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Empirical DSGE model evaluation with interest rate expectations measures and preferences over safe assets

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Empirical DSGE model evaluation with interest rate expectations measures and preferences over safe assets by Gregory de Walque, Thomas Lejeune and Ansgar Rannenberg





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Abstract

We estimate a DSGE model with Preferences Over Safe Assets (POSA) on Euro Area macroeconomic data and interest rate expectations measures. The model with POSA has much better empirical fit than the otherwise identical model without, especially once interest rate expectations are added to the data set. Including measures interest rate expectations strongly improves the model forecast of GDP and its components, with the best forecast delivered by the POSA model. Finally, with POSA, ECB forward guidance increased GDP and inflation by 1.9 % and 0.1 percentage points by 2019Q4, respectively, much less than without POSA.

Keywords: DSGE estimation with interest rate expectations in the data set, forecasting, forward guidance, preferences over safe assets.

JEL Classifications: E37, E43, E47, E52.

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Non-technical summary

The goal of this paper is to compare the empirical performance of two DSGE models using Euro Area data, and the effect forward guidance by the European central bank within them. For that purpose, we add anticipated monetary policy shocks to the monetary policy rule and identify the associated stochastic processes by including market based measures of policy rate expectations (for instance Overnight Index Swap (OIS) rates during the Euro Area period), on top of more standard macroeconomic data, following the analysis of Campbell et al. (2019) for the US. The first model is a standard Smets and Wouters (2007) model. The second model differs from the first in that household have preferences over save assets (POSA), i.e. long- and short term government bonds. As shown in Rannenberg (2019), POSA attenuate the effect of forward guidance in the model for two reasons. POSA reduces the "net weight" the household attaches to future consumption as the individual discount rate of the households exceeds the real interest rate, and creates a consumption wealth effect from government bonds.

We obtain the following results. Firstly, while the empirical fit of the POSA model is already somewhat better in absence of interest rate expectation measures from the data set, its relative performance dramatically improves once this data is included.

Secondly, as expected, with POSA, the anticipated monetary policy shocks of the estimated model have a smaller effect on GDP at all horizons than without POSA, while the effect on the nominal forward interest rate at the horizon of the anticipated shock is consistently more negative. Relatedly, if we peg the interest rate by 0.2 percentage points below its steady state value for a fixed number of periods, we find that without POSA, GDP and inflation increase very strongly exponentially in the length of the peg, in line with the existing literature (see Campbell et al. (2019), Negro et al. (2012) and Carlstrom et al. (2015)). By contrast, in the model with POSA, the effect is not only substantially muted, but also becomes concave in the length of the peg, with the "wealth effect" playing a crucial role. Hence the POSA model is not subject to the so called "Forward Guidance Puzzle".

Thirdly, we examine the out-of-sample forecasting performance of the models. We find that adding interest rate expectation measures to the NOPOSA model's data set strongly improves the forecasts of GDP, consumption and investment, and the interest rate. This improvement stems from the fact that on several occasions, the forecast by the model estimated on interest rate expectation measures of the path of real activity following a recession is less overly optimistic than the forecast generated by the model without such measures in the data set. The more pessimistic prediction is in turn related to the more persistent effect of the main demand shock in the model. Adding POSA further improves the forecast for output and consumption, and for investment at horizons exceeding nine quarters. We can relate the weaker forecasting performance of the NOPOSA model to the effect of the anticipated monetary policy shocks on the forecast.

Finally, regarding the effects of ECB forward guidance, they are found to differ between the two models. The historical decomposition of the NOPOSA model shows that post 2013Q2, the combined contribution of the anticipated monetary policy shocks became gradually more expansionary. By 2019Q4, this change had increased GDP by 8% relative to trend and year-on-year inflation by 0.4 percentage points. In the POSA model, the change in the combined contribution of the anticipated shocks amount to 2.2% and 0.1 percentage points over this period, respectively. In both models, the main driver of the low nominal policy and three year forward rate is a decline in aggregate demand rather than monetary policy. For the NOPOSA model, we find no perceptible effect of the more expansionary anticipated monetary policy shocks post 2013Q2 on the nominal three year forward rate, due to the strong stimulative effect of anticipated expansionary monetary policy shocks in the model. By contrast, with POSA, the combined contribution of the anticipated monetary policy shocks on the 3 year yield becomes more negative post 2013Q2, by 1.3 percentage points.

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1 Introduction

The goal of this paper is to compare the empirical performance of two DSGE models using Euro Area data, and the effect forward guidance by the European central bank within them. For that purpose, we add anticipated monetary policy shocks to the monetary policy rule and identify the associated stochastic processes by including market based measures of policy rate expectations (for instance Overnight Index Swap (OIS) rates during the Euro Area period), on top of more standard macroeconomic data, following the analysis of Campbell et al. (2019) for the US. The first model is a standard Smets and Wouters (2007) model. The second model differs from the first in that household have preferences over save assets (POSA), i.e. long- and short term government bonds. As shown in Rannenberg (2019), POSA attenuate the effect of forward guidance in the model for two reasons. POSA reduces the "net weight" the household attaches to future consumption as the individual discount rate of the households exceeds the real interest rate, and creates a consumption wealth effect from government bonds.

We obtain the following results. Firstly, while the empirical fit of the POSA model is already somewhat better in absence of interest rate expectation measures from the data set, its relative performance dramatically improves once this data is included. Specifically, with measures of interest rate expectations up to 8 quarters ahead in the data set, the POSA model outperforms the NOPOSA model by 33 log points. This difference grows to 42 log points if the horizon of the interest rate expectation measures included in the data set rises to 12 quarters.

Secondly, as expected, with POSA, the anticipated monetary policy shocks of the estimated model have a smaller effect on GDP at all horizons than without POSA, while the effect on the nominal forward interest rate at the horizon of the anticipated shock is consistently more negative. Relatedly, if we peg the interest rate by 0.2 percentage points below its steady state value for a fixed number of periods, we find that without POSA, GDP and inflation increase very strongly exponentially in the length of the peg, in line with the existing literature (see Campbell et al. (2019), Negro et al. (2012) and Carlstrom et al. (2015)). By contrast, in the model with POSA, the effect is not only substantially muted, but also becomes concave in the length of the peg, with the "wealth effect" playing a crucial role (the contribution of the wealth effect was already noted in the calibrated model of Rannenberg (2019)). Hence the POSA model is not subject to the so called "Forward Guidance Puzzle".

Thirdly, we examine the out-of-sample forecasting performance of the models. We find that adding interest rate expectation measures to the NOPOSA model's data set strongly improves the forecasts of GDP, consumption and investment, and the interest rate. This improvement stems from the fact that on several occasions, the forecast by the model estimated on interest rate expectation measures of the path of real activity following a recession is less overly optimistic than the forecast generated by the model without such measures in the data set. The more pessimistic prediction

is in turn related to the more persistent effect of the main demand shock in the model. Adding POSA further improves the forecast for output and consumption, and for investment at horizons exceeding nine quarters. We can relate the weaker forecasting performance of the NOPOSA model to the effect of the anticipated monetary policy shocks on the forecast.

Finally, regarding the effects of ECB forward guidance, they are found to differ between the two models. The historical decomposition of the NOPOSA model shows that post 2013Q2, the combined contribution of the anticipated monetary policy shocks became gradually more expansionary. By 2019Q4, this change had increased GDP by 8% relative to trend and year-on-year inflation by 0.4 percentage points. In the POSA model, the change in the combined contribution of the anticipated shocks amount to 2.2% and 0.1 percentage points over this period, respectively. In both models, the main driver of the low nominal policy and three year forward rate is a decline in aggregate demand rather than monetary policy. For the NOPOSA model, we find no perceptible effect of the more expansionary anticipated monetary policy shocks post 2013Q2 on the nominal three year forward rate, due to the strong stimulative effect of anticipated expansionary monetary policy shocks in the model. By contrast, with POSA, the combined contribution of the anticipated monetary policy shocks on the 3 year yield becomes more negative post 2013Q2, by 1.3 percentage points.

Our estimation is an adaption of the estimation approach of Campbell et al. (2019) to the Euro Area context, but unlike them we compare the empirical performance of the POSA and the NOPOSA model, in and out-of-sample, and investigate the shock decomposition of the model to investigate the effect of forward guidance. Another recent contribution using expectations of interest rates and other variables as observables in the estimation of a Euro Area model is Mueller et al. (2022), who draw on the Euro Area Survey of Professional Forecasters (SPF), as well as the Euro Area yield curve in some estimations. Our contribution differs from theirs in the following respects. Firstly, we construct a data set of market interest rates to measure interest rate expectations instead of the SPF, implying that our estimation observes interest rate expectation measures starting in 1990Q1. For the post 1998 period, we rely on overnight index swap (OIS) rates, while during the pre-Euro Area period we construct our series from money market, Euro Market and government bond zero coupon yields (see B.3 for details). Moreover, Mueller et al. (2022) include 1-year ahead forecasts of real GDP growth from SPF data, which guide model-implied output expectations and influence the model predictions for real GDP growth. They find no evidence of any significant difference in out-of-sample real GDP growth forecasts between models with and without interest rate expectation measures. Instead, we do not tie the output growth expectations of the model – and hence model output growth forecast – to survey data and find better out-of-sample performance for models with interest-rate expectations when it comes to predict real GDP growth. Secondly, we use interest expectations over a longer horizon, up to three years, similar to Campbell et al. (2019), who use expectations up to 10 quarters. The longer horizon may be relevant when assessing the contribution of ECB forward guidance. Thirdly, as in Campbell et al. (2019), for a given maximum horizon of interest rate expectation measures observed in an estimation (say 8 quarters), we observe the full yield curve (i.e. the average interest rate over the following quarter, the next 2 quarters from today, the next 3 quarters...,the next 8 quarters). Finally, we show that once interest rate expectation measures are included in the data set, the data strongly prefers the model with POSA. By contrast, Mueller et al. (2022) do not find evidence in favor of a related ad-hoc specification, which, like our POSA, does features "discounting" in the realized consumption Euler equation but, unlike our POSA, does not feature a "wealth effect" of government bonds on consumption.

The remainder of the paper is structured as follows. Section 2 describes the model, Section 3 describes the estimation, the estimated parameters and how they are affected by the inclusion of interest rate expectation measures in the data set, and the relative fit of the POSA and NOPOSA model(s). Section 4 discusses the IRFs and the effect of an interest rate peg. Section 5 investigates the out-of-sample forecasting performance of the models. Section 6 examines the (change of the) contribution of the anticipated monetary policy shocks to economic activity and inflation post 2013Q2.

2 The model

Many features of the model are standard and closely follow Smets and Wouters (2007). However, there is a fiscal sector levying distortionary taxes on households and firms, with expenditures and tax rates responding to economic activity and debt via estimated fiscal rules. With POSA, households have preferences over government debt. When assuming POSA, we also assume preferences over the physical capital stock, which allows us to neutralize the effect POSA otherwise would have on the steady-state capital rental rate and thus the steady-state in general. Smaller case letters denote stationarized counterparts of trended variables, i.e. $x_t = \frac{X_t}{TFP_{t-1}}$, where TFP_{t-1} denotes the deterministic component of technology, determined as $TFP_t = \gamma TFP_{t-1}$. Unless otherwise mentioned, all variables denoted as $\varepsilon_{s,t}$ denote exogenous AR(1) processes with mean zero, where the subscript s indexes the respective shock, while $\eta_{s,t}$ denote i.i.d. normally distributed shock innovations.

2.1 Firms

There is a continuum of retailers indexed as f. The production function of retailer f is given by

$$Y_{f,t} = \exp\left(\varepsilon_{a,t}\right) \left(TFP_{t-1}N_{f,t}\right)^{1-\alpha} \tilde{K}_{f,t}^{\alpha} - TFP_{t-1}\Phi \tag{1}$$

where $N_{f,t}$ denotes household labor hired by retailer f, respectively, while $\varepsilon_{a,t}$ and Φ denote a transitory technology shock and fixed costs of production, respectively, and $\tilde{K}_{f,t}$ denotes total

capital services. Retailers produce a product variety from the goods basket consumed by households. Following Smets and Wouters (2007), the basket is a Kimball (1995) aggregator. There are economy wide markets for all factors of production, implying that marginal costs are identical across firms. Furthermore, firms face nominal rigidities in the form of Calvo (1983) frictions, i.e. only a fraction $1-\omega_p$ is allowed to reoptimize its price, while, following Warne et al. (2008), the remaining fraction adjusts their prices according to the indexation scheme $P_t(f) = \prod_{t=1}^{\iota_p} \prod_{obj,t}^{(1-\iota_p)}$, with $0 \le \iota_p \le 1$, where $\prod_{obj,t}$ denotes the potentially time-varying inflation target of the central bank. As shown in Smets and Wouters (2007), up to first order these assumptions gives rise to the following New Keynesian Phillips Curve

$$\hat{\Pi}_{t} - \hat{\Pi}_{obj,t} = \frac{1}{1 + \beta \iota_{p}} \begin{pmatrix} \beta E_{t} \left(\hat{\Pi}_{t+1} - \hat{\Pi}_{obj,t+1} \right) + \iota_{p} \left(\hat{\Pi}_{t-1} - \hat{\Pi}_{obj,t} \right) \\ + \iota_{p} \beta \left(\hat{\Pi}_{obj,t+1} - \hat{\Pi}_{obj,t} \right) + \frac{(1 - \omega_{p})(1 - \omega_{p}\beta)}{\omega_{p}(\mu_{p} - 1)\epsilon_{p} + 1} \hat{m} c_{t} \end{pmatrix} + \varepsilon_{p,t}$$

$$(2)$$

where μ_p , ϵ_p and $\varepsilon_{p,t}$ denotes the gross markup, the degree of curvature in the firms demand curve and the price markup shock, respectively. $\varepsilon_{p,t}$ follows an ARMA(1,1) process:

$$\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + \eta_{p,t} - v_p \eta_{p,t-1} \tag{3}$$

The retailer's FOCs with respect labor and physical capital can be aggregated as

$$\frac{W_t}{P_t} = mc_t \left(1 - \alpha\right) \frac{Y_t + TFP_{t-1}\Phi}{N_t} \tag{4}$$

$$r_{K,t} = mc_t \alpha \frac{Y_t + TFP_{t-1}\Phi}{\tilde{K}_t} \tag{5}$$

2.2 Households

Household j derives utility from consumption $C_t(j)$, short term government bonds $\frac{B_{G,L,t}}{P_t}$, long-term government bonds $\frac{B_{G,L,t}}{P_t}$, and holdings of physical capital $\bar{K}_t(j)$, and disutility from labor $N_t(j)$:

$$\sum_{i=0}^{\infty} \beta^{i} \begin{bmatrix} ln\left(C_{t+i}\left(j\right) - hC_{S,t+i-1}\right) + \frac{N_{t+i}^{1+\sigma_{l}}\left(j\right)}{1+\sigma_{l}} \\ + \frac{\chi_{b,t+i}}{1-\sigma_{b}} \left(\frac{B_{G,t+i}\left(j\right)}{P_{t+i}} + \frac{B_{G,L,t+i}\left(j\right)}{P_{t+i}}\right)^{1-\sigma_{b}} \\ + \frac{\chi_{K,t+i}}{1-\sigma_{K}} \bar{K}_{t+i}^{1-\sigma_{K}}\left(j\right) + \left(\frac{B_{G,t+i}\left(j\right)}{P_{t+i}} + \frac{B_{G,L,t+i}\left(j\right)}{P_{t+i}}\right) \chi_{\varepsilon_{b},t+i}\varepsilon_{b,t+i} \end{bmatrix}$$

$$(6)$$

We assume $\chi_{b,t} = \frac{\chi_b}{TFP_{t-1}^{1-\sigma_b}}$ and $\chi_{K,t} = \frac{\chi_K}{TFP_t^{1-\sigma_K}}$ in order to induce a balanced growth path. Furthermore, $\varepsilon_{b,t}$ denotes a liquidity demand shock as in Fisher (2015), which increases the desirability of holding safe assets, with mean zero, while $\chi_{\varepsilon_b,t} = \frac{\theta \xi}{TFP_{t-1}}$ merely normalizes the direct impact

of $\varepsilon_{b,t}$ on the linearized household's FOCs with respect to bonds to equal the effect of a change in the short-term interest rate.

One motivation for utility from government bonds, or POSA, is liquidity preference. Krishnamurthy and Vissing-Jorgensen (2012) argue that liquidity preference may extend to assets with a positive yield if they have money-like qualities, and provide supporting evidence for the case of US government bonds. More recently, Kaplan and Violante (2018) have suggested using POSA as a simple shortcut to capture a feature of heterogeneous agent models, namely the idea that in the presence of uninsurable risk, the household sector values the existence of a safe and liquid asset due to its precautionary value. A motivation pertaining utility from all types of assets, including capital, are "Capitalist Spirit" type preferences (CSP) over wealth, meaning that households derive utility from the prestige, power and security associated with wealth. Several authors have argued that such preferences are necessary to replicate rich household's saving behavior in US data, namely the positive marginal propensity to save out of permanent-income changes (see Dynan et al. (2004) and Kumhof et al. (2015)), and the level of wealth held by rich households relative to their disposable income (see Kumhof et al. (2015) for a survey).

The households faces four constraints. The first two are the budget constraint and capital accumulation equation:

$$\frac{B_{G,t}}{P_{t}}(j) + (1 + \tau_{C,t}) C_{t}(j) + I_{t}(j) = \frac{R_{t-1}}{\Pi_{t}} \frac{B_{G,t-1}}{P_{t-1}}(j) + (1 - \tau_{w,h,t} - \tau_{N,t}) \frac{W_{t}(j)}{P_{t}} N(j)_{t} + Prof(j)_{t}
+ ((1 - \tau_{K}) (r_{K,t} Z_{t} - a(Z_{t})) + \tau_{K} \delta) \bar{K}_{t-1}(j) - T_{t}
- \frac{B_{G,L,n,t}(j)}{P_{t}} + \frac{(R_{G,L,t-1}(j) - 1 + \omega_{LTD})}{\Pi_{t}} \frac{B_{G,L,t-1}(j)}{P_{t-1}}
\bar{K}_{t}(j) = (1 - \delta) \bar{K}_{t-1}(j) + \varepsilon_{I,t} \left(1 - S\left(\frac{I_{t}(j)}{I_{t-1}(j)}\right)\right) I_{t}(j)$$
(8)

where $\frac{B_{G,t}}{P_t}(j)$, R_{t-1} , $I_t(j)$, $W_t(j)$ and $Prof(j)_t$ denote short-term government bonds, investment, the nominal wage and the profits of monopolistically competitive firms and labor unions owned by households, respectively. $r_{K,t}$, Z_t , $a(Z_t)$, δ and $S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)$ denote the capital rental, capacity utilization, convex costs of capacity utilization, the depreciation rate and the convex costs of adjusting investment, respectively. $\tau_{C,t}$, $\tau_{N,t}$ and T_t denote the consumption, labor and lump-sum tax, respectively, while $\tau_{w,h,t}$ denotes employees social security contributions.

Following Krause and Moyen (2016), we assume that long-term government bonds are re-payed in a stochastic fashion, with ω_{LTD} denoting the fraction of long-term bonds maturing each quarter. The final line of the budget constraints denotes the cash-flow associated with the households investment in long-term government bonds, where $B_{G,L,n,t}(j)$, $B_{G,L,t}(j)$, $R_{G,L,t}(j)$ denote newly

issued long-term government bonds, total government bond holdings and the average interest rate on all outstanding government bonds. $B_{G,L,t}(j)$ and $R_{G,L,t}(j)$ evolve according to

$$\frac{B_{G,L,t}(j)}{P_{t}} = (1 - \omega_{LTD}) \frac{\frac{B_{G,L,t-1}(j)}{P_{t-1}}}{\prod_{t}} + \frac{B_{G,L,n,t}(j)}{P_{t}}$$

$$(R_{G,L,t}(j) - 1) \frac{B_{G,L,t}(j)}{P_{t}} = (1 - \omega_{LTD}) \frac{(R_{G,L,t-1}(j) - 1)}{\prod_{t}} \frac{B_{G,L,t-1}(j)}{P_{t-1}} + (R_{G,L,n,t}(j) - 1) \frac{B_{G,L,n,t}(j)}{P_{t}}$$

$$(10)$$

which constitutes the third and fourth constraint faced by the household.

