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Jörg Plewka

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Fiscal Discrimination Between Consumer Groups: Tax Burden Distribution Under Price Discrimination

Abstract

In this paper it is analysed, how, under price discrimination, the tax burden is shared between the distinct consumer groups. Unit and ad valorem taxes are compared, revealing an impossibility of fiscal discrimination with regard to price changes. Contrary to conventional tax incidence analysis, it is shown that quantities traded do matter. Relative market shares are decisive for the distribution of tax burdens thereby opening up an opportunity for fiscal discrimination in choosing tax types. This discriminatory potential is limited and not caused by price discrimination per se but rather due to monopolistic supply.

JEL Classification: H22, L11

Keywords: Tax incidence, unit tax, ad valorem tax, price discrimination

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* University of Bochum and University of Dortmund. – I am grateful to Alfred Bischoff, Frauke Eckermann, Cay Folkers and Wolfram F. Richter for useful remarks on earlier drafts of this research. – All correspondence to: Jörg Plewka, Department of Economics, Chair for Public Economics, Ruhr-University of Bochum, Universitätsstr. 150, 44780 Bochum, Germany, e-mail: joerg.plewka@rub.de.

1. Introduction

The original motivation for this paper stems from the various tax proposals the aviation business is constantly faced with. For example, the governments of the developed countries have repeatedly stressed their conjoint intention to increase development policies and the additional funds needed could possibly be raised by higher taxation of the airline business (see e.g. Landau Report, 2004). Before the implementation of such a joint strategy, clearly, a lot of interstate questions are to be concerned. However, the analysis of tax effects must include characteristics that are peculiar to the industry. A prominent feature is price discrimination being virtually omnipresent in the airline business. Under price discrimination different consumers pay different prices for identical goods. Sales taxes will change each price, stirring the question how the burden is shared between consumers grouped by the distinct prices they pay. Obviously, the problem is not just one of the airline industry, many more examples for price discrimination are easily found in other service sectors and industries with high fixed costs. Theatres, swimming baths, amusement parks and others more offer a range of discounts for children, students or senior citizens and libraries pay higher prices for journals than private subscribers. Consequently, some general results on the distribution of the tax burden across consumer groups should be established here. The incidence analysis performed here is a partial equilibrium analysis and considers mainly the economic division of the tax payment.

Restricting incidence analyses to a single equilibrium price, perfectly competitive or not, public finance economists were able to focus on the burden sharing between market sides. But with different consumers paying various prices for one and the same good or service the notion of the (single) tax incidence is no longer meaningful. Rather, there are as many incidences as distinct prices and thereby consumer groups exist. Consequently, tax incidence has to be studied for each segment or consumer group separately. Next to these conventional incidence studies, though increased in number, there immediately arises the question how the tax burden is distributed over the different consumer groups. This problem is discussed here.

A precondition for price discrimination consists of some monopolistic leeway on the supply side¹ which entails the necessity to compare unit and ad valorem taxes². Will both tax types result in the same distribution of tax burdens, or can governments discriminate between consumer groups though employing identical tax rates across all segments? Suits and Musgrave (1953) nicely established the result that the effects of unit and ad valorem taxes

¹ For further prerequisites see Carlton and Perloff (2005, p. 294).

² Both tax types are being used all around the world in the airline business (see IATA,2005) and are debated for an increase.

differ in monopoly and thus carry the potential for fiscal discrimination between consumers and producers. Interest in the comparison of these tax types has never ended (see e.g. Blackorby and Murty, 2007 or Myles, 1996). Analyzing both tax types simultaneously in the conjectural variations model Delipalla and Keen (1992) show for homogenous goods that ad valorem taxes raising the same revenue as unit taxes always result in lower equilibrium prices for any number of suppliers. Allowing for many suppliers, however, there exists only one single market price which renders a comparison of tax burdens across consumer groups paying distinct prices impossible. Different prices can be found in models with vertical product differentiation where consumers no longer buy identical goods.

