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Money Is the Root of Asset Bubbles

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Money Is the Root of Asset Bubbles

Abstract

This paper examines how monetary expansion causes asset bubbles. When there is no monetary expansion, a bubbly asset is not created due to a hold-up problem. Monetary expansion increases buyers' money holdings, and then, dealers are willing to buy a worthless asset from sellers, in hopes of selling it to buyers who may not know that it is worthless—a bubble now occurs.

JEL-Codes: D820, D830, D840, E440, E520, G120, G140.

Keywords: bubbles, dealers, higher-order uncertainty, money.

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1 Introduction

As advocated by many policymakers and observers, e.g., Masaaki Shirakawa, the former governor of Bank of Japan (see Okina, Shirakawa, and Shiratsuka 2001) and Krugman (2015), bubbles occur when "too much money is chasing too few investment opportunities." While such a view is widely accepted not only among policymakers but also among the public, there has been few attempts in the economics academia to examine the exact mechanism that links too much money and bubbles.[1] Kindleberger (2000) states similar sentiments in a chapter entitled "Fueling the Flames: Monetary Expansion."

The objective of this paper is to fill this gap. We propose a new framework in which too much money causes bubbles. Specifically, we incorporate a finite-horizon asset-bubble framework into a workhorse model in monetary theory by Lagos and Wright (2005). In each period, a decentralized asset market opens where agents use money as a payment instrument. Sellers produce an asset that buyers may wish to obtain. Agents have higher-order uncertainty, that is, a lack of common knowledge. Even if all agents know that the asset is worthless, there is a situation where dealers intermediate the trade between buyers and sellers, knowing the asset is worthless, but without knowing whether buyers know that the asset is worthless. Then, dealers may have incentives to buy the asset from sellers in hopes of selling it to buyers.

In this framework, *too much money* is modeled as a consequence of an exogenous *monetary expansion*, with which the central bank issues money at the beginning of the decentralized market, and buyers receive the newly issued money as a lump-sum transfer. Hence, monetary expansion leads to a large amount of buyers' money holdings in the decentralized market, irrespective of their will.[2] The tractability of Lagos and Wright (2005) allows us to analyze such a monetary policy with a sound micro-founded

[1] Barlevy (2018) points out the importance of knowing how monetary (and other) policies affect bubbles.

[2] This is similar to the liquidity effects studied in, for example, Grossman and Weiss (1983) and Lucas (1990). Buyers, on the other hand, have no incentive to refuse to get the money.

theory of money.

Our main result is, informally,

Theorem. *An asset bubble is fueled by monetary expansion.*

We first observe that without monetary expansion, asset bubbles do not occur. This is due to a *hold-up problem*. Since buyers know that dealers do not consume the asset, dealers have to accept any terms of trade from buyers once they obtain the asset and trade with buyers. In other words, not bringing much money to the decentralized market is a commitment device for buyers to not offer what they think the asset is really worth. Anticipating this, buyers then do not have incentives to bring money to the decentralized market, and as a result, dealers are unwilling to buy the asset from sellers—bubbles never occur.

Now, how does monetary expansion cause bubbles? Too much money, induced by monetary expansion, loosens buyers' budget constraints and hence their purse strings. This makes dealers—who know that the asset is worthless but do not know that buyers know it—more optimistic about buyers' payment. Thus, dealers are willing to buy the asset from sellers even if they know that it is worthless—a bubble now occurs.

Finally, too much money can arise endogenously rather than from the exogenous monetary expansion. We show this in an extended model where we introduce another goods market. Unlike in the baseline model, this extension also shows that money can have value even if the bubbly asset is not traded.

Related Literature

This paper, of course, relates to the literature on both bubbles and money.

For the literature on bubbles, there are several approaches to studying bubbles (see, for example, Brunnermeier and Oehmke 2013 for a survey). Ours belongs to the one using higher-order uncertainty. Allen, Morris, and Postlewaite (1993) is the first to take the approach and show the existence of bubbles in a finite-horizon model. The

model is further refined and developed in a sequence of papers by Conlon (2004, 2015), Doblas-Madrid (2012), Liu and Conlon (2018), Awaya, Iwasaki, and Watanabe (2022), Dong, Jia, and Wang (2022), and Liu, White, and Conlon (2023). None of these models has money explicitly, and we use a New Monetarist framework to add money to Awaya, Iwasaki, and Watanabe (2022), henceforth referred to as AIW. On top of this, another innovation relative to this literature—except for Araujo and Doblas-Madrid (2022) who show bubbles occur in a Walrasian market where prices convey information on the quality of assets—is to show that bubbles can occur in a robust equilibrium with a finite-state space even when prices are publicly observable.

This paper also belongs to the New Monetarist literature. See Lagos, Rocheteau, and Wright (2017) and Rocheteau and Nosal (2017) for recent surveys. Among New Monetarist models, environments in which money and assets with uncertain return coexist have been studied by, for example, Rocheteau (2011), Geromichalos, Herrenbrueck, and Wang (2022). These models only consider first-order uncertainty, while higher-order uncertainty plays a key role in ours. Mattesini and Nosal (2016) and Lagos and Zhang (2019, 2020) also incorporate intermediaries into the Lagos and Wright (2005) environment. Other than the fact that they do not consider asymmetric information, another important difference is the intermediation mode. In their models, dealers are a platform that offers a marketplace to investors. In our model, dealers are *middlemen* who buy the asset from their own accounts and resell it to buyers. Monetary policy similar to ours is first introduced in Molico (2006) and then further explored by, for example, Wallace (2014). Huang and Igarashi (2023) consider such policies in the Lagos-Wright framework where agents suffer first-order uncertainty.

Finally, in the existing New Keynesian frameworks, the results on the relationship between monetary expansion and bubbles are mixed. For instance, Galí (2014, 2021) shows that a monetary policy rule that raises interest rates in response to bubbles can paradoxically lead to larger bubbles. Later, Miao, Shen, and Wang (2019) argue Galí's model contains additional equilibria in which more aggressive rules dampen bubbles.

Allen, Barlevy, and Gale (2023) show that a central bank that wants to dampen bubbles can always do so by raising interest rates aggressively enough. In contrast to these papers, we adopt a New Monetarist approach and are interested in the positive relationship between monetary expansion and the occurrence (rather than size) of bubbles. We show that such a positive relationship holds unambiguously (rather than as one possibility in multiple equilibria).

