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# Estimating the Slope of the Demand Function at Auctions for Government of Canada Bonds

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## Abstract

We use detailed data on the bids at auctions for Government of Canada bonds between 1999 and 2021 to gauge the yield sensitivity of these bonds to the issuance amount. We propose a new metric that captures the slope of the demand function at each auction by using the information in the multiple bids (quantity and yield) that each bidder submits. In the absence of an established theoretical framework, we estimate the slope of the aggregate demand function simply by weighing the slopes of the individual demand functions, where the weights are the maximum bids of each participant. We show that these slopes can provide insights into the relationship between the supply and yield of a government debt security.

*Topics: Debt management; Interest rates*

*JEL codes: G12, D44*

## Résumé

Nous utilisons des données détaillées sur les soumissions aux adjudications d'obligations du gouvernement du Canada entre 1999 et 2021 pour mesurer la sensibilité du rendement de ces obligations au montant des émissions. Nous proposons un nouvel indicateur qui rend compte de la pente de la fonction de demande de chaque adjudication au moyen de l'information tirée des multiples soumissions (quantité et rendement) de chaque soumissionnaire. En l'absence d'un cadre théorique établi, nous estimons la pente de la fonction de demande globale simplement en pondérant les pentes des fonctions de demande individuelles à partir de la soumission maximale de chaque participant. Nous montrons que ces pentes peuvent aider à mieux comprendre la relation entre l'offre et le rendement d'un titre d'État.

*Sujets : Gestion de la dette ; Taux d'intérêt*

*Codes JEL : G12, D44*

## Overview

How much does the supply of a government bond impact its yield? For example, would a government have to pay much higher or lower yields if it starts issuing a significantly larger or smaller amount of bonds in a certain maturity sector than it previously did? To estimate the potential impact on the yield from such an increase (or decrease) in the issuance amount, we propose a new metric constructed from bidding data at Government of Canada (GoC) bond auctions.

The new metric provides an alternative to existing methods, most of which estimate the term premium in the yield curve and then link it to supply. Our method can help government debt managers assess the potential impact on debt-servicing costs from changes in sector-specific issuance amounts and can assist central banks in evaluating the impact on government bond yields from quantitative easing or tightening.

Most GoC debt auctions are multiple-price auctions. This means that each bidder submits multiple bids, each of which consists of a dollar amount and a yield to indicate how much yield the bidder is willing to pay for each incremental bid amount. We use this bidding structure to try and gauge the demand function of each bidder at each auction.

Combining the individual demand functions into an overall demand function for a multiple-price auction is challenging. To our knowledge, no established theoretical framework exists for solving this problem. In the absence of such a framework, we construct an indicator that captures the sensitivity of yields to purchase amounts simply by taking the average slope—weighted by the maximum bids of each participant—of the individual demand functions.

We apply our methodology to calculate the slopes of the demand functions for auctions of 2-, 5-, 10- and 30-year GoC bonds between 1999 and 2021. To gain insights into the relationship between the supply and yield of these bonds, we then use our slope estimates to test our hypothesis that the yield sensitivity is likely to increase with the level of supply (i.e., the yield is a convex function of supply). However, we do not find any evidence that the slope of the auction demand function increases with the amount of bonds issued, contrary to our hypothesis. In fact, some evidence points to a negative relationship between the slope and bond supply (i.e., a concave supply-yield function) in some regions of the demand function.

One explanation for this puzzling finding is that the number of investors for government debt securities from a high-quality sovereign issuer like Canada tends to increase as the issuance amount rises. In this case, greater supply leads to higher demand, increasing the ability of investors to absorb a larger amount of the security with a lower incremental yield (i.e., a lower slope of the auction demand function). Anecdotal evidence about a notable improvement in auction competitiveness for 2-year GoC bonds, which has grown substantially since 2008, supports this explanation.

The paper is organized as follows. We outline and summarize [the data used](#). We then detail our [methodology](#), and then provide the [estimation results](#). A discussion of the [application](#) follows our findings before we [conclude](#).

## Data

We use the Bank of Canada’s Communication, Auction and Reporting System to obtain data on all bids submitted for GoC bond auctions between 1999 and 2021. The data include information from the multiple bids, with quantities and yields, submitted by each auction participant. For multiple-price auctions of GoC bonds, bidders can submit both competitive and non-competitive bids. We use only competitive bids to construct our metric of yield sensitivity. **Table 1** reports the summary statistics of the auction data.<sup>1</sup>

We calculate two versions of our metric. The first version uses the raw dollar amounts, and the second uses dollar amounts normalized to gross domestic product (GDP).<sup>2</sup> The GDP-normalized version of our metric is more appropriate for comparing slope estimates over a long period (e.g., 20 years) to reflect the fact that \$1 billion today is likely to have a smaller economic impact than \$1 billion many years ago.

**Table 1: Summary statistics of Government of Canada auction bidding data**

	Maturity	Periods			
		1999–2008Q2	2008Q3–2009 (global financial crisis)	2010–2020Q1	2020Q2–2021 (COVID-19 crisis)
Number of auctions each quarter	2-year	1	2	3	5
	5-year	1	2	2	4
	10-year	1	1	1	4
	30-year	1	1	1	3
Share of allotment to competitive bids	2-year	88%	93%	84%	87%
	5-year	83%	93%	83%	86%
	10-year	84%	90%	83%	87%
	30-year	83%	90%	81%	86%
Number of bidders	2-year	15	13	13	12
	5-year	15	13	13	12
	10-year	15	13	13	13
	30-year	15	13	13	13
Number of bidders with more than 1 bid	2-year	13	12	12	12
	5-year	14	12	13	12
	10-year	14	12	13	13
	30-year	13	12	12	13
Number of bids per bidder with more than 1 bid	2-year	5	5	5	5
	5-year	5	5	5	6
	10-year	5	6	6	6
	30-year	5	6	6	7

Source: Bank of Canada  
Last observation: 2021Q4

<sup>1</sup> See Hortaçsu and Kastl (2012) for a detailed description of auctions for GoC debt securities.