The full set of first order conditions is located in A.1. To illustrate the effects of the POSA and the capitalist spirit assumptions, we display the detrended first order conditions with respect to short-term bonds and capital

$$\xi_t = \beta E_t \left\{ \frac{\xi_{t+1}}{\gamma} \frac{R_t}{\Pi_{t+1}} \right\} + \chi_b \left(b_{G,L,t} \right)^{-\sigma_b} + \chi_\epsilon \varepsilon_{b,t}$$
(11)

$$Q_{t} = \beta E_{t} \left[\frac{\xi_{t+1}}{\gamma \xi_{t}} \left((1 - \tau_{K}) \left(r_{K,t+1} Z_{t+1} - a(Z_{t+1}) \right) + \tau_{K} \delta + (1 - \delta) Q_{t+1} \right) \right] + \frac{\chi_{K} \bar{k}_{t}^{-\sigma_{K}}}{\xi_{t}}$$
(12)

where ξ_t , $b_{G,L,t}$, \bar{k}_t , and Q_t denote the marginal utility of consumption, long-term government bonds, the physical capital stock and the value of an additional unit of capital. Equation (11) takes into account that we will assume that short term government bonds are in zero net supply. Linearizing these expressions yields

$$\hat{\xi}_t = \theta \left(E_t \hat{\xi}_{t+1} + \left(\hat{R}_t - E_t \hat{\Pi}_{t+1} \right) + \varepsilon_{b,t} \right) - (1 - \theta) \, \sigma_b \frac{y}{b_{C,L}} \hat{b}_{G,L,t} \tag{13}$$

$$\hat{Q}_{t} = -\left[\xi_{t} - \theta_{K}\hat{\xi}_{t+1}\right] + \frac{\beta}{\gamma}\left((1 - \tau_{K})r_{K}E_{t}\hat{r}_{K,t+1} + (1 - \delta)E_{t}\hat{Q}_{t+1}\right) - (1 - \theta_{K})\sigma_{K}\hat{k}_{t}$$
(14)

where a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with the exception of $\hat{b}_{G,L,t}$, which denotes the percentage point deviation of the government-debt-to-steady-state-GDP ratio. $\theta \equiv 1 - \frac{\chi_b(b_{G,L})^{-\sigma_b}}{\xi} = \frac{R}{\Pi} \frac{\beta}{\gamma}$, where the second equality follows from the steady-state of the model. θ represents the (steady-state) net weight that the household attaches to the t+1 marginal utility of consumption in the consumption Euler equation, which is the result of the yield the household earns for postponing consumption $\frac{R}{\Pi}$, and the discounting of the utility associated with that money flow via $\frac{\beta}{\gamma}$. Without POSA (i.e. $\chi_b = 0$), $\theta = 1$, while with POSA $\theta < 1$, which, for a given calibration target for the real interest rate $\frac{R}{\Pi}$ and trend growth γ , results in a lower calibrated value of β than without POSA. Equation (13)

shows that POSA attenuates the effect of forward guidance on consumption in two ways (see also Rannenberg (2019)). Firstly, with $\theta < 1$, the (partial equilibrium) effect of a change in the future real interest rate $E_t \left\{ \hat{R}_{t+i} - E_t \Pi_{t+1+i} \right\}$ on consumption on declines in the horizon i. Secondly, if the policy succeeds at increasing economic activity and reducing the future real interest rate trajectory, $\hat{b}_{G,L,t+i}$ will be lower than in the absence of the policy, unless the government runs a balanced budget in each quarter. The lower trajectory for $\hat{b}_{G,L,t+i}$ feeds back negatively into consumption.¹

The effects of CSP and POSA assumption on investment are captured by equation (14), with $\theta_K \equiv 1 - \frac{\chi_K \bar{k}^{-\sigma_K}}{\xi} = \frac{\beta}{\gamma} (r_K - \delta) (1 - \tau_K) + 1$. Without CSP, i.e. $\chi_K = 0$, $\theta_K = 1$ and thus the final, negative term involving the capital stock drops out. With CSP, this term tends to attenuate the response of shocks increasing investment, including forward guidance policies. Furthermore, with POSA and thus $\theta < 1$, equation (14) will be less forward looking as the weight on the expected future capital price $E_t \hat{Q}_{t+1}$ will be smaller due to a smaller β , which attenuates the response of investment to forward guidance as well as shocks with high persistence.

Following Smets and Wouters (2007), I assume that households supply their labor to labor unions owned by households. The unions differentiate the homogeneous household labor, and each supply one variety in a monopolistically competitive labor market with exactly the same characteristics as the goods market. Hence their wage setting is described by the standard New Keynesian wage Phillips Curve, and wages are subject to a wage mark-up shock analogous to the price markup shock:

$$\hat{w}_{t} = \frac{1}{1+\beta} \left(\frac{\frac{(1-\omega_{w})(1-\beta\omega_{w})}{\omega_{w}(\mu_{w}-1)\epsilon_{w}+1}} \left(\sigma_{l} \hat{N}_{t} - \hat{\lambda}_{t} + \frac{\hat{\tau}_{w,t}}{1-\tau_{w}} - \hat{w}_{t} \right) + \beta E_{t} \hat{w}_{t+1} + \hat{w}_{t+1} + \beta E_{t} \hat{\Pi}_{t+1} - (1+\beta\iota_{w}) \hat{\Pi}_{t} + \iota_{w} \hat{\Pi}_{t-1} + (1-\iota_{w}) \left(\hat{\Pi}_{obj,t} - \beta E_{t} \hat{\Pi}_{obj,t+1} \right) \right) + \varepsilon_{w,t}$$

$$(15)$$

2.3 Government and equilibrium

The monetary policy rule is given by

¹One might expect the positive effect of an increase in government debt on consumption to be weaker in the Euro Area than in the US as not all Euro Area sovereign debt is weaker since after the global financial crisis of 2007-2009 not all Euro Area sovereign debt was considered equally safe. In the context of our estimation exercise we would expect this aspect to be reflected in the estimate of curvature parameter σ_b , which governs the wealth effect, as well as in the risk premium shock $\varepsilon_{b,t}$. That being said, it is important to remember that the European Central Bank was accepting government debt of all member states as collateral as part of its of its short and long-term refinancing operations throughout the sovereign debt crisis and beyond. Furthermore, from mid 2012 onwards, it strongly reduced the interest rate spreads on the debt of Spain, Italy and other crisis countries by creating the "Outright Monetary Transactions" instrument. One would expect these policies to support the liquidity services provided by government debt.

$$\left(\hat{R}_{t} - \hat{\Pi}_{obj,t}\right) = \left(1 - \rho_{R}\right) \left(\phi_{\pi} \left(\hat{\Pi}_{t} - \hat{\Pi}_{obj,t}\right) + \phi_{y} Y G A P_{t}\right) + \phi_{\Delta y} \left(Y G A P_{t} - Y G A P_{t-1}\right) + \rho_{R} \left(\hat{R}_{t-1} - \hat{\Pi}_{obj,t-1}\right) + \varepsilon_{R,t}^{0} + \sum_{i=1}^{H} \varepsilon_{R,t-i}^{i} \tag{16}$$

$$\varepsilon_{R,t}^i = \rho_R^i \varepsilon_{R,t-1}^i + \eta_{R,t}^i \text{ for } i = 0, 1, ..., H$$

$$\tag{17}$$

$$\hat{\Pi}_{obj,t} - 0.999 \hat{\Pi}_{obj,t-1} = \rho_{obj} \left(\hat{\Pi}_{obj,t-1} - 0.999 \hat{\Pi}_{obj,t-2} \right) + \eta_{\pi_{obj},t}$$
(18)

with $YGAP_t = (\hat{y}_t - \hat{y}_t^*)$ and \hat{y}_t^* denoting flexible price output. This specification follows Smets and Wouters (2007), except for the $\sum_{i=1}^H \varepsilon_{R,t-i}^i$ and $\hat{\Pi}_{obj,t}$ terms. $\sum_{i=1}^H \varepsilon_{R,t-i}^i$ represents anticipated monetary policy shocks, which are active only in those estimations including forward rates as measures of expected future policy rates in the data set. Following Rannenberg (2020), we allow for autocorrelation of the anticipated monetary policy shock, as it dramatically improves the empirical fit of the estimated models. Finally, we specify the process for the time-varying inflation target $\hat{\Pi}_{obj,t}$ such that it captures exclusively low frequency movements of inflation, similar to Cogley et al. (2010) and Del Negro et al. (2015).

We assume that short term government bonds are in zero net supply. The government budget constraint is therefore given by

$$\frac{B_{G,L,t}}{P_t} = \frac{R_{G,L,t-1}}{\Pi_t} \frac{B_{G,L,t-1}}{P_{t-1}} + G_t - \left(T_t + (\tau_{N,t} + \tau_{w,h} + \tau_{w,f}) w_t L_t + \tau_{C,t} C_t + \tau_K Prof_t\right)$$
(19)

$$Prof_{t} = Y_{t} - (1 + \tau_{w,f,t}) \frac{W_{t}}{P_{t}} L_{t} - (\delta + a(U_{t})) \bar{K}_{t-1} - \Phi TF P_{t-1}$$
(20)

where G_t and $Prof_t$ denote government expenditure and total real profits. The various taxes are either determined as

$$\hat{u}_{tax,t} = \rho_{tax}\hat{u}_{tax,t-1} + (1 - \rho_{tax})\,\phi_{b,tax}\left(\hat{b}_{G,L,t-1}\left(\frac{y}{b_{G,L}}\right)\right) + (1 - \phi_{tax})\,\eta_{tax,t} + \phi_{tax}\eta_{tax,t-1}$$
(21)
$$\hat{tax}_t = \phi_{y,tax}\hat{y}_t + \hat{u}_{tax,t}$$

with $\phi_{b,tax} > 0$, or held constant, depending on the tax data used in the estimation, with $tax = \{\tau, \tau_N, \tau_C\}$, where $\tau_t = \frac{\frac{T_t}{TFP_{t-1}}}{y}$. The hat in \hat{tax}_t refers to a percentage point deviation from the respective steady-state value. $\eta_{g,t}$ and $\eta_{tax,t}$ denote i.i.d. normally distributed random variables.²

These fiscal rules embed the following features. Following Leeper et al. (2010a,b) they allow

The multiple $\frac{y}{b_{G,L}}$ was simply added to facilitate comparison of the coefficient $(1-\rho_g) \phi_{b,tax}$ to its counterpart in fiscal rules of other studies, which typically express the debt deviation from steady state as a percentage of its own steady-state value, whereas here $\hat{b}_{G,L,t-1}$ represents the deviation as a percentage of steady-state GDP.

for a response of all fiscal instruments to contemporaneous output in order to capture "automatic stabilizer" effects. Romer and Romer (2010) argue that accounting for the effect of economic activity on fiscal policy is important for correctly identifying discretionary fiscal policy changes. Furthermore, the rules allow exogenous fiscal policy changes to be anticipated one quarter in advance (e.g. the $(1 - \phi_{tax}) \eta_{tax,t} + \phi_g \eta_{tax,t-1}$ term), a feature whose importance is stressed in Susan Yang (2005) and Leeper et al. (2013), and used in the estimated model of Coenen et al. (2013). Finally, I allow the fiscal instruments to respond to the level of government debt in a debt-stabilizing fashion.

The resource constraint is given by

$$Y_t = C_t + I_t + G_t + EX_t \tag{22}$$

where EX_t denotes "exogenous spending" not accounted for elsewhere in the model, and in practice captures net exports and inventories. In the model it is driven by an exogenous spending shock $\varepsilon_{EX,t}$, which is expressed in units of trend GDP, and follows the process

$$\varepsilon_{EX,t} = \rho_{EX}\varepsilon_{EX,t-1} + \eta_{EX,t} + \rho_{a,EX}\eta_{a,t} \tag{23}$$

3 Estimation

3.1 Data and observation equations

We estimate the model on Euro Area data over the 1980Q1-2019Q4 period. In all estimated model variants, we use the seven data series used by Smets and Wouters (2007), i.e. the growth rates of real GDP (GDP_t) , consumption $(CONS_t)$, private fixed investment $(INVE_t)$, the real wage $(WREAL_t)$, a measure of employment $(\widehat{EMPL_t})$ in levels, the GDP deflator (YED_t) , the short term interest rate STI_t . Furthermore, we use a measure of expected inflation during the 6th-10th year from today. Furthermore, we include the government deficit-to-GDP ratio DY_t , the direct-tax-revenue (excluding social security contributions) to GDP ratio $DTXY_t$ and the implicit consumption tax rate $TAUC_t$. We construct the macroeconomic and fiscal variables using the Area Wide Model database of Fagan et al. (2005a) and the Euro Area fiscal database of Paredes et al. (2014). We obtain the measure of long-run inflation expectations from Stevens and Wauters

³We do note consider data on social security contributions in the estimation because implicit social security contribution rates display quantitatively large increase at the beginning of the 1990s before reverting back to a declining trend at the end of the 1990s, according to Coenen et al. (2013) as a result of institutional reforms. The model has difficulties to capture this feature of the data. For that reason, Coenen et al. (2013) HP filter the social security contribution data before the DSGE model estimation they perform. We do not adopt this approach because of the familiar endpoint problem that arises when using the HP filter (e.g. Hamilton (2018)), which would matter in the context of the out-of-sample forecasting exercise of Section 5.

(2018) and Camba-Mèndez and Werner (2017). Further details on data construction are located in Appendix B. Hence the measurement equations are given by

$$\begin{bmatrix} \Delta ln\left(GDP_{t}\right)*100 \\ \Delta ln\left(CONS_{t}\right)*100 \\ \Delta ln\left(INVE_{t}\right)*100 \\ \Delta ln\left(INVE_{t}\right)*100 \\ \Delta ln\left(WREAL_{t}\right)*100 \\ EMPL_{t} \\ \Delta ln\left(YED_{t}\right)*100 \\ STI_{t} \\ CPI510_{t} \\ DY_{t} \\ DTXY_{t} \\ TAUC_{t} \end{bmatrix} = \begin{bmatrix} 100\left(\gamma-1\right) \\ 100\left(\gamma-1\right) \\ 100\left(\gamma-1\right) \\ 100\left(\gamma-1\right) \\ empl \\ 100\left(\Pi-1\right) \\ 400\left(R-1\right) \\ 0.16+400\left(\Pi-1\right) \\ DY \\ DTXY \\ 100\tau_{C} \end{bmatrix} + \begin{bmatrix} \hat{y}_{t} - \hat{y}_{t-1} \\ \hat{c}_{t} - \hat{c}_{t-1} \\ \hat{u}_{t} - \hat{v}_{t-1} \\ \hat{w}_{t} - \hat{w}_{t-1} \\ empl_{t} \\ \hat{\Pi}_{t} \\ 4\hat{R}_{t} \\ 4\left(\frac{\sum_{t=21}^{40} E_{t}\hat{\Pi}_{t+t}}{20}\right) \\ \hat{b}_{G,L,t} - \frac{\hat{b}_{G,L,t}}{\Pi\gamma} + \frac{b_{G,L}}{y}\left((\gamma\Pi-1)\,\hat{y}_{t} - \hat{\Pi}_{t}\right) \\ \left(\frac{prof}{y}\tau_{K}p\hat{r}of_{t} + \frac{wN}{y}\left(\tau_{K}\left(\hat{w}_{t} + \hat{N}_{t}\right) + \hat{\tau}_{N,t}\right) \\ -\left(\tau_{K}\frac{prof}{y} + \tau_{N}\frac{wN}{y}\right)\hat{y}_{t} \end{bmatrix}$$

where DY represents the sample average of DY_t . In the $CPI510_t$ measurement equation, 0.16 represents the average difference between the annualized growth rate of the CPI and the GDP deflator over the period where data is available for $CPI510_t$, i.e. 1992Q2-2019Q4. Since employment in the data is measured in heads, the Smets and Wouters (2003) bridge equation links \hat{empl}_t to the deviation of hours from its steady state \hat{N}_t :

$$\hat{empl}_{t} - \hat{empl}_{t-1} = \beta \left(E_{t} \hat{empl}_{t+1} - \hat{empl}_{t} \right) + \frac{\left(1 - \omega_{N} \right) \left(1 - \beta \omega_{N} \right)}{\omega_{N}} \left(\hat{N}_{t} - \hat{empl}_{t} \right) \tag{24}$$

Furthermore, in some estimations we use measures of the average expected short-term interest rate over horizon i $STIEX_{i,t}$, following Del Negro et al. (2017) and Campbell et al. (2019). This gives rise to an additional H measurement equations:

$$STIEX_{i,t} = 400 (R-1) + 4 \frac{E_t \sum_{j=1}^{i} \hat{R}_{t+j}}{i} \text{ for } i = 1, 2,, H$$
 (25)

Below we report estimation results for values of H=8 and H=12.

We make sure that our choice of data for $STIEX_{i,t}$ and STI_t are as mutually consistent as possible. For example, during the post-1998 period, STI_t is the quarter t average of the Euro Overnight Index Average (EONIA), while $STIEX_{i,t}$ are EONIA Overnight Index Swap (OIS) rates from the end of quarter t. Using the end-of-quarter value implies that $STIEX_{i,t}$ can indeed be interpreted as an expectation of the average EONIA over period t+1 to t+i, and thus the average

value of STI_t . Campbell et al. (2019) adopt the same timing convention.⁴ During 1994Q1-1998Q4 period, STI_t equals the one-quarter Euribor yield from the beginning-of-quarter t.⁵ For i = 1 - 4, $STIEX_{i,t}$ equal the end-of-quarter t values of one to four quarter Euribor yield. For i = 5 - 12, we construct $STIEX_{i,t}$ from a GDP weighted average of government bond Zero Coupon Yields of the most important economies of the later Euro Area, adjusted for the difference between $STIEX_{4,t}$ and the one year government bond yield (see Appendix B.3 for further details).⁶

Our measurement equation (25) implies the expectation hypothesis of the term structure and thus abstracts from term premia. Lloyd (2021) finds that, after controlling for unpredictable events, at least over the two year horizon, OIS rates are accurate measures of expectations of the short term interest rate in the Euro Area, the United States, Japan and the UK. Furthermore, in our sample the average of the $STIEX_{i,t}$ rises only little relative to STI_t as i increases.⁷

The main reason for including measures of interest rate expectations in the data set is to identify the anticipated monetary policy shocks. Furthermore, as mentioned above, the attenuation of consumption smoothing and the "wealth effect" associated with POSA strongly attenuate the effect of changes in expected future interest rates, or forward guidance, as discussed in more detail in Rannenberg (2019). Hence one way to examine whether the data favors the mechanism POSA adds to the model is to include a measure of the expectation of the future path of the short-term interest rate in the estimation. Furthermore, incorporating such measures in the data set implies that the estimation respects the Effective Lower Bound (ELB) on the short-term interest rate by forcing not merely contemporaneous values of the short term interest rate to exceed the ELB, but also expected future ones, as pointed out by Campbell et al. (2019). The motivation for including the government deficit is that via this avenue, the estimation implicitly takes into account the dynamics of government debt, which would be expected, inter alia, to discipline the estimation of the safe asset curvature parameter σ_b , since for a given debt trajectory, different values of σ_b imply a different trajectory of the "wealth effect" of government debt on consumption. Gadatsch et al. (2016) also used the government deficit as an observable.

