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## **Optional linear input prices in vertical relations**

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# Optional linear input prices in vertical relations<sup>†</sup>

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## Abstract

This paper examines how the option of a regulated linear input price affects vertical contracting, where a monopolistic upstream supplier sequentially offers supply contracts to two symmetric downstream firms. We find that equilibrium contracts vary with production cost and regulated price level: If the regulated price is not too high, the option allows for price discrimination, but prevents foreclosure in the intermediary market. Indeed, if both cost and optional price are rather low, non-discriminatory input prices below cost may arise.

Optional input prices are socially more desirable than a flat ban on price discrimination, as consumers benefit from more intense downstream competition.

*JEL Classification:* D42, L11, L42

*Keywords:* price discrimination, vertical contracting, exclusion, regulatory outside option

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# 1 Introduction

When a monopolistic supplier of an essential input sequentially contracts with competing downstream firms, it might use price discrimination to monopolise the market or to engage in rent-shifting among these firms. As an example, consider electricity and gas retailing, or service-based competition in telecommunications. Here, the incumbent network operator might join in opportunism towards service providers when providing access. To prevent such behaviour, European legislation (Art. 82(c) TEC) and national regulatory authorities categorically prohibit discriminatory practices of market dominant firms. We question whether this approach leads to a socially desired outcome and propose potential regulation as an alternative.<sup>1</sup> Our idea is to curtail the supplier's ability to price discriminate by giving downstream firms the option of claiming a regulated linear input price. We examine how such a regulatory option affects supply contracts and evaluate its efficiency by comparing it to existent non-discrimination clauses.

There is a considerable literature dealing with opportunism in vertical contracting which also proposes solutions to the problem: E.g. Hart and Tirole (1990) suggest vertical integration and O'Brien and Shaffer (1992) non-overlapping geographic territories or a common resale price. These solutions seem rather rigorous compared to our approach of giving downstream firms an additional outside option.<sup>2</sup> In this regard, our idea is more similar to Marx and Shaffer (2004a) and Fontenay and Gans (2005). But while these look at alternative tariff options an upstream firm can offer or choose, we examine an outside option imposed by regulation. Note that additionally looking at other non-discrimination rules follows McAfee and Schwartz (1994) and, in particular, O'Brien and Shaffer (1994).

Our main insight is that the regulated option achieves higher social welfare than a flat ban on price discrimination in a dynamic market context, where downstream firms move sequentially. It prevents foreclosure, and further, results in lower retail prices than mandated uniform pricing, provided that production cost and the regulated price are not too high. In particular, consumer surplus and welfare increase, even though price discrimination is permitted.

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<sup>1</sup> This idea relates to the debate of a more economic approach to Article 82 TEC, see Gual, Hellwig et al. (2005) and Atkinson and Barnekov (2004).

<sup>2</sup> It borrows from experiences of U.S. and Australian telecommunications, see King and Maddock (1999).

These results are generated by the external threat of regulation and the sequential structure, given two-part tariffs and imperfect downstream competition: The optional regulated price restricts the supplier's per-unit price-offers and thus, prevents foreclosure if it is not too high. By sequentially settling on two-part tariff contracts, the supplier can extract profits from the downstream industry and enlarge its profit share by manipulating downstream competition. Combined, these two incentives induce the supplier to either minimize per-unit input prices and charge a fixed fee only, or to offer the second downstream firm a more favorable contract than its rival. In the latter case, the upstream firm exploits the strategic substitutability of input prices. But in principal, outcomes vary with the elasticity of final consumers' demand.

Also in McAfee and Schwartz (1994) uniform pricing in sequential vertical contracting may arise. Yet, incentives of their model differ from ours: In their setup, uniform input prices result from the fear of opportunism, given unobservable contracts. We, instead, suggest a regulatory outside option combined with contracts in two-part tariffs.

In the following, we will illustrate our reasoning in more detail: Section 2 introduces the basic model. Section 3 studies the equilibrium outcome under constrained price discrimination. Section 4 analyses the impact of non-discrimination clauses. Section 5 concludes. All proofs are relegated to the Appendix.

## 2 The Model

We consider a vertical market setup with a monopolistic upstream supplier  $N$  and two symmetric downstream firms  $S_1$  and  $S_2$ . Upstream supplier  $N$  produces a good that it sells to  $S_1$  and  $S_2$  for subsequent distribution to final consumers.  $N$  produces at marginal cost  $c$  with  $0 < c < 1$ .  $S_1$ 's and  $S_2$ 's marginal cost and all other fixed production cost are normalised to zero.

