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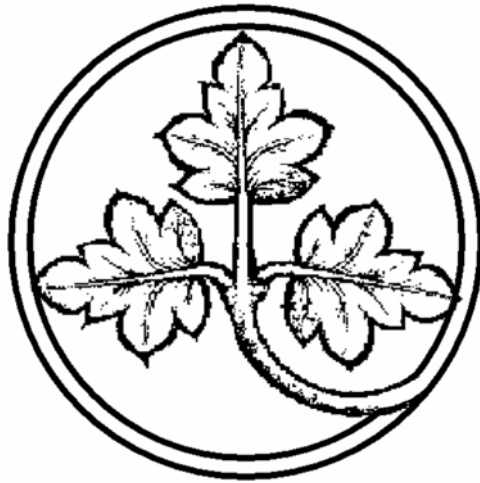
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The Welfare Economics of Adaptive Preferences

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# **The Welfare Economics of Adaptive Preferences**

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June 2005

# The Welfare Economics of Adaptive Preferences<sup>1</sup>

## Abstract

In this paper I demonstrate that a reasonable welfare theoretic concept of "progress" can be made consistent with the assumption of endogenously changing preferences as long as these preference changes correspond to the pattern of "adaptive preferences". The main theorem of the paper shows that under certain additional conditions "adaptive preferences" imply the existence of a complete pre-ordering of the consumption space in terms of "improvement paths" which allow endogenous preference changes. It is then shown that welfare economics of "improvement paths" is also possible with interpersonal influences on preferences. A conjecture is developed that results of recent empirical and experimental research into human economic behaviour corroborate the hypothesis of "adaptive preferences".

Keywords: Welfare Economics, Endogenous Preferences, Adaptive Preferences, Interpersonal Influences on Preferences, Improvement Paths, Bounded Rationality.

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## A Introduction

Traditional neo-classical economics has worked with the assumption that preferences of agents in the economy are fixed. This assumption has always been disputed, and, indeed, in the social sciences outside of neoclassical economics the assumption has never been accepted by anyone. Modern economics, especially experimental economics, has raised additional doubts about the realism of this behavioural assumption.

In this paper I do not want to discuss the empirical validity of this assumption in more detail. The purpose of the paper is a different one. I try to overcome a particular obstacle for the introduction of endogenously changing preferences into economic theory. The obstacle is the lack of an answer to the question: how can you do welfare economics, if preferences change endogenously? After all, preferences of individual agents are the basic measuring rod of economic welfare, of the performance generated in an economic system. How can we evaluate an economic system with a measuring rod that itself changes with the system?

Long ago I published a paper which tried a very preliminary answer to this question (von Weizsäcker 1971). In that paper I worked in a two-commodity world and I was able to prove a theorem which showed that – to a certain extent - in this world of endogenous preferences welfare economics is possible. But many theorists at the time believed that the results could not be generalised to more than two dimensions. So nobody in economic theory followed up the line of thought I had presented in that earlier paper. In the present paper I show that similar results to those then obtained can indeed be generalised to the  $n$ -dimensional Euclidean space of commodities. But I consider the present paper only to be a beginning for a much more extensive research programme about the possibility of welfare economics with endogenously determined tastes.

It is rather obvious that welfare economics in the traditional meaning of this concept cannot be performed with any arbitrary endogeneity of preferences. We need some "laws of motion" of preferences. If we are successful in establishing the possibility of welfare economics with certain laws of motion of endogenous preferences we may then ask the question: are these laws of motion realistic? If the laws of motion of preferences allow "no motion", i.e. fixed preferences as a special case then we certainly have raised the degree of realism of welfare economics: then the assumed laws of motion cannot be less realistic than their special case of fixed preferences.

The particular assumption which I shall pursue in this paper is the assumption of "adaptive preferences". The term has been used before in the literature, for example by Jon Elster (Elster 1982). And indeed, there is a close relation between the meaning given to the term by Elster and the meaning given to the term in this paper. Here I only discuss the meaning given to this term in this paper.

Imagine an individual in a stationary state described by an  $n$ -dimensional goods vector  $\bar{x}$ . We may then assume that tastes "adapt" to this state over time. Whatever the tastes were at the beginning they change in a certain way and converge to a stationary state "adapted" to the goods vector  $\bar{x}$ . This adaptation has a specific meaning which can best be described by a comparison of tastes adapted to two different goods vectors  $\bar{x}$  and  $\hat{x}$ . Let the corresponding tastes be described by  $q(x)$ . This simply expresses that tastes  $q$  are formed by the stationary state  $x$ . They are adapted to state  $x$ . By adaptive tastes we then mean the following: If under preferences  $q(\hat{x})$  the vector  $\bar{x}$  is preferred to vector  $\hat{x}$  then a fortiori under preferences  $q(\bar{x})$  the vector  $\bar{x}$  is preferred to vector  $\hat{x}$ . But it could well be that under preferences  $q(\hat{x})$  the vector  $\hat{x}$  is preferred to vector  $\bar{x}$  whereas under preferences  $q(\bar{x})$  the vector  $\bar{x}$  is preferred to vector  $\hat{x}$ . Preferences "adapt" to the states of the world in the sense that in comparison between the real state of the world and potential alternative states of the world people tend to value the real state of the world higher than they would if they were in a different state of the world. Elster speaks of the "sour grapes" phenomenon: you tend to discount things which you are unable to obtain anyway.

Here I refrain from discussing the realism or plausibility of this hypothesis. In my view the hypothesis of adaptive preferences is realistic and basically the outcome of the evolution of the human species.

Endogenous changes in preferences reasonably are discussed in an inter-temporal model. This is what we will do in the main part of this paper. Inter-temporal models do not only allow comparisons between different hypothetical states of the world, they also allow inter-temporal comparisons. We can ask the question: does the situation improve or does it deteriorate? This inter-temporal comparison will be the key for the welfare economics of adaptive preferences. In an a-temporal model we would be forced to compare different states of the world, but with preferences adapted to the different states of the world. This, it appears to me, is not really feasible. For example, how do we decide between states  $\bar{x}$  and  $\hat{x}$ , if people prefer  $\bar{x}$  over  $\hat{x}$  with preferences  $q(\bar{x})$  adapted to  $\bar{x}$ , but prefer  $\hat{x}$  over  $\bar{x}$  with preferences  $q(\hat{x})$  adapted to  $\hat{x}$ ?

On the other hand I see a possibility of comparison by the concept of improvement over time. Even with changing tastes it can be made precise what is meant by an increase in real income. Prices remaining the same a rising nominal income should imply a rising real income irrespective of changes in preferences. We therefore would expect that – whatever changes in preferences the person anticipates – starting from the same initial situation the person has a preference for a rising income over an income which remains the same.

Moreover, again taking a stationary state as the reference point, changes in consumption due to endogenous changes in tastes with a constant budget must be seen as superior to a stationary consumption vector. In other words, the changes occurring under a fixed budget constraint must be seen as improvements.

We thus see the possibility of defining "improvement" through time even with endogenous preferences. But so far we only have taken a local perspective of improvement. We have looked at a particular moment of time and, in comparison with a hypothetical stationary consumption vector, have seen the possibility of defining "improvement" at this moment of time. To come to a reasonable approach for welfare economics we must introduce some concept of global consistency.

