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## Working Paper Monetary asymmetries without (and with) price stickiness

ECB Working Paper, No. 2928

**Provided in Cooperation with:** European Central Bank (ECB)

*Suggested Citation:* Jaccard, Ivan (2024) : Monetary asymmetries without (and with) price stickiness, ECB Working Paper, No. 2928, ISBN 978-92-899-6676-4, European Central Bank (ECB), Frankfurt a. M., https://doi.org/10.2866/040155

This Version is available at: https://hdl.handle.net/10419/297368

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# **Working Paper Series**

Ivan Jaccard Monetary asymmetries without (and with) price stickiness



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#### Abstract

The evidence suggests that monetary policy transmission is asymmetric over the business cycle. Interacting financing frictions with a preference for liquidity provides an explanation for this fact. Our mechanism generates monetary asymmetries in a model that jointly reproduces a set of asset market and business cycle facts. Accounting for the joint dynamics of asset prices and business cycle fluctuations is key; in a variant of the model that is unable to produce realistic macro-finance implications, monetary asymmetries disappear. Our results suggest that asymmetries in the transmission mechanism critically depend on the macro-finance implications of monetary policy models, and that resorting to nonlinear techniques is not sufficient to detect monetary asymmetries.

- *Keywords:* Money Demand; Nonlinear Solution Methods; Asset Pricing in DSGE Models; Term Premium; Stochastic Discount Factor.
- **JEL:** E31, E44, E58.

#### Non-technical Summary

Asymmetries in the monetary transmission mechanism is a classical topic in macroeconomics that dates back to the Great Depression. In recent years, a renewed interest in this question has emerged, as a large body of novel empirical work has confirmed that the effects of monetary policy are not constant over time but vary over the business cycle. Understanding the root cause of this state dependence is an issue of central importance for policymakers. Indeed, the risk is that asymmetries in the transmission mechanism could reduce or amplify the effectiveness of monetary policy, thereby impairing the ability of central banks to stabilize the economy.

Given the major influence of aggregate supply in recent years, this article studies the transmission of monetary policy when supply shocks are the main source of business cycle fluctuations. In our artificial economy, the type of negative supply shocks observed in recent years gives rise to a large increase in lending rates as well as a contraction in credit. Our main conclusion is that monetary policy is particularly powerful during these periods of tight credit conditions and elevated lending rates, which are also times of subdued economic growth. Therefore, a main implication of our work is that monetary policy has been a powerful tool in the post-COVID-19 era. We also provide a formal empirical test of our model's main theoretical prediction.

We study the transmission mechanism of the type of liquidity operations that have been used by central banks since the global financial crisis. These measures, which are transmitted via the bank lending channel, rely on the impact of the aggregate liquidity conditions on the quantity of loanable funds available in the economy. According to this transmission channel, a reduction in aggregate liquidity implemented by the central bank reduces the supply of loanable funds in the banking sector. A reduction in the supply of funds then raises bank lending rates. The resulting tightening of credit conditions, in turn, forces firms to reduce employment and investment, leading to a reduction in both economic activity and inflation.

Monetary asymmetries stem from a preference for liquidity, which gives rise to a lower bound on bank interest rates. Since agents have the option to hold money in the form of cash, the deposit rate offered by banks is bounded below. The nonlinearity induced by this lower bound reduces the effectiveness of liquidity injections during periods of high credit availability because banking rates are close to the lower bound in those states of the world. In contrast, a tightening of monetary policy has larger effects on the real economy during periods of credit stress and high bank lending rates when the economy is away from the lower bound. We also study the case in which the central bank issues a digital currency and show that monetary asymmetries remain significant.

Our main takeaway is that the size of this asymmetry critically depends on the model's ability to generate interest rate risk. In a model that reproduces a term premium of the magnitude observed in recent years, the volatility of inflation, as well as the volatility and the level of nominal bank lending rates, this asymmetry significantly alters the conduct of monetary policy. The exact same monetary policy shock has an effect on output that is much greater in times of credit shortage and subdued economic growth than during periods of ample credit availability when the economy is booming.

Interestingly, this asymmetry disappears in the version of the model that fails to generate realistic amounts of interest rate risk. Without realistic fluctuations in real interest rates, the term premium falls to zero and the effects of monetary policy are symmetric over the cycle. Indeed, without real interest rate risk, lending rates become too smooth relative to the data. In this case, the nonlinearity induced by the lower bound on interest rates becomes irrelevant, and asymmetries in the transmission mechanism disappear.

Finally, for monetary asymmetries to play a significant role, we show that it is sufficient to consider a model in which the welfare cost of business cycle fluctuations remains relatively small, yet much larger than Professor Lucas' original estimate.

## 1 Introduction

Asymmetries in the monetary transmission mechanism is a classical topic in macroeconomics that dates back to the Great Depression. In recent years, a renewed interest in this question has emerged, as a large body of novel empirical work has confirmed that the effects of monetary policy are not constant over time but vary over the business cycle. Understanding the root cause of this state dependence is an issue of central importance for policymakers. Indeed, the risk is that asymmetries in the transmission mechanism could reduce or amplify the effectiveness of monetary policy, thereby impairing central banks' ability to stabilize the economy (e.g. Mishkin (2009)).

This article revisits this classical question in light of more recent developments. Indeed, the 2007-2009 Global Financial Crisis (GFC) has considerably impacted the conduct of monetary policy. First, in response to the Subprime and Sovereign Debt Crises in Europe, the major central banks implemented measures aimed at addressing impairments in credit markets with the objective to stimulate lending to the real economy (e.g. Bernanke (2009); Trichet (2009)). Second, the GFC was followed by an unusual period of very low interest rates, as monetary policy on both sides of the Atlantic was constrained by the effective lower bound. Although interest rate policies only played a limited role in the post-GFC era, monetary policy was still the main game in town as central banks resorted to liquidity injections of an unprecedented magnitude to stimulate the economy.

This article studies monetary asymmetries by analyzing the effect of liquidity injections to households on bank lending rates, credit and economic activity. Relative to the pre-GFC benchmark model (e.g. Smets and Wouters (2007)), the first main difference is that we model liquidity injections in a representative agent model in which shocks to money supply affect the quantity of loanable funds available in the economy. The second key difference is that we introduce an endogenous choice between bank deposits and cash holdings. Owing to a preference for liquidity, agents find it optimal to hold a small fraction of their wealth in the form of cash. This demand for liquidity in turn depends on the opportunity cost of holding cash instead of bank deposits. Under standard assumptions, this preference for liquidity gives rise to a nonlinear money demand. We then show that this nonlinearity produces an effective lower bound on bank lending rates, as the possibility to hoard money in the form of cash prevents banks from charging negative rates.

The third main difference is that we study monetary policy in an environment in which financing constraints per se are a source of monetary non-neutrality. As firms need to pay workers in advance of production, the key is that financing frictions introduce a distortion or wedge in the labor market. This "labor wedge" is in turn the main channel via which monetary policy is transmitted to the real economy. Given that our transmission mechanism works through this labor wedge, money is close to neutral in the absence of financing frictions. Indeed, our model reduces to a real business cycle model with money in the utility function in this limiting case.

How does monetary policy affect the real economy? An increase in money supply raises the quantity of loanable funds available in the economy. Equilibrium in the credit market in turns implies a reduction in bank lending rates, as the supply of funds increases. In a model in which firms need to finance inputs in advance of production, the key is that the size of the labor wedge is determined by the level of bank lending rates. Variations in lending rates therefore affect aggregate supply via this labor wedge stemming from the presence of financing frictions.

Why is monetary policy transmission asymmetric? Our main finding is that liquidity injections that are transmitted via this channel are particularly effective during credit crunches. Periods of credit crunches, or credit stress, are times in which credit conditions are tighter than usual. Monetary policy is particularly powerful in those states of the world because the scarcity of credit leads to elevated lending rates. High lending rates in turn imply a high opportunity cost of holding money in the form of cash. Consequently, in response to a monetary injection, agents find it optimal to channel a larger fraction of this increase in money supply into the banking sector, as they are reluctant to hold cash. In those states of the world, the shock therefore has a particularly strong effect on the supply of loanable funds. The key is that a larger increase in the supply of loanable funds in turn leads to a more pronounced decline in lending rates than in normal times. Since a decline in lending rates reduces the labor wedge, the stimulative effect on production is more pronounced during periods of credit scarcity.

Monetary policy is less effective during periods of ample credit availability because the opportunity cost of holding cash is low. In those states, credit supply is higher than usual, which in turn implies low lending rates. Since the opportunity cost of holding cash is low, and owing to their preference for liquidity, agents have little incentive to lend funds to entrepreneurs. Consequently, monetary policy becomes ineffective because liquidity injections do not reach the real economy, as agents prefer to hoard liquidity instead. In those states of the world, liquidity injections only have a muted impact on credit supply. Consequently, the decline in lending rates is smaller than during periods of credit stress. A smaller decline in lending rates in turn imply a more muted effect on the labor wedge and hence production.

We find that the size of this asymmetry critically depends on the model's ability to generate interest rate risk. In a model that reproduces a term premium of the magnitude observed in recent years, the volatility of inflation, as well as the volatility and the level of nominal bank lending rates, this asymmetry significantly alters the conduct of monetary policy. The exact same monetary policy shock has an effect on output that is much greater in times of credit shortage than during periods of ample credit availability.

Interestingly, this asymmetry disappears in the version of the model that fails to generate realistic amounts of interest rate risk. Without realistic fluctuations in real interest rates, the term premium falls to zero and the effects of monetary policy are symmetric over the cycle. Indeed, without real interest risk, lending rates become too smooth relative to the data. In this case, the nonlinearity induced by agents' preference for liquidity becomes irrelevant and asymmetries in the transmission mechanism disappear.

For asymmetries to play a significant role, we also find that it is sufficient to consider a model in which the welfare cost of business cycle fluctuations remains relatively small, yet much larger than Lucas' original estimate. To illustrate this point, we calculate the additional income that agents would demand as compensation for having to live in a world subject to business cycle fluctuations. We find that this "business cycle fluctuation premium" amounts to about 1.88% of total income per quarter in the model that matches the data. This premium falls to 0.023% in the variant of the model that is unable to produce realistic macro-finance implications and in which monetary transmission is symmetric over the cycle.<sup>1</sup>

The asymmetry documented in our study continues to play a major role if we introduce Calvo staggered contracts (Calvo, 1983), which is a widely used specification of nominal rigidities in the New Keynesian literature. The introduction of price stickiness also considerably amplifies the average effectiveness of liquidity injections. In addition, we discuss the case of a more general preference specification, provide an empirical test of the main model mechanism, and consider the case of demand shocks.

Finally, we study the case in which the central bank is able to remunerate cash. This latter experiment can be motivated by the ongoing discussions with regards to central bank digital currencies (CBDCs). Indeed, if CBDCs were introduced, central banks could remunerate cash by paying an interest rate on virtual money.

This paper is structured as follows. A literature review is provided in Section 2. The model economy is described in Section 3, whereas the calibration and results are discussed

<sup>&</sup>lt;sup>1</sup>While the methodologies are not directly comparable, in Lucas (2003) the welfare cost measure amounts to 0.05% of consumption per year.

in Sections 4 and 5, respectively. The monetary policy transmission mechanism is deconstructed in Section 6. The main result of the paper, namely the state dependence of monetary policy transmission is discussed in Section 7. The robustness analysis as well as the version of the model with price stickiness are presented in Section 8. Section 9 concludes.

## 2 Literature Review

As regards evidence on asymmetries in the transmission of monetary policy, two main conclusions emerge from the literature: (i) there is ample evidence showing that the effects of monetary policy are not symmetric over the business cycle, and (ii) whether monetary policy is more or less effective during recessions is still a widely debated question. Given this lack of consensus, Section 8.3 provides empirical support to our key model mechanism.

The work of Peersman and Smets (2005) studies asymmetries in the transmission mechanism of monetary policy in the euro area using data from 74 manufacturing industries for the period from 1980 to 1998. These authors find that monetary policy has stronger effects on output in a cyclical downturn than in a boom and attribute this asymmetry to factors related to the financial structures of firms, such as size for instance.

Lo and Piger (2005) investigate the response of output to monetary policy in the United States using a regime switching model. These authors find strong evidence that monetary policy has larger effects on output during recessions than during expansions. Garcia and Schaller (2002) also resort to regime switching techniques and show that interest rate changes have a stronger effect on output growth during recessions than during expansions. A classic explanation for the findings documented by this strand of literature is that asymmetries are due to credit market frictions (e.g. Gertler and Gilchrist (1994)).

Santoro, Petrella, Pfajfar and Gaffeo (2014) focus on the period from 1982 to 2008 and find that monetary policy has larger effects on output during recessions than during booms. At the same time, they do not find evidence of asymmetries in the response of inflation to monetary policy shocks. These authors formalize their empirical results by developing a general equilibrium model with price stickiness and loss averse consumers.

In contrast to the above-mentioned strand of literature, Tenreyro and Thwaites (2016) find evidence that monetary policy shocks have more powerful effects on output and inflation in an expansion than in a recession. These authors also find that contractionary shocks are more powerful than expansionary ones. Relative to Santoro et al. (2014), one important difference is that these authors analyze the effects of shocks occurring in the period from 1969 to 2002, and study the response of variables up to 5 years later.

The results documented by Alpanda, Granziera and Zubairy (2021) using a sample of 18 advanced economies and data until 2018 are consistent with those reported by Tenreyro and Thwaites (2016). These authors also develop a parsimonious partial equilibrium model to analyze the channels that could rationalize the empirical facts that they uncover.

Bruns and Piffer (2023) argue that estimating rather than calibrating key parameters in Smooth Transition models is crucial to study monetary asymmetries. Applying their methodologies using data from 1979 to 2020, they find that monetary policy is more effective during expansions than during recessions.<sup>2</sup>

Our work is also related to a literature that studies the effectiveness of monetary policy during periods of very low interest rates. In particular, Ahmed, Borio, Disyatat and Hofmann (2021) find that the effectiveness of monetary policy declined after the GFC, which corresponds to a period of very low interest rates. One possible explanation for their findings is that the effects of interest rates on consumption and investment could be nonlinear.

Harding, Lindé and Trabandt (2022) resolve the missing deflation puzzle by considering the effects of discount factor shocks in a model in which nominal rigidities are combined with Kimball aggregators (Kimball (1995)). The disconnect between inflation and output observed in the aftermath of the GFC suggests that nonlinearities play a key role in macroeconomics. Indeed, during the 2007-2009 financial crisis, the fall in activity was accompanied by a relatively modest decline in inflation. In contrast, during the recovery phase, inflation dynamics remained subdued (e.g. Hall (2011)).

In a related study, Andreasen, Caggiano, Castelnuovo and Pellegrino (2021) show that uncertainty shocks generate a larger contraction during periods of recessions. As in Harding et al. (2021) and Santoro et al. (2014), this strand of literature highlights the importance of price rigidities. In contrast, our main result is derived in a flexible price model in which financing constraints are the only source of non-neutrality.

Another literature focuses on the effects of the financial cycle on the monetary policy transmission mechanism. Aikman, Lehnert, Liang and Modugno (2020) study the influence of the credit cycle on the effectiveness of monetary policy. These authors find that a contractionary monetary policy shock has the usual effect on GDP and inflation during periods of low credit-to-GDP gaps. In contrast, monetary policy becomes ineffective when

<sup>&</sup>lt;sup>2</sup>A previous version of their paper was circulated under the title: "US monetary policy is more powerful in recessions: A new approach to Smooth Transition Vector Autoregressions".

credit is above trend. Alpanda and Zubairy (2019) focus on the impact of household debt on the transmission mechanism using data from 1951 to 2015 and find that monetary policy is less powerful during periods of high household debt.

One main lesson from the evidence accumulated over the last 30 years is that financing frictions, in particular frictions affecting access to credit, influence the transmission of monetary policy (e.g. Kashyap, Lamont and Stein (1994); Gertler and Gilchrist (1994)). Following the seminal contribution of Bernanke and Gertler (1989), a vast strand of literature has interpreted this line of evidence by developing models in which lending is subject to a "costly state verification" problem (Townsend (1979)). Relative to this approach, we introduce financing constraints by assuming that firms need to pay workers and rental costs in advance of production. By producing a labor wedge (e.g. Jermann and Quadrini (2012)), financing frictions create a channel of monetary policy transmission that is independent of nominal rigidities.

Since firms need to pay workers in advance, our approach also builds on the cost channel of monetary policy literature initiated by the work of Barth and Ramey (2002). Relative to the cost channel literature, a key difference is that in our environment prices increase in response to an accommodative monetary policy shock. In the flexible price version, the dynamics of prices is entirely determined by the demand for real money balances, and not by the marginal cost of firms. In the New Keynesian version, as in Christiano, Eichenbaum and Evans (2005) and Ravenna and Walsh (2006), the dynamics of prices depends on the marginal cost, which is in turn affected by borrowing costs.