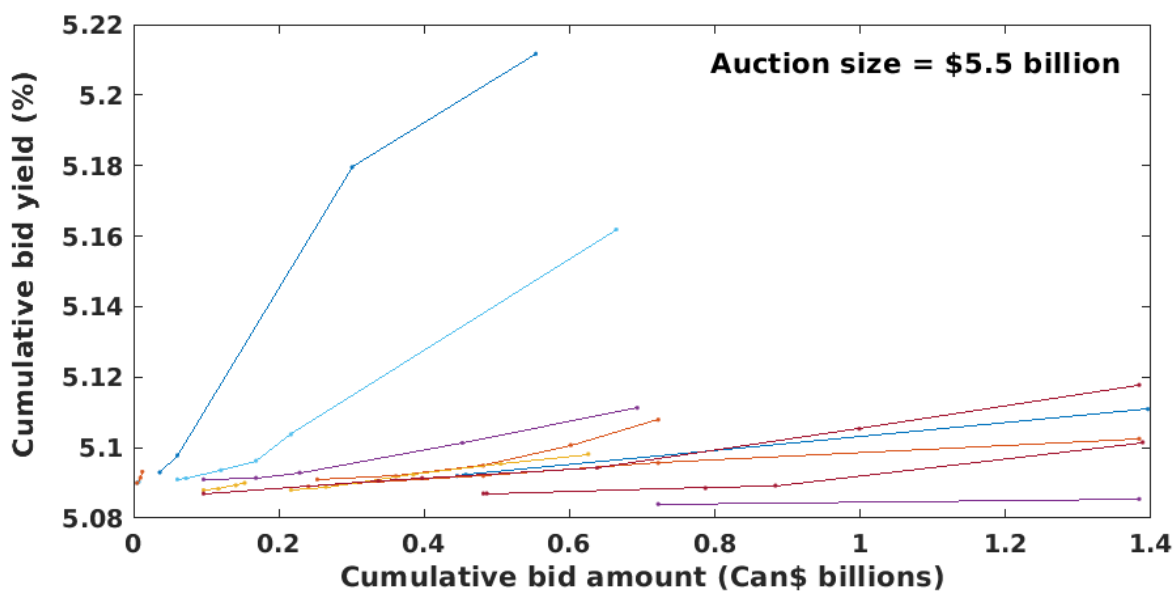
<sup>2</sup> Data on Canada’s nominal GDP come from Statistics Canada Table No. 36-10-0104-01.

## Methodology

Estimating each bidder's demand function (what yield is required at what quantity) at each auction is relatively easy since each bidder provides multiple pairs of quantities and yields when bidding. However, estimating an aggregate demand function for a multiple-price auction is much more challenging than for uniform-price auctions.<sup>3</sup> To our knowledge, no established theoretical framework exists for calculating such an aggregate demand function.<sup>4</sup>

Without using any theoretical guidance, we start by constructing a demand function for each individual auction participant (**Chart 1**). We then weigh the slopes of the individual demand functions by the maximum bids of each participant to estimate the slope of the aggregate demand function. This aggregate slope helps us gauge the sensitivity of yields to every extra dollar of bids at each auction.

**Chart 1: Sample demand functions for each participant in a Government of Canada bond auction**



Note: Each demand function is constructed from the amount and yield pairs submitted by each bidder. Each coloured line represents one bidder.

Sources: Bank of Canada and Bank of Canada calculations

<sup>3</sup> In theory, the submitted bid yields in a multiple-price auction are upward-biased estimates of the true "willingness-to-pay" yields. The bias is caused by the bid-shaving needed to compensate for the winner's curse present in the multiple-price auctions. Another reason why the bids may be higher than the willingness-to-pay yields is because the GoC debt distribution framework imposes minimum bidding requirements (both price and quantity) on dealers. When these requirements bind, the dealers will potentially need to bid higher than their willingness-to-pay. Albuquerque, Costa and Faias (2022) use a similar dataset from auctions of Portuguese treasury bonds, but their data are based on uniform-price auctions for which estimating the aggregate demand function at each auction is relatively easy.

<sup>4</sup> See Allen, Kastl and Wittwer (2020) for a potential framework.

## Notation

- GoC sectors:  $i = 1$  (2-year bond), 2 (5-year bond), 3 (10-year bond), 4 (30-year bond)
- Auctions:  $j(i) = 1, 2, \dots, N_A(i)$
- Auction participants:  $k(i, j) = 1, 2, \dots, N_P(i, j)$
- Number of competitive bids by bidder in an auction:  $N_B(i, j, k)$
- Maximum amount of competitive bids by bidder in an auction:  $X_{max}(i, j, k)$
- Bid pair (bid amount, bid yield):  $[x(i, j, k, l), y(i, j, k, l)]$  :  
 $[x(i, j, k, 1), y(i, j, k, 1)], \dots, [x(i, j, k, N_B(i, j, k)), y(i, j, k, N_B(i, j, k))]$
- Slope of individual bidder's demand function at an auction:  $b(i, j, k)$
- Slope of the aggregate demand function in an auction:  $B(i, j)$

## Estimation steps

For each auction, we first calculate the slopes of the demand function of individual bidders. We then calculate the overall demand function by weighing the slopes of the individual demand functions by the maximum bid amount of each bidder.

**Step 1:** For each sector  $i$ , auction  $j$ , bidder  $k$  with more than one competitive bid, transform the competitive bid pairs of amount and yield  $(x, y)$  into  $(xx, yy)$ , where  $xx$  is the successive cumulative bid amount and  $yy$  is the cumulative yield weighted by the bid amount:

$$xx(i, j, k, l) = \sum_{L=1}^l x(i, j, k, L)$$

$$yy(i, j, k, l) = \sum_{L=1}^l y(i, j, k, L) * \left[ \frac{x(i, j, k, L)}{xx(i, j, k, L)} \right].$$

**Step 2:** Calculate the slope of the individual bidder's demand function at each auction,  $b(i, j, k)$ , by taking the slope coefficient,  $b$ , in the ordinary least squares regression:

$$yy(i, j, k, l) = a(i, j, k) + b(i, j, k) * xx(i, j, k, l) + \varepsilon(i, j, k, l).$$

**Step 3:** Finally, for each auction, estimate the aggregate slope,  $B(i, j)$ , by weighing the slope of each bidder by their maximum bid amount:

$$B(i, j) = \sum_{k=1}^{N_P(i, j)} b(i, j, k) * \left[ \frac{X_{max}(i, j, k)}{\sum_{K=1}^{N_P(i, j)} X_{max}(i, j, K)} \right].$$



**Step 4 (optional):** Calculate a second version of the slope for applications that require a comparison of estimates over a long period (e.g., 20 years). For such applications, normalizing the bid amounts is more appropriate to reflect the fact that \$1 billion today is likely to have a smaller economic impact than \$1 billion many years ago. To adjust for this effect, multiply all the bid amounts by the ratio of quarterly GDP over the GDP of the base year, 2019.

