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Teaching DSGE models to undergraduates

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Abstract

This paper puts forward a systematic approach to teaching simple dynamic stochastic general equilibrium (DSGE) models to undergraduates. It proceeds in the following way: first, the structural model of the economy, which includes the households' and firms' problems, is presented and progressively solved. We then find the steady state and log-linearized equations of the model. Next, a productivity shock is simulated on the computer so as to tell a “story” about how the economy behaves. Finally, the model is extended by including the government and the foreign sector. We reckon instructors of macroeconomics will find this path useful in teaching their undergraduate students the basics of DSGE models.

JEL classification: A22; E32

Keywords: Dynamic stochastic general equilibrium (DSGE) modeling; Undergraduate macroeconomics; Real Business Cycle model

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1. Introduction

Dynamic stochastic general equilibrium (DSGE) modeling is a complex branch of macroeconomics. To be sure, DSGE models are generally considered to be time- and effort-consuming, which may discourage many beginners from embarking on the study of this technique. On the other hand, this type of models is viewed as a fairly good representation of how a market economy works and is accordingly used by academic researchers and professional economists working at institutions in an extensive manner. However, undergraduate macroeconomics tends to draw more on traditional static deterministic approaches, such as the IS-LM/AS-AD/Mundell-Fleming model. The main reason why instructors resort to them to teach macroeconomics is because of their simplicity yet relative efficacy at conveying the main messages and predictions. But all this comes at a cost, which may be oversimplification, and a great divide between the undergraduate and postgraduate universe. So this trade-off appears to be inevitable when it comes to teaching this subject at the undergraduate level. Or does it? Must there really be this inescapable divergence between under- and post-graduate macroeconomics worldwide? Are undergraduates taught macroeconomics the way it should

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be taught? As we argue in this paper, these two diverse views need not be incompatible. We advocate supplementing the teaching of modern static graph-based models with the simplest possible DSGE models. The latter would be taught in the last years of the undergraduate academic training. This strategy, adopted at some universities around the world, is by no means general, and a powerful reason why this may be so could lie in the fact that a methodological and straightforward way to teach DSGE models to undergraduates is lacking. This is precisely what this article intends to tackle.

Large-scale macroeconomic models, which were still the dominant paradigm in the late 1960s, used to rely on the Neoclassical Synthesis.¹ In the early 1970s these models were severely called into question by the Lucas' critique, which was reinforced by the growing awareness among macroeconomists that the divorce between microeconomics and macroeconomics (the latter's lack of microfoundations) was no longer acceptable. This increasing distrust toward the aforementioned theoretical framework has continued up to the present day.

Even though macroeconomists, by and large, seem to have lost faith in this Neoclassical Synthesis over the last 25 years, these models still prove to be the main teaching tool on which most of the undergraduate textbooks rest. It is true that a rising number of books at that level employ dynamic microfounded models to teach macroeconomics (for example, Barro, 1997; Doepke et al., 1999; Chugh and Sanjay, 2015; Gillman, 2011; Williamson, 2013), but there are still few of their kind. Similarly, there also exist some rare academic articles attempting to provide an uncomplicated theoretical framework (mostly graphical tools) with which to teach simple Real Business Cycle (RBC) and New Keynesian models (see for example, Benigno, 2015; Brevik and Gärtner, 2007; Duncan, 2015). The upshot is that dynamic general equilibrium models are overall avoided in the undergraduate segment on the grounds that there is no unified approach to walking undergraduate students through the process of learning this methodology in an orderly and simple fashion.

It is worth pointing out, however, that there is a seemingly thorough yet gradual reform underway in the undergraduate teaching of this discipline, whose inception dates back to 1996 when the outcome of the symposium on the teaching of undergraduate macroeconomics was published in *Journal of Economic Education* (volume 27, issue 2). Indeed, with a view to changing the undergraduate macroeconomics education, a lot of articles are subsequently written, such as Allsop and Vines (2000), Romer (2000),² Walsh (2002), Carlin and Soskice (2005), Bofinger et al. (2009), Chadha (2009) and Wren-Lewis (2009), to name a few, all of which constitute different undergraduate representations of the New Consensus.³ As proposed amendments to the latter, Fontana and Setterfield (2009) set up a teachable macroeconomic model that seeks to outperform both the traditional IS-LM and the New Consensus. Along the same lines, Howells (2009), Carlin and Soskice (2014) and Blanchard (2016) expand the 3-equation model to incorporate the behavior of the financial (banking) system and its interplay with the macroeconomy. Kapinos (2010), on the other hand, makes use of Excel so as to showcase several versions of the New-Keynesian model, namely, static, adaptive expectations and rational expectations, while Reingewertz (2013) advocates using flowcharts as a way to complement regular graphs and algebra in the exposition of the IS-LM and AS-AD models.⁴

Notwithstanding the fact that we agree on the main message stemming from the above references and thus we wholeheartedly support this stance that undergraduate macroeconomics needs reforming in line with those proposals, it is our belief that it is plausible – and even desirable – to go beyond that and to teach DSGE models as well in the undergraduate curriculum. The main advantage of doing so is to close the gap between how under- and graduate macroeconomics are taught, which may chiefly benefit those students with a strong interest in pursuing a masters or a PhD degree. The most obvious cost can be regarded as being the increased complexity relative to the toy models mentioned above, something that, in our opinion, should not be an insurmountable obstacle to motivated students. We believe that the simplest DSGE model is clearly accessible to well-trained undergraduates, at least to upper-level ones, as long as the entire methodology is presented in a very organized and simplified way. Generally speaking, the potential hurdles students may run into in dealing with simple DSGE models cannot be attributed to an alleged

¹ As its very name implies, this strand of macroeconomics merged traditional Keynesianism and Neoclassical Economics into a single model seeking to offer an appropriate characterization of both the short and the long run of the economy.

² Romer is credited with being the first author who groundbreakingly dispensed with the LM curve and replaced it with a monetary policy rule.

³ The New Consensus or New Neoclassical Synthesis is the blending of the two main modern macroeconomic schools of thought, New Classical macroeconomics and New-Keynesian macroeconomics, in an attempt to offer a satisfactory explanation of short-run fluctuations.

⁴ Two papers that do a splendid job of summarizing the state-of-the-art teaching of undergraduate macroeconomics nowadays are De Araujo et al. (2013) and Gärtner et al. (2013).

high degree of mathematical sophistication, as the mathematical tools required to solve those models (derivations, maximization problems, etc.) are an integral part of the economics curriculum of any university around the world. Textbooks like [Chiang and Wainwright \(2005\)](#) are extensively relied upon in the mathematics for economists courses at the undergraduate level and suffice to provide students with the mathematical prerequisites needed for that macroeconomic modeling. In addition, students are expected to have taken some courses of microeconomics before facing that advanced macroeconomics subject, so again a supposed lack of microeconomics background cannot be alluded to either to warrant these difficulties in dealing with DSGE models.

Therefore, this article sets out to offer a straightforward approach to teaching DSGE models to undergraduate students. The model studied is a stripped-down version of the canonical closed economy RBC model. Therefore, no friction, such as price and wage stickiness, adjustment costs, habit formation in consumption, etc., is considered in this simple framework. In Section 4, the basic model is enlarged with the introduction of the government and the foreign sector. The way we proceed is by first solving for the nonlinear model, searching for the steady-state and the log-linear equations and finally simulating some shocks. The remainder of the paper is organized as follows: Section 2 describes the structural model, Section 3 analyzes the effect of a productivity shock on the economy, Section 4 expands this simple model by incorporating the government and the external sector, and Section 5 concludes.

