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Supplement to “A discrete choice model for partially ordered alternatives”

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Section S1 of this supplement provides derivation of formulae used to compute price elasticities in the empirical application in Section 6 of the main text [Aristodemou and Rosen \(2022\)](#). Section S2 provides estimates of features of the distribution of elasticities using simulated data from DGP2 as described in Appendix D of [Aristodemou and Rosen \(2022\)](#).

S1. DERIVATION OF ELASTICITIES

The elasticity of the quantity sold of brand b 's product of quality y with respect to the price $p_{k\ell}$ of product (k, ℓ) is

$$\eta_{bykl} = \frac{\partial \wp_{by}}{\partial p_{k\ell}} \frac{p_{k\ell}}{\wp_{by}}.$$

We consider elasticities conditional on $Z = z$. Prices $p_{k\ell}$ can be plugged in directly and $\wp_{by} = \wp_{by}(z; \xi)$ is given in (5.3) in the text. It is then additionally necessary to compute derivatives $\frac{\partial \wp_{by}(z; \xi)}{\partial p_{k\ell}}$.

For this, we start with (5.3) with the function Δ the difference of bivariate normal CDF values defined in (5.4) and parameters $m_1^-, m_2^-, m_1^+, m_2^+$ defined in (5.6) and (5.5),

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respectively. Notation

$$g_{b,y} \equiv g_b(y; z, \theta), \quad \tilde{g}_{b,y} \equiv \sigma_b^{-1} g_{b,y}, \quad \tilde{z}_{by}^* \equiv \sigma_b^{-1} z_{by}^*$$

will also be used. There are three cases to consider, as follows, where Δ_j is used to denote the partial derivative of Δ with respect to its j th argument, $j = 1, \dots, 4$. The expressions below additionally make use of the fact that for any (k, ℓ) : $\frac{\partial m_2^-}{\partial p_{k\ell}} = \frac{\partial m_2^+}{\partial p_{k\ell}} = 0$:

1. $g_{b,y} < z_{by}^* < g_{b,y+1}$.

$$\wp_{by}(z, \zeta) = \Delta(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \Delta(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+).$$

Referring back to equations (5.1), (A.6), and (A.11) in the proof of Proposition 2, this can be equivalently written

$$\begin{aligned} \wp_{by}(z, \zeta) &= \frac{1}{\sigma_b} \int_{g_b(y; z, \theta)}^{z_{by}^*} \Phi \left(\frac{h_b(y, z, v, \theta) - \rho \frac{\sigma_d v}{\sigma_b}}{\sigma_d \sqrt{1 - \rho^2}} \right) \phi \left(\frac{v}{\sigma_b} \right) dv \\ &\quad + \frac{1}{\sigma_b} \int_{z_{by}^*}^{g_b(y+1; z, \theta)} \Phi \left(\frac{h_b(y, z, v, \theta) - \rho \frac{\sigma_d v}{\sigma_b}}{\sigma_d \sqrt{1 - \rho^2}} \right) \phi \left(\frac{v}{\sigma_b} \right) dv, \end{aligned}$$

where it follows from steps in the proof of Proposition 2 that

$$\Delta(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) = \frac{1}{\sigma_b} \int_{g_b(y; z, \theta)}^{z_{by}^*} \Phi \left(\frac{h_b(y, z, v, \theta) - \rho \frac{\sigma_d v}{\sigma_b}}{\sigma_d \sqrt{1 - \rho^2}} \right) \phi \left(\frac{v}{\sigma_b} \right) dv,$$

and

$$\Delta(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) = \frac{1}{\sigma_b} \int_{z_{by}^*}^{g_b(y+1; z, \theta)} \Phi \left(\frac{h_b(y, z, v, \theta) - \rho \frac{\sigma_d v}{\sigma_b}}{\sigma_d \sqrt{1 - \rho^2}} \right) \phi \left(\frac{v}{\sigma_b} \right) dv.$$

