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# Böhm-Bawerk and Hicks modernized

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*The paper offers a modernized Böhm-Bawerkian approach to capital theory. The Wicksell effect turns out to be a measure for the degree of vertical distribution of labor. I show that a marginal rise in the rate of interest reduces the (modernized) period of production. A ‘generalized golden rule of accumulation’ is one result of our approach. Based on these results I define a coefficient of intertemporal substitution (CIS). As opposed to the traditional elasticity of substitution between labor and capital, the CIS is also well defined for negative real rates of interest. This is important in the twenty-first century, since we observe a strong overhang of private savings over private investments (secular stagnation).*

**Keywords:** *Austrian capital theory, Wicksell effect, generalized golden rule of accumulation, coefficient of intertemporal substitution, negative real rate of interest*

**JEL codes:** *E14, E21, E22*

## 1 INTRODUCTION

The Cambridge–Cambridge controversy took little account of the Austrian tradition in capital theory. My own contribution in Weizsäcker (1971) was the exception; but it was a latecomer in this discussion. Böhm-Bawerk (1889), the founder of Austrian capital theory, was discarded by neoclassical economics after Solow’s very successful introduction of the macroeconomic production function  $y = f(k)$  with  $y$  = net output per worker and  $k$  = average capital intensity of production. It was such a convenient device for empirical research that it easily survived the critique of Cambridge UK.

Moreover, Böhm-Bawerk himself was discarded by the neo-Austrians. They could not cope with a man who, after all, had served for many years as Minister of Finance of the Habsburg Empire. Following the lead of Ludwig von Mises, the neo-Austrians fled from any kind of mathematical model building. Rather, they built on concepts like ‘Urzins’ (‘originary rate of interest’ as translated by Mises himself), which is always positive, irrespective of any empirical facts. See Mises (1940).

In his book, *Value and Capital*, Hicks (1939) modernized Böhm-Bawerk’s concept of the ‘period of production.’ He showed that the ‘period of production’ could become a useful concept by measuring it in terms of present values. Thus, the period of production  $T$  has to be seen as a function of the real rate of interest  $r$  and the ‘technique’ in use  $\theta$ . Hence  $T = T(r; \theta)$ .

The concept of a ‘technique’ was at the center of the Cambridge–Cambridge controversy. We should note at the outset that it only makes sense in the context of a steady-state analysis. In a dynamic setting outside of the steady state, the ‘techniques’ in use change all the time. Here, at any given moment in time, an opaque mixture of techniques is employed.

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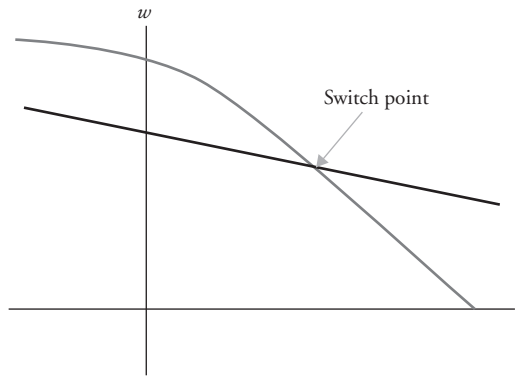


Figure 1 A wage–interest diagram as used in the Cambridge–Cambridge controversy

Cambridge UK insisted (rightly) on the fact that even for a given technique  $\theta$ , and hence – in a steady state – for a given stock of capital goods, the value of the capital stock depends on the real rate of interest. This is the ‘Wicksell effect.’

## 2 SWITCH POINT

In Figure 1 we see a wage–interest diagram of the sort that has been used in the Cambridge–Cambridge controversy. It illustrates an economy where two techniques are available, the gray one and the black one. For any given real rate of interest (= rate of profit),  $r$ , the market mechanism implements that technique, which maximizes the wage rate  $w$ . For low interest rates, the gray technique prevails. For high interest rates, the black technique prevails. We look at the ‘switch point’ where the two curves cross each other. As the rate of interest rises above the switch point, the economy switches from gray to black. As the rate of interest falls below the switch point, the economy switches from black to gray.