For isolating effects, the focus here lies on homogenous goods and price discrimination of third degree. With respect to this kind of discrimination Yang (1993) confirms all results of the seminal work by Suits and Musgrave. Yang extends the results to all segments looking on the comparison in an aggregate form rather than concerning the distribution across segments. In the same aggregate way Cheung (1998) expands the comparison of tax types to the other two forms of Pigouvian price discrimination. Perloff and Wu (2007) are to my knowledge the first trying to find a relationship between tax incidences across consumer groups. However, their result that the ratio of after- and pre-tax prices differs from segment to segment just confirms that tax induced price changes do not depend on pre-tax prices. Price changes and therefore after-tax prices depend on some kind of demand and supply elasticities. Reformulating the Perloff and Wu result shows that they ask whether price changes follow the specific distribution of pre-tax prices, whereas I answer the question how prices change in comparison to one another.

In a first step price changes are considered. Their distribution pattern is invariant to the tax type as price changes of each segment are related exclusively by the elasticities of the slopes of the inverse demand functions. Consequently, there is no opportunity for fiscal discrimination. In a second step the significance of market shares for the distribution of tax burdens across consumer groups will be pointed out. Conventional incidence analysis can truly neglect quantity aspects as there exists only a single price and only one equilibrium quantity. Looking at the distribution of tax burdens across consumer groups, however, one clearly has to account for the amounts traded as they are integral parts of each segment's tax payment. Comparing unit and ad valorem taxes raising the same revenue now displays an opportunity for fiscal discrimination between consumer groups. However, such discrimination is erratic and is not due to price discrimination as such.

In chapter two the basic model is set up which is followed by chapter three establishing the impossibility result of fiscal discrimination. In chapter four the significance of relative market shares for the distribution of tax burden is pointed out establishing the discrimination possibility result. Chapter five concludes with some general remarks on the appropriate measure for the comparison of tax burdens.

2. The basic model

To clearly identify the effects of price discrimination on the distribution of the tax burden the model is based on the standard textbook example (e.g. Silberberg and Suen, 2001, p. 71f). A monopolist is faced with n distinct consumer groups, each having its own independent inverse demand function $p_i(x_i)$. All demand functions are downward sloping and at least twice differentiable³. Other than the bulk of the literature and Perloff and Wu, the model is not restricted to constant marginal costs. Hence, it is no longer possible to look at each segment as a single independent market. By this it is unfeasible to formulate and solve separate though by structure identical maximization problems as market segments are linked by the cost function. Costs depend on the sum of all equilibrium quantities, $C(\sum_{i=1}^n x_i)$. The government interferes with this market by sales taxes. Tax rates t and τ of unit and ad valorem tax, respectively are identical across segments so that government does not directly discriminate between different consumer groups. Both tax types are considered simultaneously leading to the following monopolist's profit

$$(1) \quad \pi(x_1, \dots, x_n) = \sum_{i=1}^n [(1 - \tau)p_i(x_i) - t]x_i - C(\sum_{i=1}^n x_i).$$

The resulting FOCs for $i=1, \dots, n$ are

$$(2) \quad (1 - \tau)(p_i' x_i - 2 p_i) - t = C'$$

with a prime denoting the derivative with respect to the particular quantity. In equilibrium, marginal revenue in segment i net of taxes equals marginal costs of producing x_i . Owing to the joint production of the homogeneous good for all segments - the monopolist ultimately decides on the distribution of total output - marginal costs in each and every segment reach the same value in equilibrium. Consequently, marginal revenues net of taxes must be identical

$$(3) \quad (1 - \tau)(p_i' x_i - 2 p_i) - t = (1 - \tau)(p_j' x_j - 2 p_j) - t \quad \text{for } i, j=1, \dots, n.$$

With government not directly discriminating between consumer groups tax rates for each segment are the same and so are gross marginal revenues $MR_i = p_i' x_i - 2 p_i$. This standard result for price discrimination is not affected by uniform tax rates of either type. As MR_i is

³ Seade (1980) shows the stability conditions for Cournot competition.

constant in i , the index is dropped in what follows. Second order conditions are assumed to hold.

