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## What do reaction functions tell us about central bank preferences?

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**What Do Reaction Functions Tell Us About  
Central Bank Preferences?**

by

Steffen Elstner and Tamer Tabaković

March 2008

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# What Do Reaction Functions Tell Us About Central Bank Preferences?

Steffen Elstner and Amer Tabaković

March 11, 2008

## **Abstract**

Since Taylor's 1993 paper researchers have devoted a lot of effort to estimation of monetary policy rules. Taylor showed that a simple central bank reaction function, with the interest rate as monetary policy instrument and inflation and output gap as explanatory variables, mimics the Fed funds rate pretty well during the period from 1987 to 1992. Often, the Taylor rule coefficients are interpreted as if they reflect central bank's preferences. However, this may be misleading. In this paper we show that Taylor rule coefficients are complicated terms consisting of preference parameters as well as parameters given by the structure of the economy. We illustrate our conclusion that Taylor rule coefficients cannot be interpreted as reflecting central bank preferences by estimating standard forward-looking Taylor rules for the Bundesbank, the Fed and UK and confront these with our results obtained by a multi-equation GMM approach in order to detect central bank preferences.

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# 1 Introduction

Since Taylor's 1993 paper researchers have devoted a lot of effort to estimation of monetary policy rules. Taylor showed that a simple central bank reaction function, with the interest rate as monetary policy instrument and inflation and output gap as explanatory variables, mimics the Fed funds rate pretty well during the period from 1987 to 1992. Various forms of the so called Taylor rule – backward-looking, contemporaneous, forward-looking – have been estimated for different time periods. Often, the coefficients are interpreted as if they reflect central bank's preferences. However, this may be misleading. According to many macro models, even a strict inflation targeter may have to respond strongly to changes in the output gap, so that the estimated coefficients do not tell us anything. To understand the behavior of central banks, one would ideally have to know the preferences concerning, for example, output versus inflation stabilization.

Svensson (1997) shows that the solution of an optimization problem, where central banks minimize a social loss function subject to some specified structure of the economy, can be written in terms of a Taylor rule. This derivation reveals that the coefficients are quite complicated combinations of the structural parameters of the economy and the preference parameters specified in the social loss function. Hence, the estimated coefficients of a Taylor rule cannot be used to identify an inflation targeting versus an output stabilizing central bank. Another implication of this derivation is that even if preferences remain the same over time, the estimated Taylor rule coefficients need not since the structure of the economy may change. We apply this framework to identify central bank preferences of the Bundesbank, the Fed and the Bank of England.

There are several other studies that are related to ours. Dennis (2001) and Castelnuovo and Surico (2003), for example, use full information estimation strategies to identify central bank preferences. Their work also differs from ours in that it does not impose a finite policy horizon. The methodology applied in our paper is closely related to Favero and Rovelli (2001). However, while their objective is to analyze the underlying causes of the improved inflation performance in the US, the purpose of our study is to point out the difference between Taylor rule coefficients and central bank preferences. While all of the above mentioned studies use US data, our work covers also data from Germany and UK.

The structure of the paper is as follows. In section 2 we present a simple model which we use to derive an optimal central bank reaction function. We show that the reaction function can be written in terms of a modified

Taylor rule whose coefficients are a combination of structural and preference parameters. In section 3 we estimate a forward-looking Taylor rule using the single equation GMM and indicate the often misleading interpretation of the estimated coefficients. In section 4 we show one way to detect the "true" preferences of central banks. We therefore employ the multi-equation GMM on the first order conditions of our structural model presented in section 2. Section 5 concludes.

## 2 Central Bank Preferences and the Taylor Rule

The formal analysis of central bank's interest rate decisions is based on the assumption of certain preferences of a central bank that enter a loss function. A popular specification of central bank preferences is to assume that the central bank's objective is to minimize the expected value of a loss function that depends on the output gap, inflation fluctuations, and on interest rate fluctuations. Thus, the central bank's objective function can be formalized as

$$\min_{i_t} E_t \left[ \sum_{s=0}^{\infty} \delta^s L_{t+s}(\pi_t, \tilde{y}_t, i_t - i_{t-1}) \right], \quad (1)$$

where  $E_t$  represents expectations formed in period  $t$ , and the intertemporal discount factor satisfies  $0 < \delta < 1$ . The period loss function is given by

$$L_t(\pi_t, \tilde{y}_t, i_t - i_{t-1}) = \frac{1}{2} [(\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2 + \mu (i_t - i_{t-1})^2], \quad (2)$$

where  $\pi_t$  is inflation,  $E_t(\pi_{t+1})$  is the expected inflation in period  $t+1$  based on information available in period  $t$ ,  $i_t$  is the short term interest rate (the monetary policy instrument) and  $\tilde{y}_t$  is the output gap defined as the difference between actual output and potential output. The objective of the central bank can be described by reducing inflation fluctuations around its target value  $\pi^*$ , minimizing output gap fluctuations, and smoothing short term interest rates. The central bank puts different weights on these objectives. In order to capture these differences we denote with  $\lambda$  and  $\mu$  the central bank preferences with regard to output gap and interest rate smoothing respectively. For the sake of simplicity we assume that the preference parameter with regard to the deviation of inflation from its target value is

normalized to one. The interest rate smoothing parameter takes account of the fact that central banks change their key interest rates only gradually, in relatively small steps.<sup>1</sup> This is supposed to match several features policy-makers face in reality that are not captured in our simple specification of the economy: parameter uncertainty (Brainard (1967)) and uncertainty about the transmission mechanism of monetary policy (Goodhart (1996)) lead to cautious changes of interest rates; in addition, large changes of interest rates could cause turbulences on financial markets (Goodfriend (1991)).

