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**Working Paper**

## Labor turnover costs, workers' heterogeneity and optimal monetary policy

Kiel Working Paper, No. 1534

**Provided in Cooperation with:**

Kiel Institute for the World Economy – Leibniz Center for Research on Global Economic Challenges

*Suggested Citation:* Faia, Ester; Lechthaler, Wolfgang; Merkl, Christian (2009) : Labor turnover costs, workers' heterogeneity and optimal monetary policy, Kiel Working Paper, No. 1534, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:

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## **Working Papers**

**Kiel Institute  
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### **Labor Turnover Costs, Workers' Heterogeneity and Optimal Monetary Policy**

by Ester Faia, Wolfgang Lechthaler and  
Christian Merkl

**No. 1534** | July 2009

Web: [www.ifw-kiel.de](http://www.ifw-kiel.de)

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## **Labor Turnover Costs, Workers' Heterogeneity, and Optimal Monetary Policy\***

Ester Faia, Wolfgang Lechthaler and Christian Merkl

### **Abstract:**

We study the design of optimal monetary policy in a New Keynesian model with labor turnover costs in which wages are set according to a right to manage bargaining where the firms' counterpart is given by currently employed workers. Our model captures well the salient features of European labor market, as it leads to sclerotic dynamics of worker flows. The coexistence of those types of labor market frictions alongside with sticky prices gives rise to a non-trivial trade-off for the monetary authority. In this framework, firms and current employees extract rents and the policy maker finds it optimal to use state contingent inflation taxes/subsidies to smooth those rents. Hence, in the optimal Ramsey plan, inflation deviates from zero and the optimal volatility of inflation is an increasing function of firing costs. The optimal rule should react to employment alongside inflation.

Keywords: optimal monetary policy, hiring and firing costs, labor market frictions, policy trade-off.

JEL classification: E52, E24

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\* We thank seminar participants at Goethe University Frankfurt, the universities of Bayreuth, Dortmund and Würzburg, GREQAM, Kiel IfW, University of Rome II, and conference participants at the ECB Wage Dynamics Network, Konstanz seminars in Monetary Theory and Policy, the EES workshop "The Labor Market and the Business Cycle," the Spring Meeting of Young Economists, the Society for Economic Dynamics, the North American Summer Meeting of the Econometric Society, the ZEW conference on "Recent Developments in Macroeconomics." We thank Christian Bayer and Felix Hammermann for discussing the paper. We gratefully acknowledge financial support from the Leibniz grant. We thank Tom Schmitz for excellent research assistance.

# 1 Introduction

European labor markets are characterized by two main features. First, labor turnover costs in the form of hiring and firing costs are very high (see next section for stylized facts). Second, in most countries wages are bargained ex-ante at a centralized level and the bargaining process usually takes place between firms and employed workers. There is ample empirical evidence that large turnover costs induce much persistence in worker flows, a phenomenon which has been labeled Eurosclerosis<sup>1</sup> (see among others Kugler and Saint-Paul 2004). In addition it has been noticed that labor turnover costs coupled with the inability of wages to adjust promptly to idiosyncratic shocks, e.g., due to collective bargaining processes, induces inefficiently high levels of unemployment.<sup>2</sup> Persistence in worker flows and the inability of labor markets to adjust promptly to shocks alongside with the presence of inefficiently high unemployment rates induces severe trade-offs for monetary policy. The literature has so far neglected them in the analysis of optimal monetary policy. There is a flourishing literature studying optimal monetary policy in New Keynesian settings whose conclusions invariably lead to support the case for the optimality of price stability policy. In most cases those prescriptions are derived in an environment which lacked any significant role for labor market frictions. The analysis in this paper moves a step forward in this direction focusing on a type of labor market friction which fits well the euro area countries.

The design of optimal monetary policy is done within a dynamic stochastic general equilibrium (DSGE) model with sticky prices and labor market frictions. The labor market considered here is characterized by two main features. First, worker/firm relations are subject to idiosyncratic operating costs and turnover costs in the form of hiring and firing costs. As worker/firm pairs are heterogenous, the marginal worker is hired only when the future stream of discounted expected profits exceed hiring costs.<sup>3</sup> The same holds for firing decisions. In the absence of labor turnover costs, a worker's current employment probability is independent of whether she was previously employed or unemployed, so that her retention rate is equal to her job finding rate. In the presence of hiring and firing costs, by contrast, her retention rate exceeds her job finding rate, and thus current

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<sup>1</sup>Giersch 1985 first introduced this terminology which has then been used by several others to describe European labor markets.

<sup>2</sup>See for instance Bertola and Rogerson 1997.

<sup>3</sup>See also Lechthaler, Merkl, and Snower 2008 for a prototype of this model economy.

employment depends on past employment. Such path dependence allows the model dynamics to show additional persistence compared to standard Walrasian labor markets, something in line with empirical evidence. Second, the wage setting mechanism in our model follows a *right to manage bargaining*,<sup>4</sup> which takes place between firms and the median worker.<sup>5</sup> This type of wage bargaining has an important implication: Due to the right to manage structure hiring and firing decisions are taken only after the wage schedule has been determined; this implies that, consistently with empirical evidence, shocks have a large impact on employment.<sup>6</sup>

Our model economy features two types of distortions. On the one side, sticky prices call for optimality of zero inflation policy. On the other side, the presence of labor market frictions which generate inefficiently low levels of employment call for an active monetary policy with short-run deviations from zero inflation. Those two forces produce a trade-off for the policy maker. Importantly, the specific form of labor market frictions employed in our model allows us to highlight novel dimensions in the analysis of monetary policy trade-offs. First, hiring and firing costs reduce labor turnover at any period in time, therefore inducing a gap compared to the economy characterized by Walrasian labor markets. Second, the model features an intertemporal wedge that distorts hiring and firing decision between two subsequent periods. Indeed, once workers are inside the firm they are fired only if the discounted stream of future profits is smaller than the negative value of the firing costs. Further, firms will hire only if the discounted value of future profits is bigger than the hiring costs. As a consequence, the retention rate, defined as the mass of workers who keep their jobs, is always bigger than the firing rate. Under those circumstances current employees and firms extract some rents. Importantly it should be noticed that the wedge between the planner solution (with full employment) and our competitive economy has a time varying nature, hence it cannot be off-set with a constant fiscal subsidy but requires contingent policy responses. Third, the marginal cost in this model embeds an extra component given by the long run value of a worker: firms tend to retain workers as this allows them to save firing and hiring costs. This additional component

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<sup>4</sup>This feature captures well the reality of European labor markets in which wage schedules are typically determined ex-ante through collective bargaining agreement.

<sup>5</sup>The choice of allowing the median worker to bargain over wages simplifies the model. However, robustness checks show that alternative settings, such as individual bargaining process with marginal workers, would not change the main implications of the model and the main policy trade-offs.

<sup>6</sup>For a more thorough analysis of this issue in a related labor market framework without price adjustment costs, see Brown, Merkl, and Snower 2009.

acts as an endogenous cost push shock and prevents the marginal cost from being constant on the flexible price allocation. Finally, in the right to manage bargaining, wages are not contingent on current employment decisions, therefore they lose part of their allocative role compared to the case with efficient Nash bargaining. Ex-post unemployment rates are inefficiently high, something which calls for active policy responses.

The analysis of optimal monetary policy is carried in three steps. First, the analysis highlights the role of wedges in driving monetary policy trade-offs. Second, the design of optimal policy starts by deriving the Ramsey approach (Atkinson and Stiglitz 1976, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1991) in which the optimal path of all variables is obtained by maximizing agents' welfare subject to the relations describing the competitive economy and via an explicit consideration of all wedges that characterize both the long run and the cyclical dynamics. Recent studies apply this approach to the analysis of optimal policy in the context of New Keynesian models (Adao et al. 2003, Khan, King, and Wolman 2003, Schmitt-Grohe and Uribe 2007 and Siu 2004). Third, the design of optimal policy is completed by the characterization of an optimal operational monetary policy rule, the latter is obtained numerically by maximizing agents' welfare subject to the competitive economy conditions. Crucial in our numerical analysis is the use of second order approximations and conditional welfare, which allows to account for second order effects on mean welfare, something which acquires particular relevance in presence of large real distortions.

We find three main results. First, optimal policy implies deviations from price stability. Consider a positive productivity shock. As output and employment are below the Pareto optimal level, the monetary authority should take full advantage of the productivity improvement to push the real economy towards the Pareto frontier. In an economy with sticky prices, it can do so by increasing aggregate demand: such an increase will in turn increase labor demand, hence increase hiring and reduce firing. Second, we find that the optimal volatility of inflation is an increasing function of the firing costs. This result has two alternative interpretations. First, higher firing costs, by exacerbating inefficient unemployment fluctuations, steepen the monetary policy trade-off between stabilizing prices and reducing inefficiencies. Second, from a public finance point of view, inflation in this model act as a tax on firms' rents: the higher the hiring and firing costs, the higher are

the rents that accrue to firms, hence the larger are the fluctuations in the inflation tax. Finally, we find that the optimal monetary policy rule should target employment alongside with inflation. Once again the need for targeting the real activity alongside with inflation results from the nature of the policy trade-offs in this model.

Our paper is related to a recent literature that introduces labor market frictions in New Keynesian models. Most of the literature though has focused on search and matching frictions.<sup>7</sup> Our model presents a novel approach to modeling labor market frictions in New Keynesian models and highlights alternative monetary policy trade-offs.

