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Article

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Real exchange rate in the long run: A multi-resolution analysis

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Abstract: Findings from previous studies indicate that the long-run stationarity of the real exchange rate in different time horizons remains unclear. In order to shed light on this problem, we have adopted a new method which is widely used to analyze signals, the so-called wavelet transformation. This paper uses monthly data of the consumer price indices (CPI) and nominal exchange rates for six ASEAN countries for the 1998–2019 period. The unobserved component method is used to remove the seasonality and cyclicity of the CPI. Then, the real exchange rates are calculated. We use the unit root tests and wavelet transformation to verify if the purchasing power parity (PPP) holds in the long run in different time horizons. Key findings from the paper are as follows. *First*, the mean-reverting pattern of real exchange rate recurs itself for the period longer than 5 years. *Second*, the variation around trend using wavelet methodology can be used as a reliable indicator for future movements of the real exchange rate. *Third*, the high volatility of the exchange rate in the short run may be induced by arbitrage activities. Implications have emerged based on these findings for policy-makers and investors in relation to the behaviour of the real exchange rate.

Subjects: Economics; Political Economy; Finance

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PUBLIC INTEREST STATEMENT

The real exchange rate is driven by many factors. We are interested in its long-run dynamics which are affected by activities in relation to high-frequency changes in price. This study is conducted to investigate the validity of the purchasing power parity (PPP) in the long-run using the multi-resolution analysis (MRA). We argue that the validity of the PPP depends mostly on the definition of a long run and its measurement.

In this paper, we use a wavelet decomposition, which is widely used in digital and signal processing, and unobserved component analysis for exploring the real exchange rate patterns of the ASEAN countries using monthly data for the period from 1998 to 2019. Findings from our paper indicate that the mean-reverting pattern of the real exchange rate recurs itself for the period longer than 5 years. In addition, the MRA method provides the central banks and the governments with a convincing pattern of real exchange rate movements in the future for policy purposes.

Keywords: wavelet transformation; exchange rate; purchasing power parity; unobserved component method; ASEAN

1. Introduction

The validation of the purchasing power parity (PPP) is widely discussed in many academic papers. The law of one price (LOP) indicates that the value of any common good should be the same when any pair of currency is taken into consideration. Many classical economists refer to this idea as the real covariate (Patinkin, 1965). The PPP and its validation are distinctive phenomena of a classical dichotomy, which has attracted great attention from academics. An agreement has been achieved on the long-run validity of the PPP, while its short-run dynamics remain unclear (Salehizadeh & Taylor, 1999). This idea resounds with the efficient market hypothesis (EMH), which discusses a long-run market equilibrium. The short-run dynamic is a process of an un-correlated successive move (Malkiel, 2003). Unfortunately, the consensus has not been reached. The reason for this disagreement is mostly caused by the limited availability of historical data and limited methodologies in analyzing time series data.

Many previous studies have acknowledged that the long-run equilibrium of real exchange rate r_t is a combination of many stationary processes. This is considered as the prerequisite for the long-run holds of the PPP. This primary condition is much alike to constitute the long-run stationarity of the efficient market hypothesis (EMH), which states that the price exploitation of any financial assets includes many random processes. In turn, the price discovery process is a set of the Brownian motion and white noise which captures the change in returns and is successively uncorrelated in different time horizons. This means that the price cannot be easily deduced just by gathering past information and today's price, which is then considered the best reflection of tomorrow price. Additionally, the effect of one-lagged observation on exchange rate prediction is significant and long-lasting.

The perspective of the investment horizon differs among economic entities. Some investors who are only excited in the short-run movement in price prefer intraday trade for exploiting capital gains. International hedging managers prefer using currency market or the capital market as a tool for minimizing unsystematic risk. Policymakers who are very interested in the long-run dynamics of the real exchange rate consider using financial instruments as a tool for economic stabilization. By conducting large and infrequent trade volume, market makers act as a catalyst for providing a medium and long-term steady growth through keeping inflation bounce in a pre-determined range. In comparison, individual investors, commercial banks, mutual fund managers utilize smaller trade volume. However, they trade more frequently than the market makers. Exploiting market defections and short-run incremental change in returns are the main objectives which these investors aim to do. In sum, the perspectives of different investment time horizons vary significantly among financial market participants, and their perspectives are reflected by their interests and actions. Due to the participation of diverse economic entities and their investment time scale, what we observed from historical recorded price is a combination of low, medium- and high-frequency changes in price level corresponding to the real exchange rate structure. Because of the real exchange rate is driven by many factors, we are only interested in its long-run dynamics which are shrouded by activities with respect to high-frequency changes in price. As such, an appropriate answer for the validation of the PPP depends mostly on how we define long run and how we measure it, for example, by a number of days, months or years. The novelty of our study is summarized as follows. *First*, existing methods are powerless in the provision of the specific time horizon at which the validity of the PPP hypothesis is confirmed. Previous studies also insufficiently provide empirical evidence which supports the mean-reversion of the real exchange rate (Adler & Lehmann, 1983; Roll, 1979). There is no attempt in utilizing wavelet decomposition and unobserved component analysis for exploring the real exchange rate pattern of the Asian countries (see for example, Baqaee (2010) for New Zealand inflation rate; Vo and Vo (2019b) for six pair of the exchange rate against the dollar). For that reason, we use wavelet analysis which is widely used in digital and signal processing. *Second*, we also make use of the multi-

resolution analysis for the purpose of the projection of the real exchange rates. Our results confirm the usefulness of the approach for policymakers in formulating and implementing future monetary policies.

The paper is structured as follows. Following this introduction, Section 2 discusses relevant literature review. Next, the wavelet decomposition in analyzing time series is discussed in section 3. Section 4 presents the analyses on the validity of the PPP and our predictions for real exchange rates, followed by the conclusions in section 5.

2. Literature review

The purchasing power parity (PPP) hypothesis represents the process which comprises many price amendments. These price adjustments are deemed to be the consequence of exploiting international arbitrage. The arbitrage activities indirectly happen to rectify the price misalignments and cause real exchange rate mean-reverting (Vo and Vo, 2019b). Previous papers were conducted to examine real exchange rate dynamics. Using the augmented Dicky Fuller test and co-integration test, Salehizadeh and Taylor (1999) demonstrate that the real exchange rate is misdirected from the true value after several years. They put great efforts into stylizing two-scale analyses—the short run and the long run. Their findings demonstrate that the real exchange rates are not affected by its nominal values. In contrast, other studies do not advocate the validity of the real exchange rate in the long run. Devoting to the idea, Engel and Roger (1996) argue that the PPP is not consistent among different types of goods. They highlight the adverse impact of borderlines for rendering the price misalignment between different markets. Knetter (1994) consider the difference in the price of goods is characterized by its transportation, insurance premium and level of trade liberalization.

The violation of the PPP, in the long run, can be attributed to market defections. In turn, individual investors might “overreact” to an upswing and downswing of exchange rates which might deflect its value away from the true value. De Bondt and Thaler (1985) investigate the overreaction from historical events. The authors find that investors are sensitive to unforeseen and adverse information. Conventional tests such as stationarity and co-integration tests are widely applied in previous studies to examine the stationarity of the real exchange rate. Despite the presence of different stationary tests, the null hypothesis of those tests, time-series is non-stationary given by a sample period, is quite similar. Meanwhile, the null hypothesis should not be rejected unless the PPP’s validation is violated. If the null hypothesis cannot be rejected, then the PPP is successively uncorrelated or characterized by the Brownian motions.