In order to minimize its loss function the central bank has to consider different constraints imposed by the economy. We capture these constraints by assuming a simple model of the economy which is given by the following aggregate demand and supply curve:

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - c(i_t - E_t(\pi_{t+1})) + \eta_t \quad (3)$$

$$\pi_t = E_t(\pi_{t+1}) + a\tilde{y}_{t-1} + \epsilon_t, \quad (4)$$

where  $\eta_t$  and  $\epsilon_t$  are white noise processes affecting the demand and supply curve in period  $t$  and the coefficients  $a$  and  $c$  are positive.

We also make the simplifying assumption that expectations are formed by the following equations:

$$E_t(\tilde{y}_{t+1}) = d\tilde{y}_{t-1}, \quad (5)$$

$$E_t(\pi_{t+1}) = \pi^* + b(\pi_{t-1} - \pi^*) \quad (6)$$

where  $\pi^*$  denotes the central bank's inflation target and the parameters  $b$  and  $d$  adopt values between 0 and 1. The aggregate supply equation (4), which has the form of an expectations-augmented Phillips curve, is an increasing function of expected inflation and the past period output gap. The aggregate demand equation (3), which is an expectations-augmented IS curve, is an increasing function of expected output gap and decreasing in the pseudo real interest rate.<sup>2</sup> These two equations reveal the transmission mechanism of monetary policy in our stylized model. Namely, the central bank sets its short term interest rate  $i_t$  that has an instantaneous impact on the output

<sup>1</sup>For a survey of the arguments see Lowe & Ellis (1997) and Sack & Wieland (2000).

<sup>2</sup>The pseudo real interest rate is defined by  $i_t - E_t(\pi_{t+1})$ . For more details, see Svensson (1997).

gap of the IS curve, whereas inflation reacts to the short term interest rate change with a lag of one period.

To sum up, the central bank's problem is to minimize (1)-(2) subject to (4) and (3). For illustrative purposes we examine the case of  $\mu = 0$ . In the appendix it is shown that the solution to this problem is a central bank reaction function that can be written as

$$i_t = E_t(\pi_{t+1}) + \beta_1(E_t\pi_{t+1} - \pi^*) + \beta_2\tilde{y}_{t-1}, \quad (7)$$

with

$$\beta_1 = \frac{\delta acb}{\lambda c^2 + \delta a^2 c^2}$$

$$\beta_2 = \frac{\lambda cd + \delta a^2 c(b + d)}{\lambda c^2 + \delta a^2 c^2},$$

which is very similar to the original Taylor rule (1993):

$$i_t = r^* + \pi_t + \beta_1(\pi_t - \pi^*) + \beta_2\tilde{y}_t. \quad (8)$$

The great advantage of Taylor rules is their striking simplicity – in our case the Taylor rule consists only of two variables – and their transparency – which has been a big issue in monetary policymaking – since they provide clear implications for the setting of the short term interest rate. However, these implications of the Taylor rule coefficients have often been misinterpreted as describing central bank preferences. For instance it is often argued that a high coefficient with regard to the output gap defines a high preference of the central bank for stabilizing output. However, it simply shows a long-run relation between the interest rate and output gap. Rather, the Taylor coefficients are, even in this simple case ( $\mu = 0$ ), a combination of structural parameters of the economy ( $a, b, c, d$ ) and the parameters representing central bank preferences ( $\pi^*, \lambda, \delta$ ) which is obvious from equation (7). Note that even if we further simplified the objective function by assuming  $\lambda = 0$ , the output gap coefficient would still be a complex term.



### 3 Taylor Rules for the Bundesbank, the Fed and the Bank of England – Estimation and Interpretation

In this section we estimate Taylor rules for the abovementioned central banks for different sample periods. The objective is to show how Taylor rules have often been misinterpreted and how their correct interpretation should be. In the first subsection, we describe the data and the generalized method of moments (GMM) procedure which was employed in order to estimate the Taylor rules for the abovementioned central banks. Afterwards the estimation results are presented and their most striking features are highlighted.

