

Sunaga, Miho

**Working Paper**

## Financial development and minimum capital requirements in macroeconomic analysis

ISER Discussion Paper, No. 1202

**Provided in Cooperation with:**

The Institute of Social and Economic Research (ISER), Osaka University

*Suggested Citation:* Sunaga, Miho (2023) : Financial development and minimum capital requirements in macroeconomic analysis, ISER Discussion Paper, No. 1202, Osaka University, Institute of Social and Economic Research (ISER), Osaka

This Version is available at:

<https://hdl.handle.net/10419/296847>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Discussion Paper No. 1202

ISSN (Print) 0473-453X

ISSN (Online) 2435-0982

**FINANCIAL DEVELOPMENT AND  
MINIMUM CAPITAL REQUIREMENTS IN  
MACROECONOMIC ANALYSIS**

Miho Sunaga

March 2023

The Institute of Social and Economic Research  
Osaka University  
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

# Financial Development and Minimum Capital Requirements in Macroeconomic Analysis\*

Miho Sunaga<sup>†</sup>

February 2023

## Abstract

We develop a macroeconomic model with a moral hazard problem between financial intermediaries and households, which causes inefficient resource allocation, to make us reconsider the financial regulation according to financial development, and individual and aggregate economic activities in the short and long runs. First, we show that in an economy where financial market has not developed, raising minimum capital requirements improves resource allocation and welfare in the long run, while it reduces welfare in an economy where financial market has developed. Second, our study reveals that an economy with a minimum capital adequacy ratio of 8% has a larger drop in aggregate net worth, consumption, and output when a negative capital quality shock occurs. However, during the financial crisis, the economy recovers faster than an economy with a higher minimum capital ratio (about 10%). These results indicate that tighter bank requirements temporally mitigate crises in economies with a developed financial market; however, they do not promote their activity in the long run.

**Keywords:** Minimum Capital Requirements; Financial Development; Macroeconomic Analyses

**JEL Classification Codes:** E44, G21, G28, O11, O16

**Declarations of interest:** none

---

\*I am grateful to Koichi Futagami and Nobuhiro Kiyotaki for their thoughtful comments and suggestions since 2013 when this project started. I would like to thank Kosuke Aoki, Ryo Horii, and Tatsuro Iwaisako as well as several seminar participants for their helpful comments. I acknowledge the financial support from the Grant-in-Aid for JSPS Fellows (Grant No. 20K22120).

<sup>†</sup>Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. E-mail: [sunaga@iser.osaka-u.ac.jp](mailto:sunaga@iser.osaka-u.ac.jp); [miho.sunaga08@gmail.com](mailto:miho.sunaga08@gmail.com)

# 1 Introduction

After the 2008 financial crisis, banks worldwide began to implement tighter capital adequacy requirements under international banking standards such as Basel III. The Basel Committee sets the minimum capital requirements for banks regardless of how individual financial markets have developed across the world. However, banks operate in financial markets, and the development of financial markets varies across countries.<sup>1</sup> In this study, we are interested in the following research question: Is it desirable for countries to have tighter capital adequacy requirements irrespective of their financial development? To determine the desirability of a policy, we focus on three components: If this policy will mitigate inefficiency caused by indirect and direct financing in the long run, how well an economy recovers from a crisis, and the magnitude of the drop in aggregate economic variables. The last two components are analyses in the short runs. To approach these issues, we construct a macroeconomic model that explicitly represents the development of a financial market by reassessing these tighter minimum capital requirements. Using this model, we demonstrate how minimum capital requirements can improve welfare in an economy, depending on the development of its financial market.

Our macroeconomic model involves indirect finance (where households invest their assets through financial intermediaries) and direct finance (where households directly manage capital in the market). However, households must pay management costs due to their inexperience in managing capital compared to banks. Meanwhile, there is a moral hazard problem between depositors (households) and financial intermediaries in an economy, which causes monitoring costs. Thus, indirect and direct financing are responsible for resource allocation inefficiency in this model. An economy's efficiency (or inefficiency) is determined by the degree to which investors are protected by law in the country's financial markets or legal systems. This also determines how its financial market will

---

<sup>1</sup>Based on the World Bank's global financial development database, Čihák et al. (2013) show that the degree of financial development differs across countries. The measures are divided into 4: depth, access, efficiency, and stability. For example, in notable literature on financial development and growth, the ratio of private credit to GDP is used to measure depth (Levine (2005)). Access is determined by the percentage of accounts per thousand adults in commercial banks. Efficiency and stability are measured by lending-deposit spread and Z-score. This study focuses on access and efficiency because these measures are particularly relevant to financial crises.

develop. Our model allows analysis of the minimum capital requirements to remove inefficiency.

Gertler and Kiyotaki (GK 2010) and the following studies based on GK (2010): Gerler, Kiyotaki, Queralto (GKQ 2012), Gertler and Kiyotaki (GK 2015), Aoki et al. (2016), Gertler, Kiyotaki, and Prestipino (GKP 2016, 2020 (a), 2020 (b), 2020 (c)) also address the moral hazard problem and the role of financial intermediaries in a macroeconomic model. Other related studies on macroprudential policies are GKQ (2012) and GKP (2020 (b), 2020 (c)). However, these studies aim to replicate the U.S. economy during and after the recent financial crisis. Thus, direct finance is the main cause of inefficiency in their model. Regulations and policies on bank activities under the incentive compatibility constraint cannot improve welfare in the long run.<sup>2</sup> Compared to the literature, one of the key features of this model is that resource allocation inefficiency results from direct and indirect financing.<sup>3</sup> This study presents the optimal requirements for improving welfare of an economy by reducing inefficient resource allocation in the long run. We consider an economy where agents have to choose between the marginal costs of indirect and direct finance, both of which depend on the development of its financial market. Our model can be used by regulators to adjust the capital requirements accordingly to ensure that the banking sector remains resilient to any future economic downturns and that the economy can recover from a crisis and return to its pre-crisis growth rate. Therefore, this study contributes to the existing literature on optimal policies and financial intermediaries within macroeconomic analysis following GK (2010).

This study is relevant to Townsend (1979), and Holmström and Tirole (1997). Like them, we also include the moral hazard problem between investors and depositors and monitoring activities in our model. Furthermore, this study models depositors' monitoring cost and investor protection by legal systems, based on Townsend's (1979) costly state verification model and the empirical results of La Porta et al. (1997). As in this study and GK (2010), Holmström and Tirole (1997) examine the moral hazard problem of investors diverting a portion of their funds for private benefit. Moreover,

---

<sup>2</sup>GK (2010) and their subsequent studies demonstrate how asset price shocks contributed to the recent financial crisis in the U.S. They describe how these shocks cause bank activity inefficiency in the short run. Further, they show how these shocks spread across the entire economy. Moreover, GKQ (2012) and GKP (2020 (b), 2020 (c)) suggest optimal policies for such a crisis.

<sup>3</sup>Focusing on "access" and "efficiency" of financial development, the marginal costs of direct and indirect financing are calibrated in this model.

in this study and Holmström and Tirole (1997), the monitoring activities as agents to resolve it are incorporated. This study analyzes inefficiency due to indirect finance and direct finance, which allows us to analyze monitoring activities, requirements, and policies to improve welfare. Therefore, this device in this model gives us richer understanding of the effects of raising minimum capital adequacy ratio on an economy.

In this study, our model finds that in the long run, raising minimum capital requirements in can increase inefficiency and reduce welfare in an economy with a developed financial market. Conversely, the same policy can reduce inefficiency and improve welfare in an economy with an undeveloped financial market. These results can be interpreted as follows: In an economy with a developed financial market, welfare losses caused by inefficiency due to indirect finance are smaller than those caused by direct finance. Thus, tighter regulations on financial intermediaries will reduce the welfare of the agents in the economy because it will lead to more inefficient direct finance and suppress their activities. However, in an economy where the financial market has not developed, welfare losses caused by inefficiency due to indirect finance are larger than those caused by direct finance. Thus, financial intermediaries are suppressed by tight requirements, which shifts direct finance to indirect finance and improves the welfare of agents. Moreover, our study reveals that an economy with a minimum capital adequacy ratio of 8 % has a larger drop in aggregate net worth, consumption, and output when its financial market has developed. However, when a negative capital quality shock occurs during financial crises, the economy recovers faster than an economy with a higher minimum capital ratio (about 10%). Thus, regulators should set minimum capital requirements based on the structure of financial markets, the causes of inefficiency, and the development of the financial market. Given these findings, we should reconsider Basel III' s minimum capital requirements.

The remainder of this paper is organized as follows. Section 2 presents the baseline model, with a moral hazard problem between financial intermediaries and households, which causes inefficient resource allocation. Section 3 describes the dynamic system of an economy. Section 4 analytically investigates how minimum capital requirements affect the steady state economy in countries with

developed financial markets than those with undeveloped financial markets. Section 5 numerically analyzes how minimum capital requirements affect each country with different degrees of financial development in the long and short runs. Finally, Section 6 presents our concluding remarks.

## 2 Model

We consider a closed economy in which time is discrete. There is a representative household with a continuum of members of measure unity. The members consist of  $f$  bankers and  $1 - f$  workers, where  $f$  is constant over time. Each banker manages intermediary financial firms, while the workers supply labor to the final goods production sector. The following subsections elaborate on this final goods production sector, households, and financial intermediaries' behavior.

### 2.1 Final Goods Production Sector

With capital and labor inputs, each firm (non-financial firm) in the final goods production sector produces output using identical constant returns to scale Cobb–Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad (1)$$

with  $0 < \alpha < 1$ .  $Y_t$ ,  $K_t$ , and  $L_t$  denote the output, aggregate capital, and labor in period  $t$ , respectively.  $A$  is a productivity parameter that is constant over time. We assume that  $A \geq 1$ .

Let the final goods be the numeraire. Perfect competition prevails in this sector. The profit maximization conditions are as follows.

$$Z_t = \frac{\alpha Y_t}{K_t}, \quad (2)$$

$$w_t = \frac{(1 - \alpha)Y_t}{L_t}, \quad (3)$$

where  $Z_t$  is the rental price in period  $t$ , and  $w_t$  is the wage rate in period  $t$ .

## 2.2 Capital Market

The law of motion for capital is given by

$$K_{t+1} = I_t + (1 - \delta)K_t. \quad (4)$$

$I_t$  denotes the aggregate investment in period  $t$ , where  $\delta \in [0, 1)$  is the depreciation rate.

Capital is either mediated by banks (financial intermediate firms) or directly held by households.

$$K_t^b + K_t^h = K_t, \quad (5)$$

where  $K_t$  is the aggregate capital in period  $t$ ,  $K_t^h$  is the aggregate capital holdings of households in period  $t$ , and  $K_t^b$  is the aggregate capital holdings of banks in period  $t$ . We assume that households have a disadvantage in financing businesses relative to banks. Specifically, to manage capital in the capital market, households require to pay extra management costs, while banks do not<sup>4</sup>:

$$f^k \left( K_t^h; K_t \right) = \frac{\chi}{2} \left( \frac{K_t^h}{K_t} \right) K_t^h, \quad (6)$$

where  $\chi > 0$  is a parameter reflecting the disadvantage of households in the financing business relative to banks, which is constant over time. Moreover, we assume that management costs are proportional to the rate of investment per unit of aggregate capital, rather than its level.<sup>5</sup> Since aggregate capital consists of the capital holdings of households and banks, and households know that their assets include banks' capital as future dividends, they take care of both the level of their investment and aggregate capital. The aggregate capital given to households is  $K_t$ . Moreover, households are lenders in the capital market.<sup>6</sup> We consider a moral hazard problem in which,

---

<sup>4</sup>GKQ(2012), GK (2015), Aoki et al. (2018), and GKP (2016, 2020(a), 2020(b), 2020(c)) make the same assumption. This cost function (6) is an application of the cost of adjustment, as described in Uzawa (1969). GK(2010), GKQ(2012), Aoki et al. (2016), and GKP(2020 (a)) introduce the cost of adjustment as well as the management cost in macroeconomic models with capital goods producers.

<sup>5</sup>Turnovsky (1996) makes the same assumption and adopts the same function as Equation (6). He uses the quadratic convex adjustment cost function, which depends on the aggregate capital.