The downstream market is characterised by linear inverse demand  $p(X) = 1 - X$  with  $X = x_1 + x_2$ . Downstream firms compete in quantities. But before becoming active in this market, each downstream firm  $S_i$  has to settle on a supply contract with  $N$ . In order to look at firms' incentives to foreclose one of the downstream firms or to engage in rent shifting among them, we let contracting take place in sequential order.<sup>3</sup> Here, we assume

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<sup>3</sup> For simultaneous moves, see section 4.

that  $S_1$  is the first to negotiate its terms with the supplier. The contract itself specifies a two-part tariff, i.e. a per-unit price  $a_i$  and a fixed fee  $F_i$ , payable to the upstream supplier  $N$ .<sup>4</sup> To abstract from subsidies, we assume  $a_i \geq 0$  and  $F_i \geq 0$ .

Consistent with the market power of an upstream monopolist, we presume take-it-or-leave-it offers by supplier  $N$ . Upon receiving the offer, downstream firms may claim the regulatory outside option instead: If  $S_i$  disagrees with the offer, it can alternatively claim access at per-unit price  $a^R$ , which is exogeneously determined by regulation with  $a^R = c + \Delta$  and  $0 \leq \Delta < 1 - c$ . Thus, the option curtails the upstream firm's market power.

The timing of the game is illustrated in Figure 1:

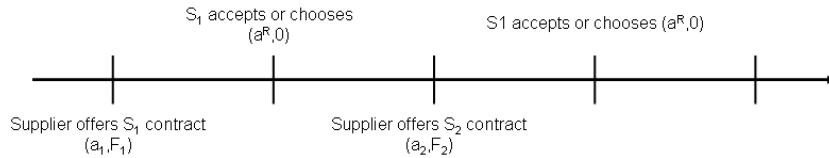


Figure 1: The Timing of the Game

First, supplier  $N$  offers firm  $S_1$  a contract  $(a_1, F_1)$  for the purchase of the good.  $S_1$  can accept this take-it-or-leave-it-offer or choose the regulated contract  $(a^R, 0)$  instead. In the second stage, after an agreement with the first downstream firm has been reached, the supplier likewise makes an offer  $(a_2, F_2)$  to firm  $S_2$ . Also  $S_2$  can accept or claim the regulated contract  $(a^R, 0)$ . In the third stage, downstream firms compete in the product market: Firms which have left the market earn zero. Firms which have stayed compete over the amount of service they deliver to final consumers and order inputs accordingly. Contracts are observable as we do not refer to issues arising with asymmetric information.

We determine the subgame perfect equilibria of the game by solving it backwards.

### 3 Equilibrium in view of optional linear access

If both downstream firms are active and compete with each other, their profits amount to

$$\Pi_i = (p - a_i) x_i - F_i. \quad (1)$$

<sup>4</sup> Non-linear pricing is a typical feature of supply contracts in network industries.

for given supply contracts  $(a_i, F_i)$  with  $i = 1, 2$ . Each downstream firm chooses its quantity  $x_i$  to maximise (1) subject to inverse demand  $P = 1 - X$  taking the competitor's quantity  $x_j$  as given. Simultaneously solving the two first-order conditions

$$p - a_i + \frac{\partial p}{\partial x_i} = 0, \quad (2)$$

we obtain equilibrium quantities

$$x_i^*(a_i, a_j) = \frac{1}{3}(1 - 2a_i + a_j) \quad (3)$$

with  $i, j = 1, 2$  and  $i \neq j$ . Hence, the optimal quantity and, therefore, individual downstream profits also, are strictly decreasing in the own input price  $a_i$ , but increasing with the rival's price  $a_j$ . This result is rather obvious since downstream firms compete in the same market and their products are perfectly substitutable. Note also that equilibrium price  $p^* > a_i$  which immediately follows from (2).

Supplier  $N$  anticipates the outcome in the downstream market when it determines the profit maximising per-unit price  $a_2$  in the preceding stage according to

$$a_2^*(a_1, a^R) = \arg \max_{a_2} \Pi_N = (a_1 - c)x_1^* + (a_2 - c)x_2^* + F_1 + F_2. \quad (4)$$

To determine  $F_2$ ,  $N$  considers  $S_2$ 's outside option to claim mandatory supply at per-unit price  $a^R$ . In this regard, let us denote downstream firm  $i$ 's sales revenues by  $\pi_i = (p - a_i)x_i$ . Then,  $S_2$  will only accept the contract, if

$$\pi_2(a_1, a_2) - F_2 \geq \pi_2(a_1, a^R).$$

This constraint becomes binding when  $N$  maximises its profits, so that

$$F_2 = \pi_2(a_1, a_2) - \pi_2(a_1, a^R). \quad (5)$$

Thus, the supplier cannot entirely shift profits towards itself, but has to concede  $\pi_2(a_1, a^R)$  to  $S_2$ .

