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Article

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Central Bank Review (CBR)

Provided in Cooperation with:

Central Bank of The Republic of Turkey, Ankara

Suggested Citation: Tanaka, Atsushi (2022) : How can a central bank exit quantitative easing without rapidly shrinking its balance sheet?, Central Bank Review (CBR), ISSN 1303-0701, Elsevier, Amsterdam, Vol. 22, Iss. 2, pp. 91-98,
<https://doi.org/10.1016/j.cbrev.2022.06.001>

This Version is available at:

<https://hdl.handle.net/10419/297949>

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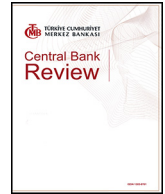
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How can a central bank exit quantitative easing without rapidly shrinking its balance sheet?



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ARTICLE INFO

Article history:

Received 24 February 2022

Received in revised form

26 May 2022

Accepted 1 June 2022

Available online 9 June 2022

JEL classification:

E52

E58

Keywords:

Central bank

Monetary base

Inflation

Quantitative easing

Exit strategy

Solvency

ABSTRACT

An important question for the major central banks of the industrialized world is how to design desirable strategies to exit quantitative easing (QE). At the exit, if a central bank needs to reduce rapidly the liquidity created by its balance sheet expansion, issuing new interest-bearing liabilities would be preferable to rapidly shrinking the balance sheet by selling existing assets, both for the stability of those assets' markets and to be able to keep capital losses from being reflected in the balance sheet under amortized-cost accounting. Given that existing assets accumulated during the quantitative easing period have low interest returns, and new liabilities to be issued in the quantitative tightening period would have high interest payouts, the central bank may run a loss that may threaten its solvency, which may force the bank to expand the monetary base above the level that is consistent with the central bank's ideal price stability path. This study considers a central bank that exits QE by issuing liabilities and examines an optimal exit strategy while maintaining the solvency by constructing a simple dynamic optimization model. The model is then applied to the Bank of Japan and the Federal Reserve to examine their possible exits.

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1. Introduction

This study constructs a simple dynamic optimization model of a central bank and examines its optimal behavior after exiting quantitative easing using interest-bearing liabilities instead of selling assets and rapidly shrinking its balance sheet. With high interest payments, the bank may be forced to expand the monetary base to maintain its solvency. The model explores when the bank faces such a situation and how it can optimally deal with it.

Some central banks are now starting tapering. That gradually reverses quantitative easing by shrinking a central bank's balance sheet, as the maturing assets are redeemed. The advantage of this strategy is that it does not impose huge losses on the bank. The disadvantage is that it can shrink its balance sheet only at the pace of asset redemption at most. If economic recovery becomes faster, the bank must absorb the reserves (i.e. must sterilize the liquidity created by QE) faster, and the bank needs another strategy.

One possible strategy is to rapidly shrink a central bank's

balance sheet by selling its assets. However, doing this imposes a large capital loss on the bank because interest rates rise and asset prices fall at and after the exit. That capital loss is recorded in the balance sheet as soon as the assets are sold. In contrast, the same capital loss is not reflected in the balance sheet if the assets in question are not sold in the amortized-cost accounting framework that many leading central banks follow. Furthermore, a large sale by the bank may shake up relevant markets. Therefore, relying on this strategy may not be desirable.

Bernanke (2009) suggests neutralizing a large part of the excess liquidity with other liabilities, such as using reverse repos or paying high interest on the excessive part of reserves while waiting for asset redemption. This type of exit strategy can sterilize a large amount of the liquidity created by QE without rapidly shrinking the balance sheet. Although this strategy does not induce any capital loss or unfavorable shakeup in the relevant markets, it continues to impose high interest payments on the bank as long as it holds liabilities.

Although they reduce the bank's profit, these interest payments are not generally considered to be large enough to jeopardize the bank's solvency. However, it is important to know how much interest burden is within the bank's ability and what actions the bank

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Peer review under responsibility of the Central Bank of the Republic of Turkey.

needs to take if the burden is beyond it. This study explores those issues.

Research on central bank solvency has been conducted to examine troubled central banks in developing countries. It can now be used to investigate the central banks that exit quantitative easing.

Central bank solvency was theoretically examined by [Reis \(2015\)](#).¹ By introducing a central bank's intertemporal budget constraint, he used a no-Ponzi game condition to ensure that the central bank remained solvent. The condition assumes that interest payments can be covered by future earnings, and states that all liabilities, except the monetary base, will be paid back in the infinite horizon. For example, [Del Negro and Sims \(2015\)](#) used this condition to check the solvency of the Federal Reserve (Fed).

If high interest payments are so large that the no-Ponzi game condition is not satisfied, a central bank may seek fiscal support from the government. However, if this option is not available or is insufficient, the bank must pay the money it creates, which is an inflationary monetary policy. Thus, solvency is important for implementing an appropriate monetary policy to stabilize inflation, as discussed by [Stella \(1997\)](#).

If the no-Ponzi game condition is not satisfied, the question is how inflationary should the monetary policy be. [Benigno and Nisticò \(2020\)](#) answered this question by presenting numerical examples of the optimal time paths of a central bank's capital, monetary base supply, and other variables while restoring its solvency.