At the switch point, the slope of the gray curve is steeper than the slope of the black curve. What is the economic meaning of the slope of the wage–interest curve? Here it is useful to understand the dimensionality of that slope. For a given technique (gray or black), we obtain the slope by differentiating  $w$  with respect to  $r$ . We introduce a variable  $T$  by defining it in the following way:

$$T = -\frac{1}{w} \frac{dw}{dr}.$$

The dimensionality of  $T$  is ‘time’: obviously the expression  $\frac{dw}{w}$  (that is, the logarithmic derivative of  $w$ ) is dimension-free, because the numerator and the denominator have the same dimension. The dimension of the interest rate  $r$  is ‘one over time,’ for example: ‘percent per year.’ Thus, the dimension of the expression  $\frac{1}{dr}$  is ‘time.’ So the right-hand side has the dimension ‘time,’ say, ‘years.’ The same must be true for the left-hand side.

Therefore, we can measure  $T$  in terms of, say, years. Moreover, we know that  $T$  is a positive stretch of time, because  $\frac{dw}{dr}$  is negative and  $w$  is positive.

We can show the economic meaning of  $T$ . It is the period of production invented by Böhm-Bawerk, but modernized by Hicks. The way we can intuitively understand this

result is the following. We imagine that the steady-state economy consists of a set of overlapping hypothetical firms, each of which is totally vertically integrated. Thus, the hypothetical firm only buys labor from outside and only sells consumption goods to the outside. For this hypothetical firm we then have a time series of labor inputs and a time series of consumption good outputs. We can compute the ‘time point of gravity’  $T_L$  of the time distribution of labor inputs and the ‘time point of gravity’ of the time distribution of consumption good outputs,  $T_C$ . If this hypothetical fully vertically integrated firm is representative of the economy at large we can define the period of production as the difference between these two time points of gravity, so that we get

$$T = T_C - T_L.$$

Indeed, by differentiating the zero-extra profit condition with respect to the rate of interest  $r$ , we obtain the equation from above: the logarithmic derivative of the wage rate with respect to the rate of interest equals the negative of the period of production defined in terms of the two ‘points of gravity.’ Here is the proof:

In equilibrium a zero extra profit condition prevails and thus we get the following equation for the hypothetical firm:

$$w \int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt = \int_{-\infty}^{+\infty} e^{-rt} \gamma(t) dt,$$

where  $\lambda(t)$  is the flow of labor inputs and  $\gamma(t)$  is the flow of consumption good outputs. Now we can differentiate this equation with respect to  $r$ , keeping the input flow and the output flow the same. We then get

$$\frac{dw}{dr} \int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt - w \int_{-\infty}^{+\infty} t e^{-rt} \lambda(t) dt = - \int_{-\infty}^{+\infty} t e^{-rt} \gamma(t) dt.$$

Rearranging provides

$$\frac{dw/dr}{w} = T_L - T_C = -T$$

with  $T_L = \frac{\int_{-\infty}^{+\infty} t e^{-rt} \lambda(t) dt}{\int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt}$  the time point of gravity of labor inputs and

$T_C = \frac{\int_{-\infty}^{+\infty} t e^{-rt} \gamma(t) dt}{w \int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt} = \frac{\int_{-\infty}^{+\infty} t e^{-rt} \gamma(t) dt}{\int_{-\infty}^{+\infty} e^{-rt} \gamma(t) dt}$  the time point of gravity of consumption good outputs

and  $T$  defined to be the difference between these two points of gravity.

Note that our approach can easily cope with fixed capital. Fixed capital implies some intertemporal complementarity of consumption good outputs. Therefore, we have a flow  $\gamma(t)$  of consumption good outputs. Nevertheless, we can construct the hypothetical firm in such a way that the two integrals  $\int_{-\infty}^{+\infty} e^{-rt} \gamma(t) dt$  and  $\int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt$  are finite. Total consumption at any given moment in time, then, is the sum of consumer good

production of the different overlapping hypothetical firms at that moment. Similarly for the labor inputs.