#### 3.1 The Estimation Procedure

In order to describe the interest rate decisions of the corresponding central banks we apply a forward-looking Taylor rule, according to which the short term interest rate depends on expected inflation, expected output gap and lagged values of the short term interest rate as explanatory variables. Hence, the reaction function can be written as:

$$i_t = \rho i_{t-1} + (1-\rho) [\bar{r} + \pi_t + (\beta_1 - 1)(E_t(\pi_{t+n}|\Omega_t) - \pi^*) + \beta_2 E_t(\tilde{y}_t|\Omega_t)], \quad (9)$$

where  $\bar{r}$  is the long run equilibrium real interest rate,  $\pi_{t+n}$  is the inflation rate between periods  $t$  and  $t+n$ . Moreover, we assume that the central bank forms expectations conditional on an information set  $\Omega_t$  with regard to the corresponding variables denoted by  $E$  at period  $t$ . Note that the forward-looking specification consists of two parts. The terms in brackets can be characterized as an expression describing the target interest rate of the central bank. The parameters  $\beta_1$  and  $\beta_2$  reflect the adjustment reaction of the central bank to the deviation of inflation from its target and to the output gap respectively. This expression looks very similar to the original Taylor rule, but contrary to the original formulation it explains the optimal interest setting behavior of central banks by means of forward looking variables. The second part of equation (9) stresses the fact that central banks seem to adjust their short term interest rates only gradually over time. Thus, we add a lagged value of the short term interest rate weighted by a parameter  $\rho$  which affects the interest rate setting behavior of the corresponding central

bank.<sup>3</sup>

A main problem which arises by estimating the coefficients of equation (9) is the point that we have to deal with expected values as explanatory variables. For this issue it is customary to employ the GMM approach which basically consists of an instrumental variables estimation of equation (9). Nevertheless, we need to get rid of the expected values of equation (9) in order to obtain an expression for which the GMM procedure can be applied. To achieve this, we define in a first step the coefficient  $\beta_0 \equiv \bar{r} - \beta_1 \pi^*$  and rewrite equation (9) as:

$$i_t = \rho i_{t-1} + (1 - \rho) [\beta_0 + \beta_1 E(\pi_{t+n} | \Omega_t) + \beta_2 E_t(\tilde{y}_t | \Omega_t)] + v_t. \quad (10)$$

Finally, we can eliminate the terms for expected inflation and expected output gap from the expression by rewriting the Taylor rule in dependence of the realized variables as follows:

$$i_t = \rho i_{t-1} + (1 - \rho) [\beta_0 + \beta_1 \pi_{t+n} + \beta_2 \tilde{y}_t] + \epsilon_t, \quad (11)$$

where the error term  $\epsilon_t \equiv -(1 - \rho)\beta_1[\pi_{t+n} - E_t(\pi_{t+n}) | \Omega] - (1 - \rho)\beta_2[\tilde{y}_t - E_t(\tilde{y}_t) | \Omega] + v_t$  is a linear combination of the forecast errors and the exogenous disturbance  $v_t$ . In order to employ the GMM procedure we need to define a vector of instrumental variables  $u(t)$  which is able to reflect the central bank's information set for the period in which the interest rate is determined. Moreover, the variables must be orthogonal to the error term  $\epsilon_t$ , so that the expectations of the central bank can be considered as rational. For this set of instrumental variables in  $u(t)$  we can set up a covariance moment condition,  $E[\epsilon_t u_t] = 0$ , which has to be fulfilled for all instrumental variables and can be written in more detail as:

$$E[(i_t - \rho i_{t-1} - (1 - \rho)\beta_0 - (1 - \rho)\beta_1 \pi_{t+n} - (1 - \rho)\beta_2 \tilde{y}_t) u_t] = 0. \quad (12)$$

In order to derive an estimate for the inflation target we use the follow-

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<sup>3</sup>We follow the approach suggested by Clarida, Gali and Gertler (1998) and others. For the Fed and the Bank of England we share the opinion of Clarida, Gali and Gertler that a Taylor rule including an additional second lag of the short term interest rate is better suited to explain the corresponding data.

ing relation between the inflation target and the equilibrium real interest rate defined by the parameters  $\beta_0$  and  $\beta_1$ :

$$\pi^* = \frac{\bar{rr} - \beta_0}{\beta_1 - 1}. \quad (13)$$

We obtain a value for the equilibrium real interest rate by calculating first the ex ante real interest rates with the Fisher equation<sup>4</sup>,  $rr_t = i_t - E_t(\pi_{t+12})$ , and then using the sample average of  $rr_t$  for providing an estimate of  $\bar{rr}$ . With the parameters of the forward looking Taylor rule  $\beta_0$  and  $\beta_1$  and  $\bar{rr}$  it is then possible to derive an estimate for the inflation target  $\pi^*$ . For the implementation of our estimation we follow basically the approach of Kamps and Pierdzioch (2002), by taking as instruments a constant, the first twelve lags of output gap, inflation rate, short term interest rate and the annualized growth rate of the monthly change of the HWWA commodity price index. It is standard to use for such an investigation the interbank lending rate for overnight loans. For each country we employ ex post monthly data of the consumer price index (CPI)<sup>5</sup> to measure inflation and an index of industrial production to measure output.<sup>6</sup> In order to receive a series for output gap, we detrend the log of industrial production employing the Hodrick-Prescott filter with the penalty parameter set to 14,400.<sup>7</sup>

Note that in our specification the number of instrumental variables or restrictions exceeds the number of coefficients to be estimated. For this case we need to set up a weighting matrix which is chosen in order to allow the GMM estimates to be robust against any kind of heteroskedasticity and serial correlation in the error term<sup>8</sup>. Nevertheless, we are faced with the problem

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<sup>4</sup>In order to obtain values for the expected inflation rates, we employ the fact that in the case of rational expectations, which is supposed for the GMM estimation of a forward-looking Taylor rule, no systematic forecasting error exists and hence  $E_t(\pi_{t+12}) = \pi_{t+12}$  holds.