The rest of the paper proceeds as follows. Section 2 shows some stylized facts relating the dynamics of selected macro variable and labor turnover costs. Section 3 presents the model and highlights the role of frictions in this economy. Section 4 presents the full-fledged Ramsey plan while section 5 presents the analysis of the optimal rule. Finally section 6 concludes.

## 2 Labor Turnover Costs and Euro Area Labor Markets

There is a vast literature looking at the importance of labor turnover costs and employment protection legislation (with the latter considered as proxy for firing costs) for unemployment dynamics, particularly for euro area labor markets. The literature dates back to Solow 1968, Sargent 1978, Nickell 1978, 1986 who introduce adjustment costs on labor demand. More recently, Bentolila and Bertola 1990 and Hopenhayn and Rogerson 1993 have shown that hiring and firing costs reduce labor turnover and make unemployment dynamics more persistent. Moreover, Hopenhayn and Rogerson 1993 and Bertola and Rogerson 1997 find that turnover costs have a sizable negative impact on unemployment, possibly leading to inefficient unemployment fluctuations. Kugler and Saint-Paul 2004 show that this is even more so if turnover costs are coupled with asymmetric information. Finally, Alvarez and Veracierto 2001 find that severance payments decrease aggregate productivity and output.

Before turning to the model's implications, it is useful to highlight some stylized facts concerning the impact of labor turnover costs on the dynamics of selected macro variables, specifically inflation and output. We focus on euro area countries. Figures 1 and 2 show that there is a

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<sup>7</sup>Several contributions exist. See, e.g., Walsh 2005 and Krause and Lubik 2007. Within this literature some authors have studied the design of optimal policy (see Blanchard and Galí 2009, Faia 2008, 2009 and Thomas 2008).

negative and significant relationship between labor turnover costs and the volatility of output and inflation. As argued before, higher labor turnover costs imply that the retention rates exceed job finding rates,<sup>8</sup> and thus current employment depends on past employment. The persistence in employment carries over to output, marginal costs and therefore to inflation. The data sample covers years from the 1999 to mid-2008. This choice is motivated by the following reasons. First, the interest in studying the recent implications of labor market regulations for macroeconomic dynamics. Second, the need to isolate the dynamics of macro variables from policy regime shifts and thereby the choice to focus on the EMU period. The volatility of real economic activity is calculated based on a quarterly output gap measure. The seasonally adjusted real GDP series (in 2000 prices) is taken from the International Financial Statistics. The output gap is calculated as percentage deviation of output from its trend, namely the Hodrick-Prescott filter with smoothing parameter  $\lambda = 1600$ . The inflation gap volatility is calculated in the same way.<sup>9</sup>

As a proxy for labor turnover costs we use the employment protection legislation index (see OECD 2004), which is a weighted average of indicators capturing protection of regular workers against individual dismissals, requirements for collective dismissals and regulation of temporary employment. We choose this index because it is a more precise measure than alternative employment protection indicators.<sup>10</sup> Figure 3 shows values of this index for various countries. It becomes immediately clear that turnover costs are much smaller in the anglo-saxon countries. European countries generally have considerably higher index values than anglo-saxon countries, although even among European countries there is a lot of heterogeneity. Southern European countries tend to have a higher index value than Northern and Central European countries or Eastern European countries.

Importantly, the model presented in this paper can account for the negative relation found in the data between output and inflation volatility and labor turnover costs (see Section 3.5). We will return to this point later on.

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<sup>8</sup>Wilke 2005 shows, for example, for Germany that the job-finding rates (i.e., the probability of finding a new job) are much lower than the retention rates (i.e., the probability of retaining an existing job).

<sup>9</sup>Note that the undetrended inflation rate delivers the same qualitative results and similar significance levels.

<sup>10</sup>Compared with other indicators, such as the Employment Legislation Index in Botero et al. (2003), or the hiring and firing costs calculated by the World Bank in its “Doing Business” studies, the OECD’s indicator both covers a larger range of relevant aspects of LTC, and has more precise and differentiated sub-indicators. Therefore, it is the best available measure for the relative importance of LTCs in different countries.



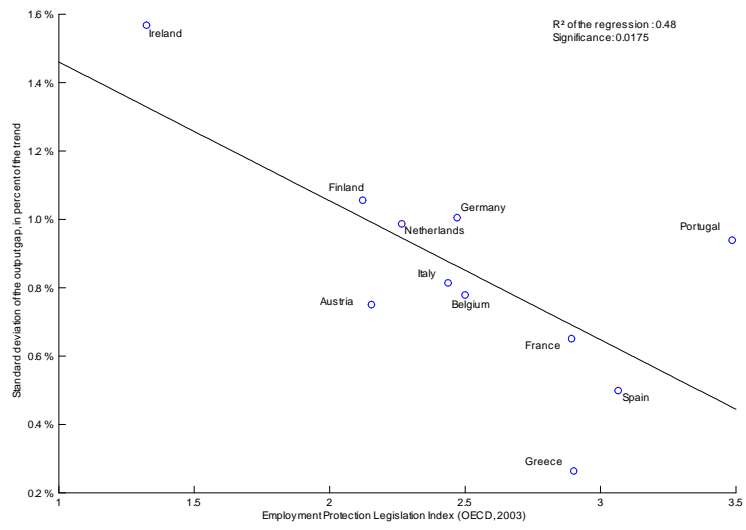


Figure 1: Output gap volatility and employment protection legislation.

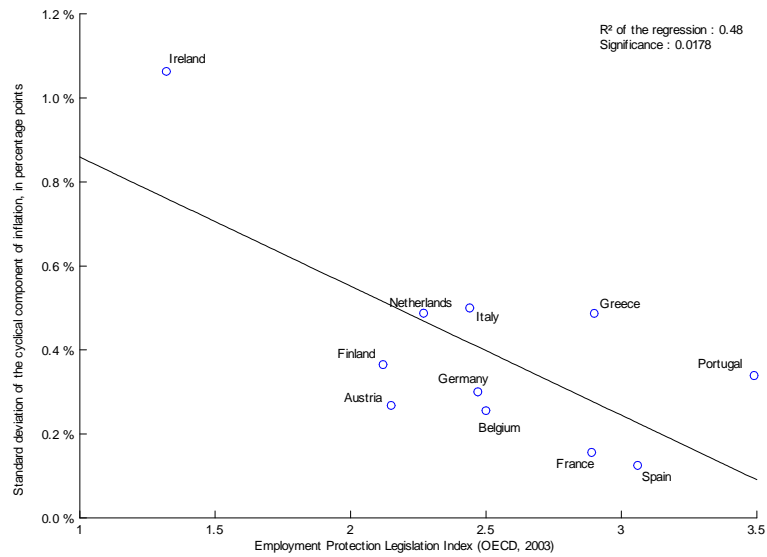


Figure 2: Inflation volatility and employment protection legislation.

	1998	2003		1998	2003
<b>Eastern Europe</b>			<b>Southern Europe</b>		
Czech Republic	1.94	1.94	Greece	3.49	2.90
Hungary	1.54	1.75	Italy	3.06	2.44
Poland	1.93	2.14	Portugal	3.66	3.49
Slovak Republic	2.20	1.70	Spain	2.96	3.06
<b>AVERAGE</b>	<b>1.90</b>	<b>1.88</b>	<b>AVERAGE</b>	<b>3.29</b>	<b>2.97</b>
<b>Northern and Central Europe</b>			<b>Anglo-saxon countries</b>		
Austria	2.38	2.15	Australia	1.47	1.47
Belgium	2.48	2.50	Canada	1.13	1.13
Denmark	1.83	1.83	Ireland	1.17	1.32
Finland	2.18	2.12	New Zealand	0.78	1.29
France	2.84	2.89	United Kingdom	0.98	1.10
Germany	2.64	2.47	United States	0.65	0.65
Netherlands	2.27	2.27	<b>AVERAGE</b>	<b>1.03</b>	<b>1.16</b>
Norway	2.72	2.62	<b>Rest of the world</b>		
Sweden	2.62	2.62	Japan	1.94	1.79
Switzerland	1.60	1.60	Korea	2.00	2.00
<b>AVERAGE</b>	<b>2.36</b>	<b>2.31</b>	Mexico	3.23	3.23
			Turkey	3.40	3.49

Figure 3: Version 2 of the EPL, including protection against collective dismissals. Source: OECD.Stat, originally published in the OECD (1999 and 2004).

### 3 The Model

Our model grafts a labor market with labor turnover costs, wage bargaining, and employed and unemployed workers onto a New Keynesian framework with Rotemberg adjustment costs. To endogenize hiring and firing decisions, it is assumed that the profitability of each worker is subject to an i.i.d. shock each period. Firms can change their price in any period but price changes are subject to quadratic adjustment costs a la Rotemberg 1982.

#### 3.1 Households

We assume that households have a standard utility function of the form:

$$U_t = \sum_{j=t}^{\infty} \beta^{j-t} E_t \frac{c_j^{1-\sigma}}{1-\sigma}, \quad (1)$$

where  $\beta$  is the household's discount factor,  $\sigma$  the elasticity of inter-temporal substitution,  $c$  a consumption aggregate (described below)<sup>11</sup> and  $E$  is the expectations operator.

<sup>11</sup>In what follows capital letters refer to nominal variables and small letters refer to real variables (i.e., de-trended by the price level).