For example, we would find it difficult to accept a process of local "improvements" as a process representing "progress", if eventually this process comes back to the initial consumption vector. The person may feel improved all the time, but he/she eventually discovers that he/she is back at the starting point. Obviously this could not happen with fixed preferences. It may of course happen with endogenously changing preferences. I want to rule this out and want to concentrate on "laws of motion" of preferences where improvement paths are non-circular. Indeed, for reasons to be explained in a moment, I want to attach the name "adaptive preferences" to the assumption that (reasonably defined) improvement paths are non-circular.

What is the relation between the adaptive character of preferences and the non-circularity of improvement paths? As discussed above, by "adaptive preferences" we mean preference changes so that a real consumption vector gets more favourably compared with hypothetical consumption vectors under preferences adapted to this real consumption vector than under preferences adapted to other consumption vectors. An improvement path from a consumption vector  $x^0$  with initial preferences adapted to  $x^0$  is a movement away from this consumption vector (otherwise it could not be an improvement). But then the induced preference changes start to "discount" the

former consumption vector  $x^0$  relative to the new consumption vectors. Thus it appears not to be possible to come back to  $x^0$  by way of steady improvements. So the improvement paths tend to be non-circular.

This is in contrast to a "law of motion" of preferences, which we may call "anti-adaptive". Here a real consumption vector gets less favourably compared with hypothetical consumption vectors under preferences "adapted" to the real vector than under preferences "adapted" to different consumption vectors. Here then the improvement, i.e. the movement away from the initial consumption vector (with preferences "adapted" to it) induces preference changes so that the initial vector starts to rise in the esteem of the person; and it is perfectly possible that an improvement path will then return to the starting point.

The person with "anti-adaptive" preferences, so to speak, is a person with a preference for "change" and thus in a sense, sees "change" as an advantage in itself. It may then be possible to lead him/her around on an "improvement path" without "real" improvement. A person with "adaptive" preferences tends to be conservative and resistant to change. On an improvement path this resistance to change must be over-compensated by more substantial improvements. Thus, since improvement (associated with change) is appreciated more selectively, improvement paths would always have to lie "above" certain improvement paths under fixed preferences; and the latter, obviously, are non-circular, are unable to return to the starting point. Thus, a fortiori, improvement paths under adaptive preferences are non-circular.

If non-circularity of improvement paths and resistance to change are different sides of the same coin there are two ways to go about axiomatisation. We can start defining "resistance to change" and derive mathematically the non-circularity property. Or we can start with non-circularity and derive mathematically the "resistance to change" property. I have chosen to start with non-circularity and to derive a particular description of "resistance to change".

I now turn to the concept of "progress". What is a reasonable concept of progress? This is reasonably clear under fixed preferences. Progress can be identified with rising (ordinal) utility. Can we maintain a similar "utilitarian" approach if preferences change endogenously? The nice thing about fixed preferences is that the utility function provides a complete pre-ordering of the commodity space. Any two commodity baskets are comparable. As we said before this is no longer the case with adaptive preferences or with other endogenously changing preferences. But, as I said before, we still can reasonably define "improvement". It now turns out – and this is the Main Theorem of this paper – that under adaptive preferences again a pre-ordering of the commodity



space can be constructed which is related to the concept of improvement. Indeed we may talk of an induced complete "preference" pre-ordering of the following meaning. We define the relation between two vectors  $x^0$  and  $x^1$ , written  $x^1 \langle \rangle x^0$  to mean:  $x^1$  can be reached from  $x^0$  with initial preferences adapted to  $x^0$  by means of an improvement path. Under the assumptions specified in the next section we then can show that this relation  $\langle \rangle$  generates a complete pre-ordering of the relevant space of commodity vectors. We can find a "quasi-utility-function"  $V(x)$  defined over the space such that  $V(x^1) > V(x^0) \Leftrightarrow x^1 \langle \rangle x^0$ . Additionally the "quasi-utility-function" induces the long run demand behaviour of the person, meaning the reaction to price and income changes taking account of the adaptation of preferences to these price and income changes.

This provides the foundation for a reasonable concept of "progress" even with adaptive preferences. Given any starting point  $x^0$  with preferences adapted to this starting point we can identify the set  $A(x^0)$  of vectors which are "superior" to  $x^0$  in the sense that they can be reached from  $x^0$  by means of an improvement path; and for any vector in this set the quasi-utility  $V(x)$  is greater than  $V(x^0)$ . Moreover, since long run demand behaviour is reflected in this function  $V(x)$ , concepts like "consumer surplus" as applied to the long run demand function, retain their useful meaning, for example for the purpose of cost-benefit analysis. The traditional analysis of Kaldor-Hicks-Scitovsky compensation is likely to remain valid, and thus the concept of efficiency improvement in all likelihood can be used in a world of adaptive preferences. "Progress" as a concept related to rising living standards can be maintained.

In the next section the reader will note that the definition of an "improvement point" rests on an asymmetric treatment of past and future. Improvement is defined to exist at a point of time, if, with preferences valid at this moment, the present situation is better than the most recent past. A comparison with the future is not made. The "technical" reason for this asymmetry is due to "constant budget" time paths. As we said above, we hold it reasonable to say that changes of consumption due to changes of preferences within a constant budget are considered improvements. But obviously with constant budgets, if evaluated at the preferences of this moment, the utility obtained at this moment is higher than the utility obtained at different moments of time. But then evaluated at these preferences utility rises as we move in time towards that moment and it falls again as we move further beyond that point.

The more fundamental or "philosophical" reason for this asymmetric treatment of past and future is that the past is determined at the present moment of decision whereas the future is open and uncertain. Adaptive preference changes may partly be anticipated by the person, but they will not be fully anticipated. The term "adaptive" is also appropriate, because I want to express a deviation from the "full rationality" approach of neo-classical economics. People adapt to their actual situation through time, because full rationality of decisions (under uncertainty perhaps "Bayesian" rationality) is patently unrealistic, given the fantastic informational and computational requirements of full rationality. Part of the "resistance to change" implied in the assumption of adaptive preferences thus may simply be the "information asymmetry" between the familiar actual state of the world and any alternative state of the world.

## B The Model

There is one person who is a consumer of commodity baskets  $x$  in the positive orthant of the  $n$ -dimensional Euclidean space  $R^n$ . We work in a continuous time model. At each moment  $t$  the consumer maximises an instantaneous utility subject to a budget constraint. But the utility function  $U(x)$  depends itself on past experience as expressed by an  $N$ -dimensional vector  $q$  which reflects past consumption.  $N$  can be smaller or larger than  $n$ ; or we can have  $N = n$ . We formalise this dependence on past consumption in the following way

$$(1) \quad \frac{dq}{dt} \equiv \dot{q} = Qx - \alpha q$$

where  $Q$  is an  $N$  times  $n$  matrix and  $\alpha$  either is a positive real number or  $\alpha$  is a positive definite  $N$  times  $N$  matrix. In this latter case an example is a nonnegative matrix with zero values outside the main diagonal and positive values on the main diagonal. That  $N$  can be substantially larger than  $n$  implies the possibility to approximate any complicated structure of influences of past consumption on present preferences. What is ruled out (but later work surely could change this) is that age of the person or other demographic factors which change with calendar time have a specific influence on present preferences.

If consumption  $\bar{x}$  remains constant over time, then  $q$  converges towards a particular value  $\frac{1}{\alpha} Q\bar{x}$ .

In the case that  $\alpha$  is an N times N matrix,  $\frac{1}{\alpha}$  is to mean the inverse of the matrix  $\alpha$ .

The utility function then can be written

$$U = U(x; q)$$

Utilities with different values of  $q$  are not comparable; they simply represent different preferences.