This study answers this question by explicitly deriving optimal time paths (as opposed to computing them numerically) of the monetary base supply and other variables of a central bank when faced with insolvency. It constructs a dynamic optimization model of a central bank and examines the case of exiting quantitative easing. This study focuses on the case of no fiscal support from the government,² since a central bank does not generally become insolvent with fiscal support as long as the government remains solvent.³

[Tanaka \(2020\)](#) theoretically examines this optimal behavior. However, his model examines banks' net assets, instead of handling assets and liabilities separately. As [Bhattarai et al. \(2015\)](#) and [Berriel and Mendes \(2015\)](#) demonstrated, a central bank may accumulate not-so-liquid assets in quantitative easing. This study allows that assets purchased by a central bank before an exit from quantitative easing may be much less liquid than liabilities. The assets bear only low returns, and the bank cannot sell them, but can only shrink their volume slowly, while the liabilities used to absorb excessive funds at the exit impose high interest payments on the bank. This situation may make the bank insolvent, and the model in the current study reveals the bank's optimal paths of liabilities and monetary base to restore solvency.

The model of this study is applied to the cases of the Bank of

Japan (BOJ) and the Fed. The simulations for the banks to exit quantitative easing in 2020 reveal a difference between them. The Fed does not face the insolvency problem and has room to raise the interest rate to absorb excessive reserves. The BOJ has relatively limited room to increase the interest rate and is faced with challenges if the need arises.⁴ The difference between the banks comes mainly from the size of high interest-bearing liabilities needed to exit quantitative easing. Comparing [Tables 1 and 2](#), which will be discussed in [Section 4](#), the BOJ has a larger L_0/A_0 , where L_0 is the high interest-bearing liabilities, and A_0 is the total assets at the exit. The BOJ needs to issue a greater quantity of high interest-bearing liabilities, which would impose a greater interest payment burden on the BOJ. The payments must not be made by printing money to avoid inflation, so they are financed by further increasing the high interest-bearing liabilities, which deteriorates the BOJ's situation.

The remainder of this paper is organized as follows. Before considering the case in which a central bank shrinks its balance sheet, [Section 2](#) examines a simple case in which the balance sheet is constant. The model is constructed, and a central bank's optimal behavior is derived and illustrated graphically. [Section 3](#) modifies the model so that the balance sheet shrinks slowly through redemption, and the bank's optimal behavior is derived. [Section 4](#) applies the model to the cases of the BOJ and the Fed, and examines the simulations of their exits under different scenarios. [Section 5](#) concludes.

2. Model with no balance sheet reduction

2.1. Model

This section considers the case in which a central bank does not shrink its balance sheet after exiting quantitative easing. The exit involves raising the interest rate and reducing the monetary base. Both, interacting with each other, affect inflation through various channels of the transmission mechanism. The model leaves out unnecessary details and considers only the monetary base growth that sets inflation.

The central bank reduces the monetary base to the normal level at the exit and attempts to limit its growth to control inflation. To realize this path of the monetary base, the bank uses high interest-bearing liabilities. The interest rate on these liabilities must be set well above the market interest rates to absorb the excessive monetary base. For simplicity, the model here assumes that the bank sets this interest rate at the exit once and for all, and keeps the rate at that level forever. Interest rates and monetary base growth would normally be closely related in practice, but they can also move independently. In this model, their relationship is left out of the model to preserve simplicity.

The balance sheet on the left in [Fig. 1](#) shows that of a central bank before an exit from quantitative easing. The bank is assumed to have only one type of assets A , which are long-term securities. They bear only a low interest rate r_A because they are purchased during quantitative easing. The bank supplies monetary base H , and is assumed to have no other liabilities. The bank has built up asset holdings A to supply a substantial amount of H . K is capital that changes via the bank's profit or loss.

At the exit, $t = 0$, the balance sheet on the right in [Fig. 1](#) shows that the bank absorbs a large amount of the monetary base via liabilities L . Liabilities can be obtained by any fund-absorbing operation, such as reverse repos or special reserves paying interest. In either case, the central bank needs to pay a high interest rate r_L ,

¹ See [Appendix A](#) in [Hall and Reis \(2015\)](#) and [Tanaka \(2021\)](#) for an extensive survey of the literature in this field.

² The Bank of Japan Act does not provide any rule regarding the government's fiscal support, and, as highlighted by [Reis \(2015, p.3\)](#), the fiscal support for the Federal Reserve and the European Central Bank is not clear.

³ Previous literature such as [Reis \(2015\)](#), [Hall and Reis \(2015\)](#), [Benigno and Nisticò \(2020\)](#), and [Berentsen et al. \(2016\)](#) compare several rules of fund transfers to and from the government.

⁴ [Fujiki and Tomura \(2017\)](#) show the simulation result of a similar exit strategy of the BOJ. In their model, they assume 1.25%–2.75% interest on the excess reserves, though it is not used as a tool to absorb the excessive funds. They do not check the solvency condition, but their result does not imply any risk of insolvency. The simulation result of this study also shows no insolvency risk when the interest rate is 2.5%. This study explores further to the case when the interest rate must be set higher, and finds the BOJ starts having the problem at 4.03%.