In all estimations, the number of shocks equals the number of observable variables. In the

 $^{^4}$ Using the end-of-period t values of the OIS rates as measures of $STIEX_{i,t}$ relies on the assumption that when making their period-t decisions, agents know all data occurring during quarter t. Note that this assumption is already implicit in the observation equations all the other variables. For instance, we assume that agents know the total value of quarter t GDP when making their decisions, regardless of how production is distributed across the three months of the quarter.

⁵Note that the using the beginning-of-quarter t value of the three-month Euribor is in fact consistent with using the quarter t average of the EONIA. The reason is the Euribor's three-month maturity, which implies that it is the beginning-of-quarter t yield which applies to funds deposited from the beginning of quarter t until the beginning of quarter t+1. By contrast, the EONIA is an overnight rate, and thus it is the quarter t average yield which applies to funds deposited from the beginning of quarter t until the beginning of quarter t+1.

⁶Dynare treats the missing values of the the interest rate expectation measures and long-run inflation expectations as unobserved states and uses the Kalman filter to infer their value (see the Dynare 4.5.7 manual).

⁷Specifically, the sample average of STI_t over the period when the $STIEX_{i,t}$ are available up to the 12 quarter horizon, i.e. 1992Q1-2019Q4, equals 2.8%, while the sample average of $STIEX_4$, $STIEX_8$ and $STIEX_{12}$ equal 2.8%, 3% and 3.1%, respectively.

absence of interest rate expectation measures from the data set, the model has 11 exogenous driving processes, namely $\varepsilon_{a,t}$, $\varepsilon_{risk,t}$, $\varepsilon_{I,t}$, $\varepsilon_{R,t}^0$, $\varepsilon_{p,t}$, $\varepsilon_{w,t}$, $\varepsilon_{x,t}$, $\eta_{\tau,t}$, η_{τ_N} , η_{τ_C} and $\hat{\Pi}_{obj,t}$. Estimations including forward rates in the data set feature H additional observables and exactly H additional exogenous driving processes, namely $\varepsilon_{R,t}^1$, $\varepsilon_{R,t}^2$, ..., $\varepsilon_{R,t}^H$, as in Del Negro et al. (2017). In that respect our approach differs somewhat from Campbell et al. (2019), who place a more complex stochastic structure on the anticipated monetary policy shocks. Their structure results in a total number of exogenous drivers related to the anticipated monetary policy shocks which exceeds the total number of forward rates included in their estimation.

3.2 Calibrated parameters and priors

We calibrate a number of parameters in advance of the estimation, displayed in Table 1. The depreciation rate, the wage and price markup and the curvature of the Kimball aggregators in the goods and labor market are set to standard values (see Lindé et al. (2016)). Since the inverse Frisch elasticity of labor supply σ_l is inherently difficult to identify, we calibrate it to 2, in line with available estimates in the literature. Following Smets and Wouters (2007), to pin down the fixed cost parameter, we assume that retailers earn zero profits in the steady state, implying that $\mu_p = \frac{\Phi + y}{y}$, with μ_p estimated. We set the steady-state distortionary tax rate τ_C to its sample average of $TAUC_t$. Following Coenen et al. (2013), we construct this tax rate as an implicit rate from the Euro Area Fiscal database of Paredes et al. (2014) and the Area Wide Model database of Fagan et al. (2005a). We set τ_K to a GDP weighted average of the estimates of the implicit tax rate on corporate income reported by the European Commission for Euro Area countries, since the Euro Area fiscal database does not distinguish between taxes on capital and labor income (see Paredes et al. (2014)).

Given these choices and the parameters to be estimated, we restrict 12 parameters in order to meet 12 steady-state targets, listed in Table 2, which unless otherwise mentioned we calculate as averages over the sample period. These parameters are reported in Table 1 (marked with a *) if their value implied by the empirical targets does not depend on the estimated parameters, and could thus be set in advance of the estimation. Regarding the steady-state labor tax ate, note that we set τ_N such that the steady-state ratio of total direct tax revenue to GDP equals the sample average of $DTXY_t$.⁸

Furthermore, to calibrate the bond utility weight χ_b , we follow Rannenberg (2019) in assuming an empirical target for the discounting wedge θ (= $\frac{\beta}{\gamma} \frac{R}{\Pi}$). This target pins down the steady-state marginal utility of save assets via $1-\theta = \frac{\chi_b(b_{G,L})^{-\sigma_b}}{\xi}$ (from equation (11)), which, given the estimate

⁸The reason for calibrating the steady-state social security contribution rates $\tau_{w,h}$ and $\tau_{w,f}$ based on the revenue-to-GDP ratios, rather than calculating an implicit tax rate, is that the steady-state labor share exceeds the sample average, because following Smets and Wouters (2007), we set retailers fixed costs such that their steady-state profits equal zero. Matching the labor-tax-to-GDP should improve the models ability to capture the feedback from an increase in the wage bill to the deficit.

of the curvature parameter σ_b , pins down the safe asset utility weight χ_b . For instance, the case without POSA corresponds to $\theta = 1 \iff \chi_b = 0$. Given the aforementioned target for $\frac{R}{\Pi}$ and γ , we can pin down β as $\beta = \frac{\gamma\theta}{R/\Pi}$. To pin down the capital utility weights χ_K , we assume that $\theta_K = \theta$, implying that the steady state return on capital $(r_K - \delta)(1 - \tau_K) + 1$ is the same as for $\theta = 1$.

With POSA, we set the discounting wedge $\theta = 0.96$ as in Rannenberg (2019), who obtains evidence on θ by drawing on 34 empirical estimates of the (time-varying) nominal individual discount rate which the household applies to future nominal income streams, $DIS_t = \frac{1}{E_t \left\{\frac{\beta \Lambda_{t+1}}{\Lambda_T \Pi_{t+1}}\right\}}$.

Table 1: Calibration

	Table 1: Cambration		
Parameter	Parameter name	Mod	lel
		NOPOSA	POSA
β	Household discount factor	0.9979*	0.9580*
σ_l	Inverse Frisch elasticity of labor supply	2.0)
γ	Quarterly gross growth rate of deterministic technology	1.003	32*
П	Quarterly gross inflation rate	1.007	^{'9*}
δ	Depreciation rate	0.02	25
μ_w	Wage markup	1.5)
ϵ_p	Kimball goods market curvature	10	
ϵ_w	Kimball labor market curvature	10	
$ au_C$	Consumption tax rate	22.3	%
$ au_N$	Labor tax rate	15.4	<u>!</u> *
$ au_{w,f}$	Employer social security contribution rate	11.59	%*
$ au_{w,h}$	Employee social security contribution rate	10.99	%*
$ au_K$	Capital tax rate	20.9	%
$\frac{\frac{B_{G,L}}{4PY}}{\frac{G}{Y}}$	Fiscal rule, target debt-to-annual GDP ratio	66.20	% *
$\frac{G}{Y}$	Fiscal rule, steady-state government expenditure share	23.59	%*
$\frac{ex}{y}$	Exogenous expenditure share	1.49	7 *
ω_{LTD}	Fraction of government debt maturing	0.037	70*

Note: Parameter values labeled with a * are calibrated such that the steady-state values of the variables listed in Table 2 correspond to their empirical counterparts. Given the target for θ and the calibration of the other parameters, the bond and capital utility weights χ_b do not matter for the linearized model dynamics and is therefore not reported.

Turning to the prior distributions, we assume that the safe asset curvature σ_b follows a normal prior distribution with mean 0.4 and standard deviation 0.05, similar to Rannenberg (2020).¹⁰ For the capital curvature parameter σ_K , where there is less guidance from the literature, we assume

⁹Given estimates of DIS_t , Rannenberg (2019) exploits the fact that for sufficiently small weights on safe assets in the utility function (i.e. θ smaller than but close to one), $\theta_t = \frac{R_t}{DIS_t}$ is approximately constant across time in the model

 $^{^{10}}$ We do not adopt a looser prior because for a looser prior, the mode finding algorithm pulls σ_b upwards, close to a region where the wealth effect from government bonds is so strong that the model solution becomes indeterminate. In that neighborhood, the mode finding algorithm has difficulties to converge and to find a mode estimate at which minus the Hessian matrix is positive definite. A non-invertible Hessian matrix at the mode implies that it is difficult to launch the Random Walk Metropolis Hastings algorithm, as we would have to find an alternative variance-covariance matrix for the proposal distribution.

	targets
	Empirical
c	N
- E	Lable

	Source	AWM database	AWM database	short term interest rate-GDP defl. inflation	AWM and EA fiscal database	AWM and EA fiscal database	AWM database	AWM and EA fiscal database	OECD and IMF fiscal monitor	AWM and EA fiscal database	AWM and EA fiscal database	AWM and EA fiscal database	See Section Rannenberg (2019).
	Data	1.3%	3.2%	2.1%	18.7%	23.5%	1.4%	86.2%	6.1	11.8%	8.2%	7.8%	0.96
	POSA				2	2		2		92			96.0
argers	NOPOSA POSA	1.3%	3.2%	2.1%	18.7%	23.5%	1.4%	86.2%	8.9	11.8%	8.2%	7.8%	1
table 2: Empirical targets	Model counterpart	$(\gamma)^4 - 1$	$(\Pi)^4 - 1$	$(\frac{R}{\Pi})^4 - 1$	<i>s</i> <i>s</i>	<i>B D</i>	$\frac{ex}{y}$	$\frac{B_{G,L}}{4PY}$	$\frac{1}{4\omega_{LTD}}$	$\frac{\tau_N w}{y} + \tau_K \frac{prof}{y}$	$\frac{\tau_{w,h}Nw}{y}$	$\frac{\tau_{w,f}Nw}{y}$	$ heta = rac{eta}{\gamma}rac{R}{\Pi}$
I	Empirical target	Annualized Real GDP growth per capita	Annualized GDP deflator inflation rate	Annualized real short-term interest rate	Private investment share	Government expenditure share	Net exports plus inventory investment GDP share	Government debt-to-GDP ratio	Average maturity of outstanding gov. debt, years	Direct tax revenue to GDP ratio	Employee-social-security-contributions-to-GDP-ratio	Employer-social-security-contributions-to-GDP-ratio	Discounting wedge

Note: All empirical targets directly obtained from the data are 1980-2019 or the longest available subsample. For more details on the data sources see B.

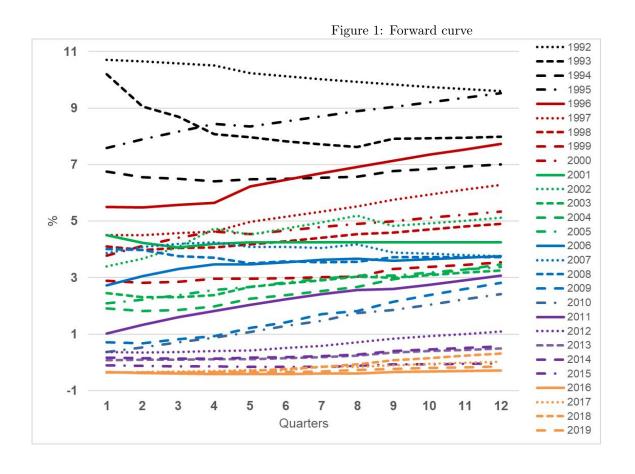
a very diffuse normal prior with a mean of one. Regarding the debt feedback coefficient in the fiscal rules, our priors have the same form but are wider than those of Zubairy (2014), Leeper et al. (2010a), Leeper et al. (2010b) and Leeper et al. (2017).¹¹ Regarding the output feedback coefficients, we assume a zero feedback for the consumption tax ($\phi_{y,\tau_C} = 0$), and fairly diffuse normal prior and a zero mean for lump-sum and labor tax feedback coefficients $\phi_{y,\tau}$ and ϕ_{y,τ_w} . The prior distributions of the parameters unrelated to the fiscal rule are close to Christoffel et al. (2020)

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3.3 Estimated parameters

Tables 3 to 6 report the posterior mean and the high probability density intervals (HPDI) of the estimated parameters for three variants of the POSA and the NOPOSA model. Columns headed "No STIEX" indicate that the respective estimation did not include data on forward rates, while "STIEX, H = 8" and "STIEX, H = 12" indicate the presence of forward rates in the set of observables, with a horizon of up to H quarters. A couple of aspects are noteworthy regarding the NOPOSA model. Regarding the fiscal rule related parameters, in line with Coenen et al. (2013), we find a strong anticipation effect for lump-sum tax shocks, but only small anticipation effects for labor and consumption tax shocks (see Table 5, line five). All taxes respond to government debt.

¹¹Note that $\hat{\tau}_t = d\tau_t$, while in the aforementioned papers the fiscal rule applies to $\frac{d\tau_t}{\sigma}$.



Secondly, adding forward rates to the set of observables increases the persistence of the risk premium shock (see Table 4) as well as the degree of price and and especially nominal wage rigidity (see Table 3). The reason for the increase in the risk premium shock persistence is presumably that the model uses the risk premium shock to jointly match the observed combination of a downward trend of the forward curve over time (see Figure 1) with an absence of an acceleration of inflation or economic activity relative to trend, and indeed a decrease during the Great Recession. By contrast, the expansionary anticipated monetary policy shocks would not be able to deliver this combination (and indeed have ambiguous effects on the forward interest rate, as we shall see). However, rendering the persistent decline of GDP relative to trend during the great recession more forecastable for wage and price setters (and thus to a lesser extent a sequence of surprises) in itself tends to increase the endogenous decline in the real wage and inflation generated by the model, which is why the estimated degree of nominal rigidity as measured by the Calvo parameters increases. The marginal cost (wage markup) coefficient of the price (wage) Phillips curve in equation (2) (in equation 15) implied by the parameter estimates in the absence of forward rates and H = 12 equals 0.0028~(0.0074) and 0.00006~(0.002), respectively. ¹² The increase in the degree of fixed costs Φ and thus the returns to scale in production also tends to flatten the Phillips curve. Furthermore, the output growth coefficient of the monetary policy rule declines.

¹²The price and wage markup coefficients are given by $\kappa_p = \frac{(1-\omega_p)(1-\omega_p\beta)}{\omega_p(\mu_p-1)\epsilon_p+1}$ and $\kappa_w = \frac{(1-\omega_w)(1-\beta\omega_w)}{\omega_w(\mu_w-1)\epsilon_w+1}$, respectively.

Table 3: Estimated parameters: Structural

					Poste	rior distril	Posterior distribution NOPOSA	OSA			Poste	erior distr	Posterior distribution POSA	3A	
	Prior dis	Prior distribution	ü	SON	STIEX	STIEX	STIEX $H = 8$	STIEX	STIEX $H = 12$	SON	NOSTIEX	STIEX	STIEX $H = 8$	STIEX	STIEX $H = 12$
Parameter name	Shape	Mean	Std.	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI
h, Habit formation	BETA	0.70	0.05	69.0	[0.64,0.74]	89.0	[0.64,0.72]	0.70	[0.67,0.74]	0.65	[0.60,0.70]	99.0	[0.62,0.70]	0.63	[0.59,0.67]
ξ_I , Inv. Adj. cost	NORMAL	4.00	0.50	4.97	[4.30,5.64]	5.29	[4.58,5.98]	4.75	[4.02,5.46]	4.36	[3.66,5.07]	4.59	[3.89,5.26]	4.44	[3.74,5.14]
$\overline{\psi}$, utilization cost	BETA	0.50	0.15	0.83	[0.74,0.92]	0.79	[0.68,0.91]	0.84	[0.75,0.93]	0.78	[0.67,0.90]	0.74	[0.60,0.87]	0.75	[0.62,0.88]
$\overline{\Phi}$, fixed cost	NORMAL	1.25	0.13	1.61	[1.47,1.76]	1.79	[1.64,1.95]	1.90	[1.80,2.04]	1.76	[1.61,1.91]	1.82	[1.67,1.97]	1.84	[1.70,1.98]
ω_w , calvo wages	BETA	0.50	0.10	0.81	[0.77,0.85]	0.85	[0.82,0.89]	0.00	[0.87,0.93]	0.83	[0.79,0.87]	0.84	[0.80,0.88]	0.83	[0.79,0.87]
ω_p , calvo prices	BETA	0.50	0.10	0.87	[0.84,0.90]	0.89	[0.87,0.92]	0.93	[0.90,0.95]	0.88	[0.85,0.91]	0.89	[0.86,0.92]	0.92	[0.90,0.95]
l_w , wage indexation	BETA	0.50	0.15	0.36	[0.23,0.49]	0.35	[0.22,0.48]	0.34	[0.22,0.47]	0.34	[0.21,0.47]	0.34	[0.21,0.46]	0.33	[0.20,0.45]
l_p , price indexation	BETA	0.50	0.15	0.20	[0.11,0.28]	0.22	[0.14,0.29]	0.23	[0.15,0.32]	0.21	[0.14,0.29]	0.22	[0.14,0.29]	0.23	[0.14,0.31]
ϕ_{π} , TR: Inflation	NORMAL	1.50	0.10	1.47	[1.30,1.63]	1.48	[1.32,1.64]	1.51	[1.34,1.67]	1.45	[1.28,1.61]	1.49	[1.33,1.65]	1.50	[1.33,1.66]
$\overline{\phi}_R$, TR: Smoothing	BETA	0.75	0.10	0.91	[0.88,0.94]	0.95	[0.94,0.96]	0.97	[0.96,0.97]	0.91	[0.88,0.93]	0.95	[0.94,0.97]	96.0	[0.96,0.97]
ϕ_y , TR: Output	NORMAL	0.13	0.02	0.22	[0.16,0.28]	0.26	[0.21,0.32]	0.28	[0.22,0.33]	0.18	[0.13,0.24]	0.22	[0.17,0.28]	0.24	[0.17,0.30]
$\overline{\phi}_{\Delta y}$, TR: Growth	NORMAL	0.13	0.05	0.18	[0.14,0.22]	0.05	[0.03,0.08]	80.0	[0.05,0.10]	0.17	[0.13,0.21]	90.0	[0.04,0.08]	0.10	[0.08,0.13]
\bar{n} , SS hours	NORMAL	0.00	2.00	-0.80	[-1.87,0.31]	-0.58	[-1.66,0.52]	0.58	[-0.92,2.04]	90.0	[-0.81,0.95]	-0.17	[-1.36,0.99]	1.15	[-0.35,2.71]
ω_n , calvo employment	BETA	0.50	0.10	0.72	[0.69,0.76]	0.75	[0.72,0.79]	0.76	[0.73,0.80]	0.71	[0.67,0.75]	0.75	[0.71,0.79]	0.71	[0.67,0.75]
σ_b , POSA curvature	NORMAL	0.40	0.05							0.40	[0.32,0.47]	0.42	[0.34,0.50]	0.41	[0.34,0.49]
σ_K , capital curvature	NORMAL	1.00	0.38							1.75	[1.39,2.11]	1.65	[1.28,2.03]	1.52	[1.16,1.86]

Note: Sample Period: 1980Q1-2019Q4. 1980Q1-1980Q4 was used as a training sample. The likelihood was evaluated over the 1981Q1-2019Q4 period. No STIEX: No interest rate expectation measures with a horizon of up to 8 quarters (i.e. H=8 in equation 25. STIEX, H=12: Dataset contains interest rate expectation measures with a horizon of up to 12 quarters (H=12 in equation 25). The estimates are based on draws from the posterior distribution obtained using the Random Walk Metropolis Hastings algorithm. We generate three chains with 500,000 draws each. The first 33% of draws of each chain are discarded as a burn-in draws. I initialize the algorithm using a posterior mode estimate obtained by directly maximizing the log posterior function using the algorithm fmincon. I use Dynare 4.6.4 throughout to conduct the estimation.

2928.8

1328.7

-425.4

2886.5

1296.0 1294.7

-439.4

Marginal Density
Mod. Harmonic Mean

La Place approx.