Before  $N$  offers  $S_2$  a supply contract, it offers  $S_1$  a supply contract. Anticipating (4) and (5) in addition to (3), the supplier fixes the per-unit price  $a_1$  according to

$$a_1^*(a^R) = \arg \max_{a_1} \Pi_N(a_1, a_2(a_1, a^R), F_1, F_2(a_1, a^R)) \quad (6)$$

subject to the participation constraint

$$\pi_1(a_1, a_2(\cdot)) - F_1 \geq \pi_1(a^R, a_2(\cdot)).$$

Again, the constraint arises due to the regulatory outside option. It is binding, when supplier  $N$  maximises its profits, so that

$$F_1 = \pi_1(a_1, a_2(\cdot)) - \pi_1(a^R, a_2(\cdot)). \quad (7)$$

By solving the entire programme we obtain different types of equilibrium constellations. The marginal production cost and mandated wholesale price, i.e.  $c$  and  $\Delta(c)$ , determine equilibrium contracts.<sup>5</sup> Let us first look at rather low marginal production cost:

**Proposition 1.** *For  $c \leq 1/7$ , there is a  $\Delta_I(c)$  and a  $\Delta_{II}(c)$  with  $0 < \Delta_I < \Delta_{II}$  such that supply contracts of firms  $i = 1, 2$  have the following properties:*

- (i) *If  $\Delta \leq \Delta_I$ , then supply contracts are non-discriminatory with  $a_1 = a_2 = 0$  and  $F_1 = F_2 > 0$ .*
- (ii) *If  $\Delta_I < \Delta \leq \Delta_{II}$ , then supply contracts are discriminatory and  $S_2$  obtains the more favorable contract, s.t.  $a_1 > a_2$  and  $F_1 < F_2$ .*
- (iii) *If  $\Delta > \Delta_{II}$ , the supplier excludes one of the downstream firms. The active downstream firm's supply contract is given by  $a_i^m = c$  and  $F_i^m = \pi_i(a_i^m) - \pi_i(a^R)$ .*

Figure 1 illustrates how production cost and regulated markup combine to induce the different types of equilibrium supply contracts for  $c \leq 1/7$ . For parameter values  $c$  and  $\Delta$  that lie in region  $I$ , both production cost and the regulated price are small and supplier  $N$  maximises its profits by raising downstream firms' profits. For this reason, it charges a fixed fee only while setting the minimum per-unit price of zero: This lowers the retail price, raises rather elastic consumers' demand and, ultimately, leads to higher retail profits. The supplier here gains more from its share of these profits than it directly loses in the upstream market from the low per-unit price, which is, indeed, below cost. The supplier's profit considerations change for parameter values which lie in region  $II$ . Here, the strategic interdependency of input prices effectively leads to price discrimination: Decreasing  $a_i$  still raises  $S_i$ 's sales revenues, but with  $\partial^2 \pi_i / \partial a_i \partial a_j$ , i.e. strategic

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<sup>5</sup> To characterise the different constellations, we define thresholds  $\Delta_I$  and  $\Delta_{II}$ , explicit expressions are given in the Appendix.



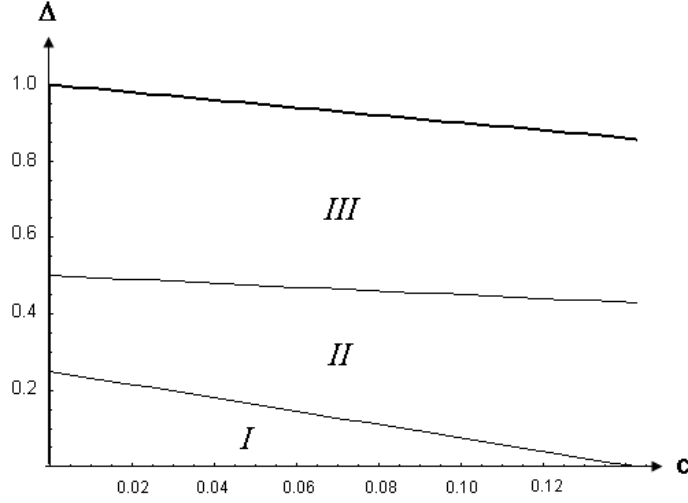


Figure 2: Equilibrium Types with respect to  $c$  and  $\Delta$  for  $c \leq 1/7$

substitutability of input prices, they increase more the higher the rival's price  $a_j$  for  $i, j = 1, 2$  and  $i \neq j$ . Actual outcomes are then dependent on the level of  $c$  and  $\Delta$  as these define how much the supplier directly foregoes in the upstream market by lower input prices.

Finally, in region *III*, the upstream supplier monopolises the market by excluding one of the downstream firms. This maximises the supplier's profits and can be implemented because the regulatory outside options seize to curtail the monopolist's market power.

