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Money, E-money, and Consumer Welfare*

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Abstract

We develop a micro-founded monetary model to inquire the role of a privately provided e-money instrument for household consumption smoothing and welfare. Different from fiat money, e-money users pay electronic transaction fees, but in turn e-money reduces spatial separation frictions and enables risk-sharing. We characterize the conditions that promotes e-money to be Pareto improving and the conditions when e-money reduces its users' welfare - despite for the consumption-smoothing it induces. We calibrate our model for the context of M-Pesa in Kenya and conduct a quantitative analysis. Since our quantitative analysis reveals a limited role for privately provided e-money, we recommend the optimality of e-money regulation.

Keywords: E-Money, M-Pesa, Risk-Sharing, Welfare, Monetary Policy

JEL Classification Numbers: E41, E44, G23, O11.

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1 Introduction

Technological innovations in electronic money products are enabling the financial integration of un-banked individuals in developing countries. In particular, a growing line of development research documents that Kenya's M-Pesa revolution helped with consumption insurance and poverty alleviation of households (Jack and Suri (2014) and Suri and Jack (2016)) and allowed credit access for small-businesses (Beck et al. (2018) and Dalton et al. (2022)).

Despite this promise, there are also some concerns related to the realized welfare effects of electronic money technologies (hereafter, e-money): e-money instruments compete with standard forms of currency as media of exchange; but the provision of e-money services are usually catered through monopolistic technology operators, as in the case of M-Pesa. The consumer welfare cost of this market structure has been highlighted as an important challenge by policy makers.[1]

A careful welfare analysis of e-money has been overlooked in the literature, as studies that aim to understand the developmental consequences of e-money products mainly build upon non-monetary foundations. Our paper aims to fill this gap in the literature and evaluate the consumer welfare effects of e-money by incorporating the stylized facts that e-money is provided by private entities and e-money and existing currency of a country are competing media of exchange. In this regard, we provide the first account of qualifying and quantifying the welfare consequences of e-money based on a monetary model, in which (public) fiat money and privately provided e-money serve as essential means of payments to settle transactions.[2]

Our paper aims to address the following questions: What are the welfare implications of introducing e-money products for consumers, who demand both fiat money and e-money payment instruments for their transactions? Should the provision of e-money be regulated by public authorities; and if so, what are the effective means of policy instruments that would improve consumer welfare? To address these questions we develop a micro-founded monetary model in which both fiat money and e-money could serve as payment instruments. We model the poten-

[1] As of 2022 M-Pesa has a 99% market share in Kenya's mobile money market. The high transaction fees charged on consumers has been a growing concern. Central Bank of Kenya is planning to introduce its own Digital Currency to reduce the cost of electronic money transactions for the final users (see Discussion Paper on Central Bank Digital Currency, 2022, by Central Bank of Kenya).

[2] For a definition of money essentiality see Williamson and Wright (2010).

tial of an e-money product in mobilizing insurance schemes among individuals who are spatially separated but socially connected:[3] in the model, e-money solves spatial separation frictions that fiat money is subject to, but its usage comes with electronic transaction fees - set by monopolistic technology providers with private profit incentives.

We build upon the frameworks by Lagos and Wright (2005), Rocheteau and Wright (2005), and Berentsen et al. (2007) and develop a search theoretic framework of money and e-money. In our model, fiat money and e-money are essential to settle transactions in a market that exhibits bilateral matching frictions. Our model's consumers (buyers) belong to a network (family) of two socially connected buyers and are subject to idiosyncratic income shocks that generate room for risk-sharing and financial integration among the members of a family: a low income shock implies demand for external support from the family member in improving consumption purchases while a high income shock implies desire to support the family member's consumption purchases - and in turn smooth consumption across different states of nature. Within-family insurance is possible, but the feasibility of the insurance arrangement is subject to a spatial separation friction: buyers within a family are spatially separated when they learn the realization of idiosyncratic income shocks and fiat money is not transferable across space. However, e-money units issued by the technology provider can travel across space and induce financial integration among spatially separated individuals.

As a benchmark solution we first characterize a stationary equilibrium where only fiat money is accepted as a medium of exchange. In this economy buyers self-insure against income shocks, which implies an ex-post inefficiency in money holdings, as in Berentsen et al. (2007): depending on the realization of the income shock, buyers are either cash-constrained when they have a low income shock or they have idle cash balances when the income shock is high. This implies that there is room for ex-post trade and ex-ante insurance arrangements, which e-money can materialize.

Building upon this benchmark, we investigate the equilibrium allocations when

[3]The introduction of e-money instruments plays an important role in reducing transaction frictions in developing countries, where formal and informal financial networks are limited and thus consumption volatility is high. A growing body of empirical research argues that the development of e-money instruments in such context have been improving risk-sharing opportunities among otherwise un-banked individuals. For instance, in their seminal paper Jack and Suri (2014) show that Kenya's M-Pesa revolution mobilized existing social networks and substantially reduced consumption volatility.

both fiat money and e-money circulate and are accepted as means of payment. In this financially more developed economy, buyers can still self-insure against the income shock with precautionary cash holdings. In addition, they can also sign an insurance contract within their family network. The insurance contract assigns the family member with the high income shock with the obligation to acquire e-money units from the technology provider and transfer them to the family member with the low income shock. This risk-sharing (insurance) agreement in-between the two members of a family, as observed in the context of M-Pesa, generates the demand for e-money.

Our first result shows that e-money could improve the net welfare of consumers by helping to mobilize their insurance agreements. However, as a surprising key finding, we also observe that the positive welfare effect could only prevail when the scope of insurance is not so large among the members of a family. The technology provider could extract all the surplus when the dispersion in income shocks is large enough by charging a large e-money transaction fee.

Second, we evaluate the consumer welfare implications of e-money introduction, comparing allocations of an economy with only fiat money against those of an economy where e-money is in place. Our analysis shows that e-money adoption has real effects on consumption allocations and improves consumer welfare when the equilibrium conversion fee is such that buyers benefit from saving idle cash balances. The reason is that buyers could always replicate the equilibrium consumption allocation by acquiring enough cash balances and not resorting to the e-money technology. When the equilibrium conversion fee makes buyers indifferent between adopting e-money and only using fiat money, consumption allocations are identical to the benchmark case (of no e-money in place). Differently, when buyers strictly prefer to make use of e-money, their consumption improves relative to the benchmark case.

What is important to highlight is that the introduction of the e-money technology has general equilibrium price effects, i.e., a pecuniary externality exists. Because the technology provider earns profits in equilibrium, the unit price of fiat money adjusts and buyers need to work more to acquire nominal balances relative to an economy without e-money. This key equilibrium property implies that the introduction of the payment technology may have adverse distributional consequences for buyers, motivating the importance of modeling the co-existence fiat money and privately provided e-money.

We show that lump-sum taxation of the technology provider can move the economy with e-money to a Pareto superior allocation if and only if the scope of insur-

ance is small enough. In that particular case, our analysis shows that it is possible to redistribute profits to increase consumer welfare and overcome the pecuniary externality due to e-money adoption. However, when the scope of insurance is large, the most that taxation can do is to achieve the same allocation efficiency of the economy with fiat money only. In this respect, our findings are highly relevant for e-money development policies that aim to stimulate financial inclusion of low-income households. We show that in a context where the scope of insurance is large, monopolistic provision of e-money may cause welfare losses and redistributive taxes are ineffective to reduce the welfare losses, arguing for the regulation of the e-money sector and influencing its degree of competitiveness.

Finally, we complete our analysis by conducting a quantitative exercise. We calibrate an economy without e-money technology using Kenyan data for the period of 2000-2007, during which there was no M-Pesa instrument in place. We then introduce e-money to this economy and study consumption smoothing (risk-sharing) and consumer welfare consequences of an e-money product, that resembles M-Pesa. Our analysis reveals that alhough M-Pesa improves risk-sharing, it leads to a relatively small welfare improvement due to the pecuniary externality implied by the monopolistic provision of the e-money product.

Our paper contributes to two recent - yet fast growing - strands of literature. On the one hand, there is a growing interest in understanding the essentiality of privately issued monies in the form of electronic money instruments (Bitcoin, Ethereum, and Paypal for instance) for aggregate allocations and macro outcomes and the optimal monetary/regulatory policy design in environments with private money.[4] In this line of research, Chiu and Wong (2015) adopt a mechanism design approach in order to explore the essentiality of e-money to implement constrained efficient allocations and show that "exclusive participation" and "discretionary participation" attributes of electronic money instruments could help to achieve this objective. Fernández-Villaverde and Sanches (2019) develop a framework to understand how competition between privately issued electronic currencies can work and the implications of this currency competition for monetary policy making.[5]

On the other hand, there is another recent literature on e-money and economic development, which almost exclusively concentrates on the mobile money (M-Pesa)

[4]In an economy with fiat money only, Berentsen (2006) inquires the feasibility and optimality of the private provision of fiat currency.

[5]The emergence of digital payments also raised privacy concerns. See Parlour et al. (2022), Ahnert et al. (2022), and Kang (2021).

revolution of Kenya from the last decade. In this research frontier, while Jack and Suri (2014) study the effect of reduced transaction costs on risk sharing, showing that income shocks lower consumption by 7% for non-M-Pesa users whereas consumption of M-Pesa-users is unaffected, Suri and Jack (2016) quantify the overall effect of M-Pesa as a 2% reduction in poverty in Kenya. Also in this strand of research, Beck et al. (2018) quantify the aggregate implications of access to finance through M-Pesa use of entrepreneurial firms in a general equilibrium model with endogenous payment instruments choice, whereas Dalton et al. (2022) design a Randomized-Controlled-Trial and explore the adoption of an M-Pesa based payment instrument among Small-and-Medium-Sized Enterprises.

The former literature is based on search theoretic models of e-money with microfoundations and does not show an interest in development relevant properties of electronic money instruments, such as the potential of e-money in reducing spatial separation constraints and inducing risk sharing. The latter literature fully abstracts from monetary microfoundations in order to study the economic development implications of e-money adoption among households and entrepreneurial firms. This paper attempts to close the gap in the literature by setting up a microfounded model of e-money that features the characteristic attributes of electronic money instruments observed in the context of developing countries to understand the welfare implications of e-money and the regulation thereof.

Our framework builds upon the New-Monetarist style of microfounded monetary economics, pioneered by Lagos and Wright (2005) and Rocheteau and Wright (2005). In this class of monetary models, frictions in a decentralized goods market give a microfounded role to monetary exchange, based on which interactions between money and financial arrangements are studied - as in our research. In our analysis we are closely related to the branch of New-Monetarist literature that studies how financial intermediaries may improve allocations and welfare by allowing to redistribute cash balances from agents with no urgency to spend to agents who are cash constrained. In particular, Berentsen et al. (2007) show that financial intermediation improves allocation by transferring money from agents with a low marginal value of consumption to those with a high valuation. In their model, the welfare gains come from the payment of interest to agents holding idle cash balances. We complement their work studying a new type of intermediation which allows insurance within a network and exhibits realistic features of the mobile money market in developing countries.

2 Institutional Context

Our model has far reaching implications for the broad e-money industry. However, since in the context of developing countries, Kenya's M-Pesa has revolutionized the financial integration of otherwise un-banked individuals, in our theoretical model and the quantitative analysis we focus on some specific aspects of M-Pesa. Therefore, in this section we provide an overview of M-Pesa's development, its usage and regulation and how we capture these elements in our theoretical model.

What is M-Pesa and how does it work? M-Pesa is a form of e-money, that circulates among its users through mobile-phone text messages; and therefore, it belongs to an e-money category broadly referred to as Mobile Money. M-Pesa technology is operated by a privately held company, Safaricom. In specialized mobile money agents (Kiosks), which are wide-spread across the country, cash (fiat money) can be converted into M-Pesa units - and stored in a ring-fenced bank account linked to the M-Pesa account of the owner - and vice versa. An individual does not need to have her own bank account to apply for M-Pesa; the ring-fenced bank account access is provided upon signing up for M-Pesa itself. In this respect, M-Pesa is specifically designed to benefit individuals who have no access to banks - either because they do not have a bank account or because they are located too far away from a bank branch.

Exchanging cash for M-Pesa deposits is free. The individual only has to visit the mobile money agent and tell the phone number that she wants to deposit money into. However, using M-Pesa comes with electronic transaction fees determined by Safaricom and applied when converting M-Pesa to cash as well as variable costs of electronic money transfers increasing in the amount sent. In addition to facilitating person to person (P2P) transfers, M-Pesa users can pay utility bills, and make purchases in stores (P2B). There are also some mobile money services through which businesses can send salaries to mobile phones of their workers, repay loans and engage in transactions with suppliers (B2B).

After its launch in 2007, M-Pesa became a popular monetary instrument in Kenya. For instance, Jack and Suri (2014) document that by 2011 70% of all households in Kenya had already adopted at least one M-Pesa account. From March 2007 to December 2014 the number of M-PESA Kiosks grew by 148% annually and reached about 124,000 (about 20% of them in Nairobi (FSP inter-active maps, 2013)), and the number of customers grew by 307% annually and reached about 25 million. In 2013, about 732.5 million transactions were conducted in total, and the total value of money transferred was 22 billion U.S dollars. Since 2007 Kenyan

households have utilized M-Pesa for not only transferring or receiving money but also for saving: 85% of Kenyan households store some money in their personal M-Pesa account according to the survey evidence provided by Jack and Suri (2011). In all these accounts the development of M-Pesa exhibits one of the most remarkable financial development experiences that the world has ever seen.

M-Pesa changed the landscape of monetary instruments in Kenya dramatically and helped with the financial integration of otherwise un-banked and spatially separated individuals. On the one hand, before the introduction of M-Pesa, access to electronic money transfer had been limited, and other forms of electronic money instruments, such as Western Union, were too costly to transfer money for the general population. On the other hand, cheap money transfer methods such as bringing cash personally or sending via friends had been common but were subject to risk of appropriation and theft.

Regulation of M-Pesa. As noted above, M-Pesa is provided by a private enterprise. Moreover, it has the monopoly power in the market: as of 2022 M-Pesa remains as the only mobile money product in Kenya, since it serves to 99% of Kenya's mobile money demand. However, M-Pesa operations are subject to government regulation and policy. In this regard, an important aspect stands out: M-Pesa deposits are ring-fenced by regulation. This means every unit of cash converted into M-Pesa is deposited in a trust account and is readily available to withdraw on demand. This safety measure, representing a form of 100% reserve banking, is imposed to ensure trust in the electronic payment instrument and financial stability.

Which e-money features does our model capture? We model a private technology operator which provides e-money tokens. E-money in our model can be transferred among economic agents (P2P), who are spatially separated from each other but socially connected, and it can also be used to make purchases (P2B) and liquidated for cash. As in the case of M-Pesa, in our model ring-fenced cash deposits are needed to create e-money units and in that respect the form of e-money that we study resembles the regulatory context of 100% reserve banking. Also, as we observe in M-Pesa context, the technology provider of our model determines the usage fees by taking the demand for its electronic money product as given. Finally, and perhaps most importantly, we will model the private provision of M-Pesa through a monopolist technology operator, and this market structure is going to be our core focus of attention with respect to policy analysis and recommendations.

3 The Model

The model builds on Rocheteau and Wright (2005), with additional features to address the specific features of e-money. Time is discrete and indexed by $t = 0, 1, 2...$ Each period is divided in two sub-periods: in the first sub-period, market trade is anonymous and - thus - decentralized (hereafter DM), whereas in the second sub-period trade takes place in a Walrasian centralized market (hereafter CM). The anonymity of exchange in DM rules out credit arrangements, so that trade must be settled immediately. Two nominal objects in the economy can be used to settle trade in DM: money and e-money. Exchange in CM is frictionless.

Government controls the supply of fiat money through periodic lump-sum transfers. In addition to the government, there are three types of agents in the economy: i) buyers, ii) sellers, and iii) technology providers of electronic money. Buyers and sellers are both infinitely lived, with a continuum of measure one population size of each type. Sellers produce the specialized good that buyers like to consume, and sell the specialized good to buyers in DM. Technology providers issue electronic money units (e-money) and extend e-money to the other agents in economy in exchange of fiat money, and redeem e-money in return for fiat money by charging an e-money conversion fee for their service. Technology providers are finitely lived, and there is a single technology provider in any given time period. Specifically, a monopolist technology provider enters the economy in each CM and lives for one period, until the next CM_{+1}, when a new technology replaces her.

Locations, Endowments, Technologies, and Preferences. In CM, all buyers and sellers meet in a centralized location, where they produce and consume a general good. Both buyers and sellers have access to a linear production technology that transforms one unit of their own labor in CM into one unit of the general (CM) good. We take the CM good as the numeraire.

In DM, buyers and sellers visit one of two symmetric, distinct, and spatially separated locations, namely location a and location b. An important feature of the model environment is that buyers belong to a family. Formally, a family is a pair of buyers $\{i, j\}$ where $i, j \in [0, 1]$, $i < j$. We assume that buyers $\{i, j\}$ belonging to the same family visit distinct locations, i.e, if i visits a, j visits b. In each location, buyers and sellers are bilaterally and randomly matched, so that the meetings are anonymous. We assume that the law of large number holds, which implies that in both locations every buyer meets a seller with certainty. In DM, sellers produce the specialized good for the buyers by utilizing a linear production technology that transforms one unit of their own labor into one unit of the specialized (DM) good.

