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Article

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On the empirical regularities of Sraffa prices

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This paper focuses on a key concern of the Cambridge capital controversies: Sraffa's theoretical demonstration that competitive relative prices, and hence the money value of aggregate capital, can vary in complex ways as the wage share (profit rate) changes. We find that, on the contrary, individual prices are usually linear or mildly curved. We develop a formal measure of curvature, and find that average price curvature does not fall with matrix size as proposed in Brody's random matrix hypothesis. Since the average curves are near-linear, it follows that aggregates such as capital, wages, and net output will exhibit the same behavior. We believe this explains the widely observed near-linearity of the wage–profit curve.

Keywords: *Sraffa, prices, production functions, input–output, wage–profit curves, labor theory of value*

JEL codes: *B51, C67, D46, D64, P16*

1 INTRODUCTION

There are two kinds of commodity prices in the classical framework. A small set comprising the prices of special commodities such as rare paintings, books, wines, artifacts, etc., that are demand-driven because their supply cannot be increased. And a vast set whose prices are ruled by production costs because an increase in demand precipitates an increase in supply (Ricardo 1951: 12). In the latter case, competition within a given industry between sellers of the same type of commodity roughly equalizes the selling price, while competition between industries roughly equalizes the profit rates. Therefore, for the vast bulk of commodities, market prices are regulated by equal-profit-rate prices. Ricardo calls these natural prices, Marx calls them prices of production. While market demand and supply, the conditions of exchange, play a role in the immediate determination of market prices, in the case of reproducible commodities market prices are themselves regulated by prices of production based on structural production conditions and a socially achieved wage share that determines the general profit rate. It should be noted that these two aspects of competition might well drive some firms out of business if the resulting price is too close to their production costs. Real competition is a Darwinian process, and survival cannot be taken for granted (Shaikh 2016: 289 and 321).

The marginalist framework shifts the focus to models of pure exchange, with utility as the foundation for smooth and continuous demand curves and similar supply curves derived from

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appropriately constructed production functions, while individual firms treat the general wage as a constituent of their unit labor costs. They are also assumed to treat the general rate of profit (labeled the interest rate) as a cost on the grounds that they are entitled to it (Baumol/Blinder 1997: 226). It is here that a ‘well-behaved’ aggregate production function comes into the picture, to determine the real wage through its marginal product of aggregate labor and the profit rate through its marginal product of aggregate capital (Cohen/Harcourt 2003: 201). The imparted lesson is clear: ‘What a social class gets is, under natural law, what it contributes to the general output of society’ (Clark 1891).

Now this is a very different position from that of the classical tradition, in which the wage share is determined through the push-and-pull between workers and their employers, while the rate of profit is determined by production conditions and the wage share. Concerning the aggregate production function hypothesis, Joan Robinson asked what ‘aggregate capital’ meant. Sraffa’s carefully planned ambush provided the answer: it was the aggregate price of production of a vector of heterogeneous materials and machines (Robinson 1953; 1961; Sraffa 1960). Sraffa argued that individual prices of production were capable of complicated patterns of movement as the wage share and corresponding rate of profit moved between their respective limits. By implication, aggregate capital could also exhibit the same behavior. Samuelson tried to counter Sraffa by developing a general model of a ‘surrogate production function’ in which the aggregate price of production of capital goods was indeed well-behaved and yielded a pseudo marginal product equal to (but not determining) the rate of profit. Garegnani then showed that Samuelson’s pseudo production function only obtained if all industries had the same capital–labor ratio, in which case relative prices would be equal to relative labor values – the simple labor theory of value! Samuelson graciously admitted defeat and retired from the field (Bhaduri 1966; Cohen/Harcourt 2003; Garegnani 1970; Harcourt 1969; Samuelson 1966). This was the core of the Cambridge capital controversies (CCC).

Most Sraffians remain within the orthodox paradigm of perfect competition and smooth adjustments, in order to advance their internal critique based on the consequences of the ‘twistiness’ of individual and aggregate relative prices. The orthodoxy remains unmoved (Chiodi/Ditta 2008; Cohen/Harcourt 2003). Our concern is somewhat different. While we begin from a Sraffian theoretical base, we wish to assess the empirical validity of the notion that individual relative prices vary in complex ways as the wage share (profit rate) changes. The paper is divided into five sections. Section 2 briefly outlines the literature that discusses the observed characteristics of Sraffa prices. In Section 3, we present the results of our own calculated prices using the database of 295 matrices we constructed from the IO benchmark tables of 2002 and 2007. We find that Sraffa prices are usually near-linear or mildly curved. In Section 4 we introduce the Curvature Index as a measure of linearity and examine whether average price curvature falls with matrix size, as proposed in Bródy’s random matrix hypothesis (Bródy 1997). Here too, the answer is that it does not. Finally, Section 5 is dedicated to concluding remarks.