We return to the switch point of the two techniques. Now we know: at the switch point, as the rate of interest rises, the economy switches from the technique with the higher period of production to the technique with the lower period of production. This is very much in the spirit of Böhm-Bawerk’s theory of capital: a higher rate of interest corresponds to a lower roundaboutness of production.

### 3 THE WICKSELL EFFECT AND THE VERTICAL DIVISION OF LABOR

The period of production of a given technique depends on the rate of interest. The curvature of the wage–interest curve of technique 1 may be different from the curvature of the wage–interest curve of technique 2. Thus, we have the theoretical possibility that the two curves cross each other more than once. This is the reswitching phenomenon. It would mean that the period of production of technique 1 is larger than the period of production of technique 2 at switch point  $A$  with the rate of interest  $r_A$ , and that the period of production of technique 1 is smaller than the period of production of technique 2 at switch point  $B$  with the rate of interest  $r_B$ .

What is the economic meaning of the curvature of the wage–interest curve? To obtain an answer we look at the second derivative of the logarithm of the wage rate as a function of the rate of interest. Let  $\ln(w)$  be the natural logarithm of the wage rate. For the first derivative we know from Section 2 above that

$$\frac{d\ln(w)}{dr} = -T = T_L - T_C.$$

Here, as before,  $T_L$  and  $T_C$  are the ‘temporal points of gravity’ of the labor inputs and consumption good outputs of the hypothetical fully vertically integrated firm. The second derivative then is this:

$$\frac{d^2\ln(w)}{dr^2} = \frac{dT_L}{dr} - \frac{dT_C}{dr}.$$

We have

$$\begin{aligned} \frac{dT_L}{dr} &= \frac{d \left\{ \frac{\int_{-\infty}^{+\infty} t e^{-rt} \lambda(t) dt}{\int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt} \right\}}{dr} = \\ &= \frac{- \left( \int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt \right) \left( \int_{-\infty}^{+\infty} t^2 e^{-rt} \lambda(t) dt \right) + \left( \int_{-\infty}^{+\infty} t e^{-rt} \lambda(t) dt \right)^2}{\left( \int_{-\infty}^{+\infty} e^{-rt} \lambda(t) dt \right)^2} \\ &= -\text{Variance}\{\lambda(t), r\}. \end{aligned}$$

By  $\text{Variance}\{\lambda(t), r\}$  we mean the variance of the of the intertemporal distribution of the present values of the labor inputs  $\lambda(t)$ .

Similarly, we obtain

$$\frac{dT_C}{dr} = -\text{Variance}\{\gamma(t), r\}.$$

This implies for the second derivative of  $\ln(w)$  the following equation:

$$\frac{d^2\ln(w)}{dr^2} = \text{Variance}\{\gamma(t), r\} - \text{Variance}\{\lambda(t), r\} = -\frac{dT}{dr}.$$

An economic interpretation of this equation is available: the division of labor, in this case, that is, the vertical division of labor. For the hypothetical fully vertically integrated firm and any given technique we expect that early on it only produces means of production, that is, intermediate products or capital goods. Only later will it combine these produced inputs (as fixed or circulating capital) together with further labor inputs to make consumption goods.

Thus, for example, assume a technique like a Solow production process  $f(k)$  with a fixed capital intensity  $k$ . Assume that the hypothetical firm produces one unit of the consumption good from time  $t = 0$  onwards until plus infinity. So  $f(k) = 1$  for  $t \geq 0$ . This implies  $\lambda(t) = 1/f(k)$  and  $\gamma(t) = 1$  for  $t \geq 0$ . Moreover, for  $t < 0$  we have  $\gamma(t) = 0$  and  $\lambda(t) = \frac{e^{\frac{f(k)t}{k}}}{f(k)}$ . For  $t < 0$  labor inputs grow exponentially through time by a growth rate equal to the capital productivity  $\frac{f(k)}{k}$ . In this example and for any  $r > 0$  it is obvious that  $\text{Variance}\{\lambda(t), r\} > \text{Variance}\{\gamma(t), r\}$ .

This inequality is a general property of any production system, which operates under the rules of the division of labor.