Organization of the Paper

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 shows that, for all parameter values, there is no equilibrium in which bubbles occur without monetary expansion, while Section 4 shows that, under certain conditions on parameters, there is an equilibrium in which bubbles occur with sufficiently large monetary expansion. Section 5 discusses some implications of our result, and Section 6 considers the extension.

2 Model

Time is discrete, continues forever, and is denoted by $t = 0, 1, \cdots$. Each period is divided into two subperiods as in the infinite-horizon monetary framework of Lagos and Wright (2005). In the first, agents interact in a decentralized asset market (DM). In the second, they interact in a frictionless centralized market (CM). These alternating markets induce an asynchronicity of expenditures and receipts crucial to any analysis of money—agents sometimes want to make purchases in the DM while their incomes accrue in the CM, so they must pay for those purchases using money acquired in the past. There are three types of infinitely lived agents: sellers, dealers, and buyers. They discount future at a common rate $\beta \in (0, 1)$.[3] Their types are fixed over time, and the measure of each type is normalized to one.

[3]For simplicity, there is no discounting between the DM and the CM.

We assume there is an intrinsically useless, durable and uncounterfeitable object—*money*. Money is assumed to be divisible, and let M_t be its supply when the DM of period t opens (before monetary expansion that will be described below).

At the beginning of the DM of each period t, the central bank issues τM_t units of money, where $\tau \geq 0$. Only buyers receive the newly issued money through a lump-sum transfer, and hence each buyer obtains τM_t units in the DM (see Figure 1).[4]

In the subsequent CM, the central bank eliminates the newly issued fraction of money supply through a lump-sum tax τM_t. In other words, temporary monetary expansion is expected to be followed by temporary shrink. Hence, $M_t = M_{t+1}$ for each t. Assume that this monetary expansion is common knowledge, and anticipated by all agents.[5]

The assumption that the central bank takes back money in the CM allows us to isolate the effect of monetary expansion from *inflation*—if the central bank did not take back the money, such monetary expansion would result in inflation. In general, inflation reduces the liquidity of buyers, which works against the original effect of monetary expansion of increasing the liquidity.

We will compare the case where $\tau = 0$ (no monetary expansion) to the case where τ is sufficiently large.

2.1 Centralized market

In the CM, money and a perishable good are traded. All agents enjoy $U(x)$ from consuming x units of the good. Assume that $U'(x) > 0$ and $U''(x) < 0$ for each $x > 0$. The good is numeraire and produced one-for-one using labor, and hence its price and the real wage are equal to one. Agents suffer disutility ℓ from working for ℓ hours.

[4]The assumption that only buyers get a transfer is for simplicity, and our result survives if sellers or dealers also obtain transfers.

[5]Our result would not hold if the monetary expansion was not expected.

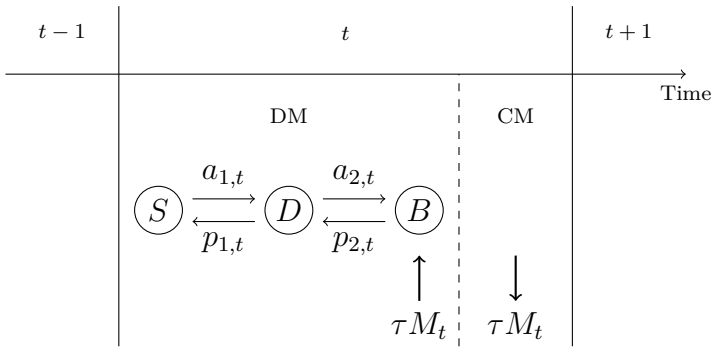

Figure 1: Timeline of the baseline model

2.2 Decentralized market

2.2.1 Economic Environment

In the DM, there exists an *indivisible* asset.[6] Each seller can produce only one unit of the asset at cost $c > 0$. Neither sellers nor dealers enjoy utility from consuming the asset, but buyers obtain utility $u > c$ with some probability and 0 with the remaining probability. Irrespective of whether it is consumed by buyers, the asset perishes after the DM.

In the DM, first, each seller meets a dealer for sure, trades the asset with the dealer, and leaves the DM. Then, the dealer meets a buyer for sure, trades the asset with the buyer, and both the dealer and the buyer leave the DM. Thus, trade between sellers and buyers is sequential and must go through dealers. For sellers and dealers, their counterparties are drawn randomly from all dealers and all buyers, respectively. Money is used as the payment instrument, and we employ generalized Nash bargaining to determine the terms of trade.[7] We denote by $\theta_1 \in (0,1)$ (resp. $\theta_2 \in (0,1)$) the

[6]Lotteries may be an efficient mechanism in the model with indivisible assets. We do not present the results with lotteries, but similar results hold with lotteries as well. Also, even if the asset is divisible, we can obtain similar implications by modifying the information structure.

[7]As in AIW, we consider a game form where a fictitious third party suggests the exchange ratio following the Nash bargaining solution, and then each agent either accepts or rejects the trade. The

bargaining power of sellers (resp. dealers) in trade between sellers and dealers (resp. dealers and buyers).

We assume that buyers *can* observe the price between sellers and dealers. We will discuss its implication in Section 5.

2.2.2 Information Structure

We will construct an information structure that induces asset bubbles. Following the notion of *strong bubbles* by Allen, Morris, and Postlewaite (1993), asset bubbles are defined as follows.

Definition 1. *An asset bubble occurs* if the asset is traded for a positive amount of money, the price of money is positive, and all agents know that the consumption value of the asset for buyers is 0.

The information structure in the DM is (a simplified version of) the finite-horizon asset-bubble model of AIW. All parameters describing utilities, costs, etc., are common knowledge except for the asset value (i.e., consumption value) for buyers. To describe the information structure, we introduce three states: ω^u_{SD}, ω^0_E, and ω^0_{SD}.[8] The superscripts indicate whether the asset value for buyers is u or 0. At ω^u_{SD}, it is u; at the other states, it is zero. The subscripts signify who knows the asset value for buyers. At ω^u_{SD} and ω^0_{SD}, only sellers and dealers know it; at ω^0_E, every agent knows it. The set of states is

$$\Omega = \{\omega^u_{SD}, \omega^0_E, \omega^0_{SD}\}.$$

trade occurs only when both agents accept. While the agents have private information, the terms of trade that the third party proposes do not depend on it. Our results do not depend on Nash bargaining, but the bargaining solution has to be Pareto efficient.