2 THEORETICAL CONSIDERATIONS ABOUT PRICE CURVATURE IN SRAFFIAN PRICES

Bródy discovered that in a random matrix with independently and identically distributed (i.i.d.) entries, the relative size of the moduli of all subdominant eigenvalues tends to zero as the matrix size approaches infinity. The resulting rank-size plot would be L-shaped: one

positive point corresponding to the dominant eigenvalue, and all other points on the horizontal axis. In the Leontief system, the smaller the ratio of the modulus of the second eigenvalue to the first, the faster the convergence between some initial vector and an equilibrium (equal-growth-rate outputs or equal-profit-rate prices) one. In the limit, any initial output vector could be transformed into a balanced growth vector in one step, or any initial price vector can be transformed into a competitive price vector in one step. In the latter case, Marx's 'transformation procedure' would be exact (Shaikh 2016: 441). On the theoretical side, Bidard/Schatteman (2001) and Sun (2008) proved the mathematical validity of Brody's conjecture in the case of random matrices. Others investigated its validity in the case of entries generated by other probability distributions (Bialas/Gurgul 1998; Gurgul/Wójtowicz 2015; Molnár/Simonovits 1998), or its implications for the shapes of Sraffian wage–profit curves (Scheffold 2013). On the empirical side, Brody's conjecture has been tested on actual input–output matrices (Gurgul/Wójtowicz 2015; Mariolis/Tsoufidis 2014; 2018; Shaikh/Nassif 2018; Torres Gonzalez 2017; Tsoufidis/Mariolis 2016).

In an earlier paper, we showed that actual matrices exhibit different properties from random matrices. We used the US¹ 2002 403-order and 2007 387-order benchmark tables of Sraffian basic commodities² and successively aggregated them to get a total of 295 matrices for these two years ranging from 15-order to the largest in each year. We then further expanded our sample with the 1977, 1982, 1987, 1992, and 1997 benchmark tables at the detailed level. We found that, as we moved to matrices of successively larger size, the rank-size curves of eigenvalue moduli approach the *same* smooth convex limiting curve in all of the 1977, 1982, 1987, 1997, 2002, and 2007 benchmark tables. We also found that this universal rank-size curve is essentially a power law, and that the frequency distribution of the eigenvalue moduli is well approximated by a Weibull probability distribution. Thus the random matrix hypothesis of an L-shaped rank-size plot was not supported by our empirical evidence (Shaikh/Nassif 2018).

The present paper uses the same 2002 and 2007 database of 295 tables, expanded in this case to include non-basic sectors (see Appendix 1) in order to assess the empirical shapes of individual Sraffian price curves and the extent to which their curvature varies as we move from the smallest to the largest matrices.

3 EMPIRICAL EVIDENCE ON PRICE SHAPES AND CURVATURE

Figures 1–2 display the Sraffian price curves for the largest 2002 and 2007 matrices as the profit rate (r) varies from zero to its maximum rate (R), that is, as $\frac{r}{R}$ varies from 0 to 1. Basic and non-basic commodities are displayed separately. Each price curve is normalized to its initial value, which is the labor value of that commodity, so that the starting point equals 1 (Sraffa 1960: 12, sec. 14). We can see that price lines generally exhibit mild curvature (Section 3 will provide a formal curvature measure) and that the curves are almost always monotonically rising or falling. For instance, in 2002 only 26 (6.1 percent) of 425 prices switch from one side of their labor value to the other. For 17 of these, the maximum deviation of prices from labor values is less than 2 percent and for the remaining nine it falls between 4–6 percent. Similar results appear in Shaikh (2012: 94). With respect to the straight line between the individual price curve endpoints, only four (0.9 percent)

1. We used US data because it is extensive, because the US economy is relatively closed (over our time span 1977–2007, almost 90 percent of goods are produced domestically) and because there are good domestic approximations to the input–output structure of imported commodities.