The preferences of buyers (B) are described as follows:

$$\sum_{t=0}^{\infty} \beta^t \left[U(X_t^B) - H_t^B + u(q_t^B) \right], \tag{1}$$

where X_t^B is buyer's CM consumption, H_t^B is her disutility from exerting labor efforts to produce the CM good, and q_t^B is her DM consumption. The utility functions $U(\cdot)$ and $u(.)$ satisfy standard properties, i.e. $u' > 0 > u''$, and $U' > 0 > U''$. Similarly, the preferences of sellers (S) are given by:

$$\sum_{t=0}^{\infty} \beta^t \left[U(X_t^S) - H_t^S - h_t^S \right], \tag{2}$$

where X_t^B is seller's CM consumption, H_t^S is the disutility from exerting labor efforts to produce the CM good, and h_t^S is the disutility from exerting labor effort to produce h_t^S units of the DM good. Finally, a technology provider (TP), who enters the economy in CM, likes to consume in CM_{t+1} according to the linear utility:

$$\mathcal{U}_t^{TP}(X_t^{TP}) = X_{t+1}^{TP}.$$

At the beginning of each DM, after the location assignments to a and b are completed, buyers receive an endowment of $\epsilon \in \{\epsilon_H, \epsilon_L\}$ units of the CM good, where $\epsilon_H > \epsilon_L = 0$. The endowment can be stored until the next sub-period (CM), and perishes then if it is not consumed. There are two possible states of nature, $\tilde{s} \in \{s_1, s_2\}$, determining the endowment profile of each family. When $\tilde{s} = s_1$, the endowment profile of family $\{i, j\}$ is $(\epsilon^i, \epsilon^j) = (\epsilon_H, 0)$, whereas when $\tilde{s} = s_2$, it is $(\epsilon^i, \epsilon^j) = (0, \epsilon_H)$.[6] We assume that the two states of nature are equally likely, such that $Prob(\tilde{s} = s_1) = Prob(\tilde{s} = s_2) = 1/2$.

Fiat Money and E-Money. In DM, since the bilateral meetings between buyers and sellers are anonymous in both a and b, credit is not feasible and payments have to be settled immediately. The payment can be settled by using the CM good endowment that a buyer receives in DM, by (fiat) money, or e-money. The government prints and introduces fiat money through lump-sum transfers to buyers

[6]The normalization $\epsilon_L = 0$ is without loss of generality. This endowment heterogeneity is going to allow us to obtain ex-post heterogeneity with respect to the use of cash balances, as in Berentsen et al. (2007).

in CM, such that

$$M_{t+1} = M_t + \tau_t, \tag{3}$$

where M_t is the aggregate stock of fiat money in t and τ_t is the money transfer to the buyers in CM_t. We will concentrate on stationary equilibria with constant fiat money growth with

$$M_{t+1} = \gamma M_t, \tag{4}$$

where γ is the money growth rate between two time periods. We assume $\gamma > \beta$; this condition ensures that in stationary equilibria the economy is away from the "Friedman-rule", so that the opportunity cost of using a nominal object as a medium of exchange is strictly positive. An important feature of fiat money is that it cannot be transferred in-between the two locations in DM, and thus it is subject to a *spatial separation friction*.

Each cohort of technology providers issues e-money tokens. For each unit of e-money issued, the technology provider must keep one unit of fiat money in reserves. We impose this 100% reserve banking property in order to resemble important real-world electronic payment counterparts, importantly the case of M-Pesa in the context of Kenya, where by regulation the provider of electronic money has to back every unit of e-money issued with one unit of fiat money kept in the reserves. In addition, the monopolist provision of e-money in our model also closely relates to the context of Kenya's M-Pesa by Safaricom, who services 99% of the country's mobile money demand as of November 2022. The electronic tokens are issued upon receipt of fiat money in both locations a and b of DM. Unlike for the case of fiat money, the transfer of e-money is not subject to the spatial separation friction. Since each technology provider exists in the economy only for one time period, e-money units cannot be stored in between time periods t and $t+1$ and therefore must be converted back into fiat money by the end of each CM.

There are five possible transactions in the economy that involve e-money: (i) acquiring e-money in exchange of fiat money (between a buyer (or a seller) and the technology provider), (ii) the transfer of e-money in-between the members of a family (between two buyers), (iii) the purchase of a specialized good in DM in exchange of e-money (between a buyer and a seller), (iv) the purchase of a general good in CM in exchange of e-money (between any two buyers or between a buyer and a seller), and (v) acquiring fiat money in exchange of e-money (between a buyer (or a seller) and the technology provider).

We assume that only the last one of these transactions, which we call as the conversion of e-money into fiat money, involve a transaction fee. The conversion

fee gets determined by the technology provider by taking the expected demand for e-money as given. Specifically, for each unit of e-money redeemed, the technology provider receives α units of fiat money

The existence of the electronic money instrument allows for financial integration, since it is not subject to the spatial separation friction, enabling the transfer of funds between buyers. In that respect, we assume that in CM the two buyers from a family could sign a family-insurance contract (an agreement). The insurance contract is a function of $(\tilde{e}^{\tilde{s}}_{i,j})$, specifying a transfer in units of e-money from member i to member j based on the realization of the state of nature. We assume that buyers within a family can commit to their promised actions, which means that family members always honor the terms of the insurance agreement with each other.

Timing. The sequence and features of transactions in each sub-period are illustrated in Figure 1. At the beginning of each time-period t, in DM, buyers get randomly allocated to locations a or location b and learn the realization of their endowment ϵ_t after the location assignment. Conditional on the realized state of nature $\tilde{s}$, they receive (transfer) e-money from (to) the other buyer within the same family according to the insurance contract they signed in CM_{t-1}. Then, the technology provider, who entered the economy in CM_{t-1}, converts fiat money units into e-money at par. After that, sellers and buyers get bilaterally matched in DM, and production of the specialized good takes place in exchange for the CM good, money, and/or e-money; and DM closes. In CM, e-money is converted back into fiat money, the government transfers fiat money balances to the buyers (τ_t), buyers and sellers produce and consume the CM good, the one-period lived technology operator exits the economy and gets replaced by a new operator, who announces the conversion fee, α_{t+1}, that will be implemented CM_{+1}. We assume that technology providers can credibly commit to the pre-announced α_{+1}.[7] Then, taking the announced conversion fee as given, the insurance contract within each family is signed and CM closes.

[7]This assumption is necessary, given that the technology provider lives for one period. The time-consistent announcement strategies of an infinitely lived (monopolist) provider of fiat currency is studied in Berentsen (2006).

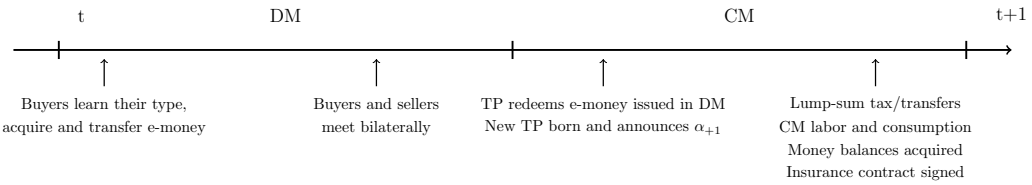

Figure 1: Timing of Events

In the next two sections we will provide solutions for two alternative economies: (i) an economy that features only fiat money, and (ii) an economy where e-money is in place and technology providers issue e-money units and thus fiat money and e-money coexist as media of exchange. Solving the model without e-money serves as a natural benchmark to evaluate welfare benefits from having an e-money instrument in place.

4 The Economy without E-money

We analyze the properties of an economy where fiat money is the only nominal object that can be used to settle transactions in DM. We focus on stationary monetary equilibria, defined as follows.

Definition 4.1. *Given a constant money growth rate γ, a stationary monetary equilibrium consists of CM decisions $\{\hat{X}^{B,k}, \hat{X}^{S,k}, \hat{H}^{B,k}, \hat{H}^{S,k}, \hat{\phi m}, \hat{\phi m}^{S}\}$, DM terms of trade $\{\hat{q}^k, \hat{d}^k_m\}$, for $k = L, H$, and fiat money transfers $\hat{\phi\tau}$, such that buyers and sellers maximize utility and markets clear.*

Note that, in the definition of stationary monetary equilibria, we only consider allocations that are independent of buyers' locations. However, it is possible to show that this is without loss of generality, as locations are symmetric and identical to each other. We adopt $\hat{V}^B(\hat{m})$ to denote the expected value for a buyer from entering DM with $\hat{m}$ units of fiat money and $\hat{W}^B(\hat{m}, \hat{\theta})$ to denote the expected value for a buyer from entering CM with $\hat{m}$ units of fiat money and $\hat{\theta}$ units of the CM good stored from the previous DM. Similarly, let $\hat{V}^S(\hat{m})$ and $\hat{W}^S(\hat{m}, \hat{\theta})$ be the corresponding value functions for a seller in DM and in CM.

The value function of a buyer who enters CM with $\hat{m}$ fiat money holdings and $\hat{\theta}$ units of the CM good stored from the previous sub-period is formalized as

$$\hat{W}^B(\hat{m}, \hat{\theta}) = \max_{\{\hat{X}^B, \hat{H}^B, \hat{m}_{+1}\}} \left\{ U(\hat{X}^B) - H^B + \beta \hat{V}^B(\hat{m}_{+1}) \right\},$$

$$s.t. \quad \hat{X}^B + \hat{\phi}\hat{m}_{+1} = \hat{\phi}\hat{m} + \hat{\phi}\tau + \hat{H}^B + \hat{\theta}.$$

where $\hat{\phi}$ is the price of money in terms of the CM good. Similarly, a seller, who enters CM with a portfolio of $(\hat{m}^S, \hat{\theta}^S)$, solves:

$$\begin{aligned}\hat{W}^S(\hat{m}^S, \hat{\theta}^S) = \max_{\{\hat{X}^S, \hat{H}^S, \hat{m}^S_{+1}\}} &\left\{U(\hat{X}^S) - H^S + \beta \hat{V}^S(\hat{m}_{+1})^S\right\}, \\ s.t. \quad \hat{X}^S + \hat{\phi}\hat{m}^S_{+1} &= \hat{\phi}\hat{m}^S + \hat{H}^S + \hat{\theta}^S.\end{aligned}$$

The first-order conditions that characterize an interior solution are[8]

$$\begin{aligned} U'(\hat{X}^B) = 1, \qquad & U'(X^S) = 1, \\ -\hat{\phi} + \beta \hat{V}^B_m(\hat{m}_{+1}) = 0, \qquad & -\hat{\phi} + \beta \hat{V}^S_m(\hat{m}^S_{+1}) \leq 0, \end{aligned} \tag{5}$$

together with the envelope conditions $\hat{W}^B_m = \hat{W}^S_{m^S} = \hat{\phi}$, and $\hat{W}^B_{\hat{\theta}} = \hat{W}^S_{\hat{\theta}^S} = 1$.

The value function of a buyer, who enters DM with $\hat{m}$ units of fiat money, is

$$\hat{V}^B(\hat{m}) = \frac{1}{2}\left[u(\hat{q}^H) + \hat{W}^B(\hat{m} - \hat{d}^H, \epsilon_H - \hat{s}^H)\right] + \frac{1}{2}\Big[u(\hat{q}^L) + \hat{W}^B(\hat{m} - \hat{d}^L, 0)\Big]. \tag{6}$$

In this formulation $\hat{q}^k$ is the quantity of the DM good traded between a seller and a buyer, where the buyer's realized endowment is ϵ_k. In this bilateral trade, $\hat{s}^k$ denotes the CM good payment and $\hat{d}^k$ denotes the monetary payment received by the seller. We assume that in the decentralized exchange of DM buyers have all the bargaining power, thus the terms of trade are determined by take-it-or-leave-it offers (TIOLI) by buyers. Using the quasi-linearity of preferences, a type-k buyer then solves the following problem:

$$\max_{\{\hat{q}^k, \hat{d}^k_m, \hat{s}^k\}} \quad \left\{u(\hat{q}^k) + \phi(\hat{m} - \hat{d}^k_m) + \epsilon_k - \hat{s}^k\right\}, \tag{7}$$

$$s.t. \quad -\hat{q}^k + \hat{\phi}\hat{d}^k_m + \hat{s}^k \geq 0, \tag{8}$$

$$\hat{d}^k_m \leq \hat{m}, \tag{9}$$

$$\hat{s}^k \leq \epsilon_k. \tag{10}$$

We observe first that the seller's participation constraint (8) binds, such that $\hat{q}^k = \hat{\phi}\hat{d}^k_m + \hat{s}^k$. Then, the problem in DM admits two solutions: either (i) $\hat{\phi}\hat{m} + \epsilon_k \geq q^*$

[8] A sufficient condition for an interior solution is $U'(\epsilon_H) > 1$.

and $\hat{q}^k = q^*$, where $u'(q^*) = 1$, or (ii) $\hat{\phi m} + \epsilon_k < q^*$, $\hat{d}_m^k = \hat{m}$, and $\hat{s}_k = \epsilon_k$. Notice that $\hat{q}^L = \hat{\phi m} < q^*$ must hold, as well as $\hat{q}^H \leq \hat{q}^*$: both conclusions follow from the assumption $\gamma > \beta$. The envelope condition for equation (6) becomes

$$\hat{V}_m^B(\hat{m}) = \frac{1}{2}\left[\phi_{+1} u'(\hat{q}^L)\right] + \frac{1}{2}\hat{V}_m^{B,H}(\hat{m}),$$

where $\hat{V}_m^{B,H}(\hat{m}) = \phi_{+1} u'(\hat{q}^H)$ if $\hat{\phi m} + \epsilon_H < q^*$, and $\hat{V}_m^{B,H}(\hat{m}) = \phi_{+1}$ otherwise. Plugging these results into (5), we conclude that a stationary monetary equilibrium can be of two types: in one case $\hat{q}^L = \hat{\phi m} < q^*$ and $\hat{q}^H = q^* < \hat{\phi m} + \epsilon_H$; and, in the second case $\hat{q}^L = \hat{\phi m} < q^*$ and $\hat{q}^H = \hat{\phi m} + \epsilon_H < q^*$. Let us now define

$$\underline{\epsilon} \equiv q^* - u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right). \tag{11}$$

We can finally partition the parameter space and characterize stationary monetary equilibria for the economy where fiat money is the only nominal object that can be exchanged in DM trade.

Proposition 4.2. *Let $\underline{\epsilon}$ be defined as in (11). Then, a stationary monetary equilibrium is characterized by CM consumption plans $\hat{X}^{B,k} = \hat{X}^S = U'^{-1}(1)$, and*

1. *If $\epsilon_H \geq \underline{\epsilon}$, then $\hat{s}^H = \min\{q^*, \epsilon^H\}$, $\hat{\phi d}_m^H = \max\{0, q^* - \epsilon^H\}$, $\hat{\phi d}_m^L = \hat{\phi m}$, $\hat{q}^H = q^*$, $\hat{q}^L = \hat{\phi m} = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right)$,*

2. *If $\epsilon_H < \underline{\epsilon}$, then $\hat{\phi d}_m^k = \hat{\phi m}$, $\hat{s}^H = \epsilon^H$, $\hat{q}^H = \hat{\phi m} + \epsilon_H$, $\hat{q}^L = \hat{\phi m}$ for $\hat{\phi m}$ solving $\frac{\gamma}{\beta} = \frac{u'(\hat{\phi m} + \epsilon^H)}{2} + \frac{u'(\hat{\phi m})}{2}$.*

In CM buyers decide on their money balances for the next period by facing a trade-off between the cost of holding excessive money balances in the state of nature with a high endowment (ϵ_H), and the benefit of relaxing the cash constraint in the state with low endowment ($\epsilon_L = 0$). In this respect, it is important to observe that buyers would benefit if they could reshuffle money holdings across family members. When $\epsilon_H > \underline{\epsilon}$, a buyer who received an endowment equal to ϵ_H units of the endowment has $\hat{\phi m} - \max\{q^* - \epsilon_H, 0\}$ units of idle cash balances, whereas the family member with no endowment is cash constrained and would benefit from receiving such idle cash balances. When $\epsilon_H < \underline{\epsilon}$, the buyer with an ϵ_H endowment has no idle cash balances. However, even in that case the buyer with a low endowment of the has a tighter cash constraint and would still benefit from a cash transfer for the family member.

5 The Economy with E-money

In this section, we assume that technology providers are active and can issue e-money as a competing medium of exchange. Figure 2 illustrates the flow of fiat money, e-money, and goods in DM and CM.

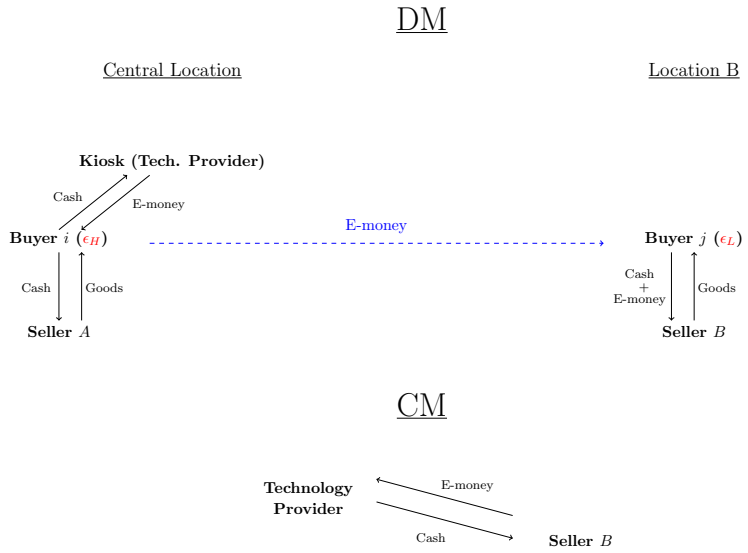

Figure 2: Flow of fiat money and e-money

In this figure, buyers i and j belong to the same family, and buyer i's realized endowment is ϵ_H, whereas buyer j's endowment is ϵ_L. In DM, buyer i converts fiat money into e-money, and then transfers e-money to her family member j. On the one hand, buyer i meets with seller A and settles her transaction using fiat money only, since paying with e-money entails a cost. On the other hand, buyer j meets with seller B and uses both fiat money and e-money to settle the ensuing transaction. Finally, in CM seller B converts e-money back into fiat money, redeeming the e-money holdings received in the previous DM, since e-money is not storable across periods.