Table 4: Estimated parameters: Exogenous processes, non-policy and contemporaneous monetary policy

Posterior distribution NOPOSA

Posterior distribution POSA

	Prior d	Prior distribution	nc	SON	NOSTIEX	STIE	STIEX $H = 8$	STIEX	STIEX $H = 12$	SON	NOSTIEX	STIE	STIEX $H = 8$	STIEX	STIEX $H = 12$
Parameter name	Shape	Mean	Std.	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI
Std. innovations															
$\eta_{a,t}$, Technology	IG	0.10	2.00	0.50	[0.42,0.58]	0.58	[0.48,0.67]	0.57	[0.48,0.67]	0.44	[0.37,0.51]	0.52	[0.43,0.60]	0.45	[0.37,0.51]
$\overline{\eta_{risk,t}}$, Risk premium	IG	0.10	2.00	0.43	[0.30,0.56]	0.17	[0.15,0.20]	0.15	[0.14,0.17]	0.33	[0.24,0.42]	0.22	[0.19,0.25]	0.19	[0.16,0.21]
$\eta_{EX,t}$, Ex. spending	IG	0.10	2.00	0.29	[0.27,0.32]	0.29	[0.27,0.32]	0.29	[0.27,0.32]	0.29	[0.27,0.32]	0.30	[0.27,0.32]	0:30	[0.27,0.32]
$\eta_{I,t}$, Investment	IG	0.10	2.00	0.55	[0.47,0.62]	0.47	[0.38,0.55]	0.51	[0.43,0.59]	09.0	[0.53,0.68]	0.58	[0.50,0.66]	0.58	[0.50,0.66]
$\eta_{p,t}$, Price markup	IG	0.10	2.00	0.13	[0.11,0.15]	0.14	[0.12,0.16]	0.14	[0.13,0.16]	0.14	[0.12,0.16]	0.14	[0.13,0.16]	0.15	[0.13,0.16]
$\eta_{w,t}$, Wage markup	IG	0.10	2.00	0.12	[0.10,0.15]	0.13	[0.11,0.15]	0.13	[0.11,0.16]	0.15	[0.13,0.17]	0.15	[0.12,0.17]	0.15	[0.13,0.17]
$\overline{\eta}_{R,t}^0$, Mpol	DI	0.10	2.00	0.10	[0.09,0.11]	0.02	[0.04,0.06]	0.05	[0.05,0.06]	0.10	[0.09,0.11]	0.05	[0.04,0.05]	90.0	[0.05,0.07]
$\eta_{obj,t}$, Infl. Target	IG	0.01	2.00	0.00	[0.00,00.01]	0.00	[0.00,00.01]	0.00	[0.00,00.00]	0.00	[0.00,00.01]	0.00	[0.00,00.01]	0.00	[0.00,00.00]
AR(1) coefficients															
ρ_a , Technology	BETA	0.75	0.10	96.0	[0.95,0.97]	0.95	[0.94, 0.97]	0.97	[0.95,0.98]	96.0	[0.94,0.97]	96.0	[0.95,0.97]	96.0	[86.0,26.0]
ρ_{risk} , Risk premium	BETA	0.75	80.0	0.84	[0.79,0.89]	0.97	[86.0, 76.0]	0.99	[0.98,0.99]	0.91	[0.86, 0.95]	96.0	[0.95, 0.98]	0.98	[0.97,0.99]
ρ_{EX} , Ex. Spending	BETA	0.75	0.10	0.98	[66.0,76.0]	0.98	[0.97,0.99]	0.98	[0.96,0.99]	0.97	[86.0,96.0]	0.97	[86.0,96,0]	0.97	[0.96,0.99]
$\overline{\rho_I}$, Investment	BETA	0.75	0.10	0.40	[0.29,0.50]	0.57	[0.45,0.69]	0.47	[0.34,0.60]	0.37	[0.26,0.47]	0.41	[0.29,0.52]	0.42	[0.31,0.52]
ρ_p , Price markup	BETA	0.75	0.10	0.83	[0.76, 0.90]	0.89	[0.86, 0.93]	0.91	[0.88,0.93]	0.89	[0.85, 0.92]	0.89	[0.86, 0.93]	0.91	[0.89,0.93]
ρ_w , Wage markup	BETA	0.75	0.10	0.88	[0.84,0.92]	0.89	[0.84,0.93]	0.87	[0.82,0.93]	0.91	[0.88,0.95]	0.92	[0.88,0.96]	0.94	[0.91,0.97]
ρ_R , Mpol	BETA	0.75	0.10	0.53	[0.44,0.62]	0.59	[0.50, 0.68]	0.61	[0.52, 0.69]	0.54	[0.46, 0.62]	0.58	[0.49, 0.68]	0.59	[0.50,0.68]
$ ho_{\pi_{obj}}$, Inflation target	BETA	0.75	0.10	0.69	[0.60, 0.77]	0.70	[0.61, 0.79]	0.72	[0.64, 0.81]	0.71	[0.65, 0.78]	0.72	[0.65, 0.80]	0.75	[0.67, 0.83]

Other

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			[00000]	9	[1000]	300	[0.010]	2	1 0 00 0		200		< > > < > < > < > < > < > < > < > < > <	
7		0		(0		0			1	1	F 4 5 4 C	
														P)
=	0.03.0.13	×	0.03.0.13	×	0.03.0.15	50.0	0.05.0.19	7	0.08.0.28	×	07.0	22.2	Α'	22. MA Wage markin
0		000		000	1			0,10	1	0,1	000	0	¥ E	717 717
-0.0			50.00		Considerated and the second	-		-			-			dansam corresponde
.04] 0.02 [0.01.0.03]	[0.01.0.04]	0.07	[0.01.0.06]	0.04	[0.00.03]	0.05	[0.01.0.04]	0.05	[0.01,0.12]	0.0	0.70	00:0	BELA	v_n . MA Fire markup
		0				000		000		0	000	1	4 6 6	

Note: See the note below Table 3 for the meaning of the labels and other details about the estimation $\rho_{a,EX}$ denotes the the effect of the transitory technology shock on exogenous spending. IG: Inverse Gamma.

Table 5: Estimated parameters: Fiscal rules

					Poste	rior distri	Posterior distribution NOPOSA	OSA			Post	erior dista	Posterior distribution POSA	SA	
	Prior o	Prior distribution	n	SON	NOSTIEX	STIE	STIEX $H = 8$	STIEX	STIEX $H = 12$	ON	NOSTIEX	STIE	STIEX $H = 8$	STIEX	STIEX $H = 12$
Parameter name	Shape	Mean	Std.	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI
Std. innov.															
$\eta_{ au,t}$, LST	JI	0.10	2.00	0.46	[0.40,0.53]	0.46	[0.39,0.53]	0.48	[0.40,0.55]	0.46	[0.39,0.53]	0.47	[0.39, 0.54]	0.46	[0.39,0.53]
$\eta_{\tau_N,t}$, LT	IG	0.10	2.00	0.31	[0.28,0.35]	0.31	[0.27,0.35]	0.31	[0.27,0.34]	0.32	[0.28,0.35]	0.31	[0.28,0.36]	0.31	[0.28,0.35]
$\eta_{\tau_C,t}$, CT	ÐI	0.10	2.00	0.16	[0.14,0.18]	0.16	[0.14,0.17]	0.15	[0.14,0.17]	0.15	[0.14,0.17]	0.15	[0.14,0.17]	0.15	[0.14,0.17]
AR(1) coef.															
ρ_{τ} , LST	BETA	0.50	0.20	0.97	[0.95,0.99]	0.97	[0.94,1.00]	0.99	[0.99,1.00]	0.99	[0.98,1.00]	96.0	[0.89,1.00]	0.99	[0.99,1.00]
ρ_{τ_N} , LT	BETA	0.50	0.20	0.98	[0.97,0.99]	0.97	[0.95,0.99]	0.99	[0.98,1.00]	0.98	[0.96,0.99]	96.0	[0.96,0.99]	0.98	[0.96,0.99]
ρ_{τ_C} , CT	BETA	0.50	0.20	96.0	[0.97,0.99]	86.0	[0.97,0.99]	0.99	[0.98,1.00]	0.99	[0.98,1.00]	0.99	[0.98,1.00]	0.99	[0.98,1.00]
Ant. innov.															
ϕ_{τ} , LST	BETA	0.50	0.20	0.83	[0.74,0.93]	0.84	[0.75,0.93]	0.82	[0.73,0.91]	0.83	[0.73,0.92]	0.83	[0.72,0.93]	0.84	[0.75,0.93]
ϕ_{τ_N} , LT	BETA	0.50	0.20	0.92	[86.0.98]	0.92	[0.86,0.98]	0.93	[0.88,0.99]	0.91	[0.85,0.98]	0.92	[0.86,0.98]	0.92	[0.86,0.98]
ϕ_{τ_C} , LT	BETA	0.50	0.20	0.77	[0.71,0.82]	0.77	[0.71,0.83]	0.77	[0.72,0.83]	0.77	[0.72,0.83]	0.77	[0.72,0.83]	0.78	[0.72,0.83]
Debt resp.															
$\phi_{b,\tau}$, LST	GAMMA	0.50	0.15	0.43	[0.24,0.62]	0.39	[0.17,0.59]	0.31	[0.15,0.48]	0.37	[0.18,0.55]	0.23	[0.08,0.38]	0.32	[0.11,0.50]
ϕ_{b,τ_N} , LT	$_{ m GAMMA}$	0.50	0.15	0.38	[0.18,0.56]	0.31	[0.12,0.49]	0.41	[0.19,0.63]	0.35	[0.17,0.53]	0.35	[0.16,0.53]	0.38	[0.18,0.57]
ϕ_{b,τ_C} , CT	$_{ m GAMMA}$	0.50	0.15	0.32	[0.14, 0.50]	0.34	[0.15, 0.52]	0.41	[0.19, 0.62]	0.35	[0.16, 0.54]	0.35	[0.14, 0.54]	0.35	[0.15,0.54]
Output resp.															

 $\frac{\phi_{y,\tau N}}{\phi_{y,\tau N}} \text{ LT NORMAL 0.00 0.20 } -0.02 \frac{1-0.09,0.05}{1-0.09,0.05} \frac{-0.07}{-0.07} \frac{1-0.04,0.01}{1-0.14,0.01} -0.03 \frac{1-0.04,0.04}{1-0.04} \frac{0.03}{1-0.12,0.03} \frac{0.04}{1-0.12,0.03} \frac{0.04}{1-0.12,0.03$

[0.03, 0.21]

0.12

0.04 [-0.05,0.13] 0.08 [-0.02,0.18]

[-0.02, 0.18]

0.07

-0.01 [-0.11,0.09] -0.07 [-0.14,0.01]

-0.04 [-0.14,0.06] -0.02 [-0.09,0.05]

0.00 0.20 0.00 0.20

NORMAL

 $\phi_{y,\tau}$, LST

Table 6: Estimated parameters: Anticipated monetary policy shocks

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			•	•	Post	erior distrib	oution NO	OPOSA	Pos	terior distri	bution P	OSA
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Prior	distribut	ion	STIE	X H = 8	STIEX	KH = 12	STIE	X H = 8	STIEX	H = 12
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Parameter name	Shape	Mean	Std.	Mean	HPDI	Mean	HPDI	Mean	HPDI	Mean	HPDI
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Std innov											
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		IG	0.10	2.00	0.05	[0.05,0.06]	0.05	[0.05,0.06]	0.05	[0.05,0.06]	0.05	[0.05,0.06
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\eta_{R,t}^2}{\eta_{R,t}^2}$	IG	0.10	2.00	0.05	[0.04,0.05]	0.05		0.05	[0.04,0.05]	0.05	[0.04,0.05
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\eta_{R,t}^{1t,t}}{\eta_{R,t}^{3}}$	IG	0.10	2.00	0.04	[0.03,0.04]	0.04	[0.03,0.04]	0.04	[0.03,0.04]	0.04	[0.03,0.04
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	η_{D}^4	IG	0.10	2.00	0.05	[0.04,0.05]	0.05	[0.04,0.06]	0.05	[0.04,0.05]	0.05	[0.04,0.06
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R.t}^5}$	IG	0.10	2.00	0.05	[0.04,0.05]	0.05	[0.04,0.06]	0.05	[0.04, 0.05]	0.05	[0.04,0.05
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R,t}^6}$	IG	0.10	2.00	0.02	[0.02, 0.02]	0.02	[0.02, 0.03]	0.02	[0.02, 0.02]	0.02	[0.02,0.02
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R,t}^7}$	IG	0.10	2.00	0.03	[0.02,0.03]	0.03	[0.02,0.03]	0.02	[0.02,0.03]	0.03	[0.02,0.03
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R.t}^8}$	IG	0.10	2.00	0.02	[0.02,0.03]	0.05	[0.04, 0.05]	0.02	[0.02, 0.02]	0.05	[0.04,0.05
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R,t}^9}$	IG	0.10	2.00			0.04	[0.03, 0.04]			0.04	[0.03,0.04
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R,t}^{10}}$	IG	0.10	2.00			0.01	[0.01,0.01]			0.01	[0.01,0.01
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\eta_{R,t}^{11}}$	IG	0.10	2.00			0.01	[0.01,0.01]			0.01	[0.01,0.01
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\eta_{R,t}^{12}$	IG	0.10	2.00			0.01	[0.01,0.01]			0.01	[0.01,0.01
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	AR(1) coef.											
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		BETA	0.50	0.20	0.06	[0.01,0.10]	0.06	[0.01,0.10]	0.06	[0.01,0.11]	0.07	[0.01,0.12
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ρ_R^2 ,AR(1) ε_{Rt}^2	BETA	0.50	0.20	0.74	[0.69,0.79]	0.74	[0.67,0.81]	0.72	[0.67,0.78]	0.70	[0.65,0.76
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ρ_R^3 , AR(1) $\varepsilon_{R,t}^3$	BETA	0.50	0.20	0.11	[0.01,0.21]	0.14	[0.01,0.30]	0.11	[0.01,0.20]	0.12	[0.01,0.23
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_R^4 , AR(1) $\varepsilon_{R,t}^4$	BETA	0.50	0.20	0.04	[0.00,0.07]	0.05	[0.00, 0.10]	0.04	[0.00,0.07]	0.05	[0.00,0.09
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_R^5 , AR(1) $\varepsilon_{R,t}^5$	BETA	0.50	0.20	0.30	[0.20, 0.41]	0.15	[0.08, 0.21]	0.28	[0.18, 0.38]	0.14	[0.08,0.20
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_R^6 , AR(1) $\varepsilon_{R,t}^6$	BETA	0.50	0.20	0.53	[0.25, 0.80]	0.90	[0.87,0.93]	0.34	[0.05, 0.66]	0.86	[0.81,0.91
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_R^7 , AR(1) $\varepsilon_{R,t}^7$	BETA	0.50	0.20	0.78	[0.68,0.88]	0.72	[0.64, 0.80]	0.80	[0.70,0.89]	0.63	[0.54,0.73
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_R^8 , AR(1) $\varepsilon_{R,t}^8$	BETA	0.50	0.20	0.76	[0.69, 0.83]	0.05	[0.00,0.09]	0.81	[0.74, 0.88]	0.04	[0.00,0.08
ρ_{R}^{10} , AR(1) ε_{R}^{10} BETA 0.50 0.20 0.88 [0.83,0.93] 0.89 [0.84,0.94]	ρ_R^9 , AR(1) $\varepsilon_{R,t}^9$	BETA	0.50	0.20			0.10	[0.05, 0.14]			0.09	[0.05,0.13
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ρ_{R}^{10} , AR(1) ε_{R}^{10}	BETA	0.50	0.20			0.88	[0.83,0.93]			0.89	[0.84,0.94
$\rho_R^{12}, {\rm AR}(1) \; \varepsilon_{R,t}^{12} {\rm BETA} 0.50 0.20 \qquad \qquad 0.95 {}_{[0.93,0.97]} \qquad \qquad 0.97 {}_{[0.94,0.96]}$	ρ_R^{11} , AR(1) $\varepsilon_{R,t}^{11}$	BETA	0.50	0.20			0.91	[0.85,0.97]			0.89	[0.82,0.96
	ρ_R^{12} , AR(1) $\varepsilon_{R,t}^{12}$	BETA	0.50	0.20			0.95	[0.93,0.97]			0.97	[0.94,0.99

Note: $\varepsilon_{R,t}^i$ denote the anticipated monetary policy shocks (see equation ??) and $\eta_{R,t}^i$ the corresponding shock innovations. See the note below Table 3 for the meaning of the labels and other details about the estimation. IG: Inverse Gamma.

The broad direction of these changes are in line with the findings of Rannenberg (2020) for the US economy, and Lindé et al. (2016), both for the US economy. Lindé et al. (2016) compare parameter estimates disregarding the ZLB with an estimation where during the ZLB period, the model is forced to match OIS rates regarding the federal funds rate in one to 12 quarters ahead. However, the increase in risk premium shock persistence that they find is even stronger (their AR(1) coefficient increases from 0.41 to 0.85) and the increase in nominal rigidity is concentrated in price setting.

With POSA, in the absence of forward rates from the data set, the parameter estimates are overall close to the NOPOSA case. The estimated persistence of the risk premium shock is larger and the degree of habit formation and investment adjust cost correspondingly lower. The effect of adding forward rates differs for some parameters from the NOPOSA model. Specifically, the increase in price and especially wage stickiness is smaller.¹³ Hence it appears that with POSA, the model relies less on these nominal rigidities to attenuate the effect of the observed forward rates. The parameters of the AR(1) processes of the anticipated monetary policy shocks ε_t^i are very close to the NOPOSA case.

Table 7: One-step-ahead prediction errors in the model with interest rate expectation measures in the data set

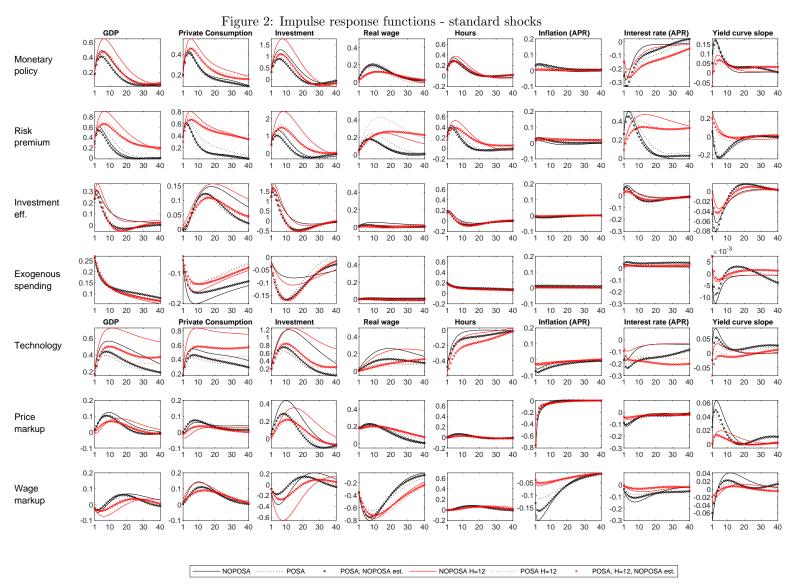
the data set			
	NOPOSA	POSA	% improvement with POSA
$\Delta ln (GDP_t) * 100$	0.58	0.55	-4.5
$\Delta ln\left(CONS_{t}\right)*100$	0.57	0.55	-3.0
$\Delta ln (INVE_t) * 100$	1.80	1.59	-11.9
$\Delta ln (YED_t) * 100$	0.26	0.26	-0.2
$\widehat{EMPL_t}$	0.64	0.53	-17.4
STI_t	2.95	2.18	-26.2
$\Delta ln \left(WREAL_t\right) * 100$	0.37	0.36	-4.2
$CPI510_t$	1.73	2.14	24.0
DY_t	0.44	0.44	-0.5
$DTXY_t$	0.19	0.19	-0.6
$TAUC_t$	0.13	0.13	0.9
$STIEX_{1,t}$	4.16	2.68	-35.6
$STIEX_{2,t}$	4.19	2.49	-40.6
$STIEX_{3,t}$	4.03	2.17	-46.1
$STIEX_{4,t}$	3.71	1.85	-50.2
$STIEX_{5,t}$	3.29	1.58	-52.2
$STIEX_{6,t}$	2.85	1.41	-50.6
$STIEX_{7,t}$	2.43	1.37	-43.8
$STIEX_{8,t}$	2.05	1.44	-29.8
$STIEX_{9,t}$	1.73	1.58	-8.6
$STIEX_{10,t}$	1.51	1.74	15.3
$STIEX_{11,t}$	1.37	1.90	38.2
$STIEX_{12,t}$	1.31	2.05	56.7
N	1 .		1 1 1 1

Note: This table reports the root mean squared in-sample-one-step-ahead-prediction errors of the models with interest rate expectation measures in the data set and H=12. The predictions are based on the posterior mode of the estimated parameters.