2. Here we only included basic industries since these are the ones responsible for the eigenvalues.

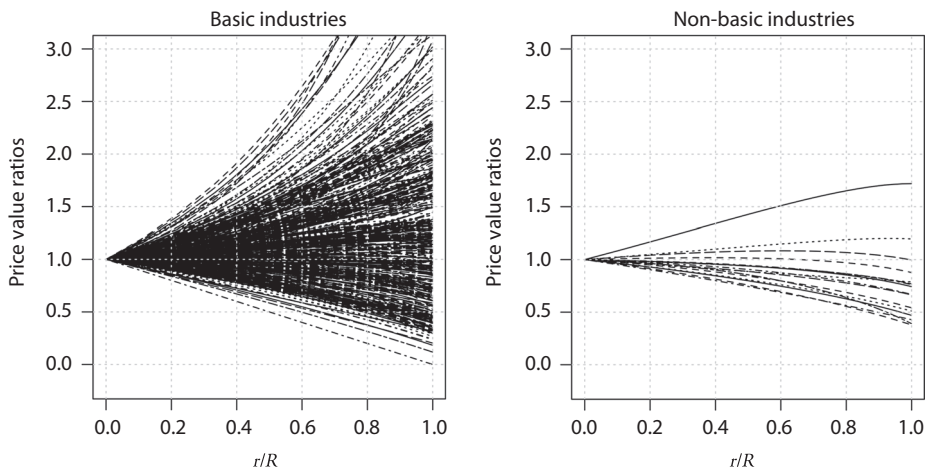


Figure 1 Sraffa prices, 2002 US input–output matrix, 425 sectors

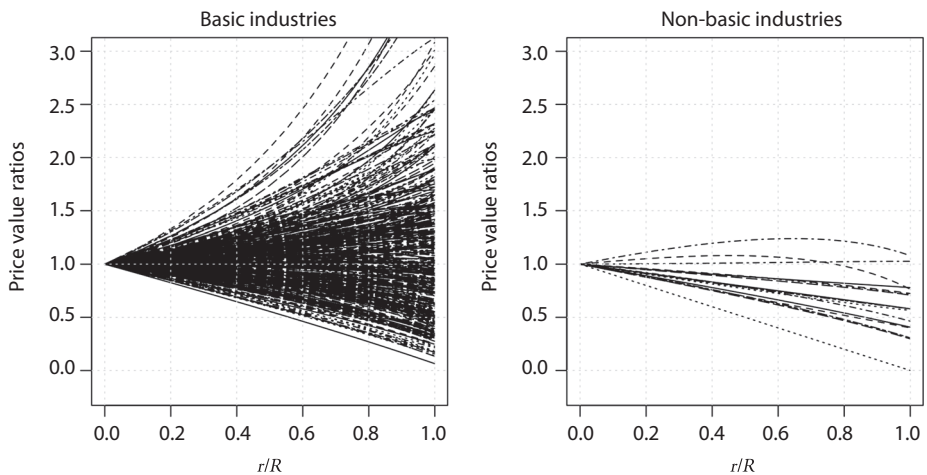


Figure 2 Sraffa prices, 2007 US input–output matrix, 383 sectors

cross over in 2002 and in every case the maximum deviations are less than 1 percent. The latter line, which we will call the Bienenfeld curve, is the shortest distance between the two endpoints of any given price curves, and plays a central role in defining the curvature measure developed in the next section.

Figures 3–4 compare the shapes of curves as we move from the smallest to the largest matrices in each year. We see that at all levels of aggregation, prices display mostly mild curvature (formal measures are developed in the next section); in addition, in every case the spread of the endpoints of the curves increases as we move to ever-larger matrices.