5.1 The problem in DM

As in the previous section, we continue to identify buyers by their endowment realizations, ϵ_k. After receiving (or transferring) e-money from (or to) her family member, a type-k buyer holds a portfolio (m, e) of fiat money and e-money balances and ϵ_k units of the CM good. The terms of trade (q^k, s^k, d_m^k, d_e^k) then consists of the quantity of DM-good, q^k, produced by the seller, and payments to the seller of s^k units of the CM good, d_m^k units of fiat money, and d_e^k units of e-money. Similar

to Section 4, we assume that the buyer has all the bargaining power in the DM meeting, so that she makes a TIOLI offer to the bilaterally matched seller. The optimal terms of trade then solves the following problem:

$$V^{B,k}(m,e) = \max_{q^k, d_m^k, d_e^k, s^k} \left\{ u(q^k) + W^{B,k}(m - d_m^k, e - d_e^k, \epsilon_k - s^k) \right\} \tag{12}$$

$$s.t. \quad -q^k + W^S(m^S + d_m^k, d_e^k, s^k) \geq W^S(m^S, 0, 0), \tag{13}$$

$$d_m^k \leq m, \qquad d_e^k \leq e, \qquad s^k \leq \epsilon_k. \tag{14}$$

where $W^{B,k}(m,e,\theta)$ and $W^S(m,e,\theta)$ are respectively the expected value functions of a buyer and that of a seller, from entering CM with m units of fiat money, e units of e-money, and θ units of the endowed CM good. Constraint (13) is the participation constraint of a seller, and constraints (14) are the feasible payments using the endowed CM good, fiat money, and e-money.

5.2 The problem in CM

A seller who enters CM with a portfolio (m,e) of fiat money and e-money, and with θ units of the CM good solves the following problem:

$$W^S(m,e,\theta) = \max_{X^S, H^S, m_{+1}^S} \left\{ U(X^S) - H^S + \beta EV^S(m_{+1}^S) \right\}, \tag{15}$$

$$s.t. \quad X^S + \phi m_{+1}^S = \phi m + \phi e(1-\alpha) + H^S + \theta. \tag{16}$$

The seller redeems her e-money balances in CM, since e-money is non-storable across periods. This conversion process requires the seller to pay a fee equal to α units of fiat money for each unit of e-money converted to fiat money.

In CM, buyers make their portfolio decision and sign an insurance contract with their family members. Specifically, buyers i and j belonging to the same family sign the insurance contract $(\tilde{e}_{i,j}^{\tilde{s}})$, specifying e-money transfers contingent on the state of nature $\tilde{s}$. There are two possible states of nature, $\tilde{s} = s_1$ and $\tilde{s} = s_2$, each occurring with probability 1/2. In each of these states, the e-money transfer from buyer i must equal to the units of e-money received by buyer j, such that $\tilde{e}_{i,j}^{\tilde{s}} = -\tilde{e}_{j,i}^{\tilde{s}}$ for all $\tilde{s} \in \{s_1, s_2\}$. Since buyers are ex-ante identical, we study symmetric insurance contracts: the transfer from buyer i to buyer j in state $\tilde{s} = s_1$ equals to the transfer from buyer j to buyer i in state $\tilde{s} = s_2$. Therefore, the transfers are a function of the realized endowment profiles, but not of buyer identities. As a result, we can reduce an insurance contract to a single object, $\tilde{e}$, which is the e-money transfer

received by a buyer from her family member, when the former experienced a low endowment shock and the latter a high endowment shock. The optimal insurance contract maximizes the ex-ante expected utility of a buyer. Thus, the type-k buyer, who enters CM with ϕm money balances, ϕe e-money balances, and θ units of the endowment stored until CM, decides on fiat money holdings m_{+1} and signs a contract $\tilde{e}$ with her family member to solve the following problem:

$$W^{B,k}(m,e,\theta) = \max_{\{X^{B,k},H^{B,k},m_{+1},\tilde{e}_{+1}\}} \Big\{ U(X^{B,k}) - H^{B,k} + \beta\Big[\frac{1}{2}V^{B,H}(m-\tilde{e}_{+1},0) + \frac{1}{2}V^{B,L}(m,\tilde{e}_{+1})\Big]\Big\} \tag{17}$$

$$s.t. \quad X^{B,k} + \phi m_{+1} = \phi m + \phi e(1-\alpha) + \phi\tau + \theta + H^{B,k} \tag{18}$$

$$\tilde{e}_{+1} \leq m_{+1}. \tag{19}$$

Constraint (18) is the buyer's budget constraint, and (19) is the feasibility on the insurance contract.

Finally, a newborn e-money technology operator takes as given buyers' demand schedule $\tilde{e}_{+1}(\alpha_{+1})$, and the ensuing $\mu_{+1}(\alpha_{+1})$ measure of agents who will convert e-money to fiat money in CM_{+1}. Before transactions take place in CM, the technology provider announces the conversion fee α^*_{+1} to maximize her consumption in CM_{+1}, i.e., $X^{TP}_{+1} = \alpha_{+1}\ \mu_{+1}(\alpha_{+1})$. Because e-money is non-storable, the e-money to fiat money conversion function becomes $\mu(\alpha_{+1}) = \frac{\tilde{e}_{+1}(\alpha_{+1})}{2}$. Thus, the announced conversion fee α^*_{+1} solves the following problem:

$$\alpha^*_{+1} \in \underset{\{\alpha_{+1}\}}{argmax} \left\{ \alpha_{+1} \frac{\tilde{e}(\alpha_{+1})}{2} \right\}. \tag{20}$$

5.3 Solving the model

We solve for DM and CM problems and then characterize the stationary monetary equilibria where fiat money and e-money may co-exist. For this purpose, at first we provide the following definition of the equilibrium concept.

Definition 5.1. *Given a constant money growth rate γ, a stationary monetary equilibrium with e-money consists of CM decisions $\{X^{B,k}, X^{S,k}, H^{B,k}, H^{S,k}, \phi m, \phi m^S, X^{TP}\}$, insurance contract $\{\tilde{e}_{+1}\}$, DM terms of trade $\{q^k, d^k_m, d^k_e\}$, fiat money transfers $\phi\tau$, e-money conversion fee α^* chosen by the technology provider, such that i) buyers and sellers maximize utility and ii) the technology providers maximize revenues.*

5.3.1 The solution in DM

Consider first the problem in DM of a type-k buyer with a portfolio (m, e). Given the quasi-linearity of preferences (q^k, d_m^k, d_e^k, s^k) must solve:

$$V^{B,k}(m,e) = \max_{\{q^k, d_m^k, d_e^k, s^k\}} \Big\{ u(q^k) + \phi(m - d_m^k) + \phi(1-\alpha)[e - d_e^k] + (\epsilon_k - s^k) + W^{B,k}(0,0,0) \Big\} \quad (12)$$

$$s.t. \quad -q^k + \phi\, d_m^k + \phi(1-\alpha) d_e^k + s^k \geq 0 \quad (13)$$

$$d_m^k \leq m, \qquad d_e^k \leq e, \qquad s^k \leq \epsilon_k. \quad (14)$$

It is easy to note that (13) should bind:

$$q^k = \phi\, d_m^i + \phi(1-\alpha) d_e^i + s^k. \quad (21)$$

Let $\phi\lambda_k$, $\phi\mu_k$, and $\phi\eta_k$ be the multipliers associated with the constraints in (14). Thus, we obtain the following necessary first-order conditions:

$$\begin{aligned} d_m^k: &\quad \phi\Big[u'(q^k) - 1 - \lambda_k\Big] = 0 \\ d_e^k: &\quad \phi\Big[u'(q^k)(1-\alpha) - (1-\alpha) - \mu_k\Big] \leq 0 \qquad = \text{if } d_e^k > 0 \\ s^k: &\quad \phi\Big[u'(q^k) - 1 - \eta_k\Big] = 0 \end{aligned}$$

Combining the first and the third equation, we obtain $\lambda_k = \eta_k = u'(q^k) - 1$.

There are two solution cases we should take into account. Let us first consider the case when $\lambda_k = \eta_k = 0$. This condition requires $q^k = q^*$, where $u'(q^*) = 1$. From the second equation we also obtain that $\mu_k = 0$, and thus $d_e^k \leq e$. From (13) holding with equality, one can note that such a solution is feasible only if $\phi m + \phi e + \epsilon_k \geq q^*$.

Consider next the case when $\lambda_k = \eta_k > 0$. In this case, $\phi d_m^k = \phi m$, $\phi d_e^k = \phi e$, and $s^k = \epsilon_k$.

Combining the results from these two cases, we can rewrite:[9]

$$s^k(m,e) = \begin{cases} q^* & \text{if } q^* - \epsilon_k \leq 0 \\ \epsilon_k & \text{if } q^* - \epsilon_k > 0 \end{cases}, \qquad \phi d_m^k(m,e) = \begin{cases} 0 & \text{if } q^* - \epsilon_k \leq 0 \\ q^* - \epsilon_k & \text{if } \phi m \geq q^* - \epsilon_k > \phi m + \epsilon_k \\ \phi m & \text{if } \phi m < q^* - \epsilon_k \end{cases} \tag{22}$$

$$\phi d_e^k(m,e) = \begin{cases} 0 & \text{if } \phi m + \epsilon_k > q^* \\ \frac{q^* - \phi m - \epsilon_k}{1-\alpha} & \text{if } \phi m + \epsilon_k + \phi e(1-\alpha) \geq q^* > \phi m + \epsilon_k \\ \phi e & \text{if } q^* \geq \phi m + \epsilon_k + \phi e(1-\alpha) \end{cases} \tag{23}$$

Next, we compute the value of the Lagrange multipliers as $\phi\eta_i(m,e) = \phi\lambda_k(m,e)$, $\phi\mu_k(m,e) = (1-\alpha)\,\phi\lambda_k(m,e)$, and

$$\phi\lambda_k(m,e) = \begin{cases} 0 & \text{if } \phi m + \epsilon_k + \phi e(1-\alpha) \geq q^* \\ \phi\left[u'\Big(\phi m + \epsilon_k + \phi e(1-\alpha)\Big) - 1\right] & \text{if } q^* \geq \phi m + \epsilon_k + \phi e(1-\alpha) \end{cases} \tag{24}$$

Finally, we characterize the envelope conditions for $V^{B,k}(m,e)$:

$$V_m^{B,k} = \phi(1+\lambda_k) \tag{25}$$

$$V_e^{B,k} = \phi(1-\alpha) + \phi\mu_k = \phi(1-\alpha)(1+\lambda_k) \tag{26}$$

5.3.2 The solution in CM

A seller who enters CM with a portfolio (m,e) of fiat money and e-money balances, and with θ units of the CM good solves the problem in equations (15)-(16). Optimality requires $U'(X^S) = 1$, $m_{+1}^S = 0$.

Consider next the problem of a type-k buyer in CM, with ϕm units of fiat money, ϕe units of e-money, and θ units of the CM good stored, which is given by (17)-(19). Using the budget constraint (18), we can rewrite this problem as follows:

$$W^{B,k}(m,e,\theta) = \phi m + \phi e(1-\alpha) + \phi\tau + \theta$$

[9]We would like to note that we also have an indeterminacy case of payment methods when $\phi m + \epsilon_k + \phi e(1-\alpha) \geq q^*$.

$$+ \max_{\{X^{B,k},m_{+1},\tilde{e}_{+1}\}} \left\{ U(X^{B,k}) - X^{B,k} - \phi m_{+1} + \beta\Big[\frac{1}{2}V^{B,H}(m-\tilde{e}_{+1},0) + \frac{1}{2}V^{B,L}(m,\tilde{e}_{+1})\Big] \right\}$$
$$s.t. \quad \tilde{e}_{+1} \leq m_{+1}. \tag{19}$$

Letting $\phi_{+1}\delta$ denote the Lagrange multiplier associated with constraint (19), the first-order conditions for optimality and the complementray slackness condition are given by the following equations:

$$\begin{aligned} m_{+1}: \quad & -\phi + \beta\Big[\frac{1}{2}V_m^{B,L}(m_{+1},\tilde{e}_{+1}) + \frac{1}{2}V_m^{B,H}(m_{+1}-\tilde{e}_{+1},0)\Big] + \phi_{+1}\delta = 0 \\ \tilde{e}_{+1}: \quad & V_e^{B,L}(m_{+1},\tilde{e}_{+1}) - V_m^{B,H}(m_{+1}-\tilde{e}_{+1},0) - \phi_{+1}\delta = 0 \\ CS: \quad & \delta\Big[m_{+1}-\tilde{e}_{+1}\Big] = 0 \end{aligned}$$

Deriving the envelope conditions:

$$\begin{aligned} W_m^{B,k}(m,e,\theta) &= \phi, \\ W_e^{B,k}(m,e,\theta) &= \phi(1-\alpha), \\ W_\theta^{B,k}(m,e,\theta) &= 1, \end{aligned}$$

and using (25) and (26), we can rewrite the first-order conditions above as follows:

$$-1 + \frac{\beta}{\gamma}\Big[1 + \frac{1}{2}\lambda_L(m_{+1},\tilde{e}_{+1}) + \frac{1}{2}\lambda_H(m_{+1}-\tilde{e}_{+1},0)\Big] + \frac{\delta}{\gamma} = 0 \tag{27}$$
$$(1-\alpha)\lambda_L(m_{+1},\tilde{e}_{+1}) - \alpha - \lambda_H(m_{+1}-\tilde{e}_{+1},0) = \delta \tag{28}$$
$$\delta\Big[m_{+1}-\tilde{e}_{+1}\Big] = 0 \tag{29}$$

Equations (27)-(28) give the optimal money demand $\phi m(\alpha)$ and the optimal insurance agreement between family members in CM, $\phi\tilde{e}(\alpha)$. We take $\underline{\epsilon}$ defined as in (11) and

$$\hat{\epsilon} \equiv u'^{-1}(\frac{\gamma}{\beta}). \tag{30}$$

For $\epsilon_H > \hat{\epsilon}$, we also define

$$\hat{\alpha} \equiv \frac{2\left(\frac{\gamma}{\beta} - u'(\epsilon^H)\right)}{2\frac{\gamma}{\beta} - u'(\epsilon^H)} \tag{31}$$

and $\tilde{\alpha}$ implicitly, based on the solution to

$$u'^{-1}\left(\frac{[1-\tilde{\alpha}]\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right) - u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right) - \epsilon_H = 0 \tag{32}$$

We can then characterize buyers' optimal money and e-money demand.

Proposition 5.2. *Let $\hat{\epsilon}$ be defined in* (30)*, $\hat{\alpha}$ in* (31)*, and $\tilde{\alpha}$ in* (32)*. An equilibrium with e-money exists only if the technology provider's service fee satisfies $\alpha \leq \overline{\alpha} \equiv \min\left\{\tilde{\alpha}, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right\}$. If $\alpha \leq \overline{\alpha}$ and*

1. *$\epsilon_H \geq q^*$, the optimal money and e-money demand by buyers satisfy:*

$$\phi\tilde{e}_{+1}(\alpha) = \phi m(\alpha) = \frac{u'^{-1}\left(\frac{1+\gamma-\frac{\beta}{2}}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha}.$$

2. *$q^* > \epsilon_H > \hat{\epsilon}$, the optimal money and e-money demand by buyers satisfy:*

$$\phi\tilde{e}_{+1}(\alpha) = \begin{cases} \phi m & \text{if } \alpha \in [0, \hat{\alpha}] \\ \frac{\epsilon_H - \left[u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right)\right]}{2-\alpha} & \text{if } \alpha \in [\hat{\alpha}, \overline{\alpha}] \end{cases}$$

$$\phi m(\alpha) = \begin{cases} \frac{u'^{-1}\left(\frac{\gamma+u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha} & \text{if } \alpha \in [0, \hat{\alpha}] \\ \frac{u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right)+(1-\alpha)(u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right)-\epsilon_H)}{2-\alpha} & \text{if } \alpha \in [\hat{\alpha}, \overline{\alpha}] \end{cases}$$

3. *$\epsilon_H < \hat{\epsilon}$, the optimal money and e-money demand by buyers satisfy:*

$$\phi\tilde{e}_{+1}(\alpha) = \frac{\epsilon_H - \left[u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right)\right]}{2-\alpha}$$

$$\phi m(\alpha) = \frac{u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right) + (1-\alpha)(u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - \epsilon_H)}{2-\alpha}$$

Proposition 5.2 relates the demand for money and e-money to two important parameters: *i)* the realization of the endowment process in the high state of nature, ϵ_H, and *ii)* the conversion fee α announced by the technology provider. Intuitively, the realization of the endowment process in the high state of nature, ϵ_H, measures the scope of insurance. The larger ϵ_H is, the looser the cash constraint (14) of a type-H buyer, and therefore the larger the scope for insurance. On the other hand, the conversion fee, α, determines the cost of insurance, i.e., the cost of transferring e-money balances between two buyers of a family. The higher the conversion fee is, the higher is the cost of within family insurance.

