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# Can they sort it out themselves? Regulating externalities between Coase and Pigou

Daniel Heyen\* and Philipp Weinschenk†

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## Abstract

How to regulate pollution externalities? According to the Pigouvian approach, the regulator should tax the polluting activity to steer the polluter to internalize negative external effects. In contrast, the Coasian Bargaining approach advises the regulator to set no pollution tax and let the polluter and victim sort it out themselves, i.e. negotiate to the efficient pollution level. We study a novel form of regulatory uncertainty and ask: how should the regulator set the pollution tax when uncertain about whether the polluter and victim are “connected”, i.e. whether they can engage in Coasian bargaining or not? We characterize how the optimal pollution tax depends on the probability that the polluter and victim are connected. When the parties can decide whether they want to connect, the regulator needs to take into account the tax level’s impact on the parties’ connection decisions. Interestingly, under such endogenous connection decisions, the optimal tax can be higher than the Pigouvian level. Overall, our work connects separate strands of literature and uncovers challenges for price-based regulatory instruments.

**Keywords:** Externalities; Pigouvian tax; Coasian Bargaining; Transaction costs; Environmental Regulation under Uncertainty

**JEL codes:** D23; D62; D82; H23

## 1 Introduction

Externalities are a major source of market failure. Consider a standard textbook problem: An industry emits a pollutant, negatively affecting a fisher downstream. Two separate strands of literature have developed fundamentally opposing perspectives and solutions to externality problems. The first, going back to Pigou (1920), suggests a centralized approach. Since the polluter maximizes their own profit (not

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taking into account the external costs imposed on others), the regulator should tax pollution at a rate equal to the marginal damage at the efficient pollution level. With this Pigouvian tax, the polluter internalizes the negative externality imposed on others and chooses efficient pollution. The second strand of literature, developed by Coase (1960), favours a decentralized approach, emphasizing the possibility of the affected parties to negotiate an efficient solution. If parties can engage in Coasian bargaining, the regulator should abstain from any centralized regulation since the parties can sort it out themselves.

While both approaches are correct in a specific environment, in practice it is unclear which perspective is more relevant. We examine the problem of a regulator who does not know whether the polluter and victim can engage in Coasian bargaining or not. To our knowledge, we are the first to analyze regulation under this fundamental form of uncertainty. The relevance is immediately apparent: If the regulator sets the Pigouvian tax, not anticipating that parties can engage in Coasian bargaining, then the affected parties will negotiate to an inefficiently low pollution level, as demonstrated by Buchanan & Stubblebine (1962) and Turvey (1963). If, on the other hand, the regulator incorrectly assumes that Coasian bargaining is possible and therefore abstains from setting a pollution tax, then the inefficient outcome of an unresolved externality problem will pertain.

We set up the simplest possible model to analyse this problem. There is one polluter (“industry”), benefitting from pollution, and one victim (“fisher”), suffering damages from pollution. The regulator’s objective is to maximize total welfare, i.e. the sum of benefits and costs of both parties. To keep the focus on the key uncertainty of whether agents can bargain or not, we assume that the regulator has full knowledge of all other relevant characteristics of the externality problem such as the polluter’s benefit function and the victim’s damage function.

We first explore, in section 2, a model in which the regulator cannot influence whether industry and fisher can solve the externality problem among themselves or not. Concretely, the regulator considers only two possibilities: Either the industry and fisher are “connected”, i.e. they can engage in Coasian bargaining; or they are “unconnected”, i.e. cannot bargain, no matter the stakes. Importantly, in this model, the regulator holds a subjective belief assigning probability  $p$  to them being connected and  $1 - p$  not being connected. We are able to identify a clear relation between connection probability  $p$  and optimal tax  $t^*(p)$ : The optimal tax monotonically decreases in connection probability  $p$ , ranging between the Pigouvian fee at  $p = 0$  – when the regulator is certain that the industry and fisher cannot negotiate – and a tax of zero at  $p = 1$  – when the regulator is certain to be in the Coasian

world where polluter and victim will resolve the externality problem among themselves. For every connection probability strictly higher than zero and strictly lower than one we find that an (ex-post) welfare loss is inevitable. The regulator sets the tax to maximize expected welfare, thus balancing the expected welfare loss.

We develop a more sophisticated model in section 3. Now, we explicitly include transaction costs that industry and fisher have to incur if they want to be connected; we therefore refer to them as “connection costs”.<sup>1</sup> Polluter and victim “connect” and engage in bargaining if the total gains from cooperation exceed the transaction costs and do not connect otherwise. In line with the focus of our paper, we study the case that the regulator does not know the true connection costs of the parties and holds a probability distribution over all non-negative transaction cost values. (The model in section 2 is the corner case where transaction costs are either zero, when connection is ensured, or so large – think infinitely – so that bargaining will never happen, under no circumstances.) In this more sophisticated model with “endogenous connection”, we show that there is a cost threshold that determines whether the parties connect or not. If actual connection costs turn out lower than the cost threshold, then industry and fisher see it advantageous to negotiate. Importantly, conditional on the parties being connected, the pollution activity the parties negotiate towards does not depend on the actual connection costs as the latter are sunk costs.

What makes this model of endogenous connection in section 4 interesting is that, in contrast to the exogenous connection model in section 2, the regulator’s choice of tax level influences the cost threshold. In other words, the tax level not only affects the pollution activity levels for connected or unconnected parties; in addition, the tax level also affects *whether* polluter and victim are connected or not. We demonstrate that the objective function – again, expected welfare – shows a more complex behaviour in the tax level than in the exogenous model. In particular, while negative tax levels can never be optimal, we demonstrate that tax levels *above the Pigouvian tax level* can indeed be optimal. This is surprising (recall that in the “exogeneous connection” model in section 2, the optimal tax was always between zero and the Pigouvian tax level). Why can it be optimal for the regulator to choose a tax level above the Pigouvian level? The intuition is the following: Setting the tax to the Pigouvian level would be ideal if the parties are not connected; if there is a small, yet sufficient chance that the parties end up connected, then it may be beneficial for the regulator to *further increase* the tax level as this reduces (under certain conditions) the connection probability. If this reduction in connection probability is sufficiently

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<sup>1</sup>The importance of transaction costs was already highlighted in Coase (1960) and has been studied in Krutilla & Krause (2011). Libecap (2014) provides an extensive review.

large, then the regulator is better off with a tax level that is always slightly off than with a tax level that is exactly right only sometimes.

In terms of general applicability, it is worth mentioning that we do not have to interpret our model literally. Instead of a fisher and an industry, the two parties in our model could also be interpreted as two regions or countries that might or might not be able to negotiate their emission problems with each other (e.g., for water pollution or greenhouse gases).

We proceed as follows. In the remainder of this section we clarify how our paper relates to the literature and highlight our contributions. Section 2 develops the basic model and in detail analyzes the case of exogenous connection probability. Section 3 augments the analysis to an endogenous decision by parties whether to connect or not (they do so for sufficiently low connection costs) and studies the expected welfare and optimal tax in this endogenous setting.

**Related literature.** We contribute to several strands of literature. Centralized approaches, in particular the idea to tax negative externalities, go back to Pigou (1920) and have received wide attention ever since. In the context of climate change and carbon prices, the most recent is Timilsina (2022). Decentralized approaches, on the other hand, go back to the famous "Coase theorem" in Coase (1960). A recent survey is Medema (2020). Applications to environmental problems are Banzhaf et al. (2013) and Deryugina et al. (2021). Our contribution is to bridge both approaches by highlighting that a regulator may face uncertainty as to whether the world she operates in is a Pigouvian or a Coasesan.

A key literature we contribute to bridges the gap between centralized and decentralized approaches and focuses on their interplay. Our paper is directly related to the classical papers Buchanan & Stubblebine (1962) and Turvey (1963), which demonstrate that Coasian bargaining in the presence of a Pigouvian fee results in inefficiencies; recent contributions in that literature are MacKenzie & Ohndorf (2016) and Kotchen & Costello (2022). Our contribution is to add uncertainty over whether the affected parties can engage in Coasian bargaining, adding a novel dimension to the literature on the interplay of centralized and decentralized approaches. In order to keep a clear focus on that novel element, we deliberately keep the rest of the model as simple as possible; in particular, we do not study free-riding incentives among victims, as does Kotchen & Costello (2022).