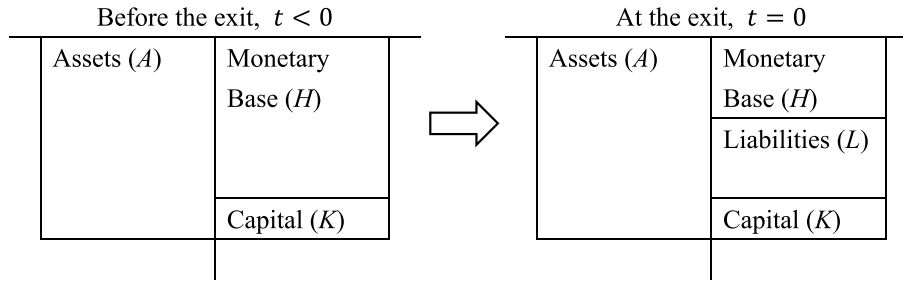


Fig. 1. Central bank's balance sheet before and at the exit.

which is higher than r_A , on the liabilities, so that private banks have an incentive to switch funds from H to L instead of using them for credit creation.⁵

The model in this study starts at the exit, $t = 0$. At $t = 0, \dots, \infty$, the central bank controls H and thus changes L , while A is assumed to be constant in this section.⁶ The central bank's profit π is defined as

$$\pi = r_A A - r_L L - C, \tag{1}$$

where C is the bank's operating cost, which is exogenous and assumed to be constant. In this study, the bank has no transfers to or from the government.⁷ Changes in the balance sheet variables satisfy

$$\dot{L} = -\dot{H} - \dot{K} = -\dot{H} - r_A A + r_L L + C, \tag{2}$$

where $\dot{L} = dL/dt$, $\dot{H} = dH/dt$, $\dot{K} = dK/dt = \pi$.

The central bank's objective is to stabilize the inflation. The price level is determined using the following equation:

$$P = \alpha_0 + \alpha_1 H + \alpha_2 \eta, \tag{3}$$

where P is the price level, η is a vector of variables affecting P . The inflation $\dot{P} = dP/dt$ is,

$$\dot{P} = \alpha_1 \dot{H} + \alpha_2 \dot{\eta}, \tag{4}$$

where $\dot{\eta} = d\eta/dt$. Because the current study focuses on the central bank's behavior and its influence on inflation, any change in η is not considered here for simplicity, and therefore,

$$\dot{P} = \alpha_1 \dot{H}. \tag{4}$$

The central bank exits quantitative easing at $t = 0$, and it attempts to make the inflation \dot{P} near the target \dot{P}^* by controlling \dot{H} at $t = 0, \dots, \infty$. It minimizes the following quadratic loss function:

⁵ In this model, L can be negative. In some cases, as shown later, L is positive at the exit, but the central bank chooses to reduce L gradually to make it negative. This means that the bank stops fund-absorbing operation, and it starts purchasing new assets that bear r_L .

⁶ Though the market price of A changes, it is assumed that the central bank uses amortized cost instead of fair value in its financial statements as in the cases of the Fed and the BOJ for the long-term assets.

⁷ Many central banks have the rule to make remittance to the government when the profit is positive. Such remittance can be easily introduced into the model in this study, but it does not make any crucial change in the result; the remittance is therefore omitted for simplicity.

$$\min_H \int_0^\infty e^{-\delta t} \left\{ \frac{1}{2} (\dot{P} - \dot{P}^*)^2 \right\} dt, \tag{5}$$

where δ is a discount factor and $\delta < r_L$. Substituting equation (2), (4'), and (5) together, we obtain

$$\min_H \int_0^\infty e^{-\delta t} \left\{ \frac{1}{2} (\alpha_1 \dot{H} - \alpha_1 \dot{H}^*)^2 \right\} dt \tag{5}$$

$$\text{s. t. } \dot{L} = -\dot{H} - r_A A + r_L L + C, \tag{2}$$

$$L = L_0 \text{ at } t = 0. \tag{6}$$

where \dot{H}^* is the target increase in the monetary base, which is consistent with the inflation target \dot{P}^* , $\dot{H}^* = \dot{P}^* / \alpha_1$. Equation (6) is the initial condition, where L_0 is the amount of funds that the central bank must absorb to end quantitative easing. Given δ , r_A , r_L , A , C , \dot{H}^* , and L_0 , the central bank sets \dot{H} for all $t \geq 0$. \dot{H} is a control variable and L is a state variable.

The central bank attempts to set \dot{H} at \dot{H}^* to hit the inflation target. Equation (2) shows that, if the bank's total loss, $r_L L + C - r_A A$, cannot be fully financed through the seigniorage, \dot{H} , the bank must finance the rest through holding more liabilities, \dot{L} .

