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The Neutral Interest Rate: Past, Present and Future. A Thematic Review.

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Abstract

The decline in safe real interest rates over the past three decades has reignited discussions on the neutral real interest rate, known as R^* . We review insights from the literature on R^* , addressing its determinants and estimation methods, as well as the factors influencing its decline and its future trajectory. While there is a consensus that R^* has declined, alternative estimation approaches can yield substantially different point estimates over time. The estimated neutral range is large and uncertain, especially in real-time and when comparing estimates based on macroeconomic data with those inferred from financial data. Evidence suggests that factors such as increased longevity, declining fertility rates and scarcity of safe assets, as well as income inequality, contribute to lowering R^* . Existing evidence also suggests the COVID-19 pandemic did not substantially impact R^* . Going forward, there is an upside risk that some pre-existing trends might weaken or reverse.

Topics: Interest rates; Monetary policy; Monetary policy framework

JEL codes: E43, E52, E62

Résumé

La baisse des taux d'intérêt réels sans risque au cours des trois dernières décennies a relancé les discussions au sujet du taux d'intérêt neutre réel, « R^* ». Nous effectuons un tour d'horizon des études publiées sur le taux neutre. Nous nous intéressons à ses déterminants et à ses méthodes d'estimation, de même qu'aux facteurs influant sur sa baisse et sa trajectoire future. Même s'il est généralement accepté que R^* a diminué, les diverses méthodes de calcul donnent des estimations ponctuelles considérablement différentes au fil du temps. La fourchette du taux neutre estimé est large et incertaine. C'est particulièrement le cas lorsqu'on la calcule en temps réel et qu'on compare les estimations fondées sur des données macroéconomiques avec les estimations déduites de données financières. Les résultats existants donnent à penser que des facteurs comme la longévité accrue, la baisse des taux de fécondité, la rareté des actifs sûrs et les inégalités de revenu contribuent à faire descendre le taux neutre. Ces résultats portent aussi à croire que la pandémie de COVID-19 n'a pas eu d'incidence substantielle sur R^* . Pour l'avenir, il existe un risque à la hausse que certaines tendances déjà présentes s'atténuent ou s'inversent.

Sujets : Cadre de la politique monétaire; Politique monétaire; Taux d'intérêt

Codes JEL : E43, E52, E62

1 Introduction

Over the past three decades, there has been a notable downward trend in safe real interest rates, in particular following the global financial crisis of 2008. This secular decline has brought back to the forefront of policy discussions a well-known concept in the monetary economics literature: the neutral (or natural) real rate of interest, R^* . Since the mid-2010s, R^* has gained prominence in central bank communications, with a rising number of speeches that mention it (see Borio, 2021). Following the monetary tightening cycle that started in 2022, policymakers and pundits have often referred to R^* as a benchmark to gauge the stance of monetary policy (IMF, 2023).

This thematic review gathers insights from the literature on R^* . It addresses four main questions: What determines R^* ? Since R^* is unobservable, how can we estimate it? Which factors can explain the decline in R^* ? And, finally, what is the future of R^* ? By addressing these questions, this review provides insights about the theoretical foundations of the neutral rate, its relationship with the existing estimation approaches, and the most important factors that shape the dynamics of R^* .

The main insights are the following:

1. There is a consensus that R^* has declined over the past 30 years. However, alternative estimation approaches can yield substantially different point estimates over time, especially when comparing estimates based on macroeconomic data with those inferred from financial data. The Bank of Canada currently uses a suite of models incorporating both reduced-form and structural estimation, and this approach helps balance out some of the risks associated with model misspecification.
2. Various forces likely contributed to the decline in R^* . There is consensus concerning the role of increased longevity, lower fertility rates, the scarcity of safe assets, and rising income inequality (especially in the United States).
3. The estimated neutral range is large and uncertain, particularly in real time. Similarly, the magnitude and impact of different drivers of R^* vary across studies. Reduced-form econometric estimates face two important challenges. First, they focus on the long-term real return on sovereign bonds, an imperfect proxy for R^* when the economy is at the zero lower bound ($R > R^*$) or during periods of sustained inflation trends. Moreover, many determinants of R^* , such as life expectancy, growth, and inequality, exhibited high degrees of comovement domi-

nated by a low-frequency component over the past half-century, resulting in multicollinearity (Kiley, 2020). Structural estimation based on general equilibrium models (e.g., Kuncl and Matveev, 2023, for Canada) provides a more transparent quantification of the drivers of R^* drivers.

4. Existing evidence suggests the COVID-19 pandemic did not have a substantial impact on R^* . However, there is an upside risk going forward. While some pre-existing trends that lowered R^* will persist over the next decade (e.g., declining fertility rates and increased longevity), other factors could exert upward pressure. These factors, among others, include a declining share of the working population, a higher supply of safe assets, the weakening of some underlying drivers of inequality, and the scaling up of the investment required to transition to a low-carbon economy. Overall, it is unlikely that R^* will fall below pre-pandemic estimates, and there is a meaningful risk that it could rise going forward (e.g., Beaudry, 2023).
5. Two main avenues for future research emerge. First, key elasticities crucial to the quantitative predictions of structural models remain weakly identified empirically. For instance, the country-premium elasticity to net foreign assets—the central parameter governing the influence of domestic versus global R^* drivers for a small open economy like Canada (e.g., Kuncl and Matveev, 2023)—is estimated imprecisely. Additionally, the use of microdata can help identify savings behavior across the age distribution (e.g., Mian, Straub, and Sufi, 2021), the impact of life expectancy on savings (Auclert, Malmberg, Martenet, and Rognlie, 2021), and wealthy households’ marginal propensity to save (e.g., the BoC HALO model). Second, further development of macro-finance estimation approaches can enhance the accuracy of R^* estimates and help reconcile the discrepancies between “macro-based” and “finance-based” estimates. Regarding the Bank of Canada’s estimation approaches, incorporating financial market information could offer fresh insights into the suite of models currently in use. For instance, the long-term component of the foreign exchange risk premium could provide information about the difference between the Canadian and U.S. neutral rates. Using exchange rate information could also aid in developing a unified framework for jointly estimating the Canadian and U.S. neutral rates.
6. Estimation uncertainty implies that R^* has limitations as a guidepost for monetary policy. While alternative measures, such as the output gap, are more suited to inform policy decisions, R^* remains an important communication tool for central banks. It can serve as a long-term

anchor for people’s expectations about the future tendency of the nominal interest rate—the sum of the neutral real rate and the central bank inflation target. A challenge going forward is to identify the most effective communication strategy to accomplish this goal.