We assume that  $U$  is continuously differentiable with respect to  $x$  and  $q$ .

We now consider consumption paths through time. Among paths  $x(t)$  which are piecewise continuous we define a subset which we call "improving paths". This description is motivated by an axiom, which allows a minimum of comparability between different preferences.

Consider a path of vector  $x$  through time such that  $x(0) = x^0$  is the starting point,  $x(T) = x^T$  is the end point and there is a set  $J$  with a finite number of moments of time, called jump points,  $t_1, t_2, \dots, t_N$  such that  $0 \leq t_1 < t_2 < \dots < t_N \leq T$ . Let  $I_T$  be the interval  $[0, T]$  of real numbers. (We admit the possibility that  $T = 0$ ). Then we consider paths  $x(t)$  such that  $x(t)$  is continuous in  $I_T - J$ . We admit the possibility that the set  $J$  of jump points is empty.

For such paths  $q(t)$  is well defined by the differential equation (1), if  $q(0) = q^0$  is given. Indeed we find the unique solution

$$q(t) = e^{-\alpha t} \left[ q^0 + \int_0^t e^{\alpha z} Qx(z) dz \right]$$

The integral is well defined for piecewise continuous functions  $x(z)$ .

For any point where the vector  $\dot{x}$  exists we can define the following expression

$$\hat{U} \equiv \sum_{i=1}^n \frac{\partial U}{\partial x_i} \dot{x}_i$$

Note that this is different from the time derivative of  $U$  which is

$$\dot{U} \equiv \sum_{i=1}^n \frac{\partial U}{\partial x_i} \dot{x}_i + \sum_{i=1}^n \frac{\partial U}{\partial q_i} \dot{q}_i$$

Given that the comparison of utility values for different values of  $q$  does not have an economic meaning the expression  $\dot{U}$  is economically meaningless. On the other hand the expression  $\hat{U}$  does have an economic meaning. If  $\hat{U}$  is positive it means that "real income" increases. For example, if  $x$  comes about by maximisation of  $U$  with respect to  $x$  subject to a budget constraint then  $\hat{U} > 0$  implies that either the budget rises or the price index (in terms of the Divisia index) falls, i.e. that real income rises. Therefore it is plausible to talk about an improvement if  $\hat{U} > 0$ .

We do not want to be restricted to differentiable paths  $x(t)$ . Thus we introduce the following definition. For a given path  $x(t)$  we consider a time point  $\bar{t}$ .

Definition 1: The point  $\bar{t}$  is an improvement point, if there exists a non-empty interval  $K(\bar{t}) = [\hat{t}, \bar{t}]$  such that for  $t \in K(\bar{t})$  we have  $U(x(t); q(\bar{t})) \leq U(x(\bar{t}); q(\bar{t}))$ .

(End of Definition).

Thus an improvement point  $\bar{t}$  is characterised by utility evaluated at preferences corresponding to this point such that for smaller  $t$  utility is not larger than utility at  $\bar{t}$ . Obviously, if  $x(t)$  is differentiable at  $\bar{t}$  and if  $\bar{t}$  is an improving point then  $\hat{U}(\bar{t}) \geq 0$ .

Remark. Note the asymmetric treatment of past and future in this definition. The utility comparison is only made between present and past, not between present and future. The basic reason for this asymmetry was given in the introduction.

This consideration motivates the following definitions of an improving and a weakly improving path.

We use the following notation for piecewise continuous paths  $x(t)$ . By  $\{x(t); q^0; T\}$  we mean a path of the piecewise continuous consumption vector  $x(t)$  in the time interval  $[0, T]$  such that preferences are determined by  $q(t) = e^{-\alpha t} \left[ q^0 + \int_0^t e^{\alpha z} Qx(z) dz \right]$  with the initial value  $q^0$ .

Definition 2. Let  $\{x(t); q^0; T\}$  be a piecewise continuous path of consumption vector  $x(t)$  with  $H$  jump points  $J = \{0 \leq t_1 < t_2 < \dots < t_H \leq T\}$  on the time interval  $I_T$ . The path is called a weakly improving path, if

for any  $\bar{t} \in I_T - 0$  the point  $\bar{t}$  is an improving point and

There exists  $\varepsilon > 0$  such that for  $0 < t < \varepsilon$  we have  $U(x(t); q(t)) \geq U(x(0); q(t))$

(End of Definition).

Note that the definition of an improving point only involves weak inequalities. Thus a constant utility path is a path to which definition 2 also applies. Therefore we talk of weakly improving paths. Note also that  $t = 0$  need not be an improving point. We want to make the definition independent of the path  $x(t)$  outside of the interval. But by condition 2 we exclude a negative utility jump point at  $t = 0$

Definition 3. Let  $\{x(t); q^0; T\}$  be a piecewise continuous path of consumption vector  $x(t)$  with  $H$  jump points  $J = \{0 \leq t_1 < t_2 < \dots < t_H = T\}$  on the time interval  $I_T$ .

The path is called an improving path if

1. the path is a weakly improving path

2. for  $T = t_H \in J$  we have the strict inequality  $\lim_{t \xrightarrow{t < t_H} t_H} U(x(t); q(t_H)) < U(x(T); q(t_H))$

(End of Definition).

In words the definition means the following. An improving path is a path on which real income never falls and in certain parts of the path actually rises. Where  $\hat{U}$  is defined it is

nonnegative (condition 1). Where there is a jump point for  $x(t)$  the jump is not "downwards" in terms of the preferences prevailing at the jump point (condition 1). And at the end there is a jump point with a jump, which is definitely "upward" (condition 2).

Remark on strictly improving paths. We require a definite jump in utility at the end if evaluated with preferences valid at the end point. As I will formulate in a conjecture later, I believe that the theorem can be extended to the case of a more general class of paths which we intuitively also would consider strictly improving.

I now introduce the improvement axiom. Since preferences change through time welfare economics would become impossible unless we had some way of normatively comparing consumption paths that do not have the same preferences. We need some kind of "meta-preferences". But I want to restrict meta-preferences to a minimum. The meta-preferences in our case are encapsulated in the following axiom.

Improvement Axiom. Let  $\{x^*(t); q^{*0}; T\}$  be an improving path. Let  $\{x(t); q^0; T\}$  be a path in which consumption remains constant. Thus  $x(t) = x(0)$  for  $0 \leq t \leq T$ . If  $x^*(0) = x(0)$  and  $q^{*0} = q^0 = \frac{1}{\alpha} Qx(0)$  then the consumer prefers  $\{x^*(t); q^{*0}; T\}$  over  $\{x(t); q^0; T\}$ .

The Improvement Axiom is highly plausible: starting from the same tastes (as represented by  $q^{*0} = q^0$ ) adapted to initial consumption and the same consumption basket the consumer prefers improvement over constant consumption even if he or she is aware that tastes adapt to the evolving situation through time.

If we accept the Improvement Axiom we can, as will be shown, maintain the concept of progress such that it is consistent with welfare economics – even with endogenously changing preferences.

Note that the Improvement Axiom is far away from a complete meta-preference pre-ordering over different preferences. In particular I want to emphasise that – beyond a general awareness that her/his preferences may change – the person may be unable to predict, even in a probabilistic sense, her/his preferences five or ten or twenty years from now.

I now show a theorem, which indicates the existence of "laws of motion" of preferences which are consistent with a reasonable concept of "progress" in the tradition of welfare economics.

