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An Application of Index Number Theory to Interest Rates: Evidence from Selected Post-Soviet Countries

Abstract: In this paper, we use index number theory to decompose changes in total interest rate due to changes in the interest rate component and the weight component. We discuss the optimal calculation of a binary index using axiomatic index number theory. Based on this theory we compare alternative indexes and as a result, we choose the Marshall-Edgeworth index because most axioms are satisfied by this index. Comparing the results of binary periods decomposition, we conclude that the differences are not significant when we apply different indices. For multiple period comparison, we suggest using the chain index because it allows accounting for the weights evolution during the whole period. In addition, we derive a formula that could be useful for explaining the differences between chain and direct indexes when we produce multiple period comparison.

Keywords: index number, interest rate, chain and direct indexes, post-Soviet countries.

JEL Classification: C43, E43

1. Introduction

In this paper, we apply the index number theory developed for price statistics to interest rate. One of the important ways for using indices is to distinguish changes in volume from changes in prices. According to this, we can note that while some economic variables for which indices

are calculated do not have a relevant meaning when expressed in absolute values (e.g. price indices, GDP growth rates, industrial production index), interest rates certainly provide useful information when expressed in absolute values. Similarly, a change in interest rate is better understood and usually communicated in absolute values rather than in ratios. For these reasons, this paper focuses on indices that keep absolute values of the change rather than expressing it as ratios. These indices were first presented in the first part of the twentieth century and they were recently been revisited by Diewert (2005).

Another important methodological issue is the approach to obtain, examine and compare alternative indices. In the index number literature, there are three approaches that can be used for comparison of alternative indices, particularly the axiomatic approach, the economic approach, and the stochastic approach. The axiomatic approach compares different indices on the basis of a number of mathematical features (axioms) with which the indices may comply or not (Fisher, 1921, 1922); the economic approach obtains the index by using economic theory and maximization techniques (Konus, 1924); the stochastic approach considers the evolution of individual prices as observations of the general inflation rate and includes stochastic factors (Diewert, 1995; Wynne, 1997; Clements, Izan and Selvanathan, 2006). In this paper, we limit ourselves to the axiomatic approach which seems very much applicable to the interest rate statistics. The economic approach is based on consumer utility functions and a basket of products and prices and does not seem to be directly applicable to the interest rates. The stochastic approach requires further study.

On the basis of the considerations above, this paper uses the work by Diewert (2005) as the starting point for the analysis. The paper is organized as follows. After the introduction, in section 2 we briefly review the literature related to the researched topic. In section 3 we draw a parallel between price indices and interest rate. In section 4 we give a theoretical background, where we present a number of possible formulas for interest rate decomposition. In this section we also provide a number of axioms and use them to decide about most appropriate formula for interest rate decomposition. Using actual interest rates and loan volumes data for some selected post-Soviet countries, in section 5 we provide a decomposition of the aggregate interest rate change into the interest rate component and the weight component. In section 6 we expand the decomposition formula also for multiple periods. In this section we derive the formula that we use for explaining the differences between direct and chain indexes. Final section concludes.

2. Literature Review

The index number theory has been developed mainly in the context of price statistics. The simplest price index is a fixed-basket index. In this approach, we are given a fixed basket of the n commodities. Then we calculate the cost of purchasing this fixed basket of commodities at the prices of period 1 (base period) and period 1 (current period). The fixed basket price index is simply the ratio of these two values where the prices vary but the quantities that are held fixed.

One of the simplest formula to measure the price change between periods 0 and 1 was to specify an approximate “representative” commodity basket quantity vector, which was to be updated every five years. According to this approach, we specify the “representative” commodity basket and calculate the price index as the ratio of the cost of buying this same basket of goods in period 1 and period 0. This fixed basket approach leaves open the following question: How exactly is the fixed basket to be chosen?

In the price statistics literature, there are two natural choices for the reference basket: the base 1 period commodity basket or the current period 1 commodity basket. These two choices lead to the Laspeyres and Paasche price indices, respectively. The problem with the Paasche and Laspeyres price indices is that they are equally plausible but they will give different answers. This suggests that we need a single estimate for the price change between two periods. For that we should take some sort of weighted average of the two indices. Examples of such average is the Fisher (1922) ideal index. At this point, the fixed basket approach to the index number theory is transformed into the test approach to the index number theory: that is, in order to determine which of these fixed indices or which of their averages might be the best, we need some criteria or tests that we would like our indices to satisfy. According to several tests, the Fisher ideal price index is the best weighted average of the Laspeyres and Paasche price indices.

The next step of developing the index number theory is related to the pure price indices. The most important point of this theory is that instead of taking a symmetric average of the Laspeyres and Paasche indices, they take a symmetric average of the baskets. For example, the average basket could be the arithmetic or geometric mean of the two baskets, leading the Marshall-Edgeworth index or the Walsh index. Diewert (2002) showed that the Walsh index is the best in this average basket framework.

In this paper we try to use principals of constructing price indexes to interest rate. There are a number of similarities between the price index theory and interest rate. In the next section we discuss all these similarities in more detail.

3. Price indexes and interest rate

As it was mentioned in the previous section, the index number theory was developed mainly in the field of price statistics. Now let us try to use principals of calculating price indexes in the context of interest rate. As a rule, to compute interest rate, the weighted average formula can be used. For example, for calculating interest rate for loans (or time deposits) we can use the following formula:

$$I_t = \sum_k i(k)_t w(k)_t \quad (1)$$

Where, I_t is an aggregate interest rate for loans at time t , $i(k)_t$ is an interest rate for the k -th currency (national currency, USD, Euro, RR and so on) loans at time t , $w(k)_t$ is a weight of the k -th currency loans in the total volume of loans at time t , $w(k)_t = v(k)_t / \sum_k v(k)_t$, where $v(k)_t$ is the absolute value of the k -th currency loans at time t .