Our paper also contributes to the literature on instrument choice, in particular the debate known as "prices vs quantities" (Weitzman 1974). There has been an intense debate about the appropriate policy instrument to tackle externality prob-

lems. Among the instruments based on economic incentives, there are quantity instruments (e.g. licenses, permits, where the quantity of pollution or the polluting activity is set) and price instruments (e.g. environmental tax, where a price is set for each unit of pollution or the polluting activity). Recent work emphasized the advantage of price over quantity instruments (Kotchen & Costello 2022, Aldy & Armitage 2022). Our findings reveal a so far unknown advantage of quantity-based over price instruments. We demonstrate that the correct price instrument level crucially depends on whether the parties can engage in Coasian Bargaining or not; the regulator uncertain about whether parties are “connected” in this sense must set a tax level to minimize expected welfare losses, thus necessarily inflicting an ex-post welfare loss. A quantity instrument, in contrast, would allow limiting the polluting activity to exactly the efficient pollution level.

Finally, we contribute to the literature on regulation under uncertainty. The existing literature emphasizes the uncertainty the regulator faces about abatement costs (Weitzman 1974, Stavins 2022) and uncertainty firms face in terms of policy and accordingly, prices (Aldy & Armitage 2022). The novel element we add to that literature is uncertainty about whether Coasian bargaining is possible or not, modelled as uncertain transaction costs.

## 2 Model

The literature studies pollution problems either in a Coasian world or in a Pigouvian world. Here, we build a model that unites both worlds.

Suppose there is a fisher and an industry. The industry emits pollution  $x \in \mathbb{R}^+$ , which harms the fisher. Let the profit of the fisher be

$$\pi_F = f(x), \tag{1}$$

where the function  $f$  is twice continuously differentiable, with  $f' < 0$  for all  $x > 0$  and  $f'' \leq 0$ . The fisher’s profit is thus decreasing and weakly concave in the pollution level  $x$ . The fisher’s profit can also be written in the form of a damage function by letting  $\omega_F := f(0)$  and  $d(x) := -f(x) + \omega_F$  such that  $\pi_F = \omega_F - d(x)$  with  $d(0) = 0$ ,  $d' > 0$  for all  $x > 0$  and  $d'' \geq 0$ .

The gross profit of the industry (or the industries) is

$$\pi_I = i(x), \tag{2}$$

where the function  $i$  is twice continuously differentiable and concave,  $i'' < 0$ . We denote by  $x^{\max} := \operatorname{argmax}_{x \in \mathbb{R}^+} i(x)$  the pollution that maximizes the gross profit and suppose that it is finite. In order to guarantee interior solutions, we suppose that  $\lim_{x \rightarrow 0} i'(x) = \infty$ .<sup>2</sup> There is thus a unique pollution level  $x^{\max} \in (0, \infty)$  that solves the first-order condition  $i'(x) = 0$ . Figure 1 illustrates the basic setting.

The regulator sets a pollution tax  $t$ . The industry's net profit is hence

$$\pi_I^{\text{net}} = i(x) - tx. \quad (3)$$

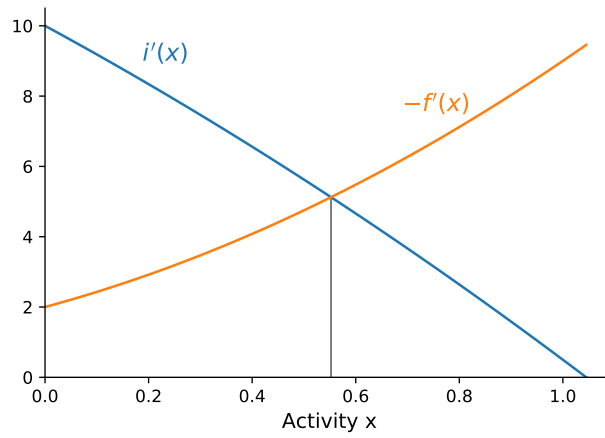


Figure 1: The polluting activity  $x$  yields marginal profit  $i'$  to the industry (blue line) and marginal damages  $-f'$  to the fisher (orange line). The two lines intersect at the efficient activity level  $x^{\text{efficient}}$ . Here and in the following figures, we use for illustration the functional forms  $i(x) = -0.5x^3 - 4x^2 + 10x$  and  $f(x) = -x^3 - 2x^2 - 2x + 10$ . This implies  $x^{\max} = 1.045$ ,  $x^{\text{efficient}} = 0.552$ ,  $t^{\text{Pigouvian}} = 5.124$ .

The parties might live in a Coasian world and thus be able to negotiate with each other or in a Pigouvian world where they cannot negotiate. To formalize this, we let the parties either be connected (such that they can negotiate) or not connected (such that they cannot negotiate).

<sup>2</sup>Alternatively, we can suppose that the marginal product  $\lim_{x \rightarrow 0} i'(x)$  is sufficiently large.

## 2.1 Efficient Pollution

We first solve for the efficient pollution level, i.e., the pollution maximizing welfare

$$W = \pi_F + \pi_I:$$

$$x^{\text{efficient}} := \arg \max_{x \in \mathbb{R}^+} f(x) + i(x). \quad (4)$$

We denote the maximal welfare by  $W^{\text{efficient}} := f(x^{\text{efficient}}) + i(x^{\text{efficient}})$ . Since welfare is concave, the efficient pollution  $x^{\text{efficient}}$  solves the first-order condition

$$f'(x) + i'(x) = 0. \quad (5)$$

By the Intermediate Value Theorem, there exists exactly one solution and the efficient pollution is positive and below the one maximizing the industry's profit:

$$x^{\text{efficient}} \in (0, x^{\text{max}}). \quad (6)$$

Note that the tax and its revenue have, per-se, no welfare consequences. However, the tax can indirectly affect welfare by influencing the industry's pollution.

## 2.2 Equilibrium with Unconnected Parties

Consider first the case where the parties are not connected. The industry then chooses the pollution level that maximizes its net profit:

$$x_{\neg \text{con.}}^*(\cdot) = \arg \max_{x \in \mathbb{R}^+} \pi_I^{\text{net}}. \quad (7)$$

The equilibrium pollution level  $x_{\neg \text{con.}}^*$  exists and is the unique solution of the first-order condition

$$i'(x) - t = 0. \quad (8)$$

By implicit differentiation, we directly obtain that

$$\frac{\partial x_{\neg \text{con.}}^*(\cdot)}{\partial t} = \frac{1}{i''(\cdot)} < 0. \quad (9)$$

Therefore, with a higher tax, the industry decides to emit less pollution.

Comparing (5) and (8), we obtain the standard result that the industry chooses the efficient pollution level,  $x_{\neg \text{con.}}^*(\cdot) = x^{\text{efficient}}$ , if and only if the pollution tax is a



Pigouvian tax, i.e., equal to the marginal harm at the efficient pollution level,<sup>3</sup>

$$t = t^{\text{Pigouvian}} := -f'(x^{\text{efficient}}). \quad (10)$$

Combining this insight with the finding that higher taxes cause lower pollution, cf. (9), we obtain that the industry emits inefficiently high pollution for lower taxes,  $x_{\neg\text{con.}}^*(\cdot) > x^{\text{efficient}}$  if  $t < t^{\text{Pigouvian}}$ , and inefficiently low pollution for higher taxes,  $x_{\neg\text{con.}}^*(\cdot) < x^{\text{efficient}}$  if  $t > t^{\text{Pigouvian}}$ .

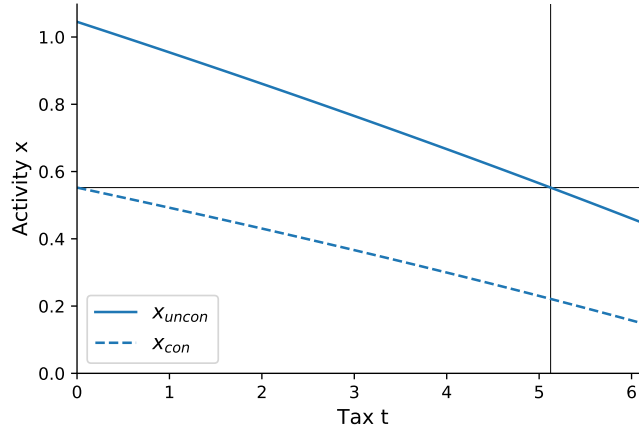


Figure 2: The activity level for unconnected parties  $x_{\neg\text{con.}}^*$  (solid line) and connected parties  $x_{\text{con.}}^*$  (dashed line) as a function of tax  $t$ . The vertical line is at the Pigouvian tax level  $t^{\text{Pigouvian}}$ , the horizontal line shows the efficient activity level  $x^{\text{efficient}}$ .