Next, let us consider equilibrium constellations in case of relatively high marginal production cost. Proposition 2 describes the results:

**Proposition 2.** *For  $c > 1/7$ , there is a  $\Delta_{II}(c) > 0$  such that supply contracts of firms  $i = 1, 2$  have the following properties:*

- (i) *If  $\Delta \leq \Delta_{II}$ , then supply contracts are discriminatory and  $S_2$  obtains the more favorable contract, s.t.  $a_1 > a_2$  and  $F_1 < F_2$ .*
- (ii) *If  $\Delta(c) > \Delta_{II}$ , the supplier excludes one of the downstream firms. The remaining supply contract takes the form  $a_i^m = c$  and  $F_i^m = \pi_i(a_i^m) - \pi_i(a^R)$ .*

Figure 3 illustrates Proposition 2 and shows whether price discrimination or exclusion occurs for given cost and regulated input price markup:

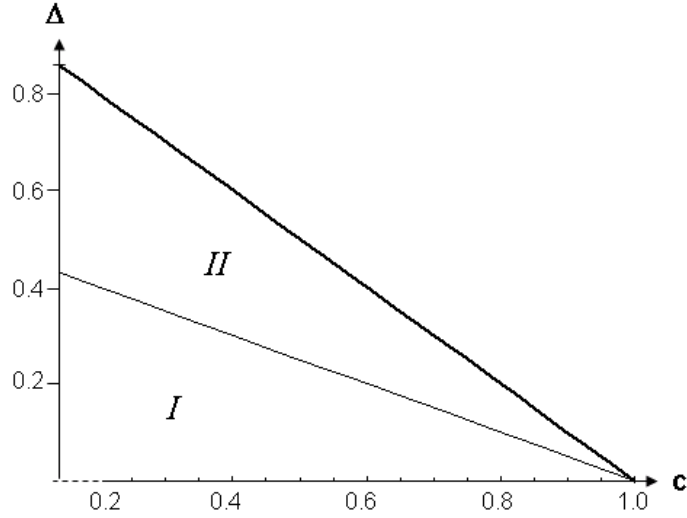


Figure 3: Equilibrium Types with respect to  $c$  and  $\Delta$  for  $c \leq 1/7$

The borderline between region  $I$  and  $II$  is defined by  $\Delta = \Delta_{II}$ . Thus, in region  $I$ , where the regulatory price is not too high, input price discrimination occurs. But in region  $II$  the regulatory price seizes to restrict the upstream supplier's market power. This leads to one downstream firm's exclusion. In contrast to the previous case, the supplier  $N$  never offers non-discriminatory contracts if  $c > 1/7$ . This is so because, at this cost level, retail profits would be lower and upstream losses higher than with low production cost, if per-unit prices were set to zero. From Proposition 1 and 2 it immediately follows:

**Corollary 1.** *If  $\Delta(c) \leq \Delta_{II}$ , the regulatory outside option prevents exclusion.*

Note here, that an unconstrained supplier would always exclude one of the downstream firms. Hence, equilibrium contracts change with a regulatory outside option if  $\Delta \leq \Delta_{II}$ .

## 4 The social optimum and non-discrimination clauses

In this section we evaluate welfare of optional regulated input prices and its various ensuing equilibrium constellations. We compare the surplus to the customary practices of banning price discrimination in principal or obliging a seller to make its best terms available to all buyers.

## 4.1 The social optimum

For our specific setup, welfare reduces to consumers' utility less production cost. By comparing welfare for varying  $\Delta$  and given  $c$ , we find:

**Proposition 3.** *For  $c \leq 1/7$ , welfare is maximised if  $\Delta \leq \Delta_I$ . Otherwise, for  $c > 1/7$ , welfare is maximised if  $\Delta = 0$ .*

Hence, the socially most desirable situation is aligned with the lowest per-unit input prices. Here, results differ with varying production cost: If marginal cost are relatively low, supply contracts comprise zero per-unit prices for the entire range of regulated input prices as long as  $\Delta \leq \Delta_I$ . But if cost are rather high, the lowest per-unit prices arise if the regulatory price option corresponds to marginal cost. In both cases, welfare is maximised since low input prices induce low prices in the final market and, therefore, raise consumer surplus. Note also that welfare is decreasing in  $\Delta$ , as higher regulated prices permit contracts with higher input prices.