The central bank cannot accumulate an infinite amount of liabilities, and it needs to satisfy the no-Ponzi game condition to stay solvent.

$$\lim_{t \rightarrow \infty} e^{-r_L t} L \leq 0. \tag{7}$$

If equation (2) with $\dot{H} = \dot{H}^*$ satisfies equation (7), the no-Ponzi game condition is not binding. The bank can always hit the target \dot{H}^* , and the loss function is equal to zero. If it does not satisfy equation (7), the no-Ponzi game condition is binding. Appendix A derives the optimal paths for L and \dot{H} as follows:

$$L = (L_0 - L_S) e^{(\delta - r_L)t} + L_S, \tag{8a}$$

$$\dot{H} = (2r_L - \delta)(L_0 - L_S) e^{(\delta - r_L)t} + \dot{H}^*, \tag{8b}$$

$$\text{where } \dot{H}_S = \dot{H}^*, L_S = \frac{\dot{H}^* + r_A A - C}{r_L}. \tag{9}$$

(\dot{H}_S, L_S) is a steady-state point. L_S is the amount that its interest payments are covered by the steady-state seigniorage plus the

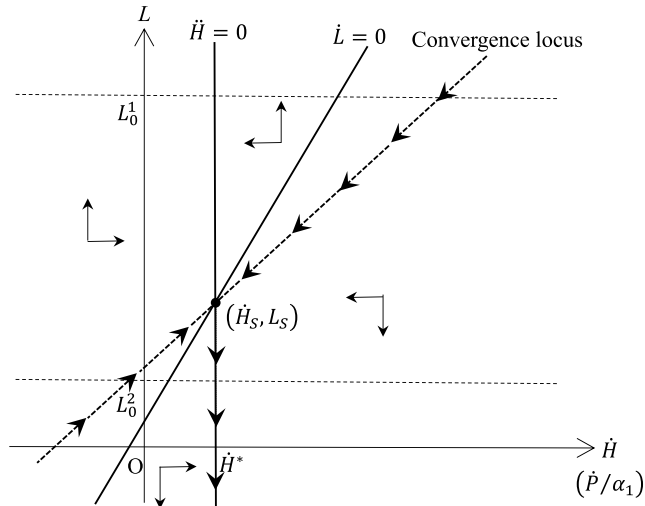


Fig. 2. Phase diagram of \dot{H} and L .

return on existing assets minus the bank's operating cost. Whether the bank moves toward the steady-state point depends on the initial condition L_0 as well as other exogenous variables. Fig. 2 illustrates a phase diagram of the variables \dot{H} and L , where $\ddot{H} = d^2H/dt^2$.

2.2. Policy implications

(a) Case with $L_0 > L_S$

In this subsection, we examine the optimal monetary base path. At the exit, the central bank holds liabilities L_0 . Suppose L_0 is substantially large, such that $L_0 > L_S$. Such L_0 is shown as L_0^1 in Fig. 2. The bank can choose any value of \dot{H} on the horizontal dotted line at $L = L_0^1$. If the bank sets $\dot{H} = \dot{H}^*$, L starts moving upward, and condition (7) does not hold. To avoid an increase in L , the bank must increase \dot{H} to reach the $\dot{L} = 0$ locus, but it is not optimal either. The bank's optimal behavior is to increase \dot{H} further to the convergence locus. The no-Ponzi game condition is binding, and only (7), which has an equal sign, holds. It is now the transversality condition for the model, and the solution is shown by equations (8a) and (8b). The bank's optimal solution only gradually moves along the convergence locus toward the steady-state points (\dot{H}_S, L_S) .⁸

If $L_0 > L_S$, the no-Ponzi game condition (7) is binding, and the central bank cannot stay solvent unless it accelerates the monetary base increase. The optimal increase in the monetary base is larger than the increase only to prevent any expansion in L ; it should be large enough to shrink L . The path of the resulting inflation is derived from equations (8b) and (4'). This shows that the bank cannot avoid considerable inflation after the exit, and inflation only gradually falls toward inflation target \dot{P}^* .

A higher r_L deteriorates the situation. This flattens the convergence locus as its slope is $1/(2r_L - \delta)$, and it makes L^S smaller; both

⁸ When the bank reaches the steady-state point, L remains unchanged, and A is constant, while H continues to expand by \dot{H}_S . This is because π is negative by the same amount, which continuously decreases K by the same amount. Negative K is allowed since it is not in the no-Ponzi game condition. If positive K is preferred, the bank can easily achieve it by accelerating \dot{H} slightly over the convergence locus. It puts the bank on a path for L to diverge downward. Negative L makes π and K positive sooner or later.

After the exit, $t \rightarrow \infty$	
New Assets (-L)	Monetary Base (H)
	Capital (K)

Fig. 3. Central Bank's balance sheet after the exit.

lead to larger \dot{H} .

(b) Case with $L_0 \leq L_S$

Suppose the central bank does not need a large amount of liabilities, $L_0 \leq L_S$, at the exit. Such L_0 is shown as L_0^2 in Fig. 2. If the bank sets $\dot{H} = \dot{H}^*$ for all $t \geq 0$, the minimal value of the loss function (5') is achieved. Equation (2) at $t = 0$ is

$$\dot{L} = r_L L_0^2 - \dot{H}^* - r_A A + C \leq r_L L_S - \dot{H}^* - r_A A + C = 0$$

as equation (9) holds. L never increases throughout $t \geq 0$, and condition (7) does not bind. Therefore, in contrast to the above case (a), the bank can always set \dot{H} at the target \dot{H}^* and thus the inflation \dot{P} at the target \dot{P}^* .

3. Model with slowly shrinking balance sheet

3.1. Model

Although a central bank does not sell the assets to avoid any capital loss or unfavorable shakeup in the relevant markets at and after an exit from quantitative easing, it can reduce asset holdings by tapering; it lets the assets gradually redeemed. In this section, the above model is modified for a central bank to shrink asset holdings at rate ρ . The rate is assumed to be given to the central bank because the redemption schedule is predetermined.