### 2.3 Equilibrium with Connected Parties

Consider next the case where the parties are connected. They then negotiate – in a Coasian way – to maximize their joint net profits:

$$x_{\text{con.}}^*(\cdot) = \underset{x \in \mathbb{R}^+}{\text{argmax}} \pi_F + \pi_I^{\text{net}}. \quad (11)$$

The equilibrium pollution level  $x_{\text{con.}}^*(\cdot)$  exists and is uniquely provided by the first-order condition

$$f'(x) + i'(x) - t = 0. \quad (12)$$

<sup>3</sup>This can also be seen from the damage form: the Pigouvian tax  $t^{\text{Pigouvian}} = d'(x^{\text{efficient}})$  implements the efficient pollution.

Implicit differentiation yields

$$\frac{\partial x_{\text{con.}}^*(\cdot)}{\partial t} = \frac{1}{f''(\cdot) + i''(\cdot)} < 0. \quad (13)$$

Thus, also with connected parties do we obtain the result that the industry emits less pollution with a higher tax.

By comparing (5) and (12), the equilibrium pollution level is efficient,  $x_{\text{con.}}^*(\cdot) = x^{\text{efficient}}$ , if and only if the tax is zero,  $t = 0$ . Together with (13), this implies that for any positive pollution tax  $t > 0$ , the equilibrium pollution is inefficiently low:  $x_{\text{con.}}^*(\cdot) < x^{\text{efficient}}$ . Intuitively, when parties are connected, we know from Coase that they implement the efficient pollution without at tax, i.e., with  $t = 0$ ; if additionally a positive tax is imposed, the parties have an incentive to avoid too much pollution.<sup>4</sup>

It is interesting to compare the equilibrium pollution with and without connection. By (8) and (12), for any tax (whether it is zero or not), equilibrium pollution is lower when the parties are connected than when they are not connected:

$$x_{\text{con.}}^*(\cdot) < x_{\text{-con.}}^*(\cdot) \text{ for all } t. \quad (14)$$

This result is intuitive. If the parties are connected, they have an incentive to lower pollution to reduce tax payments *and* raise the fisher's profit, whereas with unconnected parties only the former aspect is relevant. Figure 2 summarizes the findings of sections 2.2 and 2.3.

## 2.4 Optimal Regulation

The regulator's objective is to maximize the expected welfare over the tax. Unfortunately, the regulator does not know whether the parties are in a Coasian world or not, i.e., whether the parties are connected or not.<sup>5</sup> Denoting the probability that parties are connected (or will be connected) by  $p$ , the expected welfare is

$$E[W|t] = p [f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot))] + (1 - p) [f(x_{\text{-con.}}^*(\cdot)) + i(x_{\text{-con.}}^*(\cdot))]. \quad (15)$$

First, note that an optimum always exists. This holds true since (i) the objective function is continuous in the choice variable  $t$  and (ii) we can restrict our search for an optimal tax  $t^*$  to the closed and bounded interval  $[0, t^{\text{Pigouvian}}]$ . To see the latter,

<sup>4</sup>The latter result is known as the Buchanan-Stubblebine-Turvey Theorem, see Buchanan and Stubblebine (1962) and Turvey (1963).

<sup>5</sup>As we discuss in Section 2.6, there are alternative interpretations of this setting.

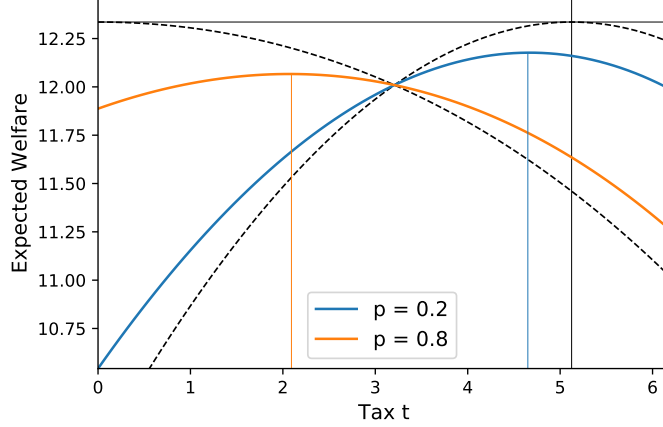


Figure 3: The regulator's objective function for exogenously given connection probability  $p$  as a function of tax  $t$ . The black vertical line indicates the Pigouvian tax level  $t^{\text{Pigouvian}}$ , the horizontal line is the efficient welfare level  $W^{\text{efficient}}$ . The dashed lines show the squared brackets in (15): The first,  $f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot))$ , is increasing in tax  $t$ , whereas the second,  $f(x_{\neg\text{con.}}^*(\cdot)) + i(x_{\neg\text{con.}}^*(\cdot))$ , is decreasing in tax  $t$  in the interval  $[0, t^{\text{Pigouvian}}]$ . The figure also illustrates that expected welfare falls short of the efficient welfare  $W^{\text{efficient}}$  whenever  $p \in (0, 1)$ , cf. Proposition 4.

note that  $f(x) + i(x)$  is concave and by (7)-(13) any tax  $t < 0$  is hence dominated by  $t = 0$  and any tax  $t > t^{\text{Pigouvian}}$  dominated by  $t = t^{\text{Pigouvian}}$ , no matter whether the parties are connected or not. Figure 3 shows expression (15) for two connection probabilities. In the optimum, the first-order condition has to be satisfied:

$$\begin{aligned} \frac{\partial E[W|t]}{\partial t} &= p [f'(x_{\text{con.}}^*(\cdot)) + i'(x_{\text{con.}}^*(\cdot))] \frac{\partial x_{\text{con.}}^*(\cdot)}{\partial t} \\ &+ (1-p) [f'(x_{\neg\text{con.}}^*(\cdot)) + i'(x_{\neg\text{con.}}^*(\cdot))] \frac{\partial x_{\neg\text{con.}}^*(\cdot)}{\partial t}. \end{aligned} \quad (16)$$

The first square bracket,  $[f'(x_{\text{con.}}^*(\cdot)) + i'(x_{\text{con.}}^*(\cdot))]$ , equals  $t$ , cf. (12), and is thus zero for  $t = 0$ , increasing in  $t$ , and hence positive for all  $t > 0$ . The second square brackets,  $[f'(x_{\neg\text{con.}}^*(\cdot)) + i'(x_{\neg\text{con.}}^*(\cdot))]$ , is zero for  $t = t^{\text{Pigouvian}}$ , increasing in  $t$ , and thus negative for all  $t < t^{\text{Pigouvian}}$ . These observations imply that for the special case  $p = 1$ , the optimal tax is  $t^* = 0$ , while in case  $p = 0$  it is  $t^* = t^{\text{Pigouvian}}$ . For all  $p \in (0, 1)$ , however, we must have that the optimal tax is  $t^* \in (0, t^{\text{Pigouvian}})$ . This holds true since for all  $p \in (0, 1)$ , (i) both square brackets have positive weight in the first-order condition (16) and (ii) for any  $t$  the square brackets cannot simultaneously be

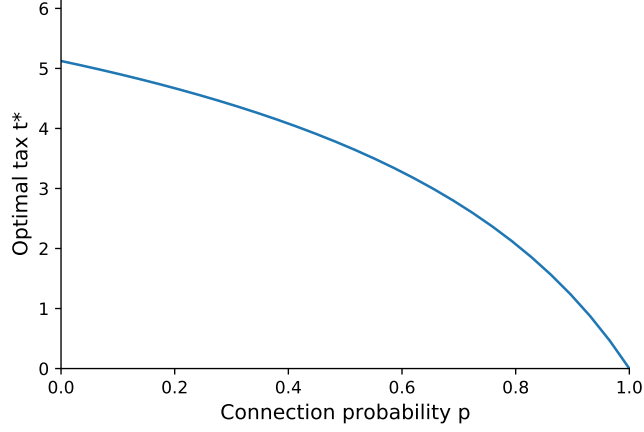


Figure 4: The optimal tax  $t^*$  decreases in the exogenously given connection probability  $p$ , cf. Proposition 3.

zero. The brackets must hence have opposite signs in order to solve the first-order condition, which can hold true only if  $t^* \in (0, t^{\text{Pigouvian}})$ .

**Proposition 1** *Whenever there is uncertainty, i.e.,  $p \in (0, 1)$ , the regulator optimally sets a positive pollution tax  $t^*$  that is below the Pigouvian tax:  $t^* \in (0, t^{\text{Pigouvian}})$ .*

Proposition 1 shows that with uncertainty, the regulator optimally sets a tax that is relatively low, compared to the Pigouvian tax. Combining Proposition 1 with (7)-(13) yields the following result.