Consequently, we see that

$$\frac{\partial \wp_{by}(z, \zeta)}{\partial z_{by}^*} = \Delta_2(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \Delta_1(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) = 0$$

and, therefore,

$$\frac{\partial \wp_{by}(z, \zeta)}{\partial p_{k\ell}} = \left[\begin{aligned} &\sigma_b^{-1} \left(\frac{\partial g_{b,y}}{\partial p_{k\ell}} \Delta_1(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \frac{\partial z_{by}^*}{\partial p_{k\ell}} \Delta_2(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) \right) \\ &\quad + \sigma_b^{-1} \left(\frac{\partial z_{by}^*}{\partial p_{k\ell}} \Delta_1(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \right. \\ &\quad \left. + \frac{\partial g_{b,y+1}}{\partial p_{k\ell}} \Delta_2(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \right) \\ &\quad + \frac{\partial m_1^-}{\partial p_{k\ell}} \Delta_3(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \frac{\partial m_1^+}{\partial p_{k\ell}} \Delta_3(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \end{aligned} \right]$$

$$= \left\{ \begin{aligned} & \sigma_b^{-1} \left(\frac{\partial g_{b,y}}{\partial p_{k\ell}} \Delta_1(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \frac{\partial g_{b,y+1}}{\partial p_{k\ell}} \Delta_2(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \right) \\ & + \frac{\partial m_1^-}{\partial p_{k\ell}} \Delta_3(\tilde{g}_{b,y}, \tilde{z}_{by}^*, m_1^-, m_2^-) + \frac{\partial m_1^+}{\partial p_{k\ell}} \Delta_3(\tilde{z}_{by}^*, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \end{aligned} \right\}$$

2. $g_{b,y} \leq g_{b,y+1} \leq z_{by}^*$. Using similar arguments as in case 1 above,

$$\begin{aligned} \wp_{by}(z, \zeta) &= \Delta(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^-, m_2^-) \\ \implies \\ \frac{\partial \wp_{by}(z, \zeta)}{\partial p_{k\ell}} &= \left\{ \begin{aligned} & \sigma_b^{-1} \left(\frac{\partial g_{b,y}}{\partial p_{k\ell}} \Delta_1(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^-, m_2^-) \right) \\ & + \frac{\partial g_{b,y+1}}{\partial p_{k\ell}} \Delta_2(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^-, m_2^-) \\ & + \frac{\partial m_1^-}{\partial p_{k\ell}} \Delta_3(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^-, m_2^-) \end{aligned} \right\} \end{aligned}$$

3. $z_{by}^* \leq g_{b,y} \leq g_{b,y+1}$. Again, following similar steps as in case 1, it follows that:

$$\begin{aligned} \wp_{by}(z, \zeta) &= \Delta(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \\ \implies \\ \frac{\partial \wp_{by}(z, \zeta)}{\partial p_{k\ell}} &= \left\{ \begin{aligned} & \sigma_b^{-1} \left(\frac{\partial g_{b,y}}{\partial p_{k\ell}} \Delta_1(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \right) \\ & + \frac{\partial g_{b,y+1}}{\partial p_{k\ell}} \Delta_2(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \\ & + \frac{\partial m_1^+}{\partial p_{k\ell}} \Delta_3(\tilde{g}_{b,y}, \tilde{g}_{b,y+1}, m_1^+, m_2^+) \end{aligned} \right\} \end{aligned}$$

Thus to compute these derivatives at a given value of parameters and a given value of observed variables, the following expressions (arguments suppressed) are needed:

$$\frac{\partial g_{b,y}}{\partial p_{k\ell}}, \quad \frac{\partial g_{b,y+1}}{\partial p_{k\ell}}, \quad \frac{\partial m_1^-}{\partial p_{k\ell}}, \quad \frac{\partial m_1^+}{\partial p_{k\ell}}, \quad \Delta_1, \Delta_2, \Delta_3.$$

Since $g_{b,y} = g_b(y; z, \theta) = \lambda_{b,y} - x_b \beta_b$ from (4.8), we have that

$$\frac{\partial g_{b,y}}{\partial p_{k\ell}} = \frac{\partial \lambda_{b,y}}{\partial p_{k\ell}}, \quad \frac{\partial g_{b,y+1}}{\partial p_{k\ell}} = \frac{\partial \lambda_{b,y+1}}{\partial p_{k\ell}}.$$