2. The model

In this section the proposed structural model of the economy is presented and solved step by step. It begins with the description of the set of agents that populate the economy (households and firms in the simplest version of the model). Then, a detailed derivation of the equilibrium conditions is shown, and, in the subsequent step, the steady state and the log-linearized equations composing the model are found.

Assumption 2.1. Our model features a closed economy where there is no government nor financial sector.

Assumption 2.2. There is no money in this economy, that is, it is an exchange economy.

Assumption 2.3. Adjustment costs are ignored in this economy.

2.1. Households

Assumption 2.4. The economy is inhabited by a continuum of infinitely-lived households indexed by $j \in [0, 1]$ whose problem is to maximize a given intertemporal welfare function. To that end, we resort to an additively separable utility function in consumption (C) and Labor (L).

Increasing consumption is assumed to raise households' utility (happiness), whereas working longer hours leads to lower utility.⁵

Assumption 2.5. There is intertemporal additive separability in consumption (i.e., no habit formation).

Assumption 2.6. Population growth is ignored.

Assumption 2.7. The labor market behaves in a perfectly competitive fashion (there is no wage rigidity).

The representative household maximizes the following welfare function:

$$\max_{C_{j,t}, L_{j,t}, K_{j,t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right) \quad (1)$$

⁵ As more leisure means higher happiness, the more the individual works, the less time she will have available to devote to leisure.

where E_t is the expectation operator, β represents the intertemporal discount factor, C denotes consumption of goods, L is the amount of hours worked, σ is the coefficient of relative risk aversion, and φ is the marginal disutility with respect to the labor supply.

The utility function⁶ must display some properties: $U_C > 0$ and $U_L < 0$, which means that consumption and work affect utility positively and negatively, respectively. By the same token, the fact that $U_{CC} < 0$ and $U_{LL} < 0$ shows that the utility function is concave,⁷ thereby indicating that as consumption rises, utility will increase as well, but at a decreasing rate.

Households maximize their welfare function subject to a sequential budget constraint which states that all sources of income must equal all uses of income within each period. In addition, households are assumed to own all factors of production in the economy (capital and labor in our simple model). Because they rent out labor and capital to firms, households are paid wages and the return to capital, respectively. They are also the owners of the firms, so they receive profits (in the form of dividends) accordingly. Thus, the households' sequential budget constraint would take the following form:

$$P_t(C_{j,t} + I_{j,t}) \leq W_t L_{j,t} + R_t K_{j,t} + \Pi_t \tag{2}$$

where P is the general price level, I denotes investment, W is the wage level, K represents the capital stock, R is the return on capital and Π stands for firms' profit (dividends).

We still need an additional equation featuring the capital accumulation process over time.

$$K_{j,t+1} = (1 - \delta)K_{j,t} + I_{j,t} \tag{3}$$

where δ is the depreciation rate of physical capital.

The representative household's problem can be solved using the following Lagrangian given by Eqs. (1)–(3):

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{j,t} \left[P_t C_{j,t} + P_t K_{j,t+1} - P_t(1-\delta)K_{j,t} - W_t L_{j,t} - R_t K_{j,t} - \Pi_t \right] \right\} \tag{4}$$

where λ is the Lagrange multiplier.

The above maximization problem yields following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -L_{j,t}^{\varphi} + \lambda_{j,t} W_t = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t+1}} = -\lambda_{j,t} P_t + \beta E_t \lambda_{j,t+1} [(1-\delta)E_t P_{t+1} + E_t R_{t+1}] = 0 \tag{7}$$

Solving for λ_t using Eqs. (5) and (6), we are left with the household's labor supply equation:

$$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t} \tag{8}$$

or,

$$\underbrace{-C_{j,t}^{\sigma} L_{j,t}^{\varphi}}_{\text{MRS consumption-leisure}} = \underbrace{-\frac{W_t}{P_t}}_{\text{Relative price consumption-leisure}}$$

⁶ The utility function most frequently used in economics to portray the choices made by households is the Constant Relative Risk Aversion (CRRA) utility function. In the literature, some other functional forms are also employed, for example, the logarithmic utility function, $U(C_t, L_t) = \ln C_t + \frac{\chi}{1-\chi} A \ln(1 - L_t)$; or a combination between a logarithmic utility function and a CRRA, $U(C_t, L_t) = \ln(C_t) - \frac{\nu}{1+\chi} L_t^{1+\chi}$.

⁷ U_C and U_L are the first-order partial derivatives of the utility function with respect to consumption and labor, respectively, while U_{CC} and U_{LL} are the second-order partial derivatives.

The labor supply equation states that the relative price leisure-consumption (real wage) must equal the marginal rate of substitution (MRS) leisure-consumption. Therefore, an increase in consumption, *ceteris paribus*, can only come about through an increase in the number of work hours (less leisure). In other words, there is a *tradeoff* between working less (more leisure) and consuming more. On the other hand, a higher real wage makes it possible for consumption to rise without sacrificing leisure.⁸

Taking into account from Eq. (5) that $\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}$ and $\lambda_{j,t+1} = \frac{C_{j,t+1}^{-\sigma}}{P_{t+1}}$, and plugging those results into Eq. (7), we arrive at the so-called Euler equation:

$$C_{j,t}^{-\sigma} = \beta E_t \left\{ C_{j,t+1}^{-\sigma} \left[(1 - \delta) + \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \right\} \quad (9)$$

The above equation determines how households make their saving decisions (in this model, saving is the acquisition of investment goods). Specifically, in deciding their level of saving, households compare the increase in utility derived from consuming an additional unit of the good today to the increase in tomorrow's utility ensuing from a marginally lower current consumption. This means that if the expected interest rate rises, consuming "today" (in t) becomes dearer, and, *ceteris paribus*, future consumption ($t + 1$) will increase.

As with the intratemporal condition, we can always rewrite the Euler equation as the typical problem in microeconomics. To make our life easier, let us assume that $\beta = 1$ and $\delta = 1$,

$$\underbrace{-E_t \left[\frac{1}{\pi_{t+1}} \left(\frac{C_{j,t+1}}{C_{j,t}} \right)^\sigma \right]}_{\text{MRS } C_t - C_{t+1}} = \underbrace{-E_t \left(\frac{r_{t+1}}{\pi_{t+1}} \right)}_{\text{Relative price } C_t - C_{t+1}}$$

where $E_t r_{t+1} = E_t \left(\frac{R_{t+1}}{P_{t+1}} \right)$ is the real rate of return on capital.

This last expression states that the marginal rate of substitution between current consumption and future consumption equals the relative price of current consumption in terms of future consumption. Or, in other words, the subjective discount rate and the market discount rate should be equated if the individuals are optimizing.

The bottom line is that the household's problem consists of two choices. The first one is an intratemporal choice between the purchase of consumption goods and leisure. The second one is an intertemporal choice, whereby the household must choose between current consumption and future consumption.

2.2. Firms

The representative firm is the economic agent producing the goods and services that will be consumed or saved (and subsequently transformed into capital) by households.

Assumption 2.8. There is a continuum of profit-maximizing firms indexed by j that operate in a perfectly competitive market, which means that their profit will tend to zero in the long run ($\Pi_t = 0$, for all t).