Now let us make a parallel between formula (1) and the following joint price formula.

$$Y_t = \sum_k p(k)_t q(k)_t \quad (2)$$

Where, Y_t joint indicator of prices at time t , which is the sum of each price at time t multiplied by the corresponding commodity volume at time t , $p(k)_t$ is a price of commodity k at time t , $q(k)_t$ is a volume of commodity k at time t .

Thus, we see that there is a similarity between formula (1) and formula (2). Indeed, instead of $p(k)_t$ we take $i(k)_t$, while instead of absolute of commodity volumes $q(k)_t$ we take the weights of k -th currency loan in the total sum of loans. Thus, we conclude that the price index theory can be easily transformed to the interest rates.

In the price index statistics, when comparing Y_t with Y_{t-1} it is required to find out which part of the difference corresponds to price developments and which part corresponds to movements in volumes. The impact of price changes is calculated by fixing the volume of two periods, that is, we use $q(k)_{t-1}$ for two periods (Laspeyres price index). The impact of quantity changes is calculated by fixing prices

for both periods, that is, we use $p(k)_t$ for two periods (Paasche quantity index). Then, by combining these two indices in one we can write the following formula for decomposition.

$$\frac{Y_t}{Y_{t-1}} = \frac{\sum_k p(k)_t q(k)_{t-1}}{\sum_k p(k)_{t-1} q(k)_{t-1}} \times \frac{\sum_k p(k)_t q(k)_t}{\sum_k p(k)_t q(k)_{t-1}} = P_L \times Q_P \quad (3)$$

Where, P_L is Laspeyres price index, Q_P is Paasche quantity index.

Using the abovementioned formulas we can compare interest rate changes in two periods. For that we need to do some changes in notation. In the case of interest rates, the change due to the development in interest rate is calculated by fixing a certain weight for two periods. Using $w(k)_{t-1}$ for both periods we obtain a Laspeyres type of index formula.

$$P_L = \frac{\sum_k i(k)_t w(k)_{t-1}}{\sum_k i(k)_{t-1} w(k)_{t-1}} \quad (4)$$

Similarly, by fixing the same interest rate $i(k)_t$ for both periods we obtain a Paasche type index.

$$Q_P = \frac{\sum_k i(k)_t w(k)_t}{\sum_k i(k)_t w(k)_{t-1}} \quad (5)$$

Finally, as in the case of prices, it is possible to combine the above calculations in one, as follows

$$\frac{I_t}{I_{t-1}} = \frac{\sum_k i(k)_t w(k)_{t-1}}{\sum_k i(k)_{t-1} w(k)_{t-1}} \frac{\sum_k i(k)_t w(k)_t}{\sum_k i(k)_t w(k)_{t-1}} \quad (6)$$

From formula (6) we conclude that changes in the aggregate interest rate from one period to another can be decomposed into a change in interest rates and weights. However, in the case of interest rate, such decomposition has a valid meaning on its own and can be considered as an intermediate step in the construction of possible indices. To construct a possible index, first we must see what kind of indices are recommended by the theory and which of these indices is the best.

4. Theoretical background

The difference index theory has been revisited and further investigated by Diewert (2005) on the basis of the work developed in the 1920s and 1930s by T.L. Benet and J.K. Montgomery. In this section we present the main principals and conclusions developed by this theory.

The interest rate difference between two periods can be presented as $\Delta I_{t,t-1} = I_t - I_{t-1}$. This difference can be decomposed into difference terms that represent the pure interest rate change **Int** and pure weight change **Wgh** and, in some cases, a mixed effect **Mix**.

$$\Delta I_{t,t-1} = I_t - I_{t-1} = Int(i_t, i_{t-1}, w_t, w_{t-1}) + Wgh(i_t, i_{t-1}, w_t, w_{t-1}) + Mix(i_t, i_{t-1}, w_t, w_{t-1}) \quad (7)$$

Now, let us examine possible formulations of the decomposition where we compare the changes in interest rate for two consecutive periods.

4.1. Laspeyres-type decomposition

First of all, for this type of decomposition we have used the term “Laspeyres” because we use the first period interest rates or weights as the appropriate weight. Now let us present the formula for 3 terms Laspeyres-type decomposition. In 3 terms formula we include interest rate effect, weight effect and mixed effect.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} w(k)_{t-1} + \sum_k i(k)_{t-1} \Delta w(k)_{t,t-1} + \sum_k \Delta i(k)_{t,t-1} \Delta w(k)_{t,t-1} \quad (8)$$

Where, $\Delta I_{t,t-1}$ is the difference in the total level of interest rates for loans (or time deposits) between time t and $t-1$, $i(k)_t$ the k -th currency loans interest rate at time t , $\Delta i(k)_{t,t-1}$ the difference in k -th currency loan interest rates between time t and $t-1$, $w(k)_t$ weight of the k -th currency loan in the total sum of loans at time t , $\Delta w(k)_{t,t-1}$ difference in k -th currency loans weight between time t and $t-1$.