In our application, with the linear specification $\alpha_{by} = \delta_b + \gamma_b p_{by}$, we have that at the estimated parameter vector $\alpha_{b2} > 2\alpha_{b1}$ for all observations. Under this inequality,

$$\lambda_{b1} = \alpha_{b1}, \quad \lambda_{b2} = \alpha_{b2} - \alpha_{b1},$$

and it follows that

$$\frac{\partial g_{b,1}}{\partial p_{k\ell}} = \frac{\partial \lambda_{b1}}{\partial p_{k\ell}} = 1[b = k, \ell = 1] \cdot \gamma_b,$$

$$\frac{\partial g_{b,2}}{\partial p_{k\ell}} = \frac{\partial \lambda_{b2}}{\partial p_{k\ell}} = (1[b = k, \ell = 2] - 1[b = k, \ell = 1]) \cdot \gamma_b,$$

If there were observations for which instead $\alpha_{b2} \leq 2\alpha_{b1}$, then for these observations we would have

$$\lambda_{b1} = \lambda_{b2} = \alpha_{b2}/2,$$

and given the linear specification $\alpha_{by} = \delta_b + \gamma_b p_{by}$, for each $y \in \{1, 2\}$,

$$\frac{\partial g_{b,y}}{\partial p_{k\ell}} = \frac{\partial \lambda_{by}}{\partial p_{k\ell}} = 1[b = k, \ell = 2] \cdot \frac{\gamma_b}{2}.$$

Now consider $\frac{\partial z_{by}^*}{\partial p_{k\ell}}$. Recall that $z_{by}^* \equiv \frac{\alpha_{d2} + \alpha_{by} - 2\alpha_{d1}}{y} - x_b \beta_b$ and, therefore, for $y \in \{1, 2\}$ and $\tilde{y} = 3 - y$:

$$\frac{\partial z_{by}^*}{\partial p_{by}} = \frac{\gamma_b}{y}, \quad \frac{\partial z_{by}^*}{\partial p_{d1}} = \frac{-2\gamma_d}{y}, \quad \frac{\partial z_{by}^*}{\partial p_{d2}} = \frac{\gamma_d}{y}, \quad \frac{\partial z_{by}^*}{\partial p_{b\tilde{y}}} = 0.$$

The variables m_1^- and m_1^+ are defined in (5.6) and (5.5). Applying the linear specification $\alpha_{by} = \delta_b + \gamma_b p_{by}$ and making explicit dependence on (b, y) yields

$$m_1^-(b, y) \equiv \frac{yx_b \beta_b + \delta_d + \gamma_d p_{d1} - \delta_b - \gamma_b p_{by} - x_d \beta_d}{\sqrt{\sigma_b^2 y^2 - 2\rho \sigma_b \sigma_d y + \sigma_d^2}}, \quad (\text{S1})$$

$$m_1^+(b, y) \equiv \frac{yx_b \beta_b + \delta_d + \gamma_d p_{d2} - \delta_b - \gamma_b p_{by} - 2x_d \beta_d}{\sqrt{\sigma_b^2 y^2 - 4\rho \sigma_b \sigma_d y + 4\sigma_d^2}}. \quad (\text{S2})$$

Thus, for $\tilde{y} \neq y$,

$$\frac{\partial m_1^-(b, y)}{\partial p_{by}} = \frac{-\gamma_b}{\sqrt{\sigma_b^2 y^2 - 2\rho \sigma_b \sigma_d y + \sigma_d^2}}, \quad \frac{\partial m_1^-(b, y)}{\partial p_{b\tilde{y}}} = 0,$$

$$\frac{\partial m_1^-(b, y)}{\partial p_{d1}} = \frac{\gamma_d}{\sqrt{\sigma_b^2 y^2 - 2\rho \sigma_b \sigma_d y + \sigma_d^2}}, \quad \frac{\partial m_1^-(b, y)}{\partial p_{d2}} = 0,$$

$$\frac{\partial m_1^+(b, y)}{\partial p_{by}} = \frac{-\gamma_b}{\sqrt{\sigma_b^2 y^2 - 4\rho \sigma_b \sigma_d y + 4\sigma_d^2}}, \quad \frac{\partial m_1^+(b, y)}{\partial p_{b\tilde{y}}} = 0,$$

$$\frac{\partial m_1^+(b, y)}{\partial p_{d2}} = \frac{\gamma_d}{\sqrt{\sigma_b^2 y^2 - 4\rho \sigma_b \sigma_d y + 4\sigma_d^2}}, \quad \frac{\partial m_1^+(b, y)}{\partial p_{d1}} = 0.$$