As common in the literature,<sup>12</sup> it is assumed that each household consists of a large number of individuals, each individual supplies one unit of labor inelastically and shares all income with the other household members. This implies that consumption does not depend on a worker's employment status. Thus, the representative household maximizes its utility subject to the budget constraint:

$$Bo_t + c_t P_t - T_t = W_t N_t + bU_t + (1 + i_{t-1})Bo_{t-1} + \Pi_{a,t}, \quad (2)$$

where  $Bo$  are nominal holdings of one period discounted bonds,  $P$  is the aggregate price level,  $T$  are tax payments,  $i$  is the nominal interest rate,  $\Pi_a$  are nominal aggregate profits, which are transferred in lump-sum manner,  $W$  is the nominal wage,  $N$  is the total household labor input,  $b$  the income of unemployed workers,<sup>13</sup> and  $U$  the number of unemployed workers. The inter-temporal utility maximization yields the standard consumption Euler equation:

$$c_t = \beta E_t c_{t+1} \left( \frac{1 + i_t}{\pi_{t+1}} \right)^{-\frac{1}{\sigma}}, \quad (3)$$

where  $\pi_{t+1}$  is the expected inflation rate.

Note that, as in a large part of the recent literature, money plays the role of nominal unit of account.<sup>14</sup> The assumption of a cashless economy implies that zero inflation will be an optimal outcome in the long-run. Departure from price stability occurs in the short-run, as the monetary authority responds to productivity and government expenditure shocks in order to reduce the impact of labor market wedges.

### 3.2 Production and the Labor Market

There are three types of firms. (i) Firms producing intermediate goods employ labor, exhibit linear labor turnover costs (i.e., hiring and firing costs) and sell their homogenous products on a perfectly competitive market to the wholesale sector. (ii) Firms in the wholesale sector transform the intermediate goods into consumption goods and sell them under monopolistic competition to the

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<sup>12</sup>See Andolfatto 1995 and Merz 1996.

<sup>13</sup>The variable  $b$  can either be interpreted as home production or as unemployment benefits provided by the government (financed by lump-sum taxes).

<sup>14</sup>See Woodford 2003, chapter 3. Thus the present model may be viewed as approximating the limiting case of a money-in-the-utility model in which the weight of real balances in the utility function is arbitrarily close to zero.

retailers. They can change their price at any time but price adjustments are subject to a quadratic adjustment cost a la Rotemberg 1982. (iii) The retailers, in turn, aggregate the consumption goods and sell them under perfect competition to the households.

### 3.2.1 Intermediate Goods Producers and Employment Dynamics

Intermediate good firms hire labor to produce the intermediate good  $z$ . Their production function is:

$$z_t = a_t N_t, \tag{4}$$

where  $a$  is technology and  $N$  the number of employed workers. They sell the product at a relative price  $mc_t = P_{z,t}/P_t$ , which they take as given in a perfectly competitive environment, where  $P_z$  is the absolute price of the intermediate good and  $P$  is the economy's overall price level. The variable  $mc_t$  in this economy plays the role of marginal costs as it represents the Lagrange multiplier on the production function.

We assume that every worker (employed or unemployed) is subject to a random operating cost  $\varepsilon$ , which follows a *logistic* probability distribution  $g(\varepsilon_t)$  over the support  $-\infty$  to  $+\infty$ .<sup>15</sup> The operating costs can be interpreted as a worker-firm pair specific idiosyncratic cost-shock. Assume an economy with a large number of firms and workers (with a lot more workers than firms).<sup>16</sup> Each period all unemployed workers are distributed randomly across firms and then draw a realization from the shock distribution. When they draw a bad realization of the shock, they stay unemployed and get assigned to another firm the next period. For simplicity, we assume that the employed workers draw realizations from the same shock distribution.

The firms learn the value of the operating costs of every worker at the beginning of a period and base their employment decisions on it, i.e., an unemployed worker with a favorable shock will be employed while an employed worker with a bad shock will be fired. Hiring and firing is not costless, firms have to pay linear hiring costs,  $h$ , and linear firing costs,  $f$ , both measured in terms of the final

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<sup>15</sup>The logistic distribution was chosen because it is very similar to the normal distribution, but in contrast to the latter has a neat expression for the cumulative density function.

<sup>16</sup>Note that we do not have to specify the number of workers and firms, as long as there a lot more workers than firms. In this case, all firms face the same shock distribution with respect to their workers (due to the law of large numbers) and thus they are completely identical.

consumption good. Wages are determined through Nash bargaining between incumbent workers and the firm. The bargaining process takes the form of a *right to manage*. This assumption leads to the following timing of events. First, the operating cost shock takes place and median workers and the intermediate goods firm bargain over the wage. Given the wage schedule, firms make their hiring and firing decisions. Thus, firms will only hire those workers who face low operating costs and fire those workers who face high operating costs.

The hiring and firing costs induce two types of distortions (a gap and a time-varying wedge). The presence of hiring and firing costs reduces labor turnover at any period in time compared to a Walrasian labor market, thereby inducing a gap between the perfectly competitive economy and our non-Walrasian labor market. Second, our model features an inter-temporal wedge that distorts hiring and firing decisions between two sub-sequent periods. Indeed, once workers are inside the firm they are fired only if the discounted stream of future profits is smaller than the firing costs, on the other side firms will hire only if the discounted value of future profits is bigger than the hiring costs. Due to the labor turnover costs, the retention rate, defined as the mass of workers who keep their jobs, is always bigger than the firing rate.

The operating costs,  $\varepsilon$ , are measured in terms of the final consumption good and are assumed to grow at the same rate as productivity.<sup>17</sup> It turns out that this ensures that technological progress does not affect the unemployment rate.

Let's now consider the real profit generated by a firm-worker pair whose operating costs are  $\varepsilon_t$ :

$$\begin{aligned} \tilde{\Pi}_{I,t}(\varepsilon_t) &= a_t m c_t - w_t - \varepsilon_t + \\ &+ E_t \left\{ \sum_{j=t+1}^{\infty} \Delta_{t,j} \left[ (1 - \phi_j)^{j-t} \left( a_j m c_j - w_j - \left( \frac{1}{1-\phi_j} \int_{-\infty}^{v_{f,j}} \varepsilon_j g(\varepsilon_j) d\varepsilon_j \right) \right) \right. \right. \\ &\quad \left. \left. - \phi_j f_j (1 - \phi_j)^{j-t-1}, \right] \right\}, \end{aligned} \tag{5}$$

where  $w$  is the real wage,  $\phi$  is the separation probability,  $\Delta_{t,j}$  is the stochastic discount factor from period  $t$  to  $j$ . To simplify the profit function let's rewrite it in recursive manner:

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<sup>17</sup>For permanent technology shocks, it can be assumed the the operating, hiring and firing costs grow at the same rate as the technological progress. This ensures that the hiring and firing rates are independent of long-run technological growth (see Lechthaler, Merkl, and Snower 2008). As we only consider mean-reverting technology shocks in this paper, we skip this assumption for analytical simplicity.

$$\tilde{\Pi}_{I,t}(\varepsilon_t) = a_t mc_t - w_t - \varepsilon_t + E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})), \quad (6)$$

where  $\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})$  are future profits.

Using the profit functions, it is possible to derive the hiring and firing decisions, hence the employment dynamics. Because of the abovementioned timing of events, the model is solved backward and hiring and firing decisions are obtained for a *given wage schedule*. Let's define the hiring and the firing threshold respectively as  $v_{h,t}$  and  $v_{f,t}$ . Unemployed workers are hired whenever their operating cost does not exceed a certain threshold such that the profitability of this worker is higher than the hiring cost. Thus, the hiring threshold,  $v_{h,t}$ , is obtained by solving the following zero profit condition:

$$h = a_t mc_t - w_t - v_{h,t} + E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})). \quad (7)$$

Unemployed workers whose operating cost is lower than this value get a job, while those whose operating cost is higher remain unemployed. The resulting hiring probability is given by:

$$\eta_t = \int_{-\infty}^{v_{h,t}} \varepsilon_t g(\varepsilon_t) d\varepsilon_t. \quad (8)$$

Similarly, the firm will fire a worker if current losses are higher than the firing cost. Again a zero profit condition defines the firing threshold as follows:

$$-f = a_t mc_t - w_t - v_{f,t} + E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})), \quad (9)$$

and the separation rate is defined as:

$$\phi_t = \int_{v_{f,t}}^{\infty} \varepsilon_t g(\varepsilon_t) d\varepsilon_t. \quad (10)$$

The change in employment ( $N_t - N_{t-1}$ ) is the difference between hirings from the unemployment pool ( $\eta U_{t-1}$ ) and firings from the employment pool ( $\phi N_{t-1}$ ), where  $U_{t-1}$  and  $N_{t-1}$  are the aggregate unemployment and employment levels:  $N_t - N_{t-1} = \eta U_{t-1} - \phi N_{t-1}$ . Letting ( $n_t = N_t/L_t$ ) be the employment rate, we assume a constant workforce,  $L_t$ , and normalize it to one. Therefore, the employment dynamics read as follows:

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t. \quad (11)$$

The unemployment rate is simply  $u_t = 1 - n_t$ .