When $\epsilon_H > q^*$, the scope for insurance is large because cash balances in the hands of type-H buyers are idle. In this case, after the state of the world realizes, type-H buyers will transfer all their cash holdings to their family members with a low endowment realization. Even though cash balances in the hands of a type-H buyer are idle, the technology provider is restricted ex-ante on the conversion fee she can announce; if α were larger than $\overline{\alpha}$, buyers' demand for e-money would equal zero, as it would be cheaper for them to self-insure using fiat money only. For intermediate values of the endowment realization, i.e., $q^* > \epsilon_H > \hat{\epsilon}$, there is moderate scope for insurance. In this parameter configuration, type-H buyers can use the cash balances in their hands to relax their cash constraint (14) or, alternatively, can transfer them to family members with a low endowment realization. Intuitively, the extent of the e-money transfer depends on the conversion fee α. When the conversion fee is below the threshold $\hat{\alpha}$, the insurance motive dominates: as in the previous case, the solution is at the corner where type-H buyers transfers all their cash holdings. If instead $\alpha > \hat{\alpha}$, the solution is interior and type-H buyers retain some of their money holdings to relax their cash constraint (14). Finally, for small values of the endowment realization, i.e., $\epsilon_H < \hat{\epsilon}$, the scope for insurance is small. In this region of the parameter space, money balances are always used by type-H buyers to relax their cash constraint (14).

Given a certain level of high endowment realization ϵ_H, a larger conversion fee α has the intuitive property of increasing the dispersion of consumption among buyers, as we summarize in the next corollary.

Corollary 5.3. *For ϵ_H given, if $0 < \alpha < \overline{\alpha}$, we have $\frac{\partial q^H}{\partial \alpha} \geq 0$, $\frac{\partial q^L}{\partial \alpha} < 0$. Moreover, for $\alpha > 0$ we have $\frac{q^H}{q^L} > 1$ and $\frac{\partial}{\partial \alpha}\left(\frac{q^H}{q^L}\right) > 0$.*

In order to conclude the equilibrium analysis, we need to determine the technology provider's revenue maximizing service charge α^*. In order to characterize α^* in closed-form, we assume that buyers' utility function over DM consumption is specified by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

We let σ_L to be defined as

$$\sigma_L \equiv \frac{4\gamma(\gamma-\beta)}{\beta(2\gamma-\beta)[2(1+\gamma)-\beta]}, \tag{33}$$

and σ_u to be implicitly defined as the unique solution to

$$\frac{\gamma-\beta}{\sigma_u(2\gamma-\beta)}-\frac{1}{1+\left(\frac{2\gamma-\beta}{\beta}\right)^{1+\frac{1}{\sigma_u}}}=0. \tag{34}$$

Moreover, for $\sigma\in(0,\sigma_L]$, we take $\epsilon_*(\sigma)$ as the unique solution to

$$\frac{2\left(\frac{\gamma}{\beta}-\epsilon_*(\sigma)^{-\sigma}\right)}{\sigma\left(2\frac{\gamma}{\beta}-\epsilon_*(\sigma)^{-\sigma}\right)}-\frac{\beta\epsilon_*(\sigma)^{-\sigma}\left[2-\beta+2\gamma\epsilon_*(\sigma)^{\sigma}\right]}{2\gamma}=0, \tag{35}$$

and for $\sigma\in(0,\sigma_u]$ take $\epsilon_{**}(\sigma)$ as the unique solution to

$$\frac{2\left(\frac{\gamma}{\beta}-\epsilon_{**}(\sigma)^{-\sigma}\right)}{\sigma\left(2\frac{\gamma}{\beta}-\epsilon_{**}(\sigma)^{-\sigma}\right)}-\frac{2}{1+\left(\frac{2\gamma\epsilon_{**}(\sigma)^{\sigma}}{\beta}-1\right)^{1+\frac{1}{\sigma}}}=0. \tag{36}$$

We then let $\Omega(\sigma)$ and $\Psi(\sigma)$ to be defined as:

$$\Omega(\sigma)=\begin{cases}\epsilon_{**}(\sigma) & \text{if } \sigma\in(0,\sigma_u]\\ \underline{\epsilon}(\sigma)+\frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}}+\frac{2\gamma-\beta}{\beta}\right] & \text{if } \sigma>\sigma_u.\end{cases} \tag{37}$$

$$\Psi(\sigma)=\begin{cases}\epsilon_*(\sigma) & \text{if } \sigma\in(0,\sigma_L]\\ q^* & \text{if } \sigma\in(\sigma_L,\sigma_u].\end{cases} \tag{38}$$

Finally, for $\sigma<\sigma_L$ α^o is defined as

$$\alpha^o=1+\sigma-\sqrt{1+\sigma^2-\beta\sigma}, \tag{39}$$

and α^{oo} solves:

$$\left(\frac{1-\alpha^{oo}/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}\left[2\left(\left(\frac{1}{1-\alpha^{oo}}\right)^{\frac{1}{\sigma}}-1\right)+\frac{\alpha^{oo}}{\sigma}\left(1+\left(\frac{1}{1-\alpha^{oo}}\right)^{1+\frac{1}{\sigma}}\right)\right]=2\epsilon_H. \tag{40}$$

In the next proposition, we prove the existence and uniqueness of a stationary monetary equilibrium with e-money. In particular, we can characterize the stationary equilibrium in terms of buyer's risk-aversion σ and income shock ϵ_H.

Proposition 5.4. *Let σ_L and σ_u be defined in* (33) *and* (34), *$\underline{\epsilon}(\sigma)$ be defined in* (11), *$\hat{\alpha}$ in* (31), *$\epsilon_*(\sigma)$ in* (35), *$\epsilon_{**}(\sigma)$ in* (36), *$\Omega(\sigma)$ in* (37), *α^o in* (39), *and α^{oo} in* (40). *There exists a unique stationary monetary equilibrium with e-money, where*

the technology provider's service fee α^* is determined as:

$$\alpha^* = \begin{cases} \frac{2(\gamma-\beta)}{2\gamma-\beta} & \text{if } \Big[\sigma \in [\sigma_L, \sigma_u) \text{ and } \epsilon_H > \Psi(\sigma)\Big] \text{ or } \Big[\sigma > \sigma_u \text{ and } \epsilon_H \geq \Omega(\sigma)\Big], \\ \alpha^o & \text{if } \sigma < \sigma_L \text{ and } \epsilon_H \geq \Psi(\sigma), \\ \hat{\alpha} & \text{if } \sigma < \sigma_u \text{ and } \epsilon_H \in \Big(\Omega(\sigma), \Psi(\sigma)\Big], \\ \alpha^{oo} & \text{if } \epsilon_H < \Omega(\sigma). \end{cases}$$

The different solutions to the technology provider's problem are depicted in Figure 3, which helps to understand the properties of the revenue maximizing conversion fee. When determining the optimal conversion fee, the technology provider considers the price and the quantity effects ensuing an increase in α, where revenues increase due to the price effect and decline due to the quantity effect. In the region of the parameter space where either *i)* $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or *ii)* $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, the price effect always dominates and the profit maximizing conversion fee equals the upper bound $\frac{2(\gamma-\beta)}{2\gamma-\beta}$. In all other regions of the parameter space, the price effect dominates for α small whereas the quantity effect does for α large and as a result, the optimal conversion fee which maximizes the technology provider revenues becomes an interior solution.

To understand the result shown in Figure 3, notice that the revenue maximizing conversion fee of the technology provider depends on the elasticity of demand for e-money. In this respect, three important parameters stand-out: the micro-level preference and income process parameters, i.e., and the coefficient of relative risk-aversion σ and the realization of the endowment shock ϵ_H, and the macro policy parameter, i.e., money growth rate γ - which determines the inflation rate.

For a given level of σ, a larger value of ϵ_H makes self-insurance relatively costly, because it increases the amount of idle cash balances in the hand of ϵ_H-buyers. As a result, when ϵ_H is large, the demand for e-money is relatively more inelastic (and the quantity effect is weaker) and the technology provider can set a high conversion fee. Similarly, holding ϵ_H constant, a higher degree of risk-aversion increases the benefits from insurance. Thus, a larger coefficient of relative risk aversion implies a more inelastic demand for e-money, and also in this situation the technology provider can set a high conversion fee α.

Finally, a high money growth rate γ raises the level of conversion fee that the technology provider can set. This is an intuitive property, because a higher money growth rate implies a higher inflation rate and lower efficiency of holding cash over time. Technology provider internalizes this opportunity cost and sets a higher conversion fee with the anticipated effects of high inflation on demand for e-money.

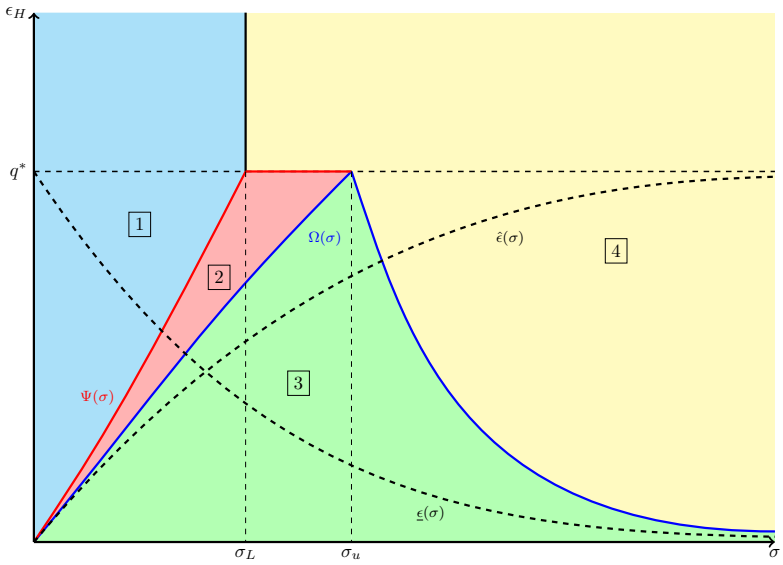

Figure 3: Stationary monetary equilibria with e-money

6 Welfare Effects of E-money

To understand the real consequences of e-money technology, we compare and contrast allocations of the economy in Section 4 (without e-money) with allocations of the economy in Section 5 (where fiat money and e-money co-exist). We would like to remind that we denoted allocations in Section 4, i.e., the economy with fiat money only, with a hat symbol ($\hat{}$), and we continue to keep that notation also in this section. We can thus establish the following result.

Lemma 6.1. *Letting $\underline{\epsilon}(\sigma)$ be defined as in (11), $\hat{\epsilon}(\sigma)$ in (30), and $\Omega(\sigma)$ in (37); the following characterizes equilibrium DM consumption:*

1. *If i) $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or ii) $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, then $\hat{q}^H = q^H > q^L = \hat{q}^L$, and $\phi m < \hat{\phi}\hat{m}$.*

2. *If $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$, then $\hat{q}_H > q_H > q_L > \hat{q}_L$.*

Lemma 6.1 provides necessary and sufficient conditions for the adoption of e-money to have real effects on risk-sharing and the allocation of consumption in DM. E-money has the potential of enabling risk-sharing among the buyers of a family. However, such risk-sharing opportunities have real effects on consumption in DM if and only if *i)* buyers are not too risk-averse, i.e. $\sigma < \sigma_L$, or *ii)* the realization of the income shock ϵ_H is relatively small, i.e., $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$. When at least one of these two conditions hold, e-money improves the allocation

of consumption; and, the transmission of income shocks through consumption is weakened and, as a result, the wedge between high and low consumption in DM is narrowed, i.e. $q^H/q^L < \hat{q}^H/\hat{q}^L$. However, if buyers are relatively risk averse, $\sigma \geq \sigma_L$, or the realization of the income shock ϵ_H is relatively large, $\epsilon_H \geq \max\{\Omega(\sigma), \Psi(\sigma)\}$, then e-money has no real effect on DM-consumption. To understand this result, we note that a buyer could always choose to hold the amount of cash that allows her to acquire the amount of consumption $\hat{q}^L$, without any transfer of e-money taking place. Relative to this strategy, e-money let buyers to economize on cash holdings. However, because in these regions of the parameter space the demand for e-money is relatively inelastic, the technology provider optimally chooses a conversion fee to extract the whole surplus from having the e-money technology in place. As a result, buyers' DM consumption in an economy with e-money remains the same as in an economy with only fiat money.

This is a highly policy relevant theoretical result. In order to evaluate further implications of this key finding, let us define buyers' welfare at the steady state as the expected lifetime utility of the representative buyer as of the beginning of a period - before location and preference shocks are realized. Specifically, buyers' welfare in the economy with fiat money only, described in Section 4, is $(1-\beta)\hat{\mathcal{W}}^B = \sum_{k\in\{L,H\}} \frac{1}{2}\left[u(\hat{q}^k) + U(\hat{X}^{B,k}) - \hat{H}^{B,k}\right]$, and buyers' welfare in the economy with fiat money and e-money, described in Section 5, is $(1-\beta)\mathcal{W}^B = \sum_{k\in\{L,H\}} \frac{1}{2}\Big[u(q^k) + U(X^{B,k}) - H^{B,k}\Big]$. After some algebra, we can prove the following result.

Proposition 6.2. *Let $\underline{\epsilon}(\sigma)$ be defined in (11), $\hat{\epsilon}(\sigma)$ in (30), and $\Omega(\sigma)$ in (37). Let also $\mathcal{U}^{TP}$ be the utility of a technology provider. Then, the following welfare properties hold:*

1. *If i) $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or ii) $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, then $(1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}^B\right) = \mathcal{U}^{TP}$.*
2. *If $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$, then $(1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}^B\right) \gtrless 0$.*

Proposition 6.2 characterizes the main result of our paper: introducing the e-money technology operated by a monopolist provider may reduce buyers' equilibrium welfare. When $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, we know from Lemma 6.1 that introducing the e-money technology operated by a monopolist provider has no real effects on buyers' DM consumption whereas it reduces the equilibrium price of fiat money, ϕ. Specifically, each unit of fiat money is worth less in terms of the CM good - compared to an economy with fiat money only. This

implies that the introduction of e-money generates a pecuniary externality on the price of money in CM and, as a result, buyers need to work more in CM to acquire fiat money balances. This is the case as buyers need to work more in order to pay for the profits accruing to the technology provider. Thus, buyers are worse off when the e-money technology is introduced. This result holds in a particular region of the parameter space. The second part of Proposition 6.2 proves that the introduction of the e-money technology may also be welfare improving. This can only occur if buyers' risk-sharing improves consumption in DM. However, this is not a sufficient condition, as the benefits from insurance may not be large enough to compensate for the cost resulting from the pecuniary externality in CM. This result proves that it is vital to assess the welfare consequences of e-money in a model that is explicit about monetary exchange and can account for the general equilibrium effects of e-money. The next result easily follows from this discussion.

Corollary 6.3. *Let $\mathcal{T}$ be a lump-sum tax levied on technology providers that the government redistributes to buyers. Let also $\underline{\epsilon}(\sigma)$ be defined as in* (11)*, $\hat{\epsilon}(\sigma)$ in* (30)*, and $\Omega(\sigma)$ in* (37)*.*

1. *If i) $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or ii) $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, there exists no feasible $\mathcal{T}$ such that $\mathcal{W}^B > \hat{\mathcal{W}}^B$.*

2. *If $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$, there exist a feasible $\mathcal{T}$ such that $\mathcal{W}^B > \hat{\mathcal{W}}^B$.*

Corollary 6.3 states that in parameter constellations of the economy where the introduction of e-money reduces buyers' welfare, a redistributive tax instrument cannot correct the welfare distortions generated by e-money technology. This is also an important finding, because it shows that fiscal policy, i.e., regulation of e-money through taxation, is ineffective in undoing the distortionary effects of e-money.

7 Quantitative Analysis

Proposition 6.2 showed that the introduction of e-money through a monopolistic technology provider with private profit incentives does not necessarily lead to welfare gains for its final users (buyers in our model). A natural question arises based on this key finding: Does e-money increase or reduce welfare in the context of a real-world economy that experienced the introduction of an electronic money instrument?

To gain a better understanding of the consequences of the private provision of e-money, and evaluate the welfare implications, we conduct a quantitative analysis by referring to Kenya's M-Pesa revolution over the last two decades. Specifically, we first calibrate our model by utilizing Kenyan micro and macro data spanning from 2000 to 2007: since this period precedes the launch of M-Pesa, we use these data to calibrate the model without e-money from Section 4. Then, we introduce the e-money provider and solve for the equilibrium allocations of the model in Section 5 and quantify the ensuing welfare effects of e-money.

To calibrate the model, we specify the functional form the CM utility function as $U(X) = \Omega \log(X)$, which based on the results in Section 5.3.2 gives $X^B = X^S = \Omega$. The model has four additional parameters to be calibrated: γ, β, Ω, ϵ_H, and σ. The first two can be directly inferred from the data: over the period of 2000-2007, the average quarterly inflation rate of Kenya was 1.96% and the average nominal interest rate on 90 days T-bill was 2.02%. The first value translates directly into $\gamma = 1.0196$. Then, assuming that government bonds are priced fundamentally, the nominal interest rate satisfies $R = \frac{\gamma}{\beta}$ and we obtain $\beta = 0.99$.