<sup>5</sup>For Germany we use for the period until the end of 1991 the CPI for West Germany and afterwards the series for Germany including East Germany. The UK changed its measure in the year 2004. However, this change from the Retail Price Index (RPIX) to CPI does not affect our results significantly. See King (2004).

<sup>6</sup>The monthly time series for output, short term interest rate and CPI were obtained from the International Financial Statistics of the IMF.

<sup>7</sup>This is in line with e.g. Kamps and Pierdzioch (2002), but stands in contrast to Clarida, Gali und Gertler (1998) who use a quadratic trend. There is a large strand of literature stressing the observation that different detrending procedures lead to different estimation results. See, for instance, Orphanides and van Norden (2002), Clausen and Meier (2005).

<sup>8</sup>In our estimation we apply a twelve lag Bartlett window in order to resolve the

that maybe some instrumental variables are not necessary and distort our estimation results. To test this, we perform a standard  $J$ -test for the validity of the overidentifying restrictions<sup>9</sup>.

### 3.2 Interpretation of the Results

We present our estimation results of the abovementioned Taylor rule for the Bundesbank, the Fed and the Bank of England for three different sample periods. The starting point of each sample is chosen such that it describes the beginning of a new monetary policy conducting in the respective country.

Our first sample period contains data from March 1979 to December 1994 for the Bundesbank and from September 1979 to December 1994 for the Fed. This is the same period as analyzed by Clarida, Gali and Gertler (1998). According to these authors the fundamental shift in the way the Bundesbank and the Fed conducted monetary policy is given by the Bundesbank's affiliation with the EMS in March 1979 and Volcker's announcement to fight inflation in October 1979 respectively. For the Bank of England our sample contains data from October 1992 to September 2005. We argue that this is a good starting point because the Bank of England left the European Exchange Rate Mechanism (ERM) in September 1992 and adopted Inflation Targeting from October 1992 onwards.

The second sample for the Bundesbank and the Fed has the same starting points as the first sample but the ending points are chosen to correspond to December 1998 for the Bundesbank, which is the last month before the ECB started operating, and to September 2005 for the Fed, which is one year prior to our latest data. The second sample for the Bank of England spans from May 1997 to September 2005, where the starting point corresponds to the date when the Bank of England was given independence.

The third sample for the Bundesbank spans from January 1985 to December 1998. From 1985 on the Bundesbank announced an inflation objective of 2%, which was defined to be the inflation rate consistent with price stability. For the Fed our third sample runs from August 1987, the date which denotes the beginning of the Greenspan era, to September 2005.

Table 1 shows the estimation results for the three sample periods for the Bundesbank. All estimated coefficients are statistically significant at the one percent level and have reasonable values for all three sample periods. The fact that the Bundesbank responds to output gap by changing interest rates does not necessarily mean that output stabilization is a goal in itself.

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autocorrelation problem.

<sup>9</sup>See Hansen (1982).

Rather, the large coefficient on output gap may indicate that output gap is regarded as a stronger leading indicator for future inflation by the Bundesbank. Consequently, it tries to close the gap in order to stabilize future inflation. Moreover, in the sample from March 1979 to December 1998 the parameter values on expected inflation and output gap are pretty close to the originally postulated values (1.5 and 0.5) by Taylor (1993). The  $J$ -statistic, which tests the overidentifying restrictions (more instruments than parameters), is insignificant and we cannot reject the overidentifying restrictions.

Next, we consider the Fed. The results are presented in Table 2. Again in all three sample periods all estimated parameters are significant at the one percent level except for the constant in the last sample. The most interesting result is that the coefficient on output gap "jumps" from below one in the first two sample periods to above two for the sample covering the Greenspan era. The validity of our instruments is again confirmed by an insignificant  $J$ -statistic.

Finally, we turn to the Bank of England (Table 3). Interestingly, our results vary extremely for the two sample periods. Probably the most striking results are obtained for the sample period from October 1992 to September 2005. All coefficients are highly significant. However, the estimated parameter value on expected inflation is -1.28 (significant at the ten percent level) whereas the one on output gap is 0.54. For our second sample from May 1997 to September 2005 the estimated parameter on expected inflation turns positive but remains below one (significant at the ten percent level) while the output parameter increases to 3.65 remaining significant at the one percent level.

The main point that we want to stress here is that the coefficient estimates of the Taylor rule have often been misinterpreted. Several researchers have inferred central bank's preferences from Taylor rule coefficients: for instance, Clarida, Gali and Gertler (1998, p. 16) estimate coefficients on inflation and output gap to be 2.04 and 0.08 for the Bank of Japan and 1.31 and 0.25 for the Bundesbank which makes them conclude that "... the Bank of Japan appears to have placed somewhat more weight on controlling inflation relative to output stabilization than the Bundesbank". As another example, consider our coefficient estimates on output gap for the Bundesbank and the Fed in the sample periods from January 1985 to December 1998 and from August 1987 to September 2005 respectively, which are given by 0.72 and 2.24. Many would conclude that the higher coefficient for the Fed implies that the Fed places more weight on output stabilization than the Bundesbank. However, this conclusion is wrong. The fact that the Fed responds to output gap by changing interest rates does not necessarily mean

that output stabilization is a goal in itself. Rather, the large coefficient on output gap may indicate that output gap is regarded as a stronger leading indicator for future inflation by the Fed. Consequently, it tries to close the gap in order to stabilize future inflation. In the following section it remains to be shown how one can determine central bank's "true" preferences.