The technology that firms are assumed to use is the standard Cobb–Douglas production function,⁹

$$Y_{j,t} = A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha} \quad (10)$$

where A_t denotes total factor productivity (TFP), a variable which can be interpreted as the level of general knowledge about the productive "arts" an economy has at its disposal, Y_t is GDP and α is the output elasticity of capital. α can

⁸ A higher real wage necessarily involves higher levels of consumption, but the same cannot be asserted about leisure. Thus, if the income effect exceeds the substitution effect, leisure will go up. On the contrary, if the latter effect dominates the former one, leisure will end up decreasing.

⁹ Even though many DSGE models opt for a Cobb–Douglas-type technology, there are several alternative specifications. A very popular production function in this literature is the CES function (*Constant Elasticity of Substitution*), which has the following form:

$$F(K_t, L_t) = \left[\alpha K_t^\rho + (1 - \alpha) L_t^\rho \right]^{\frac{1}{\rho}}$$

also be thought of as the capital’s share of output, while $(1 - \alpha)$ would be the labor’s. As is the case with the utility function, the production function must display some properties: strictly increasing ($F_K > 0$ and $F_L > 0$), strictly concave ($F_{KK} < 0$ and $F_{LL} < 0$) and twice differentiable. Besides, the production function is assumed to exhibit constant returns to scale, $F(zK_t, zL_t) = zY_t$, and to satisfy the so-called Inada conditions: $\lim_{K \rightarrow 0} F_K = \infty$; $\lim_{K \rightarrow \infty} F_K = 0$; $\lim_{L \rightarrow 0} F_L = \infty$; and $\lim_{L \rightarrow \infty} F_L = 0$.

The representative firm seeks to maximize a profit function (Π) by choosing the quantities of each input demanded (L_t, K_t):

$$\max_{L_{j,t}, K_{j,t}} \Pi_{j,t} = A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha} P_{j,t} - W_t L_{j,t} - R_t K_{j,t} \tag{11}$$

Solving the above problem yields the following first-order conditions:

$$\frac{\partial \Pi_{j,t}}{\partial K_{j,t}} = \alpha A_t K_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha} P_{j,t} - R_t = 0 \tag{12}$$

$$\frac{\partial \Pi_{j,t}}{\partial L_{j,t}} = (1 - \alpha) A_t K_{j,t}^\alpha L_{j,t}^{-\alpha} P_{j,t} - W_t = 0 \tag{13}$$

From Eqs. (12) and (13), we get:

$$\underbrace{\frac{R_t}{P_{j,t}}}_{\text{Real MCK}} = \alpha \underbrace{\frac{Y_{j,t}}{K_{j,t}}}_{\text{MPK}} \tag{14}$$

$$\underbrace{\frac{W_t}{P_{j,t}}}_{\text{Real MCL}} = (1 - \alpha) \underbrace{\frac{Y_{j,t}}{L_{j,t}}}_{\text{MPL}} \tag{15}$$

Eqs. (14) and (15) capture the demand for capital and labor, respectively, which are obtained by equating the marginal costs with the marginal products.¹⁰

It is worth noting that Eq. (15) suggests that a decrease in the real wage would lead to a higher quantity of labor demanded. The intuition behind this relationship is straightforward: at a lower real cost of hiring workers, firms will demand more labor until the marginal product of labor falls enough to ensure that the equalization of the latter with the real wage is restored¹¹ (Barro, 1997).

The productivity shock is assumed to follow a first-order autoregressive process, such that:

$$\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t \tag{16}$$

where A_{ss} is the steady-state value of TFP, ρ_A is the autoregressive parameter of TFP whose absolute value should be less than one ($|\rho_A| < 1$) to ensure the stationarity of the process, and $\epsilon_t \sim N(0, \sigma_A)$.

Assumption 2.9. TFP growth is ignored in this model.

Combining the input demand equations (14) and (15) gives:

$$-\underbrace{\frac{W_t}{R_t}}_{\text{ERS}} = -\underbrace{\frac{(1 - \alpha)K_{j,t}}{\alpha L_{j,t}}}_{\text{MRTS}}$$

The right-hand side of this equation is the marginal rate of technical substitution (MRTS) that can be defined as the rate at which labor can be exchanged with capital, holding the level of output constant. Likewise, the right-hand side

¹⁰ Real *MCK* is the real marginal cost of capital; real *MCL* is the real marginal cost of labor; *MPK* is the marginal product of capital; and *MPL* is the marginal product of labor.

¹¹ The same logic applies to capital (Eq. (14)).

is the so-called economic rate of substitution (the wage to the rental rate ratio), which measures the rate at which labor can be substituted for capital, holding the cost level constant.

Rearranging the preceding equation leads to,

$$L_{j,t} = \left(\frac{1 - \alpha}{\alpha} \right) \frac{R_t}{W_t} K_{j,t} \quad (17)$$

and by plugging Eq. (17) into the production function (Eq. (10)), we obtain the following expressions,

$$Y_{j,t} = A_t K_{j,t}^\alpha \left[\left(\frac{1 - \alpha}{\alpha} \right) \frac{R_t}{W_t} K_{j,t} \right]^{1-\alpha}$$

$$K_{j,t} = \frac{Y_{j,t}}{A_t} \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{1-\alpha} \quad (18)$$

Substituting Eq. (18) into Eq. (17) yields,

$$L_{j,t} = \frac{Y_{j,t}}{A_t} \left(\frac{1 - \alpha}{\alpha} \right) \frac{R_t}{W_t} \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{1-\alpha}$$

$$\left(\frac{1 - \alpha}{\alpha} \right) \frac{R_t}{W_t} = \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{-1}$$

$$L_{j,t} = \frac{A_t}{Y_{j,t}} \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{-\alpha} \quad (19)$$

The total cost (TC) function is given by:

$$CT_{j,t} = W_t L_{j,t} + R_t K_{j,t}$$

As before, plugging equations (18) and (19) into the aforesaid TC function gives:

$$CT_t = W_t \frac{Y_{j,t}}{A_t} \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{-\alpha} + R_t \frac{Y_{j,t}}{A_t} \left[\left(\frac{\alpha}{1 - \alpha} \right) \frac{W_t}{R_t} \right]^{1-\alpha}$$

After rearranging a little bit, we get to:

$$CT_{j,t} = \frac{Y_{j,t}}{A_t} \left(\frac{W_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha$$

And marginal cost (MC) is derived from TC¹²:

$$CM_{j,t} = \frac{1}{A_t} \left(\frac{W_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (20)$$

The fact that MC only depends on TFP and input prices means that all firms will face the same MC level ($MC_{j,t} = MC_t$). Moreover, having in mind that $P_t = MC_t$, we arrive at an expression for the general price level,

$$P_t = \frac{1}{A_t} \left(\frac{W_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (21)$$

¹² $MC_{j,t} = \frac{\partial TC_{j,t}}{\partial Y_{j,t}}$.

Table 1
Model structure.

Equation
Definition
$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t}$ (Labor supply)
$C_{j,t}^{-\sigma} = \beta E_t \left\{ C_{j,t+1}^{-\sigma} \left[(1-\delta) + \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \right\}$ (Euler equation)
$K_{t+1} = (1-\delta)K_t + I_t$ (Law of motion for capital)
$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ (Production function)
$K_t = \alpha \frac{Y_t}{R_t}$ (Demand for capital)
$L_t = (1-\alpha) \frac{Y_t}{W_t}$ (Demand for labor)
$P_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha$ (Price level)
$Y_t = C_t + I_t$ (Equilibrium condition)
$\log A_t = (1-\rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t$ (Productivity shock)

2.3. Market clearing

Once each economic agent's behavior has been described in detail, we need to study next the interaction between them so as to determine the competitive general equilibrium. Households decide how much to consume (C_t), how much to invest (I_t) and how much labor they supply (L_t), with a view to maximizing their utility, taking prices as given. Furthermore, firms decide how much to produce (Y_t) by choosing the optimal amount of labor and capital, taking input prices as given and subject to the available technology.