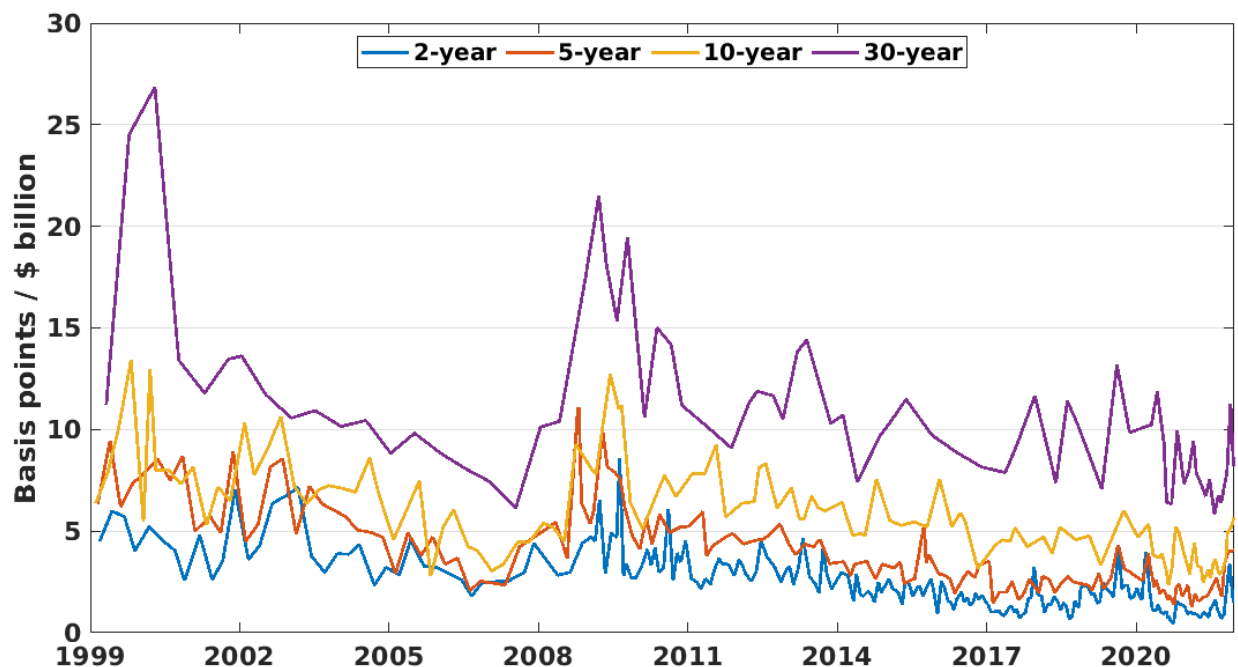
## Estimation result

Chart 2 and Chart 3 show the estimated slopes of the demand functions calculated for each auction of the four core sectors of GoC nominal bonds. In general, the magnitude of the slope is higher for longer-term bonds. This higher yield sensitivity to an additional \$1 billion bid is consistent with the fact that the level of interest rate risk borne by purchasing an extra \$1 billion of a long-maturity bond is greater than for the same amount of a short-maturity bond.

We find that the slopes across different GoC bond sectors are all positively correlated (Table 2), with higher correlations between similar maturity sectors. The correlations are 74% between the 2-year and 5-year sectors and 72% between the 10-year and 30-year sectors. The positive correlations indicate that some common, market-wide factors affect the sensitivity of yields to the supply of different GoC bonds. The results of the regression analysis reported in Table 4 show that the level of the short-term interest rate and the market volatility index can explain some of the common variations in our slope estimates across different sectors. Table 3 summarizes the average slopes by sector for the whole sample period and for subperiods.

### Chart 2: Slope of the demand function for Government of Canada bond auctions

Unadjusted bid amounts, quarterly average



Sources: Bank of Canada and Bank of Canada calculations

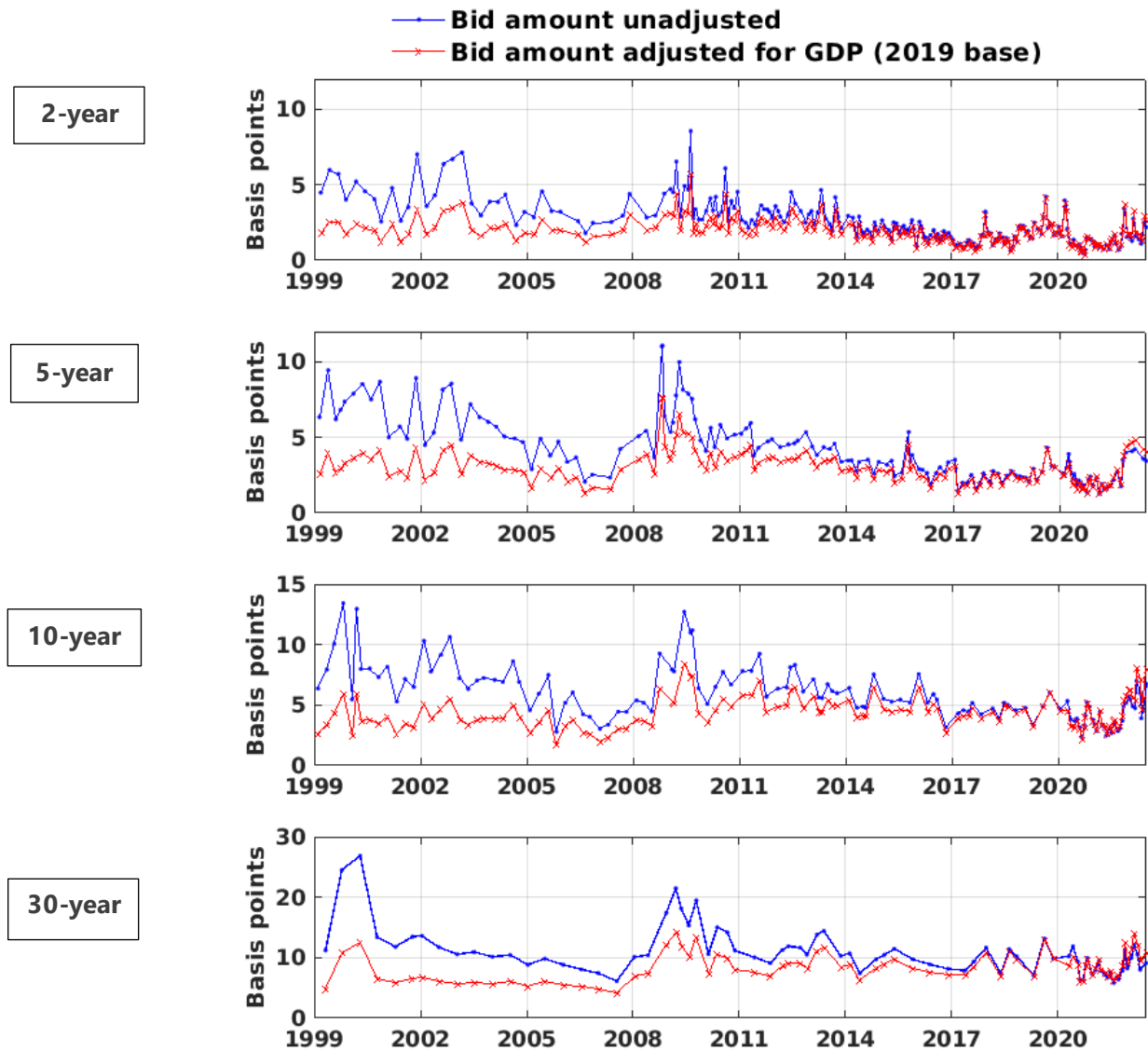
Last observation: 2021Q4

**Table 2: Correlation of the slopes of demand functions for auctions across sectors**

Government of Canada bond maturity	5-year	10-year	30-year
2-year	0.74	0.36	0.20
5-year		0.56	0.52
10-year			0.72

Sources: Bank of Canada and Bank of Canada calculations

**Chart 3: Slope of the auction demand function with and without adjusting for gross domestic product**



Sources: Statistics Canada and Bank of Canada

Last observation: 2021Q4

**Table 3: Average slope in subperiods**

Basis points per Can\$1 billion

	2-year		5-year		10-year		30-year	
	Unadj.	GDP-norm.	Unadj.	GDP-norm.	Unadj.	GDP-norm.	Unadj.	GDP-norm.
1999–2021 (whole sample)	2.5	1.9	4.1	3.0	5.9	4.3	10.8	8.3
1999–2008Q2	4.0	2.1	5.7	3.0	6.9	3.7	11.9	6.4
2008Q3–2009 (global financial crisis)	4.3	2.9	7.4	5.0	8.9	6.0	18.4	12.4
2010–2020Q1	2.3	1.9	3.4	2.9	5.8	4.8	10.6	8.9
2020Q2–2021 (COVID-19 crisis)	1.2	1.2	2.2	2.2	3.8	3.8	8.2	8.2

Note: Unadj. is unadjusted; norm. is normalized.