The 3 terms Laspeyres decomposition can be rewritten as a 2 terms if we will group the last two terms in formula (8). Indeed, let us write the formula (8) as follows

$$\begin{aligned} \Delta I_{t,t-1} &= \sum_k \Delta i(k)_{t,t-1} w(k)_{t-1} + \sum_k \Delta w(k)_{t,t-1} (i(k)_{t-1} + \Delta i(k)_{t,t-1}) = \\ &= \sum_k \Delta i(k)_{t,t-1} w(k)_{t-1} + \sum_k \Delta w(k)_{t,t-1} (i(k)_{t-1} + i(k)_t - i(k)_{t-1}) = \\ &= \sum_k \Delta i(k)_{t,t-1} w(k)_{t-1} + \sum_k \Delta w(k)_{t,t-1} i(k)_t \end{aligned} \quad (9)$$

4.2. Paasche-type decomposition

In this case, the interest rate effect is calculated in reference to the weight of the present period. In the case of the Paasche-type decomposition, the third component (mixed effect) must be subtracted rather than summed. Thus, the formula for 3 terms according to this type of decomposition can be presented as follows.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} w(k)_t + \sum_k \Delta w(k)_{t,t-1} i(k)_t - \sum_k \Delta i(k)_{t,t-1} \Delta w(k)_{t,t-1} \quad (10)$$

Also this formula can be presented in 2 terms as follows.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} w(k)_t + \sum_k \Delta w(k)_{t,t-1} i(k)_{t-1} \quad (11)$$

4.3. Marshall-Edgeworth-type decomposition

This decomposition uses the simple average of the previous and present period weights to calculate the interest rate effect. In this decomposition, the mixed effect is distributed equally between the interest rate effect and the weight effect.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{i(k)_t + i(k)_{t-1}}{2} \right) \quad (12)$$

4.4. Walsh-type decomposition

This decomposition uses geometric average of the previous and present weights to calculate the interest rate and weight effect. Walsh decomposition includes three components and particularly the third term is not easy to make explicit.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} (w(k)_t w(k)_{t-1})^{1/2} + \sum_k \Delta w(k)_{t,t-1} (i(k)_t i(k)_{t-1})^{1/2} + \text{Rest} \quad (13)$$

Thus in the above we have presented different formulas for decomposition of the aggregate interest rate. Now we should decide which decomposition and, therefore, which index built on it is the most appropriate for interest rate. For that we can apply a number of axioms developed in the index number theory. Now let us consider axioms with an application to the above mentioned type of decomposition.

1. **Exhaustiveness.** The sum of all components **Int**, **Wgh** and **Mix** must be equal to the total change in the interest rate $\Delta I_{t,t-1}$. According to this axiom, we can indicate that all above mentioned indexes in agreement with the exception of the Walsh index because here, instead of mixed component, we have residuals **Rest**, which is not a mixed effect.
2. **Scale.** Multiplying interest rate in the previous and current period by the same value must be equal if we multiply interest rate effect by this value. The same must also hold for the weight effect.

$$\begin{aligned}
 Int(z \times i(k)_t, w(k)_t, z \times i(k)_{t-1}, w(k)_{t-1}) &= z \times Int(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) \\
 Wgh(i(k)_t, z \times w(k)_t, i(k)_{t-1}, z \times w(k)_{t-1}) &= z \times Wgh(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) \quad (14)
 \end{aligned}$$

All above decomposition's methods are satisfied by this axiom.

3. **Monotonicity.** Consider two different scenarios for the present period "a" and "b" and the previous period "c" and "d". If interest rate for the k – the currency loans (or time deposits) in the present period in scenario "a" is higher than in scenario "b", then the following inequality holds:

$$Int(i(k)_t^a, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) > Int(i(k)_t^b, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) \quad (15)$$

Now let us consider the case when the interest rate in the previous period in scenario "c" is higher than in scenario "d", then the following inequality holds:

$$Int(i(k)_t, w(k)_t, i(k)_{t-1}^c, w(k)_{t-1}) < Int(i(k)_t, w(k)_t, i(k)_{t-1}^d, w(k)_{t-1}) \quad (16)$$

The same axiom can be applied to weights, switching weights and rates. All above mentioned decompositions are satisfied by this axiom.

4. **Sign consistency.** If the interest rate increases (decreases), the interest rate component also shows an increase (decrease), i.e. if $i(k)_t > i(k)_{t-1}$, then

$$Int(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) > 0 \quad (17)$$

The same must be true for the weight components, i.e., $w(k)_t > w(k)_{t-1}$, then

$$Wgh(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) > 0 \quad (18)$$

All above mentioned decompositions are satisfied by this axiom.

5. **Proportionality.** For rates, the interest rate component must increase proportionally to the weight if the weight does not change.

$$Int([i(k)_t = i(k)_{t-1} + z], [w(k)_t = w(k)_{t-1}], i(k)_{t-1}, w(k)_{t-1}) = z \times w(k)_t \quad (19)$$

Interchanging rates by weights, following the lines above, we have:

$$Whg([i(k)_t = i(k)_{t-1}], [w(k)_t = w(k)_{t-1} + z], i(k)_{t-1}, w(k)_{t-1}) = z \times i(k)_t \quad (20)$$

All the above mentioned decompositions are satisfied by this axiom.

6. **Time reversal.** If we interchange interest rates and weights of the two periods, the interest rate component should be the same in absolute terms, with a change in sign. The same is true for the weight component.

$$\begin{aligned} Int(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) &= -Int(i(k)_{t-1}, w(k)_{t-1}, i(k)_t, w(k)_t) \\ Whg(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) &= -Whg(i(k)_{t-1}, w(k)_{t-1}, i(k)_t, w(k)_t) \end{aligned} \quad (21)$$

Relating with this axiom only Marshall-Edgeworth and Walsh decompositions are satisfied.