Relative to the financial accelerator literature, another key difference is that financing constraints create a wedge that not only affects investment but also labor demand. This departure from the standard approach can be motivated by a growing literature that has uncovered a link between credit market frictions and employment (e.g. Chodorow-Reich (2014)). In particular, the evidence available for Europe strongly suggests that credit market frictions affect the real economy through their impact on the labor market (e.g. Bentolila, Jansen and Jiménez (2018); Popov and Rocholl (2018)).

Whereas credit frictions are the only source of monetary non-neutrality in the flexible price benchmark, the asymmetry that we obtain critically depends on the lower bound on interest rates. The evidence documented by Heider, Saidi and Schepens (2019) confirms that the lower bound plays an important role in the euro area economy. Indeed, banks are reluctant to pass on negative rates to depositors. As a result, bank lending rates remained in positive territory, even when monetary policy rates turned negative. The positive skewness in the distribution of lending rates is also consistent with the view that bank lending rates are bounded below. As shown by Ordoñez (2013), while stronger in economies with less developed financial systems, this positive skewness is observed in many countries.

Relative to the macroeconomic literature on monetary policy, one objective of this paper is to reproduce some empirical facts about interest rates. The work of Den Haan (1995) is probably one of the first attempts to study the term structure of interest rates in a model with production and a role for monetary policy. The model developed by the author replicates the high persistence of interest rates observed in the data. Matching the behavior of bond prices is however a challenge.

A more recent literature aims to explain the term structure of interest rates, the term premium as well as the behavior of bond prices within New Keynesian models. Hördahl, Tristani and Vestin (2008) find that a model with nominal rigidities à la Calvo and habit formation can reproduce a sizeable term premium, as well as volatile long-term yields. Rudebusch and Swanson (2008) argue that the success of models with habits comes at the cost of generating predictions for macroeconomic variables which are at variance with the facts.

In a subsequent study, Rudebusch and Swanson (2012) combine Epstein-Zin-Weil preferences (e.g. Epstein and Zin (1989); Weil (1989); Weil (1990)) with long-run risk (e.g. Bansal and Yaron (2004)) into a New Keynesian model. These authors are able to generate a large and variable term premium without compromising the model's ability to fit the main macroeconomic aggregates.

Relative to Rudebusch and Swanson (2012), Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018), henceforth AFVRR (2018), match the level and variability of the 10-year term premium with Epstein-Zin-Weil preferences using a low coefficient of relative risk aversion. This improvement is obtained by introducing a wedge akin to financial frictions into the analysis. Introducing this feedback effect allows the authors to break the macro-finance separation result typically obtained with Epstein-Zin-Weil preferences (e.g. Tallarini (2000)).

Gourio and Ngo (2020) study the effects of the zero lower bound on risk premiums in a New Keynesian model augmented with Epstein-Zin-Weil preferences. Relative to their approach, a main difference is that we study a model in which the lower bound on interest rates arises because of the presence of cash.

Relative to these studies, another key difference is that in our case interest rate risk is generated by combining habits with adjustment costs. Relative to Jermann (1998), or Campanale, Castro and Clementi (2010) who rely on Chew-Dekel preferences, this mechanism is studied in a model with composite habits (e.g. Jaccard (2014); Dimitriev (2017)) in which labor supply is endogenously determined.

As Constantinides (1990) and Boldrin, Christiano and Fisher (2001), we use a specification of internal habits, which ensures that the traditional measure of relative risk aversion is determined by the curvature parameter, which we set to 1. At the same time, and as emphasized by Campanale, Castro and Clementi (2010) and Swanson (2012), in models with different adjustment margins, the relationship between risk premiums and standard measures of local risk aversion is imperfect. As in Jermann (1998), our model with habits loses much of its ability to reproduce realistic risk premiums if we remove adjustment costs from the analysis. This confirms that increasing the curvature of the utility function is not sufficient to resolve asset pricing puzzles in models with production. The investment margin also plays a key role and this effect is independent of preferences.

Many studies have already documented real effects of a sizeable magnitude in models without nominal rigidities. In Fuerst (1992) and Christiano and Eichenbaum (1992), significant non-neutralities are obtained in a model with production in which cash-in-advance constraints are combined with limited participation. At the same time, one limitation of these models is that the non-neutralities that they generate are not sufficiently persistent (e.g. Christiano and Eichenbaum (1995)).

Relative to these studies, Cooley and Quadrini (1999) obtain more persistent real effects of monetary policy in a model with search and matching frictions in the labor market. In Khan and Thomas (2015), persistent real effects are obtained in a model in which market segmentation is endogenously determined. In De Fiore, Teles and Tristani (2011), monetary policy has real effects because debt is predetermined and issued in nominal terms. In Gomes, Jermann and Schmid (2016), the large non-neutralities that are obtained stem from the effect of inflation on long-term nominal debt contracts that are issued by firms. Bullard and DiCecio (2019) derive the optimal monetary policy in a heterogeneous agent model, without sticky prices, in which non-state contingent contracts are the source of nominal frictions.

Our approach is also related to a recent strand of literature in which the preference for liquid assets is explicitly introduced. In Piazzesi, Rogers and Schneider (2019), this preference for deposit creates a convenience yield on the policy instrument that significantly alters the transmission mechanism of an otherwise standard New Keynesian model. In Van den Heuvel (2008), agents' preference for liquidity introduces a wedge between the return on equity and the return on bank deposits. In Begenau (2020), the general equilibrium effects implied by this preference for liquidity can lead to an increase in bank lending in response to higher capital requirements. The micro-foundations that lead agents to hold money are derived and discussed in the new monetarist literature (e.g. Williamson and Wright (2010)).

The mechanism studied in this paper should play a relevant role in economies where firms are dependent on banks. We therefore choose to calibrate the model using Eurozone data. In the euro area, small and medium-sized enterprises (SMEs) represent about 99% of all firms and account for around two-thirds of the jurisdiction's workforce (e.g. Öztürk and Mrkaic (2014); European Central Bank (2014)). This large share of SMEs makes the Eurozone particularly reliant on the bank lending channel, as these firms only have a limited access to market-based financing (European Commission (2011), European Commission (2015)).

## 3 The Environment

The model is composed of a non-financial or corporate sector, a commercial banking sector, a central bank, a representative household, and a government. A role for external financing is introduced by assuming that firms in the non-financial sector need to pay workers and capital owners in advance of production.

#### 3.1 Households

The representative agent owns the economy's stock of capital and derives utility from consuming a consumption good, enjoying leisure and holding cash. All variables are detrended and the deterministic growth rate along which the economy is growing is denoted by  $\gamma$ (e.g. King and Rebelo (1999)). The period t budget constraint of the representative agent is given as follows:

$$prof_{Tt} + tr_t + r_{Kt}k_t + w_tn_t + i_{Dt}\frac{D_t}{P_t} + \frac{M_t}{P_t} + \frac{B_t}{P_t} = c_t + x_t + \gamma \frac{M_{t+1}}{P_t} + \frac{1}{1 + i_{Bt}}\gamma \frac{B_{t+1}}{P_t} \quad (1)$$

The left-hand side of equation (1) reports the various sources of income received by agents. Since they own all the sectors of the economy, households receive a dividend income paid by the financial and non-financial sectors denoted by  $prof_T$ . Each period a lump sum transfer, which is denoted by tr, is received from the government. Households own the economy's capital stock and rent it to the non-financial corporate sector. The capital stock is denoted by k and  $r_K$  is the rental rate of capital. Labor supply being endogenously determined, households divide their total time endowment, which we normalize to 1, between hours worked in the corporate sector and leisure:

$$z_t + n_t = 1 \tag{2}$$

where leisure and hours worked are denoted by z and n, respectively. The total labor income received in period t is therefore wn, where w denotes the wage rate. Relative to a real business cycle model, the difference is that households also accumulate real money balances. The nominal stock of money balances carried from the previous period is denoted by  $M_t$ , whereas money balances available during period t is denoted by  $M_{t+1}$ . Since money is a nominal asset, the real value of money holdings is obtained by dividing the nominal stock by the price level, which is denoted by P. Relative to the textbook money-in-theutility function model, the difference is that households can either keep money in the form of cash or deposit part of it in the banking sector. This portfolio decision is captured by introducing the following constraint:

$$\gamma M_{t+1} = D_t + S_t \tag{3}$$

where  $\gamma M$  denotes money holdings available during the period. The fraction that is deposited in the banking sector is denoted by D and earns a within period interest rate  $i_D$ . In real terms, the net income received from bankers is thus given by  $i_D \frac{D}{P}$ . The remaining fraction that households keep in cash is represented by S. Following the money-in-the utility view, a demand for cash is introduced by assuming that the amount of real money holdings available during the period, which is denoted by S/P, yields utility.

Following the timing adopted in models with money-in-the-utility, it is the amount available at the end of the period that yields utility (e.g. Walsh (2017)). Whereas Srepresents the fraction of money balances that is liquid and that households can access at any time, the amount deposited in the banking sector is illiquid in the sense that it cannot be converted into cash within the period. The interest rate on deposit therefore represents the opportunity cost of keeping liquid money balances in the form of cash instead of depositing them in the banking sector. Finally, households invest in a short-term riskfree bond issued by the government. The real payoff received by households depends on inflation and is given by the coupon payment B deflated by the price level P.

The right-hand side of equation (1) represents the different expenditures faced by agents in period t. Households firstly choose how to allocate their total income between consumption and investment, which are denoted by c and x, respectively. Real money balances available during period t are denoted by  $\gamma \frac{M_{t+1}}{P_t}$ , whereas  $\gamma \frac{B_{t+1}}{P_t}$  is the stock of government bonds purchased by households during the period. The price at which this one period risk-free bond is purchased is denoted by  $\frac{1}{1+i_{Bt}}$ .

Capital accumulation is subject to adjustment costs, and following Jermann (1998) and Baxter and Crucini (1993) among others, I use the following specification:

$$\gamma k_{t+1} = (1-\delta)k_t + \left(\frac{\theta_1}{1-\epsilon} \left(\frac{x_t}{k_t}\right)^{1-\epsilon} + \theta_2\right)k_t \tag{4}$$

where  $\epsilon$  measures the degree of adjustment costs and can be interpreted as the elasticity of Tobin's Q with respect to changes in the investment to capital ratio. The parameters  $\theta_1$ and  $\theta_2$  are chosen to ensure that the models with and without adjustment costs have the same deterministic steady state.

Habits are formed over the composite good consisting of the different components of utility (e.g. Jaccard (2014)). The composite good not only depends on consumption and leisure but also on the fraction of real money balances S/P that agents hold in the form of cash. This implies the following law of motion for the habit stock, which is denoted by h:

$$\gamma h_{t+1} = \tau h_t + (1-\tau)c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} \left(\psi + z_t^{\upsilon}\right)$$
(5)

where  $\tau$  is a memory parameter that affects the rate at which the habit stock depreciates over time. The weight of consumption in the utility function is denoted by the utility parameter  $\kappa$ . v and  $\psi$  are two labor supply parameters that control the Frisch elasticity, as well as the steady state allocation of time between hours worked and leisure, respectively. The objective of the representative agent is to maximize lifetime utility, which is given as follows:

$$\max_{c_t, B_{t+1}, S_t, n_t, h_{t+1}, k_{t+1}, x_t, M_{t+1}} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left(c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} (\psi + z_t^{\upsilon}) - h_t\right)^{1-\sigma}}{1-\sigma}$$

subject to constraints (1) to (5). In an infinite horizon model, the subjective discount factor is affected by the growth rate of the economy along the balance growth path (Kocherlakota (1990)) and we denote the modified discount factor by  $\hat{\beta}$ , where  $\hat{\beta} = \tilde{\beta} \gamma^{1-\sigma}$ .

#### 3.2 The non-financial or corporate sector

Firms in the non-financial sector produce a composite output good using labor n and capital k as production inputs. The final output good produced by the corporate sector is denoted by y, and the production function takes a standard Cobb-Douglas form:

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \tag{6}$$

where  $\alpha$  is the capital share parameter. The random technology shock *a* follows an autoregressive process of order one,

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{at}$$

where the random disturbance  $\varepsilon_a$  is normally distributed with mean zero and standard deviation  $\sigma_a$ . The autoregressive parameter is denoted by  $\rho_a$ , where  $0 \le \rho_a \le 1$ .

Profits at time t, which are denoted by  $prof_F$ , are given as follows:

$$prof_{Ft} = a_t k_t^{\alpha} n_t^{1-\alpha} - r_{Kt} k_t - w_t n_t - i_{Lt} \frac{L_t}{P_t}$$
(7)

Relative to the neoclassical growth model, a cost channel of monetary policy is introduced by assuming that firms need to obtain credit in order to operate. The lending decision is intratemporal, in the sense that the loan is received at the beginning of the period and needs to be reimbursed with an interest payment before the end of period t. In other words, the firm receives L/P at the beginning of the period and needs to repay  $(1+i_L)L/P$  before the period ends.

The amount of external financing needed at the beginning of period t is determined by the following loan-in-advance constraint:

$$\frac{L_t}{P_t} \ge \mu \left( w_t n_t + r_{Kt} k_t \right) \tag{8}$$

where the parameter  $\mu$  represents the fraction of total labor and capital costs, i.e.  $wn+r_Kk$ , that needs to be paid in advance and which therefore requires financing. The objective of managers in the corporate sector is to maximize the value of the firm, which is given by the infinite discounted sum of future profits:

$$\max_{n_t,k_t,L_t} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\lambda_t}{\lambda_0} prof_{Ft}$$

where  $\hat{\beta}^t \lambda_t / \lambda_0$  is the stochastic discount factor of the representative agent who owns firms in the non-financial sector, subject to constraints (6) to (8).

#### 3.3 The central bank

The central bank provides money to the private sector and any profit or loss made by the monetary authorities, which is denoted by  $rcp_t$ , is directly transferred to the government. In period t, this implies the following budget constraint:

$$rcp_t = \gamma \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} \tag{9}$$

Since money is explicitly modelled, and given that central banks use quantities to influence interest rates, the quantity of money M is the policy tool. The deposit rate  $i_D$  is therefore treated as an endogenous variable. This choice can also be motivated by the environment in which monetary policy was conducted in the post-GFC era. Indeed, until the energy price shock triggered by the war in Ukraine, interest rate policies only played a minor role. However, monetary policy remained very active as central banks around the world have used balanced sheet policies to stimulate the economy. Moreover, in a model in which the quantity of money is explicitly introduced, the extent to which monetary policy is constrained by the zero lower bound is a priori not clear. The empirical results documented in Belongia and Ireland (2018) for instance suggest that this constraint could be alleviated by using money supply rather than interest rate rules.

Monetary policy is determined by the following money supply rule:

$$\gamma M_{t+1} = \gamma \overline{M} + \rho_M \left( M_t - \overline{M} \right) - \phi_\pi (P_t - \overline{P}) + \varepsilon_{Mt} \tag{10}$$

where  $\overline{M}$  denotes the quantity of money when the economy reaches its steady state. The monetary policy shock is denoted by the random disturbance  $\varepsilon_M$ , which is normally distributed with mean zero and standard deviation  $\sigma_M$ .

Relative to steady state, the quantity of money set by the central bank firstly depends on the quantity available in the previous period, where  $\rho_M$  is a monetary policy smoothing parameter. The objective of monetary policy is to stabilize inflation and the central bank adjusts its monetary policy stance each time the price level P deviates from its steady state value, which is denoted by  $\overline{P}$ . Monetary policy therefore becomes more restrictive when Prises above  $\overline{P}$ , and more accommodative when the price level falls below its steady state value. The sensitivity of monetary policy to deviations of the price level from its target is captured by the parameter  $\phi_{\pi}$ .

#### 3.4 The commercial banking sector

As in Van den Heuvel (2008), the commercial banking sector intermediates funds between households and the non-financial sector. Banks collect deposits at the beginning of the period, which are then lent to the corporate sector.

We simplify the analysis by assuming that the lending and deposit decisions occur within the period. The timing is as follows. At the beginning of the period, bankers receive a quantity of deposit D/P from households. They are then able to extend the quantity of loan L/P to firms.

Before the period ends, bankers receive  $(1 + i_L)L/P$  from firms. Banks are then able to return the sum that households deposited along with an interest rate payment, which amounts to  $(1 + i_D)D/P$ .