The equilibrium of our model is thus composed of the following building blocks:

1. a set of prices, W_t , R_t and P_t ;
2. a set of good and factor endowments Y_t , C_t , I_t , L_t and K_t ; and
3. a production possibility frontier or constraint given by the following equilibrium condition for the good market (aggregate supply= aggregate demand).

$$Y_t = C_t + I_t \quad (22)$$

A competitive equilibrium entails finding a sequence of endogenous variables of the model such that the conditions defining the equilibrium are satisfied. In short, this economy model comprises the following equations from [Table 1](#).¹³

2.4. Steady state

After establishing the competitive equilibrium, it proves necessary to define the steady-state values. In fact, the proposed model is stationary in that there exists a time-independent value for the variables considered. In this way, an endogenous variable x_t is in its steady state, for all t, if $E_t x_{t+1} = x_t = x_{t-1} = x_{ss}$.

Some of the endogenous variables have their steady-state values set previously (in an exogenous manner). This is the case of TFP which constitutes the main source of shocks in the standard Real Business Cycle (RBC) model – in

¹³ Due to symmetry of preferences and technology, both households and firms will be dealt with by turning to the artificial construction of the representative agent (the subscript j will be eliminated).

the steady state $E(\epsilon_t) = 0$. Thus, from Eq. (16), it is not possible to know the steady-state value of TFP, so the literature largely presumes that $A_{ss} = 1$. The next step is to remove the time subscripts from the variables. Consequently, the structural model becomes:

Household block

$$C_{ss}^\sigma L_{ss}^\varphi = \frac{W_{ss}}{P_{ss}} \quad (23)$$

$$1 = \beta \left(1 - \delta + \frac{R_{ss}}{P_{ss}} \right) \quad (24)$$

$$I_{ss} = \delta K_{ss} \quad (25)$$

Firm block

$$K_{ss} = \alpha \frac{Y_{ss}}{\frac{R_{ss}}{P_{ss}}} \quad (26)$$

$$L_{ss} = (1 - \alpha) \frac{Y_{ss}}{\frac{W_{ss}}{P_{ss}}} \quad (27)$$

$$Y_{ss} = K_{ss}^\alpha L_{ss}^{1-\alpha} \quad (28)$$

$$P_{ss} = \left(\frac{W_{ss}}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_{ss}}{\alpha} \right)^\alpha \quad (29)$$

Equilibrium conditions block

$$Y_{ss} = C_{ss} + I_{ss} \quad (30)$$

The system made up of Eqs. (23)–(30) will be used to ascertain the steady-state value of 8 endogenous variables (Y_{ss} , C_{ss} , I_{ss} , K_{ss} , L_{ss} , W_{ss} , R_{ss} and P_{ss}).

Walras' law implies that, if demand equals supply in $k - 1$ markets, then demand will equal supply in the k th market. Hence, if there are K markets, only $K - 1$ relative prices are required to determine the equilibrium. This allows for the normalization of the general price level:

$$P_{ss} = 1 \quad (31)$$

One option is to find the steady state of the variables by calibrating the values for output (Y_{ss}) and for the return on capital (R_{ss}),¹⁴ and then relate the demand-side variables (C_{ss} and I_{ss}) and the capital stock (K_{ss}) to output^{15,16}:

$$C_{ss} = \phi_{C_{ss}} Y_{ss} \quad (32)$$

$$I_{ss} = \phi_{I_{ss}} Y_{ss} \quad (33)$$

$$K_{ss} = \phi_{K_{ss}} Y_{ss} \quad (34)$$

From Eqs. (25), (33) and (34), we get to the depreciation rate of the economy (δ):

$$\delta = \frac{I_{ss}}{K_{ss}} = \frac{\phi_{I_{ss}}}{\phi_{K_{ss}}}$$

The wage level (W_{ss}) is derived from Eq. (29):

$$W_{ss} = (1 - \alpha)(P_{ss})^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_{ss}} \right)^{\frac{\alpha}{1-\alpha}} \quad (35)$$

¹⁴ In this model, the return on capital is proxied by the benchmark interest rate.

¹⁵ Another possibility would be to derive all the prices in this economy and next obtain the components of aggregate demand.

¹⁶ The values of $\phi_{C_{ss}}$, $\phi_{I_{ss}}$ and $\phi_{K_{ss}}$ will also be calibrated.

Lastly, the steady-state value of labor (L_{ss}) is found from Eq. (28):

$$L_{ss} = \left(\frac{Y_{ss}}{K_{ss}^\alpha} \right)^{\frac{1}{1-\alpha}} \quad (36)$$

2.5. Log-linearization (Uhlig's method)

Dealing with nonlinear models is generally a very arduous task. When the model is very simple, it proves possible to find an approximation of the policy function by recursively solving for the value function. On the other hand, linear models are often easier to solve. The challenge is to turn a nonlinear model into a sufficiently adequate linear approximation that helps understand the behavior of the underlying nonlinear system. In this sense, a standard technique is to log-linearize the equations of the model around the steady state (there are some methods using this approach in their solution procedures: [Blanchard and Fischer, 1989](#); [Uhlig, 1999](#); [Sims, 2001](#); [Klein, 2000](#)).¹⁷

[Uhlig \(1999\)](#) recommends using a simple way of log-linearizing functions that does not call for differentiation. It is just enough to substitute out a variable X_t for $X_{ss}e^{\tilde{X}_t}$ where $\tilde{X}_t = \log X - \log X_{ss}$ stands for the log-deviation of the variable from its steady-state value. In addition, Uhlig suggests the following resolution methods:

$$e^{(\tilde{X}_t + a\tilde{Y}_t)} \approx 1 + \tilde{X}_t + a\tilde{Y}_t \quad (37)$$

$$\tilde{X}_t \tilde{Y}_t \approx 0 \quad (38)$$

$$E_t [ae^{\tilde{X}_{t+1}}] \approx a + aE_t [\tilde{X}_{t+1}] \quad (39)$$

2.6. Labor supply

Starting out with the labor supply equation,

$$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t}$$

substitute $X_t = X_{ss}e^{\tilde{X}_t}$ into each variable of the preceding equation.

$$C_{ss}^\sigma L_{ss}^\varphi e^{(\sigma\tilde{C}_t + \varphi\tilde{L}_t)} = \frac{W_{ss}}{P_{ss}} e^{(\tilde{W}_t - \tilde{P}_t)}$$

Then, use Uhlig's rule from Eq. (37),

$$C_{ss}^\sigma L_{ss}^\varphi (1 + \sigma\tilde{C}_t + \varphi\tilde{L}_t) = \frac{W_{ss}}{P_{ss}} (1 + \tilde{W}_t - \tilde{P}_t)$$

and, since in the steady state, $C_{ss}^\sigma L_{ss}^\varphi = \frac{W_{ss}}{P_{ss}}$, we arrive at:

$$\sigma\tilde{C}_t + \varphi\tilde{L}_t = \tilde{W}_t - \tilde{P}_t \quad (40)$$

2.7. Consumption Euler equation

The same procedure will be recurred to in the case of the Euler equation.

$$\frac{1}{\beta} E_t \left(\frac{C_{t+1}}{C_t} \right)^\sigma = (1 - \delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}} \right)$$

Plug $X_t = X_{ss}e^{\tilde{X}_t}$ into each variable of the preceding equation.

$$\frac{1}{\beta} \left(\frac{C_{ss}^\sigma}{C_{ss}^\sigma} \right) e^{(\sigma E_t \tilde{C}_{t+1} - \sigma\tilde{C}_t)} = (1 - \delta) + \frac{R_{ss}}{P_{ss}} e^{[E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1})]}$$

¹⁷ For further information, the reader is referred to [DeJong and Dave \(2011\)](#) and [Canova \(2007\)](#).