The balance sheet of the central bank before and at the exit is the same as that in Fig. 1. The asset holdings are A_0 at $t = 0$, and, after the exit, the bank lets the asset holdings shrink slowly as they are redeemed at the rate ρ .

$$A = A_0 e^{-\rho t}. \tag{10}$$

Fig. 3 shows the balance sheet at $t \rightarrow \infty$. L can be either positive or negative. A negative L means that the bank clears the liabilities and starts purchasing new assets that bear r_L .⁹

From the balance sheet constraint and equation (1), we have

$$\begin{aligned} \dot{L} &= \dot{A} - \dot{H} - \dot{K} = -\rho A - \dot{H} - r_A A + r_L L + C \\ &= r_L L - \dot{H} - (\rho + r_A) A_0 e^{-\rho t} + C. \end{aligned} \tag{11}$$

Thus, the model in this section is as follows:

$$\min_{\dot{H}} \int_0^{\infty} e^{-\delta t} \left\{ \frac{1}{2} (\alpha_1 \dot{H} - \alpha_1 \dot{H}^*)^2 \right\} dt \tag{5}$$

⁹ It is more realistic for a central bank to take a path of L to diverge negatively. See footnote 8.

$$\text{s. t. } \dot{L} = r_L L - \dot{H} - (\rho + r_A)A_0 e^{-\rho t} + C, \quad (11)$$

$$L = L_0 \text{ at } t = 0. \quad (6)$$

Given $\delta, r_A, r_L, A_0, \rho, C, L_0$, and H^* , the central bank sets \dot{H} for all $t \geq 0$.

If equation (11) with $\dot{H} = \dot{H}^*$ satisfies equation (7), the bank can always hit the target \dot{H}^* . If it does not, Appendix B derives the optimal paths of L and \dot{H} as follows:

$$L = (L_0 - \beta - \gamma)e^{(\delta - r_L)t} + \beta e^{-\rho t} + \gamma, \quad (12a)$$

$$\dot{H} = (2r_L - \delta)(L_0 - \beta - \gamma)e^{(\delta - r_L)t} + \dot{H}^*, \quad (12b)$$

$$\text{where } \beta = \frac{\rho + r_A A_0}{\rho + r_L}, \gamma = \frac{\dot{H}^* - C}{r_L}. \quad (13)$$

3.2. Policy implications

To see when the no-Ponzi game condition (7) is satisfied in this model, we multiply both sides of equation (11) by $e^{-r_L t}$ and sum them from $t = 0$ to ∞ to obtain

$$\int_0^{\infty} e^{-r_L t} (\dot{L} - r_L L) dt = - \int_0^{\infty} e^{-r_L t} \dot{H} dt - \int_0^{\infty} e^{-(\rho + r_L)t} (\rho + r_A) A_0 dt + \int_0^{\infty} e^{-r_L t} C dt.$$

The equation becomes the following:

$$\lim_{t \rightarrow \infty} e^{-r_L t} L = L_0 - \beta - \gamma - \int_0^{\infty} e^{-r_L t} (\dot{H} - \dot{H}^*) dt.$$

If $L_0 \leq \beta + \gamma$, the central bank achieves the target $\dot{H} = \dot{H}^*$ for all t . Condition (7) is always satisfied and is therefore not binding. If $L_0 > \beta + \gamma$, the bank must make \dot{H} larger than the target \dot{H}^* to satisfy the no-Ponzi game condition. It is binding, and the bank must follow the optimal paths (12a) and (12b) to stay solvent. The bank must accelerate the monetary base increase, causing inflation to be higher than the target $\dot{P} > \dot{P}^*$.

A change in r_L, ρ , or \dot{H}^* has the following effects on the central bank:

$$\partial(\beta + \gamma) / \partial r_L < 0, \partial(\beta + \gamma) / \partial \rho > 0, \partial(\beta + \gamma) / \partial \dot{H}^* > 0,$$

$$\partial \dot{H} / \partial r_L \geq 0, \partial \dot{H} / \partial \rho < 0, \partial \dot{H} / \partial \dot{H}^* > 0.$$

An increase in r_L , a decrease in ρ , or a decrease in \dot{H}^* pushes the bank toward $L_0 > \beta + \gamma$. The sign of $\partial \dot{H} / \partial r_L$ depends on t ; it is positive at $t = 0$, but it later turns negative. With higher r_L , the bank shrinks more L immediately after the exit to avoid the heavy interest burden, which forces the bank to accelerate the monetary

base increase \dot{H} earlier rather than later. A higher ρ allows the bank to make the monetary base increase more slowly.

4. Simulations to the Bank of Japan and the Federal Reserve

4.1. Case of the Bank of Japan

The model in Section 3 can be applied to various central banks to explore how to exit quantitative easing. This section examines some possible exit strategies for the BOJ and the Fed. Both banks have been conducting quantitative easing, but this section reveals a difference between them.

Suppose that the BOJ exits quantitative easing in April 2020. In March 2020, which is the end of fiscal year 2019, the BOJ has 604 trillion yen of total assets. Though the model assumes that a central bank holds only H and L as liabilities, the BOJ has some other liabilities, which amount to 90 trillion yen. To adjust the difference, A is the total assets minus the other liabilities, $A = 604$ trillion yen $-$ 90 trillion yen = 514 trillion yen. The BOJ has various assets, but A is assumed to consist of only long-term securities as in the model.