We calibrate the remaining three parameters, i.e., the income shock ϵ_H, the coefficient of relative risk aversion, σ, and the coefficient for the CM utility function, Ω, in two steps. First, given σ (and Ω), we parameterize ϵ_H to fit the empirical consumption variation ensuing shocks to income. Then, we choose the values of σ and Ω to minimize the distance between the empirical money demand and the model-implied money demand. In the first step we use the estimate in Jack and Suri (2014), who document that negative shocks reduce consumption among non M-Pesa users by 7 percent. We use this value to identify the income shock ϵ_H so that the DM consumption in the model without e-money matches the consumption variation documented by Jack and Suri (2014), i.e., $\hat{q}^L/\hat{q}^H = 0.93$. From Proposition 4.2, we observe that $\hat{q}^L/\hat{q}^H = 0.93$ is possible only if $\sigma \leq -\log(2\gamma/\beta - 1)/\log(0.93) \equiv \overline{\sigma}$. For $\sigma \leq \overline{\sigma}$, we define:

$$\epsilon_H(\sigma) = \frac{0.07\left(2\frac{\gamma}{\beta}\right)^{-\sigma}}{[1 + (0.93)^{-\frac{1}{\sigma}}]^{-\sigma}}. \tag{41}$$

Using Proposition 4.2, it is easy to prove that for $\sigma < \overline{\sigma}$ and $\epsilon_H = \epsilon_H(\sigma)$, we have the desired result $\hat{q}^L/\hat{q}^H = 0.93$.

Then, using the methodology in Lucas (2001) and Lucas and Nicolini (2015), we evaluate the relationship between the nominal rate i and $L \equiv \frac{M}{PY}$. This relationship represents money demand, in the sense that the desired real balances M/P are proportional to Y, with a factor of proportionality L that depends on the cost of

holding cash i. To construct L in the model, we note that in our environment $Y = Y^{CM} + Y^{DM}$, where nominal output in the centralized market is $Y^{CM} = (X^B + X^S)/\phi = \frac{2\Omega}{\phi}$ and nominal output in the decentralized market is $Y^{DM} = [\frac{1}{2}\hat{q}^H + \frac{1}{2}\hat{q}^L]/\phi$. Hence $PY = 2\Omega/\phi + [\frac{1}{2}\hat{q}^H + \frac{1}{2}\hat{q}^L]/\phi$. Also, in the equilibrium with fiat money only $M/P = \phi M = z(\hat{q}^L)$, so that:

$$L \equiv \frac{\phi M}{Y} = \frac{\hat{q}^L(1+i)}{2\Omega + \frac{1}{2}\hat{q}^H + \frac{1}{2}\hat{q}^L}. \tag{42}$$

In order to determine the values of σ and Ω, we minimize the distance between the observed liquidity services at different nominal interest rates and the implied equilibrium counterpart, which is given by equation (42). Table 1 summarizes the values that we obtain in the benchmark calibration, whereas Figure 4 plots the money demand curve implied by the model against the data for the period 2000-2007.

Parameters	Value	Target
Money growth (γ)	1.0196	Average inflation rate 2000-2007
Discount Factor (β)	0.99	Average real interest rate 2000-2007
Income shock (ϵ_H)	0.07	Non-users consumption drop from Jack and Suri (2014)
Curvature DM-utility (σ)	0.77	Money Demand 2000-2007
Coefficient CM-utility (Ω)	2.46	Money Demand 2000-2007

Table 1: Benchmark calibration.

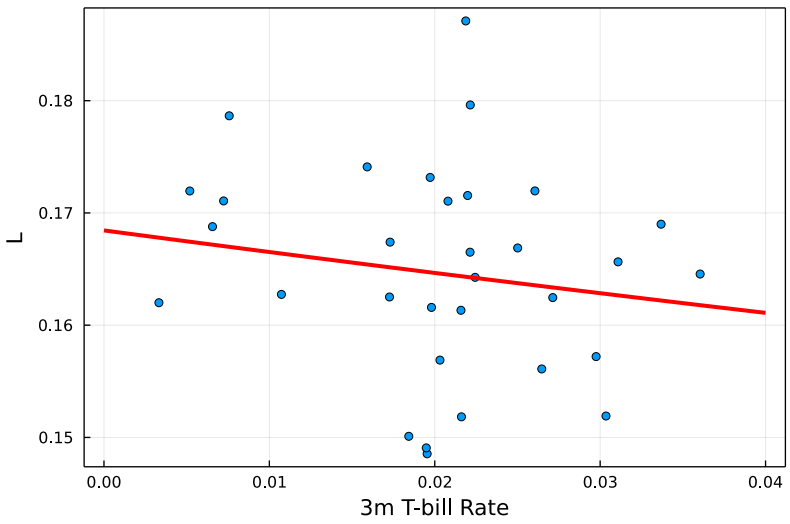

Figure 4: Money Demand

The calibrated model allows us to quantify the impact of e-money on consump-

tion smoothing and welfare. First, we compute the optimal conversion fee α^* charged by the technology provider. Our calibration suggests that the optimal conversion fee is about 2.8%, which is within ball-park figure with respect to the M-Pesa transaction fees charged by Kenya's Safaricom. More specifically, as documented by Beck et al. (2018), Safaricom's cash withdrawal fees amount to 2.9% of the average M-Pesa transaction volume in the economy - providing a good support for the external validity of our model.

As theoretically expected the model predicts that the introduction of e-money improves risk-sharing. Specifically, income shocks reduce the consumption of e-money users by 3.6%, which is about 50% less than the reduction in consumption for non-users. However, the improvement in household insurance is mitigated by the general equilibrium effect of e-money on the value of money. To quantify the welfare effects of the introduction of e-money for buyers, we compute the increase in CM-consumption in the economy (g) without e-money that makes a buyer indifferent between living in an economy with e-money and living in an economy without e-money: $(1-\beta)\hat{\mathcal{W}}^B_{(1+g)} = (1-\beta)\mathcal{W}^B$, where $(1-\beta)\hat{\mathcal{W}}^B_{1+g} = \sum_{k=L,H} \frac{1}{2}\left[u(\hat{q}^k) + U((1+g)\hat{X}^k) - \hat{H}^k\right]$.

	Economy without e-money		Economy with e-money	
Real money balances	$\hat{\phi m}$	0.929	ϕm	0.928
DM-consumption (H)	$\hat{q}^H$	0.998	q^H	0.980
DM-consumption (L)	$\hat{q}^L$	0.928	q^L	0.945
Conversion fee	-	-	α^*	0.028
E-money demand	-	-	ϕe	0.018
Consumption variability	$\hat{q}_L/\hat{q}_H$	0.930	q_L/q_H	0.964
Family insurance		$\frac{q_H/q_L}{\hat{q}_H/\hat{q}_L}$	1.0371	
Buyers' welfare e-money vs. no e-money		g	0.00004	

Table 2: Effects of e-money introduction

The coefficient g measures how much buyers benefit from the introduction of e-money, and it does so in terms of consumption of the CM good. Given the functional form assumptions, it is easy to show that $g = e^{\frac{\mathcal{W}^B - \hat{\mathcal{W}}^B}{\Omega}} - 1 = 0.00004$. Since $g > 0$, buyers gain from the introduction of e-money. However, quantitatively such welfare gains are very tiny, as they are equivalent to an increase of 0.004% of consumption in CM. The low welfare gains that we quantify are in line with the concerns by the Central Bank of Kenya, who aim to introduce a Central Bank

Digital Currency in an attempt to compete with M-Pesa and thereby reduce the cost of electronic money transactions for the final users (see "Discussion Paper on Central Bank Digital Currency", 2022, by Central Bank of Kenya).

In Table 3 we provide a sensitivity analysis for our key quantitative results. We vary two key parameters of the model; namely, money growth rate and the variation of the income shock, and present the resulting equilibrium allocations with e-money in the top panel of the table. In the lower panel we provide household insurance and buyers' welfare induced through e-money - in comparison to an economy without e-money, and in the case of the economy without e-money in this table we refer to an economy, where money growth rate and income shock take the alternative values that we specified for each column.

		$\gamma = 1.04$	$\gamma = 1.01$	$\epsilon_H = 0.10$	$\epsilon_H = 0.04$
Real money balances	ϕm	0.904	0.940	0.914	0.943
DM-consumption (H)	q^H	0.956	0.992	0.988	0.973
DM-consumption (L)	q^L	0.921	0.957	0.938	0.953
Conversion fee	α^*	0.028	0.027	0.039	0.016
E-money demand	ϕe	0.018	0.018	0.025	0.010
Consumption variability	q_L/q_H	0.963	0.931	0.949	0.979
Family insurance	$\frac{q_L/q_H}{\hat{q}_L/\hat{q}_H}$	1.039	1.036	1.024	1.021
Buyers' welfare e-money vs. no e-money	g	0.00004	-0.0004	-0.00009	0.00001

Table 3: Effects of inflation and income process

The results in Table 3 show that under the alternative parameter specifications the welfare gains from e-money continue to be negligible - at best. Raising money growth rate (and thus inflation) to $\gamma = 1.04$ does not alter e-money's net welfare gains. This is the case because the conversion fee set by the technology operator does not increase significantly compared to the benchmark. Lowering inflation reduces the net welfare gains from introducing e-money, and in fact the welfare effects become negative. The reason is that lower inflation makes the demand for e-money relatively inelastic, so the technology provider can extract a larger share of the surplus from the technology adoption. Hence, the extra effort needed by buyers in CM to acquire cash balances does not compensate for the benefits from risk-sharing. Similarly, when the size of the positive income realization increases, the demand for e-money becomes relatively more inelastic. Hence the e-money technology operator can set a higher conversion fee, causing buyers suffer net welfare losses from the introduction of e-money. Following the same line of intuition, when ϵ_H contracts

to 0.04, technology operator's conversion fee reduces to 1.6%; but at the same time the potential of consumption insurance through e-money to improve buyers' welfare also goes down - resulting in low net welfare gains in equilibrium compared to the benchmark.

8 Conclusion

We studied a microfounded monetary model to explore the interactions between fiat money and privately provided electronic money. Characterization of stationary equilibria in this framework allowed us to compare the equilibrium welfare of an economy with the e-money technology in place relative to the equilibrium welfare of an economy with only fiat money.

Since it financially integrates spatially separated individuals, in our model e-money enables risk-sharing contracts and reduces consumption volatility. However, these intuitive benefits get counteracted by a general equilibrium effect that relates to the private provision of the e-money technology: we show that as long as the technology operator is a monopolist with private profit incentives, having an e-money instrument in place may reduce the equilibrium price of fiat money. This key general equilibrium channel generates welfare losses, because compared to an economy with only fiat money, the equilibrium price effect implies that buyers have to work harder in acquiring fiat money balances to be utilized when purchasing goods from the market and e-money units from the technology provider. We uncover that the welfare losses due to the general equilibrium feedback outweigh the welfare benefits of e-money, if the heterogeneity in income shocks (i.e., the scope of insurance) is large enough in-between the two members of an insurance arrangement.

The theoretical results that we obtain are empirically plausible and policy relevant for developing countries, because our quantitative analysis reveals a potentially limited welfare impact of e-money. A recent line of empirical research has highlighted the important role of e-money instruments for risk-sharing arrangements, an effect that we capture in our framework. When microfoundations are carefully modeled, the general equilibrium macro losses may be present - arguing for improving the competitiveness of the e-money sector.

References

Ahnert, T., P. Hoffmann, and C. Monet (2022): "The digital economy, privacy, and CBDC," *ECB Working Paper*.

Beck, T., H. Pamuk, R. Ramrattan, and B. R. Uras (2018): "Payment instruments, finance and development," *Journal of Development Economics*, 133, 162–186.

Berentsen, A. (2006): "On the private provision of fiat currency," *European Economic Review*, 50, 1683–1698.

Berentsen, A., G. Camera, and C. Waller (2007): "Money, credit and banking," *Journal of Economic Theory*, 1, 171–195.

Chiu, J. and T.-N. Wong (2015): "On the essentiality of e-money," Tech. rep., Bank of Canada.

Dalton, P. S., H. Pamuk, R. Ramrattan, D. van Soest, and B. Uras (2022): "E-Payment Technology and Business Finance: A Randomized Control Trial with Mobile Money," *Forthcoming at Management Science*.

Fernández-Villaverde, J. and D. Sanches (2019): "Can currency competition work?" *Journal of Monetary Economics*, 106, 1–15.

Jack, W. and T. Suri (2014): "Risk sharing and transactions costs: Evidence from Kenya's mobile money revolution," *American Economic Review*, 104, 183–223.

Kang, K.-Y. (2021): "Digital Currency and Privacy," *Available at SSRN 3838718*.

Lagos, R. and R. Wright (2005): "A unified framework for monetary theory and policy analysis," *Journal of Political Economy*, 113, 463–484.

Lucas, R. E. (2001): "Inflation and welfare," in *Monetary Theory as a Basis for Monetary Policy*, Springer, 96–142.

Lucas, R. E. and J. P. Nicolini (2015): "On the stability of money demand," *Journal of Monetary Economics*, 73, 48–65.

Parlour, C. A., U. Rajan, and H. Zhu (2022): "When fintech competes for payment flows," *The Review of Financial Studies*, 35, 4985–5024.

Rocheteau, G. and R. Wright (2005): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73, 175–202.

Suri, T. and W. Jack (2016): "The long-run poverty and gender impacts of mobile money," *Science*, 354, 1288–1292.

Williamson, S. D. and R. Wright (2010): "New monetarist economics: Models," .

Appendix

Proof of Proposition 4.2

Consider the first-order condition (5) that characterizes the CM optimal money demand: since we assumed that the economy is away from the Friedman rule, $\gamma > \beta$, cash constraint (9) must bind in ϵ_L-states : $\hat{q}^L = \hat{\phi m} < q^*$, for q^* the efficient level of consumption, i.e., $u'(q^*) = 1$. Then, we can rewrite the first-order condition (5) as

$$-\frac{\phi}{\beta} + \frac{1}{2}\left[\phi_{+1}u'(\hat{q}_L) + \frac{\partial \hat{V}^{B,H}(m)}{\partial m}\right] = 0. \tag{43}$$

Differently, consumption in ϵ_H-states may be either such that *i)* $\hat{q}^H = q^*$ or *ii)* $\hat{q}^H < q^*$. Consider first the case where $\hat{q}^H = q^*$. Then, (43) becomes

$$-\frac{\phi}{\beta} + \frac{1}{2}\phi_{+1}\left[u'(\hat{\phi m}) + 1\right] = 0 \;\; \Rightarrow \hat{\phi m} = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right).$$

This is an equilibrium if $\hat{\phi m} + \epsilon_H \geq q^*$, or $\epsilon_H \geq q^* - u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right) = \underline{\epsilon}$.

Consider next the case when $\hat{q}^H < q^*$. Then, the cash constraint (9) and constraint (10) bind also in ϵ_H-states: $\hat{q}^H = \epsilon_H + \hat{\phi m} > \hat{\phi m} = \hat{q}_L$ and equation (5) becomes

$$-\frac{\phi}{\beta} + \frac{1}{2}\phi_{+1}\left[u'(\hat{\phi m}) + u'(\hat{\phi m} + \epsilon_H)\right] = 0 \;\; \Rightarrow u'(\hat{\phi m} + \epsilon_H) = 2\frac{\gamma}{\beta} - u'(\hat{\phi m}).$$

Since the left-hand side of this equation is decreasing and the right-hand side is increasing in $\hat{\phi m}$, there is a unique $\hat{\phi m}$ that solves the equations with equality. For this to be an equilibrium, it must be that $\hat{\phi m} + \epsilon_H < q^*$, or $\epsilon_H < q^* - u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right) = \underline{\epsilon}$.

Proof of Proposition 5.2

There are two types of equilibria that can emerge: *i)* one in which the cash constraint (19) is slack, so the multiplier is $\delta = 0$, and *ii)* one in which the cash constraint (19) is binding, so the multiplier $\delta \geq 0$. In what follows we characterize these two types of equilibria separately.

Case 1: $\delta = 0$.

Consider first the case when the cash constraint (19) is slack so that $\delta = 0$. From equation (28) we obtain

$$\lambda_H(m_{+1} - \tilde{e}_{+1}, 0) = (1-\alpha)\lambda_L(m_{+1}, \tilde{e}_{+1}) - \alpha.$$

Using this expression and equation (27), we can solve for $\lambda_L(m_{+1}, \tilde{e}_{+1})$ and $\lambda_H(m_{+1} - \tilde{e}_{+1}, 0)$:

$$\lambda_L(m_{+1}, \tilde{e}_{+1}) = \frac{\gamma/\beta}{1-\frac{\alpha}{2}} - 1, \qquad \lambda_H(m_{+1} - \tilde{e}_{+1}, 0) = (1-\alpha)\frac{\gamma/\beta}{1-\frac{\alpha}{2}} - 1. \tag{44}$$

There are two sub-cases to consider: $\lambda_H = 0$ and $\lambda_H > 0$.

Case 1a. When $\lambda^H(m_{+1} - \tilde{e}_{+1}, 0) = 0$, equation (44) gives

$$\alpha = \frac{2\left(\frac{\gamma}{\beta} - 1\right)}{2\frac{\gamma}{\beta} - 1}, \qquad \lambda_L(m_{+1}, \tilde{e}_{+1}) = 2\left(\frac{\gamma}{\beta} - 1\right).$$

From equation (24), the condition $\lambda^H(m_{+1} - \tilde{e}_{+1}, 0) = 0$ gives us that

$$\phi(m - \tilde{e}) + \epsilon_H \geq q^*, \tag{45}$$

whereas $\lambda_L(m_{+1}, \tilde{e}_{+1}) = 2\left(\frac{\gamma}{\beta} - 1\right)$ in (24) yields

$$\phi m + \frac{\beta}{2\gamma - \beta}\,\phi\tilde{e} = u^{'-1}\left(2\frac{\gamma}{\beta} - 1\right). \tag{46}$$

In this equilibrium, $\phi\tilde{e}$ and ϕm are indeterminate, as long as $\phi\tilde{e} \leq \phi m$. Expressions (46) give

$$\phi m = u^{'-1}\left(2\frac{\gamma}{\beta} - 1\right) - \frac{\beta}{2\gamma - \beta}\phi\tilde{e} \geq 0. \tag{47}$$

Replacing this value for ϕm in (45) and in the condition $\phi m \geq \phi\tilde{e}$ provide

$$\epsilon_H \geq q^* - u^{'-1}\left(2\frac{\gamma}{\beta} - 1\right) + \frac{\beta}{2\gamma}\phi\tilde{e}, \tag{48}$$

$$u^{'-1}\left(2\frac{\gamma}{\beta} - 1\right) \geq \frac{2\gamma}{2\gamma - \beta}\phi\tilde{e}. \tag{49}$$

Conditions (47), (48), and (49) are easier to satisfy when $\tilde{e}_{+1} = 0$, in which case

(47) gives $\phi m = u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$ and (48) requires $\epsilon_H \geq q^* - u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$. We summarize this equilibrium in the next lemma.