Sources: Bank of Canada and Bank of Canada calculations

Last observation: 2021Q4

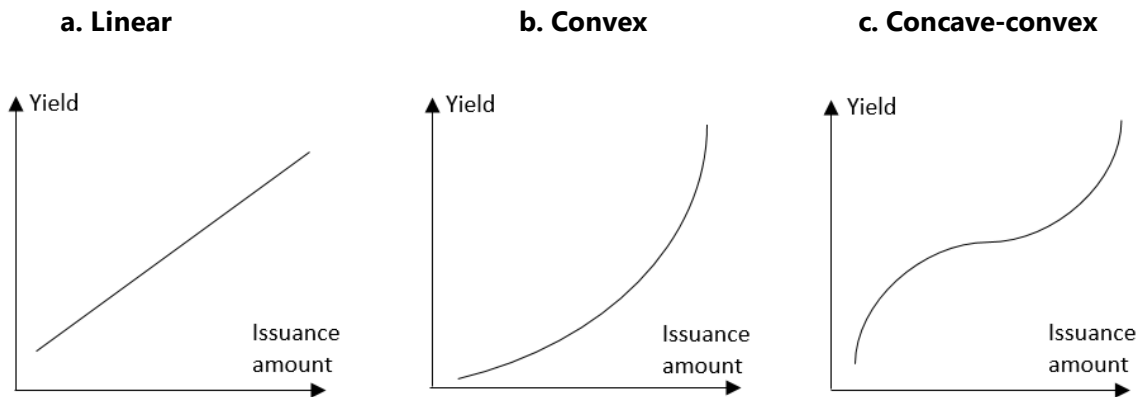
## Application

Our new metric could potentially shed some light on whether the yield of a GoC debt security becomes more sensitive to supply as the issuance amount increases. In other words, does the supply-yield function have a convex shape?

One way to answer this question is to examine the historical relationship between the issuance amount and the slope of the auction demand function. One limitation of relying on the historical data for this empirical exercise is that the range of issuance amounts could be too narrow to provide information on the shape of the demand function outside of the observed range. However, the substantial increase in issuance of the 2-year and 5-year GoC bonds since 2008 and the increase in 10-year and 30-year bond issuance following the COVID-19 crisis alleviate this limitation. We use the GDP-adjusted issuance amounts and slope to make the numbers more comparable over the 23-year time horizon of our sample.

Our hypothesis is that the supply-yield function for GoC bonds is likely to have a convex shape, implying that the sensitivity of yields increases with the size of the issuance. Our rationale for this hypothesis is that, generally, the greater balance sheet risk or inventory cost borne by a dealer to intermediate an additional dollar of a bond is likely to be much higher if the dealer is already purchasing a large amount of the same bond. Alternatively, the supply-yield function may exhibit a simple linear or a more complex concave and convex shape as shown in **Figure 1**. The concave-convex function implies that yield sensitivity decreases when supply goes from a low to a medium amount (perhaps because liquidity improves) but increases as supply becomes large. However, we limit our tests to the convexity of the supply-yield function. We leave a more comprehensive examination of the exact shape of this relationship for future research.

**Figure 1:** Possible shapes of the supply-yield function for Government of Canada bonds

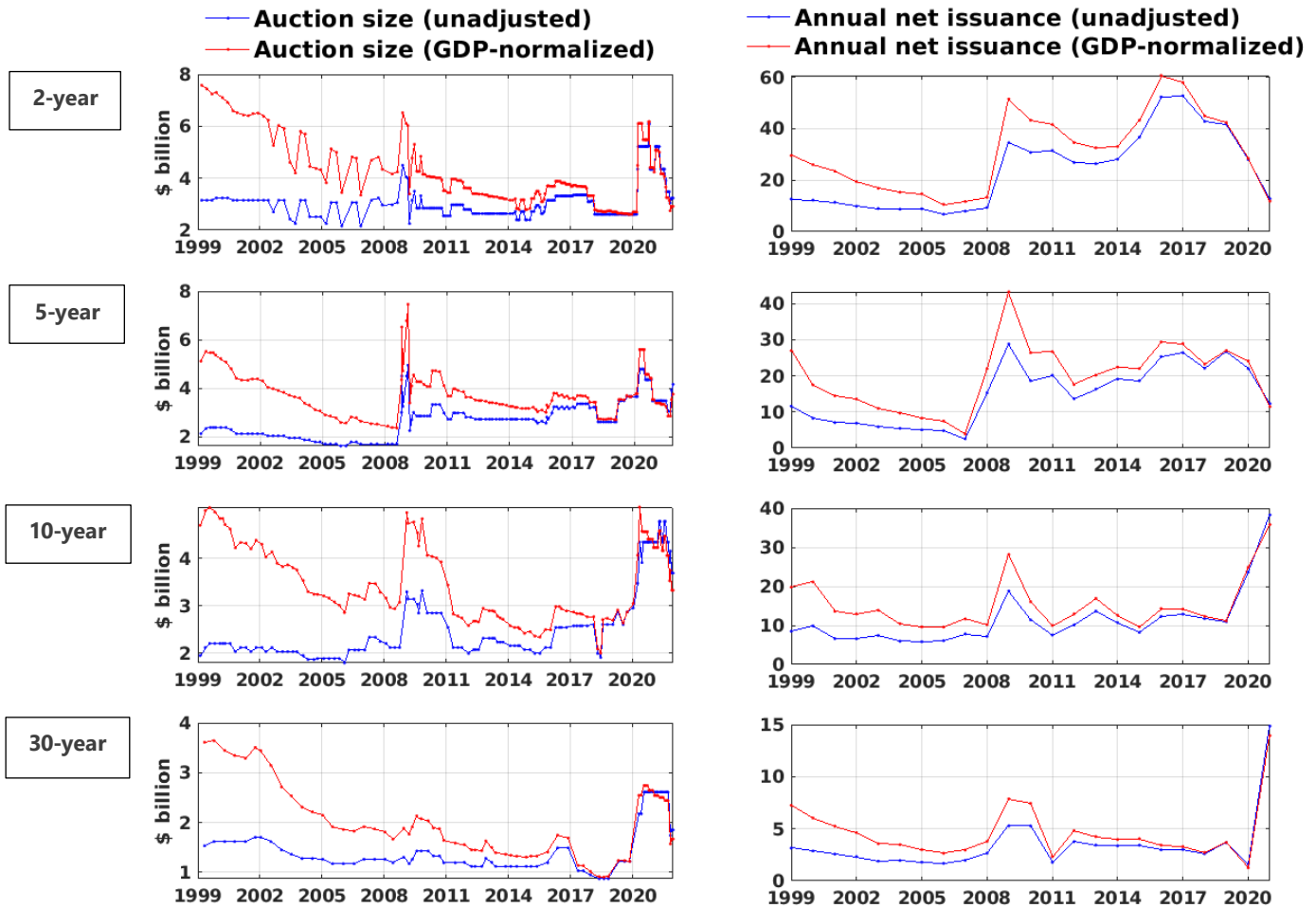


We plot the evolution of the issuance amount of GoC bonds between 1999 and 2021 in **Chart 4**. The right panel of **Chart 4** shows the annual net issuance. We observe a dramatic increase in the issuance of 2-year and 5-year GoC bonds since the global financial crisis. In contrast, the 10-year and 30-year sectors show no such increase after adjusting for GDP.