Next, use Uhlig's rule from Eq. (37),

$$\frac{1}{\beta} [1 + \sigma(E_t \tilde{C}_{t+1} - \tilde{C}_t)] = (1 - \delta) + \frac{R_{ss}}{P_{ss}} [1 + E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1})]$$

taking into account that in the steady state, $\frac{1}{\beta} = \frac{R_{ss}}{P_{ss}} + (1 - \delta)$, we have:

$$\frac{\sigma}{\beta}(E_t \tilde{C}_{t+1} - \tilde{C}_t) = \frac{R_{ss}}{P_{ss}} E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1}) \quad (41)$$

2.8. Demand for capital

Rewriting the capital demand equation,

$$R_t = \alpha \frac{Y_t}{K_t}$$

and substituting $X_t = X_{ss} e^{\tilde{X}_t}$ into each variable of the preceding equation.

$$\frac{R_{ss}}{P_{ss}} e^{(\tilde{R}_t - \tilde{P}_t)} = \alpha \frac{Y_{ss}}{K_{ss}} e^{(\tilde{Y}_t - \tilde{K}_t)}$$

Now, employ Uhlig's rule from Eq. (37),

$$\frac{R_{ss}}{P_{ss}} (1 + \tilde{R}_t - \tilde{P}_t) = \alpha \frac{Y_{ss}}{K_{ss}} (1 + \tilde{Y}_t - \tilde{K}_t)$$

having in mind that in the steady state, $\frac{R_{ss}}{P_{ss}} = \alpha \frac{Y_{ss}}{K_{ss}}$, we are left with:

$$\tilde{R}_t - \tilde{P}_t = \tilde{Y}_t - \tilde{K}_t \quad (42)$$

2.9. Demand for labor

The labor demand is:

$$\frac{W_t}{P_t} = (1 - \alpha) \frac{Y_t}{L_t}$$

Substituting $X_t = X_{ss} e^{\tilde{X}_t}$ into each variable of the preceding equation leads to:

$$\frac{W_{ss}}{P_{ss}} e^{(\tilde{W}_t - \tilde{P}_t)} = (1 - \alpha) \frac{Y_{ss}}{L_{ss}} e^{(\tilde{Y}_t - \tilde{L}_t)}$$

Then, we turn to Uhlig's rule from Eq. (37),

$$\frac{W_{ss}}{P_{ss}} (1 + \tilde{W}_t - \tilde{P}_t) = (1 - \alpha) \frac{Y_{ss}}{L_{ss}} (1 + \tilde{Y}_t - \tilde{L}_t)$$

given that in the steady state we have that $\frac{W_{ss}}{P_{ss}} = (1 - \alpha) \frac{Y_{ss}}{L_{ss}}$, then:

$$\tilde{W}_t - \tilde{P}_t = \tilde{Y}_t - \tilde{L}_t \quad (43)$$

2.10. Production function

Applying the same procedure as before to the production function:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

$$Y_{ss} e^{\tilde{Y}_t} = A_{ss} K_{ss}^\alpha L_{ss}^{1-\alpha} e^{(\tilde{A}_t + \alpha \tilde{K}_t + (1-\alpha) \tilde{L}_t)}$$

$$Y_{ss} (1 + \tilde{Y}_t) = A_{ss} K_{ss}^\alpha L_{ss}^{1-\alpha} (1 + \tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{L}_t)$$

And in the steady state, $Y_{ss} = A_{ss}K_{ss}^\alpha L_{ss}^{1-\alpha}$:

$$\tilde{Y}_t = \tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{L}_t \quad (44)$$

2.11. Law of motion for capital

The law of motion for capital is known to be:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$K_{ss}e^{\tilde{K}_{t+1}} = (1 - \delta)K_{ss}e^{\tilde{K}_t} + I_{ss}e^{\tilde{I}_t}$$

$$K_{ss}(1 + \tilde{K}_{t+1}) = (1 - \delta)K_{ss}(1 + \tilde{K}_t) + I_{ss}(1 + \tilde{I}_t)$$

Dividing both sides of the preceding equation by K_{ss} ,

$$(1 + \tilde{K}_{t+1}) = (1 - \delta) + (1 - \delta)\tilde{K}_t + \frac{I_{ss}}{K_{ss}} + \frac{I_{ss}}{K_{ss}}\tilde{I}_t$$

Because in the steady state $I_{ss} = \delta K_{ss}$, then:

$$\tilde{K}_{t+1} = (1 - \delta)\tilde{K}_t + \delta\tilde{I}_t \quad (45)$$

2.12. Equilibrium condition

It remains to find the log-linear equation for the equilibrium condition.

$$Y_t = C_t + I_t$$

$$Y_{ss}e^{\tilde{Y}_t} = C_{ss}e^{\tilde{C}_t} + I_{ss}e^{\tilde{I}_t}$$

$$Y_{ss}(1 + \tilde{Y}_t) = C_{ss}(1 + \tilde{C}_t) + I_{ss}(1 + \tilde{I}_t)$$

As in the steady state, $Y_{ss} = C_{ss} + I_{ss}$, then:

$$Y_{ss}\tilde{Y}_t = C_{ss}\tilde{C}_t + I_{ss}\tilde{I}_t \quad (46)$$

2.13. Technological shock

The law of motion for productivity is given by:

$$\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t$$

A little bit of algebraic manipulation takes us to:

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_t \quad (47)$$

Table 2 summarizes the log-linear model.

3. Productivity shock

This section aims to assess the effects of a productivity shock on the macroeconomic variables considered in this Real Business Cycle (RBC) model. The calibrated values of the parameters which will be used throughout the simulation are displayed in Table 3.

Fig. 1 showcases a positive shock to TFP.¹⁸ On impact, both marginal productivity of capital and labor rise (Eqs. (42) and (43)), thus bringing about an increased demand for those inputs, which tends to push their prices upward – wage

¹⁸ The simulation of this model was performed on the Dynare platform (for more information, see <http://dynare.org/>).

Table 2
Structure of the log-linear model.

Equation	Definition
$\sigma \tilde{C}_t + \varphi \tilde{L}_t = \tilde{w}_t$	(Labor supply)
$\frac{\sigma}{\beta} (E_t \tilde{C}_{t+1} - \tilde{C}_t) =$ $\frac{R_{ss}}{P_{ss}} E_t \tilde{r}_{t+1}$	(Euler equation)
$\tilde{K}_{t+1} = (1 - \delta) \tilde{K}_t + \delta \tilde{I}_t$	(Law of motion for capital)
$\tilde{Y}_t = \tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{L}_t$	(Production function)
$\tilde{K}_t = \tilde{Y}_t - \tilde{r}_t$	(Demand for capital)
$\tilde{L}_t = \tilde{Y}_t - \tilde{w}_t$	(Demand for labor)
$Y_{ss} \tilde{Y}_t = C_{ss} \tilde{C}_t + I_{ss} \tilde{I}_t$	(Equilibrium condition)
$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_t$	(Productivity shock)

where $\tilde{w}_t = \tilde{W}_t - \tilde{P}_t$ and $\tilde{r}_t = \tilde{R}_t - \tilde{P}_t$ represent the real wage and the real return on capital, respectively.