 $^{^{13}}$ Using the formula defined in the previous footnote, the price (wage) markup coefficients are given by 0.0026 (0.0071) and 0.0001 (0.0068), in the absence of forward rates and H = 12, respectively.

Regarding overall fit as measured by the marginal data density, without interest rate expectation measures in the set of observables, the empirical fit of the POSA model is already somewhat better than the fit of the NOPOSA model. However, the relative fit of the POSA model strongly improves once interest expectations are observed. For H=8, the difference between the two models amounts to 32.7 log points using the Harmonic mean estimator of the marginal density. Once the horizon of the included expectations increases to 12 quarters (H=12), the difference between the POSA and NOPOSA model rises to 42.2 log points (see Table 3). Using the La Place approximation to the marginal density yields very similar results.

To get a sense of how the better fit of the NOPOSA model arises, we examine the in-sample one-step-ahead prediction errors of the POSA and the NOPOSA model for the H=12 case. Specifically, we examine the prediction errors generated by the model at the posterior mode. The reason why we examine them is that the La Place approximation to the marginal data density is very close to the Harmonic mean estimate (obtained from the RWMH), and the prediction errors at the posterior mode directly affect the La Place approximation. However, performing the analysis at the posterior mean yields very similar results. Table 7 displays the root mean squared one-step-ahead-prediction errors for each observable variable and its percentage change as we move from the NOPOSA to the POSA model. The POSA model displays a better one-step ahead predictive performance for all observable variables except long-term inflation expectations $CPI510_t$, interest rate expectations measures $STIEX_{i,t}$ for the horizons 9 to 12 quarters, and the consumption tax rate $TAUC_t$. The improvement of the fit of the POSA model is the strongest for the interest rate expectation measures at horizons i = 1 - 8, but is also large for investment, employment and output.



Note: This graph displays the impulse response functions to the indicated shock, based on the parameter estimates reported in Tables 3 to 6 in the columns labeled "NOSTIEX" and "STIEX, H=12". The shock size is one standard deviation. IRFs labeled "POSA, NOPOSA est." were computed setting $\theta=0.96$ and σ_b and σ_K to their respective posterior means, but setting the remaining parameters to the estimates from the NOPOSA model. Black lines are based on the parameter values from the respective "STIEX, H=12" columns, i.e. forward rates were used in their estimation. All shocks are signed such that they generate an on-impact GDP increase, except for the wage and price markup shocks. The markup shocks display the response to a decrease in the respective markup. The yield curve slope is computed as the 3-year OIS rate minus the contemporaneous interest rate, $STIEX_{12,t} - STI_t$.

4 Impulse response functions, the effect of forward guidance and second moments

4.1 Impulse response functions

We now discuss the IRFs generated by the models estimated without interest rate expectation measures in the data set, and those estimated with interest rate expectation measures with horizon H=12. As can be seen from Figure 2, in the absence of interest rate expectation measures from the data set (the black lines), the responses of the NOPOSA and POSA models to the standard shocks known from Smets and Wouters (2007) type DSGE models are generally close. By contrast, with interest rate expectation measures in the data set, the response of GDP and its components in the NOPOSA model to the contemporaneous monetary policy shock, the risk premium shock, and the technology become considerably stronger, as a result of considerably more interest rate smoothing, much more wage rigidity, and, for the risk premium shock, a considerable increase in shock persistence. These changes all raise the persistence of the response to the shocks, and thus via anticipation effects also the on-impact response of consumption and investment. Furthermore, in response to the risk premium shock, the hump shape described by the output response becomes wider, with the output peak being delayed and output remaining about its on-impact value for 6 years or more.

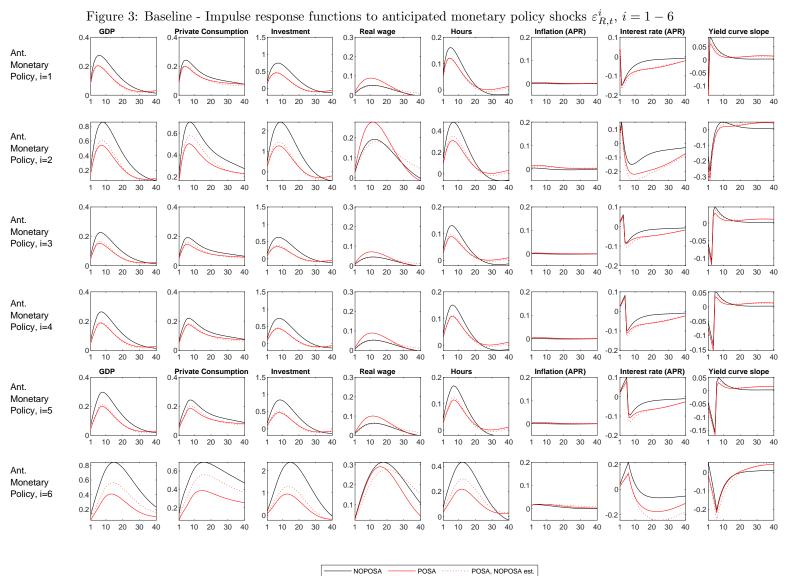
In the POSA model, the effect of these parameter changes on the impact response of consumption and investment is strongly attenuated, via the mechanisms described above (see equations 13 and 14, and the associated discussion). This attenuation persists once all other parameters are set to their NOPOSA values.

Furthermore, note that if interest rate expectation measures are included in the data set, the response to an expansionary risk premium shock involves a substantial on-impact increase in the slope of the yield curve over the horizon our estimation observes $STIEX_{12,t} - STI_t$, which remains positive for five quarters or more. Moreover, as discussed above, the increase in $STIEX_{12,t} - STI_t$ is followed by output remaining persistently above its quarter one value. The inclusion of interest rate expectation measures thus enables the IRF to replicate the well-established empirical finding that the slope of the yield curve is positively related to future economic activity (see Estrella and Mishkin (1998), Stock and Watson (2003),Ang et al. (2006), Rudebusch and Williams (2009), Berge (2015), Bauer et al. (2018) De Backer et al. (2019)). The rise in $STIEX_{12,t} - STI_t$ results from a much more gradual and persistent increase in STI_t , with the peak effect being substantially delayed, i.e. a situation where agents expect a persistent tightening of monetary policy. This more hump shaped response of STI_t is due to the aforementioned output response profile and a higher degree of interest rate smoothing in the interest feedback rule obtained with interest rate expectation measures in the data set. By contrast without such measures, the yield curve slope only increases

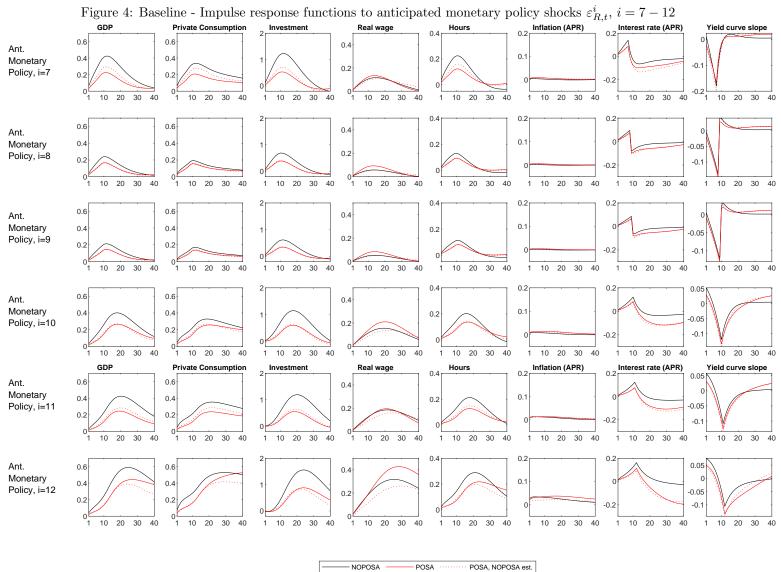
slightly on impact and then turns persistently negative.

The finding of a persistent increase in the yields curve slope in the model estimated on interest expectation measures carries over to definitions of the yield curve slope more in line with the aforementioned papers, i.e. definitions using the five year or the ten year rate as the corresponding long term rate, as well as the to the "near-term forward spread" which Engstrom and Sharpe (2019) argue statistically dominates "long-term" spreads in forecasting models. ¹⁴ The corresponding IRFs turn out to be quantitatively and qualitatively quite similar to the IRFs of the yield curve slope displayed in Figure 2.

 $^{^{14}}$ Engstrom and Sharpe (2019) define the "near-term forward spread" as the 6 quarter ahead three month rate minus the the current 3 month rate. The coutnerpart in our model is $E_t STI_{t+7} - STI_t$.



Note: See the note below Figure 2.



Note: See the note below Figure 2.

Turning to the anticipated monetary policy shocks $\varepsilon_{R,t}^i$ (Figures 3 and 4), note that their stimulative effect causes on on-impact increase in the policy rate, and a rising trajectory up until quarter i. Moreover, in the NOPOSA model, for i = 6, 7 and $i \ge 10$, the dynamic stimulative effect of the anticipated monetary policy shock is so strong that when the shock arrives in quarter i + 1, the interest rate does not actually turn negative immediately, and may remain positive for between five quarters and more than three years. The fact that an expansionary forward guidance policy may actually increase forward interest rates was already noted by de Graeve et al. (2014).

Furthermore, with POSA the response of GDP and its components is weaker for all i. The weaker response is a result of a weaker shock transmission rather than a weaker shock size, as the estimated shock standard deviations are virtually identically. This weaker response is mainly due to POSA rather than to differences in the non-POSA related estimated parameters, as the red dotted and red crossed lines are typically on top of each others. Due to the smaller stimulative effect of the anticipated monetary policy shocks, the interest rate trajectory they cause is always lower with POSA, and the effect of the shock on the interest rate remains positive for at most 4 quarters after the occurrence of the shock in quarter i.

4.2 Second moments

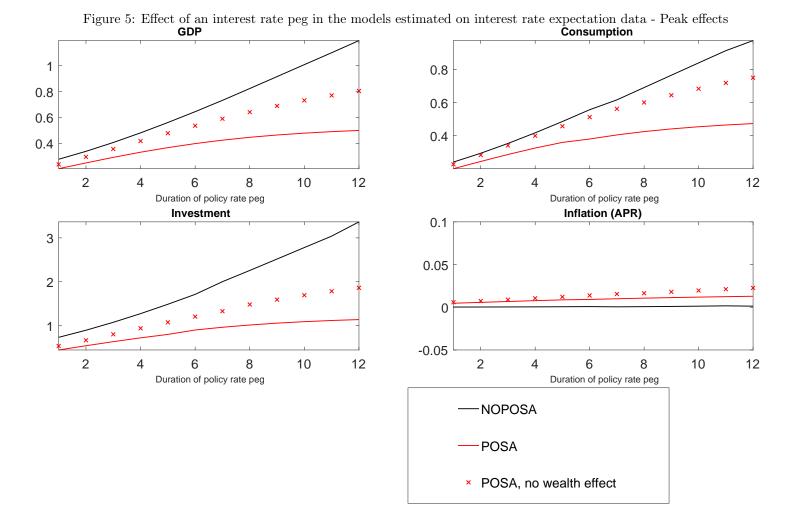
Table 8 displays the second moments of selected variables in the models estimated without and with interest rate expectation measures $STIEX_{i,t}$ in the data set. Without $STIEX_{i,t}$, NOPOSA and POSA models predict standard deviation of the growth rate of GDP and its components only somewhat higher than in the data, while the standard deviations of employment, inflation and especially the short term interest rate are much lower than in the data. Except for employment and the interest rate, the standard deviations are very close across the two models. In both models, adding $STIEX_{i,t}$ to the data set increases the volatility of all variables except inflation. However, the volatility of GDP, consumption and investment increase substantially more strongly in the NOPOSA model. This result in line with the comparison IRFs across the models reported above. Relatedly, in the variance decomposition, the anticipated monetary policy shocks, which are present only in the models estimated on a data set with $STIEX_{i,t}$, explain 38% of the variance of GDP growth in the NOPOSA model, but only 25% in the POSA model. Apart from the larger impact of the anticipated monetary policy shocks on GDP, we find evidence that the larger unconditional volatility generated by NOPOSA model is also the result of fluctuations of the risk premium shock $\varepsilon_{b,t}$ offsetting the effects of some of the anticipated monetary policy shocks when fitting the model to the data. Specifically, we find that in-sample, the innovations of some of the anticipated monetary policy shocks are correlated more strongly with the risk-premium shock in the NOPOSA model than in the POSA model. This behavior then results in a larger GDP impact of a one-standard deviation innovation (see Figure 2), which in turn increases the volatility of GDP obtained from the

calculation of the theoretical second moments, which assumes that the innovations are uncorrelated.

		Table 8	: Second	moments	
		NOST	IEX	STIEX, F	I=12
	Data	NOPOSA	POSA	NOPOSA	POSA
		Stan	dard devi	ation	
$\Delta ln \left(GDP_t\right) * 100$	0.57	0.71	0.69	1.02	0.81
$\Delta ln \left(CONS_t\right) * 100$	0.51	0.64	0.63	0.90	0.74
$\Delta ln (INVE_t) * 100$	1.71	1.96	1.84	2.93	2.16
$\Delta ln \left(WREAL_{t}\right) * 100$	0.34	0.52	0.47	0.52	0.55
\widehat{EMPL}_t	2.48	1.48	1.80	3.80	3.73
$\Delta ln \left(YED_{t} \right) * 100$	0.67	0.44	0.41	0.39	0.40
$\overline{STI_t}$	4.55	2.16	2.64	3.89	4.72

	Correlations with $\Delta ln (GDP_t) * 100$				
$\Delta ln \left(GDP_t\right) * 100$	1.00	1.00	1.00	1.00	1.00
$\overline{\Delta ln\left(CONS_t\right)*100}$	0.72	0.76	0.77	0.85	0.81
$\overline{\Delta ln \left(INVE_t\right) * 100}$	0.79	0.81	0.78	0.88	0.82
$\overline{\Delta ln\left(WREAL_{t}\right)*100}$	0.32	0.26	0.23	0.32	0.36
$\widehat{EMPL_t}$	-0.22	-0.03	0.00	0.09	0.04
$\overline{\Delta ln\left(YED_t\right)*100}$	-0.13	-0.03	-0.03	0.02	0.01
$\overline{STI_t}$	-0.07	-0.07	-0.06	0.03	-0.02

Note: This table displays the theoretical model moments and their data counterparts.

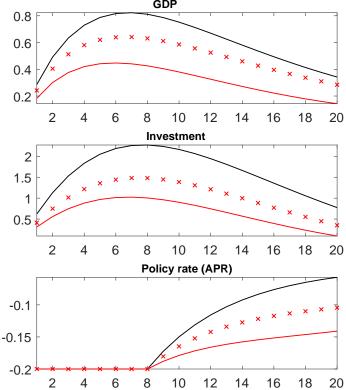


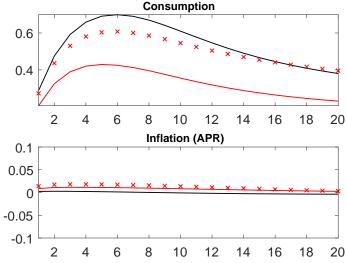
4.3 The effect of forward guidance

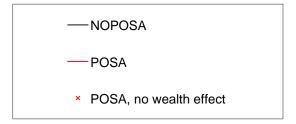
We now examine the effect of fixing the short term interest rate below its steady state value for a given number of quarters, allowing it to adjust according to the model's policy rule thereafter (see Figure 5). This type of experiment is frequently used to examine the effect of forward guidance policies in a structural model. In the NOPOSA model, the effect of forward guidance is very strong. For instance, the peak GDP effect of a 12 quarter peg of the interest rate of 0.2 percentage points below its steady state exceeds 1% (see the black dotted line). By contrast, with POSA the peak effect equals less than half of this value, with consumption and investment contributing roughly equally to the attenuation (compare the black and red solid lines).

To illustrate the importance of the estimated wealth effect of government debt for the attenuation forward guidance, the Figure also displays the response in the POSA model if we set $\sigma_b = 0$. Under this assumption, the response of GDP is roughly in the middle between the POSA and the NOPOSA model (the red cross line).

Figure 6: Effect of an interest rate peg in the models estimated on interest rate expectation data - Peak effects ${f Consumption}$







5 Out-of-sample forecasting performance

In this section, we evaluate the out-of-sample forecasting performance of the models. For GDP, consumption, private fixed investment, the real wage and the GDP deflator and employment, we use real time data following the methodology of Smets et al. (2014) and McAdam and Warne (2019). Real-time vintages start in January 2001 and are obtained from the Real-Time Data Base (RTDB) available on the website of the ECB's Statistical Data Warehouse. Most of these RTDB vintages have data starting around the mid-1990's. We therefore extend the RTDB data backwards using the annual vintages of the quarterly Area-Wide Model (AWM) data set. For the fiscal series, the interest rate and the interest rate expectation data we use the final data because we do not have vintages for the fiscal series, and financial market data is not typically subject to revisions. Our pseudo-out-of sample period ranges from 2000Q4 to 2019Q3, vielding 76 data points for 1-quarter ahead forecasts and 65 for the 12-quarter ahead forecast. We re-estimate the models once a year. We use the final vintage (2019Q4) as actual values to compute forecast errors.

Figure 7 displays RMSEs associated with model forecasts for the levels of output, consumption, investment and the nominal interest rate, expressed as ratios relative to the RMSEs of the model without POSA and interest rate expectation measures in the data set. We find that in the NOPOSA model, adding interest rate expectation measures to the data set strongly reduces the RMSEs for all variables other than inflation and at all horizons. Furthermore, in the presence of interest rate expectation measures, the POSA model outperforms the NOPOSA model for output and consumption, and for investment at horizons exceeding nine quarters.

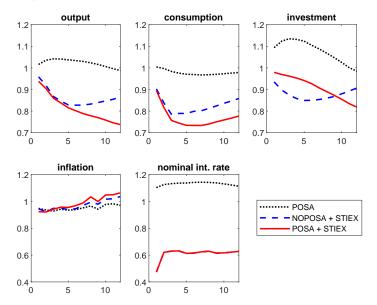
We first analyze the differences in the forecast performance in the NOPOSA model with and without interest rate expectation measures. As can be observed in Figure 8, the standard NOPOSA model generates significantly more over-optimistic three-year forecasts when predicting the period between 2006-2008 and again for the period 2012-2015 than the NOPOSA model with $STIEX_{i,t}$ in the data set. A comparison of the forecast made in 2005Q1 (predicting 2005Q2-2008Q1), performed in the top panel of Figure 9, is representative of the drivers behind the more optimistic forecast of the NOPOSA model without interest rate expectation measures around the first period. As can be obtained from the comparison of the blue bars, the NOPOSA model without $STIEX_{i,t}$ displays stronger positive contribution of the aggregate demand shocks (mainly the risk premium shock) over the forecast horizon. The reason for this difference is that while both models rely on adverse demand shocks to replicate the period of below-trend GDP growth preceding the forecast (especially

 $^{^{15}}$ https://sdw.ecb.europa.eu/browseExplanation.do?node=9689716. Details on the construction of the RTDB can be found in Giannone et al. (2012).