<sup>6</sup>We assume that the cost is infinity when households are borrowers in the capital market.



instead of investing in capital, bankers divert it . Moreover, we assume that households incur monitoring costs in the spirit of the costly state verification model as Townsend (1979) outlined.<sup>7</sup> We assume that the cost function is linear in the deposits.

$$f^d(D_t) = \theta D_t, \quad (7)$$

where  $D_t$  is the household deposits and  $\theta \in (0, 1)$ . In this model, the marginal monitoring cost  $\theta$  decreases as the financial market develops. Thus, low (or high)  $\theta$  reflects a country whose financial market has developed (or undeveloped).<sup>8</sup> Due to the moral hazard problem, households do not deposit all their assets. They lend capital in the capital market even if they incur extra capital management costs. Thus, they must choose between the marginal costs of indirect and direct finance, which cause inefficient resource allocation and welfare losses. Figure 1 depicts the capital market of the economy. Direct finance is inefficient due to the management costs, while indirect finance is inefficient due to the moral hazard problem, which diminishes as the financial market develops. In Section 4, we analyze the optimal regulation that alleviates inefficiency and improves an economy's welfare according to the development of its financial market.

### 2.3 Households' Optimizing Behavior

Each household has many members, including  $1 - f$  workers and  $f$  bankers. There is a probability that banks will return dividends to households with a probability of  $(1 - \sigma)f$  every time they retire their banking business.  $(1 - \sigma)f$  workers become new bankers in each period. Therefore, the ratio of workers to bankers is constant. Thus, the total population is 1 in every period as shown in Figure 2.<sup>9</sup> When a banker becomes a worker, he/she brings the net worth of banking to the

---

<sup>7</sup>We describe the moral hazard problem in detail in subsection 2.4.

<sup>8</sup>The degree of financial development  $\theta$  can be captured by legal protection of the capital market. A large  $\theta$  captures the poor legal protection of the capital market. Based on data from 49 countries, La porta et al. (1997) found that countries with poorer investor protection, measured by both legal rules and law enforcement, have narrower and undeveloped capital markets.

<sup>9</sup>This set up is based on Gertler and Karadi (2011), and GK (2010), GKQ (2012), Aoki et al. (2016), GKP (2020(a), 2020(b), 2020(c)) make the same assumption.

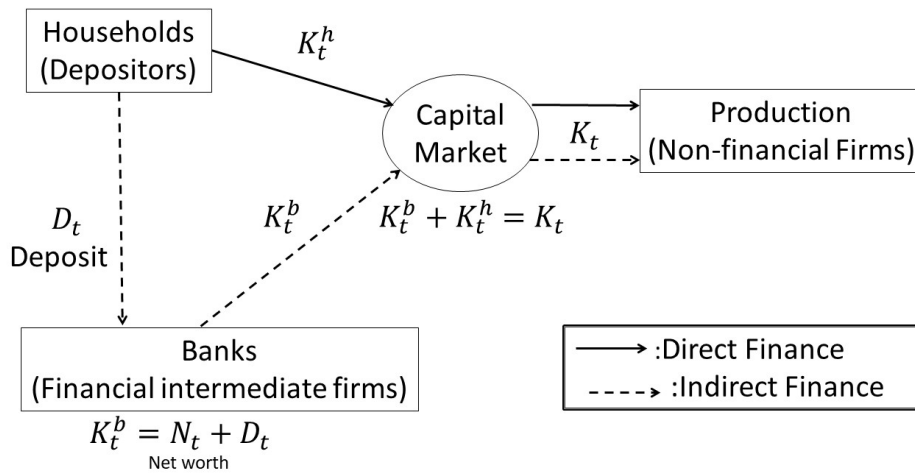


Figure 1: Direct Finance and Indirect Finance

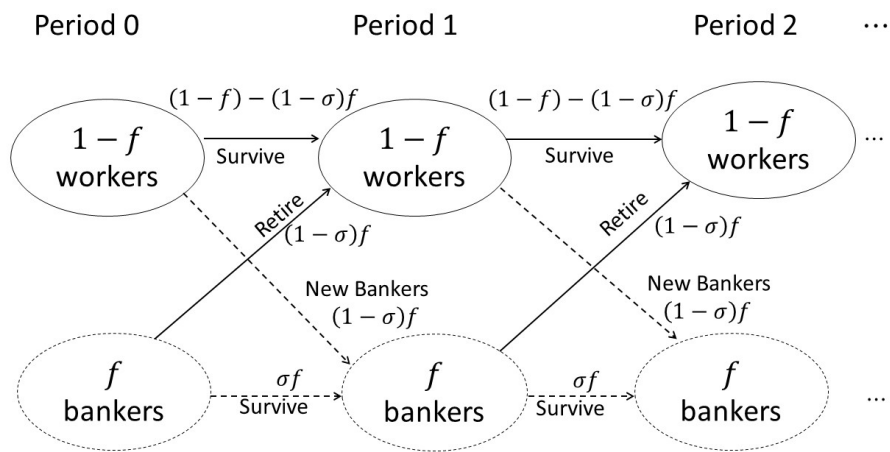


Figure 2: Each member of households

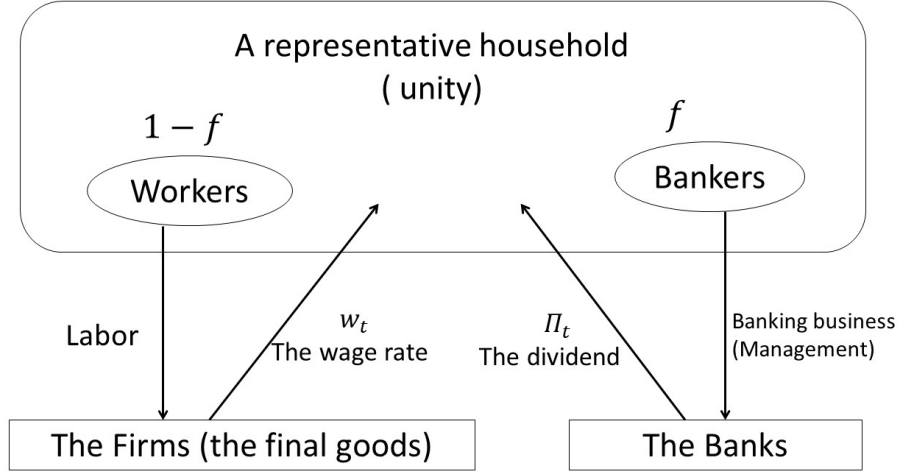


Figure 3: Structure of the economy

household. When a worker becomes a banker, the representative household provides a proportion of the total assets of the entire economy to the new banker as start-up funds. Let  $\zeta \in (0, 1)$  denote the proportion of total assets of the entire economy provided to new bankers.<sup>10</sup> Workers supply the final goods sector with one unit of labor inelastically. A representative household uses wage and asset earnings for savings and consumption. There are two ways of saving: deposit of assets with banks or manage assets in the capital market. The structure of the economy is shown in Figure 3.

The utility maximization problem of the representative household can be written as follows:

$$\max_{C_t, K_t^h, D_t} U_t = \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}, \quad (8)$$

subject to a budget constraint

$$C_t + K_t^h + D_t + f^k(K_t^h; K_t) + f^d(D_t) = w_t L + \Pi_t + R_t^k K_{t-1}^h + R_t D_{t-1}, \quad (9)$$

where

$$\Pi_t = (1 - \sigma) (R_t^k K_{t-1}^b - R_t D_{t-1}) - (1 - \sigma) f \zeta K_t, \quad (10)$$

where  $C_t$  is consumption in period  $t$ ,  $K_t^h$  is the capital investment by workers in period  $t$ ,  $D_t$  is

<sup>10</sup>GK (2010), Aoki et al. (2016) and GKP (2020 (a)) make the same assumption.

the deposits in period  $t$ , and  $\beta \in (0, 1)$  denotes the discount factor. The wage income is  $w_t L$ , where  $L_t = L = 1 - f$ .  $R_t$  represents the gross rate of return on the deposits.  $R_t^k$  is the gross interest rate in the capital market, where  $R_t^k = 1 + Z_t - \delta$ . This implies that the rental price equals the sum of the interest rate and depreciation cost such that  $Z_t = (R_t^k - 1) + \delta$ . The household solves the above optimization problem, given the net funds transferred to households  $\Pi_t$ . The first term on the right-hand side of Equation (10) is the net worth of retired bankers who return to the household as dividends. The probability of retired bankers returning to the household as dividends is  $1 - \sigma \in (0, 1)$ .  $K_t^b$  denotes the capital investment by bankers in period  $t$ .<sup>11</sup> The household provides a proportion of total assets,  $\zeta_t K_t$ , to new bankers,  $(1 - \sigma)f$ , in each period.

The first-order conditions for the utility maximization of consumption, capital investment of households, and deposits yield the following conditions:

$$1 = \Lambda_{t,t+1} \frac{R_{t+1}}{1 + \theta}; \quad (11)$$

$$1 = \Lambda_{t,t+1} \frac{R_{t+1}^k}{1 + f'(K_t^h; K_t)}, \quad (12)$$

where

$$\Lambda_{t,t+1} \equiv \frac{\beta C_t}{C_{t+1}},$$

and

$$f'(K_t^h; K_t) = \chi \left( \frac{K_t^h}{K_t} \right).$$

The transversality condition is  $\lim_{T \rightarrow \infty} \Lambda_{0,T} (D_T + K_T^h) = 0$ .

Equations (11) and (12) provide the no-arbitrage condition in which the rate of return of capital management equals the rate of return of deposits as follows:

$$\Lambda_{t,t+1} R_{t+1}^k - f'(K_t^h; K_t) = \Lambda_{t,t+1} R_{t+1} - \theta. \quad (13)$$

---

<sup>11</sup>The details of aggregating each bankers' net worth, capital investment, and deposit market clearing condition are given in subsection 2.5.

## 2.4 Bankers' Optimizing Behavior (Financial intermediate firms)

Each banker maximizes the discounted sum of the expected value of its own net worth  $V_t$ . The problem of a banker who begins the banking business in period  $t$  and exits the bank at the end of period  $\tau$  and brings net worth back to the household is

$$\max_{n_\tau} V_t = E_t \left[ \sum_{\tau=t+1}^{\infty} \Lambda_{t,\tau} \sigma^{(\tau-1)-t} (1 - \sigma) n_\tau \right], \quad (14)$$

subject to the balance sheet condition:

$$k_i^b = n_i + d_i, \text{ for } t \leq i \leq \tau, \quad (15)$$

the evolution of a continuing banker's net worth:

$$n_j = R_j^k k_{j-1}^b - R_j d_{j-1}, \text{ for } t+1 \leq j \leq \tau, \quad (16)$$

the incentive compatibility constraint (17), and the minimum capital requirement (18), where  $n_t$  denotes the net worth of each banker in period  $t$ ,  $d_t$  denotes the funds from households' deposits of each banker in period  $t$ , and  $k_t^b$  represents the investment of each banker in period  $t$ . In the objective function,  $\Lambda_{t,\tau}$  represents the stochastic discount factor of the representative household such that  $\Lambda_{t,\tau} = \frac{\beta^{\tau-t} C_t}{C_\tau}$ ,  $\sigma^{(\tau-1)-t} (1 - \sigma)$  is the probability of the banker retiring at time  $\tau$ , and  $n_\tau$  is the net worth of the banker retiring at time  $\tau$ . Equations (15) and (16) are the constraints of the flow of funds of a banker who continues banking business from  $t$  to  $\tau$ . The initial conditions for the banker who begins doing business at time  $t$  are as follows:  $n_t = k_t^b$ ,  $d_t = 0$ , and  $n_t = \zeta K_t$ , where  $\zeta$  is constant and very small for  $0 \leq t < \infty$ .

We consider the following moral hazard problem in the spirit of Holmström and Tirole (1997) and GK (2010).<sup>12</sup> After bankers collect deposits from households, they can leave the bank and divert

<sup>12</sup>GKQ (2012), GKP (2020 (a), 2020 (b), and 2020 (c)), and other related pieces of literature following GK (2010) that consider the moral hazard problem. We assume that the diversion of funds depends on households' monitoring in the spirit of costly state verification, similar to Townsend (1979) but contrasting with these studies. Moreover, each banker in this model faces not only the incentive compatibility constraint but also the minimum capital requirement.

a proportion of the funds for their private benefit. We assume each banker can divert a fraction of the funds because of depositors' protection and household monitoring. Thus, the incentive compatibility constraint can be written as

$$V_t \geq \Theta k_t^b = \Theta n_t + \Theta d_t, \quad (17)$$

where  $\Theta \equiv m\theta$  and  $0 < m < 1$ . As we mentioned, in this model, as the financial market develops,  $\theta$  decreases, and thus  $\Theta$  decreases. The left-hand side of Equation (17) is the value of the investment of bankers' funds, whereas the right-hand side is the value of diverting these funds. The last equality of (17) is given by Equation (15). Constraint (17) implies depositors' monitoring activities and financial market developments determine how bankers divert funds. Both the continuing and new bankers are subjected to (17).