## 4.2 Ban on Price Discrimination

Forbidding discriminatory supply contracts implies uniform pricing at  $a_1 = a_2 = a$  and  $F_1 = F_2 = F$ . With downstream competition given by (1) to (3), it yields equilibrium quantity  $x$  for each downstream firm. The supplier then maximises its profits by considering

$$a = \arg \max_a \Pi_N = 2(a - c)x + 2F \quad (8)$$

subject to

$$\pi_i(a, a) - F \geq 0.$$

Solving the conditions, we obtain

$$a > c \quad \text{and} \quad F = \pi_i(a, a).$$

Regarding welfare this leads to following conclusion:

**Proposition 4.** *Banning price discrimination is socially as efficient as monopolising the market, i.e.  $W^u = W^m$ .*

To see the intuition, note that prohibiting price discrimination ex-ante leads to uniform wholesale prices above marginal cost: By setting prices above marginal cost, the supplier achieves to induce a monopolistic outcome in the final market. Since social surplus sums up to consumers' utility less production cost, banning price discrimination or

a downstream firm's exclusion induce the same welfare level. Still, the two situations are different in terms of firms' profits: In case of uniform pricing, only the upstream supplier makes profits, while in case of a regulatory option, both the remaining downstream firm and the supplier obtain a profit share.

### 4.3 Renegotiation to obtain Non-Discriminatory Terms

We follow McAfee and Schwartz (1994) when considering renegotiation on non-discriminatory terms: We suppose that the upstream firm is obliged to make its best contract available to both downstream firms. Then, a discriminated buyer may replace its initially accepted contract with the more favourable offer. We find:

**Proposition 5.** *The disadvantaged downstream firm will not claim its rival's contract, even if given this option after the initial round of contracting.*

Indeed, this replicates McAfee and Schwartz (1994)'s finding even though contracts are affected by the regulatory option. To see the intuition, note that two-part tariffs are employed: By the per-unit price, the supplier determines a downstream firm's competitiveness and, accordingly, its revenues. By the fixed fee, it shifts these revenues towards itself. Now, given discrimination against  $S_1$ , suppose  $S_1$  claims its rival's contract terms: It then obtains  $S_2$ 's lower per-unit price together with its fixed fee. Due to the lower per-unit price, downstream competition becomes more intense. This effectively makes  $S_2$ 's tariff less valuable. Yet, with the original fixed fee - presuming the higher revenues under price discrimination -  $S_1$  would incur a loss instead of obtaining additional profits if it claimed the same tariff than  $S_2$ . We therefore conclude that making best terms available cannot serve to mitigate price discrimination in input markets.

## 5 Conclusion

In this paper, we regarded a monopolistic bottleneck with an upstream supplier and two symmetric downstream firms which sequentially interact with the supplier. We examined how the regulatory threat of prescribing input supply at a linear price affects the market outcome, as it permits input market price discrimination. We found that actual outcomes vary with input production cost and the regulated price. Indeed, if both production cost and the optional price are rather low, no price discrimination arises. Most importantly, the regulatory option induces input prices which lead to higher consumer surplus and welfare than a flat ban on price discrimination.

Therefore, optional linear input prices seem to be a welfare enhancing alternative to banning price discrimination per se. It complies with the presently adopted attitude of European policy claiming a more economic approach to Article 82 TEC and explicitly considers a dynamic market context. Note that outcomes vary with production cost and non-discriminatory supply contracts arise if cost as well as the regulated price are relatively low. This suggests to consider production cost when regarding price discrimination.

On a related note, one might consider downstream firms' incentive to engage in product differentiation in order to mitigate price discrimination. Referring to the example of network industries and a situation where downstream firms solely resell a network owner's product, in particular, this suggests to look at vertically differentiated entry and downstream activity of vertically integrated firms. A full analysis of this issue is left to further research.

## 6 Appendix

### Proof of Proposition 1 and 2:

*No exclusion:*

If both downstream firms are active, maximising (1), given  $p = 1 - x_i - x_j$  with  $i = 1, 2$ , yields the first-order-conditions

$$1 - 2x_i - x_j^* - a_i = 0. \quad (9)$$

Simultaneously solving these two conditions results in equilibrium quantities as stated in (3), i.e.

$$x_i^*(a_i, a_j) = \frac{1}{3}(1 - 2a_i + a_j),$$

and equilibrium retail price

$$p^*(a_1, a_2) = \frac{1}{3}(1 + a_1 + a_2). \quad (10)$$

Note that (3) implies that both downstream firms pursue market activity if and only if  $2a_2 - 1 < a_1 < (1 + a_2)/2$ .