¹⁶We warmly thank Arne Warne for his help in the collection of AWM database vintages and for sharing Matlab procedures for combining RTDB and AWM data. In-depth presentation of the AWM data can be found in Fagan et al. (2005b).

¹⁷Each vintage has data available up to the quarter that precedes the quarter of the RTDB release. For instance, the first vintage is published at the end of 2000Q4 (January 2001) with data up to 2000Q3. Therefore, at each period, the 1-quarter ahead forecast is actually a nowcast.

Figure 7: RMSEs relative to the model without POSA and interest rate expectation measures $(STIEX_{i,t})$.



Note: This graphs displays the Root Mean Squared Errors (RMSEs) of the out-sample-forecast of the model with POSA and no $STIEX_{i,t}$ in the data set, and for the NOPOSA and POSA models with $STIEX_{i,t}$ with a maximum maturity of 12 quarters (H=12). The x axis represents the forecast horizon and is expressed in quarters. Note that output, consumption and investment are expressed in cumulated growth rates over the respective forecast horizon.

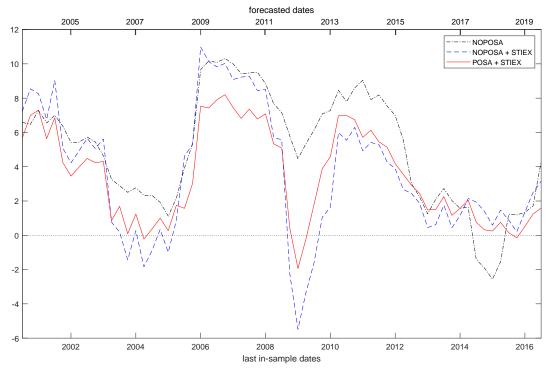
2001-2003), the estimated persistence of the risk premium shock is much lower without $STIEX_{i,t}$ in the data set, as discussed in Section 3.3. Hence the adverse risk premium shocks unwind much faster and thus generate stronger positive contributions to output forecasts. Moreover, anticipated monetary policy shocks apply an important downward pressure on output levels in the NOPOSA model with $STIEX_{i,t}$. The effects of monetary policy shocks persist in the forecast and imply a more gradual recovery in output.

The increase in the GDP forecast due to the inclusion $STIEX_{i,t}$ may be related to the well established finding (see Section 4.1) that the slope of the yield curve embeds information regarding future GDP growth. Specifically, as discussed in Section 4.1, with $STIEX_{i,t}$ in the data set, the IRF to an expansionary risk premium shock causes the coincidence of a substantial and persistent increase in the slope $STIEX_{12,t} - STI_t$ and an expectation of further GDP increases. The ability of the model to generate this coincidence may enable it to pick up the information about future real activity embedded in the yield curve and via this avenue improve the GDP forecast. Moreover, just like the superior forecasting performance, the ability of the model to generate this coincidence is related to the higher estimated risk premium shock persistence in the model estimated on $STIEX_{i,t}$ data. This connection is also suggestive of a link between the forecast performance improvement and the predictive power of the yield curve slope.

The forecast for the second forecasted period for which the NOPOSA without interest rate expectation measures significantly underperforms is decomposed in the second panel of Figure 9. A zoom is applied to forecasts of the period around 2014, predicted with in-sample data up to 2011, a period preceded by a low growth environment. Again the estimated persistence of the risk-premium shock in the model without STIEX is crucial. While the adverse risk premium shocks needed to fit the in-sample period unwind in the forecasting period and contribute to the optimistic output forecast of the NOPOSA model, they continue to contribute negatively and unwind later in the NOPOSA+STIEX model.

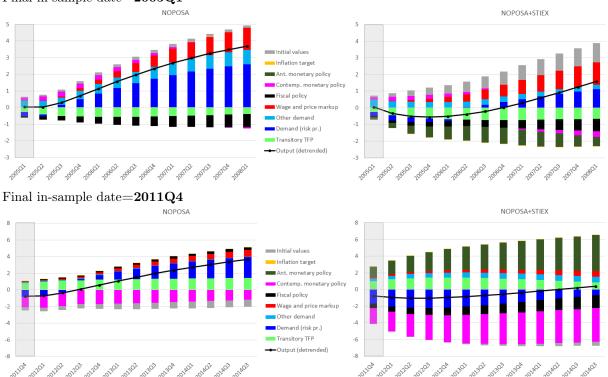
We now analyze the difference of the forecasting performance between the POSA and the NO-POSA model in the presence of STIEX in the data set. Figure 8 shows that the better performance of the POSA model arises throughout the whole period for which we calculate forecasts. Over the whole period, the forecast errors, and hence the forecast itself, appears to be significantly more volatile for the NOPOSA model than for the POSA model. Furthermore, the POSA model is less over-optimistic when predicting 2003Q3-2006Q1, 2008Q3-2011Q3 and 2017Q2-2019Q3, and less overly pessimistic in forecasting 2011Q4-2012Q3. Interestingly, we can relate the extra-volatility and the associated underperformance of the NOPOSA model to the effect of the anticipated monetary policy shocks on the forecast. The first panel in Figure 10 reports a shock decomposition of the forecast based on data up to 2007Q2 and predicting up to 2010Q2, representative of the drivers of the more optimistic forecast of the NOPOSA model around 2008Q3-2011Q3. It shows that the overoptimism in the NOPOSA model in 2010Q2 was mainly due to a reversal of the contribution

Figure 8: Output – Three-year forecasting error in the NOPOSA model without interest rate expectation measures and in the NOPOSA and POSA models with interest rate expectation measures in the data set



Note: Forecast errors are computed as model forecasts of GDP growth cumulated over three years ahead minus actual values. The final vintage (2019Q4) is used as actual values in the computation of forecasting errors. The bottom x-axis indicates the last in-sample dates, while the top x-axis shows the forecasted dates.

Figure 9: Historical decomposition of 3 year output-level forecasts Final in-sample date= $\bf 2005Q1$



Note: The graphs in the first (second) line display the GDP increase between the quarter indicated on the horizontal axis and 2004Q4 (2011Q3), and the respective shock decomposition, for the NOPOSA and the POSA model. The forecast period starts outside the grey shaded area.

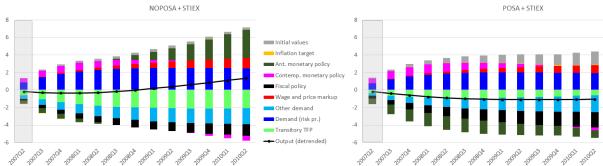
of the anticipated monetary policy shocks from negative to positive. These positive contributions come mostly from unwinding effects of restrictive anticipated monetary policy shocks generated by the NOPOSA model in the in-sample period. The effect of the smoothed shocks on the GDP growth rate is much weaker in the POSA than in the NOPOSA model, implying a smaller growth rate during the subsequent recovery. This weaker recovery effect in the POSA model then keeps the contribution of the anticipated monetary policy shocks to the forecasted GDP path negative over the forecast horizon. The smaller effect of the anticipated monetary policy shocks in the POSA model is the result of both smaller smoothed innovations and the stronger transmission of an innovation of a given size to GDP discussed above (see the IRFs in Figures 3 and 4).¹⁸

The lower panel of Figure 10 shows that the excessive pessimism of the NOPOSA model in forecasts for 2012Q3 is mainly due to a more strongly negative contribution of the anticipated monetary policy shocks. As indicated by shaded areas on the charts, the last quarters of the insample period covers the start of the Global Financial Crisis (GFC), which is interpreted differently by the two models. In the NOPOSA model, restrictive monetary policy surprises are dominant to explain the decrease in output in the GFC, and their persistent effects predict a very long-lasting downturn. In contrast, the POSA model relies heavily on adverse risk premium shocks. These negative demand factors are also persistent, yet they unwind relatively quicker, projecting a less pessimistic economic outlook in the forecasting period.

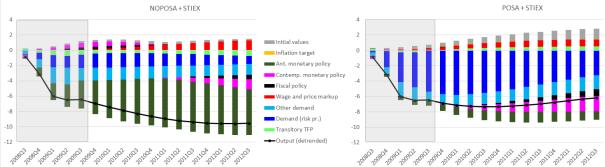
The decomposition of these two episodes reflects a finding of Sections 4.2 and 6: anticipated monetary policy shocks are more present in the historical decomposition of output fluctuations in the NOPOSA model with interest rate expectation measures relative to its POSA extension. Consequently, their long-lasting effects on output – which are moreover stronger in the NOPOSA model – affect the NOPOSA forecast significantly more, at the expense of its out-of-sample prediction performance.

¹⁸When diving into the details of this forecast decomposition (not reported here), we observe restrictive anticipated monetary policy (especially at maturities 6-7-12Q) mainly in the period 2001-2004, that contribute to these effects. In this period, innovations and contributions of those anticipated monetary policy shocks are larger for the NOPOSA model compared to its POSA extension. Coupled with the stronger effect of an anticipated shocks of a given size, a larger "boomerang effect" results in the NOPOSA configuration during the forecast period.

Figure 10: Historical decomposition of 3 year output-level forecasts Final in-sample date=2007Q2. Shaded area indicates last in-sample dates.



 $\label{eq:final in-sample date} Final \ in \textbf{-sample dates}. \ \textbf{Shaded area indicates final in-sample dates}.$



Note: The graphs in the first (second) line display the GDP increase between the quarter indicated on the horizontal axis and 2007Q1 (2008Q2), and the respective shock decomposition, for the NOPOSA and the POSA model. The forecast period starts outside the grey shaded area.

6 Contribution of forward guidance to economic activity and inflation

Figure 11 displays the historical decomposition of the deviation of output from trend for the NO-POSA model. To facilitate the exposition, we have grouped the shocks. Though there are offsetting effects of various shocks, we note that the business cycle movements of output broadly follow the shocks to private sector aggregate demand (the blue bar). The anticipated monetary policy shocks (i.e. the $\eta_{R,t}^i$ for i>0) are represented by the dark green bar. As can be obtained from Figure 11, the impact of the anticipated monetary policy shocks on economic activity is negative throughout. This may reflect the positive slope of the forward curve (see Figure 1), which the model attempts to match via expansionary shocks at shorter horizons and contractionary anticipated shocks at longer horizons. The anticipated monetary policy shock is certainly the main driver of fluctuations of the spread between the three year interest rate expectation $STIEX_{12,t}$ and STI_t (see Figure 15). At the same time, it is worthwhile noting that the main drivers of the level and the downward trend of both short and long-term interest rates is private sector aggregate demand in the form of the risk-premium shock rather than the anticipated monetary policy shocks (see Figure 13 and 14). The reason is that, unlike the monetary policy shock, the risk-premium shock can deliver the aforementioned combination of a downward trend of the forward curve over time with weak GDP growth and inflation.

Explicit forward guidance in the Euro Area started in July 2013 and was strengthened in January 2014 and January 2015. These announcements are associated with a change in the combined contribution of the anticipated monetary policy shocks to the level of GDP from -10.6% to -2.3%. Thus we observe increase of about 8.3% which occurs after the start of forward guidance by the governing council. A bit more then half of this increase is driven by investment, the remainder by consumption. The contribution of the anticipated monetary policy shocks to year-on-year inflation increases by 0.4 percentage points over the same period. However, the forward guidance does not lower the expected interest rate (see Figure 14). This result is consistent with the ambiguous effect of the anticipated monetary policy shocks on the forward interest rate discussed in Section 4.

With POSA, historical decomposition of the deviation of output from trend is similar to the NO-POSA model in that the cyclical movements of output follow broadly the contribution of the demand shock, and there are negative contributions of the anticipated monetary policy shock throughout (see 16). However, the latter have typically a much smaller magnitude than in the NOPOSA model. The increase in the combined contribution of the anticipated monetary policy shocks to GDP and inflation post 2013Q2 equals about 2.2%, mostly driven by consumption, and 0.1 percentage points, respectively. Unlike in the NOPOSA model, the impact of the forward guidance shock on $STIEX_{12}$ becomes less positive as the anticipated monetary policy shocks become more expansionary, with the change post 2013Q2 cumulating to -1.3 percentage points. This direction is in line with the

discussion of the IRFs to the anticipated monetary policy shocks in Section 4.

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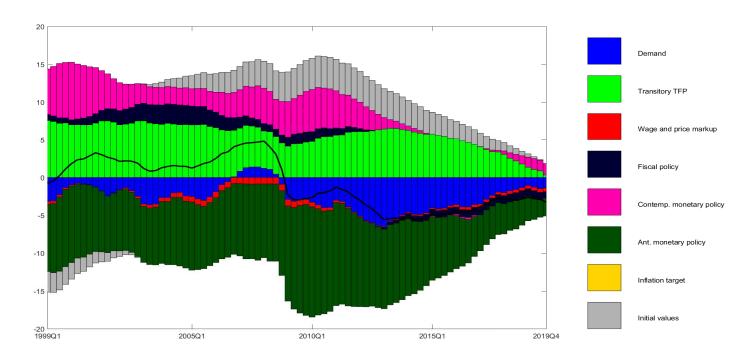
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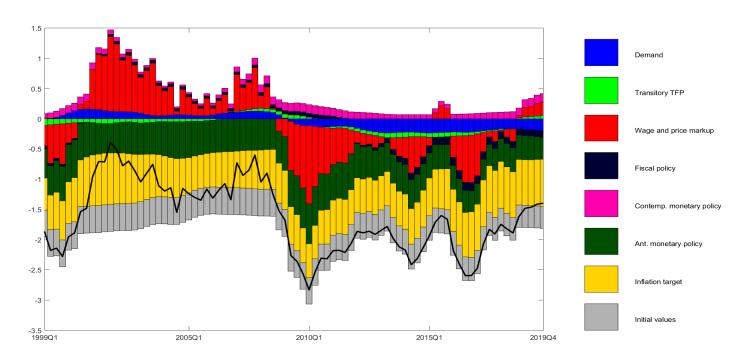
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Figure 11: Historical decomposition, NOPOSA: GDP (level)



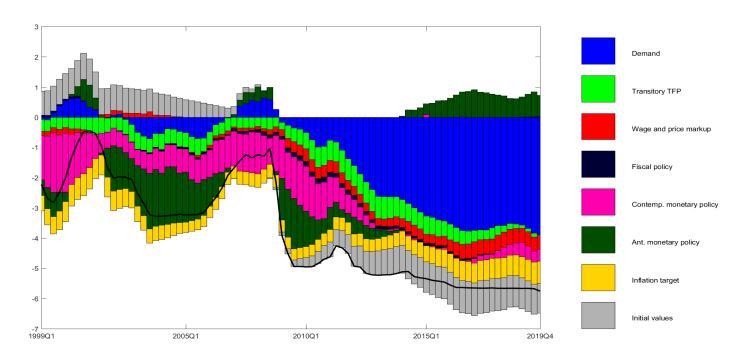
Note: This graph displays the historical decomposition of the NOPOSA model estimated with interest rate expectation measures in the data set, with H=12. The parameter estimates are as reported in Tables 3 to 6. "Demand": Risk premium $(\eta_{b,t})$, investment specific technology $(\eta_{I,t})$ and exogenous expenditure $(\eta_{EX,t})$ shocks. "Fiscal policy": lump sum tax (η_{τ_t}) , labor tax $(\eta_{\tau_w,t})$ and consumption tax $(\eta_{\tau_c,t})$ shocks. "Contemp. monetary policy": $\eta_{R,t}^0$, "Ant. monetary policy": $\eta_{R,t}^i$ for i>0.

Figure 12: Historical decomposition, NOPOSA: Year-on-year-inflation



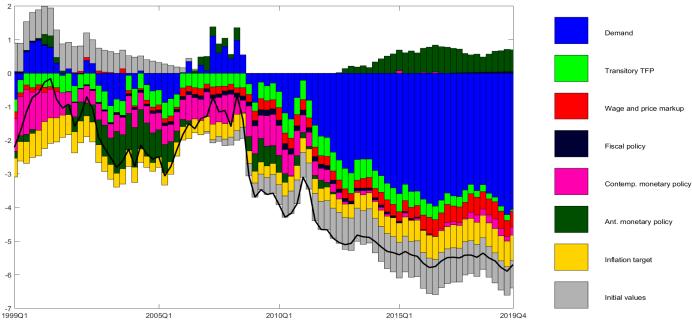
Note: This graph display the historical decomposition of the deviation of year-on-year inflation from its steady state $\hat{\Pi}_{t,t-3} = \hat{\Pi}_t + \hat{\Pi}_{t-1} + \hat{\Pi}_{t-2} + \hat{\Pi}_{t-3}$. For the definition of the shock groups and further information on the model used, see the note below Figure 11. The negative smoothed values of inflation throughout the displayed period displayed in the plot reflect the fact that the steady state inflation rate, which equals average of inflation across the sample, exceeds the inflation rate observed over the 1999-2019 period.

Figure 13: Historical decomposition, NOPOSA: STI_t



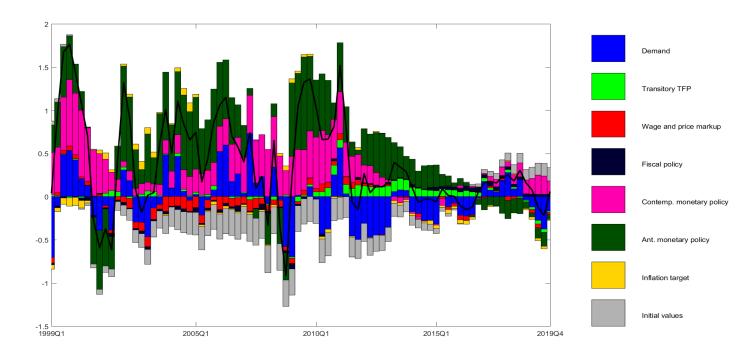
Note: For the definition of the shock groups and further information on the model used, see the note below Figure 11.

Figure 14: Historical decomposition, NOPOSA: $STIEX_{12}$



Note: For the definition of the shock groups and further information on the model used, see the note below Figure 11.

Figure 15: Historical decomposition, NOPOSA: $STIEX_{12} - STI_t$



Note: For the definition of the shock groups and further information on the model used, see the note below Figure 11.

Demand

Transitory TFP

Wage and price markup

Fiscal policy

Ant. monetary policy

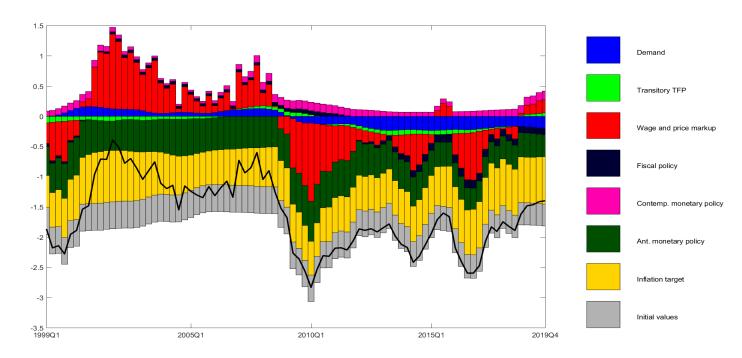
Inflation target

Initial values

Figure 16: Historical decomposition, POSA: GDP (level)

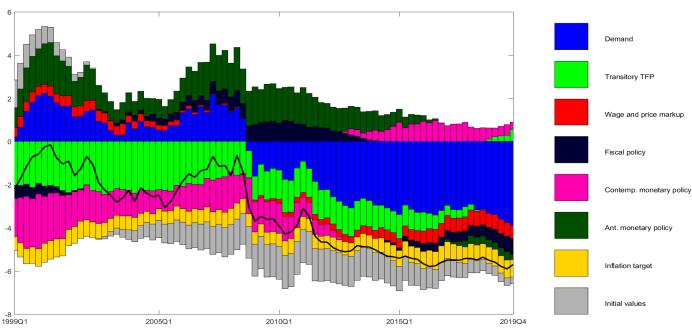
Note: This graph displays the historical decomposition of GDP obtained from the POSA model estimated with interest rate expectation measures in the data set, with H = 12. The parameter estimates are as reported in Tables Tables 3 to 6. For the definition of the shock groups, see the note below Figure 11.