### 3.2.2 Wage Bargaining

For simplicity, let the real wage  $w_t$  be the outcome of a Nash bargain between the median worker<sup>18</sup> with operating cost  $\varepsilon^I$  and her firm. The median worker faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period  $t$ . Under bargaining agreement, the median worker receives the real wage  $w_t$  and the firm receives the expected profit  $(a_t mc_t - w_t)$  in each period  $t$ . Under disagreement, the worker's fallback income is  $b_t$ , assumed for simplicity to be equal to the real unemployment benefit. The firm's fallback position is  $-s$ , where  $s$  is the cost for the firm in case of disagreement. This may be a fixed cost of non-production or a cost that is imposed due to a strike. Assuming that disagreement in the current period does not affect future surpluses, workers' surplus is  $(w_t - b_t)$  while the firm's surplus is  $a_t^I mc - w_t - \varepsilon^I + s$ , where  $\varepsilon^I$  are the operating costs of the median worker. Consequently, the Nash-product is:

$$\Theta = (w_t - b)^\gamma (a_t^I mc - w_t - \varepsilon^I + s)^{1-\gamma}, \quad (12)$$

where  $\gamma$  represents the bargaining strength of the worker relative to the firm. Maximizing the Nash-product with respect to the real wage, yields the following equation:

$$(1 - \gamma)(w_t - b) = \gamma(a_t mc_t - w_t - \varepsilon_t^I + s), \quad (13)$$

which implicitly defines the negotiated wage. Rearranging yields the following simple formula:

$$w_t = \gamma(a_t mc_t - \varepsilon_t^I + s) + (1 - \gamma)b. \quad (14)$$

It is worth noting that, due to the right to manage structures, ex-ante wages do not adjust efficiently to shocks compared to wages negotiated within efficient Nash bargaining arrangements. In efficient Nash bargaining, individual firms would have to choose wages alongside with hiring and

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<sup>18</sup>For simplicity, we allow the median worker to bargain over wages. Alternative settings, such as individual bargaining process with marginal workers, would not change the main implications of the model.

firing decisions. This would allow wages to be contingent on employment decisions and to adjust faster to shocks. Instead, in this context wages are negotiated at an aggregate level and firms make hiring and firing decisions only ex-post.<sup>19</sup> This implies, for instance, that negative shocks can affect worker flows more strongly than wages. Such a bargaining arrangement can capture well the reality of euro area labor markets in which wages are usually bargained ex-ante at an aggregate level (collectively), while individual firms make ex-post hiring and firing decision.

### 3.2.3 Marginal costs

Marginal costs in this model summarize the set of wedges that characterize the labor market. To obtain a measure of the marginal cost we should first characterize the equilibrium conditions for labor market flows. By merging equations 7 and 9 we obtain the following equilibrium condition:

$$v_{h,t} + h = v_{f,t} - f, \quad (15)$$

which states that if hiring costs happen to be higher than firing costs the hiring threshold should adjust to equilibrate the market. This condition implies that marginal costs can be equally derived from 7 or from 9. The expression for marginal costs will then read as follows:

$$mc_t = \left( w_t + v_{f,t} - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) \right) / a_t. \quad (16)$$

In this context wages lose part of their allocative role as marginal costs depend also on two additional components. The first component, which is given by  $v_{f,t} - f$  is an intra-temporal wedge which makes hiring (and firing) deviate from the ones that would arise in a Walrasian labor market at any time  $t$ . The second component, represented by  $E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))$ , is an inter-temporal wedge which distorts hiring (and firing) decisions between two sub-sequent dates. This second wedge represents the long run value of a worker as by retaining the marginal worker the firm can earn extra profits in the future. Because of this positive externality attached to the marginal worker, retention rates tend to be higher than job finding rates.

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<sup>19</sup>This is a particular case of a sequential bargaining framework proposed by Manning 1987, as firms and workers fail to internalize the consequences of today's wage decisions on future hiring and firing decisions. The scope for pre-commitment is barred as neither workers nor firms can credibly commit to a sequence of future wages and employment.



### 3.2.4 Wholesale Sector and Retail Sector

Firms in the wholesale sector are distributed on the unit interval and indexed by  $i$ . They produce a differentiated good  $y_{i,t}$  using the linear production technology  $y_{i,t} = z_{i,t}$ , where  $z_{i,t}$  is their demand for intermediate goods. They sell their goods under monopolistic competition to the retailers who use the differentiated goods to produce the final consumption good according to the Dixit-Stiglitz-aggregator:

$$y_t = \left( \int y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (17)$$

which delivers the standard price index (where  $P_{i,t}$  and  $y_{i,t}$  denote the firm specific price and output level respectively):

$$P_t = \left( \int P_{i,t}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad (18)$$

from the cost minimization problem of the aggregating firm. The implied demand function for differentiated products is

$$y_{i,t} = y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon}. \quad (19)$$

Firms in the wholesale-sector can change their prices every period, facing quadratic price adjustment costs a la Rotemberg. They maximize the following profit function:

$$\tilde{\Pi}_{W,t} = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \frac{P_{i,j}}{P_j} y_{i,j} - mc_j y_{i,j} - \frac{\Psi}{2} \left( \frac{P_{i,j}}{P_{i,j-1}} - 1 \right)^2 Y_j \right], \quad (20)$$

where  $\Psi$  is a parameter measuring the extent of price adjustment costs, which are due whenever there are price changes (i.e., whenever  $P_{i,j}/P_{i,j-1}$  is different from 1). Taking the derivative with respect to the price yields, after some manipulations, the following expectational Phillips curve:

$$\begin{aligned} 0 &= (1 - \varepsilon) + \varepsilon mc_t - \Psi (\pi_t - 1) \pi_t \\ &\quad + E_t \left\{ \Delta_{t,t+1} \Psi (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \pi_{t+1} \right\}. \end{aligned} \quad (21)$$

### 3.3 Workers' Heterogeneity and Aggregation

Let's start by deriving aggregate real profits of intermediate firms which are given by revenues minus wage payments, operating costs and labor turnover costs:

$$\begin{aligned}\tilde{\Pi}_I &= mc_t a_t n_t - w_t n_t - (1 - \phi_t) n_t \Xi_t^i - \\ &\quad (1 - n_t) \eta_t \Xi_t^e - n_t \phi_t f - (1 - n_t) \eta_t h,\end{aligned}\tag{22}$$

where  $\Xi_t^i$  is the expected value of operating costs for incumbent workers, conditional on not being fired and  $\Xi_t^e$  is the expected value of operating costs for entrants, conditional on being hired, defined by:

$$\begin{aligned}\Xi_t^e &= \frac{\int_{-\infty}^{v_h} \epsilon_t f(\epsilon_t) d\epsilon_t}{\eta_t}, \\ \Xi_t^i &= \frac{\int_{-\infty}^{v_f} \epsilon_t f(\epsilon_t) d\epsilon_t}{1 - \phi_t}.\end{aligned}$$

The real profits ( $\tilde{\Pi}_W$ ) of the wholesale sector are given by:

$$\tilde{\Pi}_W = y_t - mc_t a_t n_t - \frac{\Psi}{2} (\pi_t - 1)^2 y_t.$$

Retailers make zero-profits. Aggregate real profits in this economy therefore are given by:

$$\begin{aligned}\tilde{\Pi}_{a,t} &= y_t - w_t n_t - n_t \phi_t f - u_t \eta_t h - (1 - \phi_t) n_t \Xi_t^i - \\ &\quad (1 - n_t) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t.\end{aligned}\tag{23}$$

We can substitute this into the budget constraint, (2), and after imposing equilibrium in the bond market and using the government budget constraint ( $g_t + B_t U_t = T_t$ ) we obtain the following resource constraint:

$$\begin{aligned}g_t + c_t &= y_t - n_t \phi_t f a_t - (1 - n_t) \eta_t h - (1 - \phi_t) n_t \Xi_t^i - \\ &\quad (1 - n_t) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t.\end{aligned}\tag{24}$$

Note that final aggregate demand includes government expenditure,  $g_t$ , which follows an exogenous AR(1) process.

Inspection of the resource constraint shows that the presence of hiring and firing costs as well as of price adjustment costs induces a waste of resources. Indeed setting hiring and firing costs to zero would increase output by an amount equal to  $n_t \phi f_t a_t - (1 - n_t) \eta_t h$ , while setting inflation equal to zero would increase resources of an amount equal to  $\frac{\Psi}{2} (\pi_t - 1)^2 y_t$ .

### 3.4 Competitive Equilibrium and the Role of Wedges in This Economy

**Definition 1.** For a given nominal interest rate  $\{i_t\}_{t=0}^{\infty}$  and for a given set of the exogenous processes  $\{a_t, g_t\}_{t=0}^{\infty}$  a determinate competitive equilibrium for the distorted competitive economy is a sequence of allocations and prices  $\{c_t, \pi_t, mc_t, v_{h,t}, \eta_t, v_{f,t}, \phi_t, w_t, y_t\}_{t=0}^{\infty}$  which, for given initial  $B_0$  satisfies equations 3, 7, 8, 9, 10, 11, 14, 21, 24.

The competitive equilibrium in this economy is distorted by the wedges induced by hiring and firing costs and by the inability of wages to adjust promptly to idiosyncratic shocks. In equilibrium this makes unemployment fluctuations to deviate from the efficient ones. Hence the monetary authority faces a trade-off between stabilizing prices and reducing inefficient unemployment fluctuations.