[8]In the original AIW model, at least five states are required to generate a bubble. We thank Gadi Barlevy who pointed out later that only three states are sufficient. The specification adopted in this paper is the one pointed out by him. See also Liu, White, and Conlon (2023). We deliberately employ the simplest model to highlight the key idea.

The state of each period realizes at the beginning of the DM in that period, and drawn independently across periods.

The information is the same among the agents of each type. Sellers' and dealers' partitions are the same:

$$\mathcal{P}_{SD} = \{\{\omega_{SD}^{u}\}, \{\omega_{E}^{0}, \omega_{SD}^{0}\}\}.$$

The first element, $\{\omega_{SD}^{u}\}$, corresponds to the case where the buyers' asset value is u, and sellers and dealers know it. The second element, $\{\omega_{E}^{0}, \omega_{SD}^{0}\}$, corresponds to the case where the buyers' asset value is zero, and sellers and dealers do not know whether buyers know it. Buyers' partition is

$$\mathcal{P}_{B} = \{\{\omega_{SD}^{u}, \omega_{SD}^{0}\}, \{\omega_{E}^{0}\}\}.$$

The first element, $\{\omega_{SD}^{u}, \omega_{SD}^{0}\}$, corresponds to the case where buyers do not know whether the asset value is u or 0 for them. The second element, $\{\omega_{E}^{0}\}$, corresponds to the case where the asset value is 0, and buyers know it. An agent can distinguish any two states if those states belong to a different element of his or her partition, but cannot otherwise.

The common prior distribution over Ω does not depend on time and is denoted by μ. Assume that $\mu(\omega) > 0$ for each $\omega \in \Omega$. At states ω_{SD}^{u} and ω_{SD}^{0}, buyers believe that the asset value is u with probability

$$\psi_{B} = \frac{\mu(\omega_{SD}^{u})}{\mu(\omega_{SD}^{u}) + \mu(\omega_{SD}^{0})}.$$

At state ω_{SD}^{u}, dealers know that buyers do not know the asset value, and hence they believe that they can sell the asset to buyers for sure. At states ω_{E}^{0} and ω_{SD}^{0}, dealers believe that buyers do not know the asset value with probability

$$\psi_{D} = \frac{\mu(\omega_{SD}^{0})}{\mu(\omega_{E}^{0}) + \mu(\omega_{SD}^{0})}.$$

We summarize how each state plays a role for the existence of bubbles. State ω_{SD}^{u} creates gains from trade of the asset. State ω_{E}^{0} establishes a situation where all agents

know that the asset value for buyers is zero. State ω_{SD}^0 constructs a case where the asset value is zero and buyers do not know it—only sellers and middlemen know the fact that the value is zero. These states are necessary ingredients of bubbles.

3 No Monetary Expansion

First, we will show that

Proposition 1. *There is no equilibrium in which an asset bubble occurs if there is no monetary expansion ($\tau = 0$).*

Note that in this case, money is not used, and its price $\phi_t = 0$—there is no monetary equilibrium. Of course, this describes an extreme situation. However, in Section 6, we introduce a market of consumption goods, and show that (i) there may be an equilibrium where money has positive value even when bubbles do not occur, and (ii) a bubble can occur even without monetary expansion.

The reason for the nonexistence of bubbles is a *hold-up problem*. Note that dealers do not enjoy utility from consuming the asset. Since the bargaining between dealers and buyers is efficient, the dealers must give up the asset, *regardless of the amount of buyers' money holdings*. Since holding money across periods is costly, buyers do not have any incentives to bring money. Thus, when $\tau = 0$, buyers do not have money in the DM. Then, dealers do not have any incentives to buy the asset from sellers, and therefore there is no room for bubbles to exist.

The no-bubble result seems very different from the result of AIW who show bubbles occur for all parameter values. The key difference is the timing of the bargaining. In AIW, buyers can produce goods on the spot for dealers, and hence buyers' payments are unconstrained when buyers bargain with dealers. In the current model, buyers' payments are constrained to the amount of money that they bring to the DM when they bargain with dealers. In other words, buyers can commit smaller amounts of

payment by bringing smaller amounts of money. Anticipating this, dealers do not buy the asset from sellers.

4 Monetary Expansion

In this section, we identify a necessary and sufficient condition for which an asset bubble occurs. Throughout the section, we assume that the gains from trade are sufficiently large so that

$$\frac{\theta_2 \psi_B u}{c} \geq \underline{\tau} \equiv \max\left\{\frac{1}{\psi_D}, \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}\right\}. \tag{1}$$

As we will see later, it turns out that there is no equilibrium in which an asset bubble occurs if (1) is violated. Under (1), we have

Theorem 1. *There is an equilibrium in which asset bubbles occur at a state if and only if monetary expansion is sufficiently large ($\tau \geq \underline{\tau}$).*

Of course, Proposition 1 is a corollary of the theorem. Note that (1) is more likely to be satisfied when the interest rate $(1-\beta)/\beta$ is lower.

When these conditions are satisfied, we construct an equilibrium in which bubbles occur. This equilibrium has the following features (on-path).

- Money holdings: Sellers and buyers do not bring money from the CM, while dealers bring the minimum amount of money to make the sellers to produce the asset, c/ϕ_t, where ϕ_t is the price of money. Buyers' money holdings in the DM are τM_t. Note that this amount is given by exogenous monetary expansion, rather than their choice.

- Trade between sellers and dealers: In all states, sellers produce the asset, and dealers pay c/ϕ_t in exchange for that.

- Trade between dealers and buyers: In states ω^u_{SD} and ω^0_{SD}, dealers give the asset to buyers, and buyers pay $\min\{\tau M_t, \theta_2 \psi_B u/\phi_t\}$ units of money. In state ω^0_E, no

trade occurs.

- Occurrence of bubbles: At ω_E^0, everyone knows the asset is worthless, but dealers still buy it.

The key idea is that if buyers hold enough money, dealers speculate that buyers will pay a large amount of money. This convinces sellers to produce the asset. While buyers do not have incentives to bring money themselves due to a hold-up problem, sufficiently large monetary expansion forces them to do so.

Note that this occurs at ω_E^0 as well. There, both dealers and buyers know that the asset is worthless, but dealers do not know the fact that buyers also know that. Thus, dealers speculate.