Figure 17: Historical decomposition, POSA: Year-on-year-inflation



Note: For the definition of the shock groups and further information on the model used, see the note below Figure 16.

Figure 18: Historical decomposition, POSA: $STIEX_{12}$



Note: For the definition of the shock groups and further information on the model used, see the note below Figure 16.

A Model

A.1 Households

Households maximize (6) subject to (7), (9) and (10), by choosing $C_t(j)$ $N_t(j)$, $\frac{B_{G,L,t}(j)}{P_t}$, $\frac{B_{G,L,t}(j)}{P_t}$, $\frac{B_{G,L,n,t}}{P_t}$ and $R_{G,L,t}(j)$. The Lagrangian is given by

$$\sum_{i=0}^{\infty} \beta^{i} \begin{bmatrix} \frac{\left(C(j)_{t+i} - hC_{t+i-1}\right)^{1-\sigma_{c}}}{1-\sigma_{c}} exp\left(\frac{\sigma_{c} - 1}{1+\sigma_{t}}N_{t+i}^{1+\sigma_{t}}\left(j\right)\right) + \\ \frac{\chi_{b,t+i}}{1-\sigma_{b}} \left(\frac{B_{G,t+i}(j)}{P_{t+i}} + \frac{B_{G,L,t+i}(j)}{P_{t+i}}\right)^{1-\sigma_{b}} + \\ \frac{\chi_{K,t+i}}{1-\sigma_{k}} \bar{K}_{t-i}^{1-\sigma_{b}}\left(\frac{B_{G,t+i}(j)}{P_{t+i}} + \frac{B_{G,L,t+i}(j)}{P_{t+i}}\right) \chi_{\varepsilon_{b},t+i}\varepsilon_{b,t+i} \end{bmatrix} \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i} \begin{bmatrix} \frac{R_{t+i-1}}{\Pi_{t+i}} \frac{B_{G,t+i-1}}{P_{t+i-1}}\left(j\right) + \frac{\left(R_{L,t+i-1} - 1 + \omega_{LTD}\right)}{\Pi_{t+i}} \frac{B_{G,L,t+i-1}}{P_{t+i-1}}\left(j\right) \\ + \left(1 - \tau_{w,h,t+i} - \tau_{L,t+i}\right) \frac{W_{t+i}(j)}{P_{t+i}} N\left(j\right)_{t+i} - T_{t+i} + Prof\left(j\right)_{t+i} \\ + \left(\left(1 - \tau_{K,t+i}\right) \left(r_{K,t+i}Z_{t+i}\left(j\right) - a\left(Z_{t+i}\left(j\right)\right)\right) + \tau_{K,t+i}\delta\right) \bar{K}_{t+i-1}\left(j\right) \\ - \left(\frac{B_{G,t+i}(j)}{P_{t+i}} + \frac{B_{G,L,n+i+i}(j)}{P_{t+i}} + \left(1 + \tau_{C,t+i}\right) C_{t+i}\left(j\right) + I_{t}\left(j\right)\right) \end{bmatrix} \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i} \begin{bmatrix} \mu_{RGL,t+i}\left(j\right) \left(\left(1 - \omega_{LTD}\right) \frac{B_{G,L,t+i-1}(j) - 1}{\Pi_{t+i}} \frac{B_{G,L,t+i-1}(j)}{P_{t+i}} - \left(R_{G,L,t+i}(j) - 1\right) \frac{B_{G,L,t+i}(j)}{P_{t+i}} \right) \\ + \mu_{bGL,t+i}\left(j\right) \left(\left(1 - \omega_{LTD}\right) \frac{B_{G,L,t+i-1}(j)}{P_{t+i-1}} + \frac{B_{G,L,n,t+i}(j)}{P_{t+i}} - \frac{B_{G,L,t+i}(j)}{P_{t+i}} \right) \end{bmatrix} \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i}^{k} \left[\bar{K}_{t+i}\left(j\right) - \left(1 - \delta\right) \bar{K}_{t+i-1}\left(j\right) + \varepsilon_{I,t+i}\left(1 - S\left(\frac{I_{t+i}\left(j\right)}{I_{t+i-1}\left(j\right)}\right)\right) I_{t+i}\left(j\right) \right] \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i}^{k} \left[\bar{K}_{t+i}\left(j\right) - \left(1 - \delta\right) \bar{K}_{t+i-1}\left(j\right) + \varepsilon_{I,t+i}\left(1 - S\left(\frac{I_{t+i}\left(j\right)}{I_{t+i-1}\left(j\right)}\right)\right) I_{t+i}\left(j\right) \right] \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i}^{k} \left[\bar{K}_{t+i}\left(j\right) - \left(1 - \delta\right) \bar{K}_{t+i-1}\left(j\right) + \varepsilon_{I,t+i}\left(1 - S\left(\frac{I_{t+i}\left(j\right)}{I_{t+i-1}\left(j\right)}\right)\right) I_{t+i}\left(j\right) \right] \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i}^{k} \left[\bar{K}_{t+i}\left(j\right) - \left(1 - \delta\right) \bar{K}_{t+i-1}\left(j\right) + \varepsilon_{I,t+i}\left(1 - S\left(\frac{I_{t+i}\left(j\right)}{I_{t+i-1}\left(j\right)}\right)\right) I_{t+i}\left(j\right) \right] \\ + \sum_{i=0}^{\infty} \beta^{i} \Xi_{t+i}^{k} \left[\bar{K}_{t+i}\left(j\right) - \left(1 - \delta\right) \bar{K}_{t+i-1}\left(j\right) + \varepsilon_{I,t+i}\left(1 - S\left(\frac{I_{t+i}\left(j\right)}{I_{t+i-1}\left(j\right)}\right) \right] \right]$$

The first order conditions are given by

¹⁹The reason that the average interest rate on the households bond portfolio $R_{G,L,t}$ is a choice variable is that it is affected by the households purchases of newly issued bonds $B_{G,L,n,t}$. By contrast, the market interest rate on newly issued bonds $R_{G,L,n,t}$ is taken as given by the household (see Krause and Moyen (2016)).

$$\Xi_t = \beta E_t \left\{ \Xi_{t+1} \frac{R_t}{\Pi_{t+1}} \right\} + \chi_{b,t} \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b} + \chi_{\varepsilon_b,t} \varepsilon_{b,t}$$
 (26)

$$\Xi_t \left(\mu_{bGL,t} + \mu_{RGL,t} \left(R_{G,L,t} - 1 \right) \right) = \tag{27}$$

$$\beta \Xi_{t+1} \left[\frac{(R_{L,t} - 1 + \omega_{LTD}) + (1 - \omega_{LTD}) \left[\mu_{RGL,t+1} \left(R_{G,L,t} - 1 \right) + \mu_{bGL,t+1} \right]}{\Pi_{t+1}} \right]$$
(28)

$$+\chi_{b,t} \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t}\right)^{-\sigma_b} + \chi_{\varepsilon_b,t}\varepsilon_{b,t}$$
(29)

$$\mu_{RGL,t} = \beta \Xi_{t+1} \frac{1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD}\right)}{\Pi_{t+1}}$$
(30)

$$\mu_{bGL,t} = 1 - \mu_{RGL,t} \left(R_{G,L,n,t} - 1 \right) \tag{31}$$

Combining (30) and (27) yields

$$\left(\mu_{bGL,t} + \beta E_{t} \left\{ \frac{\Xi_{t+1}}{\Xi_{t}} \frac{1}{\Pi_{t+1}} \left[1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \right] \right\} (R_{G,L,t} - 1) \right) \\
= \beta \frac{\Xi_{t+1}}{\Xi_{t}} \left[\frac{(R_{L,t} - 1 + \omega_{LTD}) + (1 - \omega_{LTD}) \left[\mu_{RGL,t+1} \left(R_{G,L,t} - 1 \right) + \mu_{bGL,t+1} \right]}{\Pi_{t+1}} \right] \\
+ \frac{\chi_{b,t} \left(\frac{B_{G,t}}{P_{t}} + \frac{B_{G,L,t}}{P_{t}} \right)^{-\sigma_{b}}}{\Xi_{t}} + \frac{\chi_{\varepsilon_{b},t}}{\Xi_{t}} \varepsilon_{b,t}$$

or

$$\mu_{bGL,t} = \beta E_t \left\{ \frac{\Xi_{t+1}}{\Xi_t} \frac{1}{\Pi_{t+1}} \left[\omega_{LTD} + (1 - \omega_{LTD}) \mu_{bGL,t+1} \right] \right\}$$

$$+ \frac{\chi_{b,t} \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b}}{\Xi_t} + \frac{\chi_{\varepsilon_b,t}}{\Xi_t} \varepsilon_{b,t}$$

$$(32)$$

Combining (27) and (31) yields

$$\begin{split} &\Xi_{t}\left(1-\mu_{RGL,t}\left(R_{G,L,n,t}-1\right)+\mu_{RGL,t}\left(R_{G,L,t}-1\right)\right) = \\ &\beta\Xi_{t+1}\left[\frac{\left(R_{L,t}-1+\omega_{LTD}\right)+\left(1-\omega_{LTD}\right)\left[\mu_{RGL,t+1}\left(R_{G,L,t}-1\right)+1-\mu_{RGL,t+1}\left(R_{G,L,n,t+1}-1\right)\right]}{\Pi_{t+1}}\right] \\ &+\chi_{b,t}\left(\frac{B_{G,t}}{P_{t}}+\frac{B_{G,L,t}}{P_{t}}\right)^{-\sigma_{b}}+\chi_{\varepsilon_{b},t}\varepsilon_{b,t} \end{split}$$

or

$$\begin{split} &\Xi_{t}\left(1-\mu_{RGL,t}\left(R_{G,L,n,t}-R_{G,L,t}\right)\right) = \\ &\beta\Xi_{t+1}\left[\frac{\left(R_{L,t}-1+\omega_{LTD}\right)+\left(1-\omega_{LTD}\right)\left[1-\mu_{RGL,t+1}\left(R_{G,L,n,t+1}-R_{G,L,t}\right)\right]}{\Pi_{t+1}}\right] \\ &+\chi_{b,t}\left(\frac{B_{G,t}}{P_{t}}+\frac{B_{G,L,t}}{P_{t}}\right)^{-\sigma_{b}} + \chi_{\varepsilon_{b},t}\varepsilon_{b,t} \end{split}$$

or

$$\Xi_{t} \left(1 - \mu_{RGL,t} \left(R_{G,L,n,t} - R_{G,L,t} \right) \right) = \beta \Xi_{t+1} \left[\frac{\left(R_{L,t} \right) - \left(1 - \omega_{LTD} \right) \mu_{RGL,t+1} \left(R_{G,L,n,t+1} - R_{G,L,t} \right)}{\Pi_{t+1}} \right] + \chi_{b,t} \left(\frac{B_{G,t}}{P_{t}} + \frac{B_{G,L,t}}{P_{t}} \right)^{-\sigma_{b}} + \chi_{\varepsilon_{b},t} \varepsilon_{b,t}$$

Substituting (30) yields

$$\begin{split} \Xi_{t} \left(1 - \beta \Xi_{t+1} \frac{1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right)}{\Pi_{t+1}} \left(R_{G,L,n,t} - R_{G,L,t} \right) \right) = \\ \beta \Xi_{t+1} \left[\frac{\left(R_{L,t} \right) - \left(1 - \omega_{LTD} \right) \mu_{RGL,t+1} \left(R_{G,L,n,t+1} - R_{G,L,t} \right)}{\Pi_{t+1}} \right] \\ + \chi_{b,t} \left(\frac{B_{G,t}}{P_{t}} + \frac{B_{G,L,t}}{P_{t}} \right)^{-\sigma_{b}} + \chi_{\varepsilon_{b},t} \varepsilon_{b,t} \end{split}$$

or

$$\Xi_{t} \left(1 - \beta \Xi_{t+1} \frac{1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right)}{\Pi_{t+1}} \left(R_{G,L,n,t} \right) \right) = \beta \Xi_{t+1} \left[\frac{- \left(1 - \omega_{LTD} \right) \mu_{RGL,t+1} R_{G,L,n,t+1}}{\Pi_{t+1}} \right] + \chi_{b,t} \left(\frac{B_{G,t}}{P_{t}} + \frac{B_{G,L,t}}{P_{t}} \right)^{-\sigma_{b}} + \chi_{\varepsilon_{b},t} \varepsilon_{b,t}$$

or

$$\begin{split} \Xi_t &= \beta \Xi_{t+1} \left[\frac{R_{G,L,n,t} + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) R_{G,L,n,t} - \left(1 - \omega_{LTD} \right) \mu_{RGL,t+1} R_{G,L,n,t+1}}{\Pi_{t+1}} \right] \\ &+ \chi_{b,t} \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b} + \chi_{\varepsilon_b,t} \varepsilon_{b,t} \end{split}$$

or

$$\Xi_{t} = \beta E_{t} \left\{ \Xi_{t+1} \frac{R_{L,n,t} - \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \left(R_{L,n,t+1} - R_{L,n,t} \right)}{\Pi_{t+1}} \right\}$$

$$+ \chi_{b,t} \left(\frac{B_{G,t}}{P_{t}} + \frac{B_{G,L,t}}{P_{t}} \right)^{-\sigma_{b}} + \chi_{\varepsilon_{b},t} \varepsilon_{b,t}$$

$$\mu_{RGL,t} = \beta E_{t} \left\{ \frac{\Xi_{t+1}}{\Xi_{t}} \frac{1}{\Pi_{t+1}} \left[1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \right] \right\}$$
(34)

where $\mu_{RGL,t}$ and $\mu_{bGL,t}$ denotes the Lagrange multipliers on the law of motion of the average interest rate (10) and total long-term government bonds (44), respectively. These equations are identical to Krause and Moyen except for the term reflecting the marginal utility of government bonds $\chi_{b,t} \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b}$ in equations (26) and (33).

The other first order conditions are standard:

$$\begin{split} \Xi_{t}^{k} &= \beta E_{t} \left[\Xi_{t+1} \left((1 - \tau_{K,t+1}) \left(r_{K,t+1} Z_{t+1} - a(Z_{t+1}) \right) + \tau_{K,t+1} \delta \right) + \Xi_{t+1}^{k} \left(1 - \delta \right) \right] \\ &+ \chi_{K,t} \bar{K}_{t}^{-\sigma_{K}} \iff \\ Q_{t} &= \beta E_{t} \left[\frac{\Xi_{t+1}}{\Xi_{t}} \left((1 - \tau_{K,t+1}) \left(r_{K,t+1} Z_{t+1} - a(Z_{t+1}) \right) + \tau_{K,t+1} \delta + (1 - \delta) Q_{t+1} \right) \right] \\ &+ \frac{\chi_{K,t} \bar{K}_{t}^{-\sigma_{K}}}{\Xi_{t}} \\ r_{K,t} &= a' \left(Z_{t} \right) \\ \Xi_{t} \left(1 + \tau_{C,t} \right) &= \left(C \left(j \right)_{t+i} - h C_{t+i-1} \right)^{-\sigma_{c}} exp \left(\frac{\sigma_{c} - 1}{1 + \sigma_{\ell}} L_{t}^{1+\sigma_{\ell}} \left(\hbar \right) \right) \\ \Xi_{t} \frac{\left(1 - \tau_{W,h,t} - \tau_{L,t} \right) W_{t+j} \left(j \right)}{P_{t+j}} &= \left(\frac{1}{1 - \sigma_{c}} \left(C \left(j \right)_{t+i} - h C_{t+i-1} \right)^{1-\sigma_{c}} \right) \cdot exp \left(\frac{\sigma_{c} - 1}{1 + \sigma_{\ell}} L_{t} \left(\hbar \right)^{1+\sigma_{\ell}} \right) \left(\sigma_{c} - 1 \right) L_{t}^{\sigma_{t}} \left(\hbar \right) \\ \Xi_{t} &= \Xi_{t}^{k} \varepsilon_{I,t} \left(1 - S \left(\frac{I_{t}}{I_{t-1}} \right) - S' \left(\frac{I_{t}}{I_{t-1}} \right) \frac{I_{t}}{I_{t-1}} \right) \\ &+ \beta E_{t} \left[\Xi_{t+1}^{k} \varepsilon_{I,t+1} S' \left(\frac{I_{t+1}}{I_{t}} \right) \left(\frac{I_{t+1}}{I_{t}} \right)^{2} \right] \end{split}$$

A.2 Firm Cost minimization

Production function:

$$Y_t = A_t \left(TFP_t L_t \right)^{1-\alpha} K_t^{\alpha} - TFP_t \Phi$$

$$\frac{W_t}{P_t} \left(1 + \tau_{w,f,t} \right) = mc_t \left(1 - \alpha \right) \frac{Y_t + TFP_t \Phi}{L_t}$$
(35)

$$r_{K,t} = mc_t \alpha \frac{Y_t + TFP_t \Phi}{K_t} \tag{36}$$

where TFP_t denotes the technology trend (grows deterministically) in SW, with

$$\gamma_t = \frac{TFP_t}{TFP_{t-1}} \tag{37}$$

A.3 Detrending

Detrending using $\Xi_t = \frac{\xi_t}{TFP_t^{\sigma_c}}$, and assuming $\chi_{b,t} = \frac{TFP_t^{-\sigma_c}}{TFP_t^{-\sigma_b}}\chi_b$ and $\chi_{K,t} = \frac{TFP_t^{-\sigma_c}}{TFP_t^{-\sigma_c}}\chi_K$ and $\chi_{\varepsilon_b,t} = \frac{\chi_{\varepsilon_b}}{TFP_t^{\sigma_c}}$

$$\begin{split} \frac{\xi_t}{TFP_t^{\sigma_c}} &= \beta E_t \left\{ \frac{\xi_{t+1}}{TFP_{t-1}^{\sigma_c}} \frac{R_t}{\Pi_{t+1}} \right\} + \frac{TFP_t^{-\sigma_c}}{TFP_t^{-\sigma_b}} \chi_b \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b} \\ &+ \frac{\chi_\epsilon}{TFP_t^{\sigma_c}} \varepsilon_{b,t} \\ \frac{\xi_t}{TFP_t^{\sigma_c}} &= \beta E_t \left\{ \frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}} \frac{R_{L,n,t} - \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \left(R_{L,n,t+1} - R_{L,n,t} \right)}{\Pi_{t+1}} \right\} \\ &+ \frac{TFP_t^{-\sigma_c}}{TFP_t^{-\sigma_b}} \chi_b \left(\frac{B_{G,t}}{P_t} + \frac{B_{G,L,t}}{P_t} \right)^{-\sigma_b} + \frac{\chi_\epsilon}{TFP_t^{\sigma_c}} \varepsilon_{b,t} \\ \mu_{RGL,t} &= \beta E_t \left\{ \frac{\frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}}}{\frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}}} \frac{1}{\Pi_{t+1}} \left[1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \right] \right\} \\ Q_t &= \beta E_t \left\{ \frac{\frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}}}{\frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}}} \frac{1}{\Pi_{t+1}} \left[1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \right] \right\} \\ \left\{ Q_t &= \beta E_t \left\{ \frac{\xi_{t+1}}{\frac{\xi_{t+1}}{TFP_{t+1}^{\sigma_c}}} \left(r_{K,t+1} - a(Z_{t+1}) + (1 - \delta) Q_{t+1} \right) \right\} + \frac{\frac{TFP_t^{-\sigma_c}}{TFP_t^{-\sigma_c}} \chi_K \bar{K}_t^{-\sigma_K}}{\frac{\xi_t}{TFP_t^{\sigma_c}}} \right. \\ \left. \left(1 + \tau_{C,t} \right) \xi_t &= \left(c_{O,t} - \frac{h}{\gamma} \cdot c_{O,t-1} \right)^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_\ell} N_t^{1+\sigma_\ell} \right) \\ \xi_t \left(1 - \tau_{w,h,t} - \tau_{N,t} \right) w_{h,t} &= \left(\left(c_{O,t} - \frac{h}{\gamma} \cdot c_{O,t-1} \right)^{-1-\sigma_c} \right) \exp\left(\frac{\sigma_c - 1}{1 + \sigma_\ell} N_t^{1+\sigma_\ell} \right) N_t^{\sigma_l} \\ 1 &= Q_t \varepsilon_{I,t} \left(1 - S \left(\frac{I_t TFP_{t-1}}{I_{t-1} TFP_t} \frac{TFP_t}{TFP_{t-1}} \right) - S' \left(\frac{I_t TFP_{t-1}}{I_{t-1} TFP_t} \frac{TFP_t}{TFP_{t-1}} \right) \frac{I_t TFP_{t-1}}{I_{t-1} TFP_t} \frac{TFP_{t-1}}{TFP_{t-1}} \right) \\ &+ \beta E_t \left[\frac{\frac{\xi_{t+1}}{TFP_t^{\sigma_c}}}{\frac{\xi_{t+1}}{TFP_t^{\sigma_c}}} Q_{t+1} A_{I,t} \varepsilon_{I,t+1} S' \left(\frac{I_{t+1} TFP_t}{I_t TFP_{t+1}} \frac{TFP_{t+1}}{TFP_{t-1}} \right) \left(\frac{I_{t+1} TFP_t}{I_t TFP_{t+1}} \frac{TFP_{t+1}}{TFP_t} \right)^2 \right] \end{aligned}$$