**Proposition 2** *Whenever there is uncertainty, i.e.,  $p \in (0, 1)$ , the regulator sets a pollution tax  $t^*$  that never implements the efficient pollution:*

$$x_{\text{con.}}^*(t^*), x_{-\text{con.}}^*(t^*) \neq x^{\text{efficient}}.$$

The intuition for the result is that due to the uncertainty the regulator faces, she sets a pollution tax that is too low in case the parties are not connected and too high in case they are connected. As a consequence, the industry chooses an inefficient pollution level in all circumstances. We next explore how the optimal tax  $t^*$  evolves with the probability that parties are connected  $p$ .

**Proposition 3** *The optimal tax  $t^*$  is decreasing in the probability that parties are connected  $p$ .*

Proof: From (16) we get that

$$\frac{\partial^2 E[W|t]}{\partial t \partial p} < 0 \quad (17)$$

for all potentially optimal taxes  $t \in (0, t^{\text{Pigouvian}})$ . Consider two arbitrary probabilities  $p_1$  and  $p_2$  with  $p_1 < p_2$  and denote the optimally set taxes  $t_1^*$  respectively  $t_2^*$ . By (17), it cannot be optimal to set the same tax  $t_1^* = t_2^*$  for  $p_1$  and  $p_2$ , since then  $\partial E[W|t]/\partial t = 0$  cannot hold for both probabilities  $p_1$  and  $p_2$ . Finally,  $t_1^* < t_2^*$  cannot hold either, since by (17) then the additional expected welfare from setting  $t_2^*$  instead of  $t_1^*$  is smaller with probability  $p_2$  than with probability  $p_1$ :

$$\Delta E[W|p_2] = \int_{t=t_1^*}^{t_2^*} \frac{\partial E[W|t, p_2]}{\partial t} dt < \int_{t=t_1^*}^{t_2^*} \frac{\partial E[W|t, p_1]}{\partial t} dt = \Delta E[W|p_1]. \quad (18)$$

Hence, setting tax  $t_1^*$  and/or  $t_2^*$  cannot be optimal, a contradiction. Thus,  $t_1^* > t_2^*$ .  $\square$

This result is intuitive. With a higher probability  $p$ , the parties are more likely in a Coasian relationship where the tax is unhelpful in achieving efficiency. Accordingly, the regulator optimally sets a lower tax  $t^*$ . Figure 4 illustrates this relation between connection probability  $p$  and optimal tax  $t^*$ .

The former insights imply that with uncertainty, the optimal tax causes a welfare loss – compared to the welfare with efficient pollution – for sure. That is, due to the uncertainty the regulator faces, a welfare loss will realize no matter whether the parties turn out to be connected or not. This is formally described in the next proposition.

**Proposition 4** *The expected welfare  $E[W|t^*(p), p]$  is continuous and strictly convex in the probability  $p$  that parties are connected. With the optimally set tax  $t^*$ , the expected welfare  $E[W|t^*]$  is maximal and equal to the efficient level if and only if there is no uncertainty,*

$$E[W|t^*]_{p=0} = E[W|t^*]_{p=1} = W^{\text{efficient}},$$

*while it is below the maximum whenever there is uncertainty,*

$$E[W|t^*]_p < W^{\text{efficient}} \text{ for all } p \in (0, 1).$$

Proof: The value function  $E[W|t^*(p), p]$  is continuous by Berge's Maximum Theorem: the function  $(t, p) \rightarrow E[W|t, p]$  is continuous and we know that the optimal

tax, irrespective of the parameter  $p$ , always lies in the compact interval  $[0, t^{\text{Pigouvian}}]$ . The constraint correspondence is therefore compact and (as a constant correspondence) continuous, meeting the requirements of Berge's Maximum Theorem.

To show strict convexity, we calculate

$$\frac{d^2 E[W|t^*(p), p]}{dp^2} = \frac{\partial^2 E[W|t^*(p), p]}{\partial p^2} + \left[ \frac{\partial^2 E[W|t^*(p), p]}{\partial p \partial t} + \frac{\partial^2 E[W|t^*(p), p]}{\partial t^2} \cdot \frac{dt^*(p)}{dp} \right] \cdot \frac{dt^*(p)}{dp}.$$

It is  $\partial_p^2 E[W|t^*(p), p] = 0$ , due to the optimality of  $t^*$  we have  $\partial_t^2 E[W|t^*(p), p] \leq 0$ , and from proposition 3 we know  $\partial_p \partial_t E[W|t^*(p), p] < 0$  and  $\frac{dt^*(p)}{dp} < 0$ . Together, this shows  $d^2 E[W|t^*(p), p]/dp^2 > 0$  and thus strict convexity.

That the welfare levels at  $p = 0$  and  $p = 1$  correspond to the efficient level is directly apparent and intuitive: the regulator knows for certain whether to impose a Pigouvian tax (if  $p = 0$ ) or let the affected yet connected parties engage in Coasian bargaining undisturbed by any tax (if  $p = 1$ ). For intermediate probability levels  $0 < p < 1$ , strict convexity directly shows that expected welfare is strictly below the efficient welfare level  $W^{\text{efficient}}$ .  $\square$

## 2.5 Example: Linear-Quadratic Model

In the example where the fishers' profit is linear and the industry's profit is quadratic we obtain closed-form solutions of the implemented pollutions, the optimal tax, and the resulting expected welfare. We obtain the following main insights. See the Appendix for details. Our results can be summarized as follows: First, the optimal tax  $t^*$  is linear in the probability that parties are connected  $p$ . To be precise, the optimal tax is  $t^* = (1 - p)\delta$ , where  $\delta$  denotes the marginal damage. Second, we can quantify the loss of expected welfare. In a specific example, we illustrate that the loss is non-negligible and could amount up to one-quarter of the efficient level. The maximal loss is obtained when  $p = .5$ , i.e., when it is equally likely that parties are connected or not.

## 2.6 Remark on Probability $p$

The probability  $p$ , which measures how likely the parties are (or will be) in a Coasian relationship where they can negotiate with each other (i.e., are connected), has an

interesting alternative interpretation. Suppose there are many industry-fisher pairs. Some pairs are connected while others are not. The probability  $p$  can also be interpreted as the share of industry-fisher pairs that are connected. The regulator might or might not know whether a certain pair is connected or not. Our previous analysis stays completely valid: (i) in case the regulator does not have this knowledge and (ii) also in case she has the knowledge, provided that the regulator cannot impose discriminatory regulatory practices.

### 3 Endogenous Connections

#### 3.1 Optimal Decisions

We next endogenize parties' connections. To formalize this, we suppose that there are costs of connecting  $c \in \mathbb{R}^+$ . The parties can decide to invest these costs, such that they are connected, or not invest, such that they are not connected. The costs  $c$  could be interpreted as parties' costs to form a relationship and/or the costs of negotiations. We suppose that nature initially draws costs  $c$  from the cumulative distribution function  $G$ . We suppose that  $G$  is continuously differentiable, such that a density function  $g$  exists, and that the density is continuous and positive in the relevant range.