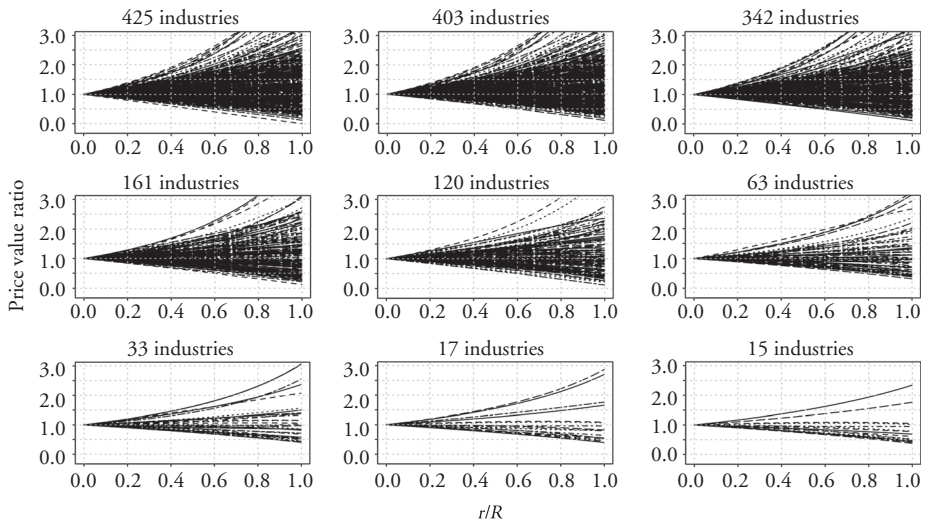


Figure 3 Sraffa prices, 2002 US input-output matrix, at various level of aggregation

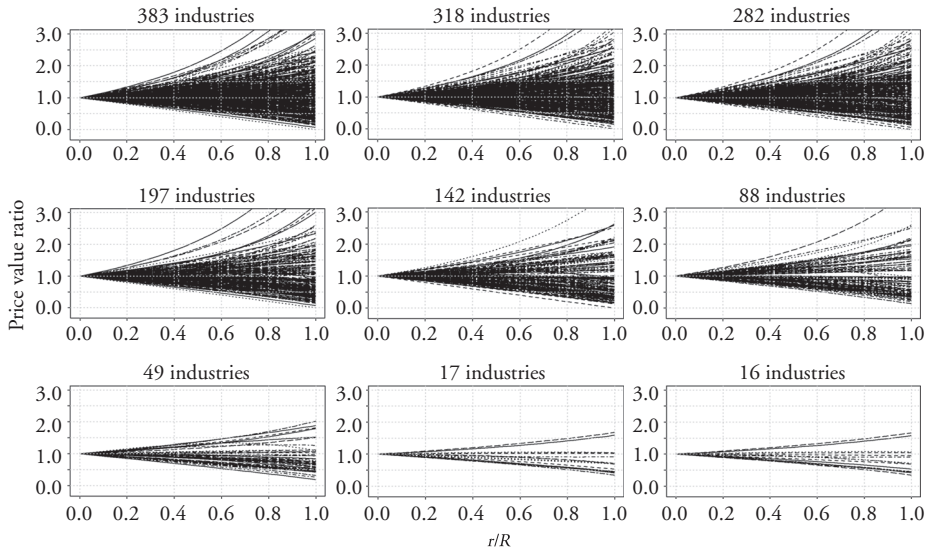


Figure 4 Sraffa prices, 2007 US input-output matrix, at various level of aggregation

4 THE CURVATURE INDEX

Biologists have long been interested in measures of the tortuosity of an animal's path between two endpoints as an indication of the efficiency of its search methods. Consider two animals moving between the same endpoints, one in a zigzag manner and the other in a relatively smooth manner. In order to compare the efficiency of their paths, one widely used measure is the straightness index (SI) the ratio of the length of a straight line (the shortest distance) between the two endpoints to the length of the actual path (the actual distance traveled) between those same points (Benhamou 2004). Then the straightness index will lie between 1 (the actual path is the shortest distance) and 0 (the actual path is infinitely long).

As it turns out, this measure has a direct correspondence to the discussion around Sraffa prices. Bienenfeld (1988: 248) proposed a linear approximation to Sraffa prices that is 'an extension of Marx's "transformation procedure" for converting values to prices ...; [and] also performs well in the empirical test.' This approximation turns out to be a straight line between any given curve at points $\frac{r}{R} = 0$ and $\frac{r}{R} = 1$. It follows that the ratio of the length of the Bienenfeld approximation to the length of an actual curve is the straightness index of that curve (Figure 5). This index approaches 1 as an actual curve approaches the Bienenfeld line, and approaches 0 the more extended is the actual. Other things being equal, the more 'twisted' is a curve, the longer is its path and the lower will be its straightness index. Since we are concerned with the curvature of prices, we will use the curvature index $CI = 1 - SI$. This will be 0 when the $SI = 1$ (linear case), and will approach 1 as the actual curve path gets longer than the shortest distance. Figure 5 illustrates two hypothetical curves: curve 1 bows smoothly between its endpoints and yields a $CI = 0.008$; curve 2 crosses back and forth across its Bienenfeld line, covering a longer relative distance and yielding a $CI = 0.128$.