## 4 An Approach to Detect Central Bank Preferences

In this section we derive the central bank preferences for the Bundesbank, the Fed and the Bank of England on the basis of the model discussed in section 2 with multi-equation GMM. In the first subsection the applied estimation procedure and the data are shortly described. After this the estimation results are presented and analyzed.

### 4.1 The Estimation Procedure

To identify the policy preference parameters  $\pi^*$ ,  $\mu$  and  $\lambda$  we perform a multi-equation GMM estimation as it was done by Favero and Rovelli (2001). To show that it is inaccurate to infer central bank's preferences from Taylor rules it is sufficient to consider our simple model discussed in section 2. For the sake of clarity we state the three key equations for our analysis once again:

$$\pi_t = E_t(\pi_{t+1}) + ay_{t-1} + \epsilon_t \quad (14)$$

$$y_t = E_t(y_{t+1}) - c(i_t - E_t(\pi_{t+1})) + \eta_t, \quad (15)$$

$$\begin{aligned} 0 = & -c\lambda [d\tilde{y}_{t-1} - c(i_t - (\pi^* + b(\pi_{t-1} - \pi^*))) \\ & + \mu(i_t - i_{t-1}) - \delta\mu(i_{t+1} - i_t) \\ & - ac\delta [b(\pi_{t-1} - \pi^*) + ad\tilde{y}_{t-1} - c(i_t - (\pi^* + b(\pi_{t-1} - \pi^*)))], \quad (16) \end{aligned}$$

where equation (16) is the Euler equation derived from the central bank's intertemporal optimization problem subject to the specification of aggregate demand and aggregate supply.

As abovementioned we use the method of multi-equation GMM in order to derive the policy preference parameters. An alternative to the joint estimation of all three equations is to apply the single-equation GMM separately to each equation. However, it is shown, for instance by Hayashi (2000),

that the estimation results differ in general<sup>10</sup> and the application of joint estimation is asymptotically more efficient. Nevertheless, neither the single-equation GMM nor the multi-equation GMM deliver consistent estimation results if the model is misspecified. This fact should be kept in mind since our model is based on quite simplifying assumptions.<sup>11</sup> The difference between the multiple-equation GMM and the single-equation GMM approach is the weighting matrix which in the latter case does not account for the possibility of correlation between the single equations. All other things of single-equation GMM estimation like the implementation of the covariance moment condition between the forecast error and the instrumental variables,  $E[\epsilon_t u_t] = 0$ , or the interpretation of the  $J$ -statistic can be done in the same way for multi-equation GMM.

In our analysis we use quarterly data for the estimation of central bank preferences. This is done because the simple structure of our model cannot be maintained when using monthly data. For the latter, more lags of output gap and inflation must be included into the model. Thus, we change the data set for the estimation of our model. The vector of instrumental variables  $u(t)$  contains a constant, the first four lags of output gap, inflation rate, quarterly average of the short term interest rate and the annualized quarterly growth rate of the HWWA commodity price index. For each country we use real time data for output.<sup>12</sup>

To receive a series for the output gap, we detrend log output by using the Hodrick-Prescott filter with the penalty parameter set to 1,600. Note that in the case of real time data an end-of-sample problem arises by applying the Hodrick-Prescott filter. We try to ease this problem by augmenting the corresponding time series with forecasts over the next 12 quarters based on an  $AR(p)$ -model. The lag length  $p$  of this time series model is respectively chosen by applying the Akaike information criterion.

To receive plausible values for the preference parameters, we follow Favero and Rovelli (2001) and set the discount factor  $\delta$  equal to 0.975.

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<sup>10</sup>In our case where the number of instrumental variables exceeds the number of coefficients to be estimated the estimation results of both procedures do not coincide. See Hayashi (2000), chapter 4.

<sup>11</sup>The reason for not augmenting our model with additional lags for the output gap and inflation is the fact that they do not provide significant values in ordinary least squares estimations of the IS and Philipps curve on the basis of quarterly data.

<sup>12</sup>The real time series for output of Germany was obtained from the Kiel Institute for the World Economy, for output of the U.S. from the Archival Federal Reserve Economic Data (ALFRED) of the Fed St. Louis and for output of the UK from the Bank of England.

## 4.2 Policy Preferences of the corresponding central banks

We estimate our model for the Bundesbank and the Fed for three different sample periods. The ranges of these sample periods are the same as in section 3.2. Only for the UK the second sample period has been augmented until the third quarter 2006. When interpreting the results of our estimations one has to be aware of the fact that with quarterly data there are not so many data points available as in the case of monthly data. Thus, a period of ten years with forty observations may not be sufficient to receive accurate estimates even in the case of asymptotically efficient estimators.