3. Pattern of price changes

Conventional incidence analysis with a single consumer group only, studies how the tax burden is split between demand and supply. Consumers bear the tax in form of the gross price change, producers in the extent of the reduction⁴ of equilibrium net price. With perfect competition this reduction equals the decrease in marginal costs, with imperfect competition, however, the producers' share in tax payments is reflected in the absolute tax amount per unit minus the gross price increase. That is why tax analyses regularly concentrate on gross price changes. Following this, gross price changes are considered as it is completely sufficient for contrasting the effects of taxation in each segment with one another. Comparing price changes reveals a systematic relationship between the changes that depends on the particular elasticity of the slope of the inverse demand functions as will be seen below. In a first step the marginal quantity change of a segment, dx_i , is established leading to

Proposition 1: Next to the rate of change of the respective marginal revenue the quantity change in a single segment additionally depends on the quantity changes in all segments.

Differentiating the system of equations (2) for tax rates t and τ and all variables x_i leads to n equations

$$(4) \quad \sum_{j=1}^n \left(\frac{\partial MR_j}{\partial x_i} + \frac{\partial MR_j}{\partial t} + \frac{\partial MR_j}{\partial \tau} \right) dx_j + \frac{\partial MR_i}{\partial t} dt + \frac{\partial MR_i}{\partial \tau} d\tau = 0.$$

Having two degrees of freedom, the change in quantity, dx_i , in reaction to a change in tax rates $d\tau$ and dt is given by

$$(5) \quad dx_i = \frac{1}{\sum_{j=1}^n \left(\frac{\partial MR_j}{\partial x_i} + \frac{\partial MR_j}{\partial t} + \frac{\partial MR_j}{\partial \tau} \right)} \left(\frac{\partial MR_i}{\partial t} dt + \frac{\partial MR_i}{\partial \tau} d\tau \right).$$

The proof is deferred to the appendix. Marginal changes in one tax rate holding the other one constant affect equilibrium quantities in the same way bearing in mind that with the ad valorem tax the marginal revenue factor is to be included⁵. The first term in the denominator reflects the rate of change of the marginal revenue. In case of perfect competition this is the rate of the price change but clearly not so in case of imperfect competition. The second term

⁴ Net prices can also rise, for an example of „overshifting“ see Delipalla and Keen (1992, p. 356).

⁵ For an application of this standard result see Delipalla and Keen (1992, p. 357f).

presents the change in marginal costs. Contrary to conventional incidence analysis with only one consumer group and a single equilibrium quantity the change in marginal costs with price discrimination must include the quantity changes in all segments or total output. If x_i changes so do all the others unless the demand functions are completely inelastic. This relationship between segment quantities is covered in the final factor.

Price changes can now be found by multiplying the marginal change of quantities as established in (5) with the derivative of the particular inverse demand function. The comparison of tax induced price changes across segments yields

Proposition 2: With regard to the pattern of price changes governments cannot discriminate between different consumer groups by the choice of sales tax types.

Instead of equation (5) the identity condition (3) is reconsidered as this offers a much smarter way to compare price changes. In equilibrium marginal revenues in each segment are the same before and after the tax change. Therefore, changes in marginal revenue are the same for all segments

$$(6) \quad dMR_i | MR_i dx_i | MR_j dx_j | dMR_j .$$

Displaying the derivative of marginal revenue explicitly and factoring out $p_i \eta_i$ leads immediately to

$$(7) \quad dp_i (\eta_i + 2) | dp_j (\eta_j + 2)$$

with $\eta_i | p_i \frac{dx_i}{x_i}$ the elasticity of the slope of the inverse demand function (see Delipalla and

Keen, 1992, p. 354 and the literature cited there). Consider, for example, segment i as the base market, then price changes in all secondary segments follow the base market price change with factor $(\eta_i + 2) / (\eta_j + 2)$. Clearly, the pattern of price changes is completely independent of the sales tax type. Therefore, it is impossible for government to discriminate between consumer groups by the choice of tax type⁶.

Moreover, the pattern of price changes is obviously not as “uneven” as suggested by Perloff and Wu⁷ but reveals some systematics.

⁶ Looking at changes of consumer rents reveals the same distribution pattern thereby confirming the impossibility result.

⁷ “(...) a tax may have an uneven effect on various parts of the price distribution (...)”, Perloff and Wu (2007).