To consider  $N$ 's contract offer to  $S_2$ , let us use  $p = 1 - x_i - x_j$  and (3) in (5). We get

$$F_2 = \frac{1}{9}(1 - 2a_2 + a_1)^2 - \frac{1}{9}(1 - 2a^R + a_1)^2. \quad (11)$$

With this and (3), the per-unit price  $a_2$  is derived according to (4). It yields Kuhn-Tucker conditions

$$\frac{1}{9}(-1 + 2a_1 - 4a_2 + 3c) \leq 0 \quad \text{and} \quad \frac{1}{9}(-1 + 2a_1 - 4a_2 + 3c)a_2 = 0$$

with  $a_2 \geq 0$ . Solving this programme yields

$$a_2^*(a_1, c) = \begin{cases} \frac{1}{4}(-1 + 2a_1 + 3c) & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\ 0 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c. \end{cases} \quad (12)$$

We obtain an expression for  $F_2$  by inserting (12) into (11), considering  $a^R = c + \Delta$ :

$$F_2^* = \begin{cases} \frac{1}{4}(1 - c)^2 - \frac{1}{9}(1 + a_1 - 2c - 2\Delta)^2 & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\ \frac{1}{9}(1 + a_1)^2 - \frac{1}{9}(1 + a_1 - 2c - 2\Delta)^2 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c. \end{cases} \quad (13)$$

$S_1$ 's contract is determined analogously to  $S_2$ 's. First, consider the participation constraint as given in (7). By (3) and (12), it leads to

$$F_1 = \frac{1}{9}(1 - 2a_2^* + a_1)^2 - \frac{1}{9}(1 - 2a^R + a_1)^2. \quad (14)$$

Considering the profit-maximising per-unit price  $a_1$  according to (6), we then obtain the Kuhn-Tucker conditions

$$\frac{1}{18}(-1 + 15c + 12\Delta - 14a_1) \leq 0 \quad \text{and} \quad \frac{1}{18}(-1 + 15c + 12\Delta - 14a_1)a_1 = 0$$

with  $a_1 \geq 0$ , if  $a_1 > \frac{1}{2} - \frac{3}{2}c$ , and

$$\frac{1}{9}(-1 + 7c + 4\Delta - 4a_1) \leq 0 \quad \text{and} \quad \frac{1}{9}(-1 + 7c + 4\Delta - 4a_1)a_1 = 0$$

with  $a_1 \geq 0$ , if  $a_1 \leq \frac{1}{2} - \frac{3}{2}c$ . Now define

$$\Delta_I \equiv \frac{1 - 7c}{4}, \quad \tilde{\Delta} \equiv \min \left[ \frac{1 - c}{2}, \frac{2}{3} - 3c \right] \quad \text{and} \quad \Delta_{II} \equiv \frac{1 - c}{2},$$

to distinguish different equilibrium types and note that  $\Delta_I \geq 0$  only if  $c \leq 1/7$ .<sup>6</sup> Then we can solve the above programme and state the results as

$$a_1^* = \begin{cases} 0 & \text{if } \Delta \leq \Delta_I \\ \frac{1}{4}(-1 + 7c + 4\Delta) & \text{if } \Delta_I < \Delta \leq \tilde{\Delta} \\ \frac{1}{14}(-1 + 15c + 12\Delta) & \text{if } \tilde{\Delta} < \Delta \leq \Delta_{II} \end{cases} \quad (15)$$

and

$$F_1^* = \begin{cases} \frac{4}{9}(a - c - \Delta)(c + \Delta) & \text{if } \Delta \leq \Delta_I \\ \frac{1}{36}(-1 + 3c)(-5 + 11c + 3\Delta) & \text{if } \Delta_I < \Delta \leq \tilde{\Delta} \\ -\frac{1}{441}(11 - 11c - 20\Delta)(-1 + c - 2\Delta) & \text{if } \tilde{\Delta} < \Delta \leq \Delta_{II}. \end{cases} \quad (16)$$

Now, inserting (15) in (12) and (13), we obtain  $S_2$ 's equilibrium supply contract given by

$$a_2^* = \begin{cases} 0 & \text{if } \Delta \leq \Delta_I \\ 0 & \text{if } \Delta_I < \Delta \leq \tilde{\Delta} \\ \frac{1}{7}(-2 + 9c + 3\Delta) & \text{if } \tilde{\Delta} < \Delta \leq \Delta_{II} \end{cases} \quad (17)$$

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<sup>6</sup> The thresholds are derived in the Addendum to Proposition 1 and 2.

and

$$F_2^* = \begin{cases} \frac{4}{9}(1-c-\Delta)(c+\Delta) & \text{if } \Delta \leq \Delta_I \\ \frac{1}{3}(1+c)(c+\Delta) & \text{if } \Delta_I < \Delta \leq \tilde{\Delta} \\ \frac{4}{441}(-1+c-2\Delta)(-17+17c+8\Delta) & \text{if } \tilde{\Delta} < \Delta \leq \Delta_{II}. \end{cases} \quad (18)$$

It is now easily verified that  $a_1^* \geq a_2^*$  for all  $\Delta \leq \Delta_{II}$ . Analogously,  $F_2 \geq F_1$  follows.