Or

$$\xi_t = \beta E_t \left\{ \frac{\xi_{t+1}}{\gamma_{t+1}^{\sigma_c}} \frac{R_t}{\Pi_{t+1}} \right\} + \chi_b \left(b_{G,t} + b_{G,L,t} \right)^{-\sigma_b} + \chi_{\epsilon} \varepsilon_{b,t}$$
(38)

$$\xi_{t} = \beta E_{t} \left\{ \frac{\xi_{t+1}}{\gamma_{t+1}^{\sigma_{c}}} \frac{R_{L,n,t} - \mu_{RGL,t+1} \left(1 - \omega_{LTD}\right) \left(R_{L,n,t+1} - R_{L,n,t}\right)}{\Pi_{t+1}} \right\}$$
(39)

$$+\chi_b \left(b_{G,t}+b_{G,L,t}\right)^{-\sigma_b}+\chi_{\epsilon}\varepsilon_{b,t}$$

$$\mu_{RGL,t} = \beta E_t \left\{ \frac{1}{\Pi_{t+1}} \frac{\xi_{t+1}}{\gamma_{t+1}^{\sigma_c} \xi_t} \left[1 + \mu_{RGL,t+1} \left(1 - \omega_{LTD} \right) \right] \right\}$$
(40)

$$= \beta E_t \left\{ \frac{\xi_{t+1}}{\gamma_{t+1}^{\sigma_c} \xi_t} \frac{1}{\Pi_{t+1}} \left[\omega_{LTD} + (1 - \omega_{LTD}) Q_{b,G,L,t+1} \right] \right\}$$
(41)

$$+\frac{\chi_{b}\left(b_{G,t}+b_{G,L,t}\right)^{-\sigma_{b}}}{\xi_{t}}+\frac{1}{\xi_{t}}\chi_{\epsilon}\varepsilon_{b,t}$$

$$Q_{t} = \beta E_{t} \left[\frac{\xi_{t+1}}{\gamma_{t+1}^{\sigma_{c}} \xi_{t}} \left((1 - \tau_{K,t+1}) \left(r_{K,t+1} Z_{t+1} - a(Z_{t+1}) \right) + \tau_{K,t+1} \delta + (1 - \delta) Q_{t+1} \right) \right] + \frac{\chi_{K} \bar{k}_{t}^{-\sigma_{K}}}{\xi_{t}}$$

$$(42)$$

$$1 = Q_t \varepsilon_{I,t} \left(1 - S\left(\frac{i_t \gamma_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1} \gamma_t}\right) \frac{i_t \gamma_t}{i_{t-1}} \right)$$

$$+ \beta E_t \left[\frac{\xi_{t+1}}{\xi_t \gamma_{t+1}^{\sigma_c}} Q_{t+1} A_{I,t} \varepsilon_{I,t+1} S'\left(\frac{i_{t+1}}{i_t} \gamma_{t+1}\right) \left(\frac{i_{t+1}}{i_t} \gamma_{t+1}\right)^2 \right]$$

$$(43)$$

From (9)

$$\frac{B_{G,L,t}}{TFP_{t}P_{t}} = (1 - \omega_{LTD}) \frac{\frac{B_{G,L,t-1}}{P_{t-1}TFP_{t}} \frac{TFP_{t-1}}{TFP_{t-1}}}{\prod_{t}} + \frac{B_{G,L,n,t}}{TFP_{t}P_{t}}$$

or

$$b_{G,L,t} = (1 - \omega_{LTD}) \frac{b_{G,L,t-1}}{\Pi_t \gamma_t} + b_{G,L,n,t}$$
(44)

From (10)

$$\left(R_{G,L,t}-1\right)\frac{B_{G,L,t}}{TFP_{t}P_{t}} = \left(1-\omega_{LTD}\right)\frac{\left(R_{G,L,t-1}-1\right)}{\Pi_{t}}\frac{B_{G,L,t-1}TFP_{t-1}}{TFP_{t}P_{t-1}TFP_{t-1}} + \left(R_{G,L,n,t}-1\right)\frac{B_{L,n,t}}{P_{t}TFP_{t}}$$

or

$$(R_{G,L,t} - 1) b_{G,L,t} = (1 - \omega_{LTD}) \frac{(R_{G,L,t-1} - 1)}{\Pi_t} \frac{b_{G,L,t-1}}{\gamma_t} + (R_{G,L,n,t} - 1) b_{L,n,t}$$
(45)

Combining (44) and (45) yields

$$\left(R_{G,L,t}-1\right)b_{G,L,t} = \left(1-\omega_{LTD}\right)\frac{\left(R_{G,L,t-1}-1\right)}{\Pi_{t}}\frac{b_{G,L,t-1}}{\gamma_{t}} + \left(R_{G,L,n,t}-1\right)\left(b_{G,L,t}-\left(1-\omega_{LTD}\right)\frac{b_{G,L,t-1}}{\Pi_{t}\gamma_{t}}\right)$$

or

$$(R_{L,t} - R_{L,n,t}) b_{G,L,t} = (1 - \omega_{LTD}) \frac{(R_{L,t-1} - R_{L,n,t})}{\prod_{t} \gamma_t} b_{G,L,t-1}$$
(46)

From (??)

$$\frac{B_{G,L,t}}{P_t} = \frac{R_{G,L,t-1}}{\prod_t} \frac{B_{G,L,t-1}}{P_{t-1}} + G_t - \left(T_t + (\tau_{w,h,t} + \tau_{w,f,t}) w_t N_t + \tau_C C_t + \tau_{K,t} Prof_t\right)$$
(47)

$$\frac{B_{G,L,t}}{TFP_{t}P_{t}} = \frac{R_{G,L,t-1}}{\Pi_{t}} \frac{B_{G,L,t-1}}{TFP_{t}P_{t-1}} + \frac{G_{t}}{TFP_{t}} - \left(\frac{T_{t}}{TFP_{t}} + \left(\tau_{w,h,t} + \tau_{w,f,t}\right) \frac{W_{t}}{TFP_{t}P_{t}} L_{t} + \tau_{C} \frac{C_{t}}{TFP_{t}} + \tau_{K,t} \frac{Prof_{t}}{TFP_{t}}\right)$$

or

$$\frac{B_{G,L,t}}{TFP_{t}P_{t}} = \frac{R_{G,L,t-1}}{\Pi_{t}} \frac{TFP_{t-1}B_{G,L,t-1}}{TFP_{t}TFP_{t-1}P_{t-1}} + \frac{G_{t}}{TFP_{t}} - \left(t_{t} + \left(\tau_{w,h,t} + \tau_{w,f,t}\right) \frac{W_{t}}{TFP_{t}P_{t}} L_{t} + \tau_{C} \frac{C_{t}}{TFP_{t}} + \tau_{K,t} \frac{Prof_{t}}{TFP_{t}}\right) + \frac{G_{t}}{TFP_{t}P_{t}} \left(t_{t} + \left(\tau_{w,h,t} + \tau_{w,f,t}\right) \frac{W_{t}}{TFP_{t}P_{t}} L_{t} + \tau_{C} \frac{C_{t}}{TFP_{t}} + \tau_{K,t} \frac{Prof_{t}}{TFP_{t}}\right) \right)$$

$$b_{G,t} = \frac{R_{L,t-1}}{\prod_{t} \gamma_{t}} b_{G,t-1} + g_{t} - (t_{t} + (\tau_{w,h,t} + \tau_{w,f,t}) w_{t} N_{t} + \tau_{C,t} l_{t} + \tau_{K,t} prof_{t})$$
(48)

$$\frac{NProf_t}{TFP_t} = \frac{Y_t}{TFP_t} - (1 + \tau_{w,f,t}) \frac{\frac{W_t}{P_t}}{TFP_t} N_t - (\delta + a(Z_t)) \frac{\bar{K}_{t-1}}{TFP_t} = y_t - (1 + \tau_{w,f,t}) w_t N_t - (\delta + a(Z_t)) \frac{\bar{K}_{t-1}}{TFP_{t-1}} \frac{TFP_{t-1}}{TFP_t}$$
(49)

$$nprof_{t} = y_{t} - (1 + \tau_{w,f,t}) w_{t} N_{t} - (\delta + a(Z_{t})) \frac{\bar{k}_{t-1}}{\gamma_{t}}$$
 (50)

Primary deficit ratio:

$$PDY_{t} = \frac{\left(g_{t} - \left(t_{t} + \left(\tau_{w,h,t} + \tau_{w,f,t}\right) w_{t} N_{t} + \tau_{C} l_{t} + \tau_{K,t} \left(r_{K,t} - \delta\right) k_{t}\right)\right)}{y_{t}}$$
(51)

Debt-to-annualized GDP ratio

$$b2gdp_t = \frac{b_{G,t}}{4Y_t}$$

Production function

$$y_t = A_t \left(N_t \right)^{1-\alpha} k_t^{\alpha} - \Phi$$

Firm FOCs

$$w_t (1 + \tau_{w,f,t}) = mc_t (1 - \alpha) \frac{y_t + \Phi}{N_t}$$
 (52)

Capital accumulation: From

$$\frac{\bar{K}_t}{TFP_t} = (1 - \delta) \frac{\bar{K}_{t-1}}{TFP_t} + \frac{I_t}{TFP_t} \Leftrightarrow
\bar{k}_t = (1 - \delta) \frac{TFP_{t-1}}{TFP_t} \frac{\bar{K}_{t-1}}{TFP_{t-1}} + i_t \Leftrightarrow
\bar{k}_t = (1 - \delta) \frac{\bar{k}_{t-1}}{\gamma_t} + i_t$$
(53)

$$\frac{K_t}{TFP_t} = \frac{\bar{K}_{t-1}}{TFP_t} Z_t \Leftrightarrow$$

$$k_t = \frac{\bar{K}_{t-1}}{TFP_{t-1}} \frac{TFP_{t-1}}{TFP_t} Z_t = \frac{\bar{k}_{t-1}}{\gamma_t} Z_t \tag{54}$$

A.4 Resource constraint

$$y_t = c_t + i_t + g_t + x_t \tag{55}$$

B Data used in the estimation and calibration

Unless otherwise mentioned, we obtained all fiscal data referred to below from the Euro Area fiscal database of Paredes et al. (2014) and all remaining macroeconomic data from the Area Wide Model database of Fagan et al. (2005a).

B.1 Macroeconomic data

- We compute GDP_t , $CONS_t$, $INVE_t$ and GOV_t as $\frac{X_t}{POP_t*\frac{YED_t}{100}}$, where
 - $-X_t$: Respective nominal data:
 - $* NGDP_t = YER_t * YED_t$
 - $*NCONS_t = CER_t * CED_t$

- * $NINVE_t = ITR_t * ITD_t GIN_t$, where GIN_t denotes government investment from the Euro Area fiscal database.
- $-POP_t$: Working age population.
- YED_t : GDP deflator.
- $WREAL_t = \frac{\frac{WIN_t}{LNN_t}}{YED_t}$, where WIN_t denotes compensation of employees and LNN_t denotes total employment.
- \widehat{LNN}_t : Detrended employment (heads). This is series is constructed by first decomposing the log of employment $\ln(LNN_t)$ into $\ln(LNN_t) = \ln\left(\frac{LNN_t}{LFN_t}\right) + \ln\left(\frac{LFN_t}{POP_t}\right)$, where LFN_t denotes the labor force (implying that $1 \frac{LNN_t}{LFN_t}$ yields the unemployment rate), and then, stationarize them separately, similar to Campbell et al. (2019). $\ln\left(\frac{LNN_t}{LFN_t}\right)$ doesn't display an obvious time trend over the sample, and using Dickey-Fuller test where the alternative hypothesis is a stationary process without a time trend, we can reject the unit root hypothesis with 98% confidence over the sample period. Therefore we only demean it. By contrast, $\ln\left(\frac{LFN_t}{POP_t}\right)$ displays a trend (though no unit root), therefore we remove a linear trend. with the detrended hours series of Campbell et al. (2019).
- STI_t : Short term interest rate (or policy rate). See B.3.
- $CPI6to10_t$: Average inflation expected for the 6th to the 10th year from today. Obtained from
 - 2005Q2-2019Q4: Expectations component of inflation linked swap rates, which we take from an ECB update of Camba-Mèndez and Werner (2017). The series is extremely close to the series published in Burban et al. (2021), with the difference never exceeding 0.08 percentage points.
 - 1990Q2-2005Q1: Average inflation expected for the 6th to 10th calendar year from today from Consensus Economics, collected by Stevens and Wauters (2018). The pre-1999 data is a GDP weighted average of the respective country values of France, Germany, Italy, Netherlands and Spain.

B.2 Fiscal data

• PY_t : Deficit-to-GDP ratio, constructed as $PDY_t = \frac{DEF_t}{YER_t*YED_t}*100$, where DEF_t denotes the headline government deficit. DEF_t differs from Paredes et al. (2014) in that, when calculating total government expenditures, we replace the nominal government consumption series in the fiscal database with the corresponding series from the Area Wide Model database, i.e. GCD_t*GCR_t .

- GGY_t : Government-demand-to-GDP ratio, constructed as $GGY_t = \frac{GG_t}{YER_t*YED_t}*100$, with $GG_t = GCD_t*GCR_t + GIN_t$, where GIN_t denotes government investment from the Euro Area fiscal database.
- $DTXY_t$: Share of direct tax revenue in GDP, constructed as $DTXY_t = \frac{DTX_t}{YER_t*YED_t}*100$, where DTX_t denotes "EA general government total direct taxes".
- $SCRY_t$: Share employer social security contributions in GDP, constructed as $SCRY_t = \frac{SCR_t}{YER_t*YED_t}*100$, where SCR_t denotes "EA general gov. social security contributions by employers".
- $SCEY_t$: Share employee social security contributions in GDP, constructed as $SCEY_t = \frac{SCE_t}{YER_t*YED_t}*100$, where SCE_t denotes "EA general gov. social security contributions by employees, self-employed and other".
- $TAUC_t$: Implicit consumption tax rate, constructed as $TAUC_t = \frac{TIN_t}{CER_t*CED_t} * 100$, where TIN_t denotes "EA general government total indirect taxes".
- Government-Debt-to-GDP ratio= $\frac{MAL_t}{YER_t*YED_t}*100$, where MAL_t denotes Euro area general government debt.
- Average maturity of outstanding government debt: Calculated as a government-debt-weighted average of the respective values of Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain. For 1992-2010, we obtained the country specific average maturities from the OECD database ("Average term to maturity for total debt, Central government"). Since the series ends in 2010, we obtained the values for the 2011-2018 period from the 2011-2018 annual issues of the IMF Fiscal Monitor. We used "General government consolidated gross debt: Excessive deficit procedure (based on ESA 2010) and former definition (linked series)" from the European Commission's database AMECO to calculate the country weights.

B.3 Short term interest rate (STI_t) and interest rate expectation measures ($STIEX_{i,t}$)

We obtained my measure of interest rate expectations $STIEX_{i,t}$ and and the short term interest rate or policy rate STI_t as described below.

- 1999Q1-2019Q4:
 - $-STI_t$: EONIA quarterly average, from the ECB's Statistical Data Warehouse (ECB-SDW)

- $STIEX_{i,t}$: OIS rates, available for i = 1 8 and i = 12, from Bloomberg. Exception: 1999Q1-2004Q4, where we also use Bund Zero Coupon Yields (ZCY) during some quarters (obtained from Deutsche Bundesbank), since the OIS curve does not always extend beyond a horizon of i = 4 or i = 8. During the 1999-2009 period, the Bund ZCY differ only marginally from OIS rates of the same maturity.
 - * 1999Q1-2000Q4: i = 8: 2 year Bund ZCY.
 - * 1999Q1-2004Q4: i = 12: 3 year Bund ZCY.
- The OIS data are averages over the final five days of the quarter. The Bund ZCY is from the final day of quarter, as the original data are end-of-month values.

• 1994Q1-1998Q4:

- STI_t: 3-month Euribor, average over the first month of the quarter, from ECBSDW.
- $STIEX_{i,t}$, i = 1, 2, 4: 3, 6 and 12 month Euribor, average over the final month of the quarter, from ECBSDW.
- $STIEX_{i,t}$, i = 8, 12: $STIEX_{i,t} = ZCY_{i,t} + (STIEX_{4,t} ZCY_{4,t})$ where the $ZCY_{i,t}$ denote the zero coupon yields on a government bond with the corresponding maturity. We calculated the $ZCY_{i,t}$ as GDP-weighted averages of the values of Belgium, Germany, France, Spain and Italy, obtained from the BIS Databank. The country specific values are averages over the final month of the quarter.

• 1990Q1-1993Q4:

- STI_t:GDP weighted average of 3-month money market rates of Austria, Spain, Germany, France, Netherlands and Italy, average over first month of the quarter. This rate is essentially identical to the Euribor 3 month rate during the period where both are available (i.e. starting 1994Q1).
- $STIEX_{1,t}$ same source as as STI_t , but average over final month of the quarter.
- $STIEX_{i,t}$, i = 2, 4: Constructed as $STIEX_{i,t} = EM_{i,t} + (EM_{1,t} STIEX_{1,t})$, with $EM_{1,t}$, $EM_{2,t}$ and $EM_{4,t}$ corresponding to Euro Area 3, 6 and 12 month Euro Market rates, obtained from the BIS Databank. We calculated $EM_{i,t}$ as GDP-weighted averages of the country specific rates of Austria, Belgium, Germany, Spain, France, Italy and the Netherlands. The resulting $STIEX_{i,t}$ series are very close to the Euribor series of the same maturity during the period where both are available (i.e. starting 1994Q1).
- From 1992Q1: $STIEX_{i,t}$, i = 8, 12: $STIEX_{i,t} = ZCY_{i,t} + (STIEX_{4,t} ZCY_{4,t})$, where $ZCY_{i,t}$ is as defined above.
- 1980Q1-1989Q4: $STI_t = STN_t$, the short term interest rate from the AWM database.

- Whenever for a given quarter t, observations for some some intra-annual horizons i are missing, we linearly interpolate them using the values of for two most adjacent horizons available. For instance, for i = 9 11 we have $STIEX_{i,t} = STIEX_{8,t} + \frac{i-8}{4} (STIEX_{12,t} STIEX_{8,t})$.
- Throughout, GDP weights are computed from 1995 PPS GDP (consistent with the weights of the AWM database), obtained from AMECO.

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