4. Pattern of tax burdens

Without any losses conventional incidence analysis can restrict itself to a comparison of mere price changes for demand and supply side. The amount traded is completely irrelevant for the distribution of the tax burden as demand equals supply. Quantities are still negligible studying the tax split between consumers and producers segment by segment. But to come to conclusions on the distribution of tax burdens across consumer groups the quantity aspect must necessarily be included⁸ unless quantities traded under the tax are all identical. The reason for this is simply that a price change only represents the absolute tax amount per unit of the good for the consumer. To arrive at the tax burden it has to be multiplied by the quantity traded under the tax. If after-tax quantities differ from segment to segment the distribution of tax burdens cannot exclusively depend on gross price changes. Instead, one can state

Proposition 3: Relative market shares determine the distribution of tax burdens next to the ratio of price changes.

The fraction of total tax revenue a particular consumer group has to bear with a unit tax imposed is

$$(8) \quad CF_i^t \mid \frac{dp_i x_i^t}{t \frac{n}{ij} x_i^t} \mid \frac{dp_i}{t} \frac{x_i^t}{\frac{n}{ij} x_i^t}.$$

On the right hand side this consumer fraction is represented by the product of the conventional incidence measure which is the gross price change per tax amount and the market share under taxation. Comparing consumer fractions of total tax revenue and plugging in the result of (7) one finds

$$(9) \quad \frac{CF_i^t}{CF_j^t} \mid \frac{dp_i x_i^t}{dp_j x_j^t} \mid \frac{(\frac{j}{i} 2 2) x_i^t}{(\frac{i}{j} 2 2) x_j^t}.$$

Next to the elasticities already explained the distribution of tax burden is determined by the relative market share after taxation.

Analogously to (8), a consumer group's fraction under ad valorem taxation is

$$(10) \quad CF_i \mid \frac{dp_i x_i}{\frac{n}{ij} p_i x_i} \mid \frac{dp_i}{\frac{n}{jl} p_j \frac{x_j}{x_i}}.$$

⁸ Folkers (1988, p. 227 ff) explicitly checks on after-tax quantities studying tax exemptions for taxpayers with different tax rates.

The right side of the equation again explicitly displays the difference to the conventional incidence measure. The tax rate is not multiplied by the after tax gross price of the particular segment p_i but by an “average” price meaning that total sales are divided by the single segment’s quantity. Identical in structure with (9) the relationship between two consumer fractions of total tax revenue is

$$(11) \quad \frac{CF_i}{CF_j} \Big| \frac{dp_i x_i}{dp_j x_j} \Big| \frac{(\frac{2}{j}) x_i}{(\frac{2}{i}) x_j} .$$

In contrast to proposition 3 which states the impossibility of fiscal discrimination, the comparison of tax types concerning relative tax burdens leads to the unapparent

Proposition 4: With regard to the relative tax burden governments can discriminate between different consumer groups by the choice of sales tax types.

Though looking similar the equivalence of equations (9) and (11) only regards the structure and not the numerical values which clearly is a limitation of the identity result of tax effects. Contrasting different tax types only reveals thorough information when both yield the same revenue in the end. From differential incidence analysis it is well known that ad valorem taxes dominate unit taxes in efficiency terms in cases of monopolistic competition. This result is confirmed for price discrimination as well (see Yang, 1993, p. 373 and Cheung, 1998, p. 1200). Therefore, with unit and ad valorem taxes resulting in different equilibrium quantities it cannot be taken for granted that relative market shares equal each other across tax types. Relative market shares and hence relative tax burdens are instead very likely to differ. For an outstanding example assume two marginal revenue curves intersecting in some point. At this point relative market share equals one. Assume equilibrium quantities before taxes to be somewhere on the right of the intersection. Now an ad valorem tax is imposed resulting in an equilibrium slightly on the right of the intersection with a relative market share just smaller (bigger) than one. The unit tax yielding the same revenue leads to greater price distortions and consequently, equilibrium quantities are found on the left of the intersection with relative market share now being bigger (smaller) than one.