We introduce the minimum capital requirements into this model.<sup>13</sup> This requirement forces bankers to keep the ratio of their net worth to risky assets, that is, a larger capital adequacy ratio than the minimum  $\bar{\kappa}$ . Thus,  $\bar{\kappa}$  is a key policy parameter in this model, which ranges from 0 to 1. Let  $\kappa_t$  denote the capital adequacy ratio, such that  $\kappa_t \equiv \frac{n_t}{k_t^b}$ . Formally, the minimum capital requirements in this model are described as follows:

$$\kappa_t \geq \bar{\kappa}. \quad (18)$$

Generally, from (14), the value of the banker who begins his or her business in period  $t$  satisfies the following Bellman equation:

$$V_t = E_t \left[ \Lambda_{t,t+1}(1 - \sigma)n_{t+1} + \Lambda_{t,t+1}\sigma V_{t+1} \right]. \quad (19)$$

---

<sup>13</sup>Holmström and Tirole (1997) introduce the financial intermediaries instead of other financial regulations to monitor the moral hazard problem between depositors and investors who divert a proportion of funds for private benefit. We introduce these capital requirements because this model has both a moral hazard problem and a problem caused by households' inefficient investment. GKP (2020 (b), 2020(c)) proposes a state-dependent time-varying capital requirement as a macroprudential policy to prevent bank runs. Instead of considering time-varying requirements and bank runs, we introduce the minimum capital requirement as a constant policy parameter to demonstrate the optimal policy to improve inefficient resource allocation in an economy in the long run.

We guess the following linear value function to solve the decision problem:

$$V_t = \iota_t k_t^b - \nu_t d_t, \quad (20)$$

where  $\iota_t > 0$  and  $\nu_t > 0$ .

Let  $\mu_t$  be defined such that  $\mu_t \equiv \iota_t - \nu_t$ . Assume that  $\mu_t < \Theta$ .<sup>14</sup> The incentive compatibility constraint (17) with Equations (20) and (15) can be rewritten as follows:

$$V_t = \mu_t k_t^b + \nu_t n_t \geq \Theta k_t^b. \quad (21)$$

We assume that Equation (21) is binding. Then, the minimum capital requirements (18) and the definition of  $\kappa_t$  yield

$$\kappa_t = \frac{\Theta - \mu_t}{\nu_t} \geq \bar{\kappa}. \quad (22)$$

As a result,  $\tilde{\kappa}_t$  is defined as the endogenous capital adequacy ratio that satisfies the incentive compatibility constraint, such that

$$\tilde{\kappa}_t \equiv \frac{\Theta - \mu_t}{\nu_t}.$$

Using the above definition of  $\tilde{\kappa}_t$ , (22) can be rewritten as

$$\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa}). \quad (22')$$

After combining the conjectured value function (21), the Bellman equation (19), and constraints on the flow funds of a continuing banker (15) and (16), we verify that the value function is linear in  $k_t^b$  and  $n_t$  such that  $V_t = \mu_t k_t^b + \nu_t n_t$  with <sup>15</sup>

$$\mu_t = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \left( R_{t+1}^k - R_{t+1} \right) \right], \quad (23)$$

---

<sup>14</sup>The incentive compatibility constraint (17) can be written as  $\nu_t n_t \geq (\Theta - \mu_t) k_t^b$ . Because  $\nu_t > 0$ ,  $n_t > 0$ , and  $k_t^b > 0$ ,  $(\Theta - \mu_t) > 0$  allows us to analyze an economy in which the incentive compatibility constraint is binding.

<sup>15</sup>See Appendix A.

$$v_t = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right], \quad (24)$$

where

$$\Omega_{t+1} \equiv (1 - \sigma) + \frac{\sigma \mu_{t+1}}{\kappa_{t+1}} + \sigma v_{t+1}, \quad (25)$$

and

$$\kappa_{t+1} = \max (\tilde{\kappa}_{t+1}, \bar{\kappa}). \quad (22')$$

$\Lambda_{t,t+1} \Omega_{t+1}$  represents the stochastic discount factor of the banker.  $\Omega_{t+1}$  is the marginal value of net worth, which is the weighted average of the marginal values for existing and continuing banks. The shadow value of a banker's capital holdings is  $\mu_t$ . The shadow value of the banker's net worth is  $v_t$ .

## 2.5 Aggregation and Equilibrium

In period  $t$ , there are  $\sigma f$  continuing bankers and  $(1 - \sigma)f$  new bankers. Each  $\sigma$  of the  $\sigma f$  continuing bankers who continue their business before  $t - 1$  has capital holdings  $k_t^b$ , deposits  $d_t$ , and the net worth  $R_t^k k_{t-1}^b - R_t d_{t-1}$  in period  $t$ . Each  $1 - \sigma$  out of the  $\sigma f$  bankers who begin their business in period  $t - 1$  have capital holdings  $k_t^b$ , deposits  $d_t$ , and the net worth  $R_t^k \zeta K_{t-1}$  in period  $t$ . Each new banker has capital holdings  $\zeta K_t$  and net worth  $\zeta K_t$  in period  $t$ . Thus, the aggregate capital holdings and aggregate net worth of bankers  $K_t^b$  and  $N_t$  are as follows:

$$K_t^b = \sigma f k_t^b + (1 - \sigma) f \zeta K_t; \quad (26)$$

$$N_t = \sigma \sigma f (R_t^k k_{t-1}^b - R_t d_{t-1}) + (1 - \sigma) \sigma f R_t^k \zeta K_{t-1} + (1 - \sigma) f \zeta K_t.$$

The deposit market clearing condition is

$$D_t = \sigma f d_t. \quad (27)$$



Using Equations (26) and (27), the aggregate net worth  $N_t$  can be rewritten as follows:

$$N_t = \sigma \left( R_t^k K_{t-1}^b - R_t D_{t-1} \right) + (1 - \sigma) f \zeta K_t. \quad (28)$$

The goods market clearing condition is as follows:

$$Y_t = C_t + I_t + f^k \left( K_t^h; K_t \right) + f^d \left( D_t \right). \quad (29)$$

Because the sum of households' total assets and bankers' total assets is equal to the aggregate capital, we obtain

$$K_t^h + D_t + N_t = K_t. \quad (30)$$

From (5), (30) can be rewritten as

$$D_t + N_t = K_t^b. \quad (31)$$

## 2.6 Summary of the Key Ingredients of This Model

In this model,  $\Theta$  and  $\chi$  are key parameters which reflect how the financial market develops. In particular,  $\Theta$  reflects a parameter that causes inefficient resource allocation through direct financing, while  $\chi$  reflects a parameter that causes inefficient resource allocation through indirect financing,  $\theta$  determines how bankers can divert the funds:  $\Theta$  in (17). In this model, we obtain the no-arbitrage condition (13), which is the key equation, from the first order conditions of utility maximization problem. (13) includes the spread determined by the market mechanism. By adjusting the optimum household behavior and the market, the key parameters of financial development in this model:  $\Theta$  and  $\chi$  play a critical role in determining resource inefficiency caused by direct and indirect financing. As a result of these mechanisms, raising minimum capital requirements (the policy in this model) might work to improve welfare in the long run and to mitigate a crisis in the short run.

### 3 Dynamic System

To describe the dynamic system of the economy, we define two variables,  $\eta_t$  and  $x_t$ , as follows: Let  $\eta_t$  be the ratio of banks' capital holdings to aggregate capital as  $\eta_t \equiv \frac{K_t^b}{K_t}$ . Then, the ratio of households' capital holdings to aggregate capital is  $1 - \eta_t \equiv \frac{K_t^h}{K_t}$ . In an economy with large (small)  $\eta_t$ , more capital is managed by direct (indirect) financing. Let  $x_t$  denote the ratio of consumption to aggregate capital as  $x_t \equiv \frac{C_t}{K_t}$ . We describe the dynamic system of the model with  $\eta_t$ ,  $x_t$ , and  $K_t$  with  $\kappa_t$ , where  $\kappa_t = \frac{n_t}{k_t^b}$  is the capital adequacy ratio, which must be larger than the minimum  $\bar{\kappa}$  because of the minimum capital requirements, as defined in Section 2.<sup>16</sup>

Combining the Euler equation (12) with management costs (6), the flow constraints of continuing bankers (15) and (16), aggregation (26), (28), and (31), and deposit market clearing (27) yields

$$\kappa_t = \frac{\sigma f \zeta [1 + \chi (1 - \eta_{t-1})]}{\frac{\beta x_{t-1}}{x_t} [\eta_t - (1 - \sigma) \zeta f]}, \quad (32)$$

where  $\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa})$ .<sup>17</sup>

From the bankers' value functions (19) and (21), as well as aggregation (26), we obtain

$$\Theta \left[ \frac{\eta_{t-1} - (1 - \sigma) f \zeta}{\eta_t - (1 - \sigma) f \zeta} \right] = \beta \frac{x_{t-1}}{x_t} [(1 - \sigma) \tilde{\kappa}_t + \sigma \Theta], \quad (33)$$

where  $\tilde{\kappa}_t$  satisfies both bankers' and workers' optimization conditions (11)–(13) and (22)–(25).<sup>18</sup>

Using the goods production function (1), the profit maximization conditions (2), and the households' utility maximization conditions (12) with management costs (6), we obtain<sup>19</sup>

$$K_t = (1 - f) \left[ \frac{\alpha}{\frac{1 + \chi(1 - \eta_{t-1})}{\Lambda_{t-1,t}} - (1 - \delta)} \right]^{\frac{1}{1 - \alpha}}, \quad (34)$$

<sup>16</sup>The aggregate flow variables in period  $t$  are  $Y_t$ ,  $Z_t$ ,  $w_t$ ,  $R_t^k$ ,  $I_t$ ,  $C_t$ , and  $R_t$ . The aggregate stock variables are  $K_t$ ,  $K_t^b$ ,  $K_t^h$ ,  $D_t$ , and  $N_t$ . All these variables are described by  $\eta_t$ ,  $x_t$ , and  $K_t$  with  $\kappa_t$ , subject to minimum capital requirements.

<sup>17</sup>See Appendix B.

<sup>18</sup>See Appendix C.

<sup>19</sup>See Appendix D.

where  $\Lambda_{t-1,t} = \frac{\beta x_{t-1} K_{t-1}}{x_t K_t}$ .

From the goods market clearing condition (29), the law of motion for capital (4), the production function (1), the Euler equation (12) with (6) and (7), the flow constraints for banks (15) and (16), as well as the aggregations (26)–(28), and (31), we obtain<sup>20</sup>

$$x_t = \frac{A[1 + \chi(1 - \eta_{t-1})]}{\alpha \Lambda_{t-1,t}} \frac{K_{t+1}}{K_t} - \frac{(1 - \delta)(A - \alpha)}{\alpha} - \frac{\chi}{2} (1 - \eta_t)^2 - \theta \sigma f \zeta \left[ \frac{1 + \chi(1 - \eta_{t-1})}{\beta \frac{x_{t-1}}{x_t}} \right] - \theta(1 - \sigma) f \zeta - \theta \eta_t, \quad (35)$$

where  $\Lambda_{t-1,t} = \frac{\beta x_{t-1} K_{t-1}}{x_t K_t}$ .

Using the definitions of  $\eta_t$ ,  $x_t$ , and  $R_t^k$ , combining the households' budget constraint (9) with (10), the goods market clearing condition (29) with (30), the aggregate net worth (28) and the law of motion for capital (4) with (5), the production function (1) with the non-financial firms' profit maximization conditions (2) and (3) yields<sup>21</sup>

$$K_{t+1} - K_t = \left[ (1 - \delta) + \alpha \left( \frac{1 - f}{K_t} \right)^{1-\alpha} \right] (K_t - K_{t-1}). \quad (36)$$

Equations (32), (33), (34), (35), (36), and (22') constitute the dynamic system of  $x_t$ ,  $\eta_t$ , and  $K_t$  with  $\kappa_t$  that describes the economy.

