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## Calculating government consumption multipliers in New Zealand using an estimated DSGE model

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# Technical Appendix: Calculating Government Consumption Multipliers in New Zealand Using an Estimated DSGE Model

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## E Model

This Section describes the model used in this paper. The model is based on the small open economy monetary-fiscal DSGE model used in Leeper et al. (2011). I add unemployment using the same approach as Galí (2011) and government investment following Baxter & King (1993). Local currency pricing is also introduced, following Adolfson et al. (2005), to better match the relative volatilities of the exchange rate and tradeables inflation. The model setup is quite general, and includes some features that I don't make explicit use of in this paper.

### E.1 The Representative Ricardian Household (Savers)

The economy has a large representative Ricardian household with a continuum of members indexed by the pair  $(j, k) \in [0, 1] \times [0, 1]$ . The first dimension  $j$ , indexes the type of labour service a family member is specialised in, while the second dimension  $k$ , determines their disutility from working. If the family member is employed, they receive disutility from working equal to  $\kappa_t k^\xi$ , and they receive zero disutility from working when they are unemployed. I use the subscript  $S$  to denote savers, when referencing variables specific to the Ricardian household. The representative Ricardian household's expected present value of their lifetime utility  $U_{S,0}(j)$ , is given by,

$$U_{S,0} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{S,t}^* - \theta \check{C}_{S,t-1}^*)^{1-\gamma}}{1-\gamma} - \kappa_t \int_0^1 \int_0^{\ell_{S,t}(j)} k^\xi dk dj \right] \right\}, \quad (25)$$

$$= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{S,t}^* - \theta \check{C}_{S,t-1}^*)^{1-\gamma}}{1-\gamma} - \kappa_t \int_0^1 \frac{\ell_{S,t}(j)^{1+\xi}}{1+\xi} dj \right] \right\}, \quad (26)$$

where  $C_{S,t}^*$  is a consumption index defined as,

$$C_{S,t}^* = C_{S,t} + \varphi_G G_t + \varphi_{G_I} G_{I,t}, \quad (27)$$

$G_t$  is public consumption,  $G_{I,t}$  is public investment, the accent over lagged consumption indicates that habits are external,  $\ell_{S,t}(j)$  is the quantity of type  $j$  labour employed and  $\kappa_t$  is a labour preference shifter that evolves according to,

$$\log \kappa_t = \rho_\kappa \log \kappa_{t-1} + (1 - \rho_\kappa) \kappa + \varepsilon_{\kappa,t}. \quad (28)$$

The representative Ricardian household is subject to the following period by period flow budget constraint,

$$\begin{aligned} (1 + \tau_{C,t}) C_{S,t} + \frac{P_{I,t}}{P_{C,t}} I_{S,t} + B_{S,t} + F_{S,t} &= \frac{R_{t-1}}{\pi_t} B_{S,t-1} \exp(A_{B,t-1}) \dots \\ &\dots + \frac{R_{t-1}}{\pi_t} F_{S,t-1} \exp(A_{B,t-1}) + \int_0^1 (1 - \tau_{\ell,t}) \frac{W_t(j)}{P_{C,t}} \ell_{S,t}(j) dj \dots \\ &\dots - \int_0^1 \frac{\phi_W}{2} \frac{W_t}{P_{C,t}} \ell_{S,t} \left[ \frac{W_t(j)}{W_{t-1}(j)} - \tilde{\pi}_{W,t} \right]^2 dj + (1 - \tau_{K,t}) R_{K,t} v_t \bar{K}_{S,t-1} \dots \\ &\dots - \left[ \psi_1 (v_t - 1) + \frac{\psi_2}{2} (v_t - 1)^2 \right] \frac{P_{I,t}}{P_{C,t}} \bar{K}_{S,t-1} + Z_{S,t} + D_{S,t} + \Phi_{S,t}, \end{aligned} \quad (29)$$

where  $\tau_{C,t}$  is the tax rate on consumption goods and  $P_{I,t}$  and  $P_{C,t}$  are the nominal investment and consumption deflators.  $I_{S,t}$  denotes private investment,  $B_{S,t}$  net public debt, and  $F_{S,t}$  net foreign debt denominated in domestic currency.  $A_{B,t}$  is a risk premium shock process which I describe in more detail below.  $R_t$  is the gross nominal short term interest rate and  $\pi_t = \frac{P_{C,t}}{P_{C,t-1}}$  is gross consumer price inflation.  $\tau_{\ell,t}$  is the average tax rate paid on labour income,  $W_t(j)$  is the nominal

wage earned by the  $j$ th variety of labour,  $W_t$  is the aggregate nominal wage and  $\tilde{\pi}_{W,t}$  is a wage inflation index described in more detail below.  $\tau_{K,t}$  is the average tax rate paid on capital income,  $R_{K,t}$  is the rental/dividend rate on capital,  $\bar{K}_{S,t}$  is the date  $t$  capital stock and  $v_t$  is the rate of capacity utilisation.  $Z_{S,t}$  denotes lump sum transfers paid to the Ricardian household,  $D_{S,t}$  is profits from domestic tradables producers and importers paid to the Ricardian household and  $\Phi_{S,t}$  is adjustment costs and shocks rebated to the Ricardian household.<sup>1</sup> The wage inflation index,  $\tilde{\pi}_{W,t}$ , is defined as,