7. **Symmetry.** If weights are interchanged along time (but rates are not) the interest rate component does not change.

$$Int(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) = Int(i(k)_t, w(k)_{t-1}, i(k)_t, w(k)_t) \quad (22)$$

If we interchange quantities for the two periods, the weight component should be the same.

$$Whg(i(k)_{t-1}, w(k)_t, i(k)_t, w(k)_{t-1}) = Whg(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) \quad (23)$$

This axiom holds only for the Marshall-Edgeworth and Walsh indexes.

8. **Transitivity.** Consider three subsequent periods, $t - 1, t, t + 1$. The property postulates that overall change in the aggregate interest rate component (between $t - 1$ and $t + 1$) equals the sum of the changes between the sub-periods $t - 1, t$ and $t, t + 1$.

$$\begin{aligned} Int(i(k)_{t+1}, w(k)_{t+1}, i(k)_{t-1}, w(k)_{t-1}) &= Int(i(k)_t, w(k)_t, i(k)_{t-1}, w(k)_{t-1}) \\ &+ Int(i(k)_{t+1}, w(k)_{t+1}, i(k)_t, w(k)_t) \end{aligned} \quad (24)$$

In other words, according to the transitivity property, the interest rate component should be the same when it is directly computed for one longer period as when it is indirectly computed as the result of two shorter periods. This axiom does not hold for any decomposition method.

5. Selection of the decomposition method

The previous section described a number of possible alternative decompositions and a set of axiomatic properties. This section analyses for each type of index whether these properties apply. The results of the mathematical properties are presented in Table 1.

Table 1: Axiomatic results for different decomposition method¹

	Exhaus	Scale	Mono	Sign	Propo	T.rever	Symmetry	Tran
Laspeyres 3 terms	Y	Y	Y	Y	Y	N	N	N
Laspeyres 2 terms	Y	Y	Y	Y	Y	N	N	N
Paasche 3 terms	Y	Y	Y	Y	Y	N	N	N
Paasche 2 terms	Y	Y	Y	Y	Y	N	N	N
Marshall-Edgeworth	Y	Y	Y	Y	Y	Y	Y	N
Walsh 3 terms	N	Y	Y	Y	Y	Y	Y	N

From Table 1 we see that all decompositions comply with the majority of the axioms. This means that whichever of the analysed decompositions is used to calculate the index, the results will be sensible. But most of the axioms as we see are satisfied by the Marshall-Edgeworth formula. On the other side, this decomposition also has several other advantages beyond only the mathematical axiomatic results (Huerga and Steklacova, 2008).

1. It clearly separates into two components the impact of changes in interest rates (first term) and the impact of changes in weights (second term), not containing the intermediate term, which is difficult to explain.
2. Second, it is symmetric between the two components in terms of the period to which they refer; the compound term of other decomposition is allocated to both components in the same way.
3. It considers the data of both periods and it is also symmetric with respect to the periods.

Now let us see whether the decomposition results are significantly different across various methods. Before we discuss the obtained results, let us present our dataset.

First, in order to keep robustness of our analysis, we apply the decomposition methods above to different countries, particularly Armenia, Georgia, Kazakhstan, and Kyrgyzstan. We choose these countries because 1) all these countries

¹ Y indicate a positive result, N is negative.

are former Soviet republics, 2) Armenia, Kazakhstan and Kyrgyzstan are members of the Eurasian Customs Union, and 3) Georgia is a neighbouring country with Armenia as one of the main trading partners. According to the Statistical Committee of the Republic of Armenia, in 2017, the trade turnover between Georgia and Armenia amounted to \$250 million, 3.9% of Armenia's total foreign trade. Second, in all mentioned countries, the interest rates are relatively volatile and in these countries the loans are classified both in national and foreign currencies. The relatively high volatility of the interest rate in developing countries is reflected by the significant volatility of the exchange rate and inflation (Kuncoro, 2020; Mukhlis, Hidayah and Retnasih, 2020; Akosah, Alagidede and Schaling, 2020). We apply the decomposition methods to the actual interest rates data both in national and foreign currency for loans up to one year. To keep comparability between selected countries, we use interest rates data in quarterly terms from 2006Q1 to 2020Q3 (59 observations in total for each country).

Thus by using the decomposition methods, we make calculations for each consecutive period of time. We do this decomposition for 2005Q4 and 2006Q1, 2006Q1 and 2006Q2 and onwards to the last pair of quarters that are 2020Q2 and 2020Q3. Relating with the calculations it should be noted that we are mainly interested in creating the interest rate component index. Taking into account this point in the process of calculations, we have considered the only Laspeyres3, Paasche3 and Marshall-Edgeworth decomposition formulas because, as we can see from the above formulas (8) - (12), the interest rate component is the same. Relating with the Walsh index, we should note that we do not include this index in our analysis because, as it was mentioned, the third component in this index cannot be explicitly computed. Thus using Laspeyres3, Paasche3 and Marshall-Edgeworth type formulas we have calculated the interest rate component values for all the above mentioned countries. The results of calculations allow us to make the following conclusions.

1. All decomposition methods show good properties and all of them can be equally useful in practice.
2. The results of calculations using Laspeyres3, Paasche3 and Marshall-Edgeworth methods are almost the same and there are no significant differences among of them. The differences are so small that it is difficult to distinguish lines on the figure.
3. Despite of that, all the methods are good and with almost no differences; nonetheless, for creating the interest rate component, we suggest using the Marshall-Edgeworth type decomposition. The reason is that, as it was shown, this method fulfills most of the axiomatic properties, it has a simpler expression, as well as several other advantages.

6. From binary to multiple period comparisons

In the previous chapter, we have discussed the comparison of two consecutive periods. According to the index number theory, the comparison can be made not only for the two periods but also for multiple periods. To compare interest rate for multiple periods, we will calculate 1) direct index and 2) chain index.