The following notation will be used:  $x^*(\succ)_q x$  means that  $x^*$  is preferred over  $x$  with preferences corresponding to past consumption vector  $q$ . It is equivalent to the expression  $U(x^*; q) > U(x; q)$ . We will consider price vectors  $p$ , which are, as usual, n-dimensional vectors of nonnegative real numbers.

I now introduce five assumptions.

Assumption 1. (Existence of a demand function). Preferences are such that a direct demand function  $x = f(p, q)$  exists, where it is always assumed that the budget  $p \cdot x = 1$ . Moreover, let the set  $R(q)$  be defined as the set of consumption vectors  $x$  for which there exists a price vector  $p$  such that  $x = f(p, q)$  plus the zero consumption vector  $x = 0$ . In a formula:  $R(q) = \{ x : \exists p \in R^n + s.t. f(p, q) = x, or : x = 0 \}$ . Then  $R(q) = R^*$  is independent of  $q$  and  $R^*$  is convex.

Discussion of Assumption 1. By assuming a unique demand vector for each  $q$  and each price vector  $p$  we restrict our analysis to convex preference structures. We need some such restriction because the process of gradual improvement which we describe with our improvement path only leads to local optimisation. We cannot expect global optimisation in the case of non-convex structures. The independence of the range of the demand function from the preference parameter  $q$  is, for example, fulfilled, if that range is equal to the positive orthant. The convexity of  $R^*$  is not a very restrictive assumption, given that convexity of preferences are already assumed. Since  $0$  is added to the set  $R^*$  the convexity of  $R^*$  implies that for any  $x \in R^* \Rightarrow \mu x \in R^*$  for  $0 \leq \mu \leq 1$ . Given the Non-Saturation Assumption this is not a very restrictive assumption.

Assumption 2. (Non-Saturation). If  $x^*_i > x_i$  for  $i = 1, 2, \dots, n$  then  $x^* (>)_q x$  for all  $q$ .

Discussion of Assumption 2. It is my guess that this assumption can be weakened. For the time being I find it convenient for my proof. It provides a uniform “direction” where to look for improvement.

For the following assumption consider a path  $\{x(t); q^0; T = \infty\}$  without a finite end. Starting with preferences  $q^0$  let  $x(t)$  be generated by maximisation of  $U(x; q(t))$  subject to a constant budget constraint  $px = 1$  with  $p$  a constant price vector. This then leads to the following definition.

Definition 4  $[x(t); q^0; p]$  is a constant budget path, if for  $t \geq 0$  we have  $x(t) = f(p; q(t))$  and  $q(0) = q^0$ . If  $x(t)$  converges to some value  $\bar{x}$  then  $\bar{x}$  is called the long run demand with respect to  $p$  and  $q^0$ .

Assumption 3. (existence of long run demand). For each budget constraint  $p > 0$  there exists a unique convergence point  $\bar{x} = F(p)$  of a constant budget constraint path; i.e. the convergence point is independent of the initial value  $q^0$ .

Discussion of Assumption 3. The independence of "asymptotic" behaviour from the initial preferences is unlikely to hold, if the preferences themselves exhibit important non-convexities. Thus, in a sense, Assumption 3 is an extension of Assumption 1. An assumption like Assumption 3 is necessary for a "global" theorem of the type to be shown here. We cannot expect global optimisation from purely local optimisation procedures as discussed in this paper, unless we make an assumption like Assumption 3.

Assumption 4. (Continuity). For each triple of vectors  $\{x^*; x; q\}$  such that  $x^* (>)_q x$  there exist neighbourhoods  $M(x^*), M(x), \hat{M}(q)$  such that  $z^* (>)_r z$  for  $z^* \in M(x^*), z \in M(x), r \in \hat{M}(q)$ .

Discussion of Assumption 4. It is an extension of the continuity of fixed preferences to changing preferences. The continuity assumption is essential for my proof. This I know from the fact that I cannot simply carry over my theorem to a model where commodity quantities are restricted to integer numbers. The theory of adaptive preferences for Non-Euclidean commodity spaces has yet to be developed.

Assumption 5 (Adaptive preferences = non-circularity of improving paths). Let  $\{x(t); q^0; T\}$  be an improving path and let  $q^0 = \frac{1}{\alpha} Qx(0)$ . Then  $x(T) \neq x(0)$ .

Discussion of Assumption 5. This is the assumption of adaptive preferences. It has already been discussed in the Introduction.

Definition 5. For any given  $x^0$  let  $A(x^0)$  be the set of vectors  $\bar{x} \in R^*$  such that there exists an improving path  $\{x(t); q^0; T\}$  with  $q^0 = \frac{1}{\alpha} Qx^0; x(0) = x^0; x(T) = \bar{x}$ .

In other words: the set  $A(x^0)$  is the set of vectors (restricted to  $R^*$ ), which can be reached from  $x^0$  by an improving path, given that initially the preferences are "adapted" to  $x^0$ .



Remark. Obviously, by Assumption 5, it follows that  $x^0 \notin A(x^0)$ .

Remark. By continuity we must have the equation  $F(p) = f(p; \frac{1}{\alpha} QF(p))$

Main Theorem 1. Part A. The "long run demand function"  $F(p)$  satisfies the strong axiom of revealed preference.

Part B. There exists a utility function  $V(x)$ , which generates the demand function  $F(p)$ . Let  $B(x^0)$  be the set of vectors such that  $V(x) > V(x^0)$ . Then  $B(x^0) = A(x^0)$ .

Proof. I split the proof in a series of lemmas.

Lemma 1. Any finite component  $\{x(t); q^0; T\}$  of a constant budget path is a weakly improving path.

Proof. A constant budget (due to a constant price vector  $p$ ) implies that at each  $\bar{t}$  utility is maximised over the same budget as at every other moment  $t$ . Thus, obviously for any  $t \in I_T$  and any  $\bar{t} \in (0, T]$  we have  $U(x(\bar{t}); q(\bar{t})) \geq U(x(t); q(\bar{t}))$ . Thus  $\bar{t}$  is an improving point according to Definition 1. Moreover, due to the same argument, Condition 2 of Definition 2 is also fulfilled. Then according to Definition 2 the path is a weakly improving path, which proves the Lemma.

Lemma 2. The demand function  $x = f(p, q)$  is continuous.

Proof. I first show continuity with respect to  $p$ , for any given  $q$ . Consider any  $q$  and  $p^0 \geq 0$  and the corresponding demand vector  $x^0 = f(p^0, q)$ . Consider now any sequence of price vectors  $p^1, p^2, \dots$  which converges to  $p^0$ . Let  $x^1, x^2, \dots$  be the corresponding sequence of demand vectors, i.e.  $x^i = f(p^i, q)$ . Because of Assumption 2 (Non-Saturation) we know that  $x^i p^i = 1$ . Because of the convergence of the sequence of price vectors to  $p^0$  for each  $\varepsilon > 0$  we know that for  $i$  sufficiently large  $0 < x^i p^0 < 1 + \varepsilon$ . Thus  $x^i$  moves on a compact subset of  $R^n$  and thus has an accumulation point which we may denote by  $\bar{x}$ . Assume for some  $\alpha > 0$  that  $\bar{x} p^0 = 1 + \alpha$ . Then the sequence  $p^i x^i$  has an accumulation point at  $p^0 \bar{x}$ . But this contradicts the budget equation  $p x = 1$ . Hence we see that  $p^0 \bar{x} \leq 1$ . But then, by revealed preference and by Assumption 1 (uniqueness of demand),  $x^0 (>)_q \bar{x}$ . By Assumption 4 (continuity) we can find a vector of equal

components  $\varepsilon > 0$  such that  $x^0 - \varepsilon(>)_q \bar{x} + \varepsilon$ . Because  $\bar{x}$  is an accumulation point we can find  $i$  such that  $x^i \leq \bar{x} + \varepsilon$ , hence by Assumption 2  $x^0 - \varepsilon(>)_q x^i$  despite the fact that  $p^i(x^0 - \varepsilon) \leq 1$  which shows that  $x^i = f(p^i, q)$  is not utility maximizing, a contradiction which disproves the assumption  $\bar{x} \neq x^0$ . Hence  $\bar{x} = x^0$  which proves continuity of demand with respect to  $p$ .