Table 3
Parameter values of the structural model.

Parameter	Meaning of the parameter	Calibrated value	Source
σ	Relative risk aversion coefficient	2	Cavalcanti and Vereda (2011)
φ	Marginal disutility of labor	1.5	Cavalcanti and Vereda (2011)
α	Output elasticity of capital	0.35	Cavalcanti and Vereda (2011)
R_{ss}	Steady-state return on capital	$1.065^{1/4}$	Bacen/Boletim/M. Finan.
Y_{ss}	Steady-state output	1.7 (trillions)	IBGE/SCN 2010 Trimestrall
C_{ss}	Steady-state consumption	$0.77 Y_{ss}$	IBGE/SCN 2010 Trimestrall
I_{ss}	Steady-state investment	$0.23 Y_{ss}$	IBGE/SCN 2010 Trimestrall
K_{ss}	Steady-state capital stock	$2.7 Y_{ss}$	Morandi and Reis (2004)
β	Discount factor	$\frac{1}{(1-\delta) + \frac{R_{ss}}{P_{ss}}}$	–
δ	Depreciation rate	I_{ss}/K_{ss}	–
ρ_A	Autoregressive coefficient for productivity	0.95	Authors
σ_A	Standard deviation of productivity	0.01	Authors

(W) and return on capital (R). With higher income, households react by acquiring more consumption and investment goods (Eq. (46)). As for inputs, initially both labor and capital go up, although over time, households seek to consume more leisure (reducing labor supply). Further, due to a higher investment, the stock of capital increases temporarily right until the third period (Eq. (45)), declining sharply thereafter. This initial rise followed by a strong decrease of this variable forms a “bell-shaped figure”. Briefly, a positive productivity shock raises consumption and investment (C and I), the demands for labor and capital (L and K) and the prices of these inputs (W and R).

4. Introducing government and external sector

In this section we present the main avenues for embedding government and external sector into DSGE models at the undergraduate level.

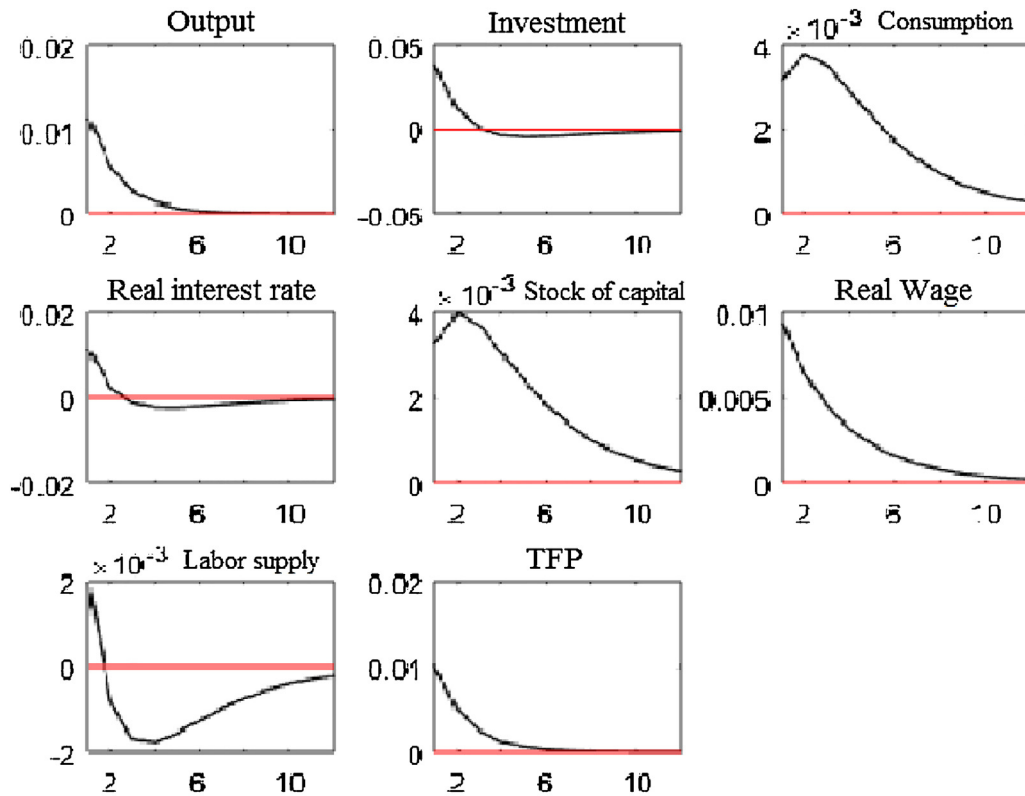


Fig. 1. Effects of a productivity shock. Results from the Dynare simulation (impulse-response functions).

4.1. Government

Overall, the DSGE literature deals with two distinct sort of taxes, *lump-sum* taxes versus distortionary taxes. The former are not relevant for policy purposes, for it proves difficult to apply them in the real world. The latter form of taxation is dubbed that way because changes in those tax rates affect good and input prices, thereby causing economic agents to alter their decisions. Distortionary taxation encompasses two different kinds of taxes, consumption taxes and (labor and capital) income taxes.

Typically, the inclusion of taxes in DSGE models requires modifying households’ budget constraints (Eq. (2)). *Lump-sum* taxes can be embedded in the following way:

$$P_t(C_{j,t} + I_{j,t}) = W_t L_{j,t} + R_t K_{j,t} + \Pi_t - Tr_t \tag{48}$$

where Tr_t is the *lump-sum* tax. As mentioned above, an alternative is to introduce distortionary taxes:

$$P_t(1 + \tau^c)(C_{j,t} + I_{j,t}) = (1 - \tau^l)W_t L_{j,t} + (1 - \tau^k)R_t K_{j,t} + \Pi_t \tag{49}$$

where τ^c , τ^l and τ^k are the consumption, labor income and capital income tax rates, respectively.

Now that we brought the government into the picture, we also need to lay out its own budget constraint.

$$\underbrace{\overbrace{\frac{B_{t+1}}{R_t^B} - B_t}^{\text{Debt issuance}} + \overbrace{T_t}^{\text{Total tax collection}} + \overbrace{M_{t+1} - M_t}^{\text{Money creation}}}_{\text{All sources of funds Government investment}} = \underbrace{\overbrace{P_t G_t}^{\text{Current expenditure}} + \overbrace{P_t I_t^G}^{\text{Government investment}} + \overbrace{P_t TRANS_t}^{\text{Income transfers}}}_{\text{Total expenditure}} \tag{50}$$

where M is the economy's money balances, G is government's current expenditure, I^G is government investment and $TRANS$ is income transfers to households. Total tax collection is given by:

$$T_t = \underbrace{\tau_t^c P_t (C_t + I_t)}_{\text{Tax revenue from consumption taxes}} + \underbrace{\tau_t^l W_t L_t}_{\text{Tax revenue from labor income taxes}} + \underbrace{\tau_t^k (R_t - \delta) K_t}_{\text{Tax revenue from capital income taxes}} + \underbrace{Tr_t}_{\text{Tax revenue from Lump-sum taxes}} \quad (51)$$

If the government issues B_t bonds in $t - 1$ maturing next period¹⁹ and with a unitary face value, B_t also represents the value of the stock of public debt that private agents hold in $t - 1$. Thus, in conducting debt-management policies, the government issues B_{t+1} new bonds in t at the price P_t^B . In practice, such operation can be considered a loan of $P_t^B B_{t+1}$ from the public to the government. The price of these bonds can be written as:

$$P_t^B = \frac{1}{R_t^B} \quad (52)$$

where R_t^B is the rate at which a currency unit is discounted over the next period as well as it is the reference interest rate set by the Central Bank in t .