The lack of precision of those tests is discussed. The augmented Dickey–Fuller (ADF) or Phillip-Perron (PP) tests fail to capture the exact order of integration, especially the first order and close to the first order. Another method for unit root testing is the KPSS test which was developed and first presented by Kwiatkowski et al. (1992). The KPSS test is more powerful than the ADF and PP tests since it includes the stationarity around trend in its null hypothesis. Moreover, the stationarity of any time series is not necessary to be presented around a constant mean, but it can embody in an accelerating mean.

A stationary series in a classical econometric definition is an untransformed series whose mean, covariance and others are considered unchanged with respect to time variation. Practically, an absence of unit root is not characterized for a non-stationarity. A time series comprises an incremental mean over time, and “white” disturbance can also constitute a stationary process. Many econometricians consider this idea as a long-range dependency, meaning that the time series demonstrates a long memory. Granger and Joyeux (1980) define a time series characterized by a long-range dependent process which will slowly have been decaying an autocorrelation function (ACF) instead of rapidly falling to 0. We can only confirm stationarity of any time series after deflating trend and correcting for its seasonality and cyclicity. Many techniques have been developed so far to address the confusion in the definitions of the so-called stationary series. Fractional Difference Process (FDP) defines a time series which have the order of integration d at lag k if its ACF satisfies the following

Table 1. Hurst’s exponents

Hurst exponent	Fractional difference parameter	The behaviour of the time series
$H \leq 0$	$d \leq \frac{-1}{2}$	Non-stationary
$0 < H < \frac{1}{2}$	$\frac{-1}{2} < d < 0$	Not persistent, mean-reverting
$H = \frac{1}{2}$	$d = 0$	No trend, Brownian motion
$\frac{1}{2} < H < 1$	$0 < d < \frac{1}{2}$	Long-range dependent structure
$H > 1$	$d > \frac{1}{2}$	Non-stationary

Source:Vo and Vo (2019a)

condition: $\rho(k) \sim k^{2d-1} \cdot c_p$; where $\rho(k)$ stands for the ACF function of obtained time series at lag k and constant term c_p . Hosking (1981) when developing the Auto Regression Fractional Integration Model (ARFIMA) mentions that d in ARFIMA (0,d,0) is not necessarily an integer number and a time series is mean reverting even when $0 \leq d \leq \frac{1}{2}$. Another method for measuring the long-run dependency of a given time series is Hurst exponent/coefficient, which is widely accepted and applicable in quantifying the long-term trend (mean-reversion) of any time process. Researchers consider Hurst exponent as a criterion for identifying the extent to which the stochastic process is dependent on its past value.

Hurst exponent was originally utilized in hydrology for determining optimal dam size for Nile’s river (Hurst, 1951). In order to gratitude the enormous contribution, the letter H by which denotes the extent of long-dependency of a stochastic process is defined by its asymptotic behaviours over a rescaled range through n obtained observations (Qian et al., 2004). When n approach indefinite, the Hurst exponent could be written as $E \left[\frac{R(n)}{S(n)} \right] \sim Cn^H$. We can calculate Hurst exponent by taking natural logarithm from both sides of the equation, then H ranges in the intervals $[0, 1]$ and requires only $\left[\frac{R(n)}{S(n)} \right]$ as a pre-determined parameter. If H equals 0.5, the stochastic process is considered to follow a Brownian motion. $R(n)$ and $S(n)$ are, respectively, range and standard deviation of the observed data. Table 1 gives reader the first glance of Hurst exponent.

The above-mentioned methods show that previous researchers have been treating time-series data in “time domain” which is given by the pre-determined sampling frequency. Technically, obtained data has been already sampled over fixed intervals (i.e., daily, weekly, monthly, annual). This method is quite convenient in analyzing behaviours of time series data (whether it increases or decreases over time). However, the method seems to be restricted when we need to understand the characteristics/components which horizontally constitute the upswings and downswings. A time series is a combination of many differential oscillations which are not well-specified and localized in time. In economics, the real exchange rate is the outcome of many economic activities in different time horizons. In contrary, technicians simply refer economic times series as an input signal which is the sum of many oscillations corresponding to different frequencies. The origin of the idea traces back to the well-known mathematician Joseph Fourier when he develops the heat equation. Fourier finds that any arbitrary function can be expressed by the sum of many trigonometric functions or sinusoids. Although Fourier transformation is capable of breaking down algebraic function, it is still inappropriate to the economic application for two fundamentals. *First*, economic time series is rarely considered periodic. *Second*, the constituents returned by the application of Fourier transformation are not well-localized in the real-time, which means, we do not have any exact information on when the economic incidences happen in the time domain. *Third*, we do not know for how long it would take for the real exchange rate to complete its cycle. These problems provide us with a physical paradox between two complementary variables which are location and momentum.

In addressing these concerns, the wavelet transformation is a more powerful technique in order to tackle the trade-off between behaviours (time-domain) and characteristics (frequency domain) of time series by shifting and scaling orthonormal wave. This trade-off follows the uncertainty theory (Heisenberg, 1971), which is articulated that the position (corresponding to the location of economic events) and velocity of any object cannot be simultaneously and accurately quantified. Despite its extensive applications in digital compression, signal processing, wavelet transformation has just been recently adopted in economic studies for a decade by a limited number of pioneers such as Cotter et al., (2011), Baqaee (2010) and Vo and Vo (2019a).

3. Methodology and data

3.1. Primer on wavelet

In this section, we provide an overview of the wavelet theory, which has been developed and catered to a wide range of audiences. Schleicher (2002) presents a concise and simplified introduction to emphasize the dominance of wavelet in its application on economics. A book by Percival and Walden (2000) presents the fundamental theorem of wavelet transform regarding statistical applications. Haar (1910) is the first to construct the first wavelet basis, which illustrates that any continuous functions belonging to $[0, 1]$ interval could be approximately broke down into a sequence of square waves. On the ground of this idea, a novel concept called *mother wavelet* in associating to shifting and scaling actions is capable of reflecting complicated function. Although Haar's definition of mother wavelet was not proposed at that time, it still acts as a cornerstone for many successive types of research on wavelet since then. An oscillation function is considered as a mother wavelet if it satisfies two basic conditions. s

$$\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1; \int_{-\infty}^{+\infty} \phi(t) dt = 0$$

The former condition means that the length of the mother wavelet, which is defined by an inner product with itself is equal to one. The latter condition implies that the oscillations in associating to mother wavelet cancel out each other over time. When two conditions are combined, wavelets are small and finite oscillations in a pre-determined interval. These vectors form an inner product space

$L^2(\mathbb{R})$. Their lengths which are defined by $\int_{-\infty}^{+\infty} |\psi(t)|^2 dt$ have the value of 1, and they are orthogonal to

each other $\int_{-\infty}^{+\infty} \psi_1(t) \cdot \psi_2(t) dt = 0$. Henceforth, the span of mother wavelet defined by $\psi_{jk}(x) =$

$2^{\frac{j}{2}} \psi(2^j x - k)$ is Hilbert space and capable of reconstructing any function by changing the value of j , $k \in \mathbb{Z}$. The wavelet transform shares the idea of breaking down any function into many sinusoids, but it seems to be more effective and predominant to Fourier series in capturing function's pattern by altering the combination of j , k . By their utilization, j is commonly called dilated coefficient because of its abilities of contraction and magnification. Likewise, k is commonly known as translated coefficient since it is capable of horizontally localizing the economic incidences in the time domain.