If the parties are connected, they negotiate to maximize their joint net profits

$$\pi_{\text{joint, con.}} = f(x) + i(x) - tx - c. \quad (19)$$

Because the connection costs  $c$  are fixed at the stage of the negotiation, they do not alter the parties' pollution decisions. Accordingly, the parties optimally negotiate to the pollution level  $x_{\text{con.}}^*(\cdot)$  characterized by (11) and the first-order condition (12). Given the optimal negotiation, the parties' joint net profits are thus

$$\pi_{\text{joint, con.}}^* = f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot)) - tx_{\text{con.}}^*(\cdot) - c. \quad (20)$$

If the parties are not connected, the industry will maximize her net profits  $\pi_I^{\text{net}}$  by choosing the pollution level  $x_{\neg\text{con.}}^*(\cdot)$  characterized by (7) and (8). Accordingly, the parties' joint net profits are then

$$\pi_{\text{joint, } \neg\text{con.}}^* := f(x_{\neg\text{con.}}^*(\cdot)) + i(x_{\neg\text{con.}}^*(\cdot)) - tx_{\neg\text{con.}}^*(\cdot). \quad (21)$$

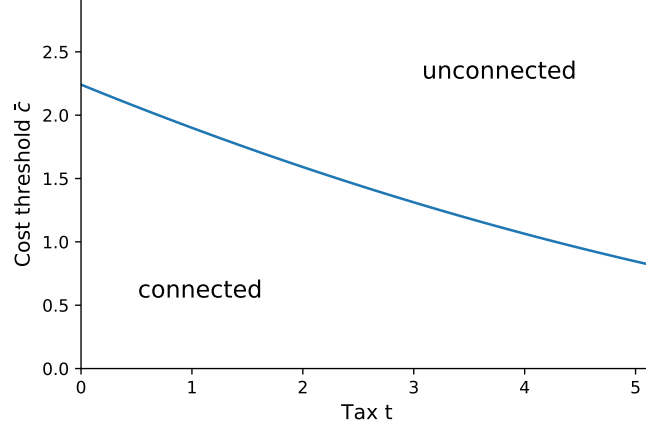


Figure 5: The cost threshold  $\bar{c}$  as a function of tax  $t$ . Polluter and victim negotiate to the activity level  $x_{\text{con}}^*$  if their connection costs are below the threshold  $\bar{c}$ ; they do not negotiate otherwise, giving rise to activity level  $x_{\neg\text{con}}^*$ . Importantly, and marking the main difference to the exogenous connection setting in section 2, whether polluter and victim are connected or not depends on the regulator's tax level.

The parties optimally decide to connect if and only if this is profitable:<sup>6</sup>

$$\pi_{\text{joint, con}}^* \geq \pi_{\text{joint, } \neg\text{con}}^* \quad (22)$$

By (20) and (21), this holds true if and only if

$$c \leq \bar{c}(\cdot) := f(x_{\text{con}}^*(\cdot)) + i(x_{\text{con}}^*(\cdot)) - tx_{\text{con}}^*(\cdot) - f(x_{\neg\text{con}}^*(\cdot)) - i(x_{\neg\text{con}}^*(\cdot)) + tx_{\neg\text{con}}^*(\cdot). \quad (23)$$

Thus, connecting is optimal for the parties as long as the connection costs do not exceed the difference in the joint net profits excluding negotiation costs. Observe that the threshold  $\bar{c}(\cdot)$  is positive, because the pollution  $x_{\text{con}}^*(\cdot)$  maximizes the parties' joint net profits, whereas the pollution  $x_{\neg\text{con}}^*(\cdot)$  does not. We can summarize.

**Proposition 5** *With connection costs  $c$ , the parties decide to connect if and only if the costs satisfy  $c \leq \bar{c}(\cdot)$ , where  $\bar{c}(\cdot)$  is defined in (23) and positive. If the parties connect, the pollution level  $x_{\text{con}}^*(\cdot)$  is implemented, while  $x_{\neg\text{con}}^*(\cdot)$  is implemented otherwise.*

Figure 5 shows for our numerical example how the cost threshold depends on the tax level  $t$ .

<sup>6</sup>We let parties connect in case of indifference. This assumption is not important for our results.



### 3.2 Cost Threshold and Tax

The tax influences the pollution levels and may, via this channel, affect the parties' connection decisions. Differentiating the cost threshold with respect to the tax and simplifying by using (8), (12), and the Envelope Theorem yields

$$\frac{\partial \bar{c}(\cdot)}{\partial t} = \underbrace{x_{\neg\text{con.}}^*(\cdot) - x_{\text{con.}}^*(\cdot)}_{>0} - \underbrace{f'(x_{\neg\text{con.}}^*(\cdot)) \frac{\partial x_{\neg\text{con.}}^*(\cdot)}{\partial t}}_{<0}. \quad (24)$$

We can see from (24) that a higher tax has two opposing effects on the cost threshold. First, a positive effect, since a higher tax increases the taxes saved from being connected due to having only pollution  $x_{\text{con.}}^*(\cdot)$  instead of  $x_{\neg\text{con.}}^*(\cdot)$ . We call this the *tax-saving effect*. Second, a negative effect, since a higher tax reduces the pollution level – and thereby the harm experienced by the fisher – also without being connected, which makes connecting less relevant. We call this the *reduced-externality effect*. The overall effect of a higher tax on the threshold, therefore, depends on whether the tax-saving effect is stronger than the reduced-externality effect. If a higher tax causes a rather mild reduction of pollution  $x_{\neg\text{con.}}^*(\cdot)$ , the reduced-externality effect is weak and dominated by the tax-saving effect such that the overall effect is positive,  $\partial \bar{c}(\cdot)/\partial t > 0$ . It is vice versa if a higher tax causes a rather strong reduction of the pollution  $x_{\neg\text{con.}}^*(\cdot)$ . Figure 6 shows the decomposition of  $\frac{\partial \bar{c}(\cdot)}{\partial t}$  into tax-saving effect and reduced-externality effect.

Can we say more about which of the two effects dominates? The Second Fundamental Theorem of Calculus allows us to write

$$x_{\neg\text{con.}}^*(t) = x_{\neg\text{con.}}^*(0) + \int_{\tau=0}^t \frac{\partial x_{\neg\text{con.}}^*(\tau)}{\partial \tau} d\tau. \quad (25)$$

For tax  $\tilde{t} = t - f'(x_{\text{con.}}^*(t))$ , where  $\tilde{t} > t$ , we have by (8) and (12) that  $x_{\text{con.}}^*(t) = x_{\neg\text{con.}}^*(\tilde{t})$ . Thus,

$$x_{\text{con.}}^*(t) = x_{\neg\text{con.}}^*(\tilde{t}) = x_{\neg\text{con.}}^*(0) + \int_{\tau=0}^{\tilde{t}} \frac{\partial x_{\neg\text{con.}}^*(\tau)}{\partial \tau} d\tau. \quad (26)$$

Hence,

$$x_{\text{con.}}^*(t) - x_{\neg\text{con.}}^*(t) = \int_{\tau=t}^{\tilde{t}} \frac{\partial x_{\neg\text{con.}}^*(\tau)}{\partial \tau} d\tau. \quad (27)$$

We can plug this into (24) to get

$$\frac{\partial \bar{c}(\cdot)}{\partial t} = - \int_{\tau=t}^{t-f'(x_{\text{con.}}^*(t))} \frac{\partial x_{\neg\text{con.}}^*(\tau)}{\partial \tau} d\tau - f'(x_{\neg\text{con.}}^*(t)) \frac{\partial x_{\neg\text{con.}}^*(t)}{\partial t}. \quad (28)$$

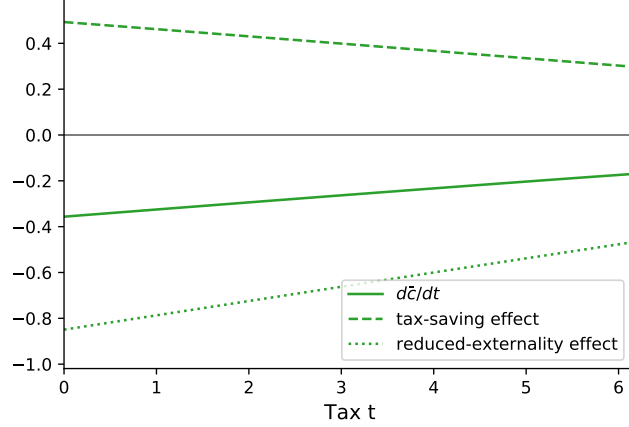


Figure 6: The decomposition of  $\frac{\partial \bar{c}(\cdot)}{\partial t}$  into tax-saving effect and reduced-externality effect, cf. expression (24). Increasing the tax has two countervailing effects: the tax-saving effect increases the cost-threshold  $\bar{c}$ , thus making Coasian bargaining more attractive, whereas the reduced-externality effect decreases the cost-threshold.

In the special case where  $i''$  is constant, also  $\partial x_{\text{con.}}^*(t)/\partial t$  is constant by (9), and we can simplify to

$$\frac{\partial \bar{c}(\cdot)}{\partial t} = \underbrace{\frac{\partial x_{\text{con.}}^*(t)}{\partial t}}_{<0} \underbrace{[f'(x_{\text{con.}}^*(t)) - f'(x_{\text{con.}}^*(t))]}_{\geq 0 \text{ (}>0 \text{ if } f'' < 0)} \leq 0. \quad (29)$$

Thus, if the second derivative of the industry's gross profit function is constant, which holds true for instance in the case of linear or quadratic profits, the cost threshold is at least weakly decreasing in the tax. It is thus weakly less likely that parties connect for higher taxes. Formally,  $G(\bar{c}(\bar{t}, \cdot)) \leq G(\bar{c}(t, \cdot))$  for all  $t < \bar{t}$ . It is readily verified that this insight stays valid also if  $i''$  is non-decreasing, i.e.,  $i''' \geq 0$ .