1. Direct index. For price indices, a direct index means that the index is calculated for each period by directly comparing the price situation at that quarter (month) with the situation at a fixed quarter (month), usually but not necessarily for the first reference period in the series. Similarly, in the case of interest rate, the index would be constructed as the interest rate component of the decomposition of the changes between each quarter (month) t and quarter (month) 0. Therefore the difference in the interest rates direct indices on the basis of Marshall-Edgeworth decomposition is calculated as follows:

$$I_d = \sum_k \Delta i(k)_{t,0} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) \quad (25)$$

Where, I_d is the direct-type decomposition difference.

2. Chain index. In the context of price indices, a chain index means that the index is calculated for each period by comparing the price situation of this period with the previous period, and then linking the results by multiplying (chaining) each individual link with the previous one. The procedure is similar in the case of difference index, with the difference that each link is chained to the previous one by adding them. Therefore, the difference chain index of interest rates on the basis of the Marshall-Edgeworth-type decomposition is as follows:

$$I_c = \sum_t \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) \quad (26)$$

Where, I_c is the difference chain type decomposition.

As a rule, there is a difference between two type of decompositions and these differences could be statistically significant. The question is which type of decomposition is more correct and, therefore, more appropriate for practical application?

In order to describe a possible relevance of the two abovementioned indices, they have been applied to the different countries interest rates data from 2006Q1 – 2020Q3 in quarterly terms. For calculations of the direct and chain indices we

choose 2005Q4 as a base year for all countries included in our analysis. Thus, we calculate the Marshall-Edgeworth-type decomposition with an application to direct and chain indices. Before presenting the results of calculations, let us present the opinions of other authors about advantages and disadvantages of the direct and chain indices.

For example, Stuevel (1989) defends the chain index since that solution solves the “index number problem” of updating the weights; furthermore, this latest approach is recommended in international statistical standards. Conversely, Von der Lippe (2001), defends the direct indices, makes clear that “it is the primarily the idea of making pure comparisons, or of comparing “like with like” which is ignored by chain index theorists, but which is on the other hand a cornerstone of index theory”.

We should note that, indeed, the main advantage of a chain index is that weights are continuously updating. In case of the direct index, we apply only two period weights, as a rule between two comparing periods. In general, we defend the chain index approach. This is explained by two reasons: first, the chain index solves the “index number problem” of updating the weights, and second, this approach is recommended by international statistical standards.

Now let us present the results of empirical calculations. Generally, when we apply the direct and chain index approaches, we get different results of calculations, and sometimes the differences could be significant. Thus using above presented formulas and interest rates data for loans, we have calculated the values for chain and direct indices for Armenia, Georgia, Kazakhstan, and Kyrgyzstan. The results of calculations are presented in Figures 1-4.

From the figures we immediately see that the largest difference between chain and direct indices is for Armenia, while the differences are relatively small for other countries. Now we will try to explain how large can differences be between chain and direct indices and what can be the reasons for such differences. For that we first derive the exact formula that can be used to calculate the size and the contribution of each period to the total difference. The complete derivation is presented in Appendix 1.

From the Appendix 1 we see that the total interest rate is calculated by using the simple weighted average formula, where as a weights we use the ratios of loans. In the Appendix 1 we also derive the formula that can be used for calculating the exact size and contribution of each period of times to the total difference between chain and direct index. We have derived the following formula:

$$\Delta I_{cd} = I_c - I_d = (i_1^1 - i_2^1) \left(\frac{w_1^T - w_1^2}{2} \right) + (i_1^T - i_2^T) \left(\frac{w_1^{T-1} - w_1^1}{2} \right) + \frac{1}{2} \text{cov}(\Delta i_{1,2}^t, \Delta w_1^{t-1,t+1}) \quad (27)$$

Figure 1: Interest rate component contribution for loans up to 1 year (Armenia)

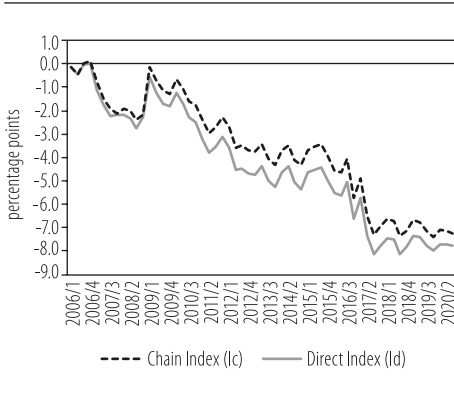


Figure 2: Interest rate component contribution for loans up to 1 year (Georgia)

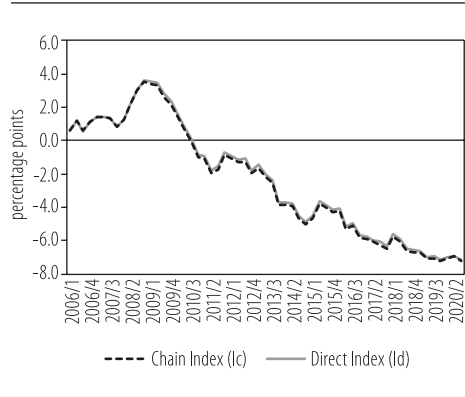


Figure 3: Interest rate component contribution for loans up to 1 year (Kazakhstan)