This clearly shows some potential for fiscal discrimination with regard to the distribution of tax burdens across consumer groups. However, this potential is limited in scope and erratic in the sense that one cannot say in general for all goods and services which consumer group identified by pre-tax prices or pre-tax market shares is discriminated against by one tax type

or the other. The discrimination is rather piecemeal and must separately be studied for any good and service that is traded in price discriminatory fashion. The reason for this is simply that the quantity traded under a tax regime does not only depend on the shape of the demand function but additionally on its position as Folkers (1988, p. 230) shows with a linear example. With the relative market share as an integral part of the tax burden distribution it is decisive whether the quantity of a segment is 100 or reaches 1000. The (relative) positions of the other demand functions clearly matter, too. The result that the government is able to discriminate between consumer groups, though only indirectly, is obviously not due to price discrimination per se but as the arguments show caused by the distinct effects of unit and ad valorem taxes in imperfectly competitive settings. Independently of a certain market segment the ad valorem tax leads to less distortion.

5. Conclusion

Whenever different consumers pay different prices for one and the same good or service it is necessary to extend conventional tax incidence analysis in two directions. Firstly, conventional analysis of how the burden is split between demand and supply has to be accomplished for each market segment separately. Secondly, the question of how the burden is shared between the different segments and hence consumer groups awaits an answer. This was given here. The analysis showed that starting from some base market tax induced price changes in all other segments follow systematically according to the particular elasticities of the slope of the inverse demand functions. Additionally, the pattern of price changes is invariant with regard to the sales tax type so that an impossibility for fiscal discrimination between consumer groups arises. Averting away from price changes towards the distribution of tax burden one has to include equilibrium quantities into the analysis. Relative market shares enter the distributional analysis as a factor next to relative price changes. With this an opportunity for fiscal discrimination opens up. However, this potential is limited and rather an effect of a sufficient monopolistic leeway on the supply side.

Contrasting both sales taxes with regard to the incidence between consumer groups could also have been done on the basis of consumer surpluses or their marginal changes. The latter would have confirmed the impossibility result, the former the possibility outcome with the limitation that without more information about the different demand functions⁹ one cannot say which consumer group is worse off by one or the other tax type. The overall result of ad valorem taxes generating less distortion persists. Which measure - be it levels or their changes

⁹ Trandel (1999) like Hines, Hlinko and Lubke (1995) study linear demand curves only.

- should underlie the study of incidence between consumer groups depends on the analyst's aims and was not discussed here. Instead, positive analysis was predominant leading to the insight that changes in prices and consumer surpluses occur in a fix systematic way. If the underlying levels price, tax burden and consumer surplus follow any different pattern or none at all, the distribution assessment must inevitably differ. From exactly this perspective the uneven result of Perloff and Wu is to reconsider. They just compare changes with the underlying levels and cannot find any systematic outcome.

6. Appendix

The proof of equation (5) is performed most conveniently in matrix form. Totally differentiating the system of FOCs (2) leads to n equations of form

$$(A1) \quad (MR_1 \dots MR_n) \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} + \begin{pmatrix} C_{11} & \dots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \dots & C_{nn} \end{pmatrix} \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} = \begin{pmatrix} dMR_1 \\ \vdots \\ dMR_n \end{pmatrix} dt.$$

Marginal revenue MR is not indexed as it is the same in all segments. In matrix form the left hand side can be displayed as the vector of quantity changes times the sum of a $-C''$ matrix with the negative of the second derivative of the cost function for every element and a diagonal matrix containing the derivatives of the marginal revenues

(A2)

$$\left(\begin{matrix} (MR_1) \\ (C_{11}) \\ \vdots \\ (C_{1n}) \\ (C_{21}) \\ \vdots \\ (C_{nn}) \\ (MR_n) \end{matrix} \right) \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} + \begin{pmatrix} (C_{11}) & \dots & (C_{1n}) \\ \vdots & \ddots & \vdots \\ (C_{n1}) & \dots & (C_{nn}) \end{pmatrix} \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} = \begin{pmatrix} dMR_1 \\ \vdots \\ dMR_n \end{pmatrix} dt.$$