*Exclusion:*

Let us now presume that one of the downstream firms is excluded. Then, the remaining firm maximises its profits according to the first-order condition

$$1 - 2x_i - a_i = 0.$$

This yields the quantity

$$x_i^*(a_i) = \frac{1 - a_i}{2}. \quad (19)$$

The supplier obtains highest profits by offering a contract according to

$$a_i^m = \arg \max_{a_i} \Pi_N(a_i, F_i) \quad (20)$$

subject to

$$F_i = \pi_i(a_i) - \pi_i(a^R). \quad (21)$$

Solving this problem results in

$$a_i^m = c, \quad (22)$$

$$F_i^m = \frac{(1-c)^2}{4} - \frac{(1-c-\Delta)^2}{4} \quad (23)$$

$$\text{and } x_i^m = \frac{1-c}{2}. \quad (24)$$

Referring to (3) exclusion of  $S_j$  is only feasible iff

$$a_j \geq \frac{1+a_i}{2} = \frac{1+c}{2} \quad (25)$$

for  $i, j = 1, 2$  and  $i \neq j$ .

q.e.d

### Addendum to Proposition 1 and 2:

Let us now look at thresholds  $\Delta_I$  and  $\tilde{\Delta}$ . Indeed,  $\Delta \leq \Delta_I$  defines the value range  $\Delta$  at



which both firms are offered zero per-unit prices. By (15) this requires

$$\frac{1}{4}(-1 + 7c + 4\Delta) \leq 0.$$

Rearranging then yields

$$\Delta \leq \frac{1 - 7c}{4} \equiv \Delta_I < \Delta_{II}.$$

Further, in case of exclusion, both firms can always claim their outside option  $a^R$ . Due to (25) exclusion effectively occurs iff  $a^R \geq \frac{1+c}{2}$ . Rearranging yields

$$\Delta \geq \frac{1 - c}{2} \equiv \Delta_{II}.$$

Finally, in order to exactly specify  $a_2$ , we check  $a_2^* \leq 0$ . This requires

$$\frac{1}{7}(-2 + 9c + 3\Delta) \leq 0.$$

Here, rearranging yields

$$\Delta \leq \frac{2}{3} - 3c \equiv \widehat{\Delta}.$$

Yet, if  $c \leq 15$ , then  $\widehat{\Delta} > \Delta_{II}$ , therefore,  $a_2^* \leq 0$  holds iff

$$\Delta \leq \min \left[ \Delta_{II}, \widehat{\Delta} \right] \equiv \widetilde{\Delta}.$$

### Proof of Corollary 1:

Corollary 1 directly follows from Proposition 1 and 2.

### Proof of Proposition 3:

Welfare in our setup is, generally, given by

$$W = \Pi_N + \Pi_1 + \Pi_2 + x_1 + x_2 - \frac{1}{2}(x_1 + x_2)^2 - p(x_1 + x_2) \quad (26)$$

$$= (1 - c)X - \frac{1}{2}X^2. \quad (27)$$

First, let us restate welfare with respect to per-unit input prices  $a_1$  and  $a_2$ . Given that both firms are active in the downstream industry, we can use (3) to rewrite aggregate demand  $X = x_1 + x_2$  as

$$X = \frac{1}{3}(2 - a_1 + a_2).$$

By (27), we then obtain

$$W = \frac{1-c}{3} (2 - a_1 - a_2)^2 - \frac{1}{18} (2 - a_1 - a_2)^2. \quad (28)$$

Taking the first derivative, we can check how welfare changes with respect to  $a_i$ . It is given by

$$\frac{\partial W}{\partial a_i} = -\frac{3(1-c)}{9} + \frac{1}{9} (2 - a_i - a_j) \quad (29)$$

with  $i, j = 1, 2$  and  $i \neq j$ . In the following, we will argue that, if both  $S_1$  and  $S_2$  are active in the market,

$$\partial W / \partial a_i < 0$$

so that a higher input price  $a_i$  always reduces welfare and only the lowest input prices can induce maximal surplus for  $i = 1, 2$ :

We first consider the equilibrium contracts as computed for Proposition 1 and 2. With our results in Proposition 1 and 2, the least upper bound of one firm  $i$ 's input price is given by  $\sup a_i = c$  for  $i = 1, 2$ . Moreover, we found that no exclusion requires  $a_i < (1 + a_j)/2$  for  $i, j = 1, 2$  and  $i \neq j$ . Therefore, the rival's input price is bound by  $\sup a_j = (1 + c)/2$  from above for  $j = 1, 2$  and  $i \neq j$ . The least upper bound of the sum of input prices is then given by

$$\sup(a_1 + a_2) = \frac{1+c}{2} + c = \frac{1+3c}{2} \quad (30)$$

and maximises (29). Here, a little rearranging leads to

$$\frac{\partial W}{\partial a_i} = -\frac{3}{18} + \frac{3}{18}c < 0$$

since  $0 < c < 1$ . We therefore conclude that the lowest per-unit prices  $a_1$  and  $a_2$  maximise welfare, in case both downstream firms participate in market activity.