The left panel of **Chart 4** shows the auction size. When investigating the relationship between the slope of the auction demand function and the supply of GoC bonds, we consider not only the amount of total issuance in a quarter (or year) but also the size of each auction. The reason why the size of an auction could play a role is because the main participants in government bond auctions are dealers that act as market intermediaries. These dealers are often constrained by their capacity to manage their inventory of bonds. For example, for the same amount of a quarterly or annual issuance of a bond, a dealer likely faces more costs and risk if the bond is issued through one large auction instead of many smaller ones.

For all sectors except that of 5-year GoC bonds, we find that the auction size had been decreasing when adjusted for GDP until the COVID-19 crisis. For the 5-year sector, the auction size increased from less than \$2 billion before the 2008–09 global financial crisis to about \$3 billion in nominal terms after the crisis, leading to a slight increase after adjusting for GDP.

**Chart 4: Issuance amount of Government of Canada bonds**



Sources: Statistics Canada and Bank of Canada

Last observation: 2021Q4

We test our hypothesis of a convex supply-yield function by first plotting the slope of the auction demand function with the issuance amount in each GoC bond sector, then conducting a series of regression analyses.

The time series plots of each slope and issuance amount in **Chart 5** do not reveal a clear positive relationship between the slope and the supply of bonds. In fact, the relationship may be negative. For example, despite a significant increase in the issuance of 2-year bonds after the global financial crisis, the slope of the 2-year sector is much lower in the five years before the COVID-19 crisis. Also, in both the 10-year and 30-year sectors where the issuance amount has been relatively constant or decreasing, we observe a significant increase in the slope after the global financial crisis.

To further investigate the relationship between the slope and the issuance amount, we run a set of ordinary least squares regressions for each sector with the slope as the dependent variable and the two metrics of issuance amount as regressors: auction size and quarterly issuance amount. We also include two control

variables: the Canadian Overnight Repo Rate Average (CORRA) to control for the interest rate level, and the Merrill Lynch Option Volatility Estimate (MOVE) index to control for the overall risk in the fixed-income market:<sup>5</sup>

$$Slope_t = \alpha + \beta_{AUCT} * AuctionSize_t + \beta_{QTR} * QuarterlyIssuance_t + \overline{CONTROLS}_t * \bar{\gamma} + \varepsilon_t. \quad (1)$$

The regression results reported in **Table 4** are consistent with our observations from the plots of the slope and the issuance amount in **Chart 5**. We find a significant negative relationship between the slope and the auction size in all sectors.

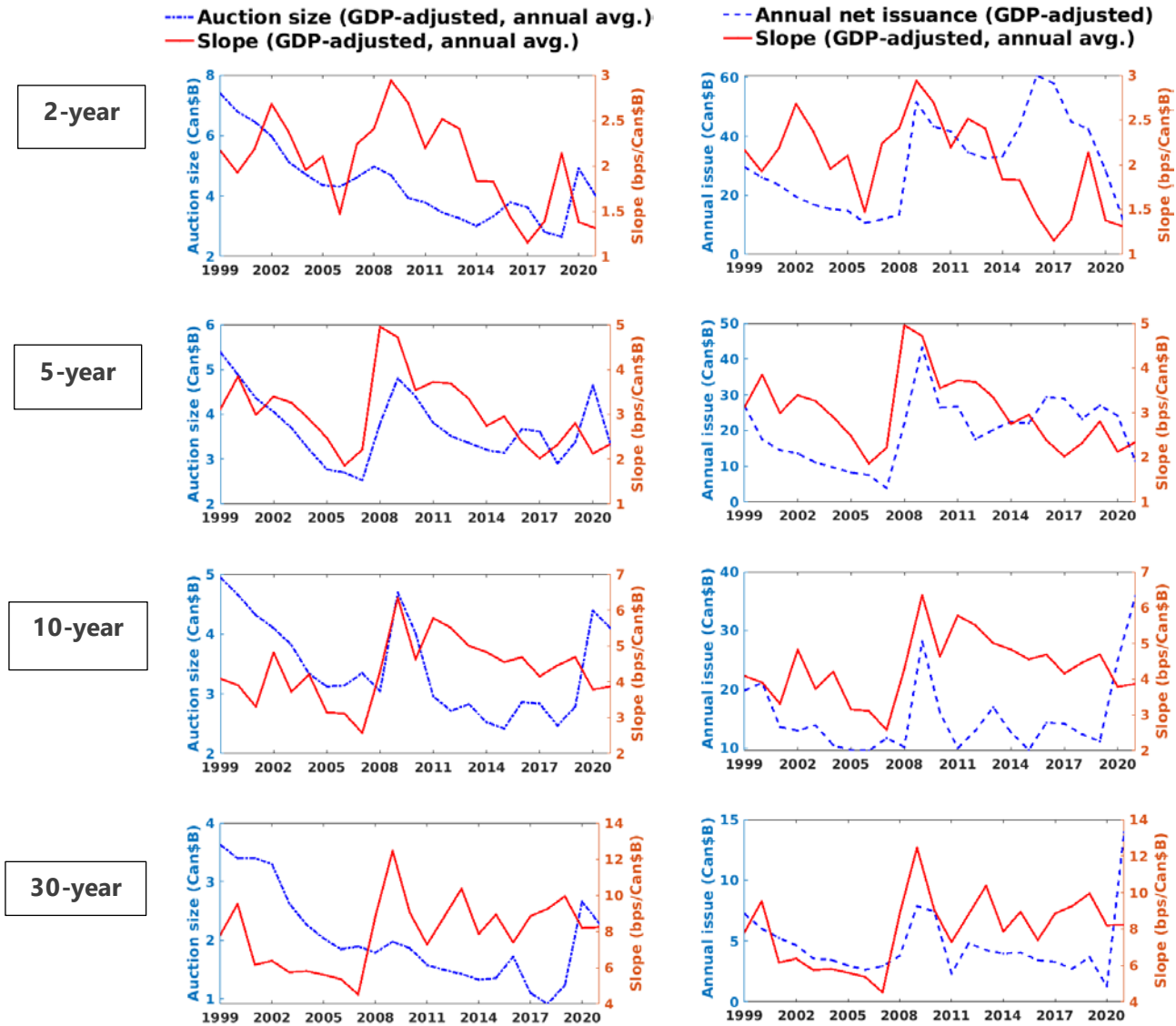
The result is weaker in the 5-year sector, with the negative coefficient on the auction size significant only when we control for the quarterly issuance. A close inspection of the regression results provides one possible explanation for this weaker relationship. We find that the two control variables capturing general market conditions (the short-term interest rate and fixed-income market volatility) explain a significantly larger proportion of the variation in the 5-year slope (60%) than in other sectors ( $R^2$  of 35–39%).