As in Goodfriend and McCallum (2007), we assume that banks are endowed with a technology that can be used to produce credit using deposits as an input. The production function is given by a linear technology that links the quantity of loans extended to the non-financial sector to the quantity of deposits raised at the beginning of the period:

$$L_t = \eta D_t \tag{11}$$

where  $\eta$  is a technology parameter measuring the efficiency of the intermediation technology operated by banks. Each period, bankers optimally choose the amount of deposits to collect from households D and the quantity of credit to extend to firms L to maximize profits, which are given as follows:

$$\max_{L_t,D_t} prof_{Bt} = i_{Lt} \frac{L_t}{P_t} - i_{Dt} \frac{D_t}{P_t}$$

subject to constraint (11).

### 3.5 The government

Government only plays a passive role in this environment. The lump sum transfer made to the representative agent is financed by issuing a short-term risk-free bond and by the receipts received from the central bank. In period t, this implies the following budget constraint:<sup>3</sup>

 $<sup>^{3}</sup>$ The assumption of lump-sum transfers ensures that money does not have any effect via the government budget constraint.

$$tr_{t} = rcp_{t} + \frac{1}{1+i_{Bt}}\gamma \frac{B_{t+1}}{P_{t}} - \frac{B_{t}}{P_{t}}$$
(12)

#### **3.6** Market clearing condition

The aggregate budget constraint can be derived by combining the budget constraints of the different sectors. Since the representative agent owns the non-financial and banking sectors, the total dividend income received is given by  $prof_{Tt} = prof_{Ft} + prof_{Bt}$ . Any loss or profit made by the central bank is transferred to the government, and since the government in turns makes a lump sum transfer to the representative agent, the economy's consolidated budget constraint is given as follows:

$$y_t = c_t + x_t \tag{13}$$

#### 3.7 Equilibrium definition

A competitive equilibrium in the economy is a sequence of prices:

$$\varpi, q, \lambda, \varphi, w, r_K, i_D, i_L, i_B, P$$

where  $\varpi$  denotes the Lagrange multipliers associated with the loan-in-advance constraint, q is Tobin's Q,  $\lambda$  is marginal utility,  $\varphi$  is the Lagrange multiplier associated with the law of motion of the habit stock, and quantities:

that satisfy agents' efficiency conditions, as well as the resource constraint (13) for all states, for t=1... $\infty$ , and given initial values for the two endogenous state variables k and h.

#### 3.8 The transmission of monetary policy to the real economy

The household optimality conditions can be analyzed to gain intuition into how a monetary policy shock is transmitted to the real economy. The optimality conditions with respect to S and c can firstly be combined to derive the following relationship between the deposit rate  $i_D$ , consumption c, the price level P, and the amount of cash held by agents S:<sup>4</sup>

$$i_{Dt} = \frac{1 - \kappa}{\kappa} \frac{P_t c_t}{S_t} \tag{14}$$

 $<sup>^{4}</sup>$ See technical appendix in Section 13.

The deposit rate represents the opportunity cost foregone by the agent when cash balances are held liquid instead of being deposited on a bank account. This creates a downward demand schedule that implies a negative relationship between interest rates and cash holdings. Given the choice to use the quantity of money as the policy instrument, equilibrium in the money market can be represented by a vertical money supply curve. An exogenous increase in money supply, which shifts this supply curve to the right, therefore has a direct impact on this opportunity cost by creating a surplus of liquidity, which in turn puts downward pressure on the deposit rate  $i_D$ .

A change in the deposit rate is then passed through to firms via the banking sector. Profit maximization in the banking sector implies the following relationship between deposit and bank lending rates:

$$i_{Lt} = \frac{1}{\eta} i_{Dt} \tag{15}$$

where the degree of pass-through depends on the efficiency of the financial intermediation technology. A change in lending rates then impacts firms in the non-financial corporate sector by modifying the cost of obtaining funds from the banking sector. The effects of a change in the cost of funds on the behavior of firms can be illustrated by deriving their optimal demand for inputs:

$$w_t = (1 - \alpha) \frac{y_t}{n_t} \frac{1}{1 + \mu i_{Lt}}$$
(16)

$$r_{Kt} = \alpha \frac{y_t}{k_t} \frac{1}{1 + \mu i_{Lt}}$$
(17)

A change in funding costs  $i_L$  therefore firstly affects the real economy through a labor wedge by moving firms' demand for labor. Secondly, since lending rates affect the marginal productivity of capital, credit frictions also distort the investment decision through an investment wedge. This latter effect can be illustrated by deriving the Euler condition associated with capital accumulation which, with adjustment costs, is given by the following equation:

$$\lambda_t q_t = \beta E_t \lambda_{t+1} q_{t+1} \left( (1-\delta) + \frac{\theta_1}{1-\epsilon} \left( \frac{x_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \theta_2 - \theta_1 \left( \frac{x_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right)$$
$$+ \beta E_t \lambda_{t+1} r_{Kt+1}$$

A change in borrowing costs therefore modifies the marginal productivity of capital, which in turn affects Tobin's Q and thus investment.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup> where, for the sake of notation, we define  $\beta = \tilde{\beta} \gamma^{-\sigma}$ 

#### 3.9 Money and the lower bound on interest rates

For realistic calibrations, the components of utility and the price level are always strictly positive in this model. Since in equilibrium the deposit rate is equal to the marginal utility of holding cash (see equation 14), this ensures that the deposit rate  $i_D$  will take values that are always strictly positive. Introducing an endogenous choice between S and D therefore captures the restriction on interest rates created by the existence of cash. If the return on bank deposits is too low, agents can always choose to hold money in cash. As will be discussed in Section 7, this lower bound on  $i_D$  is a significant source of asymmetry in the transmission mechanism of monetary policy. It is however necessary to solve the model using a nonlinear solution method to capture this effect.

Since the deposit rate always stays in positive territory, the bank lending rate  $i_L$  also takes values that are strictly positive (see equation 15). Consequently, the second advantage of introducing a bound on deposit rates is that strictly positive values for bank lending rates also rule out the case of occasionally binding constraints. Indeed, the optimality condition in the non-financial sector with respect to L implies that the Lagrange multiplier associated with constraint (8), which is denoted by  $\varpi$ , is proportional to the bank lending rate  $i_L$ :

$$i_{Lt} = \frac{\overline{\omega}_t}{\lambda_t} \tag{18}$$

Since marginal utility  $\lambda$  is always strictly positive and given the lower bound on the deposit rate implied by equation (14),  $i_L$  and thus  $\varpi$  are always strictly positive. If present, the loan-in-advance constraint is therefore always satisfied with strict equality.

#### 3.10 Equilibrium in the market for loanable funds

Combining the demand for production factors with the liquidity constraint faced by firms, and since this constraint is always binding, the following credit demand condition can be derived:

$$L_t = P_t y_t \frac{\mu}{1 + \mu i_{Lt}} \tag{19}$$

which implies a negative relationship between the amount of credit needed by firms L and the lending rate  $i_L$ . The supply of loanable funds can be derived by firstly combining the portfolio allocation constraint, *i.e.*  $\gamma M_{t+1} = D_t + S_t$ , with the demand for cash given by equation (14). Next, using the financial intermediation technology in the banking sector given by equation (11), the following supply curve implying a positive relationship between credit and lending rates can be derived:

$$L_t = \eta \gamma M_{t+1} - \frac{1 - \kappa}{\kappa} \frac{P_t c_t}{i_{Lt}}$$
(20)

#### 3.11 The dynamics of prices

The link between prices and monetary policy can be illustrated by deriving the optimality condition with respect to the demand for money M:

$$\frac{\lambda_t}{P_t} = \beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} + \frac{\lambda_t}{P_t} i_{Dt}$$
(21)

where  $\lambda$  is marginal utility. Optimality in the accumulation of real money balances implies that the cost of sacrificing one unit of consumption today has to equate the current and expected marginal benefit of doing so. Since agents can choose to deposit money in the banking sector, the marginal benefit from holding money also includes the interest rate paid on deposit  $i_D$ . To illustrate how fluctuations in the money market rate affect inflation, let us rewrite equation (21) as follows:

$$P_t = \frac{1 - i_{Dt}}{E_t \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{P_{t+1}}\right)}$$
(22)

On the denominator, the first term denotes the stochastic discount factor,  $\beta \frac{\lambda_{t+1}}{\lambda_t}$ , that agents use to evaluate future payoffs. The negative relationship between current prices and the stochastic discount factor illustrates that attitudes towards risk have an impact on inflation, a channel that is typically overlooked in the literature (e.g. Jaccard (2018b)). Everything else equal, an increase in stochastic discounting, which implies a stronger willingness to postpone current consumption in favor of future consumption, reduces the price level. In other words, when agents become more willing to save, they reduce consumption and accumulate assets, such as real money balances, that they can use to transfer wealth across time. Since  $P_t$  is the price of consumption, an increase in the degree of patience puts downward pressure on prices.

Keeping all other variables constant, this condition also shows that an increase in the short-term rate  $i_D$  reduces the price level. This effect illustrates that an increase in the deposit rate increases the benefit of holding money. As the accumulation of real money balances comes at the expense of other components of aggregate demand, such as consumption expenditures, the price level declines when  $i_D$  goes up. Finally the price level is

a forward-looking variable in this model and current prices also depend on future expected values.

#### 3.12 The term premium

The term premium can be derived by pricing a zero-coupon risk-free nominal bond that pays 1 in a given period. Following Rudebusch and Swanson (2012), this price can be expressed recursively as follows:

$$p_{Bt}^{(n)} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} p_{Bt+1}^{(n-1)}$$
(23)

where  $\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$  is the stochastic discount factors for a nominal asset. Given that the term premium is typically computed for a bond with a 10-year maturity, we set n to 40. The price of a bond that pays 1 in the current period is denoted by  $p_{Bt}^{(0)}$ . The short-term risk-free nominal rate, represented by  $i_{Bt}$ , is inversely related to the price of a bond that offers a certain payment of 1 next period:

$$p_{Bt}^{(1)} = \frac{1}{1+i_{Bt}}$$

To derive the term premium, we also need to determine the price that a risk-neutral investor would agree to pay to hold this asset. Following the notation used in Rudebusch and Swanson (2012), the risk-neutral price is denoted  $\hat{p}_{Bt}^{(n)}$  and can be obtained as follows:

$$\hat{p}_{Bt}^{(n)} = \frac{1}{1+i_{Bt}} \hat{p}_{Bt+1}^{(n-1)}$$

where  $\widehat{p}_{Bt}^{(0)} = 1$ .

For a bond with a 10-year maturity, the yield corresponding to the risk averse and risk neutral investors, respectively, are given as follows:

$$y_t^{40} = \frac{1}{p_{Bt}^{40}}$$
 and  $\hat{y}_t^{40} = \frac{1}{\hat{p}_{Bt}^{40}}$ 

Up to a first-order approximation or in the deterministic version of the model,  $\hat{p}_{Bt}^{(n)}$  and  $p_{Bt}^{(n)}$  are equivalent since certainty equivalence holds in these two particular cases. However, once the model is solved using higher-order approximations, the effect of uncertainty on the valuation drives a wedge between these two concepts, since risk averse investors will require a compensation for holding an asset whose price declines during recessions, when marginal

utility is high. In a model in which risk and stochastic discounting both matter, the price of a zero-coupon bond computed under the assumption of risk neutrality is therefore higher than the price obtained using the stochastic discount factor of a risk averse agent. Since risk adjustments reduce asset prices, risk aversion increases the yield of a long-term bond.

The mean term premium, which needs to be expressed in percent per annum, is then the difference between the yield to maturity of the bond and that obtained under the assumption of risk neutrality:

$$E(tp_t) = E\left(\left(y_t^{40}\right)^{\frac{1}{40}} - \left(\widehat{y}_t^{40}\right)^{\frac{1}{40}}\right) \cdot 400$$
(24)

and provides a measure of the effect of risk adjustments on bond yields.

### 4 Parameter selection

The data used to calibrate the model are described in Annex D. Whereas data availability for the eurozone economy can be an issue, in most cases it is possible to find series that start in the late 1990s. For the sake of parsimony, we only introduce two exogenous shocks and technology shocks are the only real source of business cycle fluctuations.

#### 4.1 Deterministic growth rate, capital share and labor supply

The deterministic growth rate of the economy is calibrated using data on population growth, which are available on an annual basis since 1960. Between 1960 and 2018, the average rate of population growth for the country group that is currently forming the eurozone is 0.45% per year, which implies a value for the quarterly growth rate  $\gamma$  of 1.00112. The capital share parameter in the production function of the final output good  $\alpha$  is set to 1/3, which implies a labor share of 2/3. With internal habits, long-term risk aversion increases with the curvature coefficient  $\sigma$  but is independent from the habit parameter (e.g. Constantinides (1990); Boldrin, Christiano and Fisher (2001)). We therefore set the curvature parameter  $\sigma$  to 1. As shown in Jaccard (2018), increasing this parameter does not help to resolve asset pricing puzzles in a production economy with endogenous labor supply.

## 4.2 Frisch elasticity of labor supply and steady state time allocation

The first labor supply parameter  $\psi$  is calibrated to ensure that in the steady state, agents spend about 20 percent of their time on work-related activities, which corresponds to a value for n of 0.2. The curvature parameter v is chosen to imply a value for the Frisch elasticity of labor supply of about 0.7 (e.g. Hall (2009); Chetty, Guren, Manoli and Weber (2011)).

### 4.3 Steady state money supply

The steady state quantity of money available in the economy, which is denoted by  $\overline{M}$ , affects the steady state price level  $\overline{P}$ . If the steady state money supply is doubled, the steady state price level and the quantity of cash holding  $\overline{S}$  double as well, but a change in money supply has no effect on the real side of the economy. Given this invariance of the real economy to the value of  $\overline{M}$ , we set the steady state money supply to 1.

#### 4.4 Matching moment procedure

The remaining 12 parameters are calibrated to match a set of 12 moments that characterize the eurozone business cycle. Since with uncertainty higher-order terms in the Taylor expansion drive a wedge between the deterministic and the stochastic versions of the model, it is necessary to simulate the model and find the combination of parameter values that minimizes the distance between the estimated and simulated moments. Table 1 below reports the combination that minimizes this distance, and the comparison between the model and the data is shown in Table 2.

Table 1: Moment matching procedure, parameter values											
$\beta$	$\kappa$	τ	$\mu$	$\phi_{\pi}$	$\eta$	$\epsilon$	δ	$\sigma_a$	$\rho_a$	$\sigma_M$	$ ho_M$
0.991	0.999	0.97	0.92	2.1	0.52	1.6	0.011	0.0069	0.978	0.0228	0.6

In Table 2,  $g_y, g_c, g_x, g_M, g_D$  and  $g_P$  denote the growth rate of output, consumption, investment, money, deposits and prices expressed in year-over-year growth rates, respectively, where for output the growth rate is computed as  $(\log(y_t) - \log(y_{t-4})) \cdot 100$ .  $\sigma(i_D)$  is the standard deviation of the nominal short-term money market rate,  $E(i_D)$  is the mean short-term rate,  $E(t_P)$  is the mean term premium and  $E(i_L - i_D)$  the mean intermediation spread. Finally, E(l/y) and E(x/y) denote the average loan to output and investment to output ratios, respectively.

#### 4.5 Solution method

The results reported in Table 2 are generated using Dynare (Adjemian et al. (2014)). We use a third-order approximation to the policy function as well as the pruning algorithm developed by AFVRR (2018) to simulate a sample of 200,000 observations.

## 4.6 Relation between structural parameters and model-implied first and second-order moments

It is difficult to associate each structural parameter with only one moment as most parameters have a significant impact on the entire system through general equilibrium effects. Some parameters do however have larger effects on a subset of model implications. In this flexible price version of the model, technology shocks are the main drivers of business cycle aggregates, and account for nearly all the variation in output. The technology shock standard deviation parameter  $\sigma_a$  can therefore be associated with the volatility of output.

The capital adjustment cost parameter  $\epsilon$  controls the supply elasticity of capital and has a first-order impact on the volatility of investment. Since capital is the main asset available to transfer wealth across time, this parameter also affects the ease at which the economy's storage technology can be used to achieve consumption smoothing. When combined with habit persistence, the degree of which is captured by the parameter  $\tau$ , the capital adjustment cost parameter also impacts the volatility of marginal utility which in turn affects how agents discount future payoffs. The model's ability to match the average term premium therefore critically depends on these two parameters.

Agents' propensity to save also depends on whether technology shocks, which are the main source of business cycle fluctuations, are perceived as temporary or permanent. The mean term premium and the volatility of consumption and investment, denoted by  $E(tp), \sigma(g_x)$  and  $\sigma(g_c)$ , can thus be associated with the capital adjustment costs, habit, and shock persistence parameters  $\epsilon$ ,  $\tau$ , and  $\rho_a$ , respectively.