The role of monetary policy trade-offs in this economy can be well understood by deriving the reduced form expression for the Phillips curve. By substituting the marginal cost expression, 16, into the Phillips curve, equation 21, we obtain the following expression:

$$0 = (1 - \varepsilon) + \frac{\varepsilon}{a_t} (w_t + v_{f,t} - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))) - \Psi (\pi_t - 1) \pi_t + E_t\{\Delta_{t,t+1} \Psi (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \pi_{t+1}\}, \quad (25)$$

which together with the wage equation, 13, leads to a reduced form Phillips curve (in log-linear terms):

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon W}{\Psi} \hat{w}_t + \frac{\varepsilon v_f}{\Psi} \hat{v}_{f,t} - \beta \frac{\varepsilon \tilde{\Pi}_I}{\Psi} E_t \hat{\Pi}_{I,t+1} - \frac{\varepsilon (\varepsilon - 1)}{\Psi} \hat{a}_t. \quad (26)$$

A comparison between the latter reduced form and the one arising in standard New Keynesian

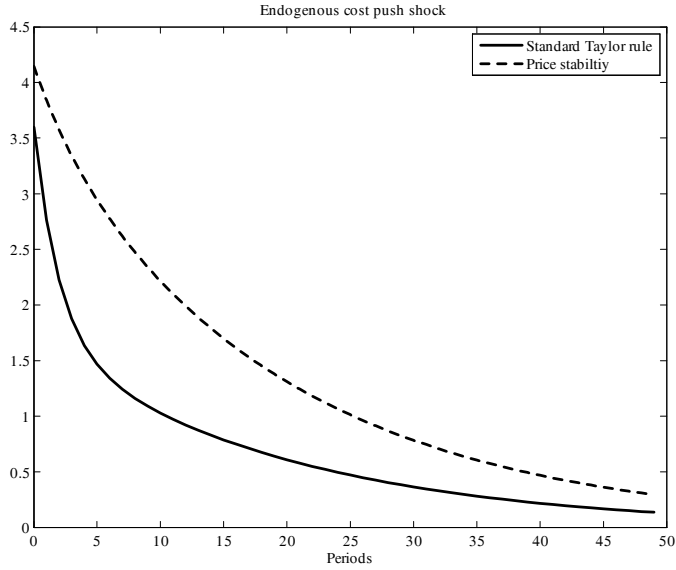


Figure 4: Reaction of the endogenous cost-push shock (in percent) due to a productivity shock under two different monetary policy regimes (standard Taylor rule and price stability).

models with Walrasian labor markets allows to highlight the nature of the monetary policy trade-offs. In standard New Keynesian models the Phillips curve takes the following form:

$$0 = (1 - \varepsilon) + \varepsilon \left( \frac{w_t}{a_t} \right) - \Psi (\pi_t - 1) \pi_t + E_t \{ \Delta_{t,t+1} \Psi (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \pi_{t+1} \}. \quad (27)$$

In response to a productivity shock the marginal cost is unaffected as  $w_t$  moves one to one with the shock. This implies that inflation is unaffected too and that monetary policy should stay passive. In this context price stability policy allows the policy maker to achieve the flexible price allocation, which also corresponds to the Pareto optimal allocation. On the other side in equation 25 the extra component represented by  $v_{f,t} - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))$  plays the role of an *endogenous cost push shock (ECPS)*. Consider a monetary policy which follows a price stability rule.<sup>20</sup> Under this policy the ECPS increases in response to positive productivity shocks (see figure 4, where the calibration of the parameters follows the one described in section 4.2).

Indeed, in response to positive productivity shocks both the firing threshold,  $v_{f,t}$ , and future profits,  $\tilde{\Pi}_{I,t+1}$ , increase. However since the firing threshold rises more than future profits, marginal

<sup>20</sup>We model this by setting the weight on inflation in the Taylor rule to 100.

costs, hence inflation, rise. In this context the flexible price allocation is simply not feasible any longer.

**Proposition 1.** *For the model economy described in Definition 1, a flexible price allocation with constant employment is not implementable and is not optimal.*

**Proof.** The monetary authority achieves the flexible price allocation by following a zero inflation policy. Imposing zero inflation on equation 25 leads to the following relation:

$$\frac{\varepsilon_t - 1}{\varepsilon_t} = \frac{(w_t + v_{f,t} - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})))}{a_t}. \quad (28)$$

Consider a reference economy in which the labor market achieves full employment. In this case, employment does not change (as it is already at the full employment level) for any increase in productivity, hence the marginal cost stays constant:

$$\frac{\varepsilon - 1}{\varepsilon} = \frac{b + v_f^{FE} - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))}{a_t}. \quad (29)$$

Lets' now consider condition 28 under the wage schedule that would arise when each worker can bargain individually with the firm. Using equations 14 and 9 the marginal cost becomes:

$$\frac{\varepsilon_t - 1}{\varepsilon_t} = \frac{(\gamma a_t + (1 - \gamma) v_{f,t} + (1 - \gamma) b - f - E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})))}{a_t}. \quad (30)$$

When employment is below the full employment level, changes in productivity lead to changes in employment, hence in marginal costs. This implies that in our model the allocation with constant mark-up is never implementable and does not lead to full employment.

The fact that employment changes in response to productivity shocks implies that the model features a time-varying wedge which cannot be off-set by a constant fiscal subsidy. Hence only a constrained Pareto efficient allocation can be achieved by a policy maker which maximizes agents' utility given the constraints of the competitive economy. Such an optimization problem leads to *shock contingent policy actions* that allow to smooth inefficient unemployment fluctuations. To understand the transmission mechanism through which the monetary authority can increase employment, let's get back to equation 5 and write the zero profit condition at the hiring threshold,  $\tilde{\Pi}_{I,t}(\varepsilon_t) = h$  as:

$$h + v_{h,t} = (a_t mc_t - w_t) + \tag{31}$$

$$E_t \left\{ \sum_{j=t+1}^{\infty} \Delta_{t,j} \left[ (1 - \phi_j)^{j-t} \left( a_j mc_j - w_j - \left( \frac{1}{1-\phi_j} \int_{-\infty}^{v_{f,j}} \varepsilon_j g(\varepsilon_j) d\varepsilon_j \right) \right) - \phi_j f_j (1 - \phi_j)^{j-t-1} \right] \right\}.$$

For given hiring costs the monetary authority can increase the hiring threshold (therefore employment) by increasing current and future firms' marginal revenues,  $a_{t+i} mc_{t+i}$ . When prices are sticky an increase in demand, achieved through monetary policy easing, reduces mark-ups and increases marginal costs, hence revenues. For given hiring costs, an increase in marginal costs leads to an increase in the hiring threshold, hence in employment.

### 3.5 Dynamic Properties of the Model

Before turning to the main focus of the paper, the design of optimal monetary policy, it is worth to verify the ability of the model to replicate the empirical evidence on labor turnover costs and macroeconomic volatilities. This makes the model suitable for policy analysis. Figure 5 shows output and inflation responses to a productivity shock with auto-correlation 0.95 under a standard Taylor rule and under three different firing cost levels ( $fc = 0.5, 0.6,$  and  $0.7$ ). The calibration of the parameters follows the one described in section 4.2. The graph shows that the model persistence (volatility) increases (decreases) with the level of firing costs.

To make our model results comparable to the empirical graphs shown further above, we calculate the model's HP-filtered standard deviation for a joint productivity and government spending shock under the different firing cost levels. Figures 6 and 7 show the results. In our model, the volatility of output and inflation, both decrease when we increase the firing costs. The absolute value of the volatilities generated by the model is very close to the absolute value shown in the data. The negative relation between the volatilities and the firing costs captures quite closely the one in the data.<sup>21</sup>

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<sup>21</sup>Note however that a direct comparison between 6 and 7 and the empirical counterparts is not possible as the measure of the hiring costs in the data is only a qualitative indicator that does not find an exact correspondence in the firing costs as calibrated in the model. See section 4.2 for a description of the calibration of the firing costs in the model.