Overall, we do not find any evidence that the slope of the auction demand function increases with the issuance amount. In fact, some evidence suggests a negative relationship, contrary to our hypothesis.

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<sup>5</sup> We also use the Chicago Board Options Exchange VIX index following the related literature (Nagel 2016) but find that the MOVE index significantly improves the explanatory power of the regressions. The Canadian equivalent of the MOVE index published by the Bank of Canada (Chang and Feunou 2013), the three-month bankers' acceptance rate (BAX), is a better proxy for risk in the Canadian fixed-income market. However, we use the MOVE index since the BAX is available beginning only in 2006. The MOVE index and the BAX-implied volatility are highly correlated with a correlation of 79%.

**Chart 5: Slope of the auction demand function compared with the issuance amount**



Note: Dollar values are billions of Canadian dollars (Can\$B). Slope is measured in basis points (bps) per billion Canadian dollars. GDP is gross domestic product; avg. is average.

Sources: Bank of Canada and Bank of Canada calculations

Last observation: 2021Q4

**Table 4: Regression of slope on bond issuance amount**

**a. 2-year Government of Canada bonds**

	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic
Constant	1.69	(6.78)	0.44	(1.72)	0.39	(1.44)	1.12	(3.91)	0.26	(0.89)	1.29	(3.19)
CORRA	0.17	(1.31)			0.08	(0.86)	0.05	(0.88)	0.10	(1.21)	0.03	(0.61)
MOVE			0.02	(7.12)	0.02	(8.02)	0.02	(12.28)	0.02	(8.11)	0.02	(11.82)
Auction size							-0.23	(-4.53)			-0.26	(-3.90)
Quarterly issue									0.01	(0.71)	-0.01	(-0.57)
R <sup>2</sup>		0.03		0.34		0.35		0.42		0.36		0.42
Adjusted R <sup>2</sup>		0.03		0.34		0.35		<b>0.41</b>		0.35		0.41

**b. 5-year Government of Canada bonds**

	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic
Constant	2.87	(7.22)	0.87	(3.73)	0.96	(3.44)	1.20	(3.25)	0.88	(3.09)	1.43	(4.05)
CORRA	0.04	(0.26)			-0.20	(-2.17)	-0.23	(-2.26)	-0.14	(-1.43)	-0.19	(-2.03)
MOVE			0.03	(11.57)	0.03	(13.34)	0.03	(11.20)	0.03	(9.37)	0.03	(10.14)
Auction size							-0.07	(-0.94)			-0.19	(-2.25)
Quarterly issue									0.04	(1.32)	0.06	(2.03)
R <sup>2</sup>		0.00		0.57		0.60		0.60		0.61		0.62
Adjusted R <sup>2</sup>		-0.01		0.56		0.59		0.59		0.60		<b>0.60</b>

**c. 10-year Government of Canada bonds**

	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic
Constant	4.73	(13.18)	3.07	(8.05)	3.25	(7.85)	5.53	(9.40)	3.94	(8.94)	5.54	(9.43)
CORRA	-0.32	(-2.15)			-0.48	(-5.29)	-0.64	(-7.35)	-0.63	(-8.03)	-0.64	(-6.58)
MOVE			0.02	(3.19)	0.02	(6.37)	0.03	(6.85)	0.02	(7.03)	0.03	(6.85)
Auction size							-0.69	(-4.88)			-0.71	(-5.32)
Quarterly issue									-0.11	(-2.62)	0.01	(0.29)
R <sup>2</sup>		0.10		0.18		0.39		0.55		0.44		0.55
Adjusted R <sup>2</sup>		0.09		0.17		0.38		<b>0.54</b>		0.42		0.53

**d. 30-year Government of Canada bonds**

	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic	Coefficient	Newey-West t-statistic
Constant	9.34	(17.72)	6.69	(8.19)	7.20	(9.73)	10.11	(10.98)	7.86	(10.67)	10.29	(10.37)
CORRA	-0.86	(-3.29)			-1.03	(-4.84)	-1.15	(-9.43)	-1.10	(-5.28)	-1.19	(-9.58)
MOVE			0.02	(2.01)	0.03	(3.81)	0.03	(5.40)	0.03	(4.07)	0.03	(5.50)
Auction size							-1.52	(-4.16)			-1.42	(-3.96)
Quarterly issue									-0.32	(-2.10)	-0.18	(-2.17)
R <sup>2</sup>		0.21		0.09		0.38		0.52		0.42		0.53
Adjusted R <sup>2</sup>		0.20		0.08		0.36		0.50		0.39		<b>0.50</b>

Note: CORRA is the Canadian Overnight Repo Rate Average. MOVE is the Merrill Lynch Option Volatility Estimate. The sample period is between 2003 (when the MOVE index starts) and 2021. The Newey-West t-statistics (with 2-year lag) in red indicate significance at 90% confidence level. The adjusted R<sup>2</sup> in bold indicates the maximum among the six regressions.



One possible explanation for this puzzling finding is that the number of investors for some government debt securities expands with increases in issuance, improving the ability of the investors to absorb a larger amount of the security with a lower incremental yield (i.e., a lower slope of the auction demand function). Increasing the supply of a security issued by high-quality sovereign issuers like Canada is well known to have a positive effect on the size of the investor base and on liquidity.

We test this possibility by comparing measures of auction competitiveness between two GoC sectors that have experienced significantly different trends in issuance amount. As we mentioned earlier, the 2-year and 5-year GoC bond sectors have experienced a tremendous amount of growth since the global financial crisis, whereas the issuance in the 10-year and 30-year sectors has stayed constant or decreased slightly. We plot three metrics of auction competitiveness in **Chart 6**. The three metrics considered are the number of bidders, auction coverage (total bid amount divided by the issuance amount) and auction tail (highest yield minus average yield). In each panel, the auction competitiveness of the 2-year sector is compared with another sector.