Fluctuations in the monetary aggregate M are mostly driven by the monetary policy shock, which illustrates that this moment helps to identify the monetary policy shock standard deviation  $\sigma_M$ . The preference parameter  $\kappa$  determines the weight of real cash balances in the utility function. The volatility of deposits  $std(g_D)$  is therefore particularly sensitive to this parameter value. In a model in which supply shocks are the main drivers of business cycle fluctuations, inflation can be a main source of risk for bondholders. As a result, whereas the inflation coefficient in the monetary rule  $\phi_{\pi}$  has a first-order impact on the volatility of prices  $g_P$ , this parameter also affects the model's asset pricing implications. The steady state value for the subjective discount factor  $\beta$  determines agents' intertemporal choices, and has a significant effect on the mean term premium and the volatility of consumption and investment. Since this parameter also has a direct impact on interest rates, it can be used to pin down the mean short-term rate  $E(i_D)$ .

		Model		
	95% confidence	Estimated empirical	Simulated moments	
	interval	moments	(3rd order)	
$std(g_y)$	[1.6, 2.1]	1.8	1.8	
$std(g_c)$	[0.9,  1.2]	1.0	0.9	
$std(g_x)$	[5.0,  6.6]	5.7	5.6	
$std(g_M)$	[2.8,  3.8]	3.3	3.3	
$std(g_D)$	[1.8, 2.4]	2.0	2.0	
$std(g_P)$	[0.8,  1.1]	0.9	0.9	
$std(i_D)$	[1.8, 2.4]	2.0	2.5	
$E(i_D)$	[2.1, 2.6]	2.4	2.4	
E(tp)	[0.5,  0.9]	0.7	0.6	
$E(i_L - i_D)$	[2.2, 2.4]	2.3	2.3	
E(l/y)	[0.88,  0.95]	0.91	0.91	
E(x/y)	[0.21,  0.22]	0.22	0.21	

Table 2: Model vs. data

Combining labor or investment frictions with habit formation also has a strong impact on the volatility of short-term rates (Jermann 1998; Boldrin et al., 2001). This moment can thus be associated with the habit formation and capital adjustment cost parameters.

As illustrated by equation (15), the magnitude of the intermediation spread  $E(i_L - i_D)$ is pinned down by the technology parameter in the production function of loans  $\eta$ . The parameter  $\mu$  measures the fraction of total costs that firms need to pay in advance. This parameter therefore determines the quantity of credit needed to operate firms, and mainly affects the steady state importance of bank-based financing in the economy. This parameter can thus be associated with the loan-to-output ratio E(l/y). The amount of output that is invested critically depends on the rate at which capital depreciates, and  $\delta$  is therefore identified by including the mean investment to output ratio in the loss function E(x/y).

The last remaining parameter, the monetary policy smoothing parameter  $\rho_M$ , is poorly identified. In particular, it is difficult to distinguish its effect on the model dynamics from that of the shock standard deviation  $\sigma_M$ . This parameter however affects the volatility of inflation, interest rates, and monetary aggregates.

## 5 Results

As will be shown in the next sections, the model's ability to generate asymmetries critically depends on the effective lower bound on interest rates. To provide a realistic quantitative assessment, it is thus important to pay particular attention to the dynamics of nominal variables.

As illustrated in Table 2, the mean as well as the volatility of the short-term nominal rate can be broadly reproduced. It should be noted that, for the period under consideration, the volatility of the short-term nominal rate is 2%. Including a longer sample, and in particular the 70s and 80s would increase the estimated value for the volatility of the nominal rate  $i_D$ . The facts documented by Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019) also suggest that risk-free rates could be more volatile than typically assumed.

At the same time, a higher risk-free volatility would amplify the asymmetry stemming from the lower bound. The rather conservative value that is targeted in Table 2 therefore ensures that the importance of our mechanism is not overstated.

Since this version of the model abstracts from nominal rigidities, it is also important to ensure that the volatility of prices is in line with the data. In this respect, the fact that the volatility of monetary aggregates  $std(g_M)$ , inflation  $std(g_P)$ , and deposits  $std(g_D)$  can be matched is reassuring.

The moment matching procedure described in Section 4 only assigns a very small value to the utility share of money, i.e.  $1 - \kappa = 0.001$ . This small value implies that agents hold on average 2% of their total liquid wealth in cash and 98% in the form of bank deposit.

The model generates a term premium of around 0.6%. This value is close to that estimated by Hördhal and Tristani (2012, 2014) using euro area data for the period from 1999 to 2018.<sup>6</sup> It is however lower than the value targeted in Rudebusch and Swanson (2012) or AFVRR (2018) for the United States.

Using data from 2005 to 2017, Wu and Xia (2017) find an average value for the term premium of 1%. Since the risk premium has been declining over time, the difference with Hördhal and Tristani can be explained by the different samples used in the two studies.

One of the key model parameters that determines the effectiveness of monetary policy is denoted by  $\mu$ . Given that this parameter measures the economy's credit dependence, it can be identified using data on credit to GDP ratios made available by the Bank for International Settlements. In line with the decentralized equilibrium described in Section 3, the empirical value for the loan-to-output ratio E(l/y) only includes credit to non-

<sup>&</sup>lt;sup>6</sup>I thank the authors for having provided an updated estimate of their results.

financial corporations obtained by banks, and excludes credit to households or other forms of non-bank source of funding.

It is also possible to match the average spread between bank lending rates and the money market rate  $E(i_L - i_D)$ . Due to a lack of data availability, it is only possible to compute this spread for a sample period that starts from the first quarter of 2000 onwards. The lending rate data used to compute this spread is the cost paid by non-financial corporations to obtain a new loan corresponding to amounts smaller than 1 million euros and for a period of less than a year. Since the representative firm is a SME, using a measure of interest rate on loans smaller than 1 mio is a better proxy of the cost of funding than the rate paid on larger loans. Finally, the fact that the investment share of output E(x/y) can be matched, also ensures that the high volatility of investment that we obtain is not due to a steady state effect.

Whereas the model is able to match the stylized facts reported in Table 2, it is important to keep in mind that it fails on some other dimensions. For instance, although the volatility of deposits can be reproduced, it is not possible to simultaneously account for the large fluctuations in credit observed in the data. Moreover, the model understates the volatility of hours worked, a limitation which suggests that incorporating both an extensive and an intensive margin of labor supply would be necessary to match the data. Since the model abstracts from unemployment, it also cannot be used to study the link between different measures of slack and inflation (e.g. Den Haan, Rendahl and Riegler (2015); Stock and Watson (2018)).

#### 5.1 What determines the term premium?

In order to isolate the contribution of inflation risk to the term premium, let us start by deriving the pricing equation that corresponds to an inflation-indexed long-term bond, the price of which is denoted by  $p_{Rt}^{(n)}$ . Relative to the formula shown in equation (23), expected inflation no longer affects the valuation of an inflation indexed long-term bonds. Consequently, we have the following pricing condition:

$$p_{Rt}^{(n)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{Rt+1}^{(n-1)}$$

where  $p_{Rt}^{(0)} = 1$ . Following the definition of term premium given by equation (24), a risk neutral measure can be obtained by firstly deriving a formula for the real risk-free rate, which we denote by  $i_R$ :

$$\lambda_t \frac{1}{1+i_{Rt}} = \beta E_t \lambda_{t+1}$$

The term premium on the inflation-indexed bond, which is denoted by  $tp^R$ , is then determined by the difference between the yield computed using the stochastic discount factor of the risk averse agent minus the risk-neutral measure. Since  $tp^R$  abstracts from inflation risk, the inflation risk premium  $\pi_{RP}$  can be defined as the difference between the term premium on a nominal and an inflation-indexed long-term bond:

$$E(\pi_{RPt}) = E(tp) - E(tp^R)$$

This measure therefore captures the impact of expected inflation on the compensation required by investors for holding a nominal asset subject to inflation risk.

1		1
	Data	Model (3rd order)
$corr(g_y, E_t(\pi_{t+3}))$	0.19	0.22
$std\left(E_t(\pi_{t+3})\right)$	0.30	0.21
$E(\pi_{RPt})$	-	-0.07

Table 3: Inflation expectations and inflation risk premium

Since the inflation risk premium critically depends on the cyclical properties of expected inflation, it is important to first understand how this measure varies over the business cycle. Table 3 uses the measure of expected inflation from the ECB survey of Professional Forecasters and reports its correlation with output growth. Since the measure reported in this survey corresponds to inflation expectations three quarters ahead, the model-based measure of inflation expectation is computed as follows:  $E_t(\pi_{t+3}) = E_t((P_{t+3}/P_{t+2}-1)\cdot 400)$ . Both in the model and in the data, the correlation between output growth and inflation expectations is shown in the first row of Table 3, whereas the second row reports the standard deviation of inflation expectations, i.e.  $std(E_t(\pi_{t+3}))$ .

Although this moment was not targeted, the first main result that stands out is that the model is able to match the correlation between inflation expectations and output growth observed in the data. Moreover, as illustrated by the second row of Table 3, it is possible to generate fluctuations in the three quarters ahead expected inflation rate of a magnitude that is broadly in line with the one observed in the data.

The average inflation risk premium predicted by the model is shown in the last row of Table 3. Interestingly, the co-movement between expected inflation and output growth observed in the data, and reproduced by the model gives rise to a negative inflation risk premium of 7 basis points. In other words, inflation contributes to reduce, rather than increase, the risk associated with an investment in long-term bonds.

Seen through the lens of this model, the fact that euro area inflation expectations are slightly procyclical is therefore a source of risk reduction. To gain intuition into this result, consider the case of a recession, a period in which marginal utility is higher than usual. This positive correlation lowers the risk of investing in long-term bonds because it implies a decline in expected inflation during periods of recession. Since a decline in expected inflation increases the value of a nominal bond, this negative co-movement implies that expected inflation acts as a hedge against consumption risk. As pointed out by some recent studies (e.g. Campbell, Sunderam and Viceira (2016); Camba-Méndez and Werner (2017)), nominal bonds can thus act as "deflation hedges".

If inflation is a source of risk reduction, what explains the model's ability to reproduce a sizable term premium? The answer is real interest risk. In contrast to expected inflation, real interest rates are countercyclical. For the calibration summarized in Table 1, the model-generated correlation between output growth and the real interest rate  $i_R$  is -0.3.

Since bond prices and real interest rates move in opposite directions, this correlation implies that bond prices fall during recessions. Long-term bonds are therefore a bad hedge against consumption risk. Indeed, in this economy, holders of long-term bonds suffer capital losses during periods of recession, precisely when marginal utility to consume is high. In spite of the negative inflation risk premium implied by expected inflation, real interest rates risk therefore explains why long-term bonds are risky in this model.

## 6 The real effects of monetary policy

A positive monetary policy shock takes the form of an exogenous increase in the innovation  $\varepsilon_M$  to the policy rule in equation (10). A positive shock increases the quantity of money in circulation and, although a fraction is absorbed by households who derive utility from holding money in cash, the supply of funds deposited in the banking sector increases. This increase in deposits is then channeled to firms in the non-financial sector in the form of short-term credit. The resulting increase in the supply of loanable funds in turn reduces the rate at which firms borrow from the banking sector. Finally, a decline in the cost of funding stimulates the demand for labor and capital through its effects on equations (16) and (17).

Since the model's state space is nonlinear, the response to shocks can be influenced by the particular point from which the impulse response is computed. To capture this potential state dependence, the impulse responses shown in this section are computed using a thirdorder approximation to the policy function. The results reported in Figure A.1 to Figure A.4 represent the average effect of a monetary policy shock. In each case, it is obtained by calculating an impulse response starting from many different points in the state space and by computing the average response following Adjemian et al. (2014). These responses correspond to the case in which monetary policy shocks are the only source of fluctuations.

As illustrated by the top left panel of Figure A.1, a positive innovation to the money supply rule increases M relative to steady state by around 1.5% on impact. Since it shifts the money supply curve to the right, equilibrium in the money market implies a reduction in the money market rate  $i_D$ , which in annualized terms declines by around one percentage point relative to its steady state value. As illustrated by the negative relationship between prices and the short-term deposit rate shown in equation (21), a decline in the nominal short-term rate has a positive impact on the price level P, which increases by around 0.4% on impact. The increase in credit L, which is shown in the bottom right panel, is less than proportional than the increase in money supply, which reflects that part of the effect of the shock is absorbed by households who increase their cash holdings S. Since the response of credit also depends on the efficiency of the financial intermediation technology, banks are only able to channel a fraction of the funds they receive to firms. This explains why a positive shock that raises money supply by 1.5% on impact only leads to an average increase in credit by about 1%.

The real effects of monetary policy are illustrated in Figure A.2, which shows the response of output y, investment x, hours worked n, and consumption c in percentage deviation from steady state. As can be seen in the top left panel, a monetary policy shock that increases money supply by 1.5% and reduces the nominal short-term interest rate relative to steady state by one percentage point raises output on impact by around 0.15%. The shock has a stronger effect on hours worked, the increase of which exceeds 0.2% on impact. This stronger effect on hours worked contrasts with the considerably more muted increase in investment.

The expansionary monetary policy shock stimulates aggregate consumption, the increase of which reaches about 0.2% on impact. The positive impact on consumption can firstly be explained by the substitution effect induced by the decline in the short-term real rate depicted in the upper right panel of Figure A.4. Moreover, since wages and the income from renting capital to firms also increase, this substitution effect is reinforced by a positive income effect that contributes to stimulate consumption on impact.

The two upper panels of Figure A.3 show the response of the real wage and marginal

productivity of capital. The increase in the price of inputs confirms that a positive monetary policy shock stimulates the demand for factors, as quantities and prices both rise.

The response of the real quantity of credit L/P and the lending spread  $i_L - i_D$  is shown on the two lower panels of Figure A.3. As shown on the lower left panel, the main effect of monetary policy is to stimulate the production of real credit. On impact, the increase in credit reaches around 0.6%. A higher quantity of credit relaxes the financing constraint (8), which in turn reduces the cost of borrowing for firms through equation (18).

Monetary policy shocks generate a negative co-movement between lending spreads and real credit. A main effect of monetary policy is therefore to reduce lending spreads. This effect of lending spreads is particularly relevant for the euro area. Indeed, in the eurozone, the intermediation spread is strongly negatively correlated with real credit growth. Periods of high credit growth are associated with low levels of the intermediation spread. In contrast, these spreads increased during the Subprime and Sovereign Debt Crises.

The response of the term premium to a positive monetary shock is shown in the upper left panel of Figure A.4. It is necessary to resort to a third-order approximation, at least, in order to generate time-variation in the term premium. This reflects that the term premium is determined by the covariance between the stochastic discount factor and the return on a long-term bond.

The very small effect of monetary policy on the term premium illustrates that it is generally difficult to generate large variations in conditional second moments within this class of models. At the same time, the decline in the term premium obtained in response to an expansionary monetary policy shock is consistent with the view that positive monetary policy shocks reduce risk premiums.

As already mentioned, the short-term real rate declines in response to a positive shock (see the upper right panel of Figure A.4). The model is therefore able to reproduce the liquidity effect documented by many empirical studies (e.g. Christiano and Eichenbaum (1995)). Since nominal rigidities are typically needed to reproduce this effect, the fact that the model passes this important test is reassuring.

As shown by the lower left panel of Figure A.4, the price of a zero-coupon nominal bond with a 10 year maturity is also sensitive to monetary policy. Relative to its steady state value, the shock increases bond prices by around 0.5% in response to an expansionary shock. Finally, the yield on a corresponding nominal long-term bond declines by around 5 basis points on impact.

#### 6.1 Credit frictions as a source of non-neutralities

In this environment, the loan-in-advance constraint given by equation (8) is the only source of monetary policy non-neutrality. Whether a change in M has real effects therefore critically depends on the credit friction parameter  $\mu$ . If  $\mu$  is set to zero, the demand for loanable funds given by equation (19) falls to zero and a monetary policy shock has no effect on the real quantity of credit demanded by firms. To illustrate this point, Figure A.6 firstly shows how the level of  $\mu$  affects the steady state of the economy. The sensitivity analysis performed in this chart shows how the loan-to-output and cash holding ratios, i.e. E(l/y)and E(S/M), respectively, vary with the financing frictions parameter  $\mu$ .

As  $\mu$  approaches zero, the loan-to-output ratio, which is depicted by the blue diamonded line, tends towards zero, the model reducing to a creditless economy in this limiting case. Since the demand for credit becomes negligible as  $\mu$  approaches zero, the degree of credit frictions also affects the allocation of money between cash S and deposits D. In Figure A.5, this is illustrated by the red crossed line, which shows how a change in  $\mu$  affects the average share of liquid wealth that households choose to hold in the form of cash. A lower dependence on credit implies a lower share of bank deposit. In the limit, the ratios D/Mand S/M tend towards 0 and 1, respectively, as  $\mu$  approaches zero, since all the money available in the economy is held in cash. As a result, in the limit, financial intermediation completely disappears and the model reduces to a version of the neoclassical growth model with money in the utility function.