This connotation forms a spine of wavelet theorem allowing any "real-world" function which can be rewritten by a sum of many translated and dilated wavelets. The interpretation is analogous to the idea that is breaking down time series into many sinusoids which are look-a-like the Fourier series. The fundamental difference is that the wavelet transformation ingeniously addresses the uncertainty principle by the combination of translated and dilated orthonormal waves. Despite the ability of approximate reflection, a sequence of mother wavelets could only account for the divergences or variations of economics in the time series. To overcome this shortcoming, we need a function which could use in conjunction with mother wavelet to capture the trend of the stochastic process. In other words, the wavelet transform requires a complementary function called *father wavelet* (denote as ϕ) to fully express the economic time series. In general, the combination of father and mother wavelets let us approximately simulate any input signal as follows:

$$f(t) \equiv \sum_{l \in \mathbb{Z}} \langle f, \phi_l \rangle \phi_l(t) + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} f, \psi_{j,k} \psi_{j,k}(t)$$

where j stands for the level of our Multi-Resolution Analysis ($j = 1, \dots, J$) and k takes the value from 1 to $\frac{n}{2^j}$ for each level. Wavelet transformation allows us to separate the “noise” $D_{j,k}$ out of the “smooth” component $S_{j,k}$. In other words, the components obtained from the wavelet transform $S_{j,k}, D_{j,k}, \dots, D_{1,k}$ are the wavelet hierarchical components which have their sum approximate the input signal. Likewise, the original signal can be reconstructed as follows:

$$f(t) \equiv S_{J,k} + D_{J,k} + D_{j-1,k} + D_{j-2,k} + \dots + D_1(t)$$

$$\text{while } S_{j,k} = \int \phi_{j,k}(t) f(t) dt; D_{j,k}(t) = \int \psi_{j,k}(t) f(t) dt \text{ and } j \in (1, 2, \dots, J); k \in \left(1, \dots, \frac{n}{2^j}\right)$$

3.1.1. A discrete wavelet transform

In the application to economics, time-series data is commonly seen not to follow any pattern and is restricted. In practice, an economic series is not a function or maybe just a part of its which are recorded over a period of time. A Discrete Wavelet Transform (DWT) carries over most advantages of wavelet transform in general. *First*, it could be sequentially performed by $O(n)$ times. *Second*, DWT results are enriched with both time and frequency information by cascading a filter bank. The only shortcoming of DWT is its time sensitivity. Mallat and Zhong (1992) argue that the dyadic sampling process is applicable for smooth components in avoidance of DWT sensitivity of time-variance. The DWT divides the stochastic sampling process into two parts, which are so-called smoothing (averaging) and detailed component by simultaneously passing through many low and high pass filters. They are directly related to each other and commonly refer to a quadrature mirror filter. It is worth noting that the DWT halves our data in the time domain and also doubles the sampling frequency. In an example, considering two times successive application of DWT on the sample with 64 observations will return us two detailed components, namely D1, D2 and one smooth component S2 with only 16 observations. As such, the mother wavelet by DWT will be dilated and scaled by the power of 2. One most common family wavelet for DWT is Daubechies. The Daubechies wavelet coefficients $h_1 = (h_{1,0}, \dots, h_{1,L-1}, 0, \dots, 0)$ requires for wavelet decomposition process strictly satisfy three conditions (Tiwari et al, 2013).

$$\sum_{i=0}^{L-1} h_{1,i} = 0; \sum_{i=0}^{L-1} h_{1,i}^2 = 1; \sum_{i=0}^{L-1} h_{1,i} h_{1,i+2n} = 0$$

The implication underneath three conditions is that a wavelet has zero-sum, unit length and orthogonal to itself even shifting. As long as three conditions are fulfilled for any h_j , the wavelet coefficients could be expressed as follows:

$$W_{i,t}(t) = 2^{\frac{j}{2}} \sum_t x_t \psi(2^j t - k)$$

while the scaling factor: $V(j, k) = 2^{\frac{j}{2}} \sum_t x_t \psi(2^j t - k)$ and ψ, ϕ are simultaneously discrete functions. j and k are shifting and scaling factors of the mother wavelet. The first parameter j represents the dilated coefficient of mother wavelet, which corresponds to refining actions. The larger j becomes, the finer mother wavelet function is. The second parameter k represents the translation which corresponds to time localizing actions or the location in time with respect to the original signal. According to Daubechies (1992), the set of functions which could be created by changing parameters j and k forms an orthonormal basis for inner product space $L^2(R)$ since its norm is defined by $\int 2^{\frac{j}{2}} \psi(2^j t - k)^2 dt = 1$ and orthogonal to each other $(\psi_{j,k} \psi_{l,m}) = 0$. At j^{th} scale, the coordinate vector which accounts for all variations is respective to the frequency band $\left(\frac{1}{2^{j+1}}, \frac{1}{2^j}\right)$. As a result, the Heisenberg’s curse can be addressed by examining these coefficients at different frequency level and capturing time-series local variations due to DWT.

In summary, a discrete wavelet transform serves as a brand-new method in treating time series at different time horizons. By changing our classical perceptions on time series, DWT refers to an

original input signal as an approximate sum of many sinusoidal functions. On that basis, this framework is often expressed as the Multi-Resolution Analysis (MRA).

3.2. Data and analysis

We use six nominal pairs of the currency against the dollar and the national consumer price indices for the calculations of the real exchange rate. The dataset is available on Thompson Reuter Datastream on a monthly basis. Because the availability of Vietnam's CPI is from 1998, our dataset, therefore, covers the interval from 1 December 1998–30 August 2019 which comprises 296 months. The real exchange rate is calculated as follows:

$$r_t = \ln(ne) + \ln P^* - \ln P$$

where ne stands for the nominal exchange rate in local currency; P^* , P are, respectively, the CPI of the United States and an investigated country. \ln is the natural logarithm operator. In general, it is reasonable to assume that CPI series has its trend, seasonality, cyclicity and noise which differ among countries. Moreover, the cyclicity and seasonality vary significantly and belong to countries specific which are necessary to rule out. Since the obtained data are specified by its corresponding country characteristics, we have to adjust for the seasonality and cyclicity using the unobserved components method which is outlined in Harvey (1985) and Clark (1987). The CPI series is the combination of many constituents which could be expressed as below:

$$P_t = Trend_t + Seasonality_t + Cyclicity_t + e_t$$

where $Trend_t$, e_t respectively denotes a trend and noise of the stochastic process. Since we do not know if the CPI of any specific country exhibits its seasonality and cyclicity, we apply the UCM uniformly for every CPI series on the basis of parsimony. After that, we remove the cyclical and seasonal factor based on the Bayesian Information Criterion with respect to each model. Figure 1 below exhibits stepwise application of UCM on Singapore CPI.