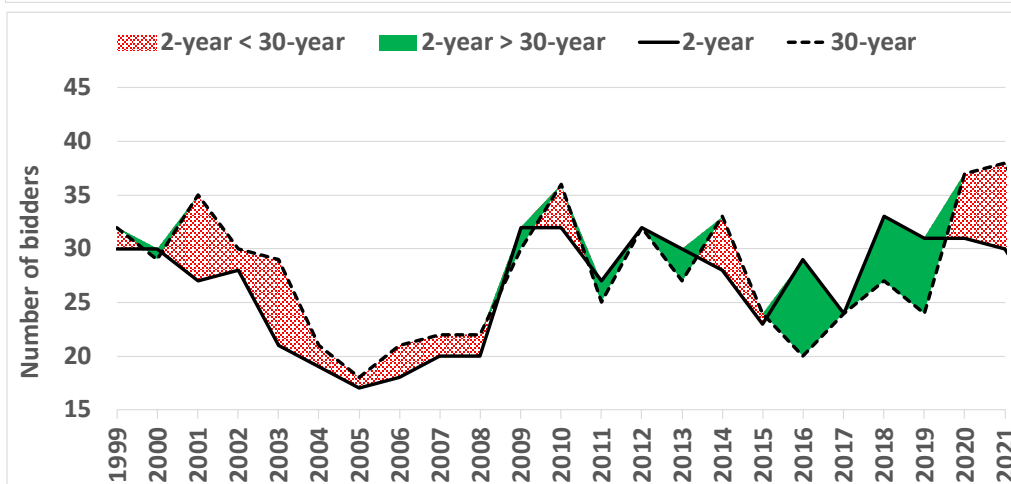
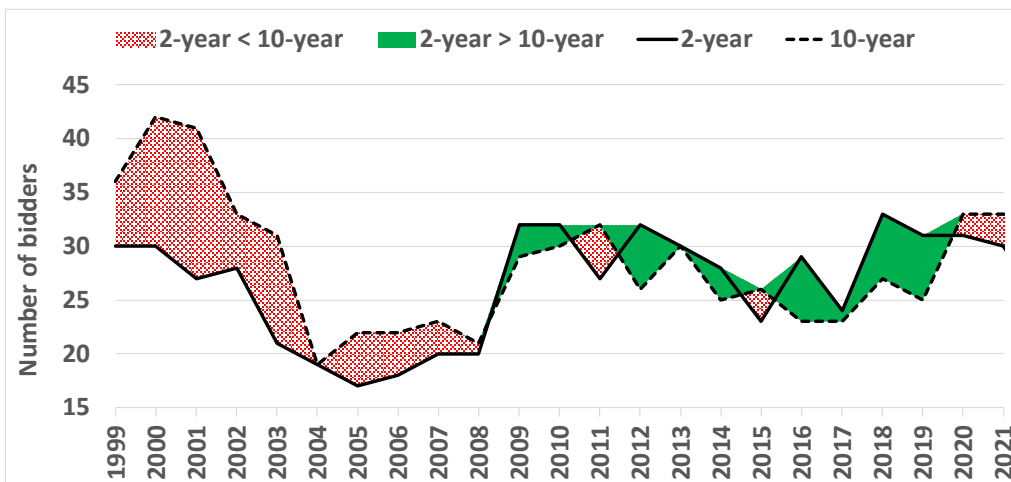
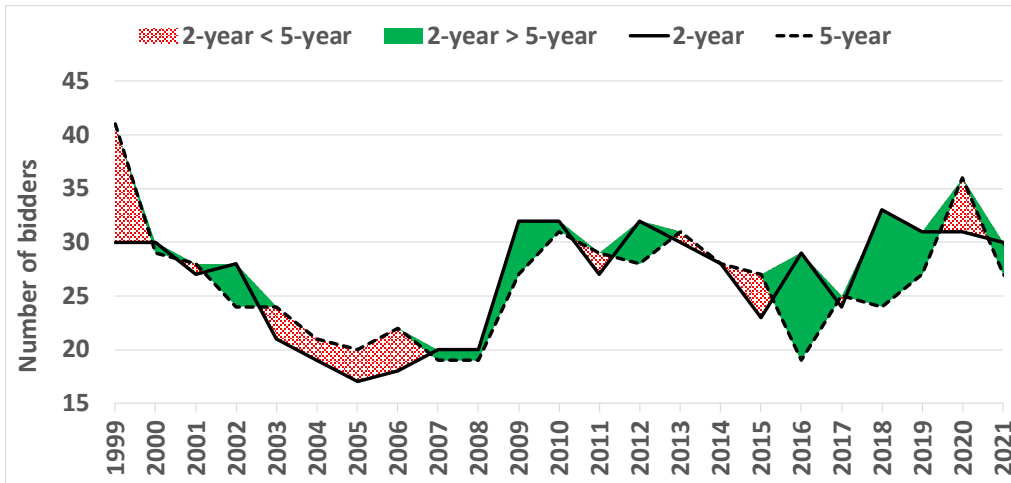
As indicated by the green areas between the two lines, we observe a significant improvement in the competitiveness of the 2-year bond auctions compared with other sectors. This observation is consistent with the explanation that a higher issuance of some government debt securities could expand the size of the investor base. An increase in the number of bidders makes auctions more competitive (higher coverage and lower tail) and potentially reduces the yield sensitivity of auction participants when bidding for extra amounts (i.e., a lower slope of the auction demand function). However, this dynamic does not explain why the negative relationship is between the slope and auction size and not between the slope and the quarterly issuance amount.

Overall, the results of our analysis point to a negative effect of the auction size on the slope of the auction demand function. Although we provide one possible explanation for this finding, we do not conduct a more extensive test of our hypothesis. The main insight from our analysis is that the supply-yield function of a government bond may have a more complex shape than can be captured by a simple convex function.