The results for the Bundesbank, the Fed and the Bank of England are listed in the Tables 4 to 6. It is striking that all estimations deliver plausible values for the inflation target of the corresponding central bank. For instance, we estimate a value for the Bundesbank inflation target of about 1.85 for the sample from the first quarter 1985 to the fourth quarter 1998. Clausen and Meier (2005) derive a value of 2 percent for the implicit inflation target which is in line with our estimate.

Moreover, our results show that the inflation target fell from 3.713 in the first sample period to 1.745 in the third sample period which is also supported by the paper of Clausen and Meier. For the inflation target of the Fed we obtain values of around 3 percent for all sample periods which corresponds to the estimate of Favero and Rovelli (2001). The inflation targets for the Bank of England seem also to be realistic at least for the first sample.

An important finding of our analysis is the fact that the preference parameters on interest rate smoothing,  $\mu$ , and on output stabilization,  $\lambda$ , of the Bundesbank and the Fed are quite small but clearly significant for nearly all samples. Recall that we set the preference parameter with respect to inflation in the intertemporal loss function equal to one. This would imply that the Bundesbank possesses at least for the last sample from the first quarter 1985 to the last quarter 1998 a hundred times larger preference in keeping inflation near to its target value than to stabilize output, since we estimate a value for  $\lambda$  of 0.01. By only considering the Taylor rule coefficients estimated in section 3.2 one may infer another preference structure. The results for the Bank of England suggest that its interest setting behavior is hardly to describe by only considering output gap and inflation gap.

To sum up, the results for the Bundesbank and the Fed underpin our statement that it is not possible to infer central bank's preferences from Taylor rule coefficients. The significant values of  $\mu$  and  $\lambda$  confirm the conjecture that these central banks possess preferences for interest rate smoothing and output stabilization. However, these preferences are much smaller than the



Taylor coefficients with respect to the lagged interest rate, expected inflation and output gap would in general suggest.

## 5 Conclusion

In this paper we derived theoretically a central bank reaction function from an intertemporal optimization problem, where the central bank minimizes a social loss function subject to an aggregate demand and supply curve given by the economy. We showed that this reaction function can be written in form of a Taylor rule. This derivation allowed us to subdivide the Taylor coefficients into structural and preference parameters, which stresses the main objective of this work: Taylor rule coefficients per se cannot be interpreted as central bank preferences as it has often been the case. We illustrate our conclusion by estimating a standard forward-looking Taylor rule for the Bundesbank, the Fed and the Bank of England and confront these with our results obtained by a multi-equation GMM approach. With these estimation results we are able to provide correct interpretations of Taylor rule coefficients and to detect central bank preferences. For instance the fact that some central banks respond strongly to changes in the output gap does not necessarily mean that output stabilization is a goal in itself. Rather, large Taylor rule coefficients with regard to the output gap may indicate that output gap is regarded as a leading indicator for future inflation by these central banks. Consequently, they try to close the output gap in order to stabilize future inflation.

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Table 1: Estimates of Taylor Rules for the Bundesbank

Sample		1979:3-1993:12	1979:3-1998:12	1985:1-1998:12
Explanatory variable	Parameter	Coefficient	Coefficient	Coefficient
Lagged interest rate	$\rho$	0.91*** (0.006)	0.93*** (0.007)	0.96*** (0.005)
Constant	$\beta_0$	2.90*** (0.241)	1.85*** (0.287)	1.79*** (0.247)
Inflation rate	$\beta_1$	1.34*** (0.068)	1.58*** (0.074)	1.61*** (0.068)
Output gap	$\beta_2$	0.46*** (0.078)	0.58*** (0.102)	0.72*** (0.094)
Real interest rate	$\bar{r}$	3.85	3.55	3.47
Inflation target	$\pi^*$	2.80	2.91	2.76
Statistics				
J-statistic		13.866 (p>0.99, df=45)	15.433 (p>0.99, df=45)	13.000 (p>0.99, df=45)
$R^2$		0.978	0.984	0.990

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 12 of the interest rate, inflation, output gap, annualized monthly growth rate of the HWWA commodity price index and a constant.

Table 2: Estimates of Taylor Rules for the Fed

Sample		1979:10-1994:12	1979:10-2005:9	1987:8-2005:9
Explanatory variable	Parameter	Coefficient	Coefficient	Coefficient
Lagged interest rate 1	$\rho_1$	1.26*** (0.012)	1.29*** (0.014)	1.38*** (0.026)
Lagged interest rate 2	$\rho_2$	-0.33*** (0.011)	-0.34*** (0.014)	-0.40*** (0.024)
Constant	$\beta_0$	1.74*** (0.616)	-0.32 (0.705)	0.18 (1.011)
Inflation rate	$\beta_1$	1.45*** (0.114)	1.82*** (0.171)	1.38*** (0.345)
Output gap	$\beta_2$	0.62*** (0.087)	0.81*** (0.136)	2.25*** (0.301)
Real interest rate	$\bar{r}$	4.02	2.96	1.77
Inflation target	$\pi^*$	5.12	4.01	4.19
Statistics				
$J$ -statistic		14.406 (p>0.99, df=44)	19.559 (p>0.99, df=44)	17.185 (p>0.99, df=44)
$R^2$		0.959	0.975	0.994

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 12 of the interest rate, inflation, output gap, annualized monthly growth rate of the HWWA commodity price index and a constant.