Let  $\Phi_x$  and  $\Phi_K$  be the functions of the variables in periods  $t - 1$  and  $t$ , respectively. Let  $\Psi_\eta$ ,  $\Psi_x$ , and  $\Psi_K$  be the dynamics of the following variables:  $\eta$ ,  $x$ , and  $K$ , which are described by variables in period  $t - 1$ , respectively. Equations (32) and (33) with (22') yield the dynamics of  $x_t$ ,  $x_{t-1}$ ,  $\eta_t$ , and  $\eta_{t-1}$ :  $x_t = \Phi_x(x_{t-1}, \eta_{t-1}, \eta_t)$ . Plugging  $\Phi_x(x_{t-1}, \eta_{t-1}, \eta_t)$  into (34) yields  $K_t = \Phi_K(x_{t-1}, \eta_{t-1}, K_{t-1}, \eta_t)$ . By combining (35) and (36), and using  $\Phi_x(x_{t-1}, \eta_{t-1}, \eta_t)$  and  $\Phi_K(x_{t-1}, \eta_{t-1}, K_{t-1}, \eta_t)$ , we obtain the dynamics of  $\eta_t$  as follows:

$$\eta_t = \Psi_\eta(x_{t-1}, \eta_{t-1}, K_{t-1}).$$

Then, the dynamics of  $x_t$  and  $K_t$  are derived from  $x_t = \Phi_x(x_{t-1}, \eta_{t-1}, \eta_t)$ ,  $K_t = \Phi_K(x_{t-1}, \eta_{t-1}, K_{t-1}, \eta_t)$ ,

<sup>20</sup>See Appendix E.

<sup>21</sup>See Appendix F.

and  $\eta_t = \Psi_\eta(x_{t-1}, \eta_{t-1}, K_{t-1})$  as follows:

$$x_t = \Phi_x(x_{t-1}, \eta_{t-1}, \eta_t) = \Phi_x(x_{t-1}, \eta_{t-1}, \Psi_\eta(x_{t-1}, \eta_{t-1}, K_{t-1})) \equiv \Psi_x(x_{t-1}, \eta_{t-1}, K_{t-1}).$$

$$K_t = \Phi_K(x_{t-1}, \eta_{t-1}, K_{t-1}, \eta_t) = \Phi_K(x_{t-1}, \eta_{t-1}, \Psi_\eta(x_{t-1}, \eta_{t-1}, K_{t-1})) \equiv \Psi_K(x_{t-1}, \eta_{t-1}, K_{t-1}).$$

After  $\eta_{t-1}$ ,  $x_{t-1}$ ,  $\eta_t$ , and  $x_t$  are determined by the above system,  $\kappa_t$  is determined by (32) with (22') and (33).

## 4 Steady State Analyses

This section examines the optimal minimum capital requirements for steady-state economies. Minimum capital requirements in this model play the role of improving inefficient resource allocation due to the marginal costs of both direct and indirect finance in an economy.<sup>22</sup> Let  $y^*$  denote the level of variable  $y$  in the steady-state. First, we compute the following four variables by evaluating Equations (32), (33), (34), and (35) as well as (22') at the steady-state: the capital adequacy ratio  $\kappa^*$ , the ratio of households' (banks') management capital  $1 - \eta^*$  ( $\eta^*$ ), the ratio of consumption to aggregate capital  $x^*$ , and aggregate capital  $K^*$  in the following subsection.<sup>23</sup>

### 4.1 Aggregate Variables in the Steady State

In the steady state, where  $\eta_{t-1} = \eta_t = \eta^*$ ,  $x_{t-1} = x_t = x^*$ , and  $\kappa_t = \kappa^*$ , evaluating Equations (22') and (33) at the steady state yields

$$\kappa^* = \max(\tilde{\kappa}^*, \bar{\kappa}); \quad (22' *)$$

$$\tilde{\kappa}^* = \frac{\Theta(1 - \beta\sigma)}{\beta(1 - \sigma)}, \quad (33 *)$$

---

<sup>22</sup>In the steady-state, inefficiency of the model is caused by household capital investment and banks' moral hazard problem. Thus, the analysis of optimal capital requirements in the steady-state economy is meaningful for considering a policy for improving the welfare of agents in an economy in the long run.

<sup>23</sup>(36) is satisfied in the steady-state where  $K_{t-1} = K_t = K_{t+1} = K^*$ .

where  $\Theta = m\theta < 1$  with parameters  $\beta, \sigma, \Theta \in (0, 1)$  and policy  $\bar{\kappa} \in [0, 1]$ . Let  $\Theta_{max}$  be such that  $\Theta_{max} \equiv \frac{\beta(1-\sigma)}{1-\beta\sigma}$  and  $\Theta_{max} < 1$  from  $\beta, \sigma \in (0, 1)$ . We then assume that  $\Theta \leq \Theta_{max}$ , allowing us to analyze an economy in which  $\bar{\kappa}^* \leq 1$ . The development of the financial market is represented by  $\theta$  and  $\Theta = m\theta < \theta$ . In this and the following sections, we focus on  $\Theta$  as a parameter of financial development. Equation (33 \*) shows the endogenous capital adequacy ratio, which increases as the financial market develops.

In the steady state,  $\eta_{t-1} = \eta_t = \eta^*$  and  $x_t = x_{t-1} = x^*$ . Using (22' \*) and (33 \*), evaluating Equation (32) in the steady state gives us

$$\eta^* = \frac{f\zeta [\sigma(1+\chi) + \kappa^*\beta(1-\sigma)]}{\kappa^*\beta + \sigma f\zeta\chi}, \quad (32 *)$$

where  $\kappa^*$  is given by (22' \*) and (33 \*) with parameters  $f, \zeta, \beta, \sigma \in (0, 1)$ , and  $\chi > 0$ . From Equation (32 \*),  $\eta^* \leq 1$  corresponds to  $\frac{f\zeta\sigma}{\beta[1-(1-\sigma)f\zeta]} \leq \kappa^*$ , the following lemma gives the policy parameter restriction of optimal requirements for  $\eta^* \in (0, 1]$ .

**Lemma 1** *Let  $\bar{\kappa}_{min}$  be such that  $\bar{\kappa}_{min} = \frac{f\zeta\sigma}{\beta[1-(1-\sigma)f\zeta]}$  with parameters  $f, \zeta, \sigma, \beta \in (0, 1)$ . Suppose  $\bar{\kappa}_{min}$  is less than one. Then, the optimal minimum capital requirement must be  $\bar{\kappa}$  such that  $\bar{\kappa} \geq \bar{\kappa}_{min}$ .*

Let  $\Theta_{min}$  be arranged so that  $\Theta_{min} \equiv \frac{f\zeta\sigma(1-\sigma)}{[1-(1-\sigma)f\zeta](1-\beta\sigma)}$ . Then, from Equations (32 \*) and (33 \*),  $\Theta_{min} \leq \Theta$  correspond to  $\eta^* \leq 1$  when  $\kappa^* = \bar{\kappa}^*$ . We suppose that  $\Theta_{min} \leq \Theta \leq \Theta_{max}$  to analyze the economy in which the endogenous capital adequacy ratio and the proportion of direct (indirect) finance in the economy are less than 1, respectively.

In the steady state,  $\eta_{t-1} = \eta_t = \eta^*$ ,  $x_t = x_{t-1} = x^*$ ,  $K_{t-1} = K_t = K_{t+1} = K^*$ ; thus,  $\Lambda_{t-1,t} = \Lambda_{t,t+1} = \beta$ . Evaluating Equations (34) and (35) in steady-state yields

$$K^* = (1-f) \left[ \frac{\alpha\beta}{1+\chi(1-\eta^*)-\beta(1-\delta)} \right]^{\frac{1}{1-\alpha}}, \quad (34 *)$$

$$x^* = \frac{A[1+\chi(1-\eta^*)]}{\alpha\beta} - 1 - \frac{(1-\delta)(A-\alpha)}{\alpha} - \frac{\chi}{2}(1-\eta^*)^2 - \frac{\Theta D^*}{m K^*}, \quad (35 *)$$

where

$$\frac{D^*}{K^*} = \frac{\sigma f \zeta}{\beta} [1 + \chi(1 - \eta^*)] + (1 - \sigma)f\zeta + \eta^*, \quad (37)$$

and  $\eta^*$  are given by (32 \*), (22' \*), and (33 \*) with parameters  $f, \alpha, \beta, \delta, \sigma, \zeta \in (0, 1)$ ,  $\chi > 0$ ,  $A \geq 1$ , and  $\frac{\Theta}{m} = \theta$ , where  $\Theta \in [\Theta_{min}, \Theta_{max}]$ ,  $m \in (0, 1)$ , and  $\theta \in (0, \frac{\Theta_{max}}{m}]$ .<sup>24</sup>

Next, we describe additional aggregate variables of the real economy in the steady state. Aggregate output  $Y^*$ , and aggregate consumption  $C^*$  are obtained from Equations (1), (4), (6), (7), (29), (22' \*), (32 \*), (33 \*), (34\*), and (37) as follows:

$$Y^* = AK^{*\alpha}L^{1-\alpha} = AK^{*\alpha}(1 - f)^{1-\alpha}, \quad (1 *)$$

$$C^* = Y^* - \delta K^* - \frac{\chi}{2}(1 - \eta^*)^2 K^* - \frac{\Theta}{m} D^*, \quad (38)$$

where  $\eta^*$ ,  $K^*$ , and  $D^*$  are given by Equations (32 \*), (33 \*), (34 \*), and (37), and (22' \*) with  $\alpha, f, \delta, m \in (0, 1)$ ,  $\chi > 0$ ,  $A \geq 1$ , and  $\frac{\Theta}{m} = \theta$ , where  $\Theta \in [\Theta_{min}, \Theta_{max}]$ . We can evaluate the welfare of all agents in the steady-state economy by using the aggregate consumption determined by (38) since the representative households consist of workers and bankers, and the utility function depends on aggregate consumption.<sup>25</sup>

## 4.2 Effects of Tighter Minimum Capital Requirements on the Aggregate Economic Activity in the Long Run

This subsection examines how minimum capital requirements relating to financial development in the long-run economy before analyzing the optimal capital requirements. Equations (32 \*) – (35 \*), (37), (38), and (22' \*) show that the minimum capital adequacy ratio  $\bar{k}$  affects aggregate variables only through the ratio of direct finance to indirect finance  $\eta^*$  when the steady-state adequacy ratio sticks to the minimum. We determined that in an unregulated economy the steady-state capital

<sup>24</sup>Equation (37) is obtained by evaluating Equation (E1') at the steady state in Appendix E.

<sup>25</sup>Van den Heuvel (2008) examines household consumption and bank deposits to determine the welfare of the bank capital requirements.

adequacy ratio is endogenous, such that  $\kappa^* = \bar{\kappa}^*$ . By contrast, the steady-state capital adequacy ratio in a regulated economy sticks to the minimum requirement:  $\kappa^* = \bar{\kappa}$  from (22' \*) and (33 \*). Then, we obtain the relationship between financial development and minimum capital requirements in the following proposition:

**Proposition 1** *Let  $\bar{\Theta}$  be such that  $\bar{\Theta} \equiv \frac{\beta(1-\sigma)}{(1-\beta\sigma)}\bar{\kappa}$ . If the financial market has developed enough such that  $\bar{\Theta} \geq \Theta$ , then the economy is regulated in the steady state, where the bankers' capital adequacy ratio is equal to the minimum such that  $\kappa^* = \bar{\kappa}$ . If the financial market has not developed enough such that  $\bar{\Theta} < \Theta$ , then the economy is unregulated in the steady state, where the bankers' capital adequacy ratio is larger than the minimum such that  $\kappa^* = \tilde{\kappa}^* > \bar{\kappa}$ .*

**Proof.** Let  $\bar{\Theta}$  be such that  $\bar{\Theta} \equiv \frac{\beta(1-\sigma)}{(1-\beta\sigma)}\bar{\kappa}$ . Then, from (22' \*) and (33 \*),  $\bar{\Theta} \geq \Theta$  correspond to  $\bar{\kappa} \geq \tilde{\kappa}^*$ .

■

Proposition 1 provides the threshold between the unregulated and regulated economy  $\bar{\Theta}$ . It depends on the discount factor  $\beta \in (0, 1)$ , the ratio of continuing bankers in the banking sector  $\sigma \in (0, 1)$ , and the minimum requirements  $\bar{\kappa}$ . To interpret Proposition 1, Figure 4 exhibits financial markets in the regulated and unregulated economy range between in the range of  $\Theta_{min} < \bar{\Theta} \leq \Theta \leq \Theta_{max}$  and the endogenous bank capital adequacy ratio, which takes the value of  $\bar{\kappa}_{min}$  to 1.<sup>26</sup> The solid line in Figure 4 represents the steady-state capital adequacy ratio, which is determined by (22' \*) with (33 \*). Because  $\frac{(1-\beta\sigma)}{\beta(1-\sigma)} > 1$ , the slope  $\tilde{\kappa}^*$  is steeper than the slope of  $\kappa^* = \Theta$ . In Figure 4, the dotted area indicates an economy with low minimum requirements and a financial market that has developed enough that  $\Theta \leq \bar{\Theta}$  is the regulated economy. In contrast, an economy without a large  $\Theta$  that  $\bar{\Theta} < \Theta$  is an unregulated economy.