We illustrate these conclusions by first discussing a benchmark theory of the determination of the neutral real rate in closed and open economies, the model of supply and demand for loanable funds (e.g., Blanchard, 2022). In this much-simplified view of the world, there is only one interest rate: the risk-free real interest rate (the nominal rate minus expected inflation). Absent any trade in goods or assets across countries (i.e., in a closed economy), the neutral rate equates saving to investment when output is at its potential level. As a result, factors that affect saving and investment propensity contribute to determining R^* . For a small open economy like Canada, the neutral rate equals the world neutral rate plus a country-specific premium. The elasticity of the country premium to changes in the country’s net foreign assets determines the importance of domestic factors for R^* . In the limiting case of an inelastic country premium (with perfect substitutability between domestic and foreign assets and when purchasing power parity holds), the small-open-economy neutral rate only depends on the global R^* .

Moving beyond this benchmark framework, we discuss recent work that investigates whether monetary policy may contribute to determining R^* . For instance, a role for monetary policy in determining R^* emerges in models with long-run money non-neutrality (e.g., Borio, Disyatat, and Rungcharoenkitkul, 2018; Fernandez-Villaverde, Marbet, Nuno, and Rachedi, 2023) or in models with multiple steady-state values for R^* (e.g., Beaudry, Kartashova, and Meh, 2023).

In the second part of the review, we discuss alternative approaches to estimate the unobservable R^* . Estimation strategies can be classified into two broad categories: empirical models and structural models. The former include (i) reduced-form models, (ii) semi-structural models, (iii) affine term-structure models, and (iv) macro-finance models. These alternative empirical approaches use different time series data and exploit different identifying assumptions to estimate R^* . Reduced-form models (either univariate or multivariate) estimate the trend component of a given long-term real rate (e.g., Del Negro, Giannone, Giannoni, and Tambalotti, 2017; Hamilton, Harris, Hatzius, and West, 2016; Lunsford and West, 2019). Semi-structural approaches estimate state-space models that exploit theoretical relationships from the benchmark New Keynesian model (e.g., Laubach and Williams, 2003). Term-structure models use financial data to infer R^* (e.g., Adrian, Crump, and Moench, 2013), while macro-finance approaches (e.g., Davis, Fuenzalida, and Taylor, 2019; Feunou and Fontaine, 2023) combine insights from approaches (ii) and (iii) above. Finally, a strand

of the literature relies on structural models that exploit general equilibrium theory to infer R^* (e.g., Cesa-Bianchi, Harrison, and Sajedi, 2022; Küncl and Matveev, 2023; Platzer and Peruffo, 2022; Rachel and Summers, 2019).

In the third part of the review, we delve into the factors that drive the dynamics of R^* . We discuss factors shifting saving and investment propensities, including demographic changes, the supply and demand of safe assets, income inequality, changes in market power, productivity growth, and the relative price of capital. We then summarize the existing evidence on the relevance of these competing forces.

In the last part of the review, we discuss the post-pandemic outlook and the most important sources of uncertainty about the future evolution of the neutral real rate.

2 Concept and Definitions

The neutral rate does not have a unique definition, as researchers and policymakers employ various concepts. Wicksell (1898) famously characterizes the natural rate of interest: “There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them.” A strand of the literature operationalized this notion by defining the neutral rate as the real interest rate consistent with the economy operating at its full potential once transitory shocks to aggregate supply or demand have abated (e.g., Laubach and Williams, 2003; Mendes, 2014). This definition takes a “longer-run” perspective, in that it refers to the level of the (short-term) real interest rate expected to prevail in the future, after the economy has emerged from any cyclical fluctuations and is expanding at its trend rate, given the economy’s current structural features.¹ This is the notion of R^* the Bank of Canada uses as an input into its monetary policy decision-making process, and it is the focus of this review.²

According to this definition, R^* is an unobservable equilibrium rate that operates in the medium to long run. It represents the real interest rate prevailing once transitory supply and demand shocks have dissipated. As a result, R^* can inform policymakers’ judgment regarding the future trajectory

¹An implication of this definition is that current estimates of R^* provide information about the real rate that will prevail when the output gap closes in the medium to long term. Other research focuses on a short-term concept of the natural rate of interest. For instance, Woodford (2003) defines the natural rate as the real interest rate that would prevail if there were no nominal rigidities (for example, price and wage stickiness). In this case, the natural rate is affected by business cycle shocks and, hence, is more volatile than the medium- to long-term neutral rate.

²Bank of Canada’s staff defines the *nominal* neutral interest rate (i^*) as “the policy rate needed to maintain output at its potential level and inflation at target after the effects of all cyclical shocks have dissipated” (Mendes, 2014). The neutral *real* rate corresponds to the difference between the nominal neutral rate and the inflation target.

of the policy rate, providing a longer-run anchor for their decisions.

At times, R^* is also used to assess the stance of monetary policy. The logic is as follows: When the policy rate is below the neutral estimate, monetary policy is considered accommodative. This suggests that there is economic slack, leading to the expectation of declining unemployment and inflation. In the opposite scenario, monetary policy is deemed restrictive. It is important to note that the medium-to-long-term concept of R^* does not inform policymakers about the real rate that is warranted by current economic conditions. The reason is that setting the current real interest rate equal to R^* does not guarantee a neutral stance in the presence of cyclical head or tailwinds.³

3 Insights from Macroeconomic Theory

The simplest framework used to understand the determination of the neutral real rate (R^*) is the supply and demand model for loanable funds. Following Blanchard (2022), consider a closed economy representing the entire world economy. Denote with R the safe rate, S the aggregate saving propensity, I the aggregate investment propensity, and Y^* the potential output. The supply of loanable funds is an upward-sloping function of the interest rate, since higher returns to saving boost the supply of savings from households, businesses, and the government. The demand for loanable funds is downward sloping, reflecting the response of borrowers to higher funding costs. The risk-free neutral real rate (R^*) is determined when desired saving and investment are equal, and world output is at its potential level. Figure 1 summarizes this equilibrium condition. Factors that affect the propensity to save or invest determine the neutral real rate and its evolution. We discuss these forces in Section 5.

Before extending the framework to an open economy, notice that, in practice, there are many different interest rates and rates of return: safe/risky, short/long, rates on corporate/government bonds, and rates of return on equities, housing, and commodities. The loanable funds model allows for a distinction between safe and risky rates, assuming that saving depends on the safe rate and investment depends on the risky rate—the safe rate plus a risk premium (Blanchard, 2022). In turn, an increase in the risk premium (due either to an increase in risk aversion or an increase in the risk itself) reduces the risk-free neutral rate by shifting the investment curve to the left.⁴

³The optimal rate is the short-term interest rate prescribed by optimal monetary policy, which is defined as the interest rate plan that maximizes a welfare criterion. In the standard version of the New Keynesian model, it corresponds to the real rate that would prevail without nominal rigidities and any real distortions (e.g., imperfect competition) that deviate allocations from their efficient level (e.g., Woodford, 2003).