Lemma A1. *If* $\epsilon_H \geq q^* - u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$ *and* $\alpha = \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$, *then* $s^H = \min\{q^*, \epsilon_H\}$, *and*

$$q^H = q^*, \qquad q^L = u'^{-1}\left(2\frac{\gamma}{\beta}-1\right),$$
$$\phi m \in \left[\, \underline{\phi m}, \overline{\phi m} \,\right], \qquad \phi\tilde{e} = \frac{2\gamma-\beta}{\beta}\left[u'^{-1}\left(2\frac{\gamma}{\beta}-1\right) - \phi m\right]$$

where $\underline{\phi m} = \left(\frac{2\gamma-\beta}{2\gamma}\right)u'^{-1}\left(2\frac{\gamma}{\beta}-1\right) + \frac{\beta}{2\gamma}\max\{q^* - \epsilon_H, 0\}$ *and* $\overline{\phi m} = u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$.

Case 1b. Consider next an equilibrium where $\delta = 0$ still holds, but with $\lambda_H(m - \tilde{e}_{+1}, 0) > 0$. From (44) we must have

$$\alpha < \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}.$$

From (24) we have $s^H = \epsilon_H$, and from (24) and (44) we obtain

$$u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right) = q^L = \phi m + \phi\tilde{e}(1-\alpha),$$
$$u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) = q^H = \epsilon_H + \phi(m - \tilde{e}).$$

Subtracting the first expression from the second we get

$$\phi\tilde{e} = \frac{\epsilon_H - \left[u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right)\right]}{2-\alpha},$$

which then gives us

$$\phi m = \frac{u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right) + (1-\alpha)\left(u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - \epsilon_H\right)}{2-\alpha}.$$

We need to check that $q^H < q^*$, $m \geq \tilde{e}_{+1}$, and that $\tilde{e}_{+1} \geq 0$. First, observe that

$$q^H < q^* \quad \text{iff} \quad \alpha < \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}. \tag{50}$$

Second, observe that

$$\phi m \geq \phi \tilde{e} \quad \text{iff} \quad u'(\epsilon_H) \geq \frac{(1-\alpha).\gamma/\beta}{1-\frac{\alpha}{2}} \tag{51}$$

Thus, conditions (50) and (51) can co-exist only if $u'(\epsilon_H) > 1$ and $\alpha \geq \hat{\alpha}$, where $\hat{\alpha}$ is defined in (31).

Finally, $\phi\tilde{e} \geq 0$ requires

$$\epsilon_H \geq u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right) - u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right).$$

It is easy to show that the right-hand side of the last equation is monotonically increasing in α, and thus we can rewrite

$$\phi\tilde{e} \geq 0 \quad \text{iff} \quad \alpha \leq \tilde{\alpha}(\epsilon_H), \tag{52}$$

where $\tilde{\alpha}(\epsilon_H)$ is defined in (32). We can show that $\tilde{\alpha}'(\epsilon_H) > 0$, $\tilde{\alpha}(0) = 0$, $\tilde{\alpha}(\underline{\epsilon}) = \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$, and for $\tilde{\alpha}(\epsilon_H) > \hat{\epsilon}$. Thus, we can conclude that when $\epsilon_H \geq q^* - u'^{-1}(2\frac{\gamma}{\beta}-1)$, then $\tilde{\alpha} \geq \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$ and therefore (52) is satisfied by (50). Contrary, when $\epsilon_H < q^* - u'^{-1}(2\frac{\gamma}{\beta}-1)$, then $\tilde{\alpha} < \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$ and therefore (50) is satisfied by (52). Thus, we have the following result:

Lemma A2. *If $\epsilon_H < q^*$ and $\alpha \in \left[\hat{\alpha}, \min\left\{\tilde{\alpha}, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right\}\right)$, for $\hat{\alpha}$ defined in (31) and $\tilde{\alpha}$ defined in (32), where $\tilde{\alpha} = \min\left\{\tilde{\alpha}, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right\}$ iff $\epsilon_H < \underline{\epsilon}$ then $s^H = \epsilon_H$ and*

$$q^H = u'^{-1}\left(\frac{(1-\alpha)\gamma/\beta}{1-\frac{\alpha}{2}}\right), \qquad q^L = u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha}{2}}\right),$$
$$\phi\tilde{e} = \frac{\epsilon_H - \left[q^H - q^L\right]}{2-\alpha}, \qquad \phi m = \frac{q^L + (1-\alpha)(q^H - \epsilon_H)}{2-\alpha}.$$

Case 2: $\delta > 0$.

Consider now equilibria with $\delta > 0$; from equation (29) we have $m_{+1} = \tilde{e}_{+1}$. Equation (28) gives us

$$\delta = (1-\alpha)\lambda_L(m_{+1}, m_{+1}) - \alpha - \lambda_H(0,0) \tag{53}$$

and equation (27) gives

$$\gamma - \beta + \alpha = \left[\frac{\beta}{2} + 1 - \alpha\right] \lambda_L(m_{+1}, m_{+1}) - \lambda_H(0,0) \left[1 - \frac{\beta}{2}\right]. \tag{54}$$

Also from (21)

$$q^H = s^H, \qquad\qquad q^L = \phi m(2-\alpha).$$

Two sub-cases are possible: one in which $q^H = q^* \leq \epsilon_H$ and $\lambda^H(0,0) = 0$, and one in which $q^H = \epsilon_H < q^*$ and $\lambda^H(0,0) > 0$.

Case 2a. Consider the first sub-case: $\epsilon_H \geq q^* = q^H$ and $\lambda^H(0,0) = 0$. Combining (53) and (54) we obtain

$$\lambda_L = \frac{\gamma - \beta + \alpha}{\frac{\beta}{2} + 1 - \alpha} > 0,$$
$$\delta = \frac{\gamma - \beta - \alpha(\gamma - \frac{\beta}{2})}{\frac{\beta}{2} + 1 - \alpha} \geq 0.$$

Easily, $\delta \geq 0$ requires

$$\alpha \leq \frac{2\left(\frac{\gamma}{\beta} - 1\right)}{2\frac{\gamma}{\beta} - 1}.$$

Finally, from (24) and $q_L = \phi m(2-\alpha)$ we have

$$\phi m = \frac{u'^{-1}\left(\frac{1+\gamma-\frac{\beta}{2}}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha}.$$

Lemma A3. *If* $\epsilon_H \geq q^*$ *and* $\alpha \in \left[0, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right)$, *then* $s^H = q^*$ *and*

$$q^H = q^*, \qquad\qquad q^L = \phi m(2-\alpha),$$
$$\phi\tilde{e} = \phi m, \qquad\qquad \phi m = \frac{u'^{-1}\left(\frac{1+\gamma-\frac{\beta}{2}}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha}.$$

Case 2b. Finally, consider an equilibrium in which $q^* > \epsilon_H = q^H$ and $\lambda^H(0,0) > 0$. From (24), we have that $\lambda_H(0,0) = u'(q_H) - 1 = u'(\epsilon_H) - 1$. Also, we have that $\lambda_L(m,m) = u'(q_L) - 1$ and $q^L = \phi m(2-\alpha)$. Combining these expressions with (54)

we obtain:

$$\phi m = \frac{u^{'-1}\left(\frac{\gamma+u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha}.$$

Replacing this value in (24) we obtain

$$\lambda_L(m_{+1}, m_{+1}) = \frac{\gamma - \beta + \alpha + \Big(u'(\epsilon_H) - 1\Big)\left[1 - \frac{\beta}{2}\right]}{\frac{\beta}{2} + 1 - \alpha} > 0.$$

Finally, we have to check that $\delta \geq 0$: from (53)

$$\delta = \frac{\gamma - \beta - \alpha(\gamma - \frac{\beta}{2}) - \beta\Big(u'(\epsilon_H) - 1\Big)(1 - \frac{\alpha}{2})}{\frac{\beta}{2} + 1 - \alpha} \geq 0$$

thus, it is easy to check that $\delta \geq 0$ if

$$\frac{\gamma}{\beta} \geq u'(\epsilon_H), \qquad \text{and} \qquad \alpha \leq \frac{2\left(\frac{\gamma}{\beta} - u'(\epsilon_H)\right)}{2\frac{\gamma}{\beta} - u'(\epsilon_H)} \equiv \hat{\alpha}.$$

where $\hat{\alpha}$ is defined in (31).

Lemma A4. *If* $u^{'-1}\left(\frac{\gamma}{\beta}\right) \leq \epsilon_H < q^*$ *and* $\alpha < \hat{\alpha}$, *for* $\hat{\alpha}$ *defined in (??), then* $s^H = q_H$ *and*

$$q^H = \epsilon_H, \qquad q^L = u^{'-1}\left(\frac{\gamma + u'(\epsilon_H)\left[1 - \frac{\beta}{2}\right]}{\frac{\beta}{2} + 1 - \alpha}\right),$$

$$\phi\tilde{e} = \phi m, \qquad \phi m = \frac{u^{'-1}\left(\frac{\gamma+u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha}\right)}{2-\alpha}.$$

Proof of Corollary 5.3

Combining equations (24) and (28) we obtain $0 \leq \delta = (1-\alpha)u'(q^L) - u'(q^H) \Leftrightarrow \frac{u'(q^H)}{u'(q^L)} \leq 1 - \alpha < 1$, hence $q^H > q^L$.

Next we show that $\frac{\partial q^H}{\partial \alpha} \geq 0$ and $\frac{\partial q^L}{\partial \alpha} < 0$. If $\epsilon_H < q^*$ and $\alpha \in \left[\hat{\alpha}, \min\left\{\tilde{\alpha}, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right\}\right)$, for $\hat{\alpha}$ defined in (31) and $\tilde{\alpha}$ defined in (32), then from Lemma A2 we have $\frac{\partial q^H}{\partial \alpha} = -\frac{2(\gamma/\beta)}{u''(q^H)(2-\alpha)^2} > 0$, and $\frac{\partial q^L}{\partial \alpha} = \frac{2\gamma/\beta}{u''(q^L)(2-\alpha)^2} < 0$. When $\epsilon_H \geq q^*$ and $\alpha \in \left[0, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}\right)$, then from Lemma A3 we have $\frac{\partial q^H}{\partial \alpha} = 0$ and $\frac{\partial q^L}{\partial \alpha} = \frac{1+\gamma-\frac{\beta}{2}}{u''(q^L)\left[\frac{\beta}{2}+1-\alpha\right]^2} < 0$. Finally, if $\frac{\gamma}{\beta} >$

$u'(\epsilon_H) > 1$, then from Lemma A4 we obtain $\frac{\partial q^H}{\partial \alpha} = 0$ and $\frac{\partial q^L}{\partial \alpha} = \frac{\gamma + u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{u''(q^L)\left[\frac{\beta}{2}+1-\alpha\right]^2} < 0$.

Proof of Proposition 5.4

Step 1: Let σ_L be defined in (33) and $\alpha^o > 0$ in (39). If $\epsilon_H \geq q^*$, then there exists a unique monetary equilibrium with e-money, where Technology Provider conversion fee is

$$\alpha^* = \begin{cases} \frac{2(\gamma-\beta)}{2\gamma-\beta} & \text{if } \sigma \geq \sigma_L, \\ \alpha^o & \text{if } \sigma < \sigma_L. \end{cases}$$

Proof. When $\epsilon_H > q^*$, the revenue function of technology provider is given by:

$$R(\alpha) = \frac{\alpha}{2-\alpha}\left[\frac{\beta + 2(1-\alpha)}{2(1+\gamma)-\beta}\right]^{\frac{1}{\sigma}}. \tag{55}$$

It is easy to compute the following expression:

$$R'(\alpha) = \frac{2}{(2-\alpha)^2}\left[\frac{\beta + 2(1-\alpha)}{2(1+\gamma)-\beta}\right]^{\frac{1}{\sigma}}\left[1 - \frac{\alpha(2-\alpha)}{\sigma[\beta + 2(1-\alpha)]}\right]. \tag{56}$$

The sign of $R'(\alpha)$ depends on the sign of the term in the last square brackets, therefore

$$R'(\alpha) \gtrless 0 \text{ if } \sigma \gtrless \frac{\alpha(2-\alpha)}{\beta + 2(1-\alpha)}. \tag{57}$$

Let $g(\alpha) = \frac{\alpha(2-\alpha)}{\beta+2(1-\alpha)}$. Notice that $g(\alpha)$ is monotone increasing in α, for $\alpha \in (0, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1})$. Thus, if $\sigma > g(\frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1})$, then $R'(\alpha) > 0$ for all $\alpha \in [0, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}]$, and the unique equilibrium corresponds to $\alpha^* = \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$. If instead there is an $\alpha^o \in (0, \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1})$ such that $R'(\alpha^o) > 0$ if $\alpha < \alpha^o$, $R'(\alpha^o) = 0$ if $\alpha = \alpha^o$, and $R'(\alpha) < 0$ for $\alpha > \alpha^o$, then the unique equilibrium corresponds to $\alpha^* = \alpha^o$. It is easy to show that $g(\frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}) = \sigma_L$, for σ_L defined in (33). Also, it is easy to show that for $\sigma < \sigma_L$ the equation $\sigma = g(\alpha)$ admits two solutions: $\alpha = 1+\sigma+\sqrt{1+\sigma^2-\beta\sigma}$ and $\alpha = 1+\sigma-\sqrt{1+\sigma^2-\beta\sigma}$. Because we should restrict our attention to values of $\alpha \leq \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1} < 1$, α^o can only correspond to second solution: $\alpha^o = 1+\sigma-\sqrt{1+\sigma^2-\beta\sigma}$. Then, we conclude that when $\epsilon_H > q^*$,

$$\alpha^* = \begin{cases} \frac{2(\gamma-\beta)}{2\gamma-\beta} & \text{if } \sigma \geq \sigma_L, \\ 1+\sigma-\sqrt{1+\sigma^2-\beta\sigma} & \text{if } \sigma < \sigma_L. \end{cases}$$

□

Step 2: Let $\hat{\epsilon}$ be defined in (30), $\hat{\alpha}$ in (31), α^o in (39), and α^{oo} in (40). If $\hat{\epsilon} < \epsilon_H < q^*$, there exists a unique monetary equilibrium with e-money, where the technology provider's haircut is

$$\alpha^* = \begin{cases} \alpha^o & \text{if } \sigma < \sigma_L \text{ and } \epsilon_H > \epsilon_*(\sigma), \\ \hat{\alpha} & \text{if } \sigma < \sigma_L \text{ and } \epsilon_H \in (\epsilon_{**}(\sigma), \epsilon_*(\sigma)), \text{ or } \sigma \in (\sigma_L, \sigma_u) \text{ and } \epsilon_H \in (\epsilon_{**}(\sigma), 1), \\ \alpha^{oo} & \text{if } \sigma < \sigma_u \text{ and } \epsilon_H < \epsilon_{**}(\sigma), \\ & \text{or } \sigma > \sigma_u \text{ and } \epsilon_H < \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right] \\ \frac{2(\gamma-\beta)}{2\gamma-\beta} & \text{if } \epsilon_H > \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right] \end{cases}$$

where α^o is defined in (39) and α^{oo} in (40).

Proof. If $\hat{\epsilon} < \epsilon_H < q^*$, the revenue function of the technology provider is

$$R(\alpha) = \begin{cases} \frac{\alpha}{2-\alpha}\left[\frac{\beta+2(1-\alpha)}{2(\epsilon_H^{-\sigma}+\gamma)-\beta\epsilon_H^{-\sigma}}\right]^{\frac{1}{\sigma}} & \text{if } \alpha < \hat{\alpha} \\ \frac{\alpha}{2-\alpha}\left[\epsilon_H + \left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}} - \left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] & \text{if } \alpha \in [\hat{\alpha}, \overline{\alpha}] \end{cases}$$

Step 2.1: If $\hat{\epsilon} < \epsilon_H < q^*$, there exists a unique monetary equilibrium with e-money.

For $\alpha < \hat{\alpha}$, it is easy to compute

$$R'(\alpha) = \frac{2}{2-\alpha}\left[\frac{\beta+2(1-\alpha)}{2(\epsilon_H^{-\sigma}+\gamma)-\beta\epsilon_H^{-\sigma}}\right]^{\frac{1}{\sigma}}\left[\frac{1}{2-\alpha} - \frac{\alpha}{\sigma}\frac{1}{\beta+2(1-\alpha)}\right].$$

The sign of $R'(\alpha)$, for $\alpha \in (0, \hat{\alpha})$, depends exclusively on the sign of the term in the last square brackets, which the same term that determined the sign of $R'(\alpha)$ in (56). Then, as in the proof following (56), let $g(\alpha) = \frac{\alpha(2-\alpha)}{\beta+2(1-\alpha)}$. This function is monotone increasing for $\alpha \in (0, \hat{\alpha})$. Thus, if $\sigma > g(\hat{\alpha})$, then $R'(\alpha) > 0$ for all $\alpha \in [0, \hat{\alpha})$, if instead $\sigma < g(\hat{\alpha})$, then for α^o defined in (39), we have that $R'(\alpha) > 0$ when $\alpha < \alpha^o$, $R'(\alpha^o) = 0$, and $R'(\alpha) < 0$ for $\alpha > \alpha^o$ (and $\alpha < \hat{\alpha}$).