I now show continuity with respect to  $q$  for given  $p$ . Consider some  $q^0$  and for a given  $p$  the demand  $x^0 = f(p, q^0)$ . Consider a sequence of vectors  $q^1, q^2, \dots$  which converges to  $q^0$ . Due to the budget equation  $xp = 1$  the corresponding sequence of demand vectors  $x^1, x^2, \dots$  moves within a compact subset of  $R^n$ , thus has an accumulation point  $\bar{x}$ . Obviously, due to  $x^i p = 1$ , we have  $\bar{x} p = 1$ . Assume  $\bar{x} \neq x^0$ . Then due to Assumption 1 (uniqueness of demand) we obtain  $x^0(>)_q \bar{x}$ . Due to Assumption 4 (continuity) there exists a vector of equal components  $\varepsilon > 0$  and a neighbourhood  $M(q^0)$  such that  $x^0 - \varepsilon(>)_q \bar{x} + \varepsilon$  for any  $q \in M(q^0)$ . For  $i$  sufficiently large  $q^i \in M(q^0)$ . Among those sufficiently large  $i$  we can find  $i$  such that  $x^i < \bar{x} + \varepsilon$ ; hence due to Assumption 2  $x^0 - \varepsilon(>)_q x^i$ , despite the fact that  $p(x^0 - \varepsilon) < 1$  so that  $x^i$  does not maximize utility, a contradiction which shows that  $\bar{x} = x^0$  which shows continuity of demand with respect to  $q$ .

By standard theorems of topology then  $x = f(p, q)$  is continuous which proves the Lemma.

Lemma 3: Define the preference relation in price space  $p^1(>)_q p^0$  to mean that  $x^1(>)_q x^0$  for  $x^1 = f(p^1, q)$  and  $x^0 = f(p^0, q)$ . The preference relation  $p^1(>)_q p^0$  is continuous in  $p^1, p^0, q$ .

Proof: Consider any triple  $p^0, p^1, q^0$  such that  $p^1(>)_q p^0$ . Then for  $x^1 = f(p^1, q^0)$  and  $x^0 = f(p^0, q^0)$  we have  $x^1(>)_q x^0$ . Due to Assumption 4 (continuity) there exist neighbourhoods  $M^1(x^1)$  of  $x^1$ ,  $M^0(x^0)$  of  $x^0$  and  $M^2(q^0)$  of  $q^0$  such that  $z^1(>)_q z^0$  for  $z^1 \in M^1, z^0 \in M^0, q \in M^2$ . Let  $N^1$  be the inverse of  $M^1$ , let  $N^0$  be the inverse of  $M^0$ , and let  $N^2 = M^2$ . Then due to continuity of the demand function (Lemma 2)  $N^1$  is a neighbourhood of  $p^1$ ,  $N^0$  is a neighbourhood of  $p^0$ . Thus we have found neighbourhoods

$N^1(p^1), N^0(p^0), N^2(q^0)$  such that for  $r^1 \in N^1, r^0 \in N^0, q \in N^2$  we have  $r^1(>)_q r^0$  which proves the Lemma.

Lemma 4: Let  $q^0, x^0 = f(p^0, q^0)$  be arbitrary initial points. Let  $p^1$  be a price vector and long run demand  $x^1 = F(p^1)$  such that under long run demand  $x^1$  is revealed preferred to  $x^0$ ; i.e.  $p^1(x^1 - x^0) \geq 0$ . Moreover let  $x = f(p^1, q^0) \neq x^0$ . Then there exists an improvement path  $\{x(t); q^0; T\}$  beginning at  $x^0$  with preferences  $q^0$  and ending at  $x^1$  within a finite period T. Moreover this path can be constructed so as to get  $q(T)$  arbitrarily close to  $\frac{1}{\alpha} Qx^1$ .

Idea of the Proof: The main idea of the proof is this: due to Lemma 1 constant budget paths are weakly improving. Due to continuity of preferences in price space (Lemma 3) we can find a price above  $p^1$  which is superior to  $p^0$ . So we can gravitate towards the long run demand of this price vector above  $p^1$  by a constant budget path and then, again due to continuity of preferences, after having come sufficiently close to that convergence point, "jump" to the long run demand of price  $p^1$ , by a final improvement.

Proof: Due to the inequality  $p^1(x^1 - x^0) \geq 0$  and due to the inequality  $x = f(p^1, q^0) \neq x^0$  we know that  $p^1(>)_q p^0$ . Then due to continuity of preferences in price space (Lemma 3) we can find  $\mu > 1$  such that  $\mu p^1(>)_q p^0$ . We now introduce the following path  $x(t)$ . At  $t=0$  we put  $x(t) = x^0$ . Then for some  $t^* > 0$  we put  $x(t) = f(\mu p^1, q(t))$  for  $0 < t < t^*$ . So within  $(0, t^*)$  the path is a constant budget path and hence due to Lemma 1 it is weakly improving. At  $t=0$  there is a jump point with an improvement. Let  $\hat{x} = F(\mu p^1)$ . Then, due to Assumption 2 (Non-saturation) we have  $p^1(>)_q \mu p^1$  for arbitrary  $q$ ; and thus  $x^1(>)_x \hat{x}$ . But then, by Assumption 4 (Continuity), there are neighbourhoods  $M^1(\hat{x})$  and  $M^2(\hat{x})$  such that  $x^1(>)_q x$  for  $x \in M^1(\hat{x})$  and  $q \in M^2(\hat{x})$ . For  $t$  sufficiently large by Assumption 3 (Convergence) we find that  $x(t) \in M^1(\hat{x})$  and  $q(t) \in M^2(\hat{x})$ . We choose  $t^*$  large enough so that  $x(t^*) \in M^1(\hat{x})$  and  $q(t^*) \in M^2(\hat{x})$ . At  $t^*$  the path  $x(t)$  jumps to  $x^1$  which is an improving jump by construction, thus satisfying Condition 2 of the definition of an improving path. (Definition 3). To satisfy the requirement that  $q(T)$  is sufficiently close to  $\frac{1}{\alpha} Qx^1$ , by Lemma 2 (continuity of the demand function) we only have to choose  $\mu > 1$  sufficiently close to 1. This proves the Lemma.

We now prove part A of the theorem.

Proof of Part A of the Main Theorem: Assume the contrary. Thus there exists a series of  $m$  price vectors  $p^1, p^2, \dots, p^m = p^0$  and the corresponding long run demand vectors  $x^1 = F(p^1), x^2 = F(p^2), \dots, x^m = x^0 = F(p^m) = F(p^0)$  such that for  $i = 0, 1, 2, \dots, m-1$  the vector  $x^{i+1}$  is revealed preferred to  $x^i$ . We thus have the following inequalities

$$p^1(x^1 - x^0) \geq 0$$

$$p^2(x^2 - x^1) \geq 0$$

.....