⁴A similar argument holds for liquidity. If we think of the asset paying the safe rate as more liquid, an increased

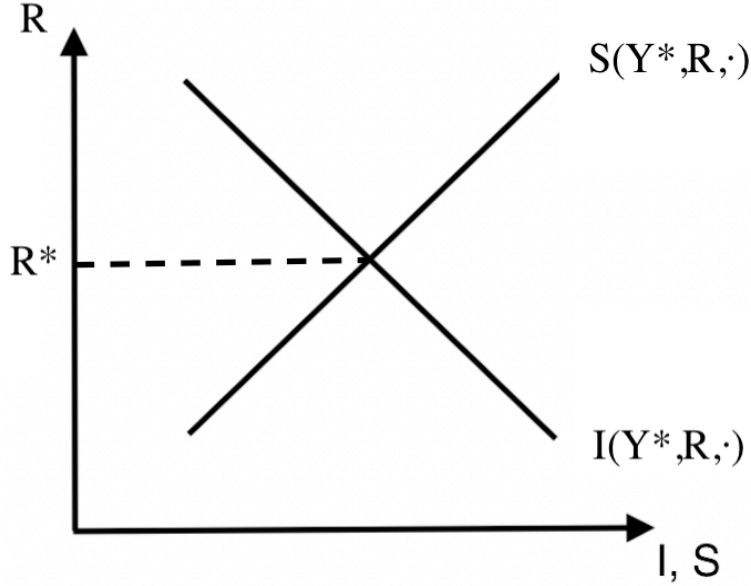


Figure 1: Loanable funds model: Equilibrium in the world economy (closed economy).

Consider now a small open economy like Canada. Mundell (1963) implies that with perfect international capital mobility, perfect substitutability between domestic and foreign assets, and purchasing power parity, the risk-free neutral real rate (R^{*C}) is equal to the risk-free neutral real rate of the world economy (R^{*W}); that is, in this case, only global factors would determine R^{*C} :

$$R^{*C} = R^{*W}. \quad (1)$$

Departure from the conditions above result in deviations from the interest parity condition (1). In this case, both domestic and global forces affect the neutral real rate in the small open economy:

$$R^{*C} = R^{*W} + \rho^C,$$

where ρ^C denotes a country-specific premium that captures deviations between the world and domestic neutral real rates. Changes in ρ^C reflect domestic trends or global developments that affect the small open economy asymmetrically relative to the rest of the world. The left-hand panel of Figure 2 represents the equilibrium in the world economy such that desired global savings

demand for liquidity will lead to a lower safe (liquid) rate and a higher rate on the less liquid one—see again Blanchard (2022).

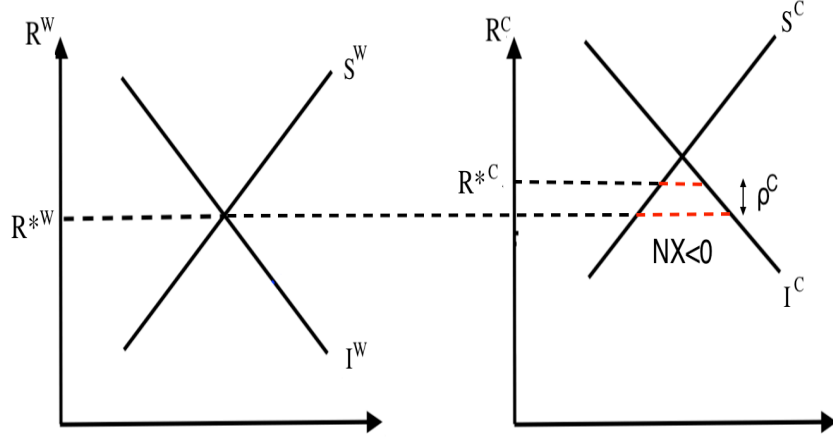


Figure 2: Loanable funds model: Equilibrium in the world economy (left-hand panel) and in a small open economy (right-hand panel).

equal desired global investment. The right-hand panel depicts the equilibrium for the small open economy, where $R^{*C} \neq R^{*W}$ because of the country-specific premium ρ^C .

The importance of domestic factors in shaping R^{*C} ultimately depends on the elasticity of the country's premium to changes in net foreign assets. When the country premium is perfectly inelastic, domestic shocks do not affect R^{*C} . In this case, the small open economy's net foreign assets adjust to preserve interest rate equalization. Consequently, a change in the world neutral rate, R^{*W} , would also have a one-for-one impact on R^{*C} . In contrast, when the country premium is elastic, domestic shocks affect ρ^C and, thus, R^{*C} . The greater the elasticity, the larger the effect.

4 Estimation Approaches

How can we estimate the unobservable R^* ? In this section, we present an overview of the main estimation approaches. These can be classified into two broad categories: empirical models and structural models. The former include (i) reduced-form models, (ii) semi-structural models, (iii) term-structure models, and (iv) macro-finance models. Both econometric and structural approaches provide valuable insights into the determination of R^* . Our goal is to highlight the role of the different identifying assumptions embedded in these alternative estimation strategies.

We note that the Bank of Canada publishes an annual assessment of the Canadian nominal neutral rate, using estimates from a set of four distinct but complementary approaches: (i) an interest rate parity equation; (ii) a reduced-form model; (iii) a risk-augmented neoclassical growth

model; and (iv) a structural model featuring overlapping generations (Kuncl and Matveev, 2023). The first approach assumes the Canadian neutral rate is equal to the world neutral rate. Approach (ii) provides a reduced-form estimate of R^{*C} , while models (iii) and (iv) are structural.⁵

Reduced-Form Models

Reduced-form models infer the neutral real rate by assuming it corresponds to the trend of short-term safe real rates. This can be achieved through the use of moving averages of the policy rate over long time spans (e.g., Hamilton, Harris, Hatzius, and West, 2016; Lunsford and West, 2019) or univariate filters.⁶ As noted by Kiley (2020), the fact that the estimation of an underlying trend requires a moving average of such a long time period underscores how the data will have trouble informing assessments of the degree of variation in the equilibrium real interest rate. In other words, the sample mean of the actual real interest rates would provide an estimate of R^* only if the neutral rate were close to constant over time. Moreover, while univariate time series methods may work well at estimating the neutral rate when inflation and economic activity are relatively stable, they are likely to be less reliable in periods of sustained high inflation (Laubach and Williams, 2016). For instance, in the U.S. during the late 1960s and much of the 1970s, de-trending methods likely underestimated R^* .