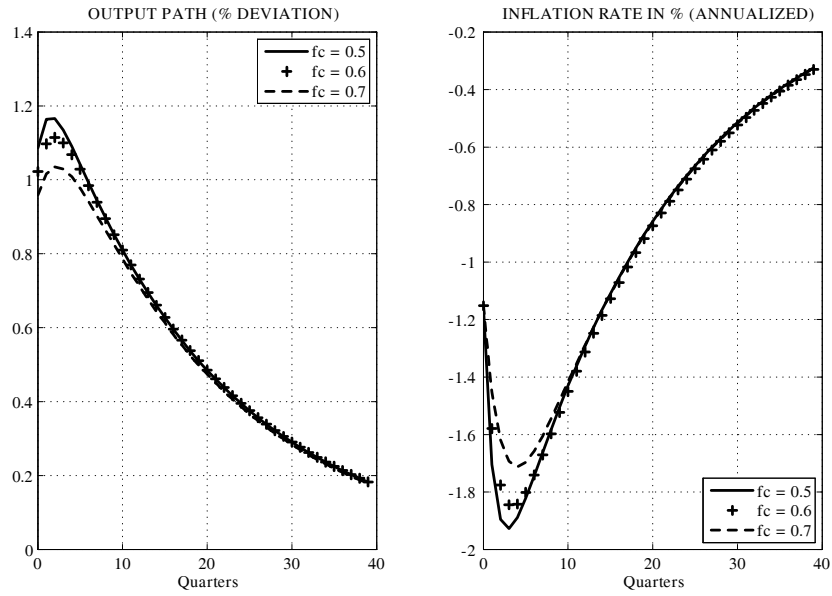


Figure 5: Model reaction under different firing costs

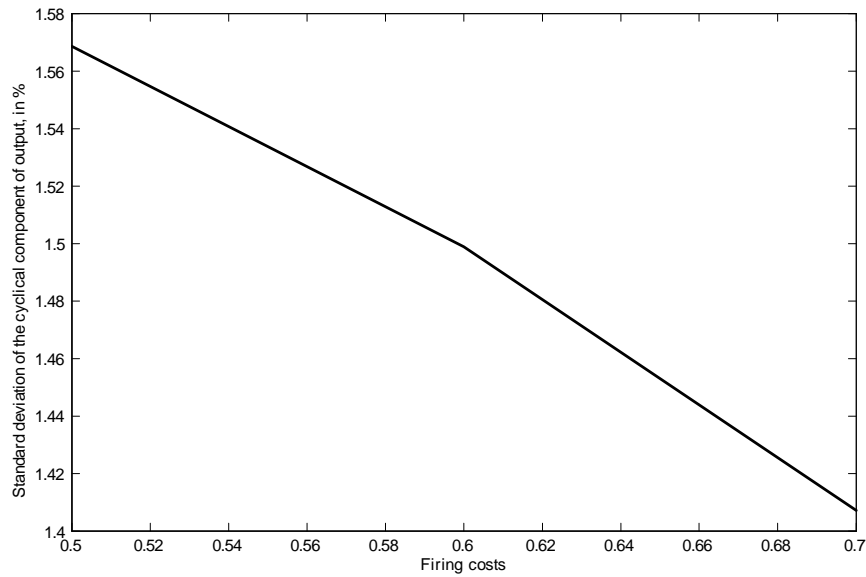


Figure 6: Changes in the volatility of output with respect to changes on the firing costs, under Taylor rules and both, productivity and government expenditure shocks.

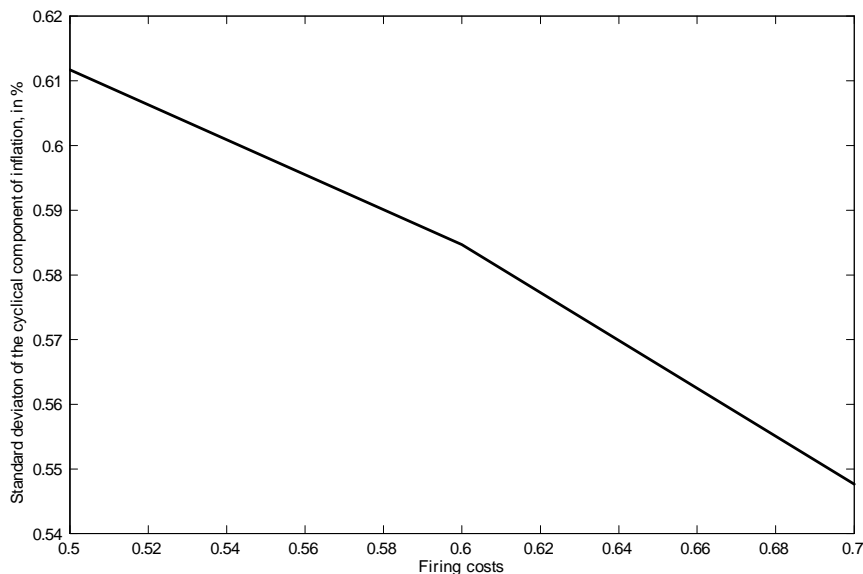


Figure 7: Changes in the volatility of inflation with respect to changes on the firing costs, under Taylor rules and both, productivity and government expenditure shocks.

## 4 Optimal Ramsey Policy

This section turns to the specification of a general set-up for the optimal policy conduct. The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983. There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe 2004 analyze a problem of joint determination of optimal monetary and fiscal policy.



$$a_t m c_t - w_t - v_{h,t} + E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) = h \quad (32)$$

$$a_t m c_t - w_t - v_{f,t} + E_t(\Delta_{t,t+1} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) = -f \quad (33)$$

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t \quad (34)$$

$$w_t = \gamma (a_t m c_t - \varepsilon_t^I) + (1 - \gamma) b \quad (35)$$

$$0 = y_t u_{c,t} [(1 - \varepsilon) + \varepsilon(m c_t) - \Psi(\pi_t - 1) \pi_t] + E_t\{u_{c,t+1} \Psi(\pi_{t+1} - 1) y_{t+1} \pi_{t+1}\} \quad (36)$$

$$\begin{aligned} c_t = & y_t - n_t \phi f_t - (1 - n_t) \eta_t h - (1 - \phi) n_t a_t \Xi_t^i - \\ & (1 - n_t) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t \end{aligned} \quad (37)$$

The government resource constraint does not need to be included among the equilibrium conditions as fiscal policy is passive (lump sum taxation).

**Definition 2.** Let  $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^\infty$  represent the sequence of Lagrange multipliers on the constraints (32), (33), (34), (35), (36), (37). Then for a given stochastic process  $\{a_t, g_t\}_{t=0}^\infty$ , plans for the control variables  $\Xi_t^n \equiv \{c_t, n_t, w_t, m c_t, \pi_t, v_{h,t}, v_{f,t}\}_{t=0}^\infty$  and for the co-state variables  $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^\infty$  represent a first best constrained allocation if they solve the following maximization problem:

$$\text{Min}_{\{\Lambda_t^n\}_{t=0}^\infty} \text{Max}_{\{\Xi_t^n\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad (38)$$

subject to (32), (33), (34), (35), (36), (37).

#### 4.1 Non-Recursivity and Initial Conditions

As a result of constraints (32), (33) and (36) exhibiting future expectations of control variables, the maximization problem as spelled out in (38) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, and then developed by Marcet and Marimon 1999, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that we denote  $\chi_{1,t}$ ,  $\chi_{2,t}$  and  $\chi_{3,t}$  for (32), (33) and (36) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the

specification of the law of motion of these Lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

$$\begin{aligned}\frac{\chi_{1,t+1}}{\beta} &= \lambda_{1,t} \\ \frac{\chi_{2,t+1}}{\beta} &= \lambda_{2,t} \\ \frac{\chi_{5,t+1}}{\beta} &= \lambda_{5,t}\end{aligned}\tag{39}$$

Using the new co-state variables so far described we amplify the state space of the Ramsey allocation to be  $\{a_t, \chi_{1,t}, \chi_{2,t}, \chi_{5,t}\}_{t=0}^{\infty}$  and we define a new saddle point problem which is recursive in the new state space. Consistently with a *timeless perspective* we set the values of the three co-state variables at time zero equal to their solution in the steady state.

## 4.2 Model Calibration

The calibration is summarized in table 1 below.

**Preferences.** The discount rate,  $\beta$ , is set to 0.99, consistently with an annual interest rate of 4 percent. The intertemporal elasticity of substitution,  $\sigma$  is set to 2. The elasticity of substitution between different product types,  $\varepsilon$ , is set to 10 (see, e.g., Galí 2008).

**Pricing.** The parameter of price adjustments,  $\Psi$ , is calibrated in line with microeconomic evidence for Europe (see Alvarez et al. 2006).

**Labor markets.** The bargaining power of workers,  $\gamma$ , is set to a benchmark value of 0.5. Taking continental Europe as reference point, the firing costs are set to 60 percent ( $f = 0.6$ ) of the annual productivity which amounts to approximately 66 percent of the annual wage<sup>22</sup> and the hiring costs are set to 10 percent ( $h = 0.1$ ) of annual productivity (see Chen and Funke 2003). The unemployment benefits is set to 65 percent of the level of productivity ( $b = 0.65$ ). This implies, that in steady state the wage replacement rate is roughly 71 percent, which is in line with evidence for continental European countries (see OECD 2007). Operating costs are assumed to follow a logistic

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<sup>22</sup>For the period from 1975 to 1986 Bentolila and Bertola 1990 calculate firing costs of 92 percent, 75 percent and 108 percent of the respective annual wage in France, Germany and Italy respectively. The OECD 2004 reports that many European countries have reduced their job security legislation somewhat from the late 1980 to 2003 (in terms of the overall employment protection legislation strictness). Therefore, we consider  $f = 0.6$  to be a realistic number for continental European countries.

distribution with zero mean. The scaling parameter of the distribution and the payments under disagreement,  $s$ , are chosen in such a way that the resulting labor market flow rates match the empirical hiring and firing rates described further below. This yields a scale parameter of 0.53 and payments under disagreement slightly below 0.28. We calibrate our flow rates using evidence for West Germany, as there are only Kaplan-Meier functions for individual countries.<sup>23</sup> Wilke's 2005 Kaplan-Meier functions indicate that about 20 percent of the unemployed leave their status after one quarter. For a steady state unemployment rate of 9 percent, a quarterly job finding rate of 2 percent is necessary. This is roughly in line with Wilke's estimated yearly risk of unemployment. The used flow numbers are in line with the OECD, 2004, numbers for other continental European countries.<sup>24</sup> Hence a quarterly job hiring rate of  $\eta = 0.20$  and a firing rate of  $\phi = 0.02$  are reasonable averages for continental European countries.