Figure 6 shows that the average degree of price curvature in the two benchmark years is small, and does not vary much across matrix size: average CIs are in the range of 0.03 to 0.06.

The frequency distribution of CIs in both years across all matrix sizes tends to cluster in a very small range: in 2002 the CIs of the bottom 50 percent fall in the 0–0.001 range, 80 percent fall below 0.022, and 90 percent below 0.158. We therefore turn to the the

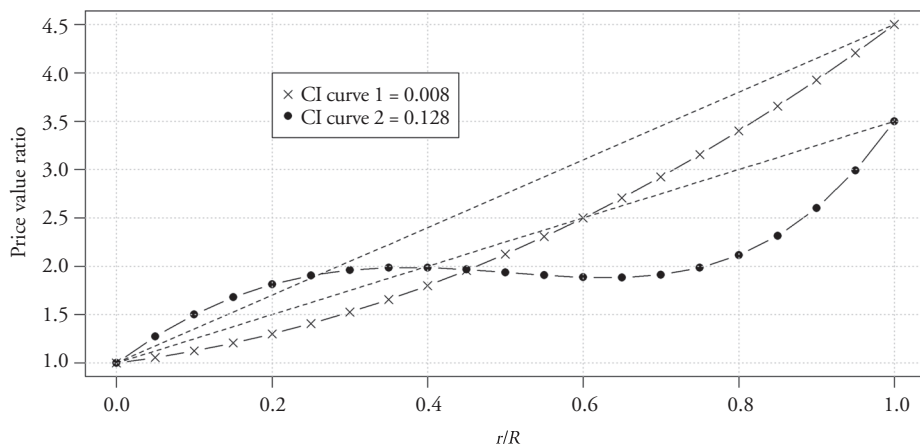


Figure 5 Curvature index for two hypothetical curves

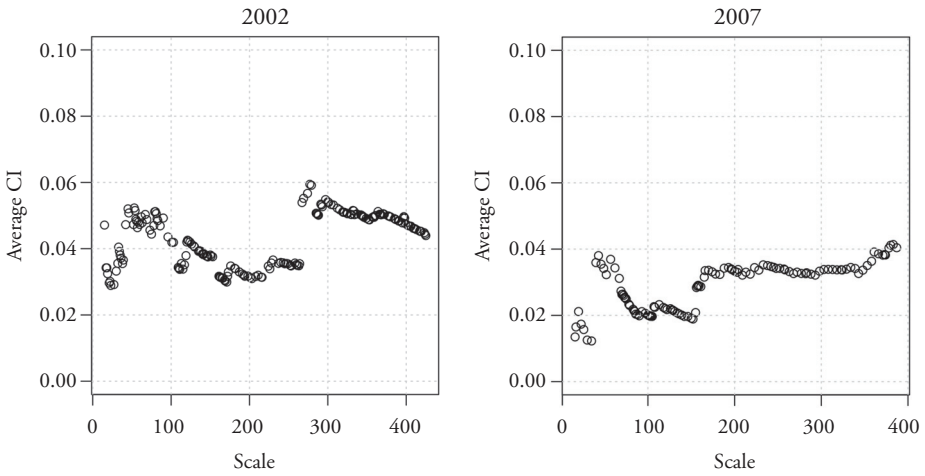


Figure 6 Average CI by matrix size, 2002 and 2007

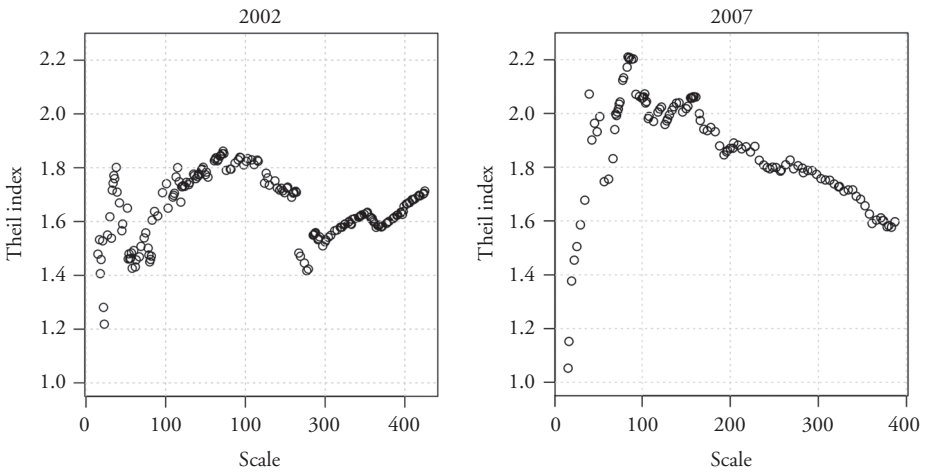


Figure 7 Theil indexes for 2002 and 2007

Theil index (Theil/Finizza: 1971) to describe their degree of variation: the lower the Theil index, the lower the degree of inequality among CIs in a given matrix. Here, we find that it varies between 1.05 and 2.2, as compared to a possible maximum value of around 6 (Figure 7).

5 SUMMARY AND CONCLUSIONS

This paper has focused on the empirical properties of competitive prices. Sraffa demonstrated that in principle relative prices can follow twisty paths as the wage share (or profit rate) varies between its limits. By implication, the price of aggregate capital can also vary in

ways that vitiate the assumptions of a well-behaved aggregate production function and its attendant marginal productivity theory of wages and profit rates. These theoretical possibilities have been conceded by prominent neoclassical economists, who then either abandoned the aggregate production function hypothesis because of its theoretical problems, or retained it as a useful empirical approximation (Cohen/Harcourt 2003: 205–206).

Our concern is with a different aspect of this issue, having to do with the empirical properties of individual prices of production. Specifically, we assess the empirical validity of the Sraffian notion that individual relative prices vary in complex ways as the wage share (profit rate) changes.

We begin from a database containing 295 tables created by successively aggregating the US 2002 425-order and 2007 383-order benchmark tables, including both basic and non-basic industries. Our first finding is that even at the highest matrix sizes, price lines generally exhibit mild curvature and mostly rise or fall monotonically. Across all levels of aggregation in both benchmark years, the average CIs range between 0.03 to 0.06 (for comparison, see Figure 5). In 2002 only 26 (6.1 percent) of 425 prices switch from one side of their labor value to the other. Only four (0.9 percent) switch with respect to the Bienenfeld line, none with a deviation greater than 1 percent. In almost all cases, the price lines have relatively modest curvatures. This confirms Bienenfeld's own finding that his linear approximation is extremely accurate. Our second finding is that as we move successively from the smallest matrix sizes to the largest, the spread of the end-points of the curves increases.