The term $(M_{t+1} - M_t)$ measures the government's resources stemming from private agents' money holdings. The literature defines this type of fiscal resource as "seigniorage", which can be thought of as an additional form of taxing. In this respect, if a government finds itself incapable of collecting revenues in an orthodox manner, it can turn to seigniorage in order to finance its expenses. Hence, the higher the inflation rate is, the more seigniorage the government collects. It should be noted that in most industrialized countries this source of revenues is negligible.

4.1.1. Public investment

Once public investment has been included into the model, the decision as to how much to invest would no longer rest solely on the private sector, but it would also be the government's responsibility. Furthermore, public capital is believed to play an important role in the production process on the grounds of the significant positive external effects that public infrastructure exerts on the private sector. And yet there are few references on this topic in the literature. Some theoretical articles from the early 1970s already considered public capital as a component of the aggregate production function (Arrow and Kurz, 1970; Weitzman, 1970; Pestieau, 1974). However, it is not until Barro (1990) that this original idea is brought back to life and a new set of academic papers incorporating public capital springs up (Barro and Sala-i-Martin, 1992; Finn, 1993; Glomm and Ravikumar, 1994; Cashin, 1995; Bajo, 2000; among others). Building on these ideas, the DSGE literature opts to use the following way of introducing public capital into the models:

$$Y_t = A_t K_t^{\alpha_1} L_t^{\alpha_2} K_t^G \alpha_3 \quad (53)$$

where K_t^G is the stock of public capital, α_1 , α_2 , α_3 are private-capital's share, labor's share and public-capital's share of output, respectively. The stock of public capital evolves according to the following law of motion:

$$K_{t+1}^G = (1 - \delta_g) K_t^G + I_t^G \quad (54)$$

where I_t^G is government investment and δ_g is the rate of depreciation of public capital.

4.1.2. Alternative forms of government in DSGE models

The previous subsections set forth the most basic ways to insert the government into DSGE models. Nevertheless, there is a vast literature on more advanced assumptions regarding public policy analysis through this modeling.

4.1.2.1. Habit formation in government consumption. In line with the work by Ravn et al. (2006), it is assumed that government spending on good i is carried out within a continuum, with $i \in (0, 1)$. So the problem that the government

¹⁹ In reality, in each period the government issues bonds at different maturities. For the sake of mathematical convenience, all bonds are assumed to be one-year securities.

chooses to solve is to minimize its expenditure on a year-by-year basis for a given price level, and subject to a basket of goods and to a habit-formation rule:

$$\min_{G_{i,t}} \int_0^1 p_{i,t} G_{i,t} di \quad (55)$$

subject to:

$$x_t^G = \left[\int_0^1 \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}} \quad (56)$$

and,

$$s_{i,t}^G = \rho_{sG} s_{i,t-1}^G + (1 - \rho_{sG}) G_{i,t} \quad (57)$$

where η is the parameter governing the elasticity of substitution between the different types of goods. The variable x_t^G captures spending on all goods lumped together $G_{i,t}$, where consumption habits for each variety of good are taken into account. Accordingly, $s_{i,t}^G$ is the government-consumption-habit for each good. Since $s_{i,t}^G$ is independent of the definition of x_t^G , government-consumption-habits considered here are external and entail that government spending is a path-dependent variable.

Using the Lagrangian to solve this problem:

$$\mathcal{L} = \int_0^1 p_{i,t} G_{i,t} di + \lambda_t \left\{ x_t^G - \left[\int_0^1 \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}} \right\}$$

The first-order condition is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial G_{i,t}} &= p_{i,t} - \lambda_t \left(\frac{1}{1-\frac{1}{\eta}} \right) \left[\int_0^1 \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{1-\frac{1}{\eta}-1} \right]^{\frac{1}{1-\frac{1}{\eta}}} \\ \left(1 - \frac{1}{\eta} \right) \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{1-\frac{1}{\eta}-1} &= 0 \end{aligned} \quad (58)$$

Because λ_t denotes marginal cost, and markets are perfectly competitive, we have that $\lambda_t = p_t$, and that $x_t^G \frac{1}{\eta} = \left[\int_0^1 \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{1-\frac{1}{\eta}} \right]^{\frac{1}{\eta-1}}$, which leads us to the following expression:

$$p_{i,t} = p_t x_t^G \frac{1}{\eta} \left(G_{i,t} - \phi_G s_{i,t-1}^G \right)^{-\frac{1}{\eta}} \quad (59)$$

Rearranging the previous expression, we find the government's demand for good i :

$$G_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\eta} x_t^G + \phi_G s_{i,t-1}^G \quad (60)$$

4.1.2.2. Public goods and services in the utility function. It is hard to put public spending into DSGE models correctly. Actually, in few models of this kind this variable is taken to affect households' utility. Nonetheless, empirical evidence suggests that government spending on goods and services appears to account for a considerable bit of the consumption basket, and that those spending decisions have an important influence over the dynamics of other variables.

The literature distinguishes two different routes when it comes to introducing public spending into DSGE models. One possible option is to embed it as an aggregate-demand component, which means that utility would not be affected by public expenditure. However, as said before, this assumption does not seem completely reasonable. Alternatively, a more adequate approach would be for public spending to get transformed into private consumption goods that would

somehow affect households’ welfare (Barro, 1981; Aschauer, 1985; Aiyagari et al., 1992). Following this latter avenue, households’ total consumption is regarded as being a linear combination of private- and public-good consumptions:

$$U_t = U(C_t, L_t) \tag{61}$$

with

$$C_t = C_t^P + \phi_G C_t^G \tag{62}$$

where ϕ_G is a parameter that measures the contribution of the public good to total consumption. The above specification illustrates that the public good can affect households’ utility when $\phi_G \neq 0$. The existence of public goods in households’ utility function implies that an increase in government spending causes a negative income effect, thereby prompting agents to supply less labor and consume fewer private goods (Aiyagari et al., 1992; Baxter and King, 1993).

4.1.2.3. *Public employment and wages.* The presence of public employment in a DSGE model requires altering the government’s budget constraint, such that:

$$\begin{aligned} & \frac{B_{t+1}}{R_t^B} - B_t + T_t + M_{t+1} - M_t \\ & = \underbrace{W_t^G L_t^G}_{\text{Spending on public employees}} + P_t G_t + P_t I_{g,t} + P_t TRANS_t \end{aligned} \tag{63}$$

The production of public goods and services, C_t^G , presupposes that the government combines public spending on goods and services, G_t , and public employment, L_t^G , by using a production function,

$$C_t^G = A_t G_t^{\alpha_G} L_t^{1-\alpha_G} \tag{64}$$

where $0 < \alpha_G < 1$ is the share of public spending in the production of goods and services supplied by the government to households.