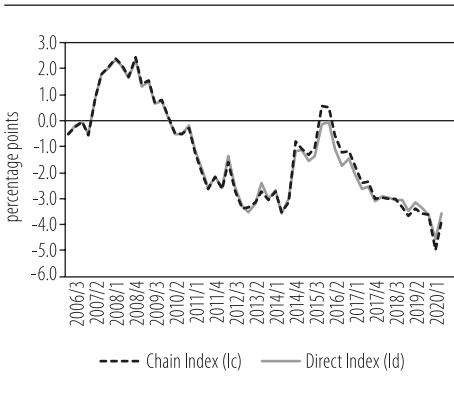
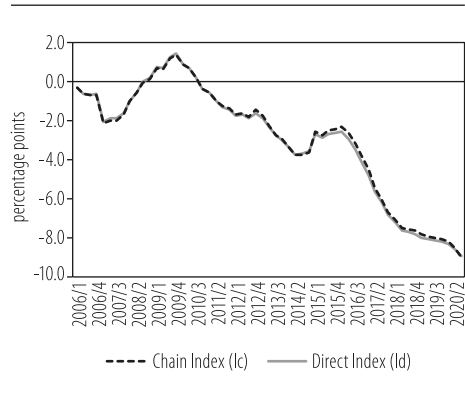


Figure 4: Interest rate component contribution for loans up to 1 year (Kyrgyzstan)



Using this formula we can explain the size of difference between chain and direct indices ΔI_{cd} . First of all, let us divid this formula in two parts. The first part is

$$\Delta I_{cd}^1 = (i_1^1 - i_2^1) \left(\frac{w_1^T - w_1^2}{2} \right) + (i_1^T - i_2^T) \left(\frac{w_1^{T-1} - w_1^1}{2} \right) \quad (28)$$

As we can see, the first part includes the difference between interest rates and weights at the end points. The second part is

$$\Delta I_{cd}^2 = \frac{1}{2} \text{cov}(\Delta i_{1,2}^t, \Delta w_1^{t-1,t+1}) \quad (29)$$

As we can see, the second part is a covariance between all the other remaining points which are located inside of the end points. Relating with the first ΔI_{cd}^1 and second ΔI_{cd}^2 parts we can consider the following four scenarios. 1. $\Delta I_{cd}^1 > 0$ and $\Delta I_{cd}^2 > 0$, 2. $\Delta I_{cd}^1 > 0$ and $\Delta I_{cd}^2 < 0$, 3. $\Delta I_{cd}^1 < 0$ and $\Delta I_{cd}^2 > 0$, 4. $\Delta I_{cd}^1 < 0$ and $\Delta I_{cd}^2 < 0$.

Now using the above mentioned scenarios we can explain the actual differences between chain and direct indexes that we observe from Figures 1-4.

Figure 1 shows the chain and direct indices for Armenian interest rates data. When we calculate the first part ΔI_{cd}^1 we obtain the positive number. This is because in Armenia, at the end of points, the interest rate for loans in national currency was larger than the interest rate for loans in foreign currency. From the other side, the differences between weights were also positive. Thus using the actual data on interest rates and weights at the end points, we have calculated $\Delta I_{cd}^1 = 0.20$. Then, using the differences in interest rates and weights that are located inside of the end points we have calculated positive covariance $\Delta I_{cd}^2 = 0.34$. Thus, for Armenia we have the first scenario where $\Delta I_{cd}^1 > 0$ and $\Delta I_{cd}^2 > 0$. The result is a positive value for the total difference $\Delta I_{cd} = 0.54$. Using the formula (27), we can also find the contributors of each period of time to total difference.

Figure 2 presents the direct and chain indices for Georgian interest rates data. First from figure we see that there is almost no differences. When we calculate the first part, we obtain $\Delta I_{cd}^1 = 0.21$. This is because the difference between interest rates and weights for the end points is positive. If we allow that the second part to be equal to zero, then the difference between chain and direct indices should be positive. Actually the second part is negative and its absolute value almost equal to the first part value. In other words, the negative covariances extinguish the positive increase that we get on the end points. Thus the case of Georgia coincides with the second scenario where we could have either zero for ΔI_{cd} or we could have a negative difference.

Figure 3 shows that at the end of period we have small differences between chain and direct indexes for Kazakhstan. When we calculate the first part, we obtain that $\Delta I_{cd}^1 = 1.36$. This is because the difference between interest rates for loans in national and foreign currency was a positive number at the end points. Differences between weights was also positive for these periods. Thus, if we allow for the

second part to equal to zero, then we conclude that there is a large positive difference between chain and direct indices. However, the actual difference is negative and it equals $\Delta I_{cd} = -0.30$. This is because the covariances between $\Delta i'_{1,2}, \Delta w_1^{t-1,t+1}$ is negative and its absolute value is higher than the value of ΔI_{cd}^1 .

Figure 4 shows that at the end of period, the differences between chain and direct indices is almost equal to zero. Again, if we calculate the first part, then we obtain $\Delta I_{cd}^1 = 2.44$. Thus if we imagine that the covariance is equal to zero, then we will have relatively large differences between chain and direct indices. Calculating the actual covariance results in a negative number and it is almost equal to the value of the first part in absolute terms. Thus, as we see, the case of Kyrgyzstan coincides with the second case, where large negative covariance eliminates this positive increase.

Thus, based on the results for different countries, we conclude that the size of differences between chain and direct indices mainly depends on the sign and size of the covariances between interest rates and weights differences. Based on the actual interest rates and loan ratios data, we have calculated that the difference between chain and direct indices for Armenia is relatively large due to the positive covariance between interest rates and weights differences. The chain and direct indices are almost the same for the Georgia and Kyrgyzstan because the covariances between interest rates and weights was negative, which eliminates the positive increase that we have at the end points. For Kazakhstan we have negative differences between chain and direct indices because the covariance is negative and it is larger in absolute terms than the values we have at the end points.