Now let us consider the case of one firm's exclusion with total demand  $x^m = (1 - c)/2$  compared to aggregate demand  $X^*$  in case both firms are active: We find  $X^* \geq x^m$  by inserting (13) and (15) into (3). Further, according to (27) welfare increases in aggregate demand at  $x^m$ , i.e.  $\partial W(x^m, c)/\partial X > 0$  at this point. Moreover, if both downstream firms are active, welfare is maximised by lowest per-unit input prices. Thus, it remains to see that the lowest  $a_1$  and  $a_2$  occur for  $c \leq 1/7$  if  $\Delta \leq \Delta_I$  and for  $c > 1/7$  if  $\Delta = 0$ .

q.e.d

**Proof of Proposition 4:**

As before, downstream competition is described by (1) to (3). Uniform pricing as described in (8) and the participation constraint in (9) lead to the first-order condition

$$-\frac{8}{9}a + \frac{2+6c}{9}.$$

This condition resolves to

$$a = \frac{1+3c}{4} \quad \text{and} \quad x = \frac{1-c}{4}.$$

Then using (23) to compute welfare we get

$$W^u = \frac{3}{8}(1-c)^2.$$

Now using (24) and (27) to compute welfare in case of exclusion shows

$$W^u = W^m.$$

q.e.d.

**Proof of Proposition 5:**

Let us assume that - contrary to Proposition 5 -  $S_1$  claims  $S_2$ 's more favorable contract if  $\Delta \leq \Delta_{II}$  without exclusion. This implies

$$\pi_1(a_2^*, a_2^*) - F_2^* > \pi_1(a_1^*, a_2^*) - F_1^* \quad (31)$$

must hold. Inserting (5) and (7) and rearranging we then obtain

$$\pi_1(a_2^*, a_2^*) - \pi_1(a^R, a_2^*) > \pi_2(a_1^*, a_2^*) - \pi_2(a_1^*, a^R).$$

Due to symmetry, i.e.  $\pi_2(a_1, a_2) = \pi_1(a_2, a_1)$ , we can rewrite this expression as

$$\pi_1(a_2^*, a_2^*) - \pi_1(a^R, a_2^*) > \pi_1(a_2^*, a_1^*) - \pi_1(a^R, a_1^*)$$

and the equivalent statement

$$\int_{a^R}^{a_2^*} \frac{\partial \pi_1(a_1, a_2^*)}{\partial a_1} da_1 > \int_{a^R}^{a_1^*} \frac{\partial \pi_1(a_1, a_1^*)}{\partial a_1} da_1. \quad (32)$$

By Proposition 1,  $a_2^* < a_1^*$  if price discrimination occurs. Further,  $\partial\pi^2(a_1, a_2)/\partial a_1\partial a_2 < 0$  holds. Therefore,

$$\frac{\partial\pi_1(\bar{a}, a_2^*)}{\partial a_1} < \frac{\partial\pi_1(\bar{a}, a_1^*)}{\partial a_1} < 0$$

for some  $a_1 = \bar{a}$ . Thus, (32) and, accordingly, (31) cannot hold. Instead,

$$\pi_1(a_1^*, a_2^*) - F_1^* > \pi_1(a_2^*, a_2^*) - F_2^*$$

is true. It immediately follows that  $S_1$  will never claim its rival's contract terms when both downstream firms participate in the market activity.

To complete the proof, let us now consider the case of exclusion. Suppose, firm  $S_i$  is excluded, but claims its rival's contract terms  $(a_j^m, F_j^m)$ . This implies

$$\pi_i(a_j^m, a_j^m) - F_j^m > 0 \tag{33}$$

for  $i, j = 1, 2$  and  $i \neq j$ . Inserting (22) and (23), this condition rewrites

$$\frac{1}{9}(1-c)^2 - \frac{1}{4}(1-c)^2 + \frac{1}{4}(1-(c+\Delta))^2 > 0. \tag{34}$$

Since exclusion occurs only if  $\Delta > \Delta_{II}$ , an upper bound on the third term of the LHS is given by  $(1-c)^2/16$ . Hence, the disadvantaged firm  $S_i$  would claim its rival's contract terms if and only if

$$\frac{1}{9}(1-c)^2 - \frac{1}{4}(1-c)^2 + \frac{1}{16}(1-c)^2 > 0.$$

Yet, this condition is never satisfied. We therefore conclude that (33) and (34) never hold. Proposition 5 summarises the results of input price discrimination without and with exclusion.

q.e.d.

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