4.2. External sector

For modeling purposes, the external sector is split into imports, exports and the equilibrium condition.

4.2.1. Imports

Imports can be modeled following Gali and Monacelli (2005). In this way, there is a continuum of households indexed by $j \in [0, 1]$. The representative household chooses consumption, savings and leisure so as to maximize lifetime utility,

$$C_{D,j,t}^D \max_{C_{F,j,t}^D, L_{j,t}, K_{j,t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{D 1-\eta}}{1-\eta} - \frac{L_{j,t}^{1+\omega}}{1+\omega} \right) \tag{65}$$

subject to the following budget constraint:

$$\begin{aligned} & \underbrace{(C_{D,j,t}^D + I_{j,t}) P_t^{C,D}}_{\text{Domestic consumption of national goods}} + \underbrace{C_{F,j,t}^D S_t P_t^{C,F}}_{\text{Domestic consumption of foreign goods}} \\ & + \underbrace{R_{t-1}^F S_t B_t^F}_{\text{Payment of foreign loans}} = W_t L_{j,t} + R_t K_{j,t} + \underbrace{S_t B_{t+1}^F}_{\text{Foreign loans}} \\ & - \frac{\chi_{BF}}{2} (B_{t+1}^F - B_{ss}^F)^2 S_t + \Pi_t \end{aligned} \tag{66}$$

where C_D^D is domestic consumption of national goods, $C_{F,j,t}^D$ is domestic consumption of foreign goods, S is the nominal exchange rate, $P^{C,D}$ is the price of the domestic good, $P^{C,F}$ is the foreign-currency price of the imported good, B^F is total foreign loans. The term $\left[\frac{\chi_{BF}}{2} (B_{t+1}^F - B_{ss}^F)^2 S_t \right]$ is used to induce stationarity in the model (Schmitt-Grohé and Uribe, 2003).

The domestic aggregate demand is:

$$C_{j,t}^D = \underbrace{(1 - \vartheta_C^D)C_{D,j,t}^D}_{\text{Domestic consumption of national goods}} + \underbrace{\vartheta_C^D C_{F,j,t}^D}_{\text{Domestic consumption of foreign goods}} \quad (67)$$

where ϑ_C^D is the foreign good’s share in the domestic consumption basket.

4.2.2. Exports

There are basically two ways to deal with exports in DSGE models: (I) by assuming that exports obey a rule that relates to the real exchange rate; (II) by modeling foreign households.

4.2.2.1. Exports following a rule regarding the real exchange rate. The foreign sector is represented by import demand, by the equilibrium condition for the balance of payments, and by the laws of motion of the foreign interest rate and the price level of imports.

Export demand follows a rule which is a function of a smoothing component, the real exchange rate and a stochastic component:

$$\frac{C_{D,j,t}^F}{C_{D,j,ss}^F} = \left(\frac{C_{D,j,t-1}^F}{C_{D,j,ss}^F} \right)^{\gamma_X} \left(\frac{S_{t-1}/P_{t-1}}{S_{ss}/P_{ss}} \right)^{(1-\gamma_X)\phi_X} \quad (68)$$

where C_D^F is foreign consumption of national goods, γ_X is a smoothing parameter, ϕ_X is the sensitivity of exports relative to the real exchange rate.

4.2.2.2. Foreign households. Exports from the Home country are assumed to be an homogeneous good before leaving the dock, but they become a differentiated good on international markets. For reasons of symmetry, export goods to the rest of the world are considered to be consumption goods as well as an input used in the rest of the world’s production process.

There is a continuum of foreign households indexed by $j \in [0, 1]$. This representative household maximizes the lifetime utility by choosing consumption:

$$\max_{C_{D,j,t}^F, C_{F,j,t}^F} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^F 1-\eta}{1-\eta} \right) \quad (69)$$

subject to the following budget constraint,

$$C_{D,j,t}^F P_t^{C,D} + C_{F,j,t}^F S_t P_t^{C,F} = Y_t^F S_t P_t^{C,F} \quad (70)$$

where C_F^F is the good consumed and produced abroad, and Y^F is the rest of the world’s income.

With the following bundling technology:

$$C_{j,t}^F = (1 - \vartheta_C^F)C_{F,j,t}^F + \vartheta_C^F C_{D,j,t}^F \quad (71)$$

where ϑ_C^F is the share of the national good in the foreign consumption basket.

4.2.3. Balance of payments

The equilibrium condition for the external sector (balance of payments) is given by:

$$S_t (B_{t+1}^F - R_{t-1}^F B_t^F) = P_t^F S_t C_{F,t}^D - P_t C_{D,t}^F \quad (72)$$

5. Conclusions

In this work we intend to offer a very systematic path to teaching simple DSGE models to undergraduate students. The proposed framework is the simplest frictionless RBC-type DSGE model. We begin by presenting the representative consumer problem in a simple way, whose resolution determines the supply of labor and capital (investment, or savings in this closed economy). Subsequently, the firm’s problem is solved, which gives the demand for labor and capital.

Box 1: Basic log-linear RBC model with Dynare.

```

//RBC model
var Y I C R K W L A;
varexo e;
parameters sigma phi alpha beta delta rhoa
phiCss phiIss phiKss Rss Pss Yss;

sigma = 2;
phi = 1.5;
alpha = 0.35;
rhoa = 0.5;
phiCss = 0.77;
phiIss = 0.23;
phiKss = 2.7;
delta = phiIss/phiKss;
Rss = 1.005^(1/4);
Pss = 1;
beta = 1/((1-delta)+(Rss/Pss));
Yss = 17.95^(1/4);

model(linear);
#Wss = (1-alpha)*(Pss^(1/(1-alpha)))*((alpha/Rss)^(alpha/(1-alpha)));
#Kss = phiKss*Yss;
#Iss = phiIss*Yss;
#Css = phiCss*Yss;
#Lss = (Yss/Kss^alpha)^(1/(1-alpha));
//1-Labor supply
sigma*C + phi*L = W;
//2-Euler equation
(sigma/beta)*(C(+1)-C)=Rss*R(+1);
//3-Law of motion for capital
K = (1-delta)*K(-1)+delta*I;
//4-Production function
Y = A + alpha*K(-1) + (1-alpha)*L;
//5-Demand for Capital
R = Y - K(-1);
//6-Demand for labor
W = Y - L;
//7-Equilibrium condition
Yss*Y = Css*C + Iss*I;
//8-Productivity shock
A = rhoa*A(-1) + e;
end;

steady;
check(qz_zero_threshold=1e-20);
model_diagnostics;
model_info;

shocks;
var e;
stderr 0.01;
end;

stoch_simul (irf = 12) Y I C R K W L A;

```

Then, the steady-state and the log-linearized equations are derived in a very organized and straightforward way. Next, a temporary shock to total factor productivity is modeled, whose dynamic effects are best understood using the impulse-response functions. Finally, we show several ways of introducing the government and how to open up this basic closed-economy model so as to include the rest of the world.

We believe that it is certainly possible to lecture a course like that within the last years of the undergraduate curriculum, as long as the student has covered before a course of mathematics for economists and a course of intermediate microeconomics. The reason why the very fact of providing students with these tools still in their undergraduate years may be worthwhile is because the benefits of doing so outweigh the associated costs. The main advantage is that it closes the gap between the two levels, thereby training students to continue their postgraduate studies. The main cost, namely the supposedly increased difficulty, we think is minimized by adopting our recommended “roadmap” (Box 1).

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