To illustrate that the value of  $\mu$  also has a crucial impact on the model's dynamics, Figure A.6 shows the response to a positive monetary policy shock in the case in which the financial friction parameter  $\mu$  is set to 0.001. As shown by the two upper panels, in this limiting case, the change in cash holding S is almost exactly proportional to the change in M. In terms of the portfolio allocation constraint (3), this implies that the shock has a negligible impact on the quantity deposited in the banking sector and thus on credit creation. As shown by the response of output in the lower right panel, the monetary policy shock is almost neutral in this case, which reflects that a change in lending conditions has essentially no real effects without financing frictions under flexible prices.

#### 6.2 The labor wedge

Whereas a variation in the cost of lending directly affects the demand for both capital and labor, the transmission mechanism operates primarily through the labor market. The importance of the labor market can be illustrated by considering the case in which the constraint only depends on capital. If wages do not need to be paid in advance, the loanin-advance constraint takes the following form:

$$\frac{L_t}{P_t} \ge \mu r_{Kt} k_t$$

and a monetary policy shock no longer affects the labor demand equation.

In Figure A.7, the red dotted line shows the response of output to a monetary policy shock in the model without a labor wedge. This comparison demonstrates that, without a labor wedge, the model loses most of its ability to generate non-neutralities.

## 7 The state dependence of monetary policy transmission

Since monetary policy is transmitted to the real economy by affecting firms' borrowing costs, the transmission mechanism critically depends on the central bank's ability to control lending rates. By varying the quantity of liquidity it supplies, the central bank has a direct impact of the deposit rate, which in turn affects banks' lending rates. This ability to control market rates however depends on how agents react to a change in the monetary policy stance. In particular, the fact that households can choose between depositing money in the banking sector or keeping money in cash interferes with the conduct of monetary policy.

The impact of this preference for liquidity on the transmission mechanism can be illustrated by analyzing the equilibrium condition in the money market implied by equation (14). Keeping P and c constant at their steady state value, and using the calibration summarized in Table 1, this relationship is plotted in Figure A.8, which shows  $i_D$  on the vertical axis as a function of cash balances S. For the calibration that reproduces the moments shown in Table 2, the average short-term nominal money market rate  $i_D$ , in annualized terms, stands at 2.4%. An average interest rate of 2.4% implies a quantity of money held in cash of about 0.02. In terms of the portfolio allocation, households therefore hold on average around 2% percent of their liquid wealth in cash and the remaining 98% in the banking sector in the form of bank deposits.

An interesting consequence of this inverse relationship between cash holdings and the deposit rate is the potential asymmetry that it entails. Indeed, for very low levels of interest rates, the demand for cash flattens and converges towards one. What this means is that households can always choose to withdraw money from their deposit account and keep
money in cash if the remuneration on their deposit account is too low. In the limit, the share of money invested in cash reaches 100%, as the deposit rate  $i_D$  approaches zero.

This nonlinear relationship formalizes the idea that the existence of cash creates a lower bound on deposit rates that constrains the transmission of monetary policy. To gain intuition into this result recall that equation (3) implies that, for a given quantity of liquidity in the system, an increase in cash holdings reduces the quantity of funds deposited in the banking sector D. In other words, the possibility to hold cash instead of bank deposits limits the room for manoeuvre of the central bank by preventing deposit rates from turning negative. Indeed, households will always strictly prefer to hold cash rather than incur a cost for depositing their money in the banking sector.

Consider for instance the case of a very aggressive injection of liquidity occurring during a period of low interest rates. Clearly, as illustrated by the nonlinear relationship between  $i_D$  and S depicted in Figure A.8, once the flatter portion of the cash demand curve is reached, any further increase in liquidity only has a small effect on deposit rates. In terms of portfolio allocation, this reflects that the supply of deposits D, and hence L, becomes insensitive to changes in the policy instrument M, when the flat portion of the demand for cash schedule is reached. Since in this case the quantity of deposits remains unaffected by the shock, a change in monetary policy only has a muted effect on the supply of credit. Indeed, when the opportunity cost of holding cash is so low, agents have no incentives to channel any further increase in M to the real economy. Consequently, at the effective lower bound, an increase in the quantity of money in circulation is absorbed by a proportional increase in cash holdings and has negligible real effects.

# 7.1 Deconstructing the mechanism

To better understand how the demand for cash affects the transmission mechanism of monetary policy, it is useful to illustrate how initial credit conditions impact the effectiveness of a monetary policy shock. The equilibrium in the credit market is represented in Figure 1 where the credit supply and credit demand curves given by equations (20) and (19) are depicted. In both panels, the red dotted lines represent the demand curves corresponding to equation (19). These two demand curves are obtained by using the value for the parameter  $\mu$  taken from Table 1 and by fixing P, c, and y at their steady state values.

In the left panel, the blue continuous line shows the initial credit supply curve in the case in which the quantity of money available in the economy M stands below its average value. When monetary conditions are tighter than usual, the credit supply curve is also steeper. This in turn implies a higher lending rate and a lower equilibrium quantity of

credit. The blue dashed line shows how a 1% increase in money supply shifts the credit supply curve when the economy is in this state of below average equilibrium quantity of credit.

In the right panel, the blue continuous line represents an initial credit supply curve that corresponds to a case in which the quantity of money available in the economy is higher than average. The initial state of the economy is therefore one in which the equilibrium quantity of credit is above average, which in turn implies lending rates that are lower than usual. The blue dashed line illustrates how a 1% increase in the money stock shifts the credit supply curve in this case.

As can be seen by comparing the blue continuous lines across the two panels of Figure 1, the first difference is that the initial supply curve is flatter in the high credit state. Second, a 1% shock to money supply, which in each panel is determined by the distance between the blue continuous and blue dashed lines, has a much smaller effect on equilibrium quantities and prices in the high money, high credit state. The much lower effect obtained in the high credit state illustrates the limits of monetary policy in a world in which agents can choose to hold cash. When the flat portion of the cash demand curve shown in Figure A.8 is reached, any further increase in money supply is hoarded in the form of cash instead of being deposited in the banking sector.



Figure 1. Equilibrium in the credit market in response to a 1% increase in money supply in the high and low credit states. x axis: Credit L. y axis: Deposit rate  $i_D \cdot 400$ .

The small effect of monetary policy on amounts deposited in the banking sector can be explained by the low opportunity cost of holding cash when the deposit rate paid to households becomes sufficiently low. Without additional funding, the banking sector is unable to increase lending to the productive sector. Therefore, the credit supply curve becomes insensitive to changes in monetary policy. As a result, and as can be seen by comparing the blue continuous and blue dashed lines shown in the right panel, a 1% change in money supply only has a negligible effect on the credit supply curve in this case.

The low effectiveness of monetary policy obtained in the high credit state contrasts with the much stronger effect depicted in the left panel, which corresponds to the low credit state. In terms of the cash demand curve depicted in Figure A.8, the difference is that the demand for cash schedule becomes steeper when monetary conditions are tighter than average. This nonlinearity reflects the notion that the demand for cash S becomes less sensitive to variations in interest rates when the opportunity cost of hoarding cash is high. In other words, any change in money supply M has a stronger effect on deposits Din this case because the demand for cash becomes less sensitive to monetary policy. Since a change in the quantity of deposits available in the economy in turn shifts the credit supply curve, the effectiveness of monetary policy increases when agents have a reduced incentive to hoard cash.

# 7.2 State dependent impulse response analysis

The next step is to provide a quantitative evaluation of the nonlinearity implied by this mechanism. In this section, we study the response of output and prices to a one standard deviation monetary policy shock by differentiating between periods of low and high credit availability. This is achieved by firstly generating 100,000 different trajectories for all the variables, including real credit L/P, by simulating the model with the two exogenous shocks using a third-order approximation as well as the pruning algorithm of AFVRR (2018). Since each trajectory depends on a sequence of random shocks, the result of this simulation can be used to generate a distribution of realized values for real credit in a given period, say period 100. This allows us to identify states of the economy in which credit is higher or lower than average. The next step is to compute an impulse response for the other variables of interest, in our case output, that corresponds to states of the economy in which credit is higher or lower than average.

Following Adjemian et al. (2014), an impulse response for any variable is computed as the difference between the initial simulated series and the same series that is subject to a one standard deviation shock. The impulse response is then computed as the difference, from period 100 onwards, between the initial simulated series, for say output, and the same series that was hit by a positive monetary policy shock. Periods of low and high credit availability can then be identified by selecting the set of impulse responses for output to a monetary policy shock that corresponds to states of the economy in which credit is below are above average.<sup>7</sup>

In Figure 2, the blue continuous line shows the impulse response of output to a positive money supply shock in the credit stress state. The credit stress state corresponds to impulse responses that were computed in states of the economy in which credit lies within the lower tercile of the distribution. In other words, the blue continuous line represents the average impulse response of output that is obtained in states of the economy in which the quantity of credit in the economy is less than or equal to 33% of all realized observations.



Figure 2. Response of output to a positive money supply shock in the low and high credit states.

Similarly, we define the high credit state as the case in which the realized value for credit is included in the upper tercile of the distribution. The red dotted line therefore depicts the average response of output to a positive monetary policy shock that corresponds to the ample credit state. The ample credit state corresponds to states of the economy for which credit takes values greater than or equal to 66% of all realizations.

When looking at Figure 2, what immediately stands out is the asymmetry in the response of output to a monetary policy shock between the high and low credit states. On

<sup>&</sup>lt;sup>7</sup>The impulse response analysis shown in this section has greatly benefited from comments and suggestions from M. Juillard whose invaluable help is gratefully acknowledged. Any remaining error is our own responsibility.

impact, the exact same monetary policy shock generates an increase in output of around 0.05% in the high credit state, as compared to an increase that is more than 3 times greater in the credit stress regime.

We also find that the magnitude of this asymmetry is less pronounced for the price level than for output. The reason is that the dynamics of prices is directly affected by the stochastic discount factor. As we discuss in Jaccard (2018b), the presence of habits gives rise to state dependence in the stochastic discount factor that attenuates the response of prices during periods of recession.

# 7.3 Monetary asymmetries in the absence of risk

To illustrate the importance of risk, we now study this asymmetry in a version of the model that cannot match the term premium observed in the data. It is well-known that standard models cannot generate term premiums of a realistic magnitude (e.g. Rudebusch and Swanson (2008)). In our case, we overcome this challenge by combining a specification of habit formation in the composite of consumption, money and leisure with capital adjustment costs.

Without habits, a case which can be obtained by setting the habit parameter to 1, the term premium falls to essentially zero. Interest rate risk disappears because nominal interest rates and inflation become too smooth relative to the data. Matching the volatility of interest rates and inflation is essential because the effective lower bound becomes irrelevant if the average interest rate produced by the model is too high or if interest rates are not sufficiently volatile. At the same time, the impact of the effective lower bound on the transmission mechanism can be overstated if interest rates are on average too low or too volatile relative to the data.

To illustrate this point, the exercise illustrated in Figure 2 is repeated using the version of the model without habits that generates a zero term premium. As shown in Figure A.9, without realistic amounts of interest rate risk, the asymmetry essentially disappears as the two lines become indistinguishable.

One main takeaway is therefore that the model's ability to generate asymmetries is tightly linked to its asset pricing predictions. Consequently, resorting to nonlinear techniques is not sufficient to detect asymmetries. Indeed, a well-documented limitation of standard models is their tendency to generate mean interest rates that are too high (e.g. Weil (1989)) and not sufficiently volatile. The risk, therefore, is to understate the importance of asymmetries in the transmission mechanism by overlooking some key macro-finance implications of monetary policy models. Our results suggest that the question of asymmetries should be addressed in models capable of matching some basic facts that not only include the mean and standard deviation of interest rates, but also a measure of risk such as the term premium.

# 8 Robustness analysis

For the sake of parsimony and to keep the analysis as transparent as possible, the main result of the paper, which is illustrated in Figure 2, is derived in the simplest possible framework. In this section we assess the extent to which our results are robust to more general assumptions. In particular, following most of the literature on monetary economics, we ask whether asymmetries are still present in a version of the model augmented with price stickiness. We then discuss the case of a more general utility specification with constant elasticity of substitution, provide empirical evidence in support of our mechanism, study additional demand shocks, and provide a measure of the welfare cost of business cycle fluctuations.

# 8.1 Introducing price stickiness

In contrast to the vast majority of models used to study monetary policy, the analysis developed so far abstracts from any source of nominal rigidities. A natural question to ask as this stage is whether our mechanism still plays a role once embedded into the workhorse New Keynesian model. Given that we introduce price stickiness using a completely standard textbook approach (i.e. Galì (2015); Walsh (2017)), the New Keynesian version of the model is shown in the online appendix.

## Parameter selection

Relative to the model analyzed in the previous section, we have two new parameters to calibrate. Since these two parameters are standard, we use values that are widely accepted in the New Keynesian literature. First, for the price elasticity of demand parameter  $\vartheta$ , we follow Galì (2015) and set it to 9. This implies a steady state markup of 12.5%. Second, the nominal rigidity parameter  $\xi$  determines the average price duration. Following Galì (2015), we set this parameter to 0.75, which implies an average price duration of four quarters.

The set of calibrated parameters are kept at the values discussed in Section 4. Following the approach described in Section 3.4, the remaining parameter values are selected to maximize the model's ability to reproduce the set of macroeconomic and financial variables shown in Table 2.

In the version with Calvo price stickiness, we find that the model is still broadly able to reproduce these moments. In this case, the loss function is minimized for the following set of parameter values:

Table 4: Moment Matching Procedure, Model with Nominal Rigidities											
$\beta$	$\kappa$	au	$\mu$	$\phi_{\pi}$	$\eta$	$\epsilon$	δ	$\sigma_a$	$ ho_a$	$\sigma_M$	$ ho_M$
0.99	0.999	0.97	0.98	1.2	0.49	1.4	0.015	0.0073	0.977	0.018	0.6

The comparison between the model implied simulated moments and the data is reported in Table B.1 in the appendix. Overall, it is fair to say that the introduction of price stickiness does not significantly impair the model's ability to reproduce these moments. Relative to the benchmark model, the first main difference is that in a model with price rigidities a lower coefficient in the monetary policy rule  $\phi_{\pi}$  is needed to match the volatility of inflation. Since price stickiness dampens the response of inflation, a less aggressive reaction to variations in prices is needed to match this moment. The second main difference is that the monetary policy shock standard deviation  $\sigma_M$  is about 25 percent lower than in the flexible price model. Since nominal rigidities are a source of monetary policy non-neutrality, the effect of monetary policy shocks are amplified and shocks of a smaller magnitude are needed to reproduce these facts.

#### The state dependence of monetary policy transmission with price stickiness

The response of output to a one standard deviation monetary policy shock in the low and high credit states is shown in Figure B.1. The takeaway is that the asymmetry implied by our mechanism is robust to the introduction of Calvo staggered contracts, which is a very widely used specification of nominal rigidities in the New Keynesian literature. Indeed, as illustrated in in Figure B.1, the response of output to a monetary policy shock remains considerably stronger in the low credit state. Relative to the response obtained in the benchmark model shown in the left panel of Figure 2, the main difference is that monetary policy has a stronger effect on output. Although the monetary policy shocks standard deviation is about 25% smaller in the New Keynesian version, the increase in output in the low credit state reaches 0.3%. In contrast, in the flexible price model the maximum impact on output stands below 0.2%.

## The persistence of real and nominal variables

A limitation of the flexible price model is that it fails to reproduce the high autocorrelation of inflation observed in the data. Indeed, the growth rate of both inflation and output, as measured by the 4-quarter change in logs, exhibits a high degree of persistence. As can be seen in Table 5 below, whereas the flexible price benchmark generates persistence in output growth, the high autocorrelation of inflation cannot be reproduced.

Introducing Calvo staggered contracts considerably improves the model's ability to reproduce this high autocorrelation coefficient. Since a fraction of firms cannot adjust prices each period, nominal rigidities considerably increase the persistence of inflation, the autocorrelation coefficient of which increases from 0.48 in the flexible price benchmark to 0.86 in the New Keynesian version.

Table 5: The persistence of real and nominal variables					
	Data	Flexible Price Model	Sticky Price Model		
$\rho(\boldsymbol{g}_{yt},\boldsymbol{g}_{yt-1})$	0.88	0.75	0.84		
$\rho(g_{Pt}, g_{Pt-1})$	0.90	0.48	0.86		

Table 5.  $\rho(g_{yt}, g_{yt-1})$  and  $\rho(g_{Pt}, g_{Pt-1})$  denote the autocorrelation coefficients of output growth and inflation, respectively.