**Proposition 6** *A higher tax has two opposing effects on the cost threshold  $\bar{c}(\cdot)$ : a tax-saving effect, which positively affects the threshold, and a reduced-externality effect, which negatively affects the threshold. If  $i''$  is constant or  $i''' \geq 0$ , the reduced-externality effect dominates,  $\partial \bar{c}(\cdot)/\partial t \leq 0$ .*

Recall that the functional form underlying Figure 6 is  $i(x) = -0.5x^3 - 4x^2 + 10x$ , i.e.  $i''' < 0$ . This demonstrates that the cost-threshold can fall in tax  $t$  even though  $i''' < 0$ . In other words, condition  $i''' \geq 0$  in Proposition 6 for the reduced-externality effect to dominate is sufficient, not necessary.

### 3.3 Optimal Regulation, Equilibrium Pollution and Welfare

With endogenous connections, welfare is

$$W = f(x) + i(x) - \mathbb{1}_{\text{con.}} c, \quad (30)$$

where  $\mathbb{1}_{\text{con.}}$  is an indicator function taking value 1 if parties connect and connection costs  $c$  accrue. The regulator seeks to maximize the expected welfare

$$\begin{aligned} E[W|t] &= G(\bar{c}(\cdot)) [f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot))] - \int_{c=0}^{\bar{c}(\cdot)} cg(c)dc \\ &+ (1 - G(\bar{c}(\cdot))) [f(x_{\neg\text{con.}}^*(\cdot)) + i(x_{\neg\text{con.}}^*(\cdot))]. \end{aligned} \quad (31)$$

We first show that setting a negative tax could never be optimal.

**Lemma 1** *The expected welfare with tax  $t = 0$  is higher than with any negative tax:  $E[W|t = 0] > E[W|\tilde{t}]$  for all  $\tilde{t} < 0$ .*

*Proof: Case 1:  $\bar{c}(t = 0) = \bar{c}(\tilde{t})$ .* Then, for every possible cost realization  $c$ , the parties' connection decisions and thus the connection costs are the same with  $t = 0$  and with  $\tilde{t}$ . Since  $i(x) + f(x)$  is concave and by (7)-(13), the tax  $t = 0$  causes a higher welfare than any tax  $\tilde{t} < 0$ , no matter whether parties are connected or not. Hence,  $W|_{t=0} > W|_{\tilde{t}}$ .

*Case 2:  $\bar{c}(t = 0) < \bar{c}(\tilde{t})$ .* For cost realizations  $c \leq \bar{c}(t = 0)$  and  $c > \bar{c}(\tilde{t})$ , the arguments from Case 1 apply and so welfare is higher with tax  $t = 0$  than with tax  $\tilde{t} < 0$ . For cost realizations  $c \in (\bar{c}(t = 0), \bar{c}(\tilde{t})]$ , welfare is

$$W|_{t=0} = f(x_{\neg\text{con.}}^*(t = 0)) + i(x_{\neg\text{con.}}^*(t = 0)) \quad (32)$$

with tax  $t = 0$ , while it is

$$W|_{\tilde{t}} = f(x_{\text{con.}}^*(\tilde{t})) + i(x_{\text{con.}}^*(\tilde{t})) - c \quad (33)$$

with tax  $\tilde{t} < 0$ . Since  $c > \bar{c}(t = 0)$  we have by (23) that

$$c > f(x_{\text{con.}}^*(t = 0)) + i(x_{\text{con.}}^*(t = 0)) - f(x_{\neg\text{con.}}^*(t = 0)) - i(x_{\neg\text{con.}}^*(t = 0)). \quad (34)$$

Moreover, since  $i(x) + f(x)$  is concave and by (7)-(13), for any  $\tilde{t} < 0$ ,

$$f(x_{\text{con.}}^*(\tilde{t})) + i(x_{\text{con.}}^*(\tilde{t})) < f(x_{\text{con.}}^*(t = 0)) + i(x_{\text{con.}}^*(t = 0)). \quad (35)$$

Combining (32)-(35) yields that

$$\begin{aligned}
W|_{\tilde{t}} &< f(x_{\text{con.}}^*(\tilde{t})) + i(x_{\text{con.}}^*(\tilde{t})) - f(x_{\text{con.}}^*(t=0)) - i(x_{\text{con.}}^*(t=0)) \\
&\quad + f(x_{\text{-con.}}^*(t=0)) + i(x_{\text{-con.}}^*(t=0)) < \\
&\quad f(x_{\text{-con.}}^*(t=0)) + i(x_{\text{-con.}}^*(t=0)) = W|_{t=0}. \quad (36)
\end{aligned}$$

*Case 3:*  $\bar{c}(t=0) > \bar{c}(\tilde{t})$ . By essentially the same arguments as in Case 2, we again have  $W|_{t=0} > W|_{\tilde{t}}$ .

Finally, since  $W|_{t=0} > W|_{\tilde{t}}$  holds true for all three possible cases and all taxes  $\tilde{t} < 0$ , as shown above, we have  $E[W|t=0] > E[W|\tilde{t}]$ .  $\square$

Lemma 1 implies that we can rule out all negative taxes as potential optima. A sufficient condition for the existence of an optimal tax  $t^*$  is that  $\lim_{x \searrow 0} f(x) + i(x) = -\infty$  or that the limit is sufficiently low.<sup>7</sup> Using Leibniz's Rule and (23), the first-order condition of the regulator's problem can be written as

$$\begin{aligned}
\frac{\partial E[W|t]}{\partial t} &= G(\bar{c}(\cdot)) [f'(x_{\text{con.}}^*(\cdot)) + i'(x_{\text{con.}}^*(\cdot))] \frac{\partial x_{\text{con.}}^*(\cdot)}{\partial t} \\
&\quad + (1 - G(\bar{c}(\cdot))) [f'(x_{\text{-con.}}^*(\cdot)) + i'(x_{\text{-con.}}^*(\cdot))] \frac{\partial x_{\text{-con.}}^*(\cdot)}{\partial t} \\
&\quad - g(\bar{c}(\cdot)) \frac{\partial \bar{c}(\cdot)}{\partial t} t (x_{\text{-con.}}^*(\cdot) - x_{\text{con.}}^*(\cdot)). \quad (37)
\end{aligned}$$

We can now explore whether the endogeneity of connections causes higher or lower taxes than in the benchmark model of Section 2. Let us define  $\tilde{p} := G(\bar{c}(t_{\text{end.}}^*, \cdot))$ , where  $t_{\text{end.}}^*$  is the optimal tax with endogenous connections. We can rewrite (37) as

$$\frac{\partial E[W|t]}{\partial t} = \frac{\partial E[W|t]}{\partial t} \Big|_{\text{ex.}} - \underbrace{g(\bar{c}(\cdot)) \frac{\partial \bar{c}(\cdot)}{\partial t} t (x_{\text{-con.}}^* - x_{\text{con.}}^*)}_{\text{same sign as } \partial \bar{c}(\cdot) / \partial t \text{ for all } t > 0}. \quad (38)$$

We hence obtain the result that if the tax lowers the parties' cost threshold, it is optimal to set a higher tax than in the base model of Section 2, i.e.,  $t_{\text{end.}}^* > t_{\text{ex.}}^*$ . In contrast, a lower tax is optimal, i.e.,  $t_{\text{end.}}^* < t_{\text{ex.}}^*$ , if the tax increases the cost threshold. To understand these results, recognize that the parties connect for a too large set of costs for any positive tax.<sup>8</sup> Intuitively, while the parties take into account the tax-saving

<sup>7</sup>The idea is that an infinitely large tax  $t \rightarrow \infty$  implements pollutions approaching zero, which is undesirable for the regulator if  $\lim_{x \searrow 0} f(x) + i(x) = -\infty$  or  $\lim_{x \searrow 0} f(x) + i(x)$  is sufficiently low.

<sup>8</sup>To see, suppose a social planner can determine whether parties connect, but not the decision process afterwards (i.e., the chosen pollution levels). The social planner compares the welfare without connection

effect of connecting, the tax effect plays no role for the regulator. By setting a relatively high [low] tax when a higher tax lowers [increases] the parties' cost threshold, the regulator discourages parties from connecting and thus mitigates this problem.