Table 2 presents the summary statistic of “de-seasonalized” CPIs and their corresponding real exchange rate. Based on D’Agostino, Belanger test for normality behaviour, it seems that real exchange rates for most of the investigated countries exhibit non-normal distributions and demonstrate a positive skewness as well as kurtosis. Moreover, there are strong dependent structures (up to lag 40) which are shown by the auto correlogram and Ljung—Box test (Ljung & Box, 1978). The depreciation of the domestic currency against the dollars in investigated countries is observed. Based on the descriptive statistics shown in the first row of Table 2, on average, the SGP dollar gains 45 per cent its value against the US dollar. In contrary, other currencies seem to lose their value against the dollars during the period.

The real exchange rate patterns are plotted in Figure 2. The dashed lines in Figure 3 represent the smooth components which are deduced by using wavelet transform to “de-noising” the high-frequency oscillations.

It is appropriate to use a wide range of stationary tests for validating the effectiveness of the PPP in the long run. Table 3 presents the test results for the real exchange rate r_t . We use three tests (augmented Dickey–Fuller (ADF) (1979), Phillips and Perron (1988) and KPSS test (1992)) which are widely used in economics for attributing whether or not an economic time series is stationary. The Bayesian Information Criterion is utilized in order to decide an optimal lag for every real exchange rate pair. Three test results confirm strong denial on the assertion that all real exchange rate series are stationary process at 99 per cent. Following the results, all investigated real exchange rates series do not exhibit a clear pattern following the law of one price (PPP). Abuaf and Jorion (1990), O’Connell (1998) consider that the power of the stationary test is slightly below unity.

Figure 1. The application of UCM in adjusting Singapore CPI.

Notes: Singapore CPI series is constructed by trend, cycle, seasonality and noise

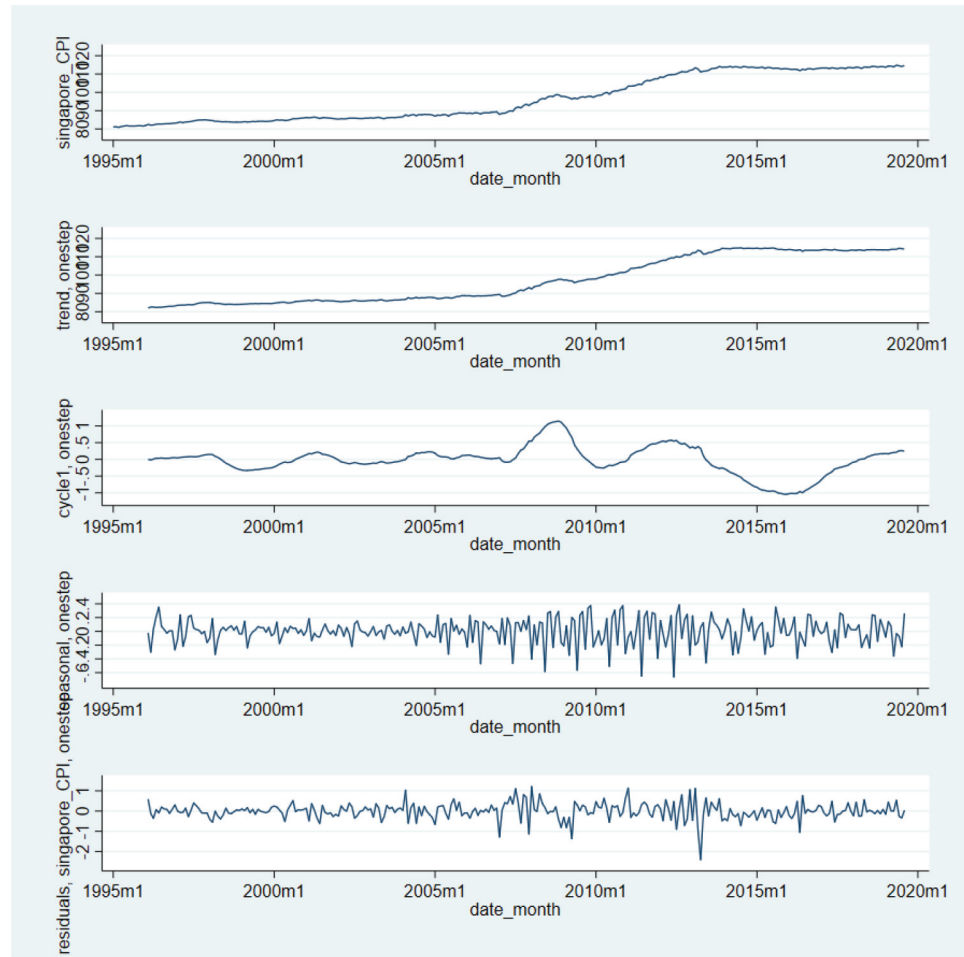


Table 2. Descriptive statistics

Country	Mean	Std	Skewness	Kurtosis	DB	LB
SGP	0.374	0.128	-0.162	1.699	.(0.0000)	7060 (0.0000)
VNM	9.893	0.174	0.162	1.418	.(0.0000)	7554 (0.000)
THA	3.584	0.136	0.378	1.922	53.37 (0.0000)	6066 (0.0000)
PHL	3.916	0.158	0.428	1.825	.(0.0000)	6636 (0.0000)
IDN	9.334	0.178	0.654	3.092	13.84 (0.0010)	4767 (0.0000)
MYS	1.288	0.103	0.272	3.887	8.056 (0.0178)	2790 (0.0000)

We utilize the panel unit root test methods which were developed by Levin, Lin and Chu (LLC) (2002), Breitung and Das (BT) (2005), Harris and Tzavalis (HT) (1999), Choi (2001) (Fisher), Hadri (Hadri) (2000), Im et al. (2003) (IPS). These tests allow us to entangle the

perplexity that the real exchange rate does exhibit stationary tendency in the long run, as presented in Table 4. The null hypothesis of the panel unit root test is the entire data set contains a unit root.

Figure 2. Real exchange rate.