From Proposition 1, Equations (22' \*), (32 \*)-(35\*), (1\*), (37), and (38), all the following steady-state variables: the ratio of capital holdings by banks to the aggregate capital  $\eta^*$ , aggregate capital  $K^*$ , aggregate output  $Y^*$ , and aggregate consumption  $C^*$  depend on how the financial market develops  $\Theta$ , but not on the minimum requirement  $\bar{\kappa}$  in an unregulated economy where financial market has not developed. Meanwhile, in a regulated economy where financial market has developed, these

<sup>26</sup>Lemma 1 ensures that  $\Theta_{min} < \bar{\Theta}$ . Because  $\bar{\kappa} < 1$  and  $\frac{\beta(1-\sigma)}{(1-\beta\sigma)} = \Theta_{max}$ ,  $\bar{\Theta} < \Theta_{max}$ , and  $\bar{\Theta} < \bar{\kappa}$  are ensured.

The bank capital adequacy ratio  $\kappa^*$ , which is endogenous

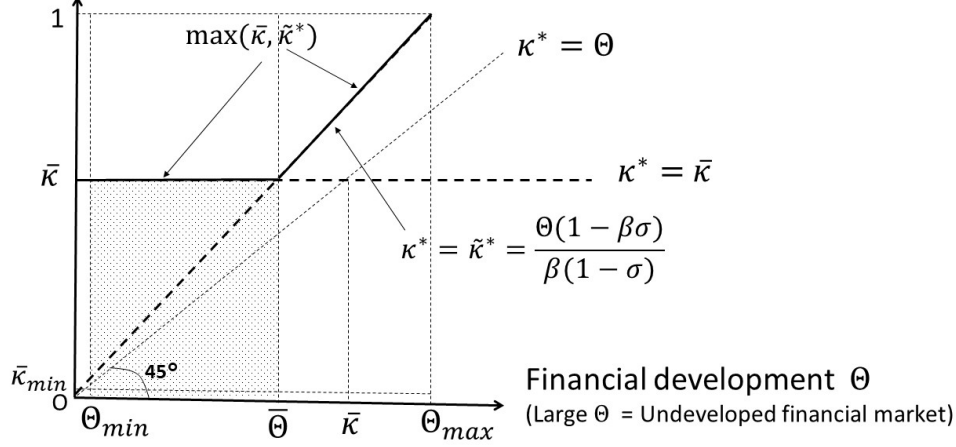


Figure 4: Relationship between financial development and minimum capital requirements variables, except for aggregate consumption (welfare), depend on  $\bar{\kappa}$  but not on  $\Theta$ . Also, in a regulated economy, welfare is affected by both the minimum capital requirements  $\bar{\kappa}$  and financial development  $\Theta$  of the economy.

Now, we examine the optimal minimum capital requirements to improve inefficient resource allocation and welfare. As we have argued before, the economic variables in only the regulated economy are affected by raising (reducing) the minimum capital requirements. For an unregulated economy, because the threshold  $\bar{\Theta}$  is affected by raising the minimum capital requirements, if a large capital adequacy ratio improves welfare, such a tight requirement raises the threshold, moves the economy from unregulated to regulated, and thus improves welfare. We first consider a regulated economy where financial market is well-developed and analyze the optimal capital requirements.

Consider an economy in which  $\Theta \leq \bar{\Theta}$  and  $\kappa^* = \bar{\kappa}$ . We examine how raising the minimum capital requirements affects the ratio of direct to indirect finance  $\eta^*$  in the regulated economy. From Equation (32 \*), differentiating  $\eta^*$  with respect to  $\bar{\kappa}$  yields

$$\frac{d\eta^*}{d\bar{\kappa}} = \frac{-f\zeta\beta\sigma}{(\bar{\kappa}\beta + \sigma f\zeta\chi)^2} \{[1 - (1 - \sigma)f\zeta]\chi + 1\} < 0. \quad (39)$$

The inequality (39) is obtained by  $(1 - \sigma)f\zeta < 1$  from  $\beta, \sigma, f, \zeta \in (0, 1), \chi > 0$ , and  $\bar{\kappa} \in [\bar{\kappa}_{min}, 1]$ .



Inefficiency in this economy is caused by households' extra management cost of capital holdings and monitoring costs for banks' moral hazard problems in terms of goods. Since the marginal management cost is  $\chi(1 - \eta^*)$ , the inequality in (39) shows that raising the minimum capital requirements enhances indirect finance more than direct finance, and thus, its policy causes more inefficient resource allocation stemming from the former causes in the economy. From (34 \*) and (39), differentiating  $K^*$  with respect to  $\bar{\kappa}$ , we obtain

$$\frac{dK^*}{d\bar{\kappa}} = \frac{dK^*}{d\eta^*} \frac{d\eta^*}{d\bar{\kappa}} = \frac{\chi [1 + \chi(1 - \eta^*) - \beta(1 - \sigma)]}{\alpha\beta(1 - \alpha)} K^* \frac{d\eta^*}{d\bar{\kappa}} < 0, \quad (40)$$

where  $1 + \chi(1 - \eta^*) - \beta(1 - \sigma) > 0$  from  $\eta^* \leq 1$ ,  $\beta(1 - \sigma) < 1$ , and  $\alpha\beta(1 - \alpha) > 0$ . From (1 \*) and (40), differentiating  $Y^*$  with respect to  $\bar{\kappa}$ , we obtain

$$\frac{dY^*}{d\bar{\kappa}} = \frac{dY^*}{dK^*} \frac{dK^*}{d\bar{\kappa}} = \alpha \frac{Y^*}{K^*} \frac{dK^*}{d\bar{\kappa}} < 0. \quad (41)$$

The inequalities in (40) and (41) show that raising the minimum capital requirements decreases aggregate capital and, thus, output in an economy where financial market has developed. Therefore, the results from the comparative statistics imply that the regulator should not raise the minimum capital requirement to attain a large output in an economy where financial market has developed such that  $\Theta \leq \bar{\Theta}$ . However, in this economy, households face a trade-off between the marginal costs of direct and indirect finance, both of which cause inefficiency. Even if an economy attains a large output by raising the minimum capital requirements, if the increase in large output is allocated inefficiently, this policy causes welfare losses. Conversely, even if the output is smaller, raising the minimum capital adequacy ratio can improve welfare through more efficient resource allocation. Therefore, we examine how raising the minimum capital requirement affects welfare in a regulated economy.

From (37) and (38), differentiating  $C^*$  and  $D^*$  with respect to  $\bar{k}$  yields

$$\frac{dD^*}{d\bar{k}} = \underbrace{\left( \frac{\beta - \sigma f \zeta \chi}{\beta} \right)}_{>0} K^* \frac{d\eta^*}{d\bar{k}} + \frac{D^*}{K^*} \frac{dK^*}{d\bar{k}} < 0; \quad (42)$$

$$\frac{dC^*}{d\bar{k}} = \underbrace{\left[ \frac{dY^*}{dK^*} - \delta - \frac{\chi}{2} (1 - \eta^*)^2 \right]}_{(*) < 0} \frac{dK^*}{d\bar{k}} + \chi (1 - \eta^*) \frac{d\eta^*}{d\bar{k}} - \underbrace{\frac{\Theta}{m} \frac{dD^*}{d\bar{k}}}_{(**) < 0}, \quad (43)$$

where  $\frac{dK^*}{d\bar{k}} < 0$  from (40),  $\frac{d\eta^*}{d\bar{k}} < 0$  from (39),  $\chi(1 - \eta^*) > 0$ , and  $\frac{dY^*}{dK^*} - \delta = Z^* - \delta = R^{k^*} - 1 = \frac{1 + \chi(1 - \eta^*) - \beta}{\beta}$ . From equations (2), (6), (12), and  $Z^* = (R^{k^*} - 1) + \delta$ . For the first term on the right-hand side of (42), we assume that  $\beta - \sigma f \zeta \chi > 0$ . For the term (\*) on the right-hand side of (43),  $\left[ \frac{dY^*}{dK^*} - \delta - \frac{\chi}{2} (1 - \eta^*)^2 \right] > 0$  is obtained from  $1 - \eta^* \leq 1$ .<sup>27</sup> The (\*) in (43) represents how raising minimum capital requirements causes inefficiency through direct finance with management costs, while the (\*\*) in (43) represents how raising minimum capital requirements causes inefficiency through indirect finance with monitoring costs. The effect of raising the minimum capital requirement on welfare depends on the magnitude of (\*) and (\*\*) in (43). When the magnitude of (\*) in (43) is larger than that of (\*\*) in (43),  $\frac{dC^*}{d\bar{k}} < 0$ , and vice versa. For the latter case, tight capital requirements help monitor activities and improve welfare. The effects (\*) and (\*\*) are larger depending on how the financial market develops. The following proposition summarizes the effects of raising minimum capital requirements on welfare according to the development of the financial market:

**Proposition 2** Consider an economy in which  $\Theta_{min} \leq \Theta \leq \bar{\Theta}$ . Let  $\bar{\bar{\Theta}}$  be such that

$$\bar{\bar{\Theta}} \equiv \frac{- \left\{ \left[ \frac{dY^*}{dK^*} - \delta - \frac{\chi}{2} (1 - \eta^*)^2 \right] \frac{dK^*}{d\bar{k}} + \chi (1 - \eta^*) \frac{d\eta^*}{d\bar{k}} \right\}}{\frac{-1}{m} \frac{dD^*}{d\bar{k}}}.$$

Suppose that  $\bar{\bar{\Theta}} < \bar{\Theta}$ . Then,  $\frac{dC^*}{d\bar{k}} \geq 0$  corresponds to  $\Theta \geq \bar{\bar{\Theta}}$ .

<sup>27</sup>The derivation of the sign of  $\left[ \frac{dY^*}{dK^*} - \delta - \frac{\chi}{2} (1 - \eta^*)^2 \right]$  in (43) in detail is given in Appendix G.

**Proof.** From (43), we obtain  $\frac{dC^*}{d\bar{k}} \geq 0$  as follows:

$$\Theta \geq \frac{-\left\{ \left[ \frac{dY^*}{dK^*} - \delta - \frac{\chi}{2}(1-\eta^*)^2 \right] \frac{dK^*}{d\bar{k}} + \chi(1-\eta^*) \frac{d\eta^*}{d\bar{k}} \right\}}{\frac{-1}{m} \frac{dD^*}{d\bar{k}}}. \quad (43')$$

■

Proposition 2 implies that when the financial market has developed sufficiently such that  $\Theta < \bar{\Theta}$ , raising the minimum capital requirements causes welfare loss. However, when the financial market has not developed enough such that  $\Theta > \bar{\Theta}$ , raising the minimum capital requirements improves welfare. Therefore, lemma 1 and Proposition 2 suggests that the regulator should reduce the minimum capital adequacy ratio to  $\bar{k}_{min}$  in an economy where financial market has developed such that  $\Theta < \bar{\Theta}$ . However, to be a regulated economy, its policy must keep  $\bar{k} \geq \bar{k}^*$ , and if  $\bar{k}^* > \bar{k}_{min}$ , the regulator cannot set the minimum to  $\bar{k}_{min}$  under minimum capital requirements (22' \*). Therefore, in an economy with well-developed financial markets, the regulator should set the following new requirement such that  $\bar{k} = \bar{k}_{min}$  for each bank, instead of the minimum capital requirements to improve welfare. From (41), the economy under this rule can attain a large output. When  $\Theta = \bar{\Theta}$ ,  $\frac{dC^*}{d\bar{k}} = 0$ , the optimal minimum capital requirements that satisfy  $\frac{dC^*}{d\bar{k}} = 0$  can attain maximum consumption and welfare.<sup>28</sup> Meanwhile, the regulator should raise the minimum capital adequacy ratio in an economy where financial market has not developed such that  $\bar{\Theta} < \Theta$  to improve welfare, although raising the minimum capital requirements reduces output. So far, we consider a regulated economy where financial market has well developed such that  $\Theta \in [\Theta_{min}, \bar{\Theta}]$ . However, the result of Proposition 2 implies that for an economy where financial market has not developed, larger welfare can be attained as the capital adequacy ratio is higher. Thus, for an unregulated economy where  $\Theta \in [\bar{\Theta}, \Theta_{max}]$ , the regulator can improve welfare in the economy by raising the minimum capital adequacy ratio, which shifts the threshold between regulated and unregulated economies. The following proposition summarizes the optimal minimum capital requirement in an economy based on how its financial market has developed.