In sum, almost all individual Sraffa prices are mildly curved, and the average curves are near-linear. It follows that aggregates such as capital, wages, and net output will exhibit the same behavior. We believe this explains the widely observed near-linearity of the wage-profit curve.

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DATA SOURCE

Benchmark Input–output Data, Bureau of Economic Analysis, URL: <https://www.bea.gov/industry/benchmark-input-output-data>.

APPENDIX 1

We constructed the US 2002 industry-by-industry direct requirements tables (A tables) in 176 different sizes that are compatible with existing aggregation correspondences that follow the North American Industry Classification System (NAICS). The 2007 tables were produced in the same manner with appropriate adjustments for the correspondence of sectors (see Torres-González and Yang (2019) for details).¹ The construction of the symmetrical tables follows standard procedure and was done using the Industry Technology Assumption (ITA) (Horowitz/Planting 2006: ch. 12; Miller/Blair 2009: ch. 5).

The NAICS description can be found at <https://www.census.gov/eos/www/naics/>. Benchmark tables are available at the following levels of aggregation: sector level (15 industries), summary level (133 industries), and detailed level (430 industries). The Bureau of Economic Analysis (BEA) input–output annual tables (available at: <https://www.bea.gov/industry/input-output-accounts-data>) contain two additional levels of aggregation: summary level (71 industries) and detailed level (389 industries). The Bureau of Labor Statistics (BLS) has one additional aggregation level with 170 industries (available at: <http://www.bls.gov/emp/classifications-crosswalks/sect300.xls>). Finally, we produce two more levels: 30 industries and 17 industries. The 30 industries level comprises all 2-digit NAICS industries along with industries 31–33 (manufacturing) which are expanded into ten industries following the 3-digit NAICS. The 17-industries-level table is the only one produced by disaggregating instead of aggregating. We start from the 15-industries-level table and disaggregate the manufacturing sector into 3 sectors: food and leather, wood, paper and petroleum, and metals. The resulting NAICS correspondences allow us to aggregate consistently from 430 sectors to 15 sectors. Further details are available on request.