Notes: The real exchange rates for six ASEAN countries

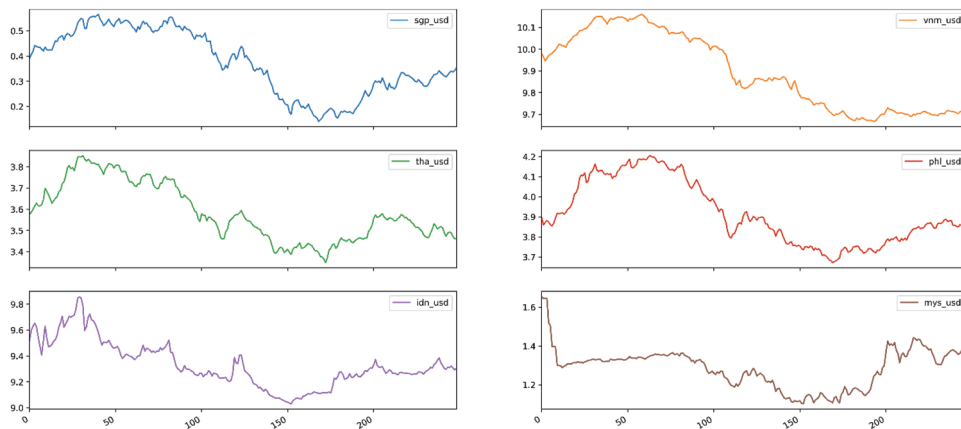
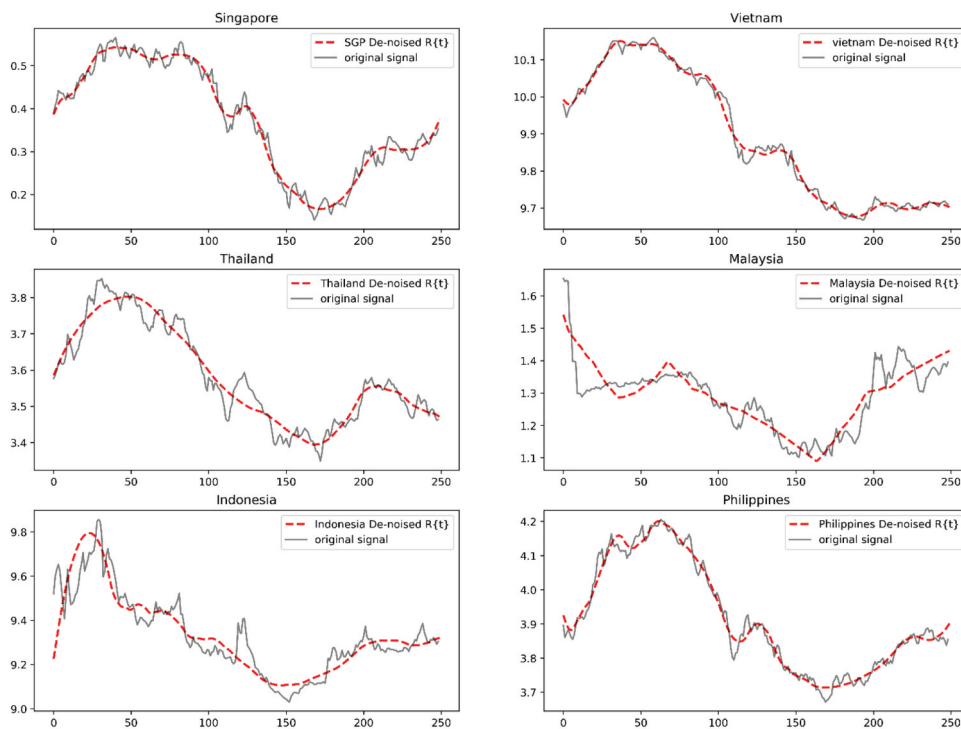


Figure 3. De-noised real exchange rate.

Notes: For each subplot, the solid black line demonstrates real exchange rate value which is calculated by the above-mentioned equation $r_t = \ln(ne) + \ln P^* - \ln P$. While P^* and P correspond to the CPI values of the US and investigated country, ne stands for the nominal exchange rate of the local currency against 1 US dollar. The red dashed line represents the unruffled (smoothed) real exchange rate value deduced from the wavelet transform method, which is highlighted in the next section.



3.3. Insights of multi-resolution analysis

Figure 4 visually explains how the multi-resolution analysis (MRA) aids in finding the long-run real exchange rate pattern in the most intuitive way. The SGP/USD real exchange rate is selected as the example for illustrating the multi-level analysis corresponding to different time horizon. It is worth considering that the MRA splits any given time series into two particular parts, to be named (i) *detailed coefficients* and (ii) *averaging coefficients*. The detailed coefficients account for the high-speed oscillations which result from arbitrage activities, whereas the averaging trends or long-term movements are

Table 3. Univariate stationary test results

	Singapore	Vietnam	Thailand	Philippines	Malaysia	Indonesia
ADF	-0.9236 (0.7801)	-0.1406 (0.9452)	-1.0102 (0.7495)	-0.9458 (0.7725)	-3.6322 (0.0052)	-1.8289 (0.3662)
PP	-0.8850 (0.7929)	-0.0966 (0.9498)	-0.8633 (0.7998)	-0.8429 (0.8062)	-3.4461 (0.0095)	-1.7084 (0.4268)
KPSS lags0	2.8485	2.1465	2.9655	2.7107	3.5414	4.1966
KPSS lags1	1.4416	1.2219	1.4888	1.3673	1.8094	2.1332
KPSS lags2	0.9707	0.8232	1.0030	0.9192	1.2285	1.4445
Critical 95	0.146	0.146	0.146	0.146	0.146	0.146
Critical 99	0.216	0.216	0.216	0.216	0.216	0.216
N	249					

Notes: Null hypothesis for the ADF and PP tests is the time series is not stationary, whereas the null hypothesis of the KPSS test is that the time series is stationary. All real exchange rate series are assumed to have trend and base level. The lag in which we choose to conduct these tests is based on the order determination method which is comprehensively outlined in Tsay (1984); Paulsen (1984); Nielsen (2001). All corresponding p-values are in parentheses. Critical values of the KPSS test are reported at 99 per cent and 95 per cent.

Table 4. Panel stationary test results

	LLC	HT	BT	IPS	Hadri	Fisher (DF)	Fisher (PP)
Statistic	-1.7549	0.9787	1.4655	1.1919	187.3906	0.3622	0.3622
Critical value	(0.0396)	(0.8524)	(0.9286)	(0.1167)	(0.0000)	(0.6414)	(0.6414)

Notes: Null hypothesis for LLC, HT, BT, IPS, Fisher DF and Fisher PP is the panel data contain unit-roots. The alternative is all panel data are stationary. The null hypothesis for the Hadri test is that all panels are stationary and the alternative hypothesis is that at least one series contains unit-roots.

reflected in the other half. In contrast, the detailed coefficients gradually approach averaging coefficients as a number of level increases. Another aspect of the MRA is the double of frequency resolution. After each level, the length of time series is cut into a half, which implies the double in our sampling frequency. We only present our application of MRA on the Singapore Dollar in Figure 4.

4. Tests for the validity of the PPP and the prediction

4.1. PPP validation

The question now is that which specific time horizons that PPP takes its effect. In addressing the relationship between two parts of MRA’s output, Table 5 incorporates our explanations which might be considered helpful for readers’ interpretation of the MRA mechanism. Table 6, in conjunction with Table 5, show the applications of the unit root test, which were previously conducted. The wavelet transformation, which is introduced in Section 3, provides a numerical embodiment of our time series under the terms of the “Detailed” coefficients and the “Averaging” coefficients. The “Detailed” coefficients explain the oscillations corresponding to the high frequencies in the frequency spectrum). The “Averaging” coefficients account for the coefficients corresponding to the low-speed oscillations or trend in the frequency spectrum). Additionally, we repeatedly apply the unit root test on each level of the MRA process, which comprises of detailed components and averaging components upon reaching maximum level obtained. The averaging coefficients which are hidden under many high-frequency fluctuations of six pairs of currency are stationary. These findings are possibly indifferent from dichotomy theory.

We report the univariate unit test results on detailed coefficients for each level of MRA in Table 6 and unit root test results on averaging coefficients for each level in Table 7. The empirical test results strongly

Figure 4. Multi-level decomposition of SGP/USD real exchange rate.