4.1 Bellman Equations

Let V_t^S, V_t^D, and V_t^B be the DM value functions of sellers, dealers, and buyers, respectively. Similarly, we denote the CM value functions by W_t^S, W_t^D, and W_t^B. For each type $i \in \{S, D, B\}$, the Bellman equation for the CM is

$$\begin{aligned} W_t^i(m_t^i) &= \max_{x_t^i, \ell_t^i, \widehat{m}_{t+1}^i} \left\{ U(x_t^i) - \ell_t^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\} \\ &\qquad \text{subject to } x_t^i = \phi_t(m_t^i - \widehat{m}_{t+1}^i) + \ell_t^i - \frac{\phi_t \tau M_t}{3}, \end{aligned}$$

where m_t^i and $\widehat{m}_{t+1}^i$ are money holdings when the CM opens and closes, and ϕ_t is the price of money. Note that since buyers receive money in the next DM (i.e., money expansion), the one they choose in the CM, $\widehat{m}_{t+1}^B$, differs from the one they actually hold in the DM. Here, we have $-\phi_t \tau M_t/3$ in the budget constraint because measure 1 of buyers receive τM_t units of money in the DM and the central bank takes back that amount of money from measure 3 of all agents through a lump-sum tax.

Assuming an interior solution for labor ℓ_t^i,

$$\begin{aligned} W_t^i(m_t^i) &= \phi_t m_t^i - \frac{\phi_t \tau M_t}{3} + \max_{x_t^i} \left\{ U(x_t^i) - x_t^i \right\} \\ &\quad + \max_{\widehat{m}_{t+1}^i} \left\{ -\phi_t \widehat{m}_{t+1}^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\}. \end{aligned}$$

The optimal consumption in the CM is pinned down by

$$U'(x_t^i) = 1.$$

The optimal money holdings when the CM closes are determined independently of money holdings when the CM opens. Thus, we have the history independence of money holdings, and all agents of each type have the same unit of money at the end of the CM. Moreover, the CM value function is linear with slope ϕ_t:

$$\frac{dW_t^i(m_t^i)}{dm_t^i} = \phi_t.$$

Trade between Dealers and Buyers

Now, we derive the terms of trade in seller-dealer meetings and dealer-buyer meetings, given money holdings of each type of agents. We start with dealer-buyer meetings and then consider seller-dealer meetings.

Consider DM trade between dealers and buyers. Let $a_{2,t}$ and $p_{2,t}$ be the amounts of the asset and money traded, respectively, that is, $p_{2,t}$ is the price of the asset. At ω_{SD}^u and ω_{SD}^0, buyers do not know the asset value. Then, from the linearity of the CM value functions, dealers' surplus is

$$W_t^D(m_t^D + p_{2,t}) - W_t^D(m_t^D) = \phi_t p_{2,t},$$

and buyers' surplus is

$$\psi_B u a_{2,t} + W_t^B(m_t^B - p_{2,t}) - W_t^B(m_t^B) = \psi_B u a_{2,t} - \phi_t p_{2,t}.$$

Therefore, the terms of trade are determined by

$$\begin{aligned} &\max_{a_{2,t} \in \{0,1\}, p_{2,t}} (\phi_t p_{2,t})^{\theta_2} (\psi_B u a_{2,t} - \phi_t p_{2,t})^{1-\theta_2} \\ &\text{subject to } a_{2,t} \leq a_t^D \text{ and } p_{2,t} \leq m_t^B, \end{aligned}$$

where a_t^D is the amount of dealers' asset holdings. Note that, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative, $\phi_t p_{2,t} \geq 0$ and $\psi_B u a_{2,t} - \phi_t p_{2,t} \geq 0$.

If $\phi_t m_t^B > 0$, the solution to the bargaining problem between dealers and buyers takes the following form:

$$\begin{aligned} a_{2,t}(a_t^D, m_t^B) &= a_t^D \\ p_{2,t}(a_t^D, m_t^B) &= \begin{cases} m_t^B & \text{if } \phi_t m_t^B < \theta_2 \psi_B u a_t^D, \\ \frac{\theta_2 \psi_B u a_t^D}{\phi_t} & \text{if } \phi_t m_t^B \geq \theta_2 \psi_B u a_t^D. \end{cases} \end{aligned}$$

Note that the amount of the asset traded is *independent* of the amount of buyers' money holdings. This is because dealers do not enjoy utility from consuming the asset, and therefore we can make buyers better off without hurting dealers by increasing the amount of the asset traded.[9] If $\phi_t m_t^B = 0$, the terms of trade $[a_{2,t}(a_t^D, m_t^B), p_{2,t}(a_t^D, m_t^B)]$ are any pairs $(a_{2,t}, p_{2,t})$ satisfying the constraints. At state ω_E^0, buyers know that the asset value for buyers is 0, and thus $a_{2,t}(a_t^D, m_t^B)$ is any of 0 and a_t^D, and $p_{2,t}(a_t^D, m_t^B)$ is 0 if $\phi_t > 0$ and any number between 0 and m_t^B if $\phi_t = 0$.

Trade between Sellers and Dealers

Next consider DM trade between sellers and dealers. For states ω_E^0 and ω_{SD}^0, let $a_{1,t}^0$ and $p_{1,t}^0$ be the amounts of the asset and money traded, that is, $p_{1,t}^0$ is the price of the asset. At states ω_E^0 and ω_{SD}^0, dealers believe that buyers do not know the asset value with probability ψ_D. Then, sellers' surplus is

$$-c a_{1,t}^0 + W_t^S(m_t^S + p_{1,t}^0) - W_t^S(m_t^S) = \phi_t p_{1,t}^0 - c a_{1,t}^0,$$

and dealers' surplus is

$$\psi_D \phi_t p_{2,t}(a_{1,t}^0, M_t^B) + W_t^D(m_t^D - p_{1,t}^0) - W_t^D(m_t^D) = \phi_t[\psi_D p_{2,t}(a_{1,t}^0, M_t^B) - p_{1,t}^0].$$

[9]The assumption that dealers do not obtain utility from assets is standard in the study of over-the-counter (OTC) markets initiated by Duffie, Gârleanu, and Pedersen (2005). Moreover, since $a_{2,t}(a_t^D, m_t^B) \in \{0, 1\}$, our results hold even when dealers obtain small utility from assets.