**Shocks.** We parameterize the shock processes in line with evidence for industrialized countries. Productivity shocks follow an  $AR(1)$  process. The autocorrelation is set to 0.95 and the standard error of the shock is 0.008. Government consumption evolves according to the following exogenous process,  $\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g$ , where the steady-state share of government consumption,  $g$ , is set so that  $\frac{g}{y} = 0.25$  and  $\varepsilon_t^g$  is an i.i.d. shock with standard deviation  $\sigma_g$ . We set  $\sigma_g = 0.0074$  and  $\rho_g = 0.9$  (see, e.g., Perotti 2004).

### 4.3 Optimal Policy in the Long run

Before turning to the analysis of the dynamic path of the optimal policy plan in response to shocks, this section characterizes the long-run optimal policy. Long-run optimal policy is represented by the steady state around which the dynamic optimal plan evolves. In analogy with the Ramsey-Cass-Koopmans model, such steady state amounts to computing the *modified golden rule* steady state<sup>25</sup>. The unconstrained optimal long-run rate of inflation (arising from the modified golden rule) is

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<sup>23</sup>We choose the Kaplan-Meier functions for Germany, as it is the largest continental European country.

<sup>24</sup>Although the numbers of the OECD outlook are not directly applicable to our model, since they are built on a monthly basis, it is possible to adjust them using a method described in Shimer 2007.

<sup>25</sup>Notice that an important distinction must be made between the optimal level of inflation characterizing the *modified golden rule* and the one characterizing the *golden rule* (See also King and Wolman 1996). In dynamic economies with discounted utility in fact the two level of inflations do not necessarily coincide. The golden rule level of inflation is the one that maximizes households' instantaneous utility under the constraint that the steady state conditions are imposed ex-ante. The impatience reflected in the rate of time preferences gives rise to a negatively sloped long run Phillips curve which by constraining the optimal policy problem maintains alive the tension between closing the inflation gap on the one side and the inefficient unemployment gap on the other.

Table 1: Parameters of the Numerical Model

Parameter	Description	Value	Source
$\beta$	Subjective discount factor	0.99	Standard value
$\sigma$	Consumption utility	2	Intertemp. elasticity of subst.
$\varepsilon$	Elasticity of subst.	10	Gali 2008
$\Psi$	Price adjustment cost	104.85	Equivalent to $\theta = 0.75$
$a$	Productivity	1	Normalization
$\gamma$	Workers' bargaining power	0.5	Standard value
$f$	Firing cost	0.6	Bentolila and Bertola 1990
$h$	Hiring cost	0.1	Chen and Funke 2003
$b$	Unemployment benefits	0.65	OECD 2007
$E(\varepsilon)$	Expected value of op. costs	0	Normalization
$sd$	Distr. scaling parameter	0.53	To match the flow rates
$s$	Payments under disagreement	0.28	To match the flow rates

the one to which the planner would like the economy to converge to if allowed to undertake its optimization unconditionally. It is obtained by imposing steady state conditions ex-post on the first order conditions of the Ramsey plan. Let's focus on the first order condition with respect to inflation which reads as follows:

$$(\lambda_{5,t} - \chi_{5,t})c_t^{-\sigma}\psi(2\pi_t - 1) - \lambda_{6,t}\psi(\pi_t - 1) = 0 \quad (40)$$

For the whole set of optimality conditions associated with the Ramsey plan to be satisfied at the steady state a necessary condition is that equation (40) is satisfied at the steady state. In that steady-state  $\lambda_{5,t} = \lambda_{5,t-1} = \chi_{5,t}$ . Hence condition (40) immediately implies:

$$\lambda_6\psi(\pi - 1) = 0 \quad (41)$$

Since  $\lambda_6 > 0$  (the resource constraint must hold with equality), and  $\psi > 0$  (we are not imposing *a priori* that the steady-state coincides with the flexible price allocation), in turn (41) must imply  $\pi = 1$ . Hence the Ramsey planner would like to generate an average (net) inflation rate of zero. The intuition for why the long-run optimal inflation rate is zero is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating the inefficiency related to the goods and labor markets. Hence the planner aims at choosing that rate of inflation that allows to minimize the cost of adjusting prices as summarized by the quadratic term  $\frac{\psi}{2}(\pi_t - 1)^2$ .

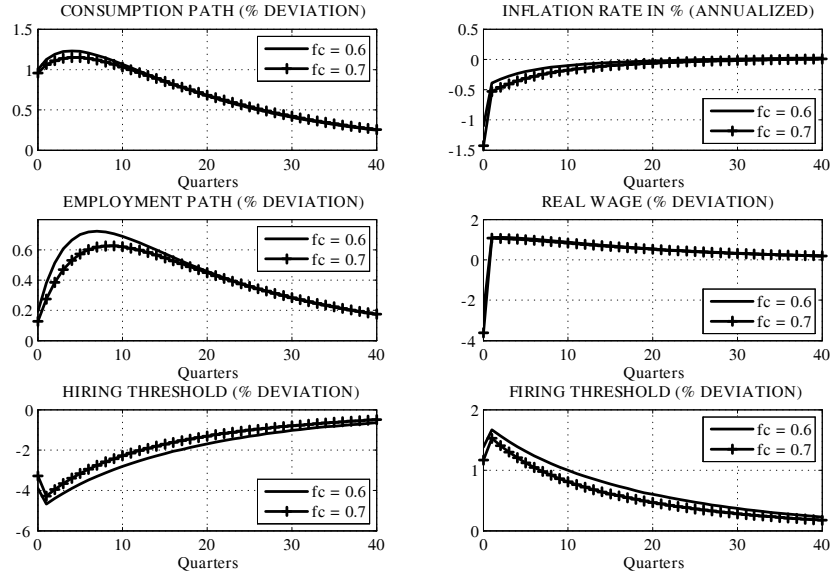


Figure 8: Impulse responses of Ramsey plan to productivity shocks (under two different firing costs).

#### 4.4 Response to Shocks and Optimal Volatility of Inflation

To compute responses of the optimal plan to shocks we resort on second order approximations<sup>26</sup> of the first order conditions of the Lagrangian problem described in definition 2. Technically, we compute the stationary allocation that characterizes the deterministic steady state of the first order conditions to the Ramsey plan. We then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady state already for a long period of time (under timeless perspective).

Figure 8 shows impulse response functions of the Ramsey plan to positive productivity shocks. In response to an increase in productivity consumption, output and employment increase. The firing threshold increases, implying a reduction in the mass of firings. On the other side the hiring threshold falls implying an increase in the mass of hiring. Most importantly inflation deviates

<sup>26</sup>Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2007) among others.

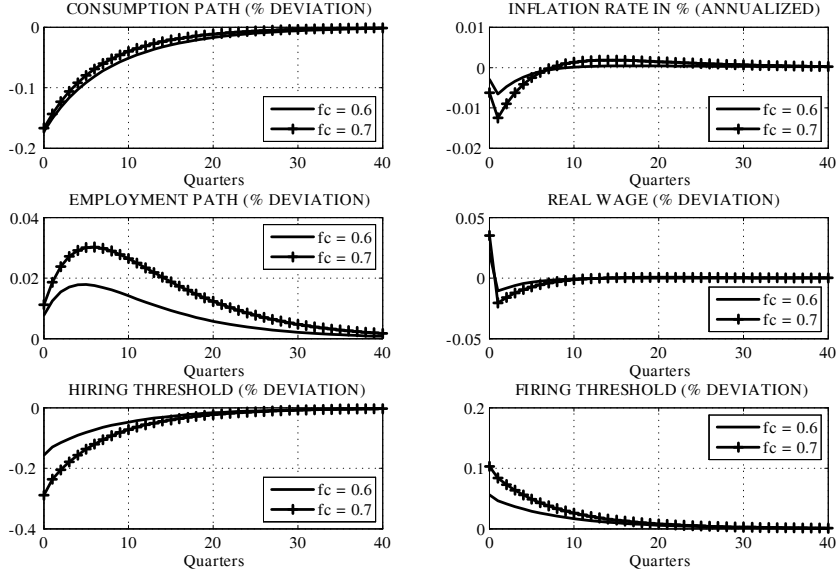


Figure 9: Impulse responses of Ramsey allocation to government expenditure shocks (under two different firing costs)

significantly from zero and falls, implying that monetary policy behaves pro-cyclically. The monetary authority in this context has a trade-off between stabilizing inflation and reducing inefficient unemployment fluctuations. The latter task can be accomplished by taking full advantage of the improved production possibilities. Therefore in response to an improvement in the production possibilities, the monetary authority reduces inflation to increase aggregate demand. Under sticky prices an increase in aggregate demand increases marginal costs, reduces the mark-ups and increases labor demand. For given hiring and firing costs, the increase in labor demand translates into an increase in the hiring threshold and a decrease of the firing threshold, as shown by conditions 7 and 9. Importantly and contrary to traditional New Keynesian models, deviations from price stability arise in this model even in response to productivity shocks. This is so since in this model the marginal costs features an extra component,  $\frac{\varepsilon}{a_t}(w_t + v_{f,t} - f - E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})))$ , which acts as an endogenous cost push shock and responds to productivity shocks (see also equation 26).