Univariate models abstract from the relationships among economic variables. Del Negro, Giannone, Giannoni, and Tambalotti (2017) estimate a VAR with common trends to extract the permanent component of the real interest rate from data on nominal bond returns and inflation and surveys on long-run expectations for the real interest rate. Their framework corresponds to a multivariate trend-cycle decomposition in which the shocks affecting the trend and the cycle are orthogonal to one another; that is, an independent trend/cycle decomposition.

Semi-Structural Models

Semi-structural models introduce identifying assumptions derived from macroeconomics theory. Seminal work by Laubach and Williams (2003)—LW henceforth—builds a multivariate model that takes into account movements in inflation, output, and interest rates to inform the estimation of R^* . The model features a parsimonious specification, assuming the neutral rate depends on two

⁵Carter, Chen, and Dorich (2019), Mendes (2014), and Kuncl and Matveev (2023) provide in-depth review of the various approaches.

⁶The reduced-form model used by the Bank of Canada links the long-term Canadian interest rate to the long-term U.S. interest rate and the growth rate of Canadian potential output (see Mendes, 2014).

unobservable variables: the trend growth rate of potential output, g_t^* , and a residual component, z_t :

$$R_t^* = cg_t^* + z_t,$$

where c is a coefficient that measures the influence of the trend growth rate on R_t^* . The term z_t captures various underlying structural factors that affect saving and investment imbalances over and beyond trend growth. Both g_t^* and z_t are assumed to follow random walks so that R^* and g^* can permanently diverge.

LW add further structure to the model by assuming (i) a backward-looking IS curve relating the output gap to its own lags and the lagged “real rate gap,” the difference between the actual real interest rate and the neutral rate; (ii) an accelerationist Phillips curve; and (iii) an underlying process for potential output growth. The IS curve informs the output gap prediction for the next period. If the output gap turns out to be lower than expected, the model reduces the estimated neutral rate. The output gap estimate, in turn, relies on the estimated Phillips curve. If inflation turns out to be lower than predicted by the existing estimates for potential output, the level of the potential output is revised up (that is, for a given level of real GDP, the output gap is revised down).

To summarize, the model aims to separate the trend component in the real rate from its transitory fluctuations, implicitly defining the neutral rate by the absence of inflationary or deflationary pressures. The model is estimated using the Kalman filter, which adjusts estimates of the unobserved variables (the neutral rate, potential output, and its trend growth rate) based on the distance between the model’s predictions for real GDP and inflation and their actual outcomes.

Despite its popularity, the semi-structural approach face some challenges. First, estimates are subject to substantial statistical uncertainty. Standard errors can span 2.5 percentage points, making it hard to ascertain whether current interest rates are above or below the neutral rate (e.g., Borio, 2021). Second, due the random walk assumption, the forecast uncertainty about R^* increases with the forecasting horizon (Feunou and Fontaine, 2023).

Third, Fiorentini, Galesi, Peez-Quiros, and Sentana (2018) show that the precision of the estimates falls when either the output gap is insensitive to the real interest rate gap (a flat IS curve), or when inflation is insensitive to the output gap (a flat Phillips curve). In these cases, which are empirically relevant according to a wide set of estimates reported in the literature, it is harder to identify the unobserved growth and non-growth components of R^* .

Fourth, the estimates are sensitive to various aspects of the model specification. On one side, estimates are potentially affected by omitted variable bias. Moreover, as discussed by Clark and Kozicki (2005), real-time estimates can be distorted by one-sided filtering (such as the Kalman filter): Time- t estimates of unobservable variables do not use information after period t , and real-time estimates can be subject to potentially large revisions, muting the practical relevance for policy application. Finally, since the model adopts a parsimonious specification regarding the factors influencing R^* (productivity growth versus unspecified other factors), there is limited insight into the drivers of R^* over time.

Term-Structure Models

Semi-structural models rely only on past macroeconomic data. An alternative approach is to exploit forward-looking financial market variables. In particular, longer-term yields contain information about the short-term real rate prevailing in the medium term. As a result, models that estimate bond risk premia using Treasury yields can be used to infer R^* .

To gain intuition, assume that R_t^* is the common trend among safe real rates of different maturities. Denote the forward nominal interest rate at horizon n on a safe asset with maturity m by $f_t^{(n,m)}$. Then the following identity holds:

$$f_t^{(n,m)} \approx R_t^* + \pi_t^* + \Gamma_t^{(n,m)},$$

where π_t^* is the (expected) trend inflation and $\Gamma_t^{(n,m)}$ is the term premium.

Models of the term structure of interest rates provide an estimate of $\Gamma_t^{(n,m)}$ (e.g., Adrian, Crump, and Moench, 2013). In turn, $\Gamma_t^{(n,m)}$ can be subtracted from the observed forward rate $f_t^{(n,m)}$ to get an estimate of the nominal neutral rate, $i_t^* \equiv R_t^* + \pi_t^*$, which can then provide an estimate of R_t^* for a given estimate of π_t^* .

Davis, Fuenzalida, and Taylor (2019) document that workhorse finance models imply dynamics of the neutral rate that are at odds with the findings in the macro literature. Estimates of i_t^* implied by term-structure models differ substantially from the estimates from the Laubach and Williams (2003) model. At the same time, bond risk premia implied by semi-structural macro models—obtained by subtracting the estimated i_t^* from the observed forward rate $f_t^{(n,m)}$ —are also at odds with the estimates obtained using interest rate term structure models (e.g., Adrian, Crump, and Moench, 2013). Davis, Fuenzalida, and Taylor (2019) label these two stylized facts as the

neutral rate puzzle.

Macro-Finance Models

The discrepancy between macro- and finance-based R^* estimates is somewhat troubling. Whom should policymakers trust? Both approaches have merits and limitations. On one hand, models of the term structure of interest rates use forward-looking data and incorporate high-frequency (daily) interest rate variations. However, they lack economic interpretation and do not exploit information from macroeconomic data and theory. On the other hand, semi-structural models rely on structural relationships but do not incorporate forward-looking information contained in financial data and can only be estimated at lower frequencies (e.g., quarterly).

Recent literature makes progress in combining the strength of the two approaches, therefore mitigating their respective shortcomings. Some important contributions include Feunou and Fontaine (2023); Davis, Fuenzalida, and Taylor (2019); Brand, Goy, and Lemke (2021). These so-called macro-finance models consist of two building blocks. Similar to semi-structural macro models (such as LW), they depict the dynamics of both the trend and cyclical components of inflation, GDP, and the real short rate. The trend components follow univariate random walk processes, while the cyclical variables adhere to stationary vector autoregressions. Additionally, the macro-finance approach uses affine term-structure models of interest rates to connect long-maturity nominal interest rates with macro factors. Intuitively, the financial block incorporates observed interest rates at different maturities to model deviations from the expectation hypothesis. As term premia are also influenced by macro factors (trends and cycles), they offer valuable additional information for deducing the dynamics of unobserved macroeconomic variables.