Finally, there is

$$\Delta(h, k, m_1, m_2) \equiv \Phi_2(k, m_1, m_2) - \Phi_2(h, m_1, m_2),$$

where $\Phi_2(x, y, \rho)$ denotes the CDF of a bivariate normal vector with standard normal marginals and correlation coefficient ρ evaluated at (x, y) . Thus the partial derivatives

of $\Delta(h, k, m_1, m_2)$ are obtained as

$$\begin{aligned}\Delta_1(h, k, m_1, m_2) &= -\frac{\partial\Phi_2}{\partial h}(h, m_1, m_2), \\ \Delta_2(h, k, m_1, m_2) &= \frac{\partial\Phi_2}{\partial k}(k, m_1, m_2), \\ \Delta_3(h, k, m_1, m_2) &= \frac{\partial\Phi_2}{\partial m_1}(k, m_1, m_2) - \frac{\partial\Phi_2}{\partial m_1}(h, m_1, m_2).\end{aligned}$$

Expressions for the relevant partial derivatives of the bivariate normal CDF are

$$\begin{aligned}\frac{\partial\Phi_2}{\partial x}(x, y, \rho) &= \phi(x)\Phi\left(\frac{y - \rho x}{\sqrt{1 - \rho^2}}\right), \\ \frac{\partial\Phi_2}{\partial y}(x, y, \rho) &= \phi(y)\Phi\left(\frac{x - \rho y}{\sqrt{1 - \rho^2}}\right),\end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal density and cumulative distribution function, respectively.

S2. ESTIMATES OF ELASTICITIES FROM SIMULATED DATA

The data used in Section 6.3 of [Aristodemou and Rosen \(2022\)](#) is not publicly available, but code used to estimate features of the distribution of household elasticities is included in our replication code. In order to demonstrate usage, the file `Compute_POR_Elasticities.R` was used to compute quantities reported in Table 6 of the main text instead using data simulated from DGP2, which is described in Appendix D.^{S1} The resulting estimates using 2000 observations generated from DGP2 are reported here in Table S1.

^{S1}In DGP2 prices have bounded support on \mathbb{R}_+ and higher-quality products are priced higher than lower-quality products.

TABLE S1. Estimated means and 0.2, 0.5, and 0.8 quantiles of household elasticities using a sample of 2000 observations from DGP2.

$\frac{\partial \log \varphi_{by}}{\partial \log p_{k\ell}}$	Price elasticities											
	p_{11}			p_{12}			p_{21}			p_{22}		
φ_{11}												
Mean	-4.764			2.984			0.782			0.423		
Quantiles	-6.966	-3.768	-2.298	1.355	2.428	4.389	0.338	0.631	1.171	0.080	0.374	0.728
φ_{12}												
Mean	1.888			-4.122			0.014			0.489		
Quantiles	0.991	1.780	2.523	-5.300	-3.790	-2.630	0.000	0.000	0.000	0.256	0.462	0.707
φ_{21}												
Mean	1.263			0.024			-7.106			5.017		
Quantiles	0.865	1.252	1.632	0.000	0.000	0.000	-10.167	-5.761	-3.773	2.620	4.302	7.256
φ_{22}												
Mean	0.236			0.402			1.464			-3.161		
Quantiles	0.051	0.216	0.395	0.204	0.379	0.589	0.674	1.331	2.098	-4.284	-2.845	-1.849
φ_0												
Mean	1.086			0.000			1.329			0.000		
Quantiles	0.707	1.067	1.466	0.000	0.000	0.000	0.807	1.377	1.848	0.000	0.000	0.000

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