We then conclude for a given technique: the logarithm of the wage rate is a declining and concave function of the rate of interest. For a given technique, the period of production is a rising function of the rate of interest.

Reswitching remains a theoretical possibility.

Our approach allows us to interpret the meaning of the Wicksell effect. We know that the Wicksell effect is absent if we use the Solow production function. The 'neoclassical' Wicksell effect means that the value of the given physical capital stock declines as the rate of interest rises. The 'anti-neoclassical' Wicksell effect means that the value of the given physical capital stock rises as the rate of interest rises.

In a stationary economy the following equation holds:

$$c = w + rv.$$

Here  $c$  stands for consumption per worker,  $w$  is the wage rate, and  $v$  is the value of the capital stock per worker. As we look at a single technique the number of consumption baskets per worker,  $c$ , is a constant. Thus, by differentiation we obtain:

$$0 = \frac{dw}{dr} + v + r \frac{dv}{dr} = -Tw + v + r \frac{dv}{dr}.$$

The term  $\frac{dv}{dr}$  is the Wicksell effect, because the physical composition of the capital stock remains unchanged. For  $r = 0$  we obtain the equation  $v = Tw$ .

This is the Böhm-Bawerk equation, as Böhm-Bawerk explained the capital requirements per worker as the wage rate times the period of production. For  $r > 0$  the Böhm-Bawerk equation need not hold; this is due to the Wicksell effect.

For  $r > 0$  we can write:

$$v = \frac{c - w}{r}.$$

We now take the special case that  $w$  is a linear function of  $r$ . Then  $\frac{dw}{dr} = -wT$  is a constant. We can write  $w = c - rwT$ . From this follows  $c - w = rwT$  and therefore

$$v = \frac{c - w}{r} = \frac{rwT}{r} = wT.$$

This is the case where the value of the given physical capital stock is independent of the rate of interest, that is, the case with a zero Wicksell effect. Note also that here the Böhm-Bawerk equation holds for all non-negative rates of interest. It is, of course, the case of the Solow model, restricted to some given capital intensity.

Because, by assumption,  $wT$  is a constant, we obtain from differentiation

$$0 = \frac{dwT}{dr} = w \frac{dT}{dr} + \frac{dw}{dr} T = w \left\{ \frac{dT}{dr} - T^2 \right\} \text{ or}$$

$$\frac{dT}{dr} = T^2 = \text{Variance}\{\lambda(t), r\} - \text{Variance}\{\gamma(t), r\}.$$

Now we look at the general case. Due to  $w(0) = c$ , we can write:

$$w(r) = c + \int_0^r \frac{dw}{d\rho} d\rho = c - \int_0^r T(\rho)w(\rho)d\rho.$$

From this follows:

$$v(r) = \frac{c - w(r)}{r} = \frac{\int_0^r T(\rho)w(\rho)d\rho}{r}.$$

The value of the capital stock is the average value of the ‘Böhm-Bawerk expression’ for the capital requirements,  $wT$ , averaged over all rates of interest from 0 to the actual value of  $r$ .

Let us then take the case that  $T(r)w(r)$  rises with rising  $r$ . This, by assumption, means that

$$T(r)w(r) > \frac{\int_0^r T(\rho)w(\rho)d\rho}{r} = v(r) \text{ for } r > 0.$$

For the Wicksell effect we then have

$$\frac{dv}{dr} = \frac{w(r)T(r) - v(r)}{r} > 0.$$

On the other hand, we have, by the same assumption,

$$\frac{d(wT)}{dr} = w \frac{dT}{dr} + \frac{dw}{dr} T = w \left\{ \frac{dT}{dr} - T^2 \right\} > 0.$$

We can write:

$$\text{Variance}\{\lambda(t), r\} - \text{Variance}\{\gamma(t), r\} = -\frac{d\ln(w)}{dr} = \frac{dT}{dr} > T^2.$$

Similarly, from the assumption

$$T(r)w(r) < \frac{\int_0^r T(\rho)w(\rho)d\rho}{r} = v(r) \text{ for } r > 0,$$

we obtain

$$\frac{dv}{dr} = \frac{w(r)T(r) - v(r)}{r} < 0$$

$$\text{Variance}\{\lambda(t), r\} - \text{Variance}\{\gamma(t), r\} = -\frac{d\ln(w)}{dr} = \frac{dT}{dr} < T^2.$$

We conclude that the Wicksell effect is a good measure of the degree of the vertical division of labor. Its sign indicates whether the degree of the vertical division of labor is larger or smaller than would be the case for the linear wage–interest curve.