.....

$$p^0(x^0 - x^{m-1}) \geq 0$$

and, by definition of revealed preference,  $x^{i+1} \neq x^i$

From  $x^{i+1} \neq x^i$  it follows by Assumption 3 that  $x = f(p^{i+1}, \frac{1}{\alpha} Qx^i) \neq x^i$ . For, if equality would hold, then obviously  $f(p^{i+1}, \frac{1}{\alpha} Qx^i) = x^i = F(p^{i+1}) = x^{i+1}$  which would contradict the inequality  $x^{i+1} \neq x^i$ . But then by the continuity of the demand function (Lemma 2) there is a neighbourhood  $M(\frac{1}{\alpha} Qx^i)$  such that for  $q \in M(\frac{1}{\alpha} Qx^i)$  the inequality  $f(p^{i+1}, q) \neq x^i$  holds.

In the rest of the proof let  $M(\frac{1}{\alpha} Qx^i)$  be defined so that this inequality holds.

We now construct the following improvement path. It starts at  $x^0$  with preferences  $q(0) = \frac{1}{\alpha} Qx^0$ .

It then reaches  $x^1$  in finite time  $T_1$  and  $q(T_1) \in M(\frac{1}{\alpha} Qx^1)$ . That such an improvement path exists, is established by Lemma 4.

We now repeat the procedure by constructing an improving path moving from  $x = x^1$  and  $q = q(T_1)$  using the price vector  $p^2$  so that the improving path ends in moment  $T_2$  at  $x^2$  and  $q(T_2) \in M(\frac{1}{\alpha}Qx^2)$  which again is possible by Lemma 4.

For each  $i$  we can repeat the procedure until we arrive at  $x^m = x^0$ . We thereby have constructed a circular improving path, in contradiction to Assumption 5. This proves Part A of the Theorem.

To prove part B we continue with a further set of Lemmas.

Lemma 5: There exists a utility function  $V(x)$ , defined on  $R^*$ , which generates the long run demand function  $F(p)$

Proof: Let the range of the long run demand function be denoted by  $R^L$ . Then  $R^L = R^*$ . For, if  $x \in R^L$ , there is  $p$  such that  $x = F(p) = f(p; \frac{1}{\alpha}QF(p))$ . Thus  $x \in R(\frac{1}{\alpha}Qx)$  and hence, by Assumption 1,  $x \in R^*$ . On the other hand, if  $x \in R^*$ , then, by Assumption 1,  $x \in R^* = R(\frac{1}{\alpha}Qx)$ .

Hence there exists  $p$  such that  $f(p; \frac{1}{\alpha}Qx) = x$  and thus  $F(p) = x$  which shows  $x \in R^L$ . But then  $R^L$  is convex, by Assumption 1. Since, by Part A of the theorem,  $F(p)$  fulfils the strong axiom of revealed preference, by well known theorems (cf. for example Sondermann (1983)) convexity of  $R^L$  implies that there exists a utility function  $V(x)$  which generates the demand function  $F(p)$ . This proves the lemma.

Lemma 6: The set  $A(x^0)$  is open.

Proof: Let  $\bar{x} \in A(x^0)$ . Then there exists an improving path  $\{x(t); q^0; T\}$  with  $x(0) = x^0$  and  $x(T) = \bar{x}$  and  $q^0 = \frac{1}{\alpha}Qx^0$  and thus an improving jump point at  $T$ . Let  $\hat{x} = \lim_{t \rightarrow T; t < T} x(t)$  and let  $\hat{q} = q(T)$ . Due to the improving jump at  $T$  we have  $\bar{x}(>)_q \hat{x}$  which by Assumption 4 (continuity) implies that there exists a neighbourhood  $M(\bar{x})$  such that  $x(>)_q \hat{x}$  for any  $x \in M(\bar{x})$ . But then any point in  $M(\bar{x})$  can be reached by an improving path from  $x^0$  with  $q^0 = \frac{1}{\alpha}Qx^0$ , simply by

using  $x(t)$  for  $t < T$  and then jumping to the point in  $M(\bar{x})$  at time  $T$ . This shows  $M(\bar{x}) \subseteq A(x^0)$  which implies that  $A(x^0)$  is an open set. This proves the Lemma.

Lemma 7: For any real number  $\mu$  such that  $0 < \mu \leq 1$  and  $\bar{x} \in R^*$  and  $\bar{x} \neq 0$ , let  $v(\mu) = V(\mu\bar{x})$ . Then  $v(\mu)$  is well defined and is strictly increasing in  $\mu$ .

Proof: Since  $R^*$  contains the zero vector and is convex, by Lemma 5, for any  $\mu$  such that  $0 < \mu \leq 1$  there exists  $p$  such that  $\mu\bar{x} = F(p)$  so that  $V(\mu\bar{x})$  is well defined. Let  $0 < \lambda < \mu$ . Then  $\lambda\bar{x}p < \mu\bar{x}p = 1$  and hence  $\mu\bar{x}$  is revealed preferred to  $\lambda\bar{x}$  which by Lemma 5 implies  $v(\lambda) < v(\mu)$  which proves the Lemma.

Lemma 8:  $B(x^0) \subseteq A(x^0)$

Proof: Let  $\bar{x} \in B(x^0)$ . Then, under the long run demand function  $\bar{x}$  is indirectly revealed preferred to  $x^0$ . By the same method as in the proof of Part A we then can construct an improving path from  $x^0$  to  $\bar{x}$  with initial preferences  $q^0 = \frac{1}{\alpha} Qx^0$ . This proves the Lemma.

Lemma 9:  $A(x^0) \subseteq B(x^0)$

Proof: Let  $\bar{x} \in A(x^0)$ . If  $V(\bar{x}) < V(x^0)$  then, by Lemma 8,  $x^0 \in A(\bar{x})$  and thus a circular improving sequence could be constructed in contradiction to Assumption 5. Thus  $V(\bar{x}) \geq V(x^0)$ . Assume  $V(\bar{x}) = V(x^0)$ . For  $\bar{x}$  now define  $v(\mu) = V(\mu\bar{x})$ . Then, by Lemma 7, for  $\mu < 1$  we have  $v(\mu) < V(\bar{x}) = V(x^0)$ . This then implies  $1 \leq \inf(\mu : \mu\bar{x} \in A(x^0))$ . But by Lemma 6  $A(x^0)$  is open which implies that for  $\lambda = \inf(\mu : \mu\bar{x} \in A(x^0))$  we have  $\lambda\bar{x} \notin A(x^0)$ . This means  $\bar{x} \notin A(x^0)$ , in contradiction to the assumption at the beginning of the proof. Thus we have shown that  $V(\bar{x}) > V(x^0)$  which proves the Lemma.

Proof of Part B of the Theorem: Obviously Lemma 8 and Lemma 9 together imply that  $B(x^0) = A(x^0)$  which together with Lemma 5 are Part B of the theorem.

Remark. Obviously, if we "widen" the definition of a strictly improving path, Assumption 5 becomes more restrictive. Thus a "widening" of this definition makes the proof of Part A of the theorem "easier". So Part A of the theorem automatically survives every "widening" of the definition of a strictly improving path. And Part A is the more fundamental part of the theorem: it

shows that a measuring rod for "progress" exists even with endogenous preferences as long as "improvement" is not circular.