Neutral rate estimates implied by macro-finance approaches are generally lower relative to semi-structural models, especially post-2008. They also exhibit lower statistical uncertainty and better statistical fit. These results reflect the inclusion of more observables and the introduction of time-varying volatility, as demonstrated by Feunou and Fontaine (2023).⁷ However, further research is still necessary to refine the integration of macro and finance approaches to R^* estimation. Important avenues for future research include the use of mixed-frequency estimation, which combines quarterly macro data with higher-frequency financial market data, as well as the incorporation of additional

⁷Models like LW assume that the variance of innovations in the trend variables is constant. The combination of constant variance and the random walk assumption implies that the forecast uncertainty around R^* increases linearly with the forecast horizon. Feunou and Fontaine (2023) show that time-varying volatility in the trend components reduces the statistical uncertainty and noise in the estimates of R^* .

financial data such as equity and corporate bond prices. Moreover, further development of macro-finance approaches should also address the modeling of the underlying forces driving R^* , which current frameworks largely treat in reduced form.

Structural Models

Structural models provide another approach to estimate R^* . They use general equilibrium theory to analyze the long-run behavior of real interest rates and to understand the fundamental factors driving R^* . Most studies focus on the global neutral rate (e.g., Cesa-Bianchi, Harrison, and Sajedi, 2022; Ferreira and Shousha, 2020; Obstfeld, 2021; Arslanalp, Lee, and Rawat, 2018) or consider a large, closed economy, such as the United States (e.g., Platzer and Peruffo, 2022; Del Negro, Giannone, Giannoni, and Tambalotti, 2017). Kuncel and Matveev (2023)—henceforth, KM—build a small open economy model for Canada, accounting for both the impact of the global neutral rate and several domestic factors.

For brevity, we discuss the features of the model by KM since it shares several ingredients common to other closed-economy structural models. The economy is populated by overlapping generations of heterogeneous households (within each cohort). Perfectly competitive firms use capital and labor to produce goods sold domestically and abroad. Purchasing power parity holds, which implies a constant real exchange rate. The government purchases goods for public consumption and makes transfer payments to households. Government spending is financed either by collecting taxes or through public borrowing. There is no aggregate or idiosyncratic uncertainty.⁸

In the model, the Canadian neutral real rate, R^{*C} , is a combination of the global neutral rate, R^{*W} , and an endogenous country premium that is determined by domestic factors that affect the small open economy’s net foreign asset position, NFA^C , relative to output, Y^C :

$$R^{*C} = R^{*W} + \rho \left(\frac{NFA^C}{Y^C} \right).$$

The country premium, $\rho(\cdot)$, arises due to imperfect asset substitutability between Canada and the rest of the world. Purchasing power parity holds and there are no barriers to international capital flows.⁹ The domestic factors affecting $\rho(\cdot)$ include total factor productivity, the relative price of capital, the labor force, longevity, inequality, and government debt. The relative importance of

⁸The suite of models used by the Bank of Canada also includes a closed-economy neoclassical growth model with a risky stochastic steady state (Carter, Chen, and Dorich, 2019). This approach recognizes that risk can affect the neutral rate as agents demand safe assets to better insure themselves.

⁹In the extreme case of perfect asset substitutability, the country premium converges to zero.

domestic factors in shaping R^{*C} depends on the elasticity of the country premium to the net-foreign-asset position. In the model simulations, KM highlight the sensitivity of the results to such an elasticity by considering a high-elasticity and a low-elasticity scenario.¹⁰

Figure 3 plots the Canadian neutral real rate implied by the KM model for the period 1980-2018. The solid line represents the scenario with a high country-premium elasticity, while the dashed line represents the low-elasticity scenario. The figure also plots the average estimate of the U.S. neutral rate used as an input by the KM model (see KM for additional details) and the official neutral range published by the Bank of Canada (shadow bars). The figure illustrates that in the low-elasticity case, R^{*C} exhibits smoother fluctuations and a stronger correlation with the U.S. neutral rate. In the high-elasticity scenario, domestic factors play a more significant role, leading to larger fluctuations.

Structural models like the one developed by KM embed several plausible transmission channels. However, structural estimation of R^* also presents challenges. First, the empirical discipline concerning key model elasticities is central to the credibility of the model estimates. The use of disaggregated data can aid in identification. For instance, recent work demonstrates how microdata can discipline saving behavior across the age distribution (Mian, Straub, and Sufi, 2021) and the impact of life expectancy on saving patterns (Auclert, Malmberg, Martenet, and Rognlie, 2021). In addition, by analyzing the marginal savings propensity of wealthy households, models like the Bank of Canada’s HALO also contribute to a better understanding of individuals’ financial choices. Second, there are additional challenges for open-economy models. As previously discussed, the elasticity of the country premium to net foreign assets plays a central role in R^{*C} dynamics. However, obtaining reliable estimates for this elasticity is challenging: Time series data provide weak identification, while cross-country estimates impose homogeneity restrictions that could bias estimates for specific economies. Furthermore, the assumption of purchasing power parity is plausible only over long horizons and less so over a five- to ten-year period. Abstracting from real exchange rate dynamics can impact the estimates of the neutral rate. Finally, several models (including KM) assume perfectly competitive firms, thus assuming that markups (and their variation over time) play no role for R^{*C} . Future research should explore these directions.

¹⁰In the former, the elasticity of the country premium to net foreign assets is \$0.016, whereas in the latter, the value is \$0.003. KM also estimate this elasticity using data on 10-year government bonds for both Canada and the United States.