Notes: This figure represents the MRA application on monthly SGP/USD real exchange rate. The investigated time horizon ranges from Dec 1998 to Aug 2019. Our original data is the sum of all detailed coefficients and the first level of averaging coefficients (Level 1). Results from the other pairs of currency are provided upon request.

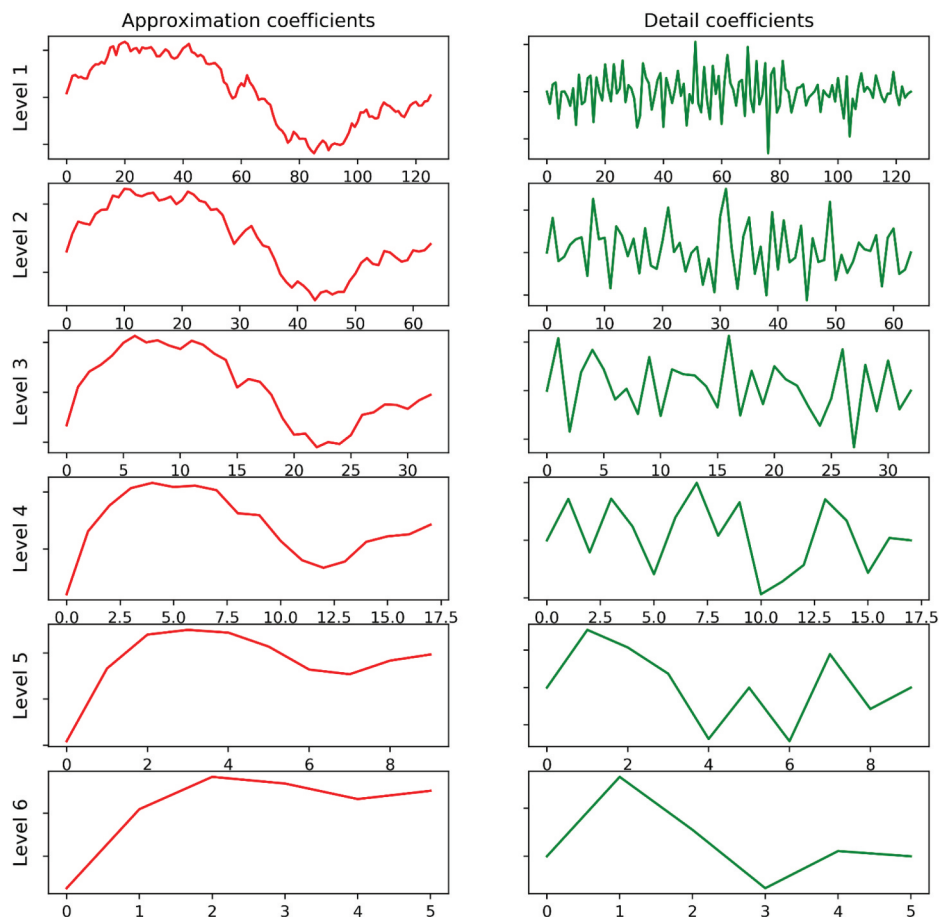


Table 5. Insights of the MRA mechanism

Decomposition level							
1	D1	S1					
2	D1	D2	S2				
3	D1	D2	D3	S3			
4	D1	D2	D3	D4	S4		
5	D1	D2	D3	D4	D5	S5	
6	D1	D2	D3	D4	D5	D6	S6

Notes: This table depicts the relationship between *Detailed coefficients* and *Averaging coefficients* as well as the MRA mechanism. The summation of each row's components will be exactly equal to the original signal.

back up our set-forth assumption that the PPP tends to hold in the long run at 1 per cent level for three well-known types of univariate unit root test (KPSS, ADF and PP).

Real exchange rates for all currencies are reported to be stationary at 5 per cent after applying MRA 5 times successively (except for Vietnam with KPSS test, Thailand with both ADF and PP test, Malaysia with KPSS test). Moreover, the strong empirical evidence on the PPP validation is recorded at 1 per cent after six times of MRA successive application. In order to acquire the certainty on PPP validation, we decide to re-

Table 6. Univariate stationary test results on six real exchange rate—Averaging coefficients

	S1	S2	S3	S4	S5	S6
1. SGD/USD						
ADF	-0.791 (0.822)	-0.871 (0.797)	-1.252 (0.651)	-2.447 (0.129)	-4.594 (0.000)	-6.500 (0.000)
PP	-0.949 (0.771)	-1.057 (0.732)	-1.569 (0.499)	-2.524 (0.110)	-3.690 (0.004)	-8.479 (0.000)
KPSS	8.721	4.226	1.815	0.432	0.199	0.330
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
2. VND/USD						
ADF	-0.047 (0.954)	-0.352 (0.918)	-0.955 (0.769)	-2.128 (0.234)	-4.050 (0.001)	-7.999 (0.000)
PP	-0.338 (0.920)	-0.577 (0.876)	-1.041 (0.738)	-2.065 (0.259)	-3.600 (0.006)	-9.662 (0.000)
KPSS	11.40	5.775	2.985	1.592	0.786	0.475
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
3. THB/USD						
ADF	-0.722 (0.841)	-0.800 (0.819)	-1.016 (0.747)	-1.504 (0.531)	-2.674 (0.079)	-4.096 (0.001)
PP	-0.999 (0.754)	-1.064 (0.729)	-1.390 (0.587)	-1.841 (0.361)	-2.554 (0.103)	-4.994 (0.000)
KPSS	8.307	4.045	1.832	0.655	0.156	0.272
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
4. PHP/USD						
ADF	-0.799 (0.820)	-1.049 (0.735)	-2.029 (0.274)	-4.295 (0.000)	-8.701 (0.000)	-21.60 (0.000)
PP	-0.998 (0.754)	-1.282 (0.638)	-2.150 (0.225)	-3.785 (0.003)	-7.142 (0.000)	-33.13 (0.000)
KPSS	7.779	3.962	2.089	1.042	0.561	0.419
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
5. IDR/USD						
ADF	-1.552 (0.508)	-1.638 (0.463)	-3.476 (0.009)	-8.204 (0.000)	-13.74 (0.000)	-16.72 (0.000)
PP	-1.502 (0.532)	-1.805 (0.378)	-3.686 (0.004)	-6.109 (0.000)	-12.36 (0.000)	-24.13 (0.000)
KPSS	7.087	3.357	0.850	0.133	0.324	0.372
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739

(Continued)

	S1	S2	S3	S4	S5	S6
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
6. MYR/USD						
ADF	-3.530	-3.589	-3.538	-3.266	-3.171	-3.386
	(0.007)	(0.006)	(0.007)	(0.017)	(0.022)	(0.011)
PP	-3.424	-3.449	-3.371	-3.240	-3.558	-4.095
	(0.010)	(0.009)	(0.012)	(0.018)	(0.007)	(0.001)
KPSS	2.574	1.452	0.928	0.638	0.544	0.465
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463

Notes: Null hypothesis for ADF and PP test is the time series is not stationary whereas the null hypothesis for the KPSS test is that time series is stationary. All corresponding p-values are in parentheses. Critical values of KPSS test are reported at 99 per cent, 97.5 per cent and 95 per cent.