The terms of trade are determined by

$$\max_{a^0_{1,t}\in\{0,1\},p^0_{1,t}} (\phi_t p^0_{1,t} - c a^0_{1,t})^{\theta_1}\{\phi_t[\psi_D p_{2,t}(a^0_{1,t}, M^B_t) - p^0_{1,t}]\}^{1-\theta_1}$$
$$\text{subject to } p^0_{1,t} \leq m^D_t.$$

Again, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative. This bargaining problem is more complicated than those in the models based on Lagos and Wright (2005) especially because dealers must take buyers' money holdings into account, when they trade with sellers.

Define $w^0(M^B_t) = \theta_1\phi_t\psi_D p_{2,t}(1, M^B_t) + (1-\theta_1)c$, which is the value of money traded in the above problem if (i) the constraint, $p^0_{1,t} \leq m^D_t$, does not bind and (ii) there are gains from trade between sellers and dealers, $\phi_t\psi_D p_{2,t}(1, M^B_t) - c > 0$. We divide the argument into three cases.

Case 1: Suppose that $\phi_t\psi_D p_{2,t}(1, M^B_t) > c$, that is, the gains from trade between sellers and dealers are positive. Then, the solution to the bargaining problem between sellers and dealers is

$$a^0_{1,t}(m^D_t, M^B_t) = \begin{cases} 1 & \text{if } \phi_t m^D_t > c, \\ 0 \text{ or } 1 & \text{if } \phi_t m^D_t = c, \\ 0 & \text{if } \phi_t m^D_t < c, \end{cases}$$

$$p^0_{1,t}(m^D_t, M^B_t) = \begin{cases} \frac{w^0(M^B_t)}{\phi_t} & \text{if } \phi_t m^D_t \geq w^0(M^B_t), \\ m^D_t & \text{if } c < \phi_t m^D_t < w^0(M^B_t), \text{ or } \phi_t m^D_t = c \text{ and } a^0_{1,t}(m^D_t, M^B_t) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Case 2: Suppose that $\phi_t\psi_D p_{2,t}(1, M^B_t) = c$, that is, there are no gains from trade

between sellers and dealers. Then,

$$a^0_{1,t}(m^D_t, M^B_t) = \begin{cases} 0 \text{ or } 1 & \text{if } \phi_t m^D_t \geq c, \\ 0 & \text{if } \phi_t m^D_t < c, \end{cases}$$

$$p^0_{1,t}(m^D_t, M^B_t) = \begin{cases} \frac{c}{\phi_t} & \text{if } a^0_{1,t}(m^D_t, M^B_t) = 1, \\ 0 & \text{if } a^0_{1,t}(m^D_t, M^B_t) = 0. \end{cases}$$

Case 3: Suppose that $\phi_t \psi_D p_{2,t}(1, M^B_t) < c$, that is, there are losses from trade between sellers and dealers. Then, $a^0_{1,t}(m^D_t, M^B_t) = 0$, and $p^0_{1,t}(m^D_t, M^B_t) = 0$ if $\phi_t > 0$ and $p^0_{1,t}(m^D_t, M^B_t)$ is any number between 0 and m^D_t if $\phi_t = 0$.

For state ω^u_{SD}, let $a^u_{1,t}$ and $p^u_{1,t}$ be the amounts of asset and money traded, respectively, and define $w^u(M^B_t) = \theta_1 \phi_t p_{2,t}(1, M^B_t) + (1-\theta_1)c$. Then, we can obtain the terms of trade $[a^u_{1,t}(m^D_t, M^B_t), p^u_{1,t}(m^D_t, M^B_t)]$ by replacing ψ_D with 1 in $[a^0_{1,t}(m^D_t, M^B_t), p^0_{1,t}(m^D_t, M^B_t)]$ derived above.

Bellman Equations for the DM

Using the linearity of the CM value functions, we define the Bellman equations for the DM. For sellers,

$$\begin{aligned} V^S_t(m^S_t) = {} & W^S_t(m^S_t) + \mu(\omega^u_{SD})[\phi_t p^u_{1,t}(M^D_t, M^B_t) - c a^u_{1,t}(M^D_t, M^B_t)] \\ & + [\mu(\omega^0_E) + \mu(\omega^0_{SD})][\phi_t p^0_{1,t}(M^D_t, M^B_t) - c a^0_{1,t}(M^D_t, M^B_t)]. \end{aligned}$$

Sellers always trade with dealers.

For dealers,

$$\begin{aligned} V^D_t(m^D_t) = {} & W^D_t(m^D_t) + \phi_t \mu(\omega^u_{SD})\{p_{2,t}[a^u_{1,t}(m^D_t, M^B_t), M^B_t] - p^u_{1,t}(m^D_t, M^B_t)\} \\ & + \phi_t\{\mu(\omega^0_{SD}) p_{2,t}[a^0_{1,t}(m^D_t, M^B_t), M^B_t] - [\mu(\omega^0_E) + \mu(\omega^0_{SD})] p^0_{1,t}(m^D_t, M^B_t)\}. \end{aligned}$$

Dealers buy the asset from sellers, and at states ω^u_{SD} and ω^0_{SD}, can sell it to buyers.

Finally, for buyers,

$$\begin{aligned} V_t^B(m_t^B) &= W_t^B(m_t^B) + \mu(\omega_{SD}^u)\{ua_{1,t}^u(M_t^D, M_t^B) - \phi_t p_{2,t}[a_{1,t}^u(M_t^D, M_t^B), m_t^B]\} \\ &\quad +\mu(\omega_{SD}^0)\{-\phi_t p_{2,t}[a_{1,t}^0(M_t^D, M_t^B), m_t^B]\}. \end{aligned}$$

Buyers purchase the asset from dealers with probability $\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)$, and then, cannot enjoy utility from the purchased asset with probability $\mu(\omega_{SD}^0)$.

4.2 Equilibrium

We will derive an equilibrium where an asset bubble occurs. Let $z_t = \phi_t M_t$, which is called the *value of money in period t*.

Note that as is usual in Lagos-Wright models, it is costly to hold money, and so agents who do not use money in the DM do not buy money in the CM. In our model, this means that neither sellers nor buyers have incentives to buy a positive amount of money in the CM, and hence each dealer has M_t units of money at the beginning of period t. Then, we have $\phi_t M_t^D = z_t$. Moreover, since buyers have τM_t units of money in the DM, $\phi_t M_t^B = \tau z_t$.