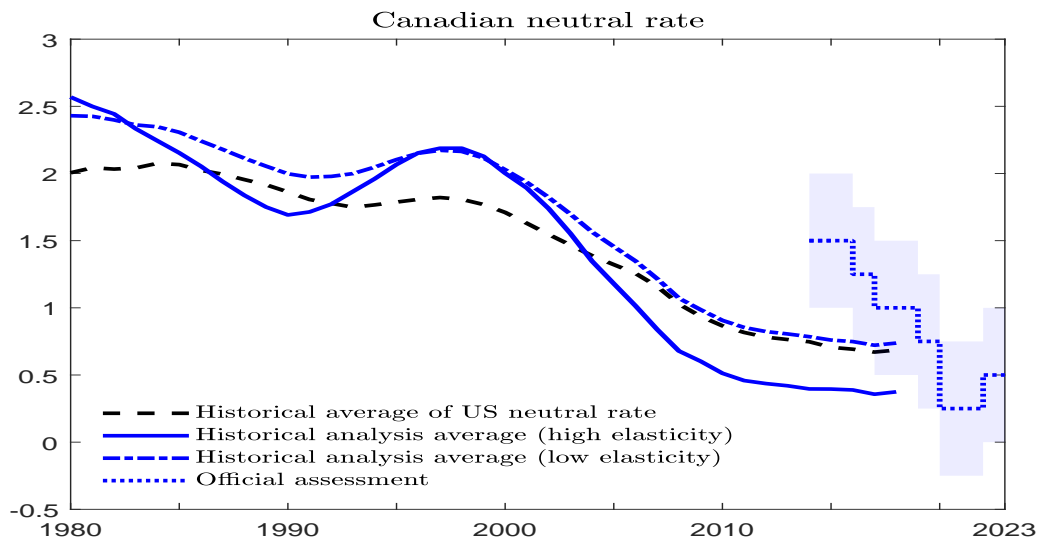


Figure 3: Estimates of the Canadian neutral real rate, high country-premium elasticity (continuous line) and low country-premium elasticity (dotted line); average estimate of the U.S. neutral rate (dashed line); official Bank of Canada neutral range (shadow bars). *Source:* Kuncel and Matveev (2023) and authors' calculations.

5 The Decline in the Neutral Rate: Driving Forces

The decline in the neutral rate has sparked a substantial amount of research. Many studies examine the neutral rate going back forty years; some even cover longer periods. Most of these studies concentrate on the United States or the global economy. Kuncel and Matveev (2023) study the evolution of the Canadian neutral rate.

The model of the supply and demand for loanable funds, discussed in Section 2, implies that a positive shift in desired saving (an outward shift in the supply of savings) leads to higher saving and investment, along with a lower neutral rate. Conversely, an inward shift in the investment curve results in a lower neutral real rate.

Blanchard (2022) points out that while R^* has declined in the past 30 years, the gross saving rate for the world economy has remained fairly stable. Since this saving rate represents an equilibrium outcome, Blanchard's analysis suggests that to the extent there were positive shifts in saving, these were accompanied by adverse shifts in investment to explain a roughly unchanged saving rate along with a decrease in R^* . Building on this insight, various approaches were employed to identify the primary drivers of R^* .

International Context

Various factors may have contributed to increasing the propensity to save and decreasing the propensity to invest at the world level. Potential candidates for higher desired savings include demographic trends and inequality dynamics. Higher longevity, which translates into higher life expectancy while holding working years constant, should raise aggregate savings in anticipation of longer retirements. At the same time, shifts in the aggregate age distribution due to the baby boom generation resulted in a larger fraction of the population in the working age group. Assuming that saving rates decline with age, this demographic shift would increase aggregate savings, all else being equal. Finally, rising inequality could boost savings relative to GDP, as the marginal propensity to consume among the rich is lower than that among other segments of the population.

A substantial amount of research addresses the importance of these potential candidates. Approaches vary, encompassing both reduced-form econometric analysis (ranging from correlation analysis to vector autoregressions) and calibrated structural models. Across studies and methodologies, there is consensus about two forces increasing the propensity to save: increased longevity (Lunsford and West, 2019; Eggertsson, Mehrotra, and Robbins, 2019; Brand, Goy, and Lemke, 2021) and rising income inequality in the United States (e.g., Rachel and Smith, 2015; Mian, Straub, and Sufi, 2021). In contrast, there is mixed evidence regarding the impact of shifts in the aggregate age distribution, such as the baby boom generation. While Lunsford and West (2019) document a significant relationship, microdata evidence from Mian, Straub, and Sufi (2021) shows that the saving propensity is fairly stable across the age distribution.

Regarding the investment propensity, potential drivers include slower economic growth, lower fertility rates, declining prices of capital goods, higher demand for safe assets, and rising markups. A slower trend in productivity growth reduces the required increases in the capital stock, consequently lowering investment demand and interest rates. Lower fertility rates reduce labor force growth (the supply of labor), which, in turn, decreases capital stock growth. A decline in the relative price of investment goods means that the same investment projects can be pursued by committing a smaller share of GDP, resulting in less investment spending competing for the same savings. A higher demand for safe assets, driven by factors such as risk aversion and regulatory requirements, leads to a higher premium on risky capital, which, all else being equal, lowers investment. Lastly, higher monopoly power (and markups) reduces the demand for production factors and investment.

Recent research (e.g., Lunsford and West, 2019; Eggertsson, Mehrotra, and Robbins, 2019;

Brand, Goy, and Lemke, 2021) supports the view that lower fertility rates contribute to a lower R^* . There is also consensus on the role played by the shortage of safe assets (e.g., Bullard, 2017; Del Negro, Giannone, Giannoni, and Tambalotti, 2017; Farhi and Gourio, 2019; Ferreira and Shousha, 2020). Results are less robust concerning productivity growth and the declining relative price of capital. Empirical work typically does not find a significant impact (e.g., Hamilton, Harris, Hatzius, and West, 2016), whereas structural models attribute a significant impact to these forces (e.g., Cesa-Bianchi, Harrison, and Sajedi, 2022). Concerning markups, structural models support the view that higher monopoly power leads to lower investment and a lower R^* , although there has not yet been an empirical investigation of this channel.

Canada

Concerning Canada, KM's model decomposes the variation in the Canadian R^* into portions driven by the global neutral rate and domestic factors. The latter consists of trend labor productivity (TLP), trend labor input (TLI), life expectancy at age 65, gross government debt to GDP, and the average after-tax income share of the bottom 80% of the lowest-income households. For the period 1980-2018, KM's baseline model calibration implies that less than 25% of the change in the Canadian neutral rate can be attributed to changes in the U.S. neutral rate. The most significant domestic factors are the increase in the TLI and life expectancy in the Canadian population. The increase in Canadian net government debt partly compensates for the decline in R^* , although the effect is estimated to be quantitatively smaller than all other factors except inequality. In contrast to the predictions of closed-economy models calibrated to U.S. data, KM's model attributes almost no role to changes in inequality in explaining the downward trend of the Canadian neutral rate. When considering the same decomposition under the assumption of a low country-premium elasticity, the contribution of changes in the U.S. neutral rate doubles.

Taking Stock

While there is qualitative consensus about the most important drivers of R^* —including increased longevity, lower fertility rates, the scarcity of safe assets, growing income inequality (particularly in the United States), and rising market power—there is no quantitative agreement regarding the relative contribution of each factor. The results depend on the specific approach, such as reduced-form analyses versus structural models.

