y e \frac{\partial w_{S}}{\partial y}-(2+r) \frac{\partial w_{U}}{\partial y}}
\end{aligned}
$$

the inequality (48) holds if and only if

$$
\frac{1}{\sigma}<\frac{R-\frac{c^{\prime}(e) e}{c(e)}\left[\frac{\tau\left(1-f^{*}-\tau^{*}\right)}{(1-\tau)(1-f)}+1\right]+\eta_{w_{S}, e}}{R-1}
$$

Given this inequality holds which is stronger than our assumption in lemma 1 and implies also the partial negative relationship between the threshold and the educational level, the total relationship between the threshold and the educational level including the indirect effect via the tax is also negative. Thus, since there are less skilled workers and the tax rate diminishes, the net skilled wage in $A$ increases. Taking $m=0, G>0$ if $e^{A}<e^{*}$. Now open the economy. The same argument as the one used for the previous proposition applies.

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[^0]:    ${ }^{1}$ The incentives for a government or an old generation to invest in internationally applicable education are analyzed in Thum and Uebelmesser (2003) and Poutvaara (2004). These questions will, however, not be included in the analysis here.

[^1]:    ${ }^{2}$ We abstain here from explicitly considering capital in the production technology. Taking the effect of education on capital into account would be interesting, but it is outside the scope of the present paper.
    ${ }^{3}$ This corresponds to empirical evidence according to which mobility increases with education. See, e.g., Ehrenberg and Smith (1993).

[^2]:    ${ }^{4}$ Education is thus considered here as a private good.

[^3]:    ${ }^{5}$ As we have just seen, increasing the threshold ability $y$ means that fewer workers attain a skilled level which raises the wage rate for skilled and decreases the wage rate for unskilled.

[^4]:    ${ }^{6}$ This can be seen from:

    $$
    \frac{\partial w_{S}}{\partial e}=\frac{f_{l l}}{L_{u}}\left(\frac{\bar{y}^{2}-\left(y^{F T}\right)^{2}}{2}\right)<0 \text { and } \frac{\partial w_{U}}{\partial e}=-l \frac{f_{l l}}{L_{u}}\left(\frac{\bar{y}^{2}-\left(y^{F T}\right)^{2}}{2}\right)>0
    $$

[^5]:    ${ }^{7}$ If the cost is linear, one has to assume that education levels are bounded: from (20) with $c^{\prime}(e)=1$, if the return to education is positive for $y$, then education should be as large as possible: the optimal level is at the upper bound.

[^6]:    ${ }^{8} y^{T}(e)$ solves $y-\frac{w_{u}(y, e)(2+r)}{w_{s}(y, e) e}=0$.(see (9)). The left-hand side is an increasing function of $y$ (see the section proof), hence $y^{T}\left(e^{*}\right) \leq y^{*}$ iff the left-hand side is larger than 0 for $y=y^{*}$ and $e=e^{*}$ which is equivalent to $-r w_{u}^{*}+c\left(e^{*}\right) \geq 0$.

[^7]:    ${ }^{9}$ A generalization to $n$ countries is straightforward but would complicate unnecessarily the notation.

[^8]:    ${ }^{10}$ Decreasing the number of skilled lowers the cost of education by $f c(e) d N$ and the tax revenues by $\tau d F$. The former effect outweighs the latter so that a surplus is generated: the tax rate decreases.

[^9]:    ${ }^{11}$ For the arbitrage condition to hold, it suffices to assume that part of the skilled workers are mobile.

[^10]:    ${ }^{12}$ The argument below works as long as the net effect is positive.