# 8.2 CES utility function

Another potential limitation of the baseline model is that we assume a Cobb-Douglas specification for the utility function. This particular utility function in turn implies a specific relationship between the deposit rate  $i_D$  and cash S (see equation (14)). In this section, we assess whether our mechanism is robust to a more general utility function with constant elasticity of substitution (CES) by considering the following specification:

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left( \left[ \kappa c_t^{1-\frac{1}{\iota}} + (1-\kappa) \left(\frac{S_t}{P_t}\right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}} (\psi + z_t^{\upsilon}) - h_t \right)^{1-\sigma}}{1-\sigma}$$

This specification in turn implies the following money demand function:

$$i_{Dt} = \frac{(1-\kappa)}{\kappa} \left(\frac{P_t c_t}{S_t}\right)^{\frac{1}{\iota}}$$
(25)

The parameter  $\iota$  denotes the constant price elasticity of demand<sup>8</sup> and the Cobb-Douglas case is obtained when  $\iota$  is set to 1. It is important to acknowledge that the elasticity parameter  $\iota$  has a major impact on the model's ability to generate asymmetries. Indeed, if  $\iota$  is set to 10, the cash demand schedule shown in Figure A.8 becomes flat. Consequently, without the curvature implied by the Cobb-Douglas specification, the model could potentially lose its ability to generate asymmetries. In contrast, for values of  $\iota$  lower than 1, the nonlinearity of the cash demand schedule is preserved.

## Using external evidence to calibrate $\iota$

Given the importance of this parameter for our results, we use available evidence to assess whether the assumption of a unit elasticity can be supported by the data. Starting from equation (25), we estimate the following equation:

$$\log i_{Dt} = \alpha_0 + \alpha_1 \log \left(\frac{P_t c_t}{S_t}\right)$$

For the deposit rate, we use data on new deposits by households with a maturity less than a year, which is the closest empirical counterpart for  $i_D$ . Nominal consumption  $P_tc_t$  is proxied by final consumption expenditures. Data on currency in circulation can be used as an empirical counterpart for  $S_t$ , which denotes cash held by households. We first estimate this equation using monthly data for the period from 2002 to 2018 and then add the COVID-19 period.<sup>9</sup>

Table 6:	Estimated money	demand elasticity
Sample	2002q1-2018q1	2002q1- $2022$ q3
$\widehat{\alpha}_1$	1.38	2.5
$95\%~{\rm CI}$	[0.87; 1.89]	[1.99; 3.0]

As shown in the first column of Table 6, our results suggest that, in the pre-COVID period, the Cobb-Douglas specification cannot be rejected by the data. Indeed, the estimated value for  $\hat{\alpha}_1$  is 1.38 and we cannot reject the hypothesis that  $\frac{1}{\iota} = 1$ . If we add the COVID-19 period, we obtain a value for  $\hat{\alpha}_1$  of 2.5, which therefore implies a value for  $\iota$  of

0.4.

<sup>8</sup>With this specification, we have that:

$$-\frac{\partial S_t}{\partial i_{Dt}}\frac{i_{Dt}}{S_t} = \iota$$

<sup>9</sup>In the euro area banknotes were introduced physically in January 2002.

The main takeaway is that a value for  $\iota$  lower than one, as suggested by the estimated value obtained for the full sample, does not impair the model's ability to generate asymmetries. To illustrate this point, the impulse response in Figure B.2 is simulated using a value for  $\iota$  of 0.4 in the version of the model with monopolistic competition and price stickiness. Clearly, the state dependence of monetary policy transmission is robust to the introduction of nominal rigidities as well as a more general utility function.<sup>10</sup>

# 8.3 Supportive empirical evidence

Having established that the model mechanism under study is robust to the introduction of nominal rigidities as well as a more general utility specification, the objective of this section is to assess whether some empirical support can be provided. This is achieved by exploiting the model structure to derive a testable implication. First, given the policy rule that we have specified in equation (10), we can construct an empirical counterpart for the monetary policy shock by estimating the following regression:

$$\widehat{M}_{1t} - \overline{M}_1 = \beta_0 + \beta_1 \left( \widehat{M}_{1t-1} - \overline{M}_1 \right) + \beta_2 (\widehat{P}_t - \overline{P}) + \widehat{\varepsilon}_{Mt}$$

where  $\widehat{M}_{1t}$  denotes the money supply, as proxied by the monetary aggregate  $M_1$ , and where  $\widehat{P}_t$  is the consumer price index in level.  $\overline{M}_1$  and  $\overline{P}$  denote the sample mean of the money supply and the price level. Data on money supply and the price level are available on a monthly basis. In this regression, the monetary policy shock therefore corresponds to  $\widehat{\varepsilon}_{Mt}$  and is estimated using monthly data by correcting for heteroskedasticity and autocorrelation. We find that the parameter  $\widehat{\beta}_2$  has the expected negative sign, as postulated by the monetary rule specified in equation (10).<sup>11</sup>

The next step is to estimate the effect of the monetary policy shock implied by our theory on financing conditions. Indeed, the asymmetry at the core of our mechanism relies on the effect of monetary policy on bank lending rates. Therefore, one possible way to formally test our hypothesis is to check whether the effect of the monetary policy shock on bank lending rates depends on the state of the economy.

Since monetary policy typically works with a delay, we estimate the cumulated effect of a monetary policy shock over a 12-month period. Indeed, as prices typically adjust after 4

<sup>&</sup>lt;sup>10</sup>Relative to the benchmark model, combining price stickiness with a value for  $\iota$  of 0.4 also allows us to increase the persistence of prices, a dynamics in closer conformity with the available evidence documented in the literature on structural VARs (e.g. Christiano et al. (2005)).

<sup>&</sup>lt;sup>11</sup>The hypothesis that the shock has a unit root can be rejected by performing an augmented Dickey-Fueller test on the estimated residual.

quarters on average, monetary policy should have its largest impact on the economy within this time frame. We then test for state dependence by introducing an interaction term that takes the form of a dummy variables. This dummy variable, which is denoted by  $D_{t1,0}$ , takes a value of one during months in which a recession was observed in the euro area and zero otherwise. We then estimate the following condition, where bank lending rates are denoted by  $\hat{i}_{Lt}$ :

$$\widehat{i}_{Lt} = \beta_0 + \beta_1 \left( \sum_{m=1}^{12} \widehat{\varepsilon}_{Mt-m} \right) + \beta_2 \cdot D_{t1,0} \cdot \left( \sum_{m=1}^{12} \widehat{\varepsilon}_{Mt-m} \right) + \nu_t$$

and where  $\nu_t$  denotes the error term. Our theory firstly implies a negative value for  $\beta_1$ , as expansionary monetary policy shocks should reduce bank lending rates. If the transmission mechanism is indeed state-dependent and monetary policy more powerful during recessions, we should observe a significant and negative value for  $\beta_2$ .

Since in the euro area, SMEs represent about 99% of all firms, and since SMEs typically contract small loans, we use data on interest rates that correspond to loans smaller than 1 million euros. Interest rates on loans with short-term maturities are also the best empirical counterparts for  $i_L$ , which captures short-term financing conditions.

Bank lending rates for the euro area as a whole are only available since January 2000 on a monthly basis. This sample period nevertheless includes 3 recessions, as defined by the Centre for Economic Policy Research (CEPR): (i) the Great Recession from the first quarter of 2008 to the second quarter of 2009; (ii) the Sovereign Debt crisis, from the third quarter of 2011 to the last quarter of 2012; and (iii) the COVID-19 crisis from the first to the second quarter of 2020.

The result of this regression is reported on the first line of Table 7 below, where the interest rate on bank lending rates is the dependent variable. It is reassuring to see that  $\hat{\beta}_1$  not only has the expected sign but is also significant at the 5% level. Importantly, and as illustrated by the second column of Table 7, the interaction term  $\hat{\beta}_2$  has the expected sign and is significant at the 5% level.<sup>12</sup>

<sup>&</sup>lt;sup>12</sup>In this regression, the estimated shock  $\hat{\varepsilon}_{Mt}$  is standardized and has mean zero and a standard deviation of 1.

Table 1. State dependent monetary transmission						
	Dependent	$\widehat{\beta}_1$		$\widehat{\beta}_2$		$R^2$
	variable	Estimated value	95% CI	Estimated value	$95\%~{ m CI}$	
	$\widehat{i}_{Lt}$	-0.098	[-0.158, -0.039]	-0.16	[-0.30, -0.02]	0.22
	$\widehat{i}_{Dt}$	-0.098	[-0.15, -0.04]	-0.18	[-0.32, -0.05]	0.30

Table 7: State-dependent monetary transmission

As illustrated by the second line of Table 7, a very similar conclusion emerges if we use the deposit rate instead of the bank lending rate. In the data, the deposit rate  $\hat{i}_{Dt}$  refers to the short-term deposit rate received by households and non-profit institutions serving households, a series which is also available on a monthly basis from January 2000. Furthermore, we checked that the results are robust if we exclude the COVID-19 period.

While imperfect, this test nevertheless provides some tentative empirical support to our mechanism. The negative value obtained for  $\hat{\beta}_1$  confirms that expansionary monetary policy shocks propagate to the economy by reducing bank lending as well as deposit rates. As shown by the larger coefficient obtained for  $\hat{\beta}_2$ , this effect of monetary policy on both lending and deposit rates is stronger during periods of recession, which in our model corresponds to periods of credit crunch.

# 8.4 Demand shocks

As illustrated by the results reported for the New Keynesian version in Table B.1, the model with technology shocks as the only source of real fluctuations is capable of matching several business cycle and financial market facts. This is also the approach followed in the classic paper of Jermann (1998). For the euro area, the other main motivation to focus on supply shocks is that the correlation between output and inflation, as measured by the core inflation index which excludes energy and processed food, is negative.<sup>13</sup>

At the same time, and as shown in Table 8 below, although the New Keynesian version with technology shocks can broadly reproduce the facts reported in Table B.1, it generates a correlation between output growth and inflation of -0.9, whereas in the data this correlation only reaches -0.2. A natural question to ask is whether asymmetries can still arise in a model capable of generating a correlation between output growth and inflation that matches the evidence.

<sup>&</sup>lt;sup>13</sup>In sticky price models, core inflation is the most suited indicator as this index excludes volatile components such as the price of energy and unprocessed food, which are not subject to the kind of price rigidities envisioned in the New Keynesian literature.

Recent findings in the literature have also highlighted the importance of demand shocks for asset pricing. In particular, Albuquerque, Eichenbaum, Luo and Rebelo (2016) have shown that introducing discount factor shocks into consumption-based models provides a potential resolution to several well-known asset pricing puzzles. In the context of a production economy, Miao, Wang and Zha (2020) find that discount factor shocks help explain the disconnect between house prices and rents, a well-established empirical regularity that models with standard demand shocks cannot reproduce.

In our New Keynesian model, introducing these demand shocks can be achieved by assuming that the modified subjective discount factor  $\beta$  is subject to exogenous disturbances. With discount shocks, the effective discount rate used by agents to evaluate future flows is now a time-varying object denoted by  $\beta_t$ :

$$\beta_t = \beta \Phi_t$$

where the preference shock  $\Phi_t$  is an autoregressive process of order one:

$$\Phi_t = \rho_\beta \Phi_{t-1} + \varepsilon_{\beta t}$$

and where  $\rho_{\beta}$  is a persistence parameter. In this process, the exogenous disturbance  $\varepsilon_{\beta t}$  is normally distributed with mean zero and standard deviation  $\sigma_{\beta}$ .

As shown in Table 8 below, we find that discount shocks not only enhance the model's ability to reproduce a sizeable term premium, these shocks also considerably reduce the correlation between output and inflation.

	Technology shocks	Demand shocks	Dat	ta
	$\sigma_a = 0.0073$	$\sigma_{\beta} = 0.004$	Estimated	95% CI
E(tp)	0.7	0.7	0.7	[0.5, 0.9]
$std(g_y)$	1.8	1.6	1.8	[1.6, 2.1]
$corr(g_u, g_{\widehat{P}})$	-0.9	-0.2	-0.2	[-0.4, 0]

Table 8: NK model with demand shocks vs. NK model with supply shocks

Since discount factor shocks are typically estimated to be persistent (e.g. Miao et al. 2020), we set the persistence parameter  $\rho_{\beta}$  to 0.99. Replacing technology by demand shocks and setting the shock standard deviation  $\sigma_{\beta}$  to 0.004 allows the extended model to perfectly match the term premium E(tp) as well as the correlation coefficient between inflation and output, which in Table 8 is denoted by  $corr(g_y, g_{\hat{P}})$ . The standard deviation of output generated by the model also lies within the estimated confidence interval. Are monetary asymmetries still present? The state dependent impulse response analysis for the demand shock model is shown in Figure B.3. It is reassuring to see that significant asymmetries can still be obtained if discount shocks are the only source of real fluctuations.

#### Interest on money

Since the welfare cost of inflation stems from the opportunity cost of holding money, we next consider the case in which the central bank offers a remuneration on cash. Of course, in practice, paying interest on cash is subject to obvious technical difficulties. At the same time, this is a classic extension in models in which money is explicitly introduced (e.g. Walsh (2017)). Moreover, at present, such an experiment could be motivated by the ongoing discussions with regards to central bank digital currencies (CBDCs). Indeed, if CBDCs were introduced, central banks could remunerate cash directly.

When cash S is remunerated, and given the assumption of a representative agent, the only difference relative to the extended model discussed in Section 8.1 is that the policy rate, which we denote by  $i_S$ , affects the money demand equation (14):

$$i_{Dt} = i_{St} + \frac{(1-\kappa)}{\kappa} \frac{P_t c_t}{S_t}$$
(26)

where we introduce the following process for the policy rate  $i_s$ :

$$i_{St} = \overline{i_S} + \rho_{i_S}(i_{St} - \overline{i_S}) + \varepsilon_{i_S t}$$

$$\tag{27}$$

In equation (27),  $\varepsilon_{i_{St}}$  is a random disturbance that is normally distributed with mean zero and standard deviation  $\sigma_{i_S}$ .  $\rho_{i_S}$  is the interest rate smoothing parameter and  $\overline{i_S}$  denotes the steady state value of the policy rate.

To gain intuition into how a change in the interest rate on cash holdings is transmitted to the economy, Figure B.4 plots the demand curve in equation (26) when the policy rate  $i_S$  increases from 0 to 0.5%. In contrast to a change in the quantity of money M, the key difference is that an increase in  $i_S$  shifts the demand for cash to the right.

As pointed out by Piazzesi, Rogers and Schneider (2019), the presence of a convenience yield dampens the effect of changes in interest rates in New Keynesian models. In our analysis, this effect can be explained by the response of S to a change in the policy rate  $i_S$ . Indeed, as can be seen from equation (26), everything else equal, an increase in cash holdings S in response to a higher remuneration rate attenuates the effect of the monetary policy shock on the deposit rate  $i_D$ . This illustrates that a concave preference for liquidity implies that banks can afford to pay a lower rate on bank deposit  $i_D$  when agents in the economy hold large amounts of cash. The pass-through of a change in the policy rate  $i_S$  to the bank deposit rate  $i_D$  therefore depends on the response of S. What does it imply for monetary policy transmission? As illustrated on Figure B.4, for a given level of S, a 0.5% increase in the policy rate  $i_S$ , which makes holding cash more attractive, has a stronger impact on the equilibrium level of the bank deposit rate  $i_D$  when agents already hold large amounts of cash. The reason is that the demand for cash is less sensitive to changes in the remuneration rate in those states of the world. Given the decreasing marginal utility of money, agents' appetite to hold even larger quantity of cash declines during periods of low bank rates and abundant cash holdings. Consequently, an increase in  $i_S$  only leads to a small increase in cash holding S in those states of the world. Therefore, the dampening effect stemming from the convenience yield discussed in Piazzesi, Rogers and Schneider (2019) is more muted when agents already hold large amounts of cash. Indeed, everything else equal, a smaller response of S in reaction to an increase in  $i_S$  leads to a larger increase in the deposit rate  $i_D$ .

In contrast, when households only hold small amounts of cash, an increase in  $i_S$  has a larger effect on the demand for S. The dampening effect is therefore particularly strong during periods of high interest rates and low cash holdings.

As can be seen on Figure B.5, the impact of the convenience yield on the pass-through from the interest rate on cash to bank rates is also a source of asymmetries. Indeed, a 0.5% increase in the rate at which the central bank remunerates cash holdings has a stronger effect on output during periods of low bank lending rates. The main takeaway, therefore, is that asymmetries could continue to play an important role if central banks were to issue CBDCs that are remunerated, a possibility that is currently under discussion.

# 8.5 Using the welfare cost of business cycle fluctuations to evaluate risk aversion

Since the seminal paper of Mehra and Prescott (1985), models are not only evaluated in terms of their ability to jointly reproduce asset pricing and business cycle facts. The level of risk aversion necessary to match these facts is also an important criteria.