Consider next the case $\alpha > \hat{\alpha}$, $R'(\alpha)$ is given by

$$R'(\alpha) = \frac{2}{(2-\alpha)^2}\left[\epsilon_H + \left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}} - \left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right]$$

$$-\frac{\alpha}{2-\alpha}\left(\frac{\beta}{2\sigma\gamma}\right)\left[\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1-\sigma}{\sigma}}+(1-\alpha)^{-2}\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1-\sigma}{\sigma}}\right] \quad (58)$$

Notice that we can rewrite (58) as follows:

$$\begin{aligned}
R'(\alpha) =& \frac{2}{(2-\alpha)^2}\left[\epsilon_H+\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}-\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] \\
&-\frac{\alpha}{(2-\alpha)}\frac{1}{2\sigma}\left[\frac{1}{1-\alpha/2}\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}+\frac{1}{1-\alpha}\left(\frac{1}{1-\alpha/2}\right)\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] \\
=& \frac{1}{(2-\alpha)^2}\Bigg\{2\left[\epsilon_H+\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}-\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] \\
&-\frac{\alpha}{\sigma}\left[\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}+\frac{1}{1-\alpha}\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right]\Bigg\} \qquad (58)
\end{aligned}$$

Notice that the sign of $R'(\alpha)$ depends exclusively on the sign of the term in the curly brackets of expression (58). Let then

$$\begin{aligned}
H_1(\alpha) &= 2\left[\epsilon_H+\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}-\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] \\
H_2(\alpha) &= \left[\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}+\frac{1}{1-\alpha}\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right]
\end{aligned}$$

so that we can rewrite

$$R'(\alpha)=\frac{1}{(2-\alpha)^2}\left\{H_1(\alpha)-\frac{\alpha}{\sigma}H_2(\alpha)\right\}.$$

Now notice that

$$H_1'(\alpha)=-\frac{2}{\sigma(2-\alpha)}\left[\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}+\frac{1}{1-\alpha}\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right]<0, \quad (59)$$

and

$$H_2'(\alpha)=\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}\left[\left(\frac{1}{1-\alpha}\right)^{2+\frac{1}{\sigma}}+\frac{1}{\sigma(2-\alpha)}\left(\left(\frac{1}{1-\alpha}\right)^{2+\frac{1}{\sigma}}-1\right)\right]>0.$$

From the last three expressions, we can conclude that if $\alpha_2>\alpha_1>\hat{\alpha}$, then $R'(\alpha_2)<R'(\alpha_1)$, whereas if $\alpha_2<\alpha_1$, then $R'(\alpha_2)>R'(\alpha_1)$. Then, we can conclude that for

$\alpha > \hat{\alpha}$, the function $R(\alpha)$ is either increasing in the whole interval, or it is first increasing and then decreasing. Hence, the function $R(\alpha)$ can achieve at most one maximum in the interval $(0, \overline{\alpha})$.

Finally, we show that when $R'(\hat{\alpha}^-) < 0$, then $R'(\hat{\alpha}^+) < 0$. Suppose that $R'(\hat{\alpha}^-) < 0$, which requires

$$\frac{\hat{\alpha}}{\sigma} > \frac{\beta + 2(1-\hat{\alpha})}{2-\hat{\alpha}}. \tag{60}$$

Next, notice that from the definition of $\hat{\alpha}$ in (31)

$$\epsilon_H = \left(\frac{1-\hat{\alpha}/2}{(1-\hat{\alpha})\gamma/\beta}\right)^{\frac{1}{\sigma}}.$$

Then,

$$R'(\hat{\alpha}^+) = \frac{1}{(2-\hat{\alpha})^2}\left\{2\left(\frac{1-\hat{\alpha}/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}} - \frac{\hat{\alpha}}{\sigma}\left[\left(\frac{1-\hat{\alpha}/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}}\left(1+\left(\frac{1}{1-\hat{\alpha}}\right)^{1+\frac{1}{\sigma}}\right)\right]\right\},$$

which implies that $R'(\hat{\alpha}^+) < 0$ if and only if

$$\frac{\hat{\alpha}}{\sigma} > \frac{2}{1+\left(\frac{1}{1-\hat{\alpha}}\right)^{1+\frac{1}{\sigma}}}. \tag{61}$$

It is easy to observe that when (60) holds, then (61) holds as well: indeed, comparing the right-hand side of (60) and (61) we have that

$$\frac{2}{1+\left(\frac{1}{1-\hat{\alpha}}\right)^{1+\frac{1}{\sigma}}} < \frac{2}{1+\left(\frac{1}{1-\hat{\alpha}}\right)} = \frac{2(1-\hat{\alpha})}{2-\hat{\alpha}} < \frac{\beta+2(1-\hat{\alpha})}{2-\hat{\alpha}},$$

proving that when $R'(\hat{\alpha}^-) < 0$, then $R'(\hat{\alpha}^+) < 0$. This result, together with the previous ones, implies that the revenue function $R(\alpha)$ can achieve a unique maximum in the interval $(0, \overline{\alpha})$. In particular, the optimal conversion fee is

$$\alpha^* = \begin{cases} \alpha^o & \text{if } R'(\hat{\alpha}-) < 0, \\ \hat{\alpha} & \text{if } R'(\hat{\alpha}-) > 0 > R'(\hat{\alpha}+), \\ \alpha^{oo} & \text{if } R'(\hat{\alpha}-) > 0 > R'(\overline{\alpha}), \\ \overline{\alpha} & \text{if } R'(\overline{\alpha}) > 0. \end{cases}$$

for α^o defined in (39), $\hat{\alpha}$ defined in (31), α^{oo} defined in (40), and $\overline{\alpha}$ defined in (**??**).

Step 2.2 If $\hat{\epsilon} < \epsilon_H < q^*$, there exists a unique monetary equilibrium with e-money,

where the technology provider's haircut is

$$\alpha^* = \begin{cases} \alpha^o & \text{if } \sigma < \sigma_L \text{ and } \epsilon_H > \epsilon_*(\sigma), \\ \hat{\alpha} & \text{if } \sigma < \sigma_u \text{ and } \epsilon_H \in \Big(\Omega(\sigma), \Psi(\sigma)\Big], \\ \alpha^{oo} & \text{if } \epsilon_H < \Omega(\sigma). \\ \frac{2(\gamma-\beta)}{2\gamma-\beta} & \text{if } R'(\hat{\alpha}^+) > 0 \end{cases}$$

where α^o is defined in (39) and α^{oo} in (40).

First, consider the case when $\alpha = \alpha^o$. From the previous steps, we know that $\alpha = \alpha^o$ if $R'(\hat{\alpha}-) < 0$, which is the same as $\sigma > g(\hat{\alpha})$. Using the expression in (31), it is easy to compute that

$$g(\hat{\alpha}) = \frac{4\gamma\big(\gamma - \beta\epsilon_H^{-\sigma}\big)}{\beta\big[2\gamma - \beta\epsilon_H^{-\sigma}\big]\big[2\big(\epsilon_H^{-\sigma} + \gamma\big) - \beta\epsilon_H^{-\sigma}\big]}.$$

Define then the implicit function $\epsilon_*(\sigma)$ as the solution to

$$\frac{4\gamma\big(\gamma - \beta\epsilon_*(\sigma)^{-\sigma}\big)}{\beta\big[2\gamma - \beta\epsilon_*(\sigma)^{-\sigma}\big]\big[2\big(\epsilon_*(\sigma)^{-\sigma} + \gamma\big) - \beta\epsilon_*(\sigma)^{-\sigma}\big]} - \sigma = 0. \tag{62}$$

Notice that, when this function exists, this is the function $\epsilon_*(\sigma)$ defined in (35). Assuming for the moment that $\epsilon_*(\sigma)$ is well-defined, it is easy to observe that $\sigma > g(\hat{\alpha})$ if and only if $\epsilon_H < \epsilon_*(\sigma)$. After some manipulation, we can rewrite

$$\frac{\partial g'(\hat{\alpha})}{\partial \sigma} = \frac{4\gamma\epsilon_H^{-\sigma}\log(\epsilon_H)}{(2\gamma - \beta\epsilon_H)(\gamma + 2\epsilon_H^{-\sigma} - \beta\epsilon_H^{-\sigma})}\left(\frac{\gamma}{2\gamma - \beta\epsilon_H^{-\sigma}} + \frac{(\gamma - \beta\epsilon_H^{-\sigma})(2-\beta)}{\beta(\gamma + 2\epsilon_H^{-\sigma} - \beta\epsilon_H^{-\sigma})}\right) < 0,$$

where the conclusion follows from $1 \equiv q^* > \epsilon_H > \hat{\epsilon}$. Then, if a solution $\epsilon_*(\sigma)$ exists, from (62) the solution is such unique. Notice that when $\sigma = 0$, then $\epsilon_*(\sigma) = \hat{\epsilon}$ which also equals 0 when $\sigma = 0$. Moreover, from (62) it is easy to compute

$$\frac{4\gamma\epsilon_*(\sigma)^{-\sigma}\left[\log(\epsilon_*(\sigma)) + \frac{\sigma\epsilon_*'(\sigma)}{\epsilon_*(\sigma)}\right]}{(2\gamma - \beta\epsilon_*(\sigma))(\gamma + 2\epsilon_*(\sigma)^{-\sigma} - \beta\epsilon_*(\sigma)^{-\sigma})}\left(\frac{\gamma}{2\gamma - \beta\epsilon_*(\sigma)^{-\sigma}} + \frac{(\gamma - \beta\epsilon_*(\sigma)^{-\sigma})(2-\beta)}{\beta(\gamma + 2\epsilon_*(\sigma)^{-\sigma} - \beta\epsilon_*(\sigma)^{-\sigma})}\right) - 1 = 0$$

hence $\epsilon_*'(\sigma) > 0$. Thus, the function $\epsilon_*(\sigma)$ is a well defined monotone increasing function for $\sigma \in (0, \sigma')$, where σ' is the largest σ such that (62) holds for $\epsilon_*(\sigma') = 1$.

Replacing this value in (62) we obtain

$$\frac{4\gamma\big(\gamma-\beta\big)}{\beta\big[2\gamma-\beta\big]\big[2\big(1+\gamma\big)-\beta\big]}-\sigma'=0 \;\;\Rightarrow \sigma'=\frac{4\gamma\big(\gamma-\beta\big)}{\beta\big[2\gamma-\beta\big]\big[2\big(1+\gamma\big)-\beta\big]}=\sigma_L$$

for σ_L defined in (33). Moreovoer, it is easy to check that, for $\sigma<\sigma_L$, $\epsilon^*(\sigma)>\hat{\sigma}$:

$$\frac{4\gamma\big(\gamma-\beta\epsilon_*(\sigma)^{-\sigma}\big)}{\beta\big[2\gamma-\beta\epsilon_*(\sigma)^{-\sigma}\big]\big[2\big(\epsilon_*(\sigma)^{-\sigma}+\gamma\big)-\beta\epsilon_*(\sigma)^{-\sigma}\big]}=\sigma\geq 0 \text{ iff } \epsilon_*(\sigma)\geq\left(\frac{\gamma}{\beta}\right)^{-\frac{1}{\sigma}}=\hat{\epsilon}.$$

Therefore, for σ_L defined in (33), $\epsilon_*(\sigma)$ in (35), and α^o in (39), we conclude that the optimal haircut is $\alpha^*=\alpha^o$ if $\sigma<\sigma_L$ and $\epsilon_H>\epsilon_*(\sigma)$.

Next, consider the case for $\alpha^*=\hat{\alpha}$: we know that $\alpha^*=\hat{\alpha}$ if and only if $R'(\hat{\alpha}-)>0>R'(\hat{\alpha}+)$. Since $R'(\hat{\alpha}-)>0$ when $\sigma<g(\hat{\alpha})$, for $g(\alpha)=\frac{\alpha(2-\alpha)}{\beta+2(1-\alpha)}$, from the discussion above we know when $\sigma<\sigma_L$, $R'(\hat{\alpha}-)>0$ requires $\epsilon_H<\epsilon_*(\sigma)$. When $\sigma>\sigma_L$, $R'(\hat{\alpha}-)>0$ for all ϵ_H satisfying $1>\epsilon_H>\hat{\epsilon}$. On the other hand, equation (61) must hold true for $R'(\hat{\alpha}+)<0$.

Replacing the value of $\hat{\alpha}$, we can rewrite (61) as follows:

$$\frac{\left(\frac{\gamma}{\beta}-\epsilon_H^{-\sigma}\right)}{\sigma[2\frac{\gamma}{\beta}-\epsilon_H^{-\sigma}]}>\frac{1}{1+\left(\frac{2\gamma\epsilon_H^{\sigma}-\beta}{\beta}\right)^{1+\frac{1}{\sigma}}}.$$

Let then

$$h_1(\epsilon)=\frac{\left(\frac{\gamma}{\beta}-\epsilon^{-\sigma}\right)}{\sigma[2\frac{\gamma}{\beta}-\epsilon^{-\sigma}]},$$
$$h_2(\epsilon)=\frac{1}{1+\left(\frac{2\gamma\epsilon^{\sigma}-\beta}{\beta}\right)^{1+\frac{1}{\sigma}}}.$$

Notice that $\epsilon_{**}(\sigma)$ defined in (36) satisfies $h_1(\epsilon_{**})=h_2(\epsilon_{**})$. Also, $h_1'(\epsilon)>0$ and $h_2'(\epsilon)<0$. Hence, the function $\epsilon_{**}(\sigma)$ is well defined, in the sense that if an $\epsilon_{**}(\sigma)$ that satisfies $h_1(\epsilon_{**}(\sigma))=h_2(\epsilon_{**})(\sigma)$ exists, this is unique. Moreover,, if $\epsilon_H>\epsilon_{**}(\sigma)$, then (61) holds and $R'(\hat{\alpha}+)<0$. Also, we can show that $\epsilon_{**}(\sigma)\to 0$ and $\sigma\to 0$. Moreover, by comparing (35) and (36), we have that ($\sigma\leq\sigma_L$) $\epsilon_*(\sigma)>\epsilon_{**}(\sigma)$, and by comparing (30) and (36), $\epsilon_{**}(\sigma)>\hat{\epsilon}$. Finally, we can show that $\epsilon_*(\sigma_u)=q^*$, for σ_u defined in (34). Therefore, we conclude that $\alpha=\hat{\alpha}$ if $\sigma\leq\sigma_L$ and $\epsilon_H\in\{\epsilon_{**}(\sigma),\epsilon_*(\sigma)\}$, or $\sigma_L<\sigma\leq\sigma_u$ and $\epsilon_H\in\{\epsilon_{**}(\sigma),q^*(\sigma)\}$.

Next, consider the case when $\alpha^* = \alpha^{oo}$: this is the optimal solution if $R'(\hat{\alpha}+) > 0$ and $R'(\overline{\alpha}) < 0$. From the previous discussion, we know that $\epsilon_H < \epsilon_{**}(\sigma)$ must hold when $\sigma_H < \sigma_u$. Consider then $R'(\overline{\alpha}-)$: if $\epsilon_H < \underline{\epsilon}$, so that $\overline{\alpha} = \tilde{\alpha}$, for $\tilde{\alpha}$ defined in (32), then

$$R'(\tilde{\alpha}) = -\frac{\alpha}{\sigma(2-\alpha)^2}\left[\left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}} + \frac{1}{1-\alpha}\left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right] < 0.$$

If instead $\epsilon_H < \underline{\epsilon}$

$$R'(\overline{\alpha}-) = -\frac{1}{(2-\alpha)^2}\left[\epsilon_H + \left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} - 1 - \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left(\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right)\right],$$

and therefore $R'(\overline{\alpha}-) < 0$ then $1 < \epsilon_H < \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$. It is easy to show that then $\sigma = \sigma_u$ this threshold equals $q^* = 1$, and that this function is monotone decreasing in σ. Hence, we conclude that when $1 < \epsilon_H < \hat{\epsilon}(\sigma)$, $\alpha^* = \alpha^{oo}$ if $\epsilon_H < \epsilon_{**}(\sigma)$ when $\sigma < \sigma_u$ and $\epsilon_H < \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$ when $\sigma > \sigma_u$, and that $\alpha^* = \frac{2\left(\frac{\gamma}{\beta}-1\right)}{2\frac{\gamma}{\beta}-1}$ if $\epsilon_H > \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$.

□

Step 3: If $\epsilon_H < \hat{\epsilon}$, there exists a unique monetary equilibrium with e-money, where the technology provider's haircut is $\alpha^* = \alpha^{oo}$ if $\epsilon_H < \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$, and $\alpha^* = \frac{2(\gamma-\beta)}{2\gamma-\beta}$ if $\epsilon_H > \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$.

Proof. In this case the revenue function of the Technology Provider is given by:

$$R(\alpha) = \frac{\alpha}{2-\alpha}\left[\epsilon_H + \left(\frac{1-\alpha/2}{\gamma/\beta}\right)^{\frac{1}{\sigma}} - \left(\frac{1-\alpha/2}{(1-\alpha)\gamma/\beta}\right)^{\frac{1}{\sigma}}\right]. \tag{63}$$

Consider the first derivative of the revenue function (63): this is the same expression as (58). Evaluate $R'(\alpha)$ at $\alpha = 0$:

$$R'(\alpha)\Big|_{\alpha=0} = \frac{\epsilon_H}{2},$$

which shows that $R'(\alpha)\Big|_{\alpha=0} > 0$ is always positive. This means the revenue function

of the technology provider, $R(\alpha)$, is increasing for low values of α.