The BOJ has 505 trillion yen of monetary base, of which 120 trillion yen is the cash and required reserves, and 385 trillion yen is the excess reserves. Since the excess reserves used to be negligible before quantitative easing, it is assumed that the BOJ absorbs all excess reserves with some interest-bearing liabilities L at the exit in April. Thus, the balance sheet at the exit, $t = 0$, is set as the upper part of Table 1. The figures $r_A = 0.36\%$ and $C = 0.199$ trillion yen are from the fiscal year 2019, and δ is assumed to be lower than r_L by 1%.

The lower part of Table 1 presents the simulation results. Case (1) assumes that r_L is 2.5%¹⁰ and the BOJ targets 0% inflation, $\dot{P}^* = \alpha_1 \dot{H}^* = 0$. The majority of the assets held by the BOJ are Japanese government bonds (JGBs), and the average maturity of those purchased by the bank is approximately nine years.¹¹ In case (1), the BOJ stops purchasing JGBs, and the asset holdings are assumed to decrease at $\rho = 1/9 = 11.1\%$ due to the redemption. Then, $L_0 - \beta - \gamma = -40.70 < 0$, and the no-Ponzi game condition (7) is therefore not binding; the BOJ can hit the target $\dot{P}^* = \alpha_1 \dot{H}^* = 0$. In case (2), the minimum value of r_L is calculated to satisfy $L_0 - \beta - \gamma > 0$. If $r_L = 2.5\%$ is too low and r_L must be raised to 4.03% to absorb the excess reserves, $L_0 - \beta - \gamma$ becomes positive, and the BOJ starts having difficulty in hitting the target. Thus, the BOJ does not face the solvency problem in the normal circumstances, but it is vulnerable to a rise in the interest rate.

It may not be appropriate to assume that the BOJ stops purchasing JGBs completely at and after the exit. In fiscal year 2019, the government issued 46 trillion yen of JGBs, excluding refunding bonds, and the BOJ's holdings of JGBs increased by 16 trillion yen. At the end of the fiscal year 2019, the total outstanding of JGBs is 988 trillion yen, 486 trillion yen of which is held by the BOJ. In such a situation, reducing the BOJ's holdings by $486 \times 11.1\% = 54$ trillion yen in one year may have a large impact on the JGB market, and the BOJ may want to avoid such a shakeup.

As the extreme case is opposite to no purchase, suppose the BOJ purchases some JGBs to keep the asset holdings constant, as

¹⁰ Iwata and the Japan Center for Economic Research (2014) assume the interest rate on a part of the excess reserves after exiting quantitative easing in Japan to be 2.5% or 3.0% in its simulation. In their simulation, Fujiki and Tomura (2017) assume the short-term interest rate to be 2.75% in the long run.

¹¹ Cabinet Office of Japan (2015) calculated the average maturity of the JGBs purchased by the BOJ. It gradually became longer and reached approximately nine years in December 2014–May 2015.

Table 1
Some simulations of the Bank of Japan's exit.

At the exit, $t = 0$					
$A_0 = 514^a$					$H_0 = 120$
					$L_0 = 385$
					$K_0 = 10$
$r_A = 0.36\%^b, C = 0.199, \delta = r_L - 1\%$					
Case	r_L	\dot{H}^*	ρ	$L_0 - \beta - \gamma$	\dot{H}_0
(1)	2.5%	0	11.1%	-40.70	0
(2)	4.03%	0	11.1%	0.08	0.00
(3)	2.5%	0	0.0%	319.25	11.17
(4)	2.5%	2.40	0.0%	223.16	7.81

Notes: Trillion yen unless otherwise stated.

^a A_0 is equal to the total assets minus the liabilities other than H_0 and L_0 .

^b r_A is equal to the sum of the BOJ's profit and C divided by A_0 .

Source: The BOJ's figures are from its financial statements and annual review.

Table 2
Some simulations of the Federal Reserve's exit.

At the exit, $t = 0$					
$A_0 = 3348^a$					$H_0 = 1975$
					$L_0 = 1333$
					$K_0 = 39$
$r_A = 1.88\%^b, C = 7.44, \delta = r_L - 1\%$					
Case	r_L	\dot{H}^*	ρ	$L_0 - \beta - \gamma$	\dot{H}_0
(1)	2.05%	0	14.3%	-1615.8	0
(2)	25.42%	0	14.3%	0.0	0.01
(3)	2.05%	0	0.0%	-1371.9	0
(4)	2.05%	39.5	0.0%	-3298.6	0

Notes: Billion dollars unless otherwise stated.

^a A_0 is equal to the total assets minus the liabilities other than H_0 and L_0 .

^b r_A is equal to the sum of the Fed's profit and C divided by A_0 .

Source: The Fed's figures are from its financial statements.

assumed in Section 2. Case (3) in Table 1 shows that $L_0 - \beta - \gamma$ is positive and the monetary base should expand by 11.17 trillion yen at $t = 0$, which corresponds to a $\dot{H}_0/H_0 = 11.17/120 = 9.31\%$ increase. If the BOJ targets a $2\% \times H_0 = 2.40$ trillion yen increase, case (4) shows that the monetary base should expand by 7.81 trillion yen, which corresponds to a 6.50% increase. Thus, the BOJ's success in exiting quantitative easing in April 2020 crucially depends on the fiscal deficit and JGB market conditions. The more JGBs the BOJ must purchase for market stability, the more likely it is that the bank has no option but to accelerate the monetary base increase and thus inflation.