Table 7. Univariate stationary test results on six real exchange rate detailed coefficients

	D1	D2	D3	D4	D5	D6
1. SGD/USD						
ADF	-14.970	-10.379	-9.420	-3.887	-2.729	-1.774
	(0.000)	(0.000)	(0.000)	(0.002)	(0.069)	(0.393)
PP	-16.145	-11.178	-9.779	-3.880	-2.757	-1.763
	(0.000)	(0.000)	(0.000)	(0.002)	(0.065)	(0.399)
KPSS	0.041	0.130	0.131	0.158	0.229	0.190
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
2. VND/USD						
ADF	-14.458	-11.533	-7.395	-6.125	-3.854	-2.942
	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.041)
PP	-16.055	-12.383	-7.326	-6.401	-3.839	-3.112
	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.026)
KPSS	0.107	0.043	0.149	0.048	0.150	0.099
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
3. THB/USD						
ADF	-14.549	-11.549	-10.581	-4.166	-2.690	-2.595
	(0.000)	(0.000)	(0.000)	(0.001)	(0.076)	(0.094)
PP	-15.234	-12.303	-10.512	-4.175	-2.672	-2.739
	(0.000)	(0.000)	(0.000)	(0.001)	(0.079)	(0.067)
KPSS	0.115	0.038	0.040	0.146	0.078	0.095
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739

(Continued)

Table 7. (Continued)

	D1	D2	D3	D4	D5	D6
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
4. PHP/USD						
ADF	-14.285	-13.011	-7.717	-5.243	-3.451	-2.898
	(0.000)	(0.000)	(0.000)	(0.000)	(0.009)	(0.046)
PP	-14.891	-13.610	-8.362	-5.206	-3.514	-2.850
	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.051)
KPSS	0.036	0.046	0.092	0.131	0.160	0.134
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
5. IDR/USD						
ADF	-14.374	-9.723	-6.964	-3.185	-2.927	-2.095
	(0.000)	(0.000)	(0.000)	(0.021)	(0.042)	(0.247)
PP	-15.442	-9.913	-7.154	-3.183	-2.952	-2.093
	(0.000)	(0.000)	(0.000)	(0.021)	(0.040)	(0.247)
KPSS	0.045	0.074	0.114	0.155	0.192	0.167
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463
6. MYR/USD						
ADF	-14.176	-10.783	-9.239	-3.448	-3.537	-2.869
	(0.000)	(0.000)	(0.000)	(0.009)	(0.007)	(0.049)
PP	-14.605	-11.306	-10.168	-3.405	-3.612	-2.785
	(0.000)	(0.000)	(0.000)	(0.011)	(0.006)	(0.060)
KPSS	0.048	0.076	0.045	0.129	0.137	0.156
Critical Value 1%	0.739	0.739	0.739	0.739	0.739	0.739
Critical Value 2.5%	0.574	0.574	0.574	0.574	0.574	0.574
Critical Value 5%	0.463	0.463	0.463	0.463	0.463	0.463

Notes: Null hypothesis for ADF and PP test is the time series is not stationary whereas the null hypothesis for the KPSS test is that the time series is stationary. All corresponding p-values are in parentheses. Critical values of KPSS test are reported at 99 per cent, 97.5 per cent and 95 per cent.

apply panel tests which mentioned in the previous section (LLC, HT, BT, IPS, Hadri, Fisher) on the average coefficients. The test's results are reported in Table 8 below.

The PPP validity seems independent of the variation in stationary tests. Following the consensus of results of the univariate stationary test, the confirmation on the validation of PPP remains in the LLC, IPS, HT, Hadri, Fisher test at 5 per cent confidence level. Overall, the mean-reverting pattern of the real exchange rate is consistent with the test results found in both Tables 7 and 8. We conclude that at the time horizon approximately equal $2^6 = 64$ months, the real exchange rate takes 5.3 years for completing its cycle which is in agreement to the most precedented studied related to PPP (Rogoff, 1996; Vo and Vo, 2019b).

4.2. Prediction of the exchange rates

In this section, we use the averaging coefficients acquired from the MRA for prediction. Wavelet method not only filters out the high-pitch frequencies in accordance with arbitraging activities but also returns the

Table 8. The multivariate test results on six averaging coefficients and detailed coefficients

1. Averaging coefficients

	S1	S2	S3	S4	S5	S6
LLC	-1.080 (0.140)	-0.835 (0.202)	-0.061 (0.476)	0.557 (0.711)	-3.727 (0.000)	.
HT	0.854 (0.804)	0.336 (0.632)	-1.943 (0.026)	-3.184 (0.001)	-1.072 (0.142)	0.906 (0.818)
IPS	-1.045 (0.146)	-1.395 (0.085)	-3.411 (0.003)	-7.232 (0.000)	-10.527 (0.016)	-8.181
BT	1.275 (0.899)	0.831 (0.797)	0.528 (0.701)	0.562 (0.713)	0.799 (0.788)	1.091 (0.862)
Hadri	90.643 (0.000)	40.369 (0.000)	13.519 (0.000)	4.210 (0.000)	3.092 (0.001)	1.611 (0.054)
Fisher DF	0.238 (0.594)	0.804 (0.789)	0.872 (0.808)	-0.408 (0.342)	-1.186 (0.118)	.
Fisher PP	0.621 (0.733)	0.118 (0.547)	-1.826 (0.034)	-5.015 (0.000)	-9.472 (0.000)	-15.861 (0.000)

2. Detailed coefficients

	D1	D2	D3	D4	D5	D6
LLC	-27.538 (0.000)	-15.201 (0.000)	-7.215 (0.000)	-5.150 (0.000)	-6.878 (0.000)	.
HT	-75.122 (0.000)	-38.554 (0.000)	-20.529 (0.000)	-7.407 (0.000)	-4.985 (0.000)	-2.939 (0.002)
IPS	-36.966 (0.000)	-27.285 (0.000)	-19.888 (0.000)	-7.728 (0.000)	-5.420 (0.000)	-20.999
BT	-21.404 (0.000)	-15.489 (0.000)	-11.022 (0.000)	-6.409 (0.000)	-4.647 (0.000)	-3.271 (0.001)
Hadri	-2.141 (0.984)	-2.543 (0.995)	-1.411 (0.921)	-0.349 (0.636)	-1.082 (0.860)	-1.393 (0.918)
Fisher DF	-16.417 (0.000)	-12.338 (0.000)	-6.733 (0.000)	-4.821 (0.000)	-3.705 (0.000)	.
Fisher PP	-19.904 (0.000)	-19.904 (0.000)	-17.940 (0.000)	-8.228 (0.000)	-5.084 (0.000)	-3.088 (0.001)

Notes: Null hypothesis for LLC, HT, BT, IPS, Fisher DF and Fisher PP is the panel data contain unit-roots. The alternative is that all panel data are stationary. The null hypothesis for the Hadri test is that all panels are stationary. The alternative hypothesis is that at least one series contains unit-roots. Time series are presumed to have trend, and the lag specifications are based on the AIC.

pattern or the long-run movements of the real exchange rate. First, we reuse our original real exchange rate data which spans over 242 months (12/1998–08/2019) for comparative purpose. Next, the last 12 months (12 observations) are used as the baseline for our projection. ARIMA model is used to make a comparison to the baseline observations. The lags order $AR(p)$ used for ARIMA is determined by the Bayesian Information Criteria. The details of ARIMA given by the equation as follows:

$$\left(1 - \sum_{i=1}^p \phi_i L^i\right) r_t = \left(1 + \sum_{i=1}^q \theta_i L^i\right) \epsilon_t$$

where ϕ_i θ_i corresponds to the parameters of the autoregressive part and moving average part of the ARIMA model. One step ahead ($t + 1$) projection as $\hat{r}_{t+1} = \hat{c} + \sum_{i=1}^p \hat{\phi}_i \hat{r}_{t-i+1}$ is computed. Root mean squared errors (RMSE) is considered as our measurements of effectiveness for projection which is calculated using the following equation:

$$\sqrt{\sum_{j=1}^{12} (\hat{r}_{t+j} - r_{t+j})^2}$$

Figure 5 below presents the one-step projection. The method yields a significant accurate projection on real exchange rate movements by means of RMSE.