Lemma 1. *Dealers' money holdings satisfy $m_t^D \leq c/\phi_t$ in any equilibrium with $\phi_t > 0$.*

This is because in all the cases in the previous section, choosing $m_t^D > c/\phi_t$ does not increase $a_{1,t}^0$ or $a_{1,t}^u$ and hence bringing more money only incurs additional cost.

For an asset bubble to occur, we must have $a_{1,t}^0(M_t, \tau M_t) = 1$, and this can occur only in Cases 1 or 2 in the previous section. The condition for which $a_{1,t}^0(M_t, \tau M_t) = 1$ can be rewritten as

$$\frac{\phi_t p_{2,t}(1, \tau M_t)}{c} \geq \frac{1}{\psi_D}. \tag{2}$$

With this condition, we also have $a_{1,t}^u(M_t, \tau M_t) = 1$ because $\psi_D < 1$.

Observe that in order for $a_{1,t}^0(M_t, \tau M_t) = 1$ to occur, we must have $m_t^D \geq c/\phi_t$ and thus $m_t^D = c/\phi_t$. Thus, $z_t = c$, which means $\phi_t = c/M_t$, and $p_{1,t}^0(M_t, \tau M_t) =$

$p^u_{1,t}(M_t, \tau M_t) = c/\phi_t$ for each t. Note that the prices are the same in all states, and thus buyers do not learn anything about the state from prices. This implies that our bubble equilibrium is robust even when prices are observable. See Section 5 for more on the observability of prices.

Now,

$$p_{2,t}(1, \tau M_t) = \begin{cases} \tau M_t & \text{if } \tau < \frac{\theta_2 \psi_B u}{c}, \\ \frac{\theta_2 \psi_B u}{\phi_t} & \text{if } \tau \geq \frac{\theta_2 \psi_B u}{c}. \end{cases}$$

If $\tau < \theta_2 \psi_B u/c$, then (2) is rewritten as

$$\tau \geq \frac{1}{\psi_D}.$$

If $\tau \geq \theta_2 \psi_B u/c$, it is

$$\frac{\theta_2 \psi_B u}{c} \geq \frac{1}{\psi_D}.$$

Therefore, (2) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\psi_D}.$$

Finally, dealers must have incentives to bring money to the DM, that is, we must have the following condition:

$$\phi_t M_{t+1} \leq \beta\{\phi_{t+1} M_{t+1} + [\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1}) - \phi_{t+1} M_{t+1}\}. \quad (3)$$

The left-hand side, $\phi_t M_{t+1}$, is the cost of bringing money to the DM. In the right-hand side, $\phi_{t+1} M_{t+1}$ is the resale value of money, $[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1})$ is the benefit from trade using money, and $-\phi_{t+1} M_{t+1}$ is the payment. We have $M_t = M_{t+1}$ and $\phi_t M_t = c$, and hence, (3) is rewritten as

$$\frac{\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1})}{c} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

Thus, (3) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

This inequality is of course violated when $\tau = 0$, and Proposition 1 follows.

Occurrence of Asset Bubbles

We will explain how an asset bubble occurs in this equilibrium. Consider state ω_E^0. At this state, the asset value for buyers is 0, and every agent knows it. Hence, the fundamental value of the asset is 0. However, since dealers' knowledge is

$$\mathcal{P}_{SD} = \{\{\omega_{SD}^u\}, \{\omega_E^0, \omega_{SD}^0\}\},$$

dealers do not know that buyers know that the asset value is 0. Moreover, with monetary expansion, buyers have too much money in the sense that they can buy the same unit of the asset with any smaller amount of money than the one injected by the monetary expansion. To obtain such money from buyers, dealers buy the asset from sellers in hopes of selling it to buyers, but buyers do not purchase it from dealers because buyers know that the asset is worthless. That is, an asset bubble occurs in trade between sellers and dealers, and bursts in trade between dealers and buyers.

This completes the proof of the theorem.

5 Discussion

Observability of Prices

Prices in trade between sellers and dealers are the same across all the states, and therefore buyers do not learn the asset value from its price. Thus, our result survives irrespective of whether past prices are observable or not. This is in sharp contrast to AIW who need to assume that past prices are unobservable to establish robust bubbles. More precisely, in their Appendix A, AIW show that bubbles occur when past prices are observable but this is true only for some knife-edge parameter values. In the current paper, bubbles occur in an open, nonempty set of parameter values.

The fact that one cannot infer states from prices echoes policymakers' difficulty. For example, Bernanke and Gertler (2012) write: "Trying to stabilize asset prices per se is

problematic for a variety of reasons, not the least of which is that it is nearly impossible to know for sure whether a given change in asset values results from fundamental factors, non-fundamental factors, or both."

Of course, the robustness result of the current model depends on many assumptions. One of them is that the cost incurred to produce the asset is the same across states. The price is just enough to cover sellers' cost, and that is independent of the state by this assumption.

Intermediation Mode

In our model, intermediaries (i.e., dealers) do trade using their own accounts and make profits by flipping. There is another mode of intermediaries.[10] In particular, platforms (or brokers) just connect buyers and sellers, or investors and interdealer markets, and make profits by brokerage fees. This difference is crucial.

In our model, dealers must hold the asset when they trade with buyers and thus their holding of the asset is sunk. This opens the room for the hold-up problem—buyers do not bring money and hence there is no bubble without monetary expansion. Such a hold-up would never occur if intermediaries were brokers. When there is monetary expansion, bubbles occur because dealers *buy* the asset even when they know that it is worthless. If intermediaries are brokers, trade never occurs if buyers know that the asset is worthless.

Policy Implications

Is it better to burst bubbles? In our model, creation of worthless assets—and therefore bubbles—is just a waste, but *ex ante* (without knowing which state will be realized) creation of asset is welfare-improving. Hence, a policy to curb bubbles by setting a small τ regardless of the state is welfare deteriorating. On the other hand, if the central

[10]See Gautier, Hu, and Watanabe (2023) for more details.

bank can choose τ *after* observing the state, then the optimal monetary policy would be to choose sufficiently large τ when the asset has value ($\omega = \omega^u_{SD}$), and sufficiently small τ (to prevent bubbles) when the asset has no value ($\omega \in \{\omega^0_{SD}, \omega^0_E\}$).[11] In this sense, our model echoes the view of the wait-and-see approach advocated by, for example, Bernanke and Gertler (2012).[12]

6 Extension

The baseline model explains how monetary expansion causes asset bubbles in a simple manner. In this section, we introduce another market for goods and demonstrate that exogenous monetary expansion is just one way to induce bubbles, and too much money and bubbles can result from buyers' endogenous decision. This extension also allows us to show that the price of money is positive even without asset bubbles.