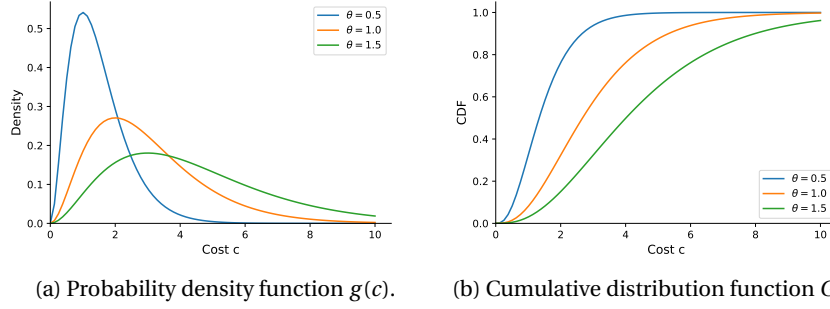


Figure 7: Gamma distribution. Density  $g(c) = \frac{1}{\Gamma(k) \cdot \theta^k} \cdot c^{(k-1)} \cdot e^{-c/\theta}$  (left panel) and cumulative distribution function (right panel). The shape parameter is  $k = 3$  in all plots and the scale parameter  $\theta$  varies.

We next evaluate the derivative at tax zero. In case  $t = 0$ , we can use (5) and (12) to simplify (37) to

$$\left. \frac{\partial E[W|t]}{\partial t} \right|_{t=0} = (1 - G(\bar{c}(\cdot))) f'(x_{\text{con.}}^*(\cdot)) \frac{\partial x_{\text{con.}}^*(\cdot)}{\partial t}, \quad (39)$$

which is positive. Thus, the regulator could improve by increasing the tax such that a tax of zero could never be optimal.

We finally explore whether setting the Pigouvian tax can be optimal. For  $t^{\text{Pigouvian}}$ , we can use (12) to simplify (37) to

$$\begin{aligned} \left. \frac{\partial E[W|t]}{\partial t} \right|_{t=t^{\text{Pigouvian}}} &= \underbrace{G(\bar{c}(\cdot)) t^{\text{Pigouvian}} \frac{\partial x_{\text{con.}}^*(\cdot)}{\partial t}}_{<0} \\ &\quad - \underbrace{g(\bar{c}(\cdot)) \frac{\partial \bar{c}(\cdot)}{\partial t} t^{\text{Pigouvian}} (x_{\text{con.}}^*(\cdot) - x_{\text{con.}}^*(\cdot))}_{\text{same sign as } \partial \bar{c}(\cdot) / \partial t}. \end{aligned} \quad (40)$$

In case  $\partial \bar{c}(\cdot) / \partial t \geq 0$ , the derivative is negative such that the Pigouvian tax cannot be optimal. Together with (38) and (39), this directly implies that with endogenous connections, the regulator optimally sets a positive tax  $t_{\text{end}}^*$  that is weakly below the

$f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot))$  to the welfare with connection  $f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot)) - c$ . Connecting is thus socially optimal if and only if  $c \leq \bar{c}^{\text{social}} := f(x_{\text{con.}}^*(\cdot)) + i(x_{\text{con.}}^*(\cdot)) - f(x_{\text{con.}}^*(\cdot)) - i(x_{\text{con.}}^*(\cdot))$ . Applying (23) directly yields that  $\bar{c} > \bar{c}^{\text{social}}$  for all  $t > 0$  since  $x_{\text{con.}}^* > x_{\text{con.}}^*$ .

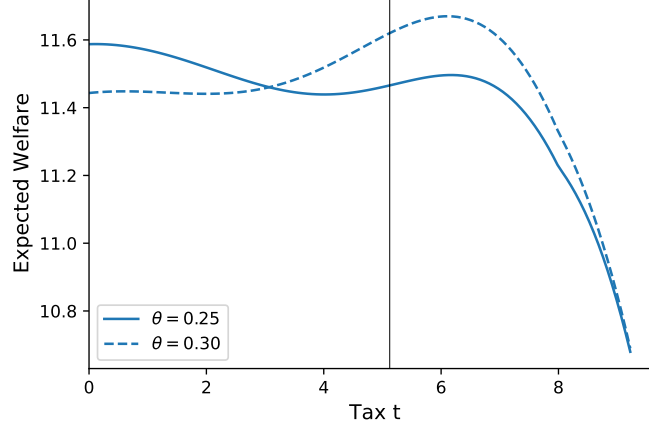


Figure 8: Expected welfare as a function of tax level  $t$  when polluter and victim can endogenously decide whether to connect or not. The connection cost is Gamma-distributed with scale parameter  $\theta$  and shape parameter  $k = 3$ , see Figure 7. The optimal tax for  $\theta = 0.25$  is  $t^* = 0.11$ , the optimal tax for  $\theta = 0.30$  is  $t^* = 6.11$ . See Figure 9 for a summary how the optimal tax depends on the distribution of connection costs.

optimal tax with exogenous connections  $t_{\text{ex}}^*$  for  $\tilde{p} := G(\bar{c}(\cdot))$ , i.e.,  $t_{\text{end}}^* \in (0, t_{\text{ex}}^*)$ .

In case  $\partial \bar{c}(\cdot) / \partial t < 0$ , the derivative provided by (40) is generically not zero. This holds true since a zero derivative requires that the fraction  $g(\bar{c}(\cdot)) / G(\bar{c}(\cdot))$  exactly equals the specific value  $\frac{\partial x_{\text{con}}^*(\cdot)}{\partial t} / \left( \frac{\partial \bar{c}(\cdot)}{\partial t} (x_{-\text{con}}^*(\cdot) - x_{\text{con}}^*(\cdot)) \right)$ . Thus, generically, the optimal tax does not equal the Pigouvian tax, i.e.,  $t_{\text{end}}^* \neq t^{\text{Pigouvian}}$ . Moreover, by (38) the regulator optimally sets a higher tax with endogenous than with exogenous connections, i.e.,  $t_{\text{end}}^* > t_{\text{ex}}^*$  for  $\tilde{p} := G(\bar{c}(\cdot))$ .

These insights together with the results of Sections 2.2 and 2.3 imply that the implemented equilibrium pollutions are generically inefficient:

$$x_{\text{con}}^*(t_{\text{end}}^*), x_{-\text{con}}^*(t_{\text{end}}^*) \neq x^{\text{efficient}}.$$

Accordingly, the realized welfare and thus also the expected welfare is below the efficient level:

$$E[W|t_{\text{end}}^*] < W^{\text{efficient}}. \quad (41)$$

**Proposition 7** *With endogenous connections, the regulator optimally sets a tax that is positive and generically unequal to the Pigouvian tax:  $t_{\text{end}}^* > 0$  and  $t_{\text{end}}^* \neq t^{\text{Pigouvian}}$ .*

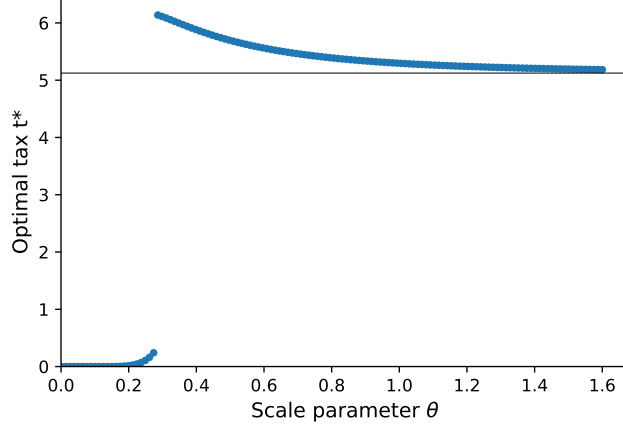


Figure 9: The optimal tax when polluter and victim can endogenously decide whether to connect or not.