However, evaluating risk aversion in dynamic general equilibrium models with production is a complex task. In contrast to endowment economies, the process for consumption is endogenously determined within a general equilibrium system. As a result, the ease with which agents can insure against shocks, and hence the extent to which they care about uncertainty cannot be summarized by one single measure of curvature of the utility function.

#### Local measures of risk aversion and the term premium

To illustrate this point, Figure B.6 reports a sensitivity analysis showing how the term premium varies with the adjustment cost parameter  $\epsilon$ , while all other parameter values are kept at the values reported in Table 1. Clearly, even in the presence of habits, the model loses much of its ability to generate a realistic term premium if we reduce adjustment costs. This illustrates the challenge at hands. In a production economy, local measures of risk aversion that solely focus on preferences do not tell the full story. In equilibrium, the amount of risk agents are exposed to critically depends on general equilibrium effects.

To provide a measure of risk exposure, we indirectly evaluate attitudes towards risk by exploiting the tight connection between risk aversion and the measure of welfare cost proposed by Lucas (2003) (e.g. Tallarini (2000)). One possible way to measure risk aversion is to ask how much agents would be willing to pay to completely eliminate business cycle fluctuations. This measure is therefore akin to a risk premium that agents would be ready to pay to be able to live in a certain world. While it cannot be demonstrated analytically, such a measure is necessarily linked to the curvature of the value function, and hence risk aversion. Indeed, a risk neutral agent would require a very small compensation, whereas very risk averse agents would be ready to abandon a large share of their income to completely eliminate uncertainty.

#### The cost of business cycle fluctuations as a fraction of GDP

To perform this calculation, the first step is to compute lifetime utility, which is given by the value function of our representative agent, denoted by  $u_t$ :

$$u_t = \frac{\left(c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} \left(\psi + z_t^{\upsilon}\right) - h_t\right)^{1-\sigma}}{1-\sigma} + \widehat{\beta} E_t u_{t+1}$$

Since risk averse agents dislike fluctuations, welfare in the stochastic economy that reproduces the moments shown on Table 2 is lower than what would be obtained in the same economy where the standard deviation of all shocks is set to zero, a case which can be thought of as the certainty equivalent. Denoting by  $E(u_t)$  welfare in the stochastic economy that generates a 0.6% term premium, and  $\overline{u}$  welfare in the deterministic economy without shocks, and hence zero risk premiums, we therefore have that  $E(u_t) < \overline{u}$ .

But how to convert the distance between E(u) and  $\overline{u}$  into a measure that can be interpretable? We ask how much additional revenue agents would need to receive to be indifferent between the stochastic and the deterministic economies. Starting from the allocation that matches the facts on Table 2, we therefore need to determine the compensation required to make the agent indifferent between the two allocations. Expressing this compensation as a fraction of output, the compensation received by the agent living in the risky economy is denoted  $\varkappa y_t$ , where  $\varkappa$  is the compensation parameter.<sup>14</sup> The goal of the exercise is then to find a value of  $\varkappa$  such that  $E(u_t) = \overline{u}$ . In other words, starting from the benchmark economy that reproduces a realistic term premium, by how much should we increase  $\varkappa$  to make agents indifferent between the risky and the deterministic allocations?

We obtain a value for  $\varkappa$  of 0.0188. This implies a cost of business cycle fluctuations amounting to 1.88% of total GDP per quarter. In comparison, this measure, which is reminiscent to a risk premium, falls from 1.88% to 0.023% if we remove habits from the analysis.

Without habits, a welfare cost of 0.023% implies that an agent with a total disposable income of 20,000 EUR would only ask to receive 4.6 EUR per year or about 40 cents per month to completely eliminate business cycle fluctuations. In this case, the risk premium demanded by agents for having to live in a world subject to business cycles is thus negligible. In equilibrium, agents are therefore close to risk neutral. Of course, the model without habits is also unable to generate a realistic term premium since uncertainty is irrelevant.

In contrast, in the benchmark model that matches both asset pricing and business cycle facts, for that same household, the cost of business cycle fluctuations represents around 378 EUR per year, or 31.5 EUR per month.

Is the magnitude of this business cycle fluctuation premium plausible? Whereas the cost of business cycle fluctuations is much larger in the model that matches the data and produces asymmetries in the transmission mechanism, we argue that the value we obtain is plausible. Indeed, many European economies do allocate a non-negligible fraction of their resources to alleviate the effects of business cycle fluctuations. For example, according to OECD data on public unemployment spending, as an average over the period from 1999 to 2019, Spain spent around 2.17% of GDP on unemployment insurance. In France and Germany, this measure reaches 1.6% and 1.26% of GDP, respectively.<sup>15</sup>

# 9 Conclusion

Interacting financing frictions with a preference for liquidity provides a potential explanation for the time-varying effectiveness of monetary policy documented in many studies.

<sup>&</sup>lt;sup>14</sup>The compensation parameter therefore affects the entire system. It can also be interpreted as an exogenous increase in productivity.

<sup>&</sup>lt;sup>15</sup>Spending on unemployment insurance increased dramatically during the COVID-19 crisis.

This asymmetry is quantitatively important, as monetary policy is considerably more effective during periods of credit stress. The main policy implication is that this asymmetry could reduce the effectiveness of monetary policy, or amplify its impact, thereby impairing central banks' ability to achieve their price stability mandate.

With regards to model development, one main takeaway is that the magnitude of this asymmetry crucially depends on the size of the term premium. Our results therefore suggest that monetary asymmetries are more likely to play a role in models able to produce realistic amounts of interest rate risk.

A first main limitation is that this study does not consider the issue of firm heterogeneity. Since SMEs are typically more likely to be credit constrained than large corporations, the effect of monetary policy could potentially vary across firms. This heterogeneity could in turn generate a reallocation of inputs across firms in response to monetary policy shocks. Whether this reallocation channel attenuates or amplifies the asymmetric effect of monetary policy would be an interesting question for future work.

A second main limitation is that the model cannot reproduce the humped-shaped response of inflation to monetary policy shocks documented in the literature that uses structural VARs.<sup>16</sup> Jointly reproducing this fact in a framework also able to reproduce realistic asset pricing implications remains a formidable challenge.

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<sup>&</sup>lt;sup>16</sup>As shown by Altig, Christiano, Eichenbaum and Lindé (2011), the challenge is to simultaneously explain why an inertial response of inflation is not observed in response to technology shocks.

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# 10 Annex A



Figure A.1. Response of the monetary aggregate, the price level, the short-term deposit rate, and nominal credit to a one standard deviation monetary policy shock. x axis: quarters after the shock. y axis: percentage deviation from steady state and annualized percent for short-term rate.



Figure A.2. Response of output, investment, hours and consumption. x axis: quarters after the shock. y axis: percentage deviation from steady state.



Figure A.3. Response of the real wage, marginal productivity of capital, real credit, and the lending spread to a one standard deviation monetary policy shock. x axis: quarters after the shock. y axis: percentage deviation from steady state and annualized percent for lending spread.



Figure A.4. Response of the term premium, the short-term real rate, nominal bond prices, and the nominal yield to a one standard deviation monetary policy shock. x axis: quarters after the shock. y axis: annualized percent and percentage deviation from steady state for bonds prices.



Figure A.5. Sensitivity to credit friction parameter  $\mu$ .



Figure A.6. Response of money, cash holdings, credit, and output to a one standard monetary policy shock when  $\mu$  is set to 0.0001.



Figure A.7. Response of output to a positive shock in the model with and without a labor wedge.



Figure A.8. Cash demand curve.



Figure A.9. Response of output in the low and high credit states in the model with low interest rate risk. x axis: Quarters after the shock. y axis: Percentage deviation from steady state.

# 11 Annex B: Robustness

		Model	
	95% confidence	Estimated empirical	Simulated moments
	interval	moments	(3rd order)
$std(g_y)$	[1.6, 2.1]	1.8	1.7
$std(g_c)$	[0.9,  1.2]	1.0	0.9
$std(g_x)$	[5.0,  6.6]	5.7	5.8
$std(g_M)$	[2.8, 3.8]	3.3	3.3
$std(g_D)$	[1.8, 2.4]	2.0	2.1
$std(g_P)$	[0.8,  1.1]	0.9	1.0
$std(i_D)$	[1.8, 2.4]	2.0	2.6
$E(i_D)$	[2.1, 2.6]	2.4	2.2
E(tp)	[0.5,  0.9]	0.7	0.7
$E(i_L - i_D)$	[2.2, 2.4]	2.3	2.3
E(l/y)	[0.88,  0.95]	0.91	0.86
E(x/y)	[0.21,  0.22]	0.22	0.21
ſ	3	Output	
0			

## Table B.1: Model vs. Data



Figure B.1. Response of output in the low and high credit states. Model with price stickiness. x axis: Quarters after the shock. y axis: Percentage deviation from steady state.



Figure B.2. Response of output in the low and high credit states. Model with price stickiness and CES utility function. x axis: Quarters after the shock. y axis: Percentage deviation from steady state.



Figure B.3. Demand shock model when E(tp) = 0.7,  $\sigma_{\beta} = 0.004$ ,  $corr(g_y, g_{\widehat{P}}) = -0.2$ . Response of output in the low and high credit states. x axis: Quarters after the shock. y axis: Percentage deviation from steady state.



Figure B.4. Equilibrium in the money market before and after an increase in the interest rate on cash of 0.5 percentage point.



Figure B.5. Response output in the low and high interest rate environment to a 0.5 percentage point increase in the interest rate on cash  $i_S$ . Model with Calvo price stickiness. x axis: Quarters after the shock. y axis: Percentage deviation from steady state.



Figure B.6. Sensitivity of the term premium E(tp) to the capital adjustment cost parameter  $\epsilon$ .

# 12 Annex C: Data Appendix

Variables	Description	Source	
Output, y	Real GDP	Stat. Office of the EC	
	EA 19, mio of chained 2010 Euros	1995q1-2018q4	
Consumption, $c$	Final consumption expenditures	Stat. Office of the EC	
	EA 19, mio of chained 2010 Euros	1995q1-2018q4	
Investment, $x$	Gross capital formation	Stat. Office of the EC	
	EA 19, mio of chained 2010 Euros	1995q1- $2011$ q4	
Money supply, $M$	M1 Money supply adjusted for	ECB	
	reclassifications, EA 11-19	1997q3-2018q4	
Deposit, $D$	Total deposits of euro area households	ECB	
	amount outstanding	1997q3-2018q4	
Price level, $P$	Harmonized index of consumer prices	ECB	
	2015=100, EA 11-19	1997 q 1 - 2018 q 4	
Monetary policy rate, $i_D$	3-month deposit (EURIBOR)	ECB	
	EA 11-19	1995q1-2018q4	
Long-term rate, $i_{10Y}$	10-year government benchmark	ECB	
	bond yield, EA 11-19	1995q1-2018q4	
Term premium, $E(tp)$	Term structure model for the	Hördahl and Tristani	
	euro area and the U.S.	1999q1m7-2018q4	
Intermediation spread, $i_L - i_D$	Rate on new loans to non-financial	ECB	
	corporations (less than $1 \mod 1$ keys	2000q1-2018q4	
	than a year) minus 3-month deposit		
Loan-to-output ratio, $l/y$	Credit to non-financial corporations	BIS	
	% of GDP, market value	1997q3-2018q4	
# 13 Online Technical Appendix (not for publication)

# 13.1 The competitive equilibrium

## Households

$$\max_{c_t, S_t, B_{t+1}, n_t, M_{t+1}, x_t, h_{t+1}, k_{t+1}} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left(c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} \left(\psi + z_t^{\upsilon}\right) - h_t\right)^{1-\sigma}}{1-\sigma}$$

where:

$$\widehat{\beta} = \widetilde{\beta} \gamma^{1-\sigma}$$

such that:

$$\gamma \frac{M_{t+1}}{P_t} = \frac{S_t}{P_t} + \frac{D_t}{P_t}$$
$$1 = n_t + z_t$$

$$tr_{t} + prof_{Tt} + w_{t}n_{t} + i_{Dt}\frac{D_{t}}{P_{t}} + r_{Kt}k_{t} + \frac{B_{t}}{P_{t}} = c_{t} + x_{t} + \gamma\frac{M_{t+1}}{P_{t}} - \frac{M_{t}}{P_{t}} + \frac{1}{1 + i_{Bt}}\gamma\frac{B_{t+1}}{P_{t}}$$
$$\gamma k_{t+1} = (1 - \delta)k_{t} + \left(\frac{\theta_{1}}{1 - \epsilon}\left(\frac{x_{t}}{k_{t}}\right)^{1 - \epsilon} + \theta_{2}\right)k_{t}$$
$$\gamma h_{t+1} = \tau h_{t} + (1 - \tau)c_{t}^{\kappa}\left(\frac{S_{t}}{P_{t}}\right)^{1 - \kappa}(\psi + z_{t}^{\upsilon})$$

The Lagrangian:

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left(c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} \left(\psi + z_t^{\upsilon}\right) - h_t\right)^{1-\sigma}}{1-\sigma} \right\}$$

$$+\sum_{t=0}^{\infty}\widehat{\beta}^{t}\lambda_{t}\left[\begin{array}{c}tr_{t}+prof_{Tt}+w_{t}n_{t}+i_{Dt}\left(\gamma\frac{M_{t+1}}{P_{t}}-\frac{S_{t}}{P_{t}}\right)+r_{Kt}k_{t}+\frac{B_{t}}{P_{t}}-c_{t}-x_{t}\\-\left(\gamma\frac{M_{t+1}}{P_{t}}-\frac{M_{t}}{P_{t}}\right)-\frac{1}{1+i_{Bt}}\gamma\frac{B_{t+1}}{P_{t}}\end{array}\right]$$

$$+\sum_{t=0}^{\infty}\widehat{\beta}^{t}q_{t}\lambda_{t}\left[(1-\delta)k_{t}+\left(\frac{\theta_{1}}{1-\epsilon}\left(\frac{x_{t}}{k_{t}}\right)^{1-\epsilon}+\theta_{2}\right)k_{t}-\gamma k_{t+1}\right]$$
$$+\sum_{t=0}^{\infty}\widehat{\beta}^{t}\varphi_{t}\left[\tau h_{t}+(1-\tau)c_{t}^{\kappa}\left(\frac{S_{t}}{P_{t}}\right)^{1-\kappa}(\psi+z_{t}^{\upsilon})-\gamma h_{t+1}\right]\right\}$$

## Banks

Banks maximize profits, which are given by:

$$prof_{Bt} = i_{Lt} \frac{L_t}{P_t} - i_{Dt} \frac{D_t}{P_t}$$

such that:

$$L_t = \eta D_t$$

Firms

$$\max_{k_t, n_t, L_t} prof_{Ft} = a_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_{Kt} k_t - i_{Lt} \frac{L_t}{P_t}$$

such that:

$$\frac{L_t}{P_t} \ge \mu \left( w_t n_t + r_{Kt} k_t \right)$$

The Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\lambda_t}{\lambda_0} \left\{ \begin{array}{c} a_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_{Kt} k_t - i_{Lt} \frac{L_t}{P_t} \\ + \frac{\varpi_t}{\lambda_t} \left[ \frac{L_t}{P_t} - \mu \left( w_t n_t + r_{Kt} k_t \right) \right] \end{array} \right\}$$

Market clearing condition

$$a_t k_t^{\alpha} n_t^{1-\alpha} = c_t + x_t$$

# 13.2 The dynamic system

$$\left\{ \left[ c_t^{\kappa} \left( \frac{S_t}{P_t} \right)^{1-\kappa} (\psi + z_t^{\upsilon}) - h_t \right]^{-\sigma} + (1-\tau)\varphi_t \right\} \kappa c_t^{\kappa-1} \left( \frac{S_t}{P_t} \right)^{1-\kappa} (\psi + z_t^{\upsilon}) = \lambda_t$$
$$\left\{ \left[ c_t^{\kappa} \left( \frac{S_t}{P_t} \right)^{1-\kappa} (\psi + z_t^{\upsilon}) - h_t \right]^{-\sigma} + (1-\tau)\varphi_t \right\} c_t^{\kappa} \left( \frac{S_t}{P_t} \right)^{1-\kappa} \upsilon z_t^{\upsilon-1} = w_t \lambda_t$$

$$\begin{split} i_{Dt} \frac{S_t}{P_t} &= \frac{1-\kappa}{\kappa} c_t \\ \lambda_t q_t &= \beta E_t \lambda_{t+1} q_{t+1} \left[ \left(1-\delta\right) + \frac{\theta_1}{1-\epsilon} \left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \theta_2 - \theta_1 \left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon} \right] + \beta E_t \lambda_{t+1} r_{Kt+1}^{17} \\ 1 &= q_t \theta_1 \left(\frac{x_t}{k_t}\right)^{-\epsilon} \\ \frac{\lambda_t}{P_t} &= \beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} + \frac{\lambda_t}{P_t} i_{Dt} \\ \frac{\lambda_t}{P_t} \frac{1}{1+i_{Bt}} &= \beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} \\ \gamma M_{t+1} &= S_t + D_t \\ \varphi_t &= \tau \beta E_t \varphi_{t+1} - \beta E_t \left( c_{t+1}^{\kappa} \left(\frac{S_{t+1}}{P_{t+1}}\right)^{1-\kappa} \left(\psi + z_{t+1}^{\upsilon}\right) - h_{t+1} \right)^{-\sigma} \end{split}$$

$$(1-\delta)k_t + \left(\frac{\theta_1}{1-\epsilon} \left(\frac{x_t}{k_t}\right)^{1-\epsilon} + \theta_2\right)k_t - \gamma k_{t+1} = 0$$
  
$$\tau h_t + (1-\tau)c_t^{\kappa} \left(\frac{S_t}{P_t}\right)^{1-\kappa} (\psi + z_t^{\upsilon}) - \gamma h_{t+1} = 0$$

$$L_t = \eta D_t$$

$$i_{Lt} = \frac{1}{\eta} i_{Dt}$$

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha}$$

<sup>17</sup>where for the sake of notation, I define:

 $\beta = \widetilde{\beta} \gamma^{-\sigma}$ 

$$\begin{aligned} \frac{L_t}{P_t} &= y_t \frac{\mu}{1 + \mu \frac{\varpi_t}{\lambda_t}} \\ w_t &= (1 - \alpha) \frac{y_t}{n_t} \frac{1}{1 + \mu \frac{\varpi_t}{\lambda_t}} \\ r_{Kt} &= \alpha \frac{y_t}{k_t} \frac{1}{1 + \mu \frac{\varpi_t}{\lambda_t}} \\ i_{Lt} &= \frac{\varpi_t}{\lambda_t} \\ y_t &= c_t + x_t \\ \gamma M_{t+1} &= \gamma \overline{M} + \rho_M \left( M_t - \overline{M} \right) - \phi_\pi (P_t - \overline{P}) + \varepsilon_M \\ \log a_t &= \rho_a \log a_{t-1} + \varepsilon_a \end{aligned}$$

## 13.3 Introducing Price Stickiness

This section describes how the introduction of price stickiness affects the production side of the economy.