#### 4 MARGINALISM

Classical economics did not use first and second derivatives of functions. Neoclassical economics discovered marginal rates. It developed price theory as ‘marginalism.’ Even though important pioneers of marginalism, such as Böhm-Bawerk, were not familiar with calculus, their ‘philosophy’ was tied to the concepts of marginal cost, marginal utility, marginal rates of substitution, etc. Böhm-Bawerk saw the real rate of interest as a price signal of the percentage marginal productivity of roundaboutness. He measured roundaboutness by means of the period of production.

If, following Newton and physics ever since, we take advantage of the elegance of continuously differentiable functions, we turn to an approach with an ‘infinity’ of techniques. In a Böhm-Bawerkian steady-state analysis, this means that the period of production  $T(r, \theta)$  (see Section 1 above) is not only a continuously differentiable function of the rate of interest, but also of the technique in use  $\theta$ . But what does that mean? If only a finite number of techniques are available we can give each technique an integer number as its ‘name.’ How do we cope with an infinity of available techniques?

Here it is useful to restrict ourselves to steady-state analysis. For a given set of available techniques,  $T\theta$ , we may look at different steady states, each with a different steady-state rate of interest  $r$ . We can define a set  $R$  of feasible rates of interest. It is the set of interest rates which are compatible with some steady state, that is: compatible with a non-negative real wage rate. Then, describing the choice of technique, we define a mapping  $\theta(r)$  from the set of feasible rates of interest  $R$  into the set of available techniques



$\Theta$ . We may go one step further by defining a sub-set  $\Theta \subset \Theta$ , by means of the condition:  $\theta \in \Theta$ , if  $\exists r \in R$ , such that  $\theta = \theta(r)$ . The set  $\Theta$  consists of those available techniques  $\theta$  for which there exists a feasible steady-state rate of interest, which induces the use of technique  $\theta$ .

Now we can use a naming convention for the techniques  $\theta \in \Theta$ . By construction there exists a rate of interest  $r$  such that  $\theta = \theta(r)$ . We then can give  $\theta$  the ‘name’  $r$ . This means that we have the equation  $\theta(r) = r$ . Obviously, if there is only a finite number of techniques in  $\Theta$ , many of these techniques will be used by more than one steady-state rate of interest. In that case, our naming convention would be confusing, because a given technique may have many different ‘names.’ In that case, our naming procedure would not be advisable. But it would still be possible, because we are free to give the same technique different names, each of which depends on the rate of interest in which it is employed.

Before continuing, we introduce the real rate of growth of our system. We denote it by  $g$ . We assume that it is non-negative:  $g \geq 0$ .

We start with:

**Assumption 1** *The set  $R$  is a connected sub-set of the set of real numbers. For  $r_1 \in R$  and  $r_2 \in R$  with  $r_1 \neq r_2$  the technique  $\theta(r_1)$  is different from technique  $\theta(r_2)$ . Furthermore we assume  $g \in R$ .*

With this assumption, the naming convention  $\theta(r) = r$  makes sense. However, in order to take advantage of the marginalist philosophy we need to make an additional ‘smoothness’ assumption. This then is:

**Assumption 2** *For  $r \in R$  and for  $\theta \in \Theta$  the values  $w(r; \theta)$  and  $v(r; \theta)$  are continuously differentiable functions of  $r$  and  $\theta$ .*

Here, as in the preceding section, the variable  $w$  is the real wage rate; and the variable  $v$  is the market value of the capital stock per worker.