Table 3: Estimates of Taylor Rules for the Bank of England

Sample		1992:10-2005:9	1997:5-2005:9
Explanatory variable	Parameter	Coefficient	Coefficient
Lagged interest rate 1	$\rho_1$	0.42*** (0.024)	0.36*** (0.016)
Lagged interest rate 2	$\rho_2$	0.40*** (0.025)	0.59*** (0.015)
Constant	$\beta_0$	8.52*** (0.627)	2.80** (1.140)
Inflation rate	$\beta_1$	-1.28* (0.214)	0.77* (0.445)
Output gap	$\beta_2$	0.54*** (0.132)	3.65*** (0.796)
Real interest rate	$\bar{r}$	2.75	2.63
Inflation target	$\pi^*$	2.53	0.80
Statistics			
$J$ -statistic		11.948 (p>0.99, df=44)	7.685 (p>0.99, df=44)
$R^2$		0.755	0.836

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 12 of the interest rate, inflation, output gap, annualized monthly growth rate of the HWWA commodity price index and a constant.

Table 4: Structural and preference parameters of the Bundesbank

Sample	1979:1-1993:4	1979:1-1998:4	1985:1-1998:4
Parameter	Coefficient	Coefficient	Coefficient
a	0.026*** (0.003)	0.029*** (0.003)	0.027*** (0.002)
b	0.952*** (0.004)	0.958*** (0.009)	0.948*** (0.005)
c	0.055*** (0.004)	0.052*** (0.006)	0.043*** (0.002)
d	0.841*** (0.010)	0.801*** (0.012)	0.797*** (0.006)
$\pi^*$	3.713*** (0.052)	2.446*** (0.040)	1.745*** (0.014)
$\lambda$	0.002*** (0.000)	0.010*** (0.001)	0.010*** (0.001)
$\mu$	0.0007*** (0.000)	0.002*** (0.000)	0.001*** (0.000)
Statistics			
J-statistic	15.986 (p=0.100, df=10)	20.136 (p=0.028, df=10)	15.278 (p=0.122, df=10)

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 4 of the interest rate, inflation, output gap, annualized quarterly growth rate of the HWWA commodity price index and a constant.

Table 5: Structural and preference parameters of the Fed

Sample	1979:3-1994:4	1979:3-2005:3	1987:3-2005:3
Parameter	Coefficient	Coefficient	Coefficient
a	0.042*** (0.009)	0.071*** (0.013)	0.114*** (0.010)
b	0.972*** (0.007)	0.970*** (0.009)	0.925*** (0.011)
c	0.008*** (0.002)	0.009*** (0.002)	0.022*** (0.003)
d	0.898*** (0.005)	0.886*** (0.013)	0.869*** (0.016)
$\pi^*$	2.751*** (0.022)	3.321*** (0.054)	3.056*** (0.030)
$\lambda$	0.013*** (0.002)	0.009*** (0.002)	-0.004 (0.004)
$\mu$	0.000 (0.000)	0.000 (0.000)	0.002*** (0.000)
Statistics			
J-statistic	16.366 (p=0.090, df=10)	21.069 (p=0.021, df=10)	18.844 (p=0.042, df=10)

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 4 of the interest rate, inflation, output gap, annualized quarterly growth rate of the HWWA commodity price index and a constant.



Table 6: Structural and preference parameters of the Bank of England

Sample	1992Q3-2005Q3	1997Q2-2006Q3
Parameter	Coefficient	Coefficient
a	0.262*** (0.005)	0.311*** (0.013)
b	0.808*** (0.004)	0.767*** (0.011)
c	-0.026*** (0.001)	-0.008*** (0.001)
d	0.798*** (0.006)	0.755*** (0.008)
$\pi^*$	2.038*** (0.014)	2.506*** (0.024)
$\lambda$	-0.186*** (0.007)	-0.298*** (0.019)
$\mu$	-0.001*** (0.000)	0.001*** (0.000)
Statistics		
J-statistic	14.591 (p=0.148, df=10)	10.665 (p=0.384, df=10)

Notes: SEs are given in parentheses below the estimated values (\*, \*\*, \*\*\* indicating significance at the 10, 5, 1 percent level),  $p$ -values are given in parenthesis below the  $J$ -statistic (df=degrees of freedom). The  $J$ -statistic of the Hansen test for overidentifying restrictions is  $\chi^2$ -distributed. The GMM instrument set includes lags 1 to 4 of the interest rate, inflation, output gap, annualized quarterly growth rate of the HWWA commodity price index and a constant.