Reduced-form econometric estimates face significant challenges. First, many potential determi-

nants of the equilibrium real interest rate, such as life expectancy, underlying growth, and inequality, exhibited high degrees of comovement dominated by a low-frequency component over the past half-century (Kiley, 2020). As a result, empirical assessments are plagued by multicollinearity. Second, most studies identify the determinants of R^* by studying the drivers of long-term real returns on sovereign bonds (e.g., the difference between 10-year nominal rates and 10-year inflation forecasts). Such a measure is an imperfect proxy for the neutral real rate when the economy is at the zero lower bound ($R > R^*$) or during periods of sustained inflation trends. Third, there could be potentially omitted variables, especially when financial variables are not considered. Finally, the steady-state or long-run relationships highlighted by the loanable funds model can result in complicated dynamics playing out over decades. This reflects, for example, life-cycle effects associated with demographic transitions and the slow movement in the capital stock in response to persistent shifts in saving and investment. The use of low-frequency aggregate data, available only for relatively short time periods, can hinder the correct identification of the complex effects at play.

Structural estimation based on general equilibrium models (e.g., Küncl and Matveev, 2023, for Canada) provides a more transparent quantification of the drivers of R^* . However, structural models face the possible risk of misspecification, and key elasticities crucial to their quantitative predictions remain weakly identified empirically. For instance, the country-premium elasticity to net foreign assets—the central parameter governing the influence of domestic versus global drivers of R^* for a small open economy like Canada—is estimated imprecisely. Additionally, the use of microdata can help identify savings behavior across the age distribution (e.g., Mian, Straub, and Sufi, 2021), the impact of life expectancy on savings (Auclert, Malmberg, Martenet, and Rognlie, 2021), and wealthy households’ marginal propensity to save (e.g., the BoC HALO model).

Beyond the Conventional View: The Role of Monetary Policy

So far, we have reviewed studies that focus on real factors determining desired savings and investment. More-recent literature discusses the potential role of monetary policy. For instance, Beaudry, Kartashova, and Meh (2023) show that if the long-run asset demand is C-shaped—meaning that below a certain interest rate level, lower interest rates motivate households to increase their asset-to-income ratios—multiple steady-state real interest rates emerge (see Figure 4). In this context, monetary policy can influence the long-run evolution of real rates, even though money is neutral in the long run. The framework highlights how an inflationary episode followed by a strong monetary response can have long-term implications for real interest rates.

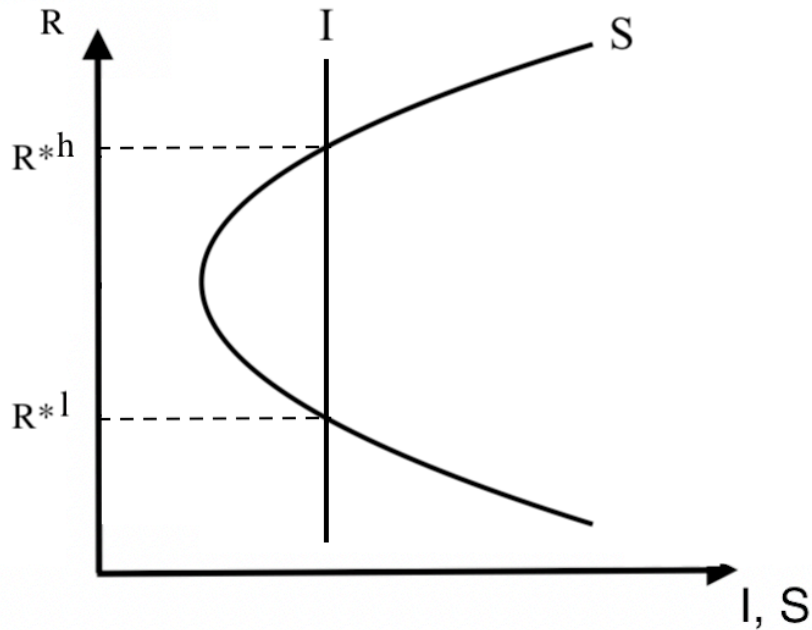


Figure 4: Loanable funds model: Equilibrium with a C-shaped asset demand.

A second explanation for why monetary policy can affect R^* is long-run money non-neutrality. Borio, Disyatat, and Rungcharoenkitkul (2018) discuss how loose monetary policy can fuel inefficient credit growth and resource misallocation. Fernandez-Villaverde, Marbet, Nuno, and Rachedi (2023) show that in the presence of households' heterogeneity and inequality, a drop in the inflation target reduces the long-run real interest rate, as households increase precautionary saving against the higher risk of ZLB events.

The possibility that R^* is endogenous to monetary policy over the relevant policy horizon raises practical questions about the use of the natural interest rate as a monetary policy guidepost, questioning its ability to act as a policy anchor. While more empirical research is warranted, existing work with macroeconomics and financial data supports the view that R^* is primarily driven by real factors (e.g., Feunou and Fontaine, 2023; Schmitt-Grohe and Uribe, 2022).

6 The Future of R^*

Will R^* remain low in the post-pandemic world? This question has garnered significant attention in policy debates since the start of the monetary tightening cycle in 2022. Existing evidence suggests that the COVID-19 pandemic did not significantly impact the neutral real rate. For instance, the International Monetary Fund reported no evidence of changes to the neutral rates of advanced economies compared with pre-pandemic estimates (IFM, 2023). Updated R^* estimates from the VAR model in Del Negro, Giannone, Giannoni, and Tambalotti (2017) trend down slightly after the pandemic.¹¹ However, given the difficulties in estimating R^* , the future outlook remains uncertain.

Ultimately, future R^* dynamics will depend on structural developments that influence saving and investment decisions. Some pre-COVID trends are unlikely to reverse, given the persistence of increasing longevity, declining fertility rates, and rising markups. However, some forces may change course, exerting upward pressure on R^* . Overall, it is unlikely that the neutral real rate will fall below pre-pandemic estimates, and there is a meaningful risk that it could increase going forward (e.g., Beaudry, 2023). We now briefly discuss some forces that may exert upward pressure on R^* .

Demographic Trends

Two demographic trends that exert downward pressure on the neutral rate are expected to persist over the next decade. Figures 5 and 6 indicate that global life expectancy is projected to rise while fertility rates are projected to decline further.