Thus, the differences between direct and chain indices depend on the size and sign of covariances between interest rates and weights differences. Based on our calculations, we found that the differences between direct and chain indexes are relatively small. Therefore, we can use both direct and chain indices in practice. On the other side, depending on the size and sign of the covariances, the difference can also be relatively large. Taking into account this point, we suggest using the Marshall-Edgeworth-type decomposition formula with an application to the chain index rather than to the direct index. This is because in the case of large differences, using the chain index would allow accounting for the evolution of the index number during the whole period and also accounting for changes in the weights.

7. Conclusions

This paper is an application of the index number theory to interest rates statistics on the basis of the work by Diewert (2005) using difference rather than ratios. This approach seems more appropriate for interest rates, for which changes are usually measured in absolute rather than in relative terms.

Following a building blocks approach, it has first examined the possible decompositions applicable to interest rates on the basis of adaptations of the axiomatic theory of index numbers. Additive decompositions that separate interest rate components and weight components were proposed, adapting the Laspeyres, Paasche, Marshall-Edgeworth and Walsh indices.

On the basis of the axiomatic analysis it has been concluded that the preferred decomposition is the Marshall-Edgeworth index formula, which separates increase in aggregate interest rates into an interest rate component and a weight component. We choose this index because this index is satisfied by most axioms. Also this index has several other advantages that we have presented in the text.

Thus, using Marshall-Edgeworth-type index we can decompose the changes in aggregate interest rates caused by interest rate component and a weight component. This decomposition can be done not only for two consecutive period of times but also for multiple period comparison. For that we consider chain and direct indexes for multiple period comparison. When we calculate these two indexes we see that there is a small differences, but in some cases the difference also could be relatively large. We calculate the chain and direct indexes using actual data on interest rates and loans volume for four post Soviet countries, particularly Armenia, Georgia, Kazakhstan and Kyrgyzstan. To explain the possible differences between chain and direct indexes we derive the exact formula, that can be used to calculate the size of difference and contribution of each period to the total difference. Based on this formula we conclude that the size of differences between chain and direct indices depend mainly on the covariances between interest rates and weights differences. Finally, we conclude that despite the fact that both direct and chain indices are reasonable and can be equally useful, we suggest using the Marshall-Edgeworth formula with chain index for multiple period comparison purposes.

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Appendix 1

For each corresponding time periods we have the volume of loans and their interest rates both in national and foreign currencies. Using this data, we can calculate the aggregate interest rate for loans using the following formula.

$$I_t = i_t^1 w_t^1 + i_t^2 w_t^2$$

Where I_t is an aggregate interest rate for loans (or time deposits) at time t ; i_t^1 and i_t^2 is the interest rates for loans in national and foreign currency at time t ; w_t^1, w_t^2 are the weights for loans in national and foreign currency at time t , where $w_t^1 = \frac{v_t^1}{v_t^1 + v_t^2}, w_t^2 = \frac{v_t^2}{v_t^1 + v_t^2}$.

Using the same formula we can calculate an average interest rate for $t-1$ period of time: $I_{t-1} = i_{t-1}^1 w_{t-1}^1 + i_{t-1}^2 w_{t-1}^2$. Then, we can calculate the change in interest rate as $\Delta I_{t/t-1} = I_t - I_{t-1}$. According to the Marshall-Edgeworth formula the change in total interest rate can be decomposed on the change in interest rate and weight components, that is $\Delta I_{t/t-1} = \Delta Int_{t/t-1} + \Delta Whg_{t/t-1}$, where

$$\begin{aligned} \Delta Int_{t/t-1} &= (i_t^1 - i_{t-1}^1) \left(\frac{w_t^1 + w_{t-1}^1}{2} \right) + (i_t^2 - i_{t-1}^2) \left(\frac{w_t^2 + w_{t-1}^2}{2} \right) \\ \Delta Whg_{t/t-1} &= (w_t^1 - w_{t-1}^1) \left(\frac{i_t^1 + i_{t-1}^1}{2} \right) + (w_t^2 - w_{t-1}^2) \left(\frac{i_t^2 + i_{t-1}^2}{2} \right) \end{aligned}$$

We are interested mainly in creating the interest rate component index $\Delta Int_{t/t-1}$. Hence, for this component we can write the following direct and chain index formulas.

$$\begin{aligned} I_d &= (i_1^T - i_1^1) \left(\frac{w_1^1 + w_1^T}{2} \right) + (i_2^T - i_2^1) \left(\frac{w_2^1 + w_2^T}{2} \right) \\ I_c &= (i_1^2 - i_1^1) \left(\frac{w_1^1 + w_1^2}{2} \right) + (i_1^3 - i_1^2) \left(\frac{w_1^2 + w_1^3}{2} \right) + \dots + (i_1^T - i_1^{T-1}) \left(\frac{w_1^{T-1} + w_1^T}{2} \right) + \\ & (i_2^2 - i_2^1) \left(\frac{w_2^1 + w_2^2}{2} \right) + (i_2^3 - i_2^2) \left(\frac{w_2^2 + w_2^3}{2} \right) + \dots + (i_2^T - i_2^{T-1}) \left(\frac{w_2^{T-1} + w_2^T}{2} \right) = \\ & \sum_{t=2}^T \sum_{k=1}^2 (i_k^t - i_k^{t-1}) \left(\frac{w_k^t + w_k^{t-1}}{2} \right) \end{aligned}$$