Larger hiring and firing costs lead to larger time-varying wedges and larger inefficient unemployment fluctuations, hence they boost the incentives of the policy maker to deviate from the zero

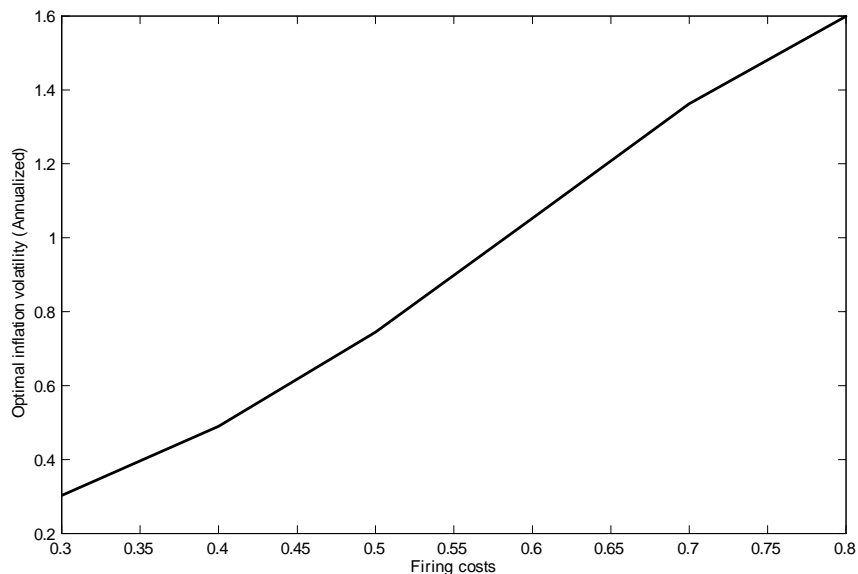


Figure 10: Optimal inflation volatility in response to the two shocks

inflation policy. This can be seen from figure 10 which shows that the optimal volatility of inflation, in response to both productivity and government expenditure shocks, increases when firing costs increase. In our model time-varying wedges are mapped into firms' rents, hence the monetary authority uses the only available instrument, inflation, to tax firms' rents.

Finally, figure 9 shows impulse responses of the Ramsey plan in response to government expenditure shocks. An increase in government expenditure crowds out consumption demand. However because of the increase in aggregate demand employment increases, the mass of firings shrinks and the mass of hiring rises. Once again deviations from price stability arise. This result is consistent with the literature, as Adao, Correia and Teles 2003 and Khan, King and Wolman 2003 have shown that shocks to government expenditure cause fluctuations in the ratio between aggregate demand and output which prevent implementability of the flexible price allocation with constant mark-ups. Notice however that consistently with previous studies deviations from zero inflation are rather small under this shock.

## 5 Optimal Operational Rules

So far our analysis indicated what should be the optimal path of variables if the economy had been run by a Ramsey planner. Now we follow a Tinbergen approach and ask what type of operational<sup>27</sup> rules central bankers should follow in the context of our model if they want to maximize agents' welfare. We therefore solve the problem of a monetary authority which maximizes households welfare subject to the competitive equilibrium conditions and the class of monetary policy rules represented by:

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_n \ln\left(\frac{n_t}{n}\right)\right) + \phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) \quad (42)$$

where  $\phi_\pi$  represents a response to the CPI inflation rate,  $\phi_y$  represents a response to the output gap,  $\phi_n$  represents the response to unemployment and  $\phi_r$  represents interest rate smoothing. Within this class of rules, we look for the coefficients which deliver the highest level of welfare. Note that we allow interest rates to react to unemployment alongside with inflation. The reason for that is as follows. In our model the policy maker faces a trade-off between inflation and unemployment stabilization due to the inefficiencies associated with the labor market. Because of this we expect a rule targeting unemployment alongside with inflation to perform better than a rule neglecting labor market variables.

Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. Indeed in an economy with a distorted steady state, stochastic volatility affects both first and second moments of those variables that are critical for welfare. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions.<sup>28</sup> Additionally one needs to focus on the *conditional* expected discounted utility of the representative agent. This allows to account for the transitional effects from the deterministic to the different stochastic steady states respectively implied by each

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<sup>27</sup>The word operational indicates two things: a) a rule that responds to variables which can be easily observed and b) a rule that delivers real determinacy.

<sup>28</sup>See Kim and Kim 2003 for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.



alternative policy rule. Define  $\Omega$  as the fraction of household's consumption that would be needed to equate conditional welfare  $\mathcal{W}_0$  under a generic interest rate policy to the level of welfare  $\widetilde{\mathcal{W}}_0$  implied by the optimal rule. Hence  $\Omega$  should satisfy the following equation:

$$\mathcal{W}_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1 + \Omega)c_t) \right\} = \widetilde{\mathcal{W}}_0$$

Under a given specification of utility one can solve for  $\Omega$  and obtain:

$$\Omega = \exp \left\{ \left( \widetilde{\mathcal{W}}_0 - \mathcal{W}_0 \right) (1 - \beta) \right\} - 1$$

In our simulations, we search for the rule that maximizes welfare<sup>29</sup>. In response to both productivity and government expenditure shocks the optimal rule features the following coefficients:  $\phi_r = 0.9, \phi_\pi = 3, \phi_y = 0, \phi_n = 0.2$ . Several results arise. First, the optimal rule must respond to employment, alongside with inflation, to smooth inefficient fluctuations. The response to output is welfare detrimental. As shown in previous literature (see Schmitt-Grohe and Uribe 2004, Faia and Monacelli 2004) responding to output might be welfare detrimental if an appropriate measure of the output gap is not available. Third, the optimal rule is characterized by a significant degree of interest rate smoothing: the excess volatility generated by the labor market frictions requires the monetary authority to take over a stabilization role.

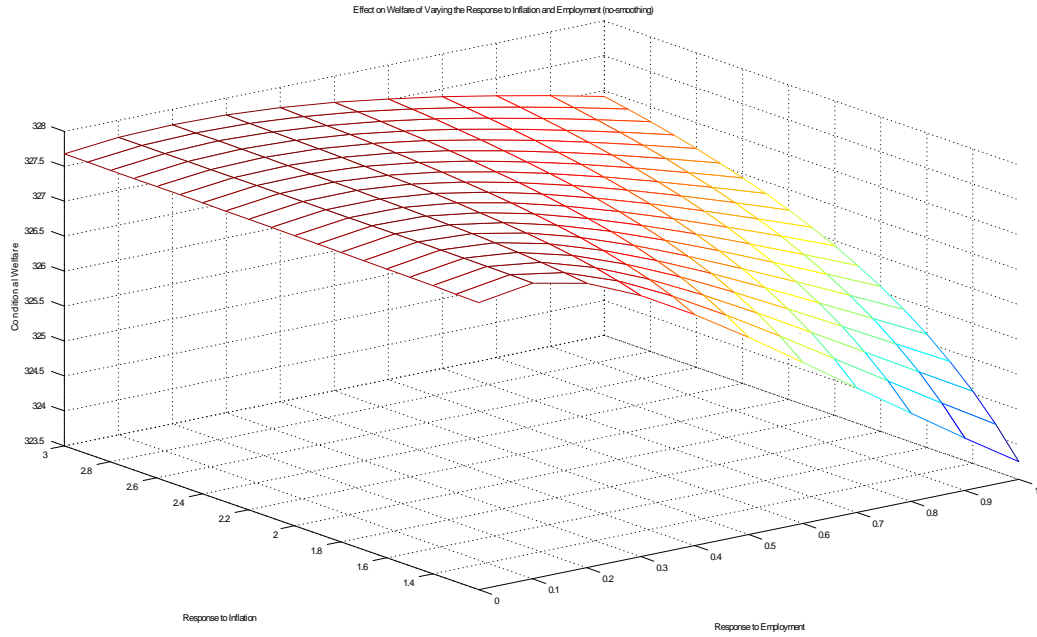
Figure 5 shows the welfare gain of responding to inflation and employment: the gains reach a maximum when the response to employment is at 0.2 and decrease after that. The graph also shows that, for any level of the response to employment, it is always welfare improving an aggressive response to inflation. Hence stabilizing prices remains an important goal to achieve within the set defined by the operational rules.

## 6 Conclusions

The design of optimal monetary policy is derived in a DSGE model with sticky prices, labor turnover costs and right-to-manage bargaining. The model assumptions are meant to capture the reality

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<sup>29</sup>The search grid is set as follows:  $\phi_r = [0, 0.9], \phi_\pi = [0, 3], \phi_n = [0, 1]$ . The coefficient on output is set to zero, as any positive value decreases the welfare gains for any combination of the other parameters.



of Euro area labor markets. The type of labor market frictions considered give rise to non-trivial trade-offs for the monetary authority. Optimal policy features deviations from price stability and those deviations are larger the larger the size of firing costs. The optimal operational rule should respond to unemployment alongside with inflation.

This paper provides a theoretical and a policy contribution. From a theoretical point of view our analysis shows that the case for price stability can be challenged if one considers a model with a significant role for real frictions. Optimal monetary policy in the presence of real frictions can be usefully characterized by applying a Ramsey-type analysis. Our analysis carries also some important policy implications. Before the Lisbon agenda is brought to completion and structural reforms in the labor market are implemented, an active role for the monetary authority is needed to overcome some of the welfare costs generated by turnover costs and bargaining by incumbent workers.

A natural extension of this analysis is to consider the role of labor turnover costs in a DSGE model for a currency area model. Euro area countries face significant differences in terms of labor market institutions, particularly turnover costs and employment protection indices. An analysis of

those differences could shed light on the differential response of output and inflation to common monetary policy actions.

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