---

<sup>28</sup>We show it numerically in the next section.

**Proposition 3** Consider an economy in which the financial market has developed  $\Theta_{min} \leq \Theta \leq \Theta_{max}$ . Let  $\bar{\kappa}_{op}$  be the optimal capital requirement such that  $\bar{\kappa} \in [\bar{\kappa}_{min}, 1]$ . Then,

$$\bar{\kappa}_{op} = \begin{cases} \bar{\kappa}_{min} & \text{for } \Theta_{min} \leq \Theta < \bar{\Theta} \\ \bar{\kappa}_{op}^* & \text{for } \Theta = \bar{\Theta} \\ 1 & \text{for } \bar{\Theta} < \Theta \leq \Theta_{max}, \end{cases}$$

where  $\bar{\kappa}_{op}^*$  is  $\bar{\kappa}$ , such that  $\Theta = \bar{\Theta}$ , where  $\frac{dC^*}{d\bar{\kappa}} = 0$ .

Propositions 2 and 3 imply that the regulator should raise the minimum capital adequacy ratio for an economy where financial market has not developed, whereas it should be reduced for an economy where financial market has developed. We now interpret the intuition behind the optimal minimum capital requirement described by Propositions 2 and 3. In an economy where financial market has developed, the economy is not affected much by the problems caused by undeveloped financial markets through indirect finance but is affected much by households' inefficient investment through direct finance. Thus, in this economy, reducing minimum capital requirements promotes investment through more efficient indirect financing through financial intermediaries. In such an economy, if the regulator raises the minimum capital requirements, its policy hinders a large output and leads to less consumption, thus reducing welfare. Meanwhile, in an economy where financial market has not developed, the economy is greatly affected by the problems caused by undeveloped financial markets through indirect finance with moral hazard problems. Thus, in this economy, raising minimum capital requirements, such as Basel III, promotes investment by more direct finance but suppresses indirect finance through financial intermediaries, and then, the policy improves welfare. Therefore, Propositions 2 and 3 show that optimal capital requirements should be set according to the development of the financial market.

## 5 Numerical Analyses in the Long and Short Runs

This section numerically analyzes the effects of tighter minimum capital requirements on the aggregate economic activity in the long and short runs. In the long run analyses, we investigate how this policy might work to mitigate the inefficient resource allocation caused by distortions of the financial market through indirect and direct financing and promote the aggregate economic activity, according to the degree of financial development. In the short run analyses, focusing on economies with developed financial markets, such as the U.S. economy, we examine whether this policy might mitigate a financial crisis regarding the speed of recovery and the magnitude of a drop in aggregate economic variables. We first start with the calibration.

Table 1: Parameter Values

Parameter	Value	Description	Source
Households (workers and bankers)			
$\beta$	0.99	Discount factor (impatience)	Literature
$\zeta$	0.00289	Transfer to entering bankers (Startup net worth)	Literature: GKQ (2012)
$f$	0.2	Ratio of bankers to workers	Literature: GK (2010)
$\sigma$	0.968	Proportion of continuing bankers	Literature: GKQ (2012)
Production Sector			
$\alpha$	0.33	Capital share	Literature
$\delta$	0.025	Depreciation rate	Literature
$A$	1	Productivity	Literature: GK (2010)

There are eleven parameters for which we need to assign values. Four are standard preference and technology parameters. These include the discount factor  $\beta$ , the capital share parameter  $\alpha$ , the depreciation rate  $\delta$  and the productivity of the production of the final goods  $A$ . We use reasonably conventional values for these parameters as reported in Table 1. The other three parameters reported in Table 1 are specific to the models based on GK (2010). The proportion of total assets to new bankers  $\zeta$  is minimal (0.00289) as in GKQ (2012) and the ratio of bankers to workers  $f$  is 0.2 as in GK (2010). The proportion of continuing bankers  $\sigma$  is 0.968, implying that bankers survive for eight years on average.

The other four parameters are specific to this model, including the key three parameters in this model: financial market development  $\Theta$  and  $\chi$ , and minimum capital requirement (a policy)  $\bar{k}$ . We consider the steady state leverage ratio before a crisis  $\frac{K^b}{N}$  is as follows: 12.5 when the minimum capital adequacy ratio  $\bar{k} = 0.08$ ; 10.5 when  $\bar{k} = 0.095$ ; and 10 when  $\bar{k} = 0.10$ . We choose  $\chi$  and

$\Theta$  to hit the following two targets: an average credit spread of one hundred basis points per year and an economy-wide leverage ratio of four, as in GK (2010), and the ratio of total housing stock to aggregate capital:  $K^H/K = 1/3$  as in the U.S economy in 2008.<sup>29</sup> We finally set the monitoring costs  $\theta$  as half of asset diversion.<sup>30</sup> Table 2 summarizes the above calibrated parameters.

Table 2: Calibration

Parameter	Description	Target
$\chi$	Management cost: Financial development (access)	The ratio of total housing stock to aggregate capital: $K^H/K = 1/3$
$\Theta$	Asset diversion: Financial development (efficiency)	1% annual external finance premium: $E[R^K] - R = (1.01)^{1/4} - 1$ (Literature: GK (2010)) Leverage $K^b/N = \{12.5, 10.5, 10\}$
$\bar{\kappa}$	Minimum capital adequacy ratio	A policy $\{0.08, 0.095, 0.10\}$ Leverage $K^b/N = \{12.5, 10.5, 10\}$
$\theta$	Monitoring costs	$\theta = 2\Theta$

## 5.1 Financial Development and Minimum Capital Requirements in the Long Run Economy

This subsection presents numerical analyses of our main results in Propositions 2 and 3 outlined in the previous section. We consider three economies, corresponding to Proposition 3: a well-developed financial market, a developed financial market, and an undeveloped financial market as shown in Table 3 and Figure 5.

Figure 5 summarizes the results of Proposition 2 and shows how raising minimum capital requirements from 0.08 to 0.10, as recommended by Basel requirements, affects economies whose financial market has developed shown in Table 3. A tight policy requirement reduces welfare in an economy with a developed financial market (the upper two lines in Figure 5). By contrast, a tight policy improves welfare in an economy with an undeveloped financial market (the lowest line in Figure

<sup>29</sup>GKQ (2012) noted that the total housing stock accounted for about a third of the overall capital stock before the beginning of the 2008 financial crisis. As a result, a leverage ratio of 4 is reasonable as described by GKQ(2012).

<sup>30</sup>As long as we consider the monitoring cost function is a linear function of deposits in Equation (7), the other settings of  $\theta$  do not have a significant effect on the numerical results shown in the following subsections. There may be a difference in illustration if monitoring costs are used in other shapes ; however, the substantial elements for the mechanism of this model is depositors' monitoring activities, that is, the existence of monitoring costs, not the shapes of the cost functions.

Table 3: Financial Development and Minimum Capital Requirements in the Long Run Economy

Classification	Financial development (access) $\chi$ , Financial development (efficiency) $\Theta$	Description
a well-developed financial market	0.2580, 0.0841	Low $\chi$ and Low $\Theta$
a developed financial market	0.2597, 0.0841	High $\chi$ and Low $\Theta$
an undeveloped financial market	0.2597, 0.3000	High $\chi$ and High $\Theta$

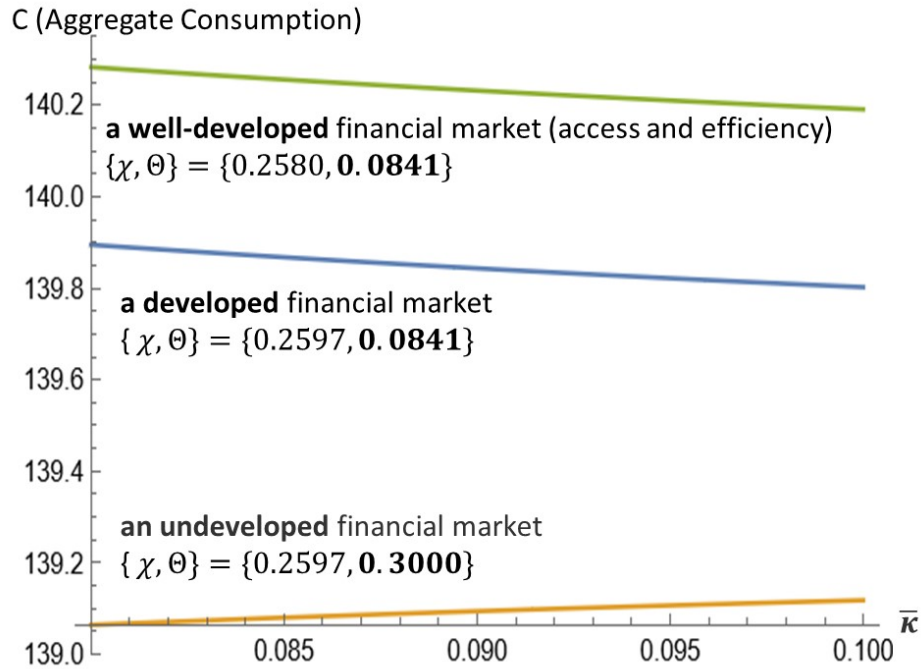


Figure 5: Effects of raising minimum capital requirements in the steady state economy in the long run according to the degree of financial development

5). In Figure 5,  $\Theta$ , is the difference between the upper and lowest lines, which causes inefficient resource allocation from indirect financing. Thus, Figure 5 demonstrates that the minimum capital requirements should be set according to the degree of financial development, especially measured by efficiency, to improve welfare in the long run. Moreover, in an economy where  $\chi = 0.2597$  shown in the lowest and middle lines in Figure 5, the optimal minimum capital requirement  $\bar{\kappa}_{op}^*$  is 8% when the degree of financial development (access) is 0.2203, that is,  $\bar{\Theta}$  in Proposition 2 is 0.2203. For this economy, the present minimum requirement, 0.08 is optimal, and to avoid welfare losses, regulators should not change its minimum requirements.

## 5.2 Tighter Minimum Capital Requirements and Aggregate Economic Activity in a Country with a Financial Crisis

This subsection presents numerical results how raising minimum capital adequacy ratio might work to mitigate the crisis. Figure 6 shows the impulse responses of the key aggregate economic variables to a negative shock to capital quality in an economy whose financial market has developed.<sup>31</sup> We suppose that the capital quality shock obeys a first order autoregressive process. The dotted red (solid blue) line reports a case with minimum capital requirements is 0.08 (0.10). We set the initiating disturbance as a five percent unanticipated decline in capital quality with an autoregressive factor of 0.66 for both economies with different policies. We fix the size of the shock as in GK (2010). This negative capital quality shock captures an exogenous force that triggers a decline in the value of assets, as explained by GK (2010) and GKQ (2012).

Despite its impact on financial development in the short run, both values indicate high financial development. The dotted line illustrates a condition with financial development  $\chi$  (access: a friction of direct financing) and  $\Theta$  (efficiency: a friction of indirect financing) is  $\{0.2580, 0.0841\}$ , while solid line is  $\{0.1974, 0.0633\}$ .<sup>32</sup> Both cases experience drops in expected spreads due to this negative shock, and the path is nearly identical. However, this shock has different effects on the gross rate of deposits  $R$ , and the gross interest rate in the capital market  $R^k$ .  $R$  in the red 8 % requirements case has a larger drop than the blue 10% requirements case. This makes a difference in responses of the variables in the financial and real sectors between the 8% and 10% requirements cases. Thus, Figure 6 indicates that inefficient resource allocation is caused by both direct and indirect financing through households' behaviors and interest rates in the market.

---

<sup>31</sup>All small letters in Figure 6 shows the aggregate variables. For example,  $y$  in Figure 6 implies the aggregate variable  $Y$ , as described in the previous sections.

<sup>32</sup>In both economies,  $\chi$  is so small that we analytically obtain  $\frac{d\eta^*}{d\bar{\kappa}} < 0$  even if higher  $\bar{\kappa}$  might change  $\chi$  and  $\Theta$ . Moreover, we obtain  $\frac{dC^*}{d\bar{\kappa}} < 0$  even if higher  $\bar{\kappa}$  might change  $\chi$  and  $\Theta$  for both economies in the long run, using the other parameters in Tables 1 and 2.