### 13.3.1 Imperfect competition

Price rigidities imply that firms are able to set prices and therefore enjoy some degree of market power. Following Galì (2015), among others, this is achieved by firstly introducing a final goods sector that produces the final output good  $y_t$ . The final output good is obtained by combining the inputs produced by each intermediate good producer. Each intermediate good producer j produces a variety of the intermediate good, which is denoted by  $y_{jt}$ .

Every period, the final goods producer chooses the quantity of inputs that minimizes costs:

$$\min_{y_{jt}} \int_{0}^{1} P_{jt} y_{jt} dj$$

where  $P_{jt}$  represents the price of variety j, subject to a Dixit-Stiglitz aggregator that converts intermediate goods into the final good  $y_t$ :

$$\left(\int\limits_{0}^{1} y_{jt}^{\frac{\vartheta-1}{\vartheta}} dj\right)^{\frac{\vartheta}{\vartheta-1}} \ge y_{t}$$

where we assume the existence of a continuum of goods. The quantity of goods is fixed and normalized to 1. The outcome of this optimization problem is a demand schedule for the variety j:

$$y_{jt} = \left(\frac{P_t}{P_{jt}}\right)^\vartheta y_t \tag{28}$$

The Dixit-Stiglitz parameter  $\vartheta$  can be interpreted as the price elasticity of demand. The higher this parameter, the more elastic the demand for variety j and the lower the market power of intermediate good producer j. Optimality in turn implies that the aggregate price level  $P_t$  is determined as follows:

$$P_t = \left(\int_{0}^{1} P_{jt}^{1-\vartheta} dj\right)^{\frac{1}{1-\vartheta}}$$
(29)

### 13.3.2 Intermediate goods producers

In the New Keynesian version, the loan-in-advance constraint appears in the problem of each intermediate good producer j. Each period, intermediate good producer j chooses the optimal combination of inputs that minimizes costs:

$$\min\frac{W_t}{P_t}n_{jt} + \frac{R_t}{P_t}k_{jt} + i_{Lt}\frac{L_{jt}}{P_t}$$

where  $n_{jt}$ ,  $k_{jt}$  and  $L_{jt}$  denote the number of hours worked to hire, the quantity of capital to rent, and the amount of credit obtained from banks, respectively.<sup>18</sup>

Firms minimize costs subject to the production function:

$$y_{jt} = a_t k^{\alpha}_{jt} n^{1-\alpha}_{jt}$$

where  $a_t$  is total factor productivity, which is common across all intermediate good producers, and the loan-in-advance constraint:

<sup>&</sup>lt;sup>18</sup>Since labor, capital and credit are perfectly mobile across intermediate good producers, wages  $W_t$ , the rental rate of capital  $R_t$ , and the bank lending rate  $i_{Lt}$  are identical across firms and we drop the index j to denote these variables.

$$\frac{L_{jt}}{P_t} \ge \mu \left( \frac{W_t}{P_t} n_{jt} + \frac{R_t}{P_t} k_{jt} \right)$$

As in the baseline model, the optimization condition with respect to credit gives rise to the following condition:

$$i_{Lt} = \frac{\varpi_t}{\lambda_t}$$

where  $\varpi_t$  is the Lagrange multiplier associated with the loan-in-advance constraint and  $\lambda_t$  marginal utility.<sup>19</sup>

Relative to the baseline case, the difference is that the presence of monopolistic competition introduces a time-varying markup, which is denoted by  $\chi_t$ , and which is identical across intermediate goods producers.<sup>20</sup> Therefore, the labor and capital demand schedules are given as follows:

$$\left(1 + \frac{\varpi_t}{\lambda_t}\mu\right)\frac{W_t}{P_t} = \chi_t(1-\alpha)\frac{y_{jt}}{n_{jt}}$$
$$\left(1 + \frac{\varpi_t}{\lambda_t}\mu\right)\frac{R_t}{P_t} = \chi_t\alpha\frac{y_{jt}}{k_{jt}}$$

### 13.3.3 Optimal pricing

After having determined the optimal quantity of inputs, and since they are monopolistic competitors, intermediate good producers choose the optimal price. Following Galì (2015), we focus the analysis on the case of Calvo pricing (e.g. Calvo, 1983), which is commonly used in the New Keynesian literature. The Calvo friction implies that each period only a fraction  $1 - \xi$  of intermediate good producers are able to adjust prices. When setting its price, an intermediate good producer j thus needs to take into account that with probability  $\xi$ , it will not be able to reoptimize its price for several periods. When able to adjust its price, the firm therefore needs to anticipate that it may be unable to reoptimize for several periods. Consequently, the optimization problem takes the following form:

<sup>&</sup>lt;sup>19</sup>As the banking sector charges the same lending rate to each firm, the tightness of the loan-in-advance constraint is the same for all intermediate good producers. We therefore drop the index j for the Lagrange multiplier.

<sup>&</sup>lt;sup>20</sup>Marginal costs depend on wages, the rental rate of capital, bank lending rates and total factor productivity, which are all common across firms. We can therefore drop the index j to denote marginal costs.

$$\max_{P_{jt}^*} E_t \sum_{i=0}^{\infty} \widehat{\beta}^i \xi^i \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}} \left( \frac{P_{jt}^*}{P_{t+i}} y_{t,t+i} - \chi_{t+i} y_{t,t+i} \right)$$

where  $\hat{\beta} \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}}$  is the stochastic discount factor.  $P_{jt}^*$  is the optimal price chosen by a firm that is able to adjust its price in the current period. The assumption of monopolistic competition in turn implies that firms take the demand for their variety j, given by equation (28), as an additional constraint.

After a few manipulations, we obtain the following condition for the optimal price chosen by firm j. Since firms that are able to adjust in the current period choose the same price, we have that (e.g. Walsh (2017)):

$$\frac{P_t^*}{P_t} = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{i=0}^{\infty} \beta^i \xi^i \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}} \chi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\vartheta} y_{t+i}}{E_t \sum_{i=0}^{\infty} \beta^i \xi^i \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}} \left(\frac{P_{t+i}}{P_t}\right)^{\vartheta - 1} y_{t+i}}$$
(30)

#### 13.3.4 Aggregate dynamics and price dispersion term in nonlinear models

Using the definition of the aggregate price level in equation (29), and given that all firms resetting prices will choose  $P_t^*$ , the Calvo pricing assumption in turn implies the following dynamics for the aggregate price level:

$$P_t^{1-\vartheta} = (1-\xi)P_t^{*1-\vartheta} + \xi P_{t-1}^{1-\vartheta}$$
(31)

When  $\xi = 0$ , equations (31) and (30) therefore imply a constant markup given by the following expression:

$$\chi = \frac{\vartheta - 1}{\vartheta} \tag{32}$$

Since the model is solved using a nonlinear solution methods, we also need to analyze how the price dispersion term, which is the source of inefficiency in sticky price models, affects aggregate variables. Starting from the production function of intermediate good producer j:

$$y_{jt} = a_t k_{jt}^{\alpha} n_{jt}^{1-\alpha}$$

and given the demand for variety j in equation (28), the economy's aggregate production function is given as follows:

$$y_t = \frac{a_t k_t^{\alpha} n_t^{1-\alpha}}{\Delta_t}$$

where:

$$\Delta_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\vartheta} dj \tag{33}$$

and where  $\Delta_t$  denotes the price dispersion term, which can be expressed recursively as follows<sup>21</sup>:

$$\Delta_t = (1 - \xi) \left(\frac{P_t^*}{P_t}\right)^{-\vartheta} + \left(\frac{P_{t-1}}{P_t}\right)^{-\vartheta} \xi \Delta_{t-1}$$

## 13.4 The New Keynesian version with CES utility

Households

$$\begin{cases} \left[ \left[ \kappa c_t^{1-\frac{1}{\iota}} + (1-\kappa) \left( \frac{S_t}{P_t} \right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}} (\psi + z_t^{\upsilon}) - h_t \right]^{-\sigma} + (1-\tau)\varphi_t \\ \left[ \kappa c_t^{1-\frac{1}{\iota}} + (1-\kappa) \left( \frac{S_t}{P_t} \right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}-1} (\psi + z_t^{\upsilon})\kappa c_t^{-\frac{1}{\iota}} \\ = \lambda_t \end{cases}$$
(34)

$$\begin{cases}
\left[ \left[ \kappa c_t^{1-\frac{1}{\iota}} + (1-\kappa) \left( \frac{S_t}{P_t} \right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}} (\psi + z_t^{\upsilon}) - h_t \right]^{-\sigma} + (1-\tau)\varphi_t \\
\left[ \kappa c_t^{1-\frac{1}{\iota}} + (1-\kappa) \left( \frac{S_t}{P_t} \right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}} \upsilon z_t^{\upsilon - 1} \\
= \frac{\chi_t}{1+i_{Lt}\mu} (1-\alpha) \frac{y_t}{n_t} \Delta_t \lambda_t$$
(35)

 $^{21}$ To obtain this expression, we first need to express equation (33) as follows:

$$\Delta_t = \int_0^{1-\xi} \left(\frac{P_t^*}{P_t}\right)^{-\vartheta} dj + \int_{1-\xi}^1 \left(\frac{P_{jt-1}}{P_t}\right)^{-\vartheta} dj$$

Then since the distribution of prices among firms not adjusting corresponds to the distribution of prices in period t - 1 (e.g. Gali, 2015), we obtain:

$$\Delta_t = (1-\xi) \left(\frac{P_t^*}{P_t}\right)^{-\vartheta} + \left(\frac{P_{t-1}}{P_t}\right)^{-\vartheta} \xi \int_0^1 \left(\frac{P_{jt-1}}{P_{t-1}}\right)^{-\vartheta} dj$$

$$i_{Dt} = \frac{1 - \kappa}{\kappa} \left(\frac{c_t}{S_t/P_t}\right)^{\frac{1}{\iota}}$$
(36)

$$\frac{\lambda_t}{P_t} = \beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} + \frac{\lambda_t}{P_t} i_{Dt}$$
(37)

$$\frac{\lambda_t}{P_t} \frac{1}{1+i_{Bt}} = \beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} \tag{38}$$

$$\varphi_{t} = \tau \beta E_{t} \varphi_{t+1} - \beta E_{t} \left( \left[ \kappa c_{t+1}^{1-\frac{1}{\iota}} + (1-\kappa) \left( \frac{S_{t+1}}{P_{t+1}} \right)^{1-\frac{1}{\iota}} \right]^{\frac{1}{1-\frac{1}{\iota}}} (\psi + z_{t+1}^{\upsilon}) - h_{t} \right)^{-\sigma}$$
(39)

$$\gamma h_{t+1} = \tau h_t + (1 - \tau) \left[ \kappa c_t^{1 - \frac{1}{\iota}} + (1 - \kappa) \left( \frac{S_t}{P_t} \right)^{1 - \frac{1}{\iota}} \right]^{\frac{1}{1 - \frac{1}{\iota}}} (\psi + z_t^{\upsilon})$$
(40)

$$1 = z_t + n_t \tag{41}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} q_{t+1} \left( (1-\delta) + \frac{\theta_1}{1-\epsilon} \left( \frac{x_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \theta_2 - \theta_1 \left( \frac{x_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right)$$
(42)

$$+\beta E_t \lambda_{t+1} \frac{\chi_{t+1}}{1+i_{Lt+1}\mu} \alpha \frac{y_{t+1}}{k_{t+1}} \Delta_{t+1}$$
(43)

$$1 = q_t \theta_1 \left(\frac{x_t}{k_t}\right)^{-\epsilon} \tag{44}$$

$$(1-\delta)k_t + \left(\frac{\theta_1}{1-\epsilon} \left(\frac{x_t}{k_t}\right)^{1-\epsilon} + \theta_2\right)k_t - \gamma k_{t+1} = 0$$
(45)

$$\gamma M_t = S_t + D_t \tag{46}$$

Optimal Pricing:

$$\frac{P_t^*}{P_t} = \frac{\vartheta}{\vartheta - 1} \frac{z_{Nt}}{z_{Dt}} \tag{47}$$

$$P_t^{1-\vartheta} = (1-\xi)P_t^{*1-\vartheta} + \xi P_{t-1}^{1-\vartheta}$$
(48)

$$\Delta_t = (1 - \xi) \left(\frac{P_t^*}{P_t}\right)^{-\vartheta} + \left(\frac{P_{t-1}}{P_t}\right)^{-\vartheta} \xi \Delta_{t-1}$$
(49)

$$z_{Nt} = \chi_t P_t^{\vartheta} y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} z_{Nt+1}$$
(50)

$$z_{Dt} = P_t^{\vartheta - 1} y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} z_{Dt+1}$$
(51)

Financial frictions:

$$\frac{L_t}{P_t} = \mu \frac{\chi_t}{1 + i_{Lt}\mu} y_t \tag{52}$$

$$L_t = \eta D_t \tag{53}$$

$$i_{Lt} = \frac{\overline{\omega}_t}{\lambda_t} \tag{54}$$

$$i_{Lt} = \frac{1}{\eta} i_{Dt} \tag{55}$$

Production function:

$$y_t = \frac{a_t k_t^{\alpha} n_t^{1-\alpha}}{\Delta_t} \tag{56}$$

Market clearing:

$$y_t = c_t + x_t \tag{57}$$

Monetary policy rule:

$$\gamma M_t = \gamma \overline{M} + \rho_M (M_{t-1} - \overline{M}) + \phi_\pi (P_t - \overline{P}) + \varepsilon_{Mt}$$
(58)

TFP shock:

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{at} \tag{59}$$

## 13.5 Replication files

The main results of the paper can be replicated by using Dynare. Dynare is a free software that can be downloaded online:

https://www.dynare.org/download/

Replication files are available on my webpage:

https://sites.google.com/site/ivanjaccard/replication-files

#### Acknowledgements

The views expressed in this article are my own and do not represent the views of the ECB or the Eurosystem. This draft as well as previous versions of this project have benefited from comments and suggestions from D. Krueger, 4 anonymous referees, O. Tristani, V. Quadrini, W. den Haan, G. Rousselet, and from participants at the 7th Asset Pricing Workshop of the University of York, the Workshop in Empirical and Theoretical Macroeconomics of King's College, the T2M conference and the Conference Computing in Economics and Finance. The impulse response analysis performed in Section 7 has benefited from comments and suggestions from M. Juillard. Any remaining errors are my own responsibility.

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2DF ISBN 978-92-899-6676-4	ISSN 1725-2806	doi:10.2866/040155	
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