Now calculating $\Delta I_{cd} = I_c - I_d$ it will be possible to show how large are the differences between these indexes and at the same time we can get the contribution of each period of time in the total difference. Substituting instead of I_c and I_d we will get

$$\begin{aligned} \Delta I_{cd} = I_c - I_d &= (i_1^2 - i_1^1) \left(\frac{w_1^1 + w_1^2}{2} \right) + (i_1^3 - i_1^2) \left(\frac{w_1^2 + w_1^3}{2} \right) + \dots + (i_1^T - i_1^{T-1}) \left(\frac{w_1^T + w_1^{T-1}}{2} \right) + \\ &(i_2^2 - i_2^1) \left(\frac{w_2^1 + w_2^2}{2} \right) + (i_2^3 - i_2^2) \left(\frac{w_2^2 + w_2^3}{2} \right) + \dots + (i_2^T - i_2^{T-1}) \left(\frac{w_2^T + w_2^{T-1}}{2} \right) - (i_1^T - i_1^1) \left(\frac{w_1^1 + w_1^T}{2} \right) - \\ &(i_2^T - i_2^1) \left(\frac{w_2^1 + w_2^T}{2} \right) = i_1^2 \left(\frac{w_1^1 + w_1^2}{2} \right) - i_1^1 \left(\frac{w_1^1 + w_1^2}{2} \right) + (i_1^3 - i_1^2) \left(\frac{w_1^2 + w_1^3}{2} \right) + \dots + \\ &i_1^T \left(\frac{w_1^T + w_1^{T-1}}{2} \right) - i_1^{T-1} \left(\frac{w_1^T + w_1^{T-1}}{2} \right) + i_2^2 \left(\frac{w_2^1 + w_2^2}{2} \right) - i_2^1 \left(\frac{w_2^1 + w_2^2}{2} \right) + (i_2^3 - i_2^2) \left(\frac{w_2^2 + w_2^3}{2} \right) + \dots + \\ &i_2^T \left(\frac{w_2^T + w_2^{T-1}}{2} \right) - i_2^{T-1} \left(\frac{w_2^T + w_2^{T-1}}{2} \right) - i_1^T \left(\frac{w_1^1 + w_1^T}{2} \right) + i_1^1 \left(\frac{w_1^1 + w_1^T}{2} \right) - i_2^T \left(\frac{w_2^1 + w_2^T}{2} \right) + i_2^1 \left(\frac{w_2^1 + w_2^T}{2} \right) = \\ &i_1^1 \left(\frac{w_1^T - w_1^1}{2} \right) + i_1^2 \left(\frac{w_1^1 + w_1^2}{2} \right) + (i_1^3 - i_1^2) \left(\frac{w_1^2 + w_1^3}{2} \right) + \dots + (i_1^{T-1} + i_1^{T-2}) \left(\frac{w_1^{T-1} + w_1^{T-2}}{2} \right) - \\ &i_1^{T-1} \left(\frac{w_1^{T-1} + w_1^T}{2} \right) + i_1^T \left(\frac{w_1^{T-1} - w_1^1}{2} \right) + i_2^1 \left(\frac{w_2^T - w_2^1}{2} \right) + i_2^2 \left(\frac{w_2^1 + w_2^2}{2} \right) + (i_2^3 - i_2^2) \left(\frac{w_2^2 + w_2^3}{2} \right) + \dots + \\ &(i_2^{T-1} + i_2^{T-2}) \left(\frac{w_2^{T-1} + w_2^{T-2}}{2} \right) - i_2^{T-1} \left(\frac{w_2^{T-1} + w_2^T}{2} \right) + i_2^T \left(\frac{w_2^{T-1} - w_2^1}{2} \right) \end{aligned}$$

We know that $w_2^1 = 1 - w_1^1, w_2^2 = 1 - w_1^2, \dots, w_2^T = 1 - w_1^T$. Substituting these values instead of $w_2^1, w_2^2, \dots, w_2^T$ and rearranging the terms we will obtain the following results for chain and direct index differences decomposition.

$$\begin{aligned} \Delta I_{cd} = I_c - I_d &= (i_1^1 - i_2^1) \left(\frac{w_1^T - w_1^1}{2} \right) + (i_1^2 - i_2^2) \left(\frac{w_1^1 - w_1^3}{2} \right) + (i_1^3 - i_2^3) \left(\frac{w_1^2 - w_1^4}{2} \right) + \dots + \\ &(i_1^{T-1} - i_2^{T-1}) \left(\frac{w_1^{T-2} - w_1^T}{2} \right) + (i_1^T - i_2^T) \left(\frac{w_1^{T-1} - w_1^1}{2} \right) \end{aligned}$$

From the last equation we can see that

$$\frac{1}{2} \text{cov}(\Delta i_{1,2}^t, \Delta w_1^{t-1,t+1}) = (i_1^2 - i_2^2) \left(\frac{w_1^1 - w_1^3}{2} \right) + (i_1^3 - i_2^3) \left(\frac{w_1^2 - w_1^4}{2} \right) + \dots + (i_1^{T-1} - i_2^{T-1}) \left(\frac{w_1^{T-2} - w_1^T}{2} \right)$$

Taking into account this identity, the last equation can be rewritten as

$$\Delta I_{cd} = I_c - I_d = (i_1^1 - i_2^1) \left(\frac{w_1^T - w_1^1}{2} \right) + (i_1^T - i_2^T) \left(\frac{w_1^{T-1} - w_1^1}{2} \right) + \frac{1}{2} \text{cov}(\Delta i_{1,2}^t, \Delta w_1^{t-1,t+1})$$

From this formula we see that the size and sign of the ΔI_{cd} depends on the sign and size of covariance between $\Delta i_{1,2}$ and $\Delta w_1^{t-1,t+1}$. Also, it depends on the size and sign of the differences between interest rates and weights at the end points.