Then, the expression for $R'(\alpha)$ is given in (58), and using the same argument to the one used in the proof following (58) we can easily prove that $R'(\alpha)$ is monotonically decreasing. Since $R'(0) > 0$, from the step above we know that $R'(\overline{\alpha}-) < 0$ if and only if $\epsilon_H < \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$, in which case revenues are maximized at $\alpha^* = \alpha^{oo}$, for α^{oo} defined in (40). If instead $\epsilon_H > \underline{\epsilon}(\sigma) + \frac{\gamma-\beta}{\sigma(2\gamma-\beta)}\left[\left(\frac{\beta}{2\gamma-\beta}\right)^{\frac{1}{\sigma}} + \frac{2\gamma-\beta}{\beta}\right]$, then revenues are maximized at $\overline{\alpha} = \frac{2(\gamma-\beta)}{2\gamma-\beta}$. □

Proof of Lemma 6.1

1. Consider first the solution for the economy with fiat money only: since $\epsilon_H > \underline{\epsilon}(\sigma)$ holds, from Proposition 4.2 we have $\hat{q}^H = q^*$ and $\hat{q}^L = \hat{\phi m} = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right)$. Consider then the economy with fiat money and e-money: from Proposition 5.4, we have that $\alpha^* = \frac{2(\gamma-\beta)}{2\gamma-\beta}$. Moreover, from Proposition 5.2, if we have that $q^H = q^*$, $q^L = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right)$, and

$$\phi m = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right) - \frac{\beta}{2\gamma-\beta}\phi\tilde{e} < \hat{\phi m}.$$

Therefore, $\hat{q}^H = q^H > q^L = \hat{q}^L$, and $\phi m < \hat{\phi m}$.

2. Consider next the case where $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$.

   (a) If $\epsilon_H > \underline{\epsilon}(\sigma)$, for $\underline{\epsilon}(\sigma)$ defined in (11), from Proposition 4.2 we have that in the economy with only fiat money $\hat{q}^H = q^*$ and $\hat{q}^L = \hat{\phi m} = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right)$. In the economy with fiat money and e-money we have the following observations.

      i. If $\epsilon_H > q^*$, which can occur only if $\sigma < \sigma_L$, from Proposition 5.4 we have that $\alpha^* = \alpha^o$, for α^o defined in (39), and from Proposition 5.2 we have $q^H = q^*$ and $q^L = u'^{-1}\left(\frac{1+\gamma-\frac{\beta}{2}}{\frac{\beta}{2}+1-\alpha^o}\right)$. Since $\alpha^o < \frac{2(\gamma-\beta)}{2\gamma-\beta}$, it follows that $q_L > u'^{-1}\left(\frac{1+\gamma-\frac{\beta}{2}}{\frac{\beta}{2}+1-\frac{2(\gamma-\beta)}{2\gamma-\beta}}\right) = u'^{-1}\left(2\frac{\gamma}{\beta} - 1\right)$. Hence, for this parameter configuration, the following holds: $\hat{q}^H = q^H$ and $q^L > \hat{q}^L$.

      ii. If $\epsilon_H < q^*$, then from Proposition 5.4 we have that $\alpha^* \in \{\alpha^o, \hat{\alpha}, \alpha^{oo}\} < \frac{2(\gamma-\beta)}{2\gamma-\beta}$, for α^o defined in (39), $\hat{\alpha}$ in (31), and α^{oo} in (40). From Proposition 5.2 we have that $q^H = \epsilon_H$ if $\epsilon_H > \Omega(\sigma)$ (thus $\alpha^* \in \{\alpha^o, \hat{\alpha}\}$), and $q^H = u'^{-1}\left(\frac{(1-\alpha^{oo})\gamma/\beta}{1-\frac{\alpha^{oo}}{2}}\right)$ if $\epsilon_H < \Omega(\sigma)$ (thus $\alpha^* = \alpha^{oo}$). Moreover, $q^L = u'^{-1}\left(\frac{\gamma+u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha^*}\right)$ if $\epsilon_H > \Omega(\sigma)$ (thus $\alpha^* \in \{\alpha^o, \hat{\alpha}\}$), and

$q^L = u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha^{oo}}{2}}\right)$ if $\epsilon_H < \Omega(\sigma)$ (thus $\alpha^* = \alpha^{oo}$). When $\epsilon_H > \Omega(\sigma)$ (thus $\alpha^* \in \{\alpha^o, \hat{\alpha}\}$), since $\alpha^* < \frac{2(\gamma-\beta)}{2\gamma-\beta}$ and $\epsilon_H < q^* = u'^{-1}(1)$, then $q^L > u'^{-1}\left(\frac{\gamma+1-\frac{\beta}{2}}{\frac{\beta}{2}+1-\frac{2(\gamma-\beta)}{2\gamma-\beta}}\right) = u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$. When $\epsilon_H < \Omega(\sigma)$ (thus $\alpha^* = \alpha^{oo}$), since $\alpha^{oo} < \frac{2(\gamma-\beta)}{2\gamma-\beta}$, then $q^H < u'^{-1}\left(\frac{(1-\frac{2(\gamma-\beta)}{2\gamma-\beta})\gamma/\beta}{1-\frac{\frac{2(\gamma-\beta)}{2\gamma-\beta}}{2}}\right) = u'^{-1}(1) = q^*$, and $q^L > u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\frac{2(\gamma-\beta)}{2\gamma-\beta}}{2}}\right) = u'^{-1}\left(2\frac{\gamma}{\beta}-1\right)$. Hence, for this parameter configuration, the following holds: $q^H < \hat{q}^H$ and $q^L > \hat{q}^L$.

(b) If $\epsilon_H < \underline{\epsilon}(\sigma)$, for $\underline{\epsilon}(\sigma)$ defined in (11), from Proposition 4.2 we have that in the economy with only fiat money $\hat{q}^H = \hat{\phi m} + \epsilon_H$ and $\hat{q}^L = \hat{\phi m}$, for $\hat{\phi m}$ solving $\frac{\gamma}{\beta} = \frac{u'(\hat{\phi m}+\epsilon_H)}{2} + \frac{u'(\hat{\phi m})}{2}$. Consider next the economy with fiat money and e-money.

i. If $\epsilon_H \geq \Omega(\sigma)$, then it follows that $\epsilon_H > \hat{\epsilon}(\sigma)$ hold as well, and from Proposition 5.4 we have $\alpha^* \in \{\alpha^o, \hat{\alpha}\} \leq \hat{\alpha}$, for α^o defined in (39) and $\hat{\alpha}$ defined in (31). From Lemma 5.2, we have that $q^H = \epsilon_H$ and $q_L = u'^{-1}\left(\frac{\gamma+u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha^*}\right)$. Since $\alpha^* \leq \hat{\alpha}$, we have that

$$q_L \geq u'^{-1}\left(\frac{\gamma + u'(\epsilon_H)\left[1-\frac{\beta}{2}\right]}{\frac{\beta}{2}+1-\alpha^*}\right) = u'^{-1}\left(2\frac{\gamma}{\beta} - u'(\epsilon_H)\right).$$

Then, easily $q^H = \epsilon_H < \epsilon_H + \hat{\phi m} = \hat{q}^H$. Moreover, to show that $q^L > \hat{q}^L$, define the function

$$F(x) = \frac{u'(x+\epsilon_H)}{2} + \frac{u'(x)}{2}.$$

Notice that this function is monotonically decreasing in x and that $F(\hat{q}_L) = \frac{\gamma}{\beta}$, for $\hat{q}_L$ consumption in the low state of nature with fiat money only. Moreover, note that

$$F\left(u'^{-1}\left(2\frac{\gamma}{\beta} - u'(\epsilon_H)\right)\right) = \frac{\gamma}{\beta} + \frac{u'(u'^{-1}\left(2\frac{\gamma}{\beta} - u'(\epsilon_H)\right) + \epsilon_H)}{2} - \frac{u'(\epsilon_H)}{2} < \frac{\gamma}{\beta}$$

Since $F(\cdot)$ is monotone decreasing in x, we conclude that $u'^{-1}\left(2\frac{\gamma}{\beta} - u'(\epsilon_H)\right) > \hat{q}_L$. Thus for this parameter configuration we have $q_H < \hat{q}_H$ and $q_L > \hat{q}_L$.

ii If $\epsilon_H < \Omega(\sigma)$, from Proposition 5.4 we have $\alpha^* = \alpha^{oo}$, for α^{oo} defined in

(40). Moreover, from Proposition 5.2 we have $q^H = u'^{-1}\left(\frac{(1-\alpha^{oo})\gamma/\beta}{1-\frac{\alpha^{oo}}{2}}\right)$ and $q^L = u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\alpha^{oo}}{2}}\right)$. Also, from Corollary 5.3, since $\alpha^{oo} < \tilde{\alpha}$, for $\tilde{\alpha}$ defined in (32), we have that $q^H < u'^{-1}\left(\frac{(1-\tilde{\alpha})\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)$ and $q_L > u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)$. Also, note that $u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right) = u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)+\epsilon_H$, and that $\frac{1}{2}u'\left(u'^{-1}\left(\frac{(1-\tilde{\alpha})\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)\right)+\frac{1}{2}u'\left(u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)\right) = \frac{\gamma}{\beta}$. Thus, $u'^{-1}\left(\frac{(1-\tilde{\alpha})\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right) = \hat{q}^H$ and $\hat{q}^L = u'^{-1}\left(\frac{\gamma/\beta}{1-\frac{\tilde{\alpha}}{2}}\right)$, therefore also for this parameter configuration we have $q_H < \hat{q}_H$ and $q_L > \hat{q}_L$.

Proof of Proposition 6.2

Consider first buyers' welfare in the economy with fiat money only of Section 4:

$$\begin{aligned}
(1-\beta)\hat{\mathcal{W}}^B &= \sum_{k=L,H}\frac{1}{2}\left[u(\hat{q}^k)+U(\hat{X}^k)-\hat{H}^k\right]\\
&= \sum_{k=L,H}\frac{1}{2}\left[u(\hat{q}^k)+(\hat{\phi m}-\hat{\phi}\hat{d}^k_m)+\epsilon_k-\hat{s}_k\right]+U(\hat{X})-\hat{X}-\hat{\phi m}_{+1}+\hat{\phi}\tau\\
&= \frac{u(\hat{q}^H)+\epsilon_H-(\hat{\phi}\hat{d}^H_m-\hat{s}^H)}{2}+\frac{u(\hat{q}^L)-\hat{\phi}\hat{d}^L_m}{2}+U(\hat{X})-\hat{X}\\
&= \frac{u(\hat{q}^H)-\hat{q}^H+\epsilon_H}{2}+\frac{u(\hat{q}^L)-\hat{q}^L}{2}+U(\hat{X})-\hat{X}.
\end{aligned}$$

Consider next buyers' welfare in the economy with fiat money and e-money:

$$\begin{aligned}
(1-\beta)\mathcal{W}^B &= \sum_{k=L,H}\frac{1}{2}\left[u(q^k)+U(X^k)-H^k\right] =\\
&+\frac{u(q^H)+U(X^H)-X^H+\phi(m-\tilde{e}-d^H_m)+\left(\epsilon_H-s^H\right)-\phi m_{+1}+\phi\tau}{2}\\
&+\frac{u(q^L)+U(X^L)-X^L+\phi(m-d^L_m)+(1-\alpha)\phi(\tilde{e}-d^L_e)-\phi m_{+1}+\phi\tau}{2}\\
&= \frac{u(q^H)+\epsilon_H-(\phi d^H_m+s^H)}{2}+\frac{u(q^L)-\phi d^L_m-\phi d^L_e(1-\alpha)-\alpha\phi\tilde{e}}{2}+U(X)-X\\
&= \frac{u(q^H)-q^H+\epsilon_H}{2}+\frac{u(q^L)-q^L}{2}+U(X)-X-\alpha\frac{\phi\tilde{e}}{2}.
\end{aligned}$$

1. Consider first an economy where *i)* $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or *ii)* $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$. From Proposition 5.4, the technology provider's optimal conversion fee is $\alpha^* = \frac{2(\gamma-\beta)}{2\gamma-\beta}$. Also, from Lemma 6.1, $\hat{q}^H = q^H > q_L = \hat{q}^L$. Using equation (20) and the fact that the measure of sellers converting e-money into money in equilibrium is $\mu(\alpha) = \frac{1}{2}$, the conclusion follows since $\mathcal{U}^{TP} = \frac{\alpha^*}{2}\phi e(\alpha^*)$.

2. Consider next the parameter configuration where $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$. By continuity, we know that by continuity, when $\sigma \geq \sigma_L$, there exists an ϵ_H close enough to $\max\{\Omega(\sigma), \Psi(\sigma)\}$ such that the conclusion $(1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}\right) > 0$ holds. Thus, we have to show that there exist parameter configurations such that e-money adoption improves buyers' welfare: $(1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}\right) < 0$ holds. Without loss of generality, suppose that $\epsilon_H < \min\{\underline{\epsilon}(\sigma), \hat{\epsilon}(\sigma)\}$. From Proposition 4.2 we have that $\hat{q}^H = \hat{q}^L + \epsilon_H$ and $0 = -\frac{\gamma}{\beta} + \frac{u'(\hat{q}^L+\epsilon_H)}{2} + \frac{u'(\hat{q}^L)}{2}$, whereas from Preposition 5.4 we have that $\alpha^* = \alpha^{oo} < \tilde{\alpha}$, from α^{oo} defined in (40) and $\tilde{\alpha}$ defined in (32). Moreover, from Proposition 5.2 we have $q^H = \left(\frac{2(1-\alpha)}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma}$, $q^L = \left(\frac{2}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma}$, $\phi\tilde{e} = \frac{\epsilon_H - \left[q^H - q^L\right]}{2-\alpha}$. For $\alpha < \tilde{\alpha}$, consider the function:

$$\begin{aligned}\Gamma(\alpha) = &\frac{1}{2}\left[\left(\frac{(\hat{q}^H)^{1-\sigma}}{1-\sigma} - \hat{q}^H\right) + \left(\frac{(\hat{q}^L)^{1-\sigma}}{1-\sigma} - \hat{q}^L\right)\right] \\ &- \frac{1}{2}\left[\left(\frac{\left(\frac{2(1-\alpha)}{2-\alpha}\frac{\gamma}{\beta}\right)^{-\frac{1-\sigma}{\sigma}}}{1-\sigma} - \left(\frac{2(1-\alpha)}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma}\right) + \left(\frac{\left(\frac{2}{2-\alpha}\frac{\gamma}{\beta}\right)^{-\frac{1-\sigma}{\sigma}}}{1-\sigma} - \left(\frac{2}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma}\right)\right] \\ &+ \frac{\alpha}{2(2-\alpha)}\left[\epsilon_H - \left(\left(\frac{2(1-\alpha)}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma} - \left(\frac{2}{2-\alpha}\frac{\gamma}{\beta}\right)^{-1/\sigma}\right)\right].\end{aligned}$$

Notice that $\Gamma(\alpha^{oo}) = (1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}\right)$, and that

$$\Gamma(0) = \frac{1}{2}\left[\left(\frac{(\hat{q}^H)^{1-\sigma}}{1-\sigma} - \hat{q}^H\right) + \left(\frac{(\hat{q}^L)^{1-\sigma}}{1-\sigma} - \hat{q}^L\right)\right] - \left[\frac{\left(\frac{\gamma}{\beta}\right)^{-\frac{1-\sigma}{\sigma}}}{1-\sigma} - \left(\frac{\gamma}{\beta}\right)^{-\frac{1-\sigma}{\sigma}}\right] < 0,$$

where the inequality follows from concavity of $u(\cdot)$ and the fact that $\frac{u'(\hat{q}^H)}{2} + \frac{u'(\hat{q}^L)}{2} = \frac{\gamma}{\beta}$. Since $\Gamma(\alpha)$ is continuous in α, there exists a α^u such that $\Gamma(\alpha) < 0$ for $\alpha < \alpha^u$. Since $\Gamma(\alpha^{oo}) = (1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}\right)$, it is enough to show that there exist a parameter configuration such that $\tilde{\alpha} < \alpha^u$, to conclude that $\Gamma(\alpha^{oo}) < 0$, and thus $(1-\beta)\left(\hat{\mathcal{W}}^B - \mathcal{W}\right) < 0$. Note that from the definition of $\tilde{\alpha}$ in (32), for α^u given, there exists an $\mathcal{E}_H > 0$ small enough such that $\tilde{\alpha} < \alpha^u$ whenever $\epsilon_H < \mathcal{E}_H$. Thus, we conclude that when $\epsilon_H < \mathcal{E}_H$, then $\Gamma(\alpha^{oo}) < 0$ and therefore $\mathcal{W} > \hat{\mathcal{W}}^B$ holds true, which concludes our proof.

Proof of Corollary 6.3

The conclusion follows directly from Proposition 6.2, and in particular from the fact that $\mathcal{U}^{TP} + (1-\beta)\left(\mathcal{W}^B - \hat{\mathcal{W}}^B\right) = 0$ if *i)* $\sigma \in [\sigma_L, \sigma_u)$ and $\epsilon_H > \Psi(\sigma)$ or *ii)* $\sigma > \sigma_u$ and $\epsilon_H \geq \Omega(\sigma)$, whereas $\mathcal{U}^{TP} + (1-\beta)\left(\mathcal{W}^B - \hat{\mathcal{W}}^B\right) > 0$ if $\sigma < \sigma_L$ or $\epsilon_H < \max\{\Omega(\sigma), \Psi(\sigma)\}$.

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