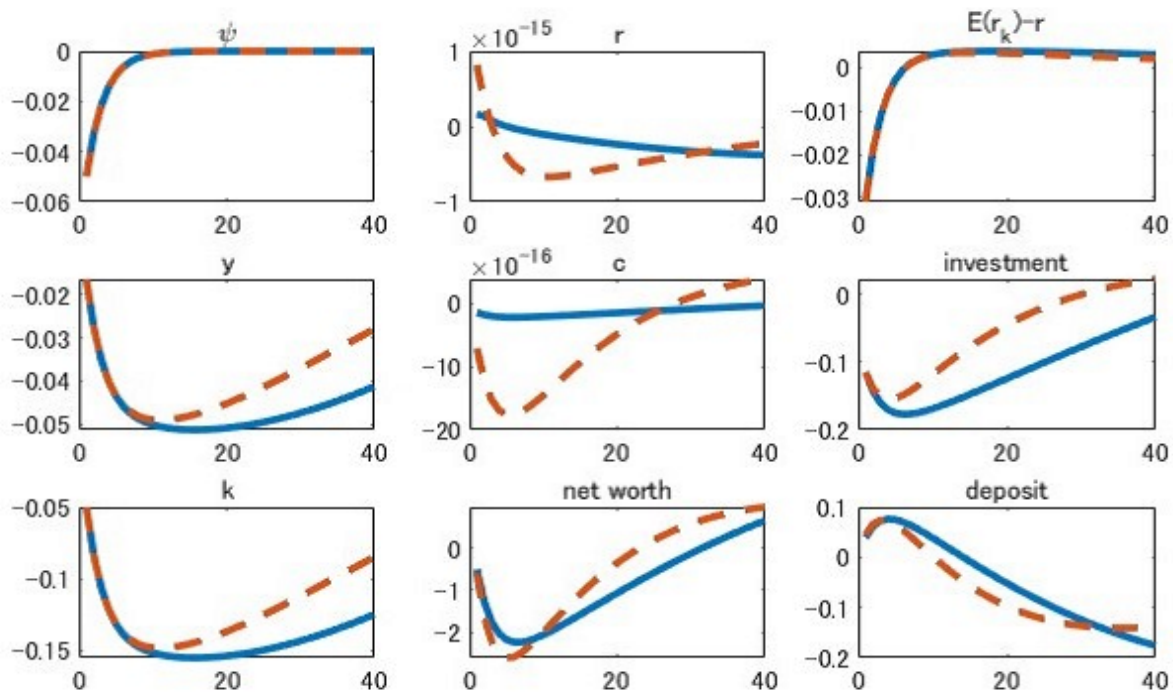


Figure 6: Effects of raising minimum capital adequacy ratio on aggregate economic activities in an economy where financial market has developed

## 6 Discussion: Interpretation of Results

The effects of inefficiency caused by indirect finance  $\Theta$  are larger than those by direct finance  $\chi$  in the long run. As a result, this minimum capital requirements should be determined based on the degree of financial development (efficiency). However, in the short run, an increase in the minimum capital ration makes the drop in aggregate output in a recession (in response to a capital quality shock) smaller. Its policies (tighter requirements) slow down the recovery process in the U.S. (a country with a developed financial market).

Tighter regulation suppress banks' activities when financial markets are developed ( $\Theta$  is low), which harms households wanting to deposit more assets. The households manage more capital on the market (direct financing), taking account of balance sheet (15). This causes resource inefficiency due to  $\chi$ , and decreases their consumption in the steady state (in the long run). Further, a negative shock to capital quality results in a moderate response of consumption and output. These effects imply that both considering the significance of balance sheet condition and the difference sources of inefficiency resource allocation: direct and indirect financing is important for setting of minimum capital requirements during a financial crisis as well as in the long run.<sup>33</sup> In a country with an undeveloped financial market ( $\Theta$  is high), when households deposit more assets, the resource inefficiency caused by indirect financing decreases their consumption. What can policies (requirements) do? Raising minimum capital requirements can mitigate the resource inefficiency caused by indirect financing, suppressing the banks' activities.<sup>34</sup>

---

<sup>33</sup>This study insists that both the significance of balance sheet condition and the importance of an imperfect financial market is important for considering a policy for a financial crisis in a large literature based on GK (2010). This study's model incorporates the difference sources of inefficiency resource allocation: direct and indirect financing.

<sup>34</sup>The other research subjects (policies and requirements) include: (capital) taxes and policies regarding the activities of banks and depositors that might resolve the problems caused by such inefficiency. In this study, we focus on the minimum capital requirements which is one of international standards regardless of the difference in financial development.

## 7 Concluding Remarks

This study aims to determine the optimal minimum capital requirements for an economy based on its financial market development. We construct a macroeconomic model in which both indirect and direct finance cause inefficiency due to moral hazard problems between financial intermediaries and households. We analytically and numerically show that raising minimum capital requirements improves resource allocation and welfare in an economy where financial market has not developed, whereas it reduces welfare in an economy where financial market has developed. Our model shows that raising minimum capital requirements in an economy with undeveloped financial markets increases resource allocation and welfare. By contrast, it reduces welfare in an economy with developed financial markets. Our numerical analyses allow us to calculate each economy's optimal minimum capital requirements based on how much its financial market has developed. We also show that raising the minimum capital requirements can mitigate the crisis; however, there is a choice of its purposes between the high speed of recovery from a financial crisis and a small magnitude of a drop in aggregate variables when the crisis occurs, through inefficiency resource allocation caused both by indirect and direct financing. Our findings imply that, when setting the minimum capital requirements, regulators should consider how the financial market of each economy has developed.

For future research, there are some directions. First, this model may be extended to models of an open economy to analyze the capital requirement, considering the international financial market. Second is the time-varying capital requirements. This will contribute to a large body of literature based on GK (2010).<sup>35</sup> Third, the dynamic equilibrium model in which endogenous fluctuations in the asset market prices can be extended, as in Kiyotaki and Moore (1997), especially focusing on more complex contracts between banks and depositors and on the assumption of rational expectation. Finally, the results of this model suggest that we should adopt different measures of financial development based on their intended purposes, such as minimum capital requirements. The implications of this finding could contribute to a large body of empirical research and calibration regarding financial development, economic growth, and economic crises.

---

<sup>35</sup>GKP (2020 (b), 2020 (c)) recently conducted analyses on the time-varying capital requirements.

# Appendix

## A Derivations of Equations (23), (24), and (25)

Substituting Equation (21) into  $V_t$  on the left-hand side of Equation (19) and  $V_{t+1}$  on the right-hand side of Equation (19), we obtain

$$\mu_t k_t^b + v_t n_t = E_t \left[ \Lambda_{t,t+1} (1 - \sigma) n_{t+1} + \Lambda_{t,t+1} \sigma \mu_{t+1} k_{t+1}^b + \Lambda_{t,t+1} \sigma v_{t+1} n_{t+1} \right],$$

where  $\mu_t = \iota_t - v_t$ . Substituting the definition of  $\kappa_t$  into the above equation yields:

$$\mu_t k_t^b + v_t n_t = E_t \left[ \Lambda_{t,t+1} (1 - \sigma) n_{t+1} + \Lambda_{t,t+1} \sigma \mu_{t+1} \frac{n_{t+1}}{\kappa_{t+1}} + \Lambda_{t,t+1} \sigma v_{t+1} n_{t+1} \right], \quad (\text{A1})$$

with

$$\kappa_{t+1} = \max(\bar{\kappa}, \tilde{\kappa}_{t+1}). \quad (22')$$

Let  $\Omega_{t+1}$  be defined as

$$\Omega_{t+1} \equiv (1 - \sigma) + \frac{\sigma \mu_{t+1}}{\kappa_{t+1}} + \sigma v_{t+1}. \quad (25)$$

Then, (A1) can be rewritten as follows:

$$\mu_t k_t^b + v_t n_t = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} n_{t+1} \right], \quad (\text{A1}')$$

with (25) and (22').

Combining (15) with (16) for  $i = t$  and  $j = t + 1$ , we obtain

$$n_{t+1} = R_{t+1}^k k_t^b - R_{t+1} (k_t^b - n_t) = (R_{t+1}^k - R_{t+1}) k_t^b + R_{t+1} n_t. \quad (\text{A2})$$

Substituting (A2) into (A1'), we obtain

$$\mu_t k_t^b + v_t n_t = E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} \left[ \left( R_{t+1}^k - R_{t+1} \right) k_t^b + R_{t+1} n_t \right] \right\}, \quad (\text{A1}'')$$

where

$$\Omega_{t+1} \equiv (1 - \sigma) + \frac{\sigma \mu_{t+1}}{\kappa_{t+1}} + \sigma v_{t+1}, \quad (24)$$

with

$$\kappa_{t+1} = \max (\bar{\kappa}, \tilde{\kappa}_{t+1}). \quad (22')$$

Because the coefficients on the left-hand side of Equation (A1'') are equivalent to the coefficients on the right-hand side of (A1''), if the guess is correct, we obtain the following equations:

$$\mu_t = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \left( R_{t+1}^k - R_{t+1} \right) \right], \quad (23)$$

$$v_t = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right], \quad (24)$$

where

$$\Omega_{t+1} \equiv (1 - \sigma) + \frac{\sigma \mu_{t+1}}{\kappa_{t+1}} + \sigma v_{t+1}, \quad (25)$$

and

$$\kappa_{t+1} = \max (\tilde{\kappa}_{t+1}, \bar{\kappa}). \quad (22')$$

## B Derivation of Equation (32)

Using (26), Equation (28) can be rewritten as follows:

$$N_t = \sigma \sigma f \left( R_t^k k_{t-1}^b - R_t d_{t-1} \right) + (1 - \sigma) \sigma f R_t^k \zeta K_{t-1} + (1 - \sigma) f \zeta K_t \dots \quad (\text{B1})$$

Substituting (16) into (B1), we obtain

$$N_t = \sigma \sigma f n_t + (1 - \sigma) \sigma f R_t^k \zeta K_{t-1} + (1 - \sigma) f \zeta K_t. \quad (\text{B1}')$$

Combining Equation (31) with Equations (26) and (27) yields

$$N_t = K_t^b - D_t = \sigma f k_t^b + (1 - \sigma) f \zeta K_t - \sigma f d_t = \sigma f (k_t^b - d_t) + (1 - \sigma) f \zeta K_t.$$

Substituting (15) into the above equation, we obtain

$$N_t = \sigma f n_t + (1 - \sigma) f \zeta K_t. \quad (\text{B2})$$

Substituting Equation (B2) into the left-hand side of Equation (B1'), we obtain

$$n_t = R_t^k \zeta K_{t-1}. \quad (\text{B3})$$

By dividing both sides of Equation (B3) by  $k_t^b$  and using the definition of  $\kappa_t$ , we obtain

$$\kappa_t \frac{k_t^b}{K_t} = R_t^k \zeta \frac{K_{t-1}}{K_t}, \quad (\text{B3}')$$

with  $\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa})$  (22'). Equation (26) and the definition of  $\eta_t$  yields

$$\frac{k_t^b}{K_t} = \frac{\eta_t - (1 - \sigma) f \zeta}{\sigma f}. \quad (\text{B4})$$

Combining Equation (B3') with Equation (B4) yields

$$\kappa_t = \frac{\sigma f \zeta R_t^k \frac{K_{t-1}}{K_t}}{[\eta_t - (1 - \sigma) \zeta f]}, \quad (\text{B5})$$

with  $\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa})$  (22'). Taking one lag of Equation (12) with Equation (6) and the definition of  $\eta_t$  yields

$$R_t^k = \frac{1 + f' \left( K_{t-1}^h; K_{t-1} \right)}{\Lambda_{t-1,t}} = \frac{1 + \chi (1 - \eta_{t-1})}{\Lambda_{t-1,t}}, \quad (\text{B6})$$

where  $\Lambda_{t-1,t} = \frac{\beta \frac{C_{t-1}}{\bar{K}_{t-1}} K_{t-1}}{\frac{C_t}{K_t}} = \frac{\beta x_{t-1}}{x_t} \frac{K_{t-1}}{K_t}$ , where  $\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa})$ , (22'). Combining (B5) with (B6), we obtain:

$$\kappa_t = \frac{\sigma f \zeta [1 + \chi (1 - \eta_{t-1})]}{\frac{\beta x_{t-1}}{x_t} [\eta_t - (1 - \sigma) \zeta f]}, \quad (\text{32})$$

with  $\kappa_t = \max(\tilde{\kappa}_t, \bar{\kappa})$  (22').