Let  $y$  be value added per worker. We then can write:

$$y = I + c,$$

with  $I$  being net investment per worker, and  $c$ , as before, consumption per worker. We then introduce an additional assumption. Thereby limiting our analysis to the case that in any steady state with a given rate of interest  $r$ , the capital–consumption ratio remains constant through time. This amounts to:

**Assumption 3** *For any given steady-state rate of interest  $r$ , net investment  $I(r; \theta) = gv(r; \theta)$ .*

We then can write:

$$y = gv(r; \theta) + c(r, \theta).$$

Next, we make the usual assumption in capital theory: for any given rate of interest  $r$ , the wage rate is maximized. In a formula:

**Assumption 4** *For each  $r \in R$  the following inequality holds:  $w(r; \theta(r)) \geq w(r; \theta)$  for all  $\theta \in \Theta$ .*

This, together with the smoothness assumption (Assumption 2), implies:

$$\frac{\partial w}{\partial \theta} = 0 \quad \text{for} \quad \theta = \theta(r) = r.$$

Further, we have the income distribution equation

$$y = w(r; \theta(r)) + rv(r; \theta)$$

so that we can write

$$gv(r; \theta) + c(r; \theta) = w(r; \theta(r)) + rv(r; \theta) \text{ or}$$

$$c(r, \theta) = w(r; \theta) + (r - g)v(r; \theta).$$

Partial differentiation with respect to  $\theta$  yields

$$\frac{\partial c}{\partial \theta} = \frac{\partial w}{\partial \theta} + (r - g) \frac{\partial v}{\partial \theta}.$$

By Assumption 4, for  $\theta = \theta(r) = r$  we then have

$$\frac{\partial c}{\partial \theta} = (r - g) \frac{\partial v}{\partial \theta}.$$

Now we introduce:

**Assumption 5**  $\frac{\partial v}{\partial \theta} \leq 0$ .

We can assume that  $\frac{\partial v}{\partial \theta} \leq 0$ . A higher value of  $\theta$  means that the allocation  $\theta$  is induced by a higher cost of capital  $r$ . Keeping the price basis  $r$  the same (and thus neutralizing the Wicksell effect) one can expect that fewer capital goods will be employed.

We then conclude:

$$\frac{\partial c}{\partial \theta} (r - g) \leq 0.$$

Moreover, as we already discussed in the preceding section, we can set  $\frac{\partial c}{\partial r} = 0$  so that we can write:

$$\frac{dc}{dr} (r - g) = \left( \frac{\partial c}{\partial r} + \frac{\partial c}{\partial \theta} \right) (r - g) = \frac{dc}{dr} (r - g) \leq 0.$$

In words: if the rate of interest is below the steady-state rate of growth, *raising* the interest rate raises consumption per head. If the rate of interest is above the steady-state rate of growth, *reducing* the interest rate raises consumption per head. If the rate of interest equals the rate of growth, steady-state consumption reaches its maximum.

This is what I want to call: 'the generalized golden rule of accumulation.'

## 5 THE COEFFICIENT OF INTERTEMPORAL SUBSTITUTION

In the special case of the Solow production function, neoclassical economics uses the elasticity of substitution between labor and capital. What is the measure of substitution in the neo-Böhm-Bawerkian approach? To answer this question we first have to show the relation between the rate of interest (that is, the 'cost of capital') and the degree of roundaboutness of production.

Take any rate of interest  $\bar{r} \in R$ . Because of Assumption 4, we know that for any  $r \in R$

$$w(r; \theta(r)) \geq w(r; \theta(\bar{r}))$$

and

$$\frac{dw}{dr} = \frac{\partial w}{\partial r} + \frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial r} \text{ at } r = \bar{r},$$

which implies

$$\frac{\partial^2 w}{\partial^2} \leq 0 \text{ at } r = \bar{r} \text{ and } \theta = \theta(\bar{r}) \text{ and } \frac{d^2 w}{dr^2} \geq \frac{\partial^2 w}{\partial r^2} \text{ (envelope theorem).}$$