Now, each period is divided into three subperiods. In the first, agents interact in a decentralized goods market (GM). The second and third subperiods are the same as the DM and the CM in the baseline model, respectively. In particular, the state is determined at the beginning of the DM. There are additional measure 1 of agents, and we call them goods sellers. They enter only the GM and the CM, and do not trade in the DM.

The GM opens with probability $\alpha \in [0, 1]$, and the probability is serially independent. The GM has perishable indivisible goods. Each goods seller can produce only one unit

[11] Other bubble-bursting policies can be announcing the state, or prohibiting creation or trading of assets. Of course, similar arguments hold and their welfare implications heavily depend on whether the policy can depend on states.

[12] Some papers point out that bubbles are detrimental. For example, Dong, Jia, and Wang (2022) show that bubbles create misallocation of talent. Grossman and Yanagawa (1993) and Guerron-Quintana, Hirano, and Jinnai (2023) demonstrate that asset bubbles crowd out investment. In Allen, Barlevy, and Gale (2022), bubbles cause costly default. See Barlevy (2018) for further discussion. None of these channels are present in the current paper.

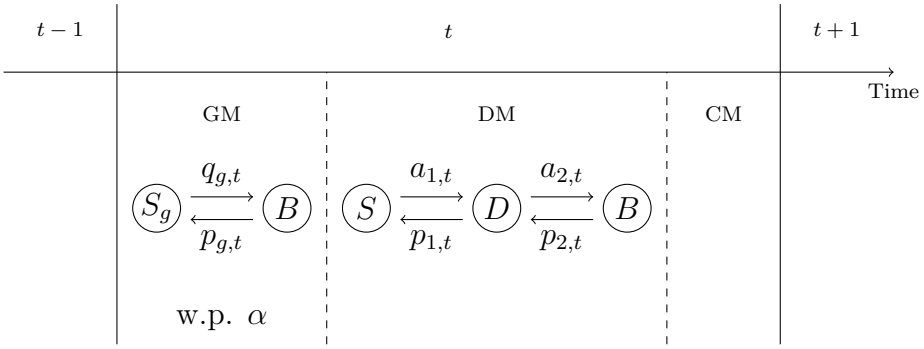

Figure 2: Timeline of the extended model

of them at a cost $c_g > 0$, and buyers enjoy utility $u_g > c_g$ from consuming it. In the GM, there are only meetings between goods sellers and buyers. If $\alpha = 0$, the extended model would be exactly the same as the baseline one. As in the DM, money is used as the payment instrument, and the terms of trade are determined by generalized Nash bargaining, where $\theta_g \in (0, 1)$ is the bargaining power of goods sellers in trade between goods sellers and buyers. To simplify the model, we assume that there is no monetary expansion, that is, $\tau = 0$. We also assume that, at the beginning of the DM, all agents know whether or not the GM opened. See Figure 2.

Our result of this section, whose proof is relegated to the Appendix A, is the following.

Theorem 2. *In the extended model with $\tau = 0$, there is an equilibrium in which asset bubbles occur with positive probability at a state if and only if*

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2 \psi_B u}{c}\right\} \geq \max\left\{\frac{1}{\psi_D}, \frac{1-\alpha\beta}{(1-\alpha)\beta[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]}\right\}$$

and

$$\frac{\alpha u_g}{c_g} + (1-\alpha)\left\{1 - [\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)] \min\left\{1, \frac{\theta_2 \psi_B u}{c_g}\right\}\right\} \geq \frac{1}{\beta}.$$

The first condition is similar to that in the baseline model and requires that the gains from trade between (asset) sellers and dealers should be sufficiently large and dealers

should have incentives to bring money to the DM. Note that this condition does not hold if $\alpha = 1$. This is because, when $\alpha = 1$, the amount of buyers' money holdings after the GM is equal to zero and dealers do not have incentives to bring money to the DM. Thus, for the existence of bubbles, we need $\alpha < 1$.

The second condition means that buyers must have incentives to bring money to the GM. Note that this condition does not hold if $\alpha = 0$ since, in that case, the GM never opens and buyers do not bring money to the GM. Hence, a bubble occurs only if $\alpha > 0$. As long as $\alpha > 0$, the condition is satisfied for sufficiently large gains from trade of GM goods, u_g/c_g.

Observe that even though we assume $\tau = 0$, asset bubbles occur with positive probability. If $\tau > 0$, buyers have a larger amount of money in the DM, and bubbles are more likely to occur. However, what is essential for the existence of bubbles is not monetary expansion but too much money. Observe also that even when asset bubbles do not occur, buyers obtain money in the CM to purchase goods in the GM as long as the GM opens with positive probability and the gains from trade of GM goods are sufficiently large. Therefore, money can have a positive value without asset bubbles.

A Appendix: Details of the Extended Model

Let $G_t^{S_g}$ and G_t^B be the GM value functions of goods sellers and buyers, respectively, and we denote the CM value function of goods sellers by $W_t^{S_g}$ (hereafter, superscript S_g indicates goods sellers). For goods sellers, the Bellman equation for the CM is

$$\begin{aligned} W_t^{S_g}(m_t^{S_g}) &= \max_{x_t^{S_g}, \ell_t^{S_g}, \widehat{m}_{t+1}^{S_g}} \left\{ U(x_t^{S_g}) - \ell_t^{S_g} + \beta G_{t+1}^{S_g}(\widehat{m}_{t+1}^{S_g}) \right\} \\ &\quad \text{subject to } x_t^{S_g} = \phi_t(m_t^{S_g} - \widehat{m}_{t+1}^{S_g}) + \ell_t^{S_g}. \end{aligned}$$

For buyers, the Bellman equation for the CM is

$$\begin{aligned} W_t^B(m_t^B) &= \max_{x_t^B, \ell_t^B, \widehat{m}_{t+1}^B} \left\{ U(x_t^B) - \ell_t^B + \beta G_{t+1}^B(\widehat{m}_{t+1}^B) \right\} \\ &\quad \text{subject to } x_t^B = \phi_t(m_t^B - \widehat{m}_{t+1}^B) + \ell_t^B. \end{aligned}$$

Since buyers enter the GM, V_{t+1}^B is replaced with G_{t+1}^B. For (asset) sellers and dealers, the Bellman equations for the CM are the same as those in the baseline model.