However, some argue that the global economy may experience a “demographic reversal” in the coming decades, potentially reversing the decline in real interest rates. The reversal pertains to a new phase characterized by an increasing dependency ratio—the ratio of individuals not in the labor force (e.g., ages 0 to 14 and 65+) to those in the labor force (e.g., ages 15 to 64). The dependency ratio has been declining in advanced economies since the 1960s, reflecting the entry of baby boomers into the labor force and the decrease in fertility rates. In other words, the ratio of working-age individuals increased significantly in relation to the dependent young (due to falling birth rates) and the elderly (because the population was still growing rapidly and longevity increased more slowly). This ratio is now projected to increase in both advanced and emerging countries. Goodhart and Pradhan (2020) argue that such a reversal will ultimately reduce aggregate savings because of

¹¹See <https://libertystreeteconomics.newyorkfed.org/2023/08/the-post-pandemic-r>. Estimates based on their DSGE model display an upward trend. In the latter case, short-run fluctuations have a more significant impact on R^* , whereas in the trendy VAR (which is based on a trend-cycle decomposition), this is not the case.

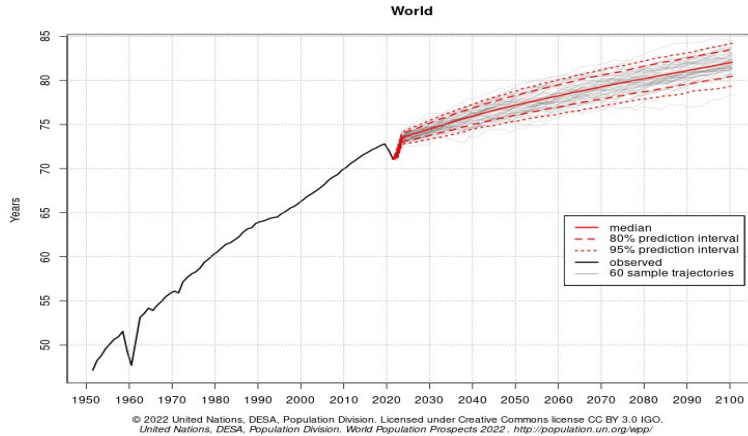


Figure 5: World life expectancy, United Nations.

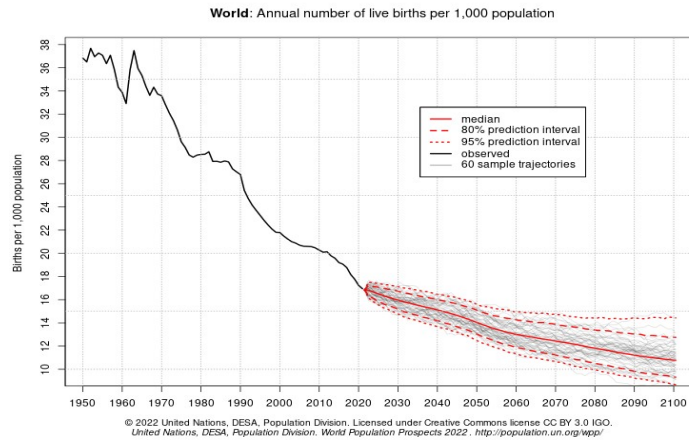


Figure 6: World fertility rate, United Nations.

increasing health-related expenses (financed with new public debt or transfers from the working-age population) and because of elderly dissaving. However, others argue that higher longevity will remain the most important demographic force, increasing precautionary savings and the demand for safer portfolios (e.g., Blanchard, 2022). Moreover, Auclert, Malmberg, Martenet, and Rognlie (2021) argue that aging will lower desired investment and, consequently, R^* .

Supply and Demand of Safe Assets

Since the COVID-19 pandemic there has been a significant and enduring increase in public debt in advanced economies. Theoretically, higher public debt raises R^* by increasing the supply of safe

assets.¹² Using estimates of the impact of deficits on interest rates, Rachel and Summers (2019) calculate that an increase in the public debt-to-GDP ratio from 18% to 68% in advanced economies over the last four decades should have raised real rates by between 1.5 and 2 percentage points. However, structural models are less conclusive on the quantitative importance of public debt on interest rates. Model predictions are sensitive to specific model assumptions, such as insurance considerations in the presence of idiosyncratic income risk.

Population aging and China’s focus on consumption-led growth may also reduce the saving propensity and the demand for safe assets (consistent with the observed lower growth of international reserves in several emerging market economies). Additionally, geopolitical pressures may hinder some countries, making them less inclined or capable of directing aggregate savings into the global financial system.

Transition to a Low-Carbon Economy

Another factor that may contribute to increasing the neutral rate over the next year is green investment. Climate transition demands a substantial expansion of renewable energy sources to replace coal and gas, and clean-energy investments have largely been delayed. Recent data indicates that a threefold increase is necessary by 2030 to scale up successful decarbonization and electrification (IMF, 2023). A well-timed green investment plan can potentially boost interest rates, as discussed by Blanchard (2022). However, counteracting this shift towards green investments is a concerning reduction in “brown investment” and precautionary motives associated with the risks involved in the transition.

Inequality Trends

Some of the underlying drivers of inequality could also be waning. For example, globalization may be stalling relative to the pace we saw in the 2000s and geopolitical pressures could even send it into reverse. This could lead to less inequality within advanced economies, where the benefits of globalization have generally not been shared evenly (e.g., Beaudry, 2023). Population aging could further decrease inequality—as more and more people retire, labour becomes scarce relative to capital.

¹²The analysis in Beaudry, Kartashova, and Meh (2023) also suggests that a large public debt increase can result in a discontinuous jump in long-run real interest rates.

7 Conclusions

A large body of work shows that the long-run neutral real rate, R^* , declined over the past 30 years. However, the estimated neutral range remains large and uncertain, especially in real time. Increased longevity, lower fertility rates, the scarcity of safe assets, increasing income inequality (especially in the United States), and rising market power have been identified as key factors driving the downward trend in R^* . Nevertheless, the precise magnitude and impact of these factors vary considerably across studies, reflecting researchers' use of different data and estimation approaches.

The future trajectory of the neutral real rate remains highly uncertain. Existing evidence suggests the COVID-19 pandemic did not have a substantial impact on R^* . However, there is an upside risk going forward. While some pre-existing trends will persist in the next decade (e.g., declining fertility rates and increased longevity), other factors could exert upward pressure on R^* (e.g., a declining share of the working population, a higher supply of safe assets, the weakening of some underlying drivers of inequality, and the scaling up of green investment). Overall, there is a meaningful risk that R^* could increase in the future (e.g., Beaudry, 2023).

The thematic reviews highlights two main avenues for future research. First, further development of macro-finance estimation approaches can enhance the accuracy of R^* estimates and help reconcile the discrepancies between macro-based and finance-based estimates. Second, the use of microdata can help identify key model elasticities, including saving behavior across the age distribution, the impact of life expectancy on saving, and wealthy households' marginal propensity to save.

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