On the other hand,

$$\begin{aligned} \frac{d^2 w}{dr^2} &= \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial r \partial \theta} + \frac{\partial^2 w}{\partial \theta \partial r} + \frac{\partial^2 w}{\partial \theta^2} \\ \frac{\partial^2 w}{\partial \theta \partial r} &= \frac{\partial^2 w}{\partial r \partial \theta} = -w \frac{\partial T}{\partial \theta} - \frac{\partial w}{\partial \theta} T = -w \frac{\partial T}{\partial \theta}. \end{aligned}$$

We can write:

$$\frac{d^2 w}{dr^2} - \frac{\partial^2 w}{\partial r^2} - \frac{\partial^2 w}{\partial \theta^2} \geq 0.$$

Or, which is the same,

$$2 \frac{\partial^2 w}{\partial r \partial \theta} = -2w \frac{\partial T}{\partial \theta} \geq 0.$$

Hence

$$\frac{\partial T}{\partial \theta} \leq 0.$$

Keeping the price-weighting system the same (that is, neutralizing the Wicksell effect), the degree of roundaboutness declines with a higher rate of interest.

Using this result, I introduce the coefficient of intertemporal substitution, which I denote by  $\psi$ . As in the case of the elasticity of substitution, we want the coefficient of intertemporal substitution to be dimension-free. Therefore, I define it to be

$$\psi \equiv \frac{\partial \frac{1}{T}}{\partial \theta}.$$

Obviously,  $\psi$  is non-negative, because  $-\frac{\partial T}{\partial \theta}$  has been shown to be non-negative. Note that  $\psi$  is also well defined, when the rate of interest is zero or even negative. This is of interest in the context of actual policy discussions, where one has to cope with negative interest rates.

We can write the equation in a different way:

$$\frac{\partial T}{\partial \theta} = -\psi T^2.$$

As an example for the application of the coefficient of intertemporal substitution, I report the following result. As already shown, steady-state consumption per worker is maximized at the rate of interest equal to the rate of growth. We can ask: how large is the welfare loss in terms of steady-state consumption at a rate of interest  $r \neq g$ ? For this we can get an approximate answer by means of a second-order Taylor approximation around the optimal rate of interest equal to the rate of growth. The formula is this:

$$c(r) \approx c(g) \left\{ 1 - \frac{1}{2} \psi T^2 (r - g)^2 \right\}.$$

Or, which is the same:

$$c(r) \approx c(g) \left\{ 1 + \frac{1}{2} \frac{\partial T}{\partial \theta} (r - g)^2 \right\}.$$

Proof is in Weizsäcker 2020. In that paper I also show that this Taylor approximation works quite well for values of  $r - g$  between  $-5$  percent and  $+5$  percent per annum. The proportionate loss in steady-state consumption rises linearly with  $\psi$  and with the square of the period of production and with the square of the difference between the rate of interest and the rate of growth.

Intuitively the formula is very plausible: the more sensitively the production system reacts to a change in the rate of interest ( $\frac{\partial T}{\partial r}$ ), the more sensitive is the steady-state consumption as a function of the rate of interest. Moreover, the expression should be invariant with respect to a change in the time unit of measurement. And this is the case for the expression  $T(r - g)$ : as we switch from the year to the month as a time unit of measurement,  $T$  is 12 times higher and  $r - g$  is smaller by a factor of 12. Being dimension-free,  $\psi$  is not affected by such a change in the time unit of measurement.

Note that this Taylor approximation is correct – irrespective of the specific properties of the underlying capital theoretic model. We did not have to specify the model further than by Assumptions 1 to 5, given in Section 4.

## 6 CONCLUSION

The Cambridge–Cambridge controversy half a century ago fostered the understanding of capital theory. Due to the decline of interest in Marxian theory and due to the rapidly rising potential for empirical research, the neoclassical school lost interest in the subtleties of that controversy. It followed the lead of Solow's approach in aggregating capital into a single unidimensional variable. In this paper, I show that a return to the neoclassical approach of Böhm-Bawerk avoids some of the pitfalls of the Solow approach. Moreover we thereby obtain a way to handle capital–labor substitution even in a world with negative real interest rates. And this is important for an understanding of 'secular stagnation' in the twenty-first century.

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