At the beginning of the DM, all agents know whether or not the GM opened. If it opened, the amount of buyers' money holdings in the DM is equal to the amount that they bring from the CM minus the payment for goods sellers in the GM; otherwise, the amount of buyers' money holdings in the DM is the same as the amount that they bring from the CM. All agents know this fact about buyers' money holdings. Other than this feature, there is no change in the DM.

For trade in the GM, let $q_{g,t}$ and $p_{g,t}$ be the amounts of goods and money traded, respectively. Since goods sellers do not enter the DM, their surplus is

$$-c_g q_{g,t} + W_t^{S_g}(m_t^{S_g} + p_{g,t}) - W_t^{S_g}(m_t^{S_g}) = \phi_t p_{g,t} - c_g q_{g,t}.$$

Buyers' surplus is

$$u_g + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B)$$

because they trade in the DM after the GM. Unlike the difference in $W_t^{S_g}$, the difference in $V_t^{S_g}$ takes a complicated form, and we leave it as it is. The terms of trade are determined by

$$\begin{aligned} &\max_{q_{g,t} \in \{0,1\}, p_{g,t}} (\phi_t p_{g,t} - c_g q_{g,t})^{\theta_g} [u_g + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B)]^{1-\theta_g} \\ &\text{subject to } p_{g,t} \leq m_t^B. \end{aligned}$$

As in the baseline model, we also have incentive constraints that agents' surpluses must be nonnegative. Each solution to the bargaining problem depends on a buyer's money holdings in this trade, m_t^B, in addition to the other buyers' money holdings, M_t^B, and

dealers' money holding, M_t^D. Define the Bellman equations for the GM as follows. For goods sellers, it is

$$G_t^{S_g}(m_t^{S_g}) = W_t^{S_g}(m_t^{S_g}) + \alpha[p_{g,t}(M_t^B, M_t^B, M_t^D) - c_g q_{g,t}(M_t^B, M_t^B, M_t^D)].$$

For buyers, it is

$$G_t^B(m_t^B) = \alpha\{u_g q_{g,t}(m_t^B, M_t^B, M_t^D) + V_t^B[m_t^B - p_{g,t}(m_t^B, M_t^B, M_t^D)]\} + (1-\alpha)V_t^B(m_t^B).$$

We will derive an equilibrium where, with probability α, asset bubbles occur ($a_{1,t}^0 = a_{1,t}^u = 1$, $p_{1,t}^0 = p_{1,t}^u > 0$, and $\phi_t > 0$) and buyers purchase goods if the GM opens ($q_{g,t} = 1$ and $p_{g,t} > 0$). As in the baseline model, goods sellers and (asset) sellers do not have incentives to obtain money in the CM, and dealers' money holdings must satisfy $\phi_t M_t^D = c$, which implies that $p_{1,t}^0 = p_{1,t}^u$. Buyers still do not have incentives to bring money to the DM due to the hold-up problem. However, they may have incentives to bring money to the GM to purchase goods. Note that if buyers still have money at the end of the GM, they may have to pay some fraction of the money to dealers in the DM. Hence, we have

$$V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B) \geq -\phi_t p_{g,t},$$

and, as a result, there are always gains from GM trade:

$$u_g - c_g + \phi_t p_{g,t} + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B) > 0.$$

By a similar argument about dealers' money holdings, buyers' money holdings must satisfy $\phi_t M_t^B = c_g$. Therefore, we obtain

$$z_t = \phi_t M_t = \phi_t(M_t^D + M_t^B) = c + c_g.$$

If the GM did not open, buyers have c_g/ϕ_t units of money in the DM. In this case, as in the baseline model, for $a_{1,t}^0 = a_{1,t}^u = 1$ to hold, we must have

$$\frac{\phi_t p_{2,t}\left(1, \frac{c_g}{\phi_t}\right)}{c} \geq \frac{1}{\psi_D}.$$

Now,

$$p_{2,t}\left(1, \frac{c_g}{\phi_t}\right) = \begin{cases} \frac{c_g}{\phi_t} & \text{if } c_g < \theta_2 \psi_B u, \\ \frac{\theta_2 \psi_B u}{\phi_t} & \text{if } c_g \geq \theta_2 \psi_B u. \end{cases}$$

The above condition is rewritten as

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\psi_D}.$$

Dealers buy the asset from (asset) sellers exactly when the GM does not open. Hence, dealers' incentive constraint is

$$\phi_t M_{t+1}^D \leq \beta\left\{\alpha\phi_{t+1}M_{t+1}^D + (1-\alpha)[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\phi_{t+1}p_{2,t+1}\left(1, \frac{c_g}{\phi_{t+1}}\right)\right\}.$$

Since $\phi_t M_t^D = c$, the above incentive constraint holds if and only if

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1-\alpha\beta}{(1-\alpha)\beta[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]}.$$

Now, in addition to dealers, we need to consider buyers' incentive constraint:

$$\begin{aligned} \phi_t M_{t+1}^B \quad \leq \quad & \beta\Big\{\phi_{t+1}M_{t+1}^B + \alpha(u_g - \phi_{t+1}M_{t+1}^B) \\ & -(1-\alpha)[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\phi_{t+1}p_{2,t+1}\left(1, \frac{c_g}{\phi_{t+1}}\right)\Big\}. \end{aligned}$$

The left-hand side, $\phi_t M_{t+1}^B$, is the cost of bringing money to the GM. In the right-hand side, $\phi_{t+1}M_{t+1}^B$, is the resale value of money, and $\alpha(u_g - \phi_{t+1}M_{t+1}^B)$ is the expected value of the benefit minus the payment in GM trade. If the GM does not open, buyers may have to pay some fraction of their money holdings to dealers in the DM. The remaining term captures this cost. Since $\phi_t M_t^B = c_g$, the incentive constraint is rewritten as

$$\frac{\alpha u_g}{c_g} + (1-\alpha)\left\{1 - [\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\min\left\{1, \frac{\theta_2 \psi_B u}{c_g}\right\}\right\} \geq \frac{1}{\beta}.$$

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