But with a "reasonable" widening of the definition of strictly improving paths it should be possible to also maintain Part B of the theorem. By a reasonable definition I mean something like the following:

Definition 3a ( as a substitute for Definition 3): Let  $\{x(t); q^0; T\}$  be a piecewise continuous path of consumption vector  $x(t)$  with jump points  $J = \{0 \leq t_1 < t_2 < \dots < t_N \leq T\}$  on the time interval  $I_T$ .

The path is called an improving path if

1. the path is a weakly improving path

2. Either there is at least one  $t_j \in J$  such that

$$\lim_{t \xrightarrow{t < t_j} t_j} U(x(t); q(t_j)) < \lim_{t \xrightarrow{t > t_j} t_j} U(x(t); q(t_j))$$

or there exist  $t^{low}, t^{high}$  such that  $0 \leq t^{low} < t^{high} \leq T$  and  $\hat{U} > 0$  for  $t \in (t^{low}, t^{high})$

In words: an improving path must be weakly improving and also have a part in which strict improvement takes place either by an "upward jump" in utility or by an interval in which "real income" rises steadily. It should be possible to show that under this definition the set  $A(x^0)$  is open. In that case the rest of the proof goes through. I thus formulate the following

Conjecture: The Main Theorem remains correct, if Definition 3 is replaced by Definition 3a.

There is a corollary to the Main Theorem : Adaptive preferences imply that long run demand is "more elastic" than the short run demand. The precise formulation is the following:

Corollary to the Main Theorem: The Assumptions of the Main Theorem hold. Let

$$U(x; \frac{1}{\alpha} Qx^0) > U(x^0; \frac{1}{\alpha} Qx^0). \text{ Then } V(x) > V(x^0).$$

In words: The set of consumption vectors which are preferred to  $x^0$  under the short run preferences adapted to  $x^0$  is part of the set of consumption vectors which are "preferred" under the "preferences", which generate the long run demand function.

Proof of the Corollary: If  $U(x; \frac{1}{\alpha} Qx^0) > U(x^0; \frac{1}{\alpha} Qx^0)$  it is obvious that  $x \in A(x^0)$ : A jump at time zero from  $x^0$  to  $x$  is an improving path. Hence by the Main Theorem we have  $V(x) > V(x^0)$ .

We may express this corollary in this way: the indifference surface going through a point  $x^0$  with preferences adapted to  $x^0$  lies "above" the indifference surface corresponding to "long run preferences." This then means that the reaction of "compensated demand" in the long run is stronger than in the short run. Adaptation of preferences helps the consumer to react more elastically on price changes than reaction under given preferences. Or, we can express the same thing the other way round: adaptation of preferences means resistance to change. Thus a position different from today's position may look better than today's position, if the person were adapted to this other position, but looks inferior from the perspective of today's position.

## C Extension: Interpersonal Adaptation of Preferences

The welfare economics of adaptive preferences is not restricted to intra-person influences on tastes. It can be extended to inter-person influences on preferences. In this section I sketch how this can be done. I do this by way of example, but I believe this approach to be quite general. As an example I set up a model of Bergson (1938) welfare functions or social indifference curves. Within this set-up I then generalise the Main Theorem which I proved in section B.

The first step is to set up the welfare function for fixed tastes. We look at an economy with  $n$  goods (as before) and  $m$  persons. Each of these  $m$  persons has a utility function  $U_i(x_{i1}, x_{i2}, \dots, x_{in})$  where the goods vector  $x^i = (x_{i1}, x_{i2}, \dots, x_{in})$  designates the consumption basket of person  $i$ . Consider now a "macro" goods vector

$$\bar{x} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) = \left( \sum_{i=1}^m \bar{x}_{i1}, \sum_{i=1}^m \bar{x}_{i2}, \dots, \sum_{i=1}^m \bar{x}_{in} \right) = \sum_{i=1}^m \bar{x}^i.$$

Assume, it is distributed among the members of the society in a Pareto-optimal way. We may then ask: which other macro vectors would enable the economy to provide at least the same utility to each member of the society? This question then leads to the construction of a "welfare function" of the following kind.<sup>2</sup> Let  $\bar{x}$  be an initial "macro" vector and let  $(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^m)$  be a distribution  $\bar{d}$  of this "macro" vector among people which is Pareto-optimal. We give this initial macro vector the welfare index  $W = 1$ , and

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2 This is basically a "Bergson welfare function".



we call it and the corresponding distribution  $\bar{d}$  the "anchor" of the welfare function. Now for any  $\lambda > 0$  we look at  $\lambda\bar{x}$  and at the distribution  $\lambda\bar{d} = (\lambda\bar{x}^1, \lambda\bar{x}^2, \dots, \lambda\bar{x}^m)$ . We interpret this distribution as an "initial allocation" in a Walras general equilibrium setting of pure exchange. Let  $(\lambda\bar{z}^1, \lambda\bar{z}^2, \dots, \lambda\bar{z}^m)$  be the (unique)<sup>3</sup> Walras equilibrium resulting from this initial allocation. The Walras equilibrium is of course a Pareto- optimal distribution. We now give the macro vector  $\lambda\bar{x}$  the Welfare index  $\lambda$ . Thus, on a ray of vectors going through the "anchor" vector we have defined welfare.

Now we look at any macro vector  $x$ . For any distribution  $(x^1, x^2, \dots, x^m)$  of this macro vector among people we define  $\mu(x^1, x^2, \dots, x^m) = \frac{\min}{i=1,2,\dots,m} (\lambda_i : U_i(x^i) = U_i(\lambda_i \bar{z}^i))$ . In words: for any distribution the associated  $\mu$  is defined as the minimum of all  $\lambda$  values defined for each person by the vector on the anchor ray which provides the same utility as the person obtains under this distribution. Let then the welfare index of the macro vector  $x$  be defined as

$$W(x; \bar{x}, \bar{d}) = \max \mu \left\{ \mu(x^1, x^2, \dots, x^m) : \sum_{i=1}^m x^i \leq x \right\}$$

In words: the welfare index of any macro vector  $x$  is equal to the welfare index of that macro vector on the "anchor" ray which is "Pareto-equivalent" to it. In this way we have constructed a welfare function for macro goods vectors for fixed preferences of the members of the society. It is, of course, the case that this welfare function depends on the choice of the "anchor" vector  $\bar{x}$  and the initial distribution  $\bar{d}$  of that "anchor" vector.

We now move to adaptive preferences. I transform the multi- person problem formally into a single person problem. I construct a utility function of one "super-person" . Assume that individual utility functions  $U_i(x^i; q^1, q^2, \dots, q^i, \dots, q^m)$  are like in the one person model, except that there may be influences not only of past consumption of person i herself, but also of the consumption by other persons on person i's preferences. Assumptions 1 through 5 (existence of a demand function, non-saturation, existence of long run demand, continuity, non-circularity of improvement paths) can be carried over.

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3 If there are several Walras equilibria a selection mechanism would have to be defined with certain continuity properties with respect to changes in  $\lambda$ .