*Equilibrium pollutions are generically inefficient:  $x_{\text{con.}}^*(t_{\text{end.}}^*), x_{\neg\text{con.}}^*(t_{\text{end.}}^*) \neq x^{\text{efficient}}$ . The expected welfare is below the efficient level:  $E[W|t_{\text{end.}}^*] < W^{\text{efficient}}$ . If  $\partial\bar{c}(\cdot)/\partial t \geq 0$ , the optimal tax is below the one with exogenous connections:  $t_{\text{end.}}^* < t_{\text{ex.}}^*$ , where  $\bar{p} := G(\bar{c}(\cdot))$ , while it is vice versa if  $\partial\bar{c}(\cdot)/\partial t < 0$ .*

What can we say about comparative statics? It is noteworthy that a cost distribution where high-cost realizations are more likely – and thus all else equal connections are less likely – does not necessarily cause a higher tax to be optimal. Formally, consider two distributions  $\tilde{G}$  and  $G$ , where  $\tilde{G}$  first-order stochastically dominates  $G$ , i.e.,  $\tilde{G}(c) \leq G(c)$  for all  $c$  and  $\tilde{G}(c) < G(c)$  for some  $c$ . The optimal tax  $t_{\text{end.}}^*$  need not be higher with  $\tilde{G}$  than with  $G$ . This can directly be seen from (37): if at the cost threshold  $\bar{c}$ , the distributions are relatively similar, i.e.,  $\tilde{G}(\bar{c}(\cdot)) \approx G(\bar{c}(\cdot))$ , while the density  $\tilde{g}(\bar{c}(\cdot))$  is sufficiently lower [higher] than  $g(\bar{c}(\cdot))$  in case  $\partial\bar{c}(\cdot)/\partial t < [>]0$ , the marginal effect of a higher tax on the expected welfare is lower with distribution  $\tilde{G}$  than with  $G$ . Formally,  $\left. \frac{\partial E[W|t]}{\partial t} \right|_{\tilde{G}} < \left. \frac{\partial E[W|t]}{\partial t} \right|_G$  holds such that the regulator sets a lower tax with  $\tilde{G}$  than with  $G$ .

Finally, we want to stress an interesting observation. Under specific circumstances, with endogenous connections the optimally set tax could exceed the Pigouvian tax. Consider a setting where, first,  $\partial\bar{c}(\cdot)/\partial t < 0$  such that the derivative in (40) has a chance to be positive. Second,  $G(\bar{c}(\cdot))$  is small, meaning that low-cost realizations and thus connections are rather unlikely (and so the optimal tax also with

exogenous connections is near the Pigouvian tax). Third,  $g(\bar{c}(\cdot))$  is sufficiently large, such that it is sufficiently important to discourage parties from connecting by setting a high tax.

We are able to demonstrate such a case with our numerical example,  $i(x) = -0.5x^3 - 4x^2 + 10x$  and  $f(x) = -x^3 - 2x^2 - 2x + 10$ . Recall that  $\partial \bar{c}(\cdot) / \partial t < 0$ , see Figure 5. Figure 9 shows the optimal tax in the endogenous connection model when connection costs are Gamma-distributed (see Figure 7 for the influence of scale parameter  $\theta$ ). The optimal tax is positive but near zero for low values of scale parameter  $\theta$ , i.e. when there is a lot of probability mass on low cost values so that it is very likely that the parties will connect. Once enough probability mass is on higher cost values, the optimal tax jumps above the Pigouvian level  $t^{\text{Pigouvian}}$ . Figure 8 demonstrated that there are typically several local optima. It is interesting to note that the optimal tax strongly exceeds the Pigouvian level for intermediate values of scale parameter  $\theta$  but converges to the Pigouvian tax as more probability weight is on large cost values. The reason is that high values of cost parameter  $c$  make it very likely that the parties are not connected; then the Pigouvian tax level is optimal. However, when the probability that parties are connected is sufficiently high, then it can make sense to apply a very high tax to discourage connection (recall: the cost threshold  $\bar{c}$  here decreases in tax level  $t$ ).

It is important to note that this effect is not always present. Indeed, for some functional forms, the optimal tax is always between zero and the Pigouvian level. One example is that the fisher's profit is linear and the industry's profit is quadratic, see the appendix. In this case, the cost threshold  $\bar{c}$  does not depend on the tax level  $t$  and the model is essentially again one of exogenous connection.

## 4 Conclusion

What is the right framework to think about emission problems or externalities in general? Is it the Coasian or the Pigouvian approach? Our novel contribution is to consider a model where a regulator is uncertain – as we economists are – about the right approach. To formalize this, we suppose that a regulator does not know whether parties can solve their externality problem themselves through Coasian bargaining or not. If they can – we say, if they are “connected” – then the regulator should not interfere, i.e. set a tax of zero; otherwise, the regulator should tax the polluting activity.

We studied two models: In the first model with “exogenous connection”, the reg-



ulator's action does not influence whether polluter and victim are connected or not. In contrast, our second model features "endogenous connection", i.e. polluter and victim decide whether they want to connect, i.e. whether the costs of connection – these can be seen as transaction costs – justify the gains from Coasian bargaining or not.

Our main findings are as follows: unless uncertainty is degenerate, optimal regulation in terms of maximizing *expected* welfare cannot avoid an ex-post welfare loss. In the model with exogenous connection, the regulator sets a tax between zero and the Pigouvian level: the higher the belief that polluter and victim are connected, the lower the tax. An interesting twist emerges with endogenous connection: the regulator's choice of tax level (to some extent) *determines* whether the parties are connected or unconnected. This lever can lead the regulator to set a tax *higher* than the Pigouvian level, because the efficiency loss from the too-high tax can be more than compensated by the gain from increasing the likelihood of the case favorable to high tax levels, namely that polluter and victim are not connected.

Our work adds a novel dimension to the extensive literature on external effects and bridges the two often isolated strands of literature on centralized approaches – in the tradition of Pigou (1920) – and decentralized approaches – in the tradition of Coase (1960). It has implications for instrument choice as well: The difficulty in setting the best tax level is restricted to the price instrument and does not occur with a quantity instrument: under the assumptions of our model at least, the regulator has perfect knowledge of the efficient activity level and can therefore easily choose the correct quantity instrument (e.g. the right number of permits).

It is worth mentioning that our model has an interesting alternative interpretation. Instead of uncertainty about the transaction costs of one specific polluter-victim-pair, think of a continuum of polluter-victim-pairs (without any interaction between different pairs). If a regulator cannot regulate every pair individually but rather needs to choose one uniform tax level for all of them, then our model directly applies to that situation with a fraction of  $p$  pairs connected and a fraction  $1 - p$  unconnected (section 2) or re-interpreting the probability distribution function as a distribution function of transaction costs in the continuum of polluter-victim-pairs.

In terms of future research, we see several extensions of our model worth studying. In this paper we have focused on taxing an activity with negative externalities; future research can uncover whether this directly applies to subsidizing activities with positive externalities. Another valuable model extension is to include uncertainty about pollution benefits and damages as in Weitzman (1974) and Kotchen & Costello (2022). Our finding that the quantity instrument fares better than the price

instrument has been derived in a setting deliberately simple: in particular, the regulator has perfect information about the marginal benefits and marginal damages of pollution. Future research should study whether the advantage of a quantity instrument persists in more realistic settings.

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## Appendix

### Analysis of Linear-Quadratic Model

Let the fisher's profit be linear

$$\pi_F = f(x) = -\delta x$$

and the industry's profit be quadratic

$$\pi_I = i(x) = \alpha x - \frac{1}{2}\beta x^2,$$

where  $\alpha, \beta, \delta > 0$  and  $\alpha \geq 2\delta$  in order to have interior solutions. Efficient pollution is

$$x^{\text{efficient}} = \frac{\alpha - \delta}{\beta}.$$

Equilibrium pollution in case parties are connected is

$$x_{\text{-con.}}^*(\cdot) = \frac{\alpha - t}{\beta} \text{ if } t \leq \alpha$$

and zero otherwise. The Pigouvian tax equals the marginal harm, i.e.,  $t^{\text{Pigouvian}} = \delta$ . Equilibrium pollution in case parties are not connected is

$$x_{\text{con.}}^*(\cdot) = \frac{\alpha - \delta - t}{\beta} \text{ if } t \leq \alpha - \delta$$

and zero otherwise. To maximize the expected welfare, the regulator optimally sets

$$t^* = (1 - p)\delta.$$

The optimal tax is thus linear in the probability that parties are connected. The implemented pollutions are hence  $x_{\text{-con.}}^*(t^*) = \frac{\alpha - (1-p)\delta}{\beta}$  and  $x_{\text{con.}}^*(t^*) = \frac{\alpha - \delta - (1-p)\delta}{\beta}$ . Only for  $p = 0$  and  $p = 1$  is the efficient pollution implemented and welfare maximal. To illustrate the loss of welfare, we examine the case where  $\alpha = \beta = 1, \delta = 1/2$ . The expected welfare is then

$$E[W|t] = .125 - .125p + .125p^2. \quad (42)$$

While the expected welfare equals the efficient level for  $p = 0$  and  $p = 1$ , it is minimal for  $p = 1/2$ . In the latter case, the expected welfare loss amounts to one-quarter of the efficient level.