4.2. Case of the Federal Reserve

Suppose that the Fed exits quantitative easing in January 2020. In December 2019, the end of the fiscal year 2019, the Fed had a monetary base of 3308 billion dollars. The excess reserves used to be 4% of the required reserves before quantitative easing, so it is assumed that the Fed reduces the monetary base to the sum of the cash, the required reserves, and the excess reserves that is equal to 4% of the required reserves. The sum amounted to 1975 billion dollars, and 1333 billion dollars of excess reserves are assumed to be absorbed. Thus, the balance sheet at the exit, $t = 0$, is set as the upper part of Table 2. The figures $r_A = 1.88\%$ and $C = 7.44$ billion dollars are from the fiscal year 2019, and δ is assumed to be lower

than r_L by 1%.

The lower part of Table 2 presents some simulations, and the cases are equivalent to those for the BOJ. In case (1), r_L is assumed to be 2.05%.¹² The average maturity of Treasury Securities purchased by the Fed is approximately seven years.¹³ The Fed stops purchasing Treasury Securities, and asset holdings are assumed to decrease at $\rho = 1/7 = 14.3\%$ due to redemption. Since $L_0 - \beta - \gamma = -1615.8 < 0$, the no-Ponzi game condition (7) is not binding; the Fed can hit the target. In case (2), the minimum value of r_L satisfying $L_0 - \beta - \gamma > 0$ is 25.42%. In contrast to the BOJ, the Fed has much room to raise r_L to absorb excess reserves. Cases (3) and (4) show that $L_0 - \beta - \gamma < 0$, and the no-Ponzi game condition (7) does not bind.

In contrast to the BOJ, the Fed can hit the target with no risk of insolvency. Even if $r_L = 2.05\%$ is not high enough to absorb excessive reserves, it can raise r_L up to 25.42%. Thus, the Fed's situation is much better than that of the BOJ. This is mainly because of the

¹² Carpenter et al. (2015) assume the interest rate for fund-absorbing operation is the IOER+0.5% for their simulations. The IOER is 1.55% in December 2019, and so r_L is set to 2.05%.

¹³ The Fed releases its monthly purchase allocations of Treasury Securities. In August 2021, the bank allocated 31% of purchases for the maturity range 0–2.25 years, 21% for the range 2.25–4.5 years, and so on. The average maturity is calculated as $31\% \times (2.25 - 0)/2 + 21\% \times (4.5 - 2.25)/2 + \dots = 7\%$.

following three differences. First, the Fed only needs to absorb a smaller portion of its balance sheet; L_0/A_0 is smaller. Second, a higher r_A helps make the situation better since a higher interest rate on assets brings more profit. Third, the Fed's ρ is larger because of the shorter average maturity of A_0 , and a larger ρ means that a larger amount is absorbed by redemption.

5. Conclusion

This study examined a central bank's optimal behavior to exit quantitative easing using interest-bearing liabilities instead of rapidly shrinking its balance sheet. It constructed a dynamic optimization model of a central bank and derived the optimal paths of its monetary base supply and liabilities. This study investigated the model with an unreduced balance sheet and then modified the model to allow the balance sheet to shrink slowly by redemption (tapering). The analysis revealed the following findings:

The condition for a central bank to face the insolvency problem depends on various variables, including interest rates, the rate of asset redemption, asset holdings, the size of funds to be absorbed at an exit, targets of monetary base increase and inflation, and the bank's operating costs.

When this condition is not satisfied, the central bank faces insolvency. It forces the bank to accelerate the monetary base increase, which should be large enough to shrink the liabilities used to exit quantitative easing and, therefore, cause higher inflation. The situation deteriorates with higher interest in liabilities or slower asset redemption.

The model was applied to the BOJ and the Fed. The simulations for the banks to exit quantitative easing reveal a difference between them. The Fed does not face the insolvency problem and has room to raise the interest rate to absorb excessive reserves. In contrast, the BOJ has limited room and is vulnerable to a sizeable increase in the interest rate. If the interest rate that is needed to ensure price stability is high, the bank may have no choice but to be inflationary.

This study explicitly derived the optimal path of monetary base supply, making it easy to examine how the path changes according to other variables. However, some details are left out in the model. The model assumes a one-time shift at the exit, such as setting a higher interest rate only once. It includes no transmission channel of the interest rate on inflation, and it does not assume that a central bank holds various assets.

These issues should be addressed in future studies.

Declaration of Competing interest

None.

Acknowledgements

I would like to thank Takuma Kunieda, Kenjiro Hirayama, the editor, and the anonymous referees for their helpful comments and suggestions. The responsibility for any remaining errors is mine alone.