Multiplying out on the left hand side and summing the dx_j leads to

$$(A3) \quad \begin{pmatrix} (MR_1) \\ (C_{11}) \\ \vdots \\ (C_{1n}) \\ (C_{21}) \\ \vdots \\ (C_{nn}) \\ (MR_n) \end{pmatrix} \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} + \begin{pmatrix} (C_{11}) & \dots & (C_{1n}) \\ \vdots & \ddots & \vdots \\ (C_{n1}) & \dots & (C_{nn}) \end{pmatrix} \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} = \begin{pmatrix} dMR_1 \\ \vdots \\ dMR_n \end{pmatrix} dt.$$

Adding the first summand on both sides and multiplying with the inverse of the diagonal matrix results in

$$(A4) \begin{pmatrix} dx_1 \\ \vdots \\ dx_n \end{pmatrix} \begin{pmatrix} \frac{1}{(14)MR_i} & 0 & 3 & 0 \\ 0 & 6 & 4 & 4 \\ 4 & 6 & 0 & 4 \\ 0 & 3 & 0 & \frac{1}{(14)MR_n} \end{pmatrix} \begin{pmatrix} MR_i d^2 dt^2 C_{j1}^p dx_j \\ \vdots \\ MR_i d^2 dt^2 C_{j1}^p dx_j \end{pmatrix}.$$

The quantity change in a single segment then is

$$(A5) \quad dx_i \left| MR_i \frac{1}{(14)MR_i} d^2 \frac{1}{(14)MR_i} dt^2 \frac{C_{j1}^p dx_j}{(14)MR_i} \frac{1}{dx_i} dx_i \right.$$

Subtracting the final term on both sides, factoring out dx_i and rearranging terms again gives

$$(A6) \quad dx_i \left| MR_i \frac{1}{(14)MR_i} d^2 \frac{1}{(14)MR_i} dt^2 \right.$$

References

Blackorby, C. and S. Murty, 2007, Unit versus ad valorem taxes: monopoly in general equilibrium, *Journal of Public Economics* 91, p. 817 – 822.

Carlton, D. W. and J. M. Perloff, 2005, *Modern Industrial Organization*, 4th ed., Boston.

Cheung, F. K., 1998, Excise taxes on a non-uniform pricing monopoly: ad valorem and unit taxes compared, *Canadian Journal of Economics* 31, p. 1192 – 1203.

Delipalla, S. and M. Keen, 1992, The comparison between ad valorem and specific taxation under imperfect competition, *Journal of Public Economics* 49, p. 351 – 367.

Folkers, C., 1988, Wer wird durch Steuervergünstigungen eigentlich begünstigt? Zu einem Problem der materiellen Inzidenz, *Finanzarchiv N.F.* 46, p. 214 – 235.

Hines, J. R., J. C. Hlinko and T. J. F. Lubke, 1995, From each according to his surplus: Equi-proportionate sharing of commodity tax burdens, *Journal of Public Economics* 58, p. 417 – 428.

IATA, International Air Transport Association, 2005, *IATA list of ticket and airport taxes and fees*, Montreal and Geneva: IATA.

Landau Report, 2004, *Working on new international contributions to finance development*, report to president Chirac.

Myles, G. D., 1996, Imperfect competition and the optimal combination of ad valorem and specific taxation, *International Tax and Public Finance* 3, p. 29 – 44.

Perloff, J. M. and X. Wu, 2007, Tax incidence varies across the price distribution, *Economics Letters* 96, p. 116 – 119.

Seade, J., 1980, The stability of Cournot revisited, *Journal of Economic Theory* 23, p. 15 – 27.

Silberberg, E. and W. Suen, 2001, *The structure of economics*, 3rd ed., McGraw-Hill.

Suits, D. B. and R.A. Musgrave, 1953, Ad valorem and unit taxes compared, *Quarterly Journal of Economics* 67, p.598 – 604.

Trandel, G. A., 1999, Producers lose: The relative percentage reductions in surplus due to an ad valorem commodity tax, *Public Finance Review* 27,1, p.96 – 104.

Yang, C. W., 1993, Taxation and the output and welfare of third-degree price discrimination, *Southern Economic Journal* 59, p. 371 – 384.