Now, the "vector" of N-dimensional vectors  $(q^1, q^2, \dots, q^m)$  can be interpreted as one m times N dimensional vector. Moreover we can redefine "goods" in a similar way: the quantity of good ij now is  $x_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n$ . The "pre-utility function" of the "super-person" can be found by the following procedure:  $Y(x_{11}, x_{12}, \dots, x_{1n}, x_{21}, x_{22}, \dots, x_{2n}, \dots, x_{mn}; q^1, q^2, \dots, q^m; \bar{x}, \bar{d}) =$

$$= \mu(x^1, x^2, \dots, x^m) = \frac{\min}{i=1, 2, \dots, m} (\lambda_i : U_i(x^i; q^1, q^2, \dots, q^m) = U_i(\lambda_i \bar{z}^i; q^1, q^2, \dots, q^m)) \text{ and then, going}$$

back to the n-dimensional commodity space we define in the usual way the welfare function

$$W(x; q^1, q^2, \dots, q^m; \bar{x}, \bar{d}) = \max Y \left\{ \mu(x^1, x^2, \dots, x^m) : \sum_{i=1}^m x^i \leq x \right\}$$

which then is the utility function of the "super-person". Now, it can be shown that the function  $W$ , as defined, satisfies all five assumptions of the Main Theorem, if the functions  $U_i$  do. Therefore the Main Theorem can be applied to the function  $W$ . Thus we can find a "long run utility function"  $W^*(x; \bar{x}, \bar{d})$  such that for preferences adapted to initial point  $\hat{x}$  there exists an improving path to point  $\tilde{x}$  if and only if  $W^*(\tilde{x}; \bar{x}, \bar{d}) > W^*(\hat{x}; \bar{x}, \bar{d})$ . In this way we have found a "long run Bergson welfare function"  $W^*$  for the case interpersonal influences on tastes as long as these taste interdependencies are "adaptive". The "welfare" comparison here is to be understood in terms of "Pareto-improvement paths". For, by construction of  $W$  as the maximum of alternative minima, the  $\lambda$  value of each person is the same along the improvement path, which means that each  $\lambda$  rises along an improvement path.

The "long run welfare function" remains of course "anchored" in  $\bar{x}$  and  $\bar{d}$  as was the welfare function with fixed preferences.

We should be aware that in the multi-person setting the Improvement Axiom may not as easily be accepted as in the single-person setting. In the single-person setting the Improvement Axiom simply says that awareness of changing preferences does not prevent the consumer from saying: I prefer improvement to stationarity. In the multi-person setting using the Pareto criterion, as I did above, improvement means that each person is on an improvement path. A path in which my improvement is much smaller than my neighbour's improvement is also an improvement path. So then here the Improvement Axiom means that I prefer improvement over stationarity even if my improvement goes together with a much greater improvement of my neighbour. In a sense, in the interpersonal setting the Improvement Axiom includes a "no envy assumption".

Much more work, I believe, can fruitfully be done to develop a welfare economics for the case of interpersonal adaptation of preferences.

## D Outlook

This paper proposes to be a beginning for a new field of research: the welfare economics of adaptive preferences. In this last section of the paper I want to indicate some ideas how to develop this field further.

I start with a small theoretical construction linking my approach to the well known welfare theorems of general equilibrium theory. In traditional theory it can be shown that a Walras equilibrium is Pareto-optimal. We may then ask: given a stationary Walras equilibrium of consumers with adaptive preferences, can we show some efficiency property? We may introduce the following definitions.

Definition 6. Consider an economy with a fixed number of consumers each consuming a flow of consumption goods. A set of consumption paths for these consumers is an improving path for the economy, if it is weakly improving for all consumers and strongly improving for at least one consumer.

Definition 7. A steady state of an economy is called steady state efficient, if there exists no feasible improving path starting from this steady state and with initial preferences adapted to the steady state.

We now can announce the following

Conjecture: If a steady state economy is a Walras equilibrium then it is steady state efficient.

Outline of a proof: In the steady state equilibrium consumers maximise their long run utility  $V(x)$ , because their long run demand behaviour corresponds to this maximisation. The Walras equilibrium then is also a Walras equilibrium for the "as if case" that fixed preferences exist which are characterised by the utility function  $V(x)$ . By the standard theory of general equilibrium the Walras equilibrium of this "as if case" is Pareto-optimal in terms of the "as if fixed" utility function  $V(x)$ . An improving path of the economy starting from this steady state would lead to higher values in terms of the utility function  $V(x)$  for those consumers who experience a strictly improving path and would lead to at least as high values of  $V(x)$  for all other consumers (who experience a weakly improving path). This is so due to the Main Theorem

$(x \in A(x^0) \Rightarrow V(x) > V(x^0))$ . Thus this improving path for the economy is Pareto-superior to the steady state in terms of the "as if fixed" utility function  $V(x)$ . But then it is not feasible because the steady state is Pareto-optimal for the "as if fixed" utility function  $V(x)$ . Thus the steady state is steady state efficient.

An important goal must be to find out which of the theories of human behaviour which offer an alternative to the traditional neoclassical theory of utility maximisation are consistent with a reasonable concept of "adaptive preferences". In this paper I have adhered to the neo-classical model of utility maximisation for the "short run preferences". But I believe that I do not need all the behavioural implications of this full rationality model. Thus, for example, the proof of the Main Theorem requires Lemma 1: that constant budget paths are weakly improving paths. It should be possible to set up a framework in which this proposition can be obtained from other behavioural assumptions and definitions. What is important is the maintenance of the "adaptive" feature or a certain resistance to change.

Thus, for example, the theory of satisficing by Herbert Simon (Simon 1955) has a similar structure to our adaptive preferences. Starting from an initial state the person sets himself/herself certain goals. These goals are influenced strongly by the starting point. Even though they may be different from the state achieved they are unlikely to deviate drastically from the status quo. There is then a strong positive correlation between the initial state and the goals to be achieved. Moreover, the goals, if they are achieved, are an improvement relative to the existing state. So the path to be observed under the satisficing hypothesis has a remarkable similarity to the improvement paths discussed in this paper. Indeed, as Simon states, if goals can be reached easily new goals will be set which are more ambitious, which extrapolate from the previous goals, but which also take account of the amount of effort needed for their achievement. Goals and thereby behaviour are adapted to the opportunities offered by the environment. This is in full accordance with the theory of adaptive preferences.

Modern experimental economics emphasises "reciprocity" of human behaviour (Fehr and Schmidt 2002). Interpersonal influences on preferences were discussed in section C. If they take the form of reciprocity this is likely to lead to stabilisation of mutual interaction and to resistance to change, once a satisfactory equilibrium has been achieved. At least at first sight, this seems to be consistent with the hypothesis of adaptive preferences.

Another line of investigation could be the relation between the concept of "progress" and happiness. Happiness research has become quite extensive (Frey and Stutzer 2002). One of the results which stand out is the weak correlation between material wealth and happiness. Happiness, it seems, is much more related to an embeddedness and active participation in a social environment than to material wealth. But this again points in the direction of strong adaptive forces; and these findings may thus be consistent with the hypothesis of adaptive preferences.

A word about social philosophy. Modern contributions to the theory of justice, from Buchanan to Rawls to Sen to Habermas emphasise the procedural aspect of justice. There are good reasons for this trend. But the rationality of procedures and the justice of procedures cannot be evaluated without a view on the "global" consistency of outcomes from these procedures. If such procedures would lead to "improvements" which eventually are circular we would have doubts about the rationality of these procedures. Thus the non-circularity of improvement paths may be another and a powerful test for rational procedures.

## **E Conclusion**

In this paper I have demonstrated that a reasonable concept of progress can be made consistent with the assumption of endogenously changing preferences as long as these preference changes correspond to the pattern of "adaptive preferences". I hope that I can generate enough interest in this topic to induce others to develop further the theory of welfare under adaptive preferences. There exists the conjecture that many theories of non-neoclassical behaviour turn out to be consistent with the hypothesis of adaptive preferences and thus are able to be combined with the tradition of welfare economics.

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