**Chart 6: Competitiveness of 2-year Government of Canada bond auctions compared with other sectors**

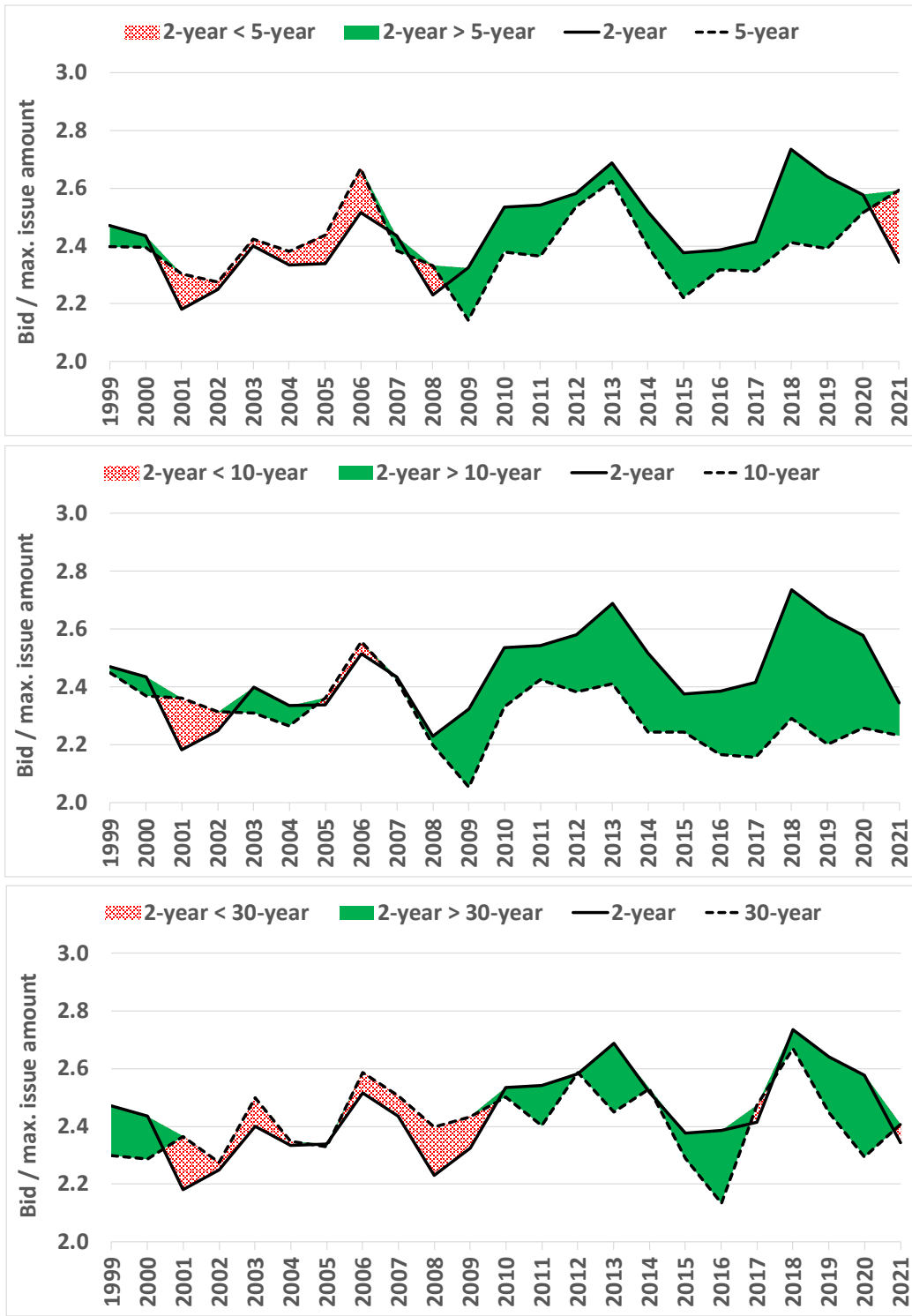
**a. Number of bidders at auctions**

Green/Red: 2-year sector is more/less competitive



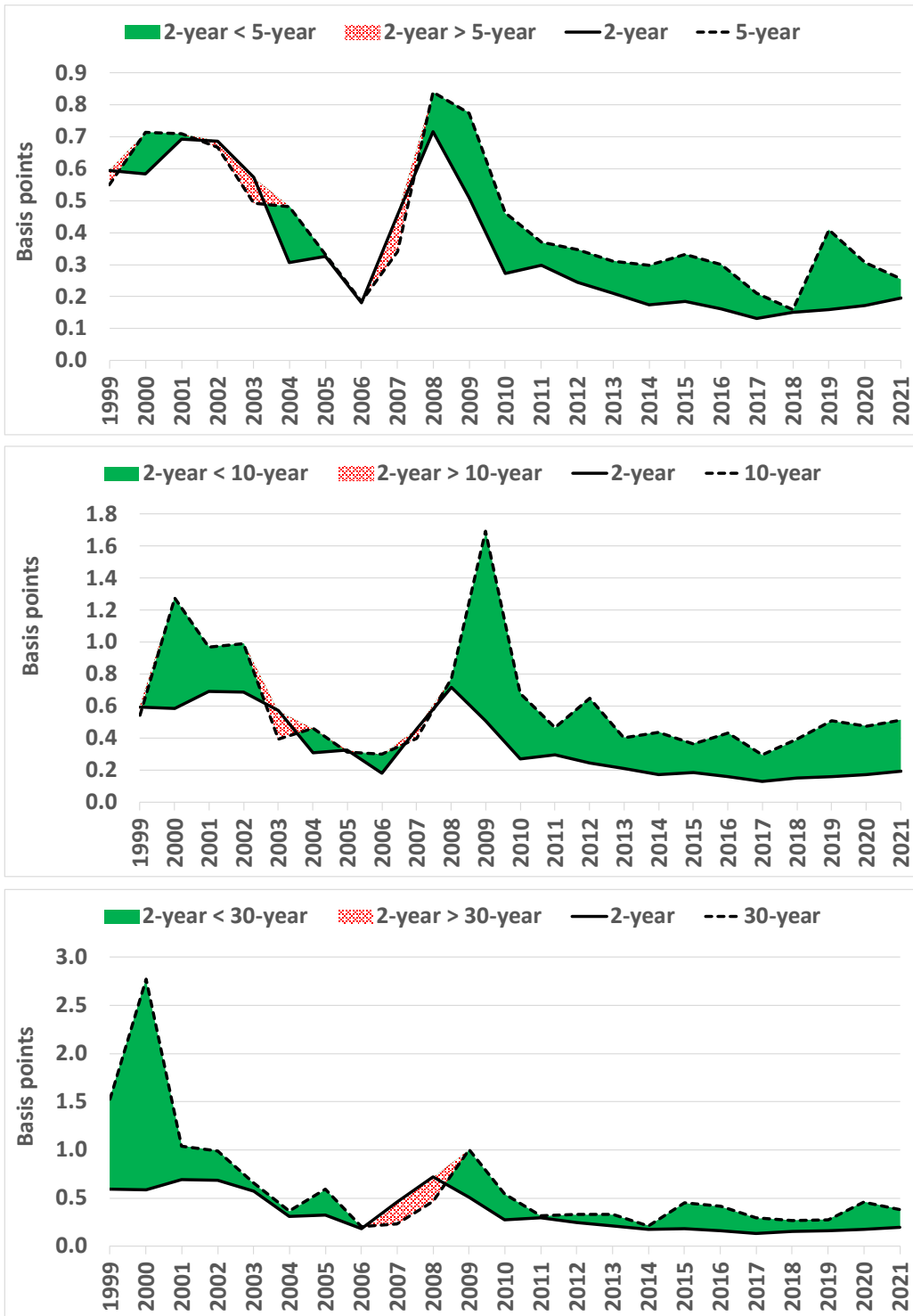
**b. Auction coverage**

Green/Red: 2-year sector is more/less competitive



**c. Auction tail**

Green/Red: 2-year sector is more/less competitive



## Conclusion

We use detailed data on the bids at GoC bond auctions between 1999 and 2021 to gauge the yield sensitivity of these bonds to the issuance amount. Using information on quantity and yield in the multiple bids submitted by participants, we propose a new method for estimating the slope of the demand function of each bidder at each auction.

We use these estimated slopes to shed some light on the relationship between the supply and yield of a government debt security. We test the hypothesis that the relationship between supply and yield is convex. Contrary to our hypothesis, we find some evidence of a negative relationship between the auction size and the slope, implying that the supply-yield function is concave in some regions of the issuance amount. Our finding is consistent with the improved competitiveness of auctions for 2-year GoC bonds following a significant increase in issuance since the global financial crisis.

Many questions remain about both the methodology for measuring the sensitivity of yields to supply using bid data and the relationship between the supply and yield of government bonds. For instance, our method for calculating the slope of the aggregate demand function at a multiple-price auction is not based on any established theoretical framework. We leave the work of devising such a framework and of conducting a more comprehensive analysis of the relationship between supply and yield using our new metric as topics for future research.

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