Figure 5. ARIMA on six real exchange rate series.

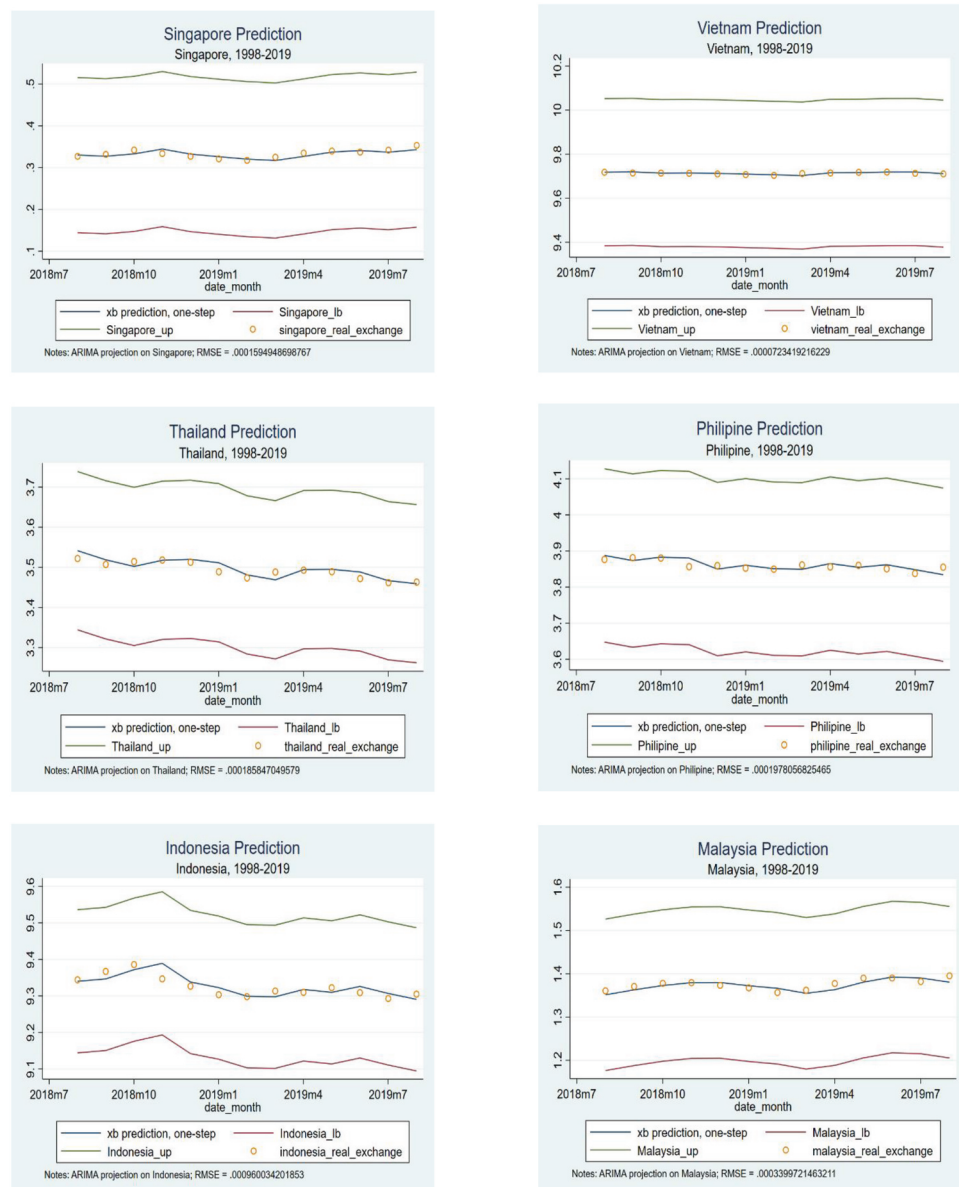


Figure 6. The estimated alpha and beta coefficients.



For the next step, we take S as the representation of the trend in real exchange rate series. The averaging coefficients S_1 is de-noised and characterized by many turning points. The reason behinds this step is for the policy implication, which might attract many significant attentions. Visually, we call the S_1 averaging series as the r_s and measure the magnitude of variation in respect to time-lapse using the following equation:

$$\hat{r}_{t+h} - r_t = \alpha_h + \beta_h(r_t - r_s) + u_{t+h}$$

Presumably, the coefficients α is expected to be zero and β_h is expected to be -1 . Hence, the robustness of our projection is also the robustness of slope and intercept of the above equation. Figure 6 below demonstrates the test results for two coefficients α and β . The test results are all significant at the 5 per cent significance level, but the power of the estimated value remains relatively low. The results are shown in Figure 6.

5. Conclusions and policy implications

This study is conducted to investigate the validity of the purchasing power parity (PPP) in the long-run using the multi-resolution analysis (MRA). The MRA has its root related to wavelet methodology, which has been widely utilized in signals processing, but the analysis is relatively new in applications in economic studies. Monthly data for the period from 1998 to 2019 for six key ASEAN countries are used in this study. Empirical findings from this paper indicate that the predicted real exchange rates exhibit good real-time properties because of the significant correlation between one-step ahead values and the wavelet measurements. Three significant insights corresponding to the interpretation of these dynamics can be summarized as follows. *First*, the mean-reverting pattern of the real exchange rate recurs itself for the period longer than five years. In this paper, the univariate and multivariate analyses are utilized to confirm this important finding. *Second*, the application of MRA provides many turning points of the real exchange rate, which could be utilized as a preference for an effective monetary system and an counter-cyclical fiscal policy. Moreover, the MRA method provides the central banks and the governments with a convincing pattern of real exchange rate movements in the future for policy purposes. *Third*, the validity of the real exchange rate forms a concrete basis for the efficiency of the currency markets in six ASEAN countries, meaning that the currency fair value is justified by the constant high-pitch activities.

Wavelet transformation is useful in physics and engineering. However, its applications and implications appear to be still unfamiliar to many economists. In addition to the capability of decomposition, the wavelet transformation also has the predictive ability which transforms the data into many turning points over the time domain. Counter-cyclical policies should be considered and initiated for stabilizing macroeconomic conditions when the values of the real exchange rates are off-track from their long-term patterns. Moreover, investors can consider the MRA method for predicting the price movement of their financial assets at different time horizons. On the ground of the findings obtained from this study, we recommend the use of the wavelet approach because of its credibility and applications in providing important insights into the financial markets for policymakers and practitioners.

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