## C Derivation of Equation (33)

Combining the Bellman equation (19) and the binding incentive compatibility constraint (21), we obtain

$$\Theta k_t^b = E_t \left[ \Lambda_{t,t+1} (1 - \sigma) n_{t+1} + \Lambda_{t,t+1} \sigma \Theta k_{t+1}^b \right].$$

Taking one lag of the above equation and dividing both sides of it by  $k_t^b$  yields

$$\Theta \frac{k_{t-1}^b}{k_t^b} = \Lambda_{t-1,t} (1 - \sigma) \frac{n_t}{k_t^b} + \Lambda_{t-1,t} \sigma \Theta.$$

Using (B4) and the definitions of  $\tilde{\kappa}_t$ ,  $x_t$ , and  $\Lambda_{t-1,t}$ , the above equation can be rewritten as follows:

$$\Theta \left[ \frac{\eta_{t-1} - (1 - \sigma) f \zeta}{\eta_t - (1 - \sigma) f \zeta} \right] = \beta \frac{x_{t-1}}{x_t} [(1 - \sigma) \kappa_t + \sigma \Theta]. \quad (\text{33})$$

## D Derivation of Equation (34)

Combining Equations (1) and (2) with  $Z_t = R_t^k - 1 + \delta$  and  $L = 1 - f$  yields

$$R_t^k = (1 - \delta) + \frac{\alpha Y_t}{K_t} = (1 - \delta) + \alpha \left( \frac{1 - f}{K_t} \right)^{1-\alpha}. \quad (\text{D1})$$

From Equations (12) and (6), we obtain

$$R_t^k = \frac{1 + \chi(1 - \eta_{t-1})}{\Lambda_{t-1,t}}, \quad (12')$$

where  $\Lambda_{t-1,t} = \frac{\beta x_{t-1} K_{t-1}}{x_t K_t}$ . Plugging Equation (12') into Equation (D1), we obtain

$$\frac{1-f}{K_t} = \left[ \frac{\frac{1+\chi(1-\eta_{t-1})}{\Lambda_{t-1,t}} - (1-\delta)}{\alpha} \right]^{\frac{1}{1-\alpha}}. \quad (D2)$$

(D2) can be rewritten as follows:

$$K_t = (1-f) \left[ \frac{\alpha}{\frac{1+\chi(1-\eta_{t-1})}{\Lambda_{t-1,t}} - (1-\delta)} \right]^{\frac{1}{1-\alpha}}, \quad (34)$$

where  $\Lambda_{t-1,t} = \frac{\beta x_{t-1} K_{t-1}}{x_t K_t}$ .

## E Derivation of Equation (35)

Dividing the goods market clearing condition (29) by  $K_t$ , we obtain

$$\begin{aligned} \frac{Y_t}{K_t} &= \frac{C_t}{K_t} + \frac{I_t}{K_t} + \frac{f^k(K_t^h; K_t)}{K_t} + \frac{f^d(D_t)}{K_t} \\ &= x_t + \frac{I_t}{K_t} + \frac{\chi}{2} (1 - \eta_t)^2 + \theta \frac{D_t}{K_t}. \end{aligned} \quad (29')$$

The last equality is given by the definitions of  $x_t$  and  $\eta_t$  as well as Equations (6) and (7). Dividing both sides of (31) with the definition of  $\eta_t$  yields

$$\frac{D_t}{K_t} = \frac{N_t}{K_t} + \eta_t. \quad (E1)$$



Combining (B2) and (B3), we obtain

$$\frac{N_t}{K_t} = \sigma f R_t^k \zeta \frac{K_{t-1}}{K_t} + (1 - \sigma) f \zeta.$$

Using (B6) and the definition of  $x_t$ , we can rewrite the above equation as follows:

$$\frac{N_t}{K_t} = \sigma f \zeta \left[ \frac{1 + \chi (1 - \eta_{t-1})}{\beta^{\frac{x_{t-1}}{x_t}}} \right] + (1 - \sigma) f \zeta. \quad (\text{E2})$$

Using (E2), Equation (E1) can be rewritten as

$$\frac{D_t}{K_t} = \sigma f \zeta \left[ \frac{1 + \chi (1 - \eta_{t-1})}{\beta^{\frac{x_{t-1}}{x_t}}} \right] + (1 - \sigma) f \zeta + \eta_t. \quad (\text{E1}')$$

Substituting the law of motion for capital (4) and (E1') into Equation (29'), we obtain

$$\frac{Y_t}{K_t} = x_t + \frac{K_{t+1}}{K_t} - (1 - \delta) + \frac{\chi}{2} (1 - \eta_t)^2 + \theta \sigma f \zeta \left[ \frac{1 + \chi (1 - \eta_{t-1})}{\beta^{\frac{x_{t-1}}{x_t}}} \right] + \theta (1 - \sigma) f \zeta + \theta \eta_t. \quad (\text{E3})$$

Equation (1) yields

$$\frac{Y_t}{K_t} = A \left( \frac{L}{K_t} \right)^{1-\alpha}. \quad (1')$$

Substituting Equation (1') into the left-hand side of (E3), we obtain

$$x_t = -\frac{K_{t+1}}{K_t} + (1 - \delta) + A \left( \frac{1 - f}{K_t} \right)^{1-\alpha} - \frac{\chi}{2} (1 - \eta_t)^2 - \theta \sigma f \zeta \left[ \frac{1 + \chi (1 - \eta_{t-1})}{\beta^{\frac{x_{t-1}}{x_t}}} \right] - \theta (1 - \sigma) f \zeta - \theta \eta_t. \quad (\text{E4})$$

Note that  $L = 1 - f$ . Plugging (D2) into (E4) yields

$$x_t = \frac{A[1 + \chi(1 - \eta_{t-1})]}{\alpha \Lambda_{t-1,t}} - \frac{K_{t+1}}{K_t} - \frac{(1 - \delta)(A - \alpha)}{\alpha} - \frac{\chi}{2} (1 - \eta_t)^2 - \theta \sigma f \zeta \left[ \frac{1 + \chi (1 - \eta_{t-1})}{\beta^{\frac{x_{t-1}}{x_t}}} \right] - \theta (1 - \sigma) f \zeta - \theta \eta_t. \quad (35)$$

## F Derivation of Equation (36)

Substituting (10) into Equation (9) yields

$$C_t + D_t + K_t^h + f^k \left( K_t^h; K_t \right) + f^d (D_t) = w_t L + (1 - \sigma) \left( R_t^k K_{t-1}^b - R_t D_{t-1} \right) - (1 - \sigma) f \zeta K_t + R_t^k K_{t-1}^h + R_t D_{t-1}. \quad (9')$$

Substituting (28) and (30) into (9'), we obtain:

$$C_t + K_t + f^k \left( K_t^h; K_t \right) + f^d (D_t) = w_t L + R_t^k K_{t-1}^b + R_t^k K_{t-1}^h. \quad (F1)$$

Combining (29) and (5) with (F1) yield

$$K_t + Y_t - I_t = w_t L + R_t^k K_{t-1}. \quad (F2)$$

From (2) and (3),

$$w_t L = Y_t - Z_t K_t = Y_t - \left( R_t^k - 1 + \delta \right) K_t. \quad (F3)$$

Substituting (F3) and (4) into (F2), we obtain

$$K_{t+1} - K_t = R_t^k (K_t - K_{t-1}). \quad (F4)$$

Substituting (D1) into (F4), we obtain

$$K_{t+1} - K_t = \left[ (1 - \delta) + \alpha \left( \frac{1 - f}{K_t} \right)^{1 - \alpha} \right] (K_t - K_{t-1}). \quad (36)$$

## G The sign of $\left[\frac{dY^*}{dK^*} - \delta - \frac{\chi}{2}(1 - \eta^*)^2\right]$ in (43)

Now, we examine the sign of  $\left[\frac{dY^*}{dK^*} - \delta - \frac{\chi}{2}(1 - \eta^*)^2\right]$  in (43). From Equations (2) and (12), and  $Z^* = (R^{k^*} - 1) + \delta$ , we obtain

$$\frac{dY^*}{dK^*} - \delta - \frac{\chi}{2}(1 - \eta^*)^2 = Z^* - \delta - \frac{\chi}{2}(1 - \eta^*)^2 = (R^{k^*} - 1) - \frac{\chi}{2}(1 - \eta^*)^2 = \left[\frac{1 + \chi(1 - \eta^*) - \beta}{\beta}\right] - \frac{\chi}{2}(1 - \eta^*)^2. \quad (\text{G1})$$

Let  $\Gamma(\eta^*)$  on the right-hand side of Equation (G1). Taking the derivative of  $\Gamma(\eta^*)$ , we obtain

$$\frac{d\Gamma(\eta^*)}{d\eta^*} = -\frac{\chi [1 - \beta(1 - \eta^*)]}{\beta} < 0,$$

where  $\beta < 1$  and  $1 - \eta^* \leq 1$ . By computing  $\Gamma(1)$ , we obtain:

$$\Gamma(1) = \left[\frac{1 - \beta}{\beta}\right] > 0. \quad (\text{G2})$$

Because  $\Gamma(\eta^*)$  is a decreasing function of  $\eta^*$  and its maximum value is positive from (G2) and  $\eta^* \in [0, 1]$ , we obtain  $\Gamma(\eta^*) > 0$  for all  $\eta^* \in [0, 1]$ , which implies that  $\left[\frac{dY^*}{dK^*} - \delta - \frac{\chi}{2}(1 - \eta^*)^2\right] > 0$  for all  $\eta^* \in [0, 1]$ .

## References

- [1] Aoki, K., Benigno, G., Kiyotaki, N. 2018. Monetary and financial policies in emerging markets. *Mimeo, Princeton University*.
- [2] Bernanke, B., Gertler, M. 1989. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79 (1), 14-31.
- [3] Čihák, M., Demirgüç-Kunt, A., Feyen, E., Levine, R. 2013. Financial development in 205 economies, 1960 to 2010, *NBER Working Paper*, No. 18946, National Bureau of Economic Research, Cambridge, Massachusetts.

- [4] Gertler, M., Karadi, P. 2011. Model of unconventional monetary policy. *Journal of Monetary Economics*, 58 (1), 17–34.
- [5] Gertler, M., Kiyotaki, N. 2010. Financial intermediation and credit policies in business cycle analysis. In *Handbook of Monetary Economics*, 3A, 547–599. Elsevier.
- [6] Gertler, M., Kiyotaki, N. 2015. Banking liquidity and bank runs in an infinite horizon economy. *American Economic Review*, 105 (7), 2011–2043.
- [7] Gertler, M., Kiyotaki, N., Prestipino, A., 2016. Wholesale banking and bank runs in the macroeconomic modeling of financial crises. In *Handbook of Macroeconomics*, 2, 1345–1425. Elsevier.
- [8] Gertler, M., Kiyotaki, N., Prestipino, A. 2020a. Macroeconomic model with financial panics. *Review of Economic Studies*, 87(1), 240–288.
- [9] Gertler, M., Kiyotaki, N., Prestipino, A., 2020b. Credit booms, financial crises, and macroprudential policies. *Review of Economic Dynamics*, 37, S8–S33.
- [10] Gertler, M., Kiyotaki, N., Prestipino, A. 2020c. Banking panics as endogenous disasters and welfare gains from macroprudential policy. *American Economic Review Papers and Proceedings*, 110, 463–469.
- [11] Gertler, M., Kiyotaki, N. & Queralto, A. 2012. Financial crises, bank risk exposure, and government financial policy. *Journal of Monetary Economics* 59, S17–S34.
- [12] Holmström, B. & Tirole, J. 1997. Financial intermediation, loanable funds, and real sector. *The Quarterly Journal of Economics*, 112 (3), 663–691.
- [13] Kiyotaki, N. & Moore, J. 1997. Credit cycles. *Journal of Political Economy*, 105 (2), 211–248.
- [14] La Porta, R., Lopez-De-Silanes, F., Shleifer, A., Vishny, R. W. (1997). Legal determinants of external finance. *Journal of Finance*, 52 (3), 1131–1150.

- [15] Townsend, R.M. 1979. Optimal contracts and competitive markets with cost state verification. *Journal of Economic Theory*, 21 (2), 265–293.
- [16] Turnovsky, S. J. 1996. Fiscal policy, growth, and macroeconomic performance in a small open economy. *Journal of International Economics*, 40, 41–66.
- [17] Uzawa, H. 1969. Time preference and the penrose effect in a two-class model of economic growth. *Journal of Political Economy*, 77 (4), 628–652.
- [18] Van den Heuvel, S. J. 2008. The welfare cost of bank capital requirements. *Journal of Monetary Economics*, 55, 298–320.