The first step of our process is to exclude the following industries: owner-occupied dwelling; scrap; used and secondhand goods; non-comparable imports; and rest of the world adjustment. For owner-occupied dwelling, we exclude both the commodity and the industry from the use-and-make tables and reallocate the value of its intermediate consumption into final consumption. For the remaining four sectors, we exclude the industries from the use-and-make tables and redistribute the commodities to other industries according to their market share using the ITA. At the end of these adjustments, we have reduced our original use-and-make tables by five industries and one commodity. We end up with the following levels of aggregation: 425, 342, 161, 120, 63, 30, 17, and 15.

We then construct vectors of correspondence that describe which sectors to aggregate in order to go from the 425 industries level to the 342 industries level, from the 342 to the 161, and so on. The next step is to aggregate industries in the use-and-make tables, according to the correspondence, and produce A tables from those. Every time a set of industries is aggregated in the use-and-make tables, we calculate a new A table. We proceed iteratively until we reach the 342 industries table. We then start again from the 342 industries tables, aggregating iteratively to construct one A table at each step until we end up with the 161 industries table, and so on until the 15 industries table has been constructed.

APPENDIX 2

We utilize a pure circulating capital model of prices of production characterized by $n \times n$ single product industries, a uniform rate of profit r , and a uniform wage rate w paid *post festum*

1. We thank Luis Torres-González for making his data available to us.

as in Sraffa. We create skill-adjusted sectoral labor inputs by using the compensation of employee entries in the use matrices published by the BLS, normalizing them by the ratio of aggregate employee compensation to aggregate employment (Shaikh 2012: 98). For an input–output matrix \mathbf{A} with entries $a_{ij} \geq 0$, prices \mathbf{p} , labor coefficients \mathbf{l} , wages w , and profit rates r , the circulating capital Sraffa price system is:

$$\mathbf{p}(r) = w\mathbf{l} + (1 + r)\mathbf{p}(r)\mathbf{A}. \tag{A1}$$

Writing (A1) as $\mathbf{p}(r)(\mathbf{I} - \mathbf{A}) = w\mathbf{l} + r\mathbf{p}(r)\mathbf{A}$, we can represent (A1) in terms of the sum of direct and indirect labor inputs $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ and the total capital coefficients matrix $\mathbf{H} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}$. Setting $w = 1 - \frac{r}{R}$ to normalize prices by Sraffa’s standard net output, where R is the maximum profit rate, gives

$$\mathbf{p}(r) = \left(1 - \frac{r}{R}\right)\mathbf{v} + r\mathbf{p}(r)\mathbf{H}. \tag{A2}$$

At $r = 0$, $\mathbf{p}(0) = \mathbf{v}$ so that standard prices always ‘begin’ from labor values and the i th price–value ratio $\frac{p_i}{v_i}$ begins from 1. At the other end, with $w = 0$, $r = R =$ the maximum profit rate, $\mathbf{p}(R) = R\mathbf{p}(R)\mathbf{H}$ so that standard prices end in $\mathbf{p}(R)$ which is the dominant left-hand eigenvector associated with the dominant eigenvalue of \mathbf{H} which is $\lambda_1 = \frac{1}{R}$.

Bienenfeld (1988: 248) derives a powerful linear approximation to Sraffa prices that is ‘an extension of Marx’s “transformation procedure” for converting values to prices ... [and] also performs well in the empirical test.’

$$\mathbf{p}(r) = \mathbf{v} + \frac{r}{R}(R\mathbf{p}(R)\mathbf{H} - \mathbf{v}) \tag{A3}$$

At $r = 0$ the Bienenfeld line yields $\mathbf{p}(0) = \mathbf{v}$ and at $r = R$ it yields $\mathbf{p}(R) = R\mathbf{p}(R)\mathbf{H}$. These are the same endpoints as in the Sraffa price equation (A2), which means that the Bienenfeld linear approximation is the straight line between the endpoints of any given Sraffa price curve. We calculate prices via equation (A2) and the Bienenfeld linear approximation via equation (A3), letting $\frac{r}{R}$ vary between 0 and 1 (that is, letting r vary between 0 and R).