## 6 Appendix

Assume that the macroeconomic environment is described by the following two equations

$$\pi_t = E_t(\pi_{t+1}) + a\tilde{y}_{t-1} + \epsilon_t, \quad (17)$$

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - c(i_t - E_t(\pi_{t+1})) + \eta_t, \quad (18)$$

with

$$E_t(\pi_{t+1}) = \pi^* + b(\pi_{t-1} - \pi^*)$$

$$E_t(\tilde{y}_{t+1}) = d\tilde{y}_{t-1}.$$

The parameters  $a, c$  are constrained to be positive whereas  $0 < b < 1$  and  $0 < d < 1$  holds,  $\tilde{y}_t$  represents the output gap, i.e. the deviation of actual output from its potential level,  $i_t$  is the central bank's policy instrument and represents short term interest rate,  $\pi_t$  is inflation and  $\pi^*$  denotes the inflation target. The error terms  $\epsilon_t$  and  $\eta_t$  are supposed to be white noise and affect the supply and demand side of the economy. Substituting expectations by the abovementioned expressions we can rearrange the aggregate supply and demand curve to get

$$\pi_t = (1 - b)\pi^* + b\pi_{t-1} + a\tilde{y}_{t-1} + \epsilon_t \quad (19)$$

$$\tilde{y}_t = d\tilde{y}_{t-1} - c(i_t - (\pi^* - b(\pi_{t-1} - \pi^*))) + \eta_t \quad (20)$$

The central bank's preferences can be described by the following objective function:

$$\min_{i_t} E_t \left[ \sum_{s=0}^{\infty} \beta^s L_{t+s}(\pi_{t+s}, y_{t+s}, i_{t+s} - i_{t+s-1}) \right], \quad (21)$$

where the period loss function  $L_t$  is given by

$$L_t(\pi_t, y_t, i_t - i_{t-1}) = \frac{1}{2} [(\pi_t - \pi^*)^2 + \lambda y_t^2 + \mu(i_t - i_{t-1})^2].$$

Iterating the aggregate supply curve one period ahead yields

$$\pi_{t+1} = E_{t+1}(\pi_{t+2}) + a\tilde{y}_t + \epsilon_{t+1}.$$

Substituting for expectations and aggregate demand leaves us with

$$\begin{aligned} \pi_{t+1} = & (1 + b + ac)(1 - b)\pi^* + b(b + ac)\pi_{t-1} + a(b + d)\tilde{y}_{t-1} \\ & - aci_t + a\eta_t + b\epsilon_t + \epsilon_{t+1}. \end{aligned} \quad (22)$$

Imposing  $\mu = 0$ , the intertemporal optimization problem of the central bank looks as follows:

$$\min_{i_t} L = \frac{1}{2} [(\pi_t - \pi^*)^2 + \lambda\tilde{y}_t^2] + \frac{1}{2} [(E_t(\pi_{t+1}) - \pi^*)^2 + \lambda\tilde{y}_{t+1}^2] \quad (23)$$

subject to

$$\begin{aligned} \pi_{t+1} = & (1 + b + ac)(1 - b)\pi^* + b(b + ac)\pi_{t-1} + a(b + d)\tilde{y}_{t-1} \\ & - aci_t + a\eta_t + b\epsilon_t + \epsilon_{t+1} \end{aligned} \quad (24)$$

$$\tilde{y}_t = d\tilde{y}_{t-1} - c(i_t - (\pi^* + b(\pi_{t-1} - \pi^*))) + \eta_t. \quad (25)$$

The first derivative with respect to  $i_t$  yields

$$\begin{aligned} 0 = & \lambda\tilde{y}_t \frac{\delta\tilde{y}_t}{\delta i_t} + (\pi_t - \pi^*) \frac{\delta(\pi_t - \pi^*)}{\delta i_t} \\ & + \delta\lambda\tilde{y}_{t+1} \frac{\delta\tilde{y}_{t+1}}{\delta i_t} \\ & + ((E_t(\pi_{t+1}) - \pi^*)) \frac{\delta(E_t(\pi_{t+1}) - \pi^*)}{\delta i_t}. \end{aligned} \quad (26)$$

We know that

$$\begin{aligned}\frac{\delta \tilde{y}_t}{\delta i_t} &= -c \\ \frac{\delta \tilde{y}_{t+1}}{\delta i_t} &= 0 \\ \frac{\delta(E_t(\pi_t) - \pi^*)}{\delta i_t} &= 0 \\ \frac{\delta(E_t(\pi_{t+1}) - \pi^*)}{\delta i_t} &= -ac\end{aligned}$$

Substituting these inner derivatives into equation (26) and rearranging terms we end up with

$$\begin{aligned}0 &= -(\lambda cd + \delta a^2 c(b + d))\tilde{y}_{t-1} + (\lambda c^2 + \delta a^2 c^2)i_t \\ &\quad -(\lambda c^2 + \delta a^2 c^2)E_t(\pi_{t+1}) - \delta acb(E_t(\pi_{t+1}) - \pi^*)\end{aligned}\quad (27)$$

which can be written as a Taylor Rule:

$$i_t = E_t(\pi_{t+1}) + \frac{\delta acb}{\lambda c^2 + \delta a^2 c^2} ((E_t \pi_{t+1}) - \pi^*) + \frac{\lambda cd + \delta a^2 c(b + d)}{\lambda c^2 + \delta a^2 c^2} \tilde{y}_{t-1}. \quad (28)$$

We are now able to compare our optimal monetary policy reaction function in equation (28) with the original Taylor rule stated below:

$$i_t = rr + \pi_t + \theta_\pi(\pi_t - \pi^*) + \theta_y \tilde{y}_t \quad (29)$$

One can see that the coefficients  $\theta_\pi$  and  $\theta_y$  are dependent on the structural parameters of the economy.