Appendix A. Derivations of the Model with the Balance Sheet Constant

The current value Hamiltonian \mathcal{H} is

$$\mathcal{H} = \frac{1}{2}(\alpha_1 \dot{H} - \alpha_1 \dot{H}^*)^2 + m(-\dot{H} - r_A A + r_L L + C), \quad (A1)$$

where m is the current-value multiplier. The first-order conditions are as follows:

$$\partial \mathcal{H} / \partial \dot{H} = \alpha_1^2 (\dot{H} - \dot{H}^*) - m = 0, \quad (A2a)$$

$$\dot{m} = \delta m - \partial \mathcal{H} / \partial L = (\delta - r_L)m, \quad (A2b)$$

where $\dot{m} = dm/dt$. Equation (A2a), (A2b), and (2) are reduced to the following differential equations:

$$\dot{L} = r_L L - \dot{H} - r_A A + C, \quad (A3a)$$

$$\ddot{H} = (\delta - r)(\dot{H} - \dot{H}^*), \quad (A3b)$$

where $\ddot{H} = d^2H/dt^2$. A steady-state (\dot{H}_S, L_S) is

$$\dot{H}_S = \dot{H}^*, L_S = \frac{\dot{H}^* + r_A A - C}{r_L}. \quad (9)$$

The steady-state of inflation is

$$\dot{P}_S = \dot{P}^* = \alpha_1 \dot{H}^*. \quad (A4)$$

Fig. 2 illustrates the phase diagram of variables \dot{H} and L . The $\dot{L} = 0$ locus is derived from equation (A.3a), and its slope is $1/r_L$. L increases in the area to the left of $\dot{L} = 0$ and decreases in the area to its right. The $\ddot{H} = 0$ locus is derived from equation (A.3b) and is vertical at \dot{H}^* . \dot{H} is decreasing if $\dot{H} > \dot{H}^*$ and increases if $\dot{H} < \dot{H}^*$ since $\delta < r_L$. The steady-state (\dot{H}_S, L_S) is a saddle point. A central bank needs to satisfy the following no-Ponzi game condition:

$$\lim_{t \rightarrow \infty} e^{-r_L t} L \leq 0, \quad (7)$$

to stay solvent.

From equations (A.3a) and (A.3b), a second-order differential equation of L is derived as follows:

$$\ddot{L} - \delta \dot{L} + r_L(\delta - r_L)L = (\delta - r_L)(r_A A + \dot{H}^* - C), \quad (A5)$$

where $\ddot{L} = d^2L/dt^2$. Solving differential equation (A.5) derives the optimal paths of L .

$$L = k_1 e^{r_L t} + k_2 e^{(\delta - r_L)t} + L_S. \quad (A6)$$

As $1 > r_L > \delta > 0$ is assumed, only the path with $k_1 = 0$ can reach L_S .

With the initial condition (6), the optimal paths of L and \dot{H} are derived as follows:

$$L = (L_0 - L_S)e^{(\delta - r_L)t} + L_S, \quad (8a)$$

$$\dot{H} = (2r_L - \delta)(L_0 - L_S)e^{(\delta - r_L)t} + \dot{H}^*. \quad (8b)$$

The resulting inflation path is derived by substituting equation (8b) into (4'). The convergence loci shown by equations (8a) and (8b) are illustrated in Fig. 2. Its slope is $1/(2r_L - \delta)$, which is smaller than the slope of $\dot{L} = 0$ locus.

Appendix B. Derivations of the Model with Shrinking the Balance Sheet Slowly

The current value Hamiltonian \mathcal{H} is

$$\mathcal{H} = \frac{1}{2}(\alpha_1 \dot{H} - \alpha_1 \dot{H}^*)^2 + m\{r_L L - \dot{H} - (\rho + r_A)A_0 e^{-\rho t} + C\}, \quad (\text{B1})$$

where m is the current–value multiplier. The first-order conditions are as follows:

$$\partial \mathcal{H} / \partial \dot{H} = \alpha_1^2 (\dot{H} - \dot{H}^*) - m = 0, \quad (\text{B2a})$$

$$\dot{m} = \delta m - \partial \mathcal{H} / \partial L = (\delta - r_L)m. \quad (\text{B2b})$$

Equations (B.2a), (B.2b), and (11) are reduced to the following differential equations:

$$\dot{L} = r_L L - \dot{H} - (\rho + r_A)A_0 e^{-\rho t} + C, \quad (\text{B3a})$$

$$\ddot{H} = (\delta - r_L)(\dot{H} - \dot{H}^*). \quad (\text{B3b})$$

From these equations, a second-order differential equation of L is derived;

$$\ddot{L} - \delta \dot{L} + r_L(\delta - r_L)L = (\rho + r_A)(\delta - r_L + \rho)A_0 e^{-\rho t} + (\delta - r_L)(\dot{H}^* - C). \quad (\text{B4})$$

Solving the differential equation (B4) derives the optimal path of L .

$$L = (L_0 - \beta - \gamma)e^{(\delta - r_L)t} + \beta e^{-\rho t} + \gamma, \quad (\text{12a})$$

$$\text{where } \beta = \frac{\rho + r_A}{\rho + r_L} A_0, \quad \gamma = \frac{\dot{H}^* - C}{r_L}. \quad (\text{13})$$

From equations (12a) and (11), the optimal path of \dot{H} is

$$\dot{H} = (2r_L - \delta)(L_0 - \beta - \gamma)e^{(\delta - r_L)t} + \dot{H}^*. \quad (\text{12b})$$

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