$$\tilde{\pi}_{W,t} = \pi_{W,t}^{\omega_W} \pi_{W,t-1}^{1-\omega_W}, \quad (30)$$

where  $\pi_{W,t} = \frac{W_t}{W_{t-1}}$  is gross wage inflation. The risk premium shock process evolves according to,

$$A_{B,t} = \rho_{A_B} A_{B,t-1} + \tilde{\varepsilon}_{A_B,t}. \quad (31)$$

As per Section 4,  $\tilde{\varepsilon}_{A_B,t}$  is an endogenous risk premium shock. To incorporate parameter uncertainty into the analysis, I normalise the interest rate tracks when calculating government consumption multipliers at the endogenously binding lower bound. I use the endogenous risk premium shock to condition on a common interest rate track across all model parameterisations. An additional endogenous variable requires adding an equation to the system. I add the following equation to the system of model equations,

$$\mathbf{X}_t (R_t - \mathbb{R}_t) + (1 - \mathbf{X}_t) (\tilde{\varepsilon}_{A_B,t} - \varepsilon_{A_B,t}) = 0, \quad (32)$$

where  $\mathbf{X}_t$  is an indicator function that takes the value 1 in the periods the model is constrained to match the forecast conditions and 0 in all other periods and  $\mathbb{R}_t$  is an exogenous interest rate track. In all other settings, including estimation and simulation,  $\mathbf{X}_t$  is set to 0 in all periods. The adjustment cost on capacity utilisation costs take the form,

$$a(v_t) = \psi_1 (v_t - 1) + \frac{\psi_2}{2} (v_t - 1)^2, \quad (33)$$

where,

$$a'(u_t) = \frac{\partial a(u_t)}{\partial v_t} = \psi_1 + \psi_2 (v_t - 1), \quad (34)$$

and,

$$a''(v_t) = \frac{\partial^2 a(u_t)}{\partial v_t^2} = \psi_2, \quad (35)$$

so that,

$$\frac{a''(1)}{a'(1)} \equiv \chi = \frac{\psi_2}{\psi_1}, \quad \text{and} \quad \psi_2 = \chi \psi_1. \quad (36)$$

The representative Ricardian household is also constrained by capital's law of motion,

$$\bar{K}_{S,t} = (1 - \delta) \bar{K}_{S,t-1} + A_{I,t} \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 \right] I_{S,t}, \quad (37)$$

where  $A_{I,t}$  is an investment efficiency shock process which evolves according to,

$$A_{I,t} = A_{I,t-1}^{\rho_{A_I}} \exp(\varepsilon_{A_I,t}). \quad (38)$$

Setting up the Lagrangean for the representative Ricardian household,

$$\mathcal{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \dots - \lambda_t \left[ \begin{aligned} & \frac{(C_{S,t}^* - \theta \check{C}_{S,t-1}^*)^{1-\gamma}}{1-\gamma} - \kappa_t \int_0^1 \frac{\ell_{S,t}(j)^{1+\xi}}{1+\xi} dj \dots \\ & (1 + \tau_{C,t}) C_{S,t} + \frac{P_{I,t}}{P_{C,t}} I_{S,t} + B_{S,t} \dots \\ & \dots + F_{S,t} - \frac{R_{t-1}}{\pi_t} B_{S,t-1} \exp(A_{B,t-1}) \dots \\ & \dots - \frac{R_{t-1}}{\pi_t} F_{S,t-1} \exp(A_{B,t-1}) - \int_0^1 (1 - \tau_{\ell,t}) \frac{W_t(j)}{P_{C,t}} \ell_{S,t}(j) dj \dots \\ & \dots + \int_0^1 \frac{\phi_W}{2} \frac{W_t}{P_{C,t}} \ell_{S,t} \left[ \frac{W_t(j)}{W_{t-1}(j)} - \tilde{\pi}_{W,t} \right]^2 dj \dots \\ & \dots - (1 - \tau_{K,t}) R_{K,t} v_t \bar{K}_{S,t-1} \dots \\ & \dots + \left[ \psi_1 (v_t - 1) + \frac{\psi_2}{2} (v_t - 1)^2 \right] \frac{P_{I,t}}{P_{C,t}} \bar{K}_{S,t-1} \dots \\ & \dots - Z_{S,t} - D_{S,t} - \Phi_{S,t} \dots \\ & \dots + Q_t \left[ \begin{aligned} & \bar{K}_{S,t} - (1 - \delta) \bar{K}_{S,t-1} \dots \\ & \dots - A_{I,t} \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 \right] I_{S,t} \end{aligned} \right] \end{aligned} \right] \right\}, \quad (39)$$

<sup>1</sup>Coenen et al. (2013) makes a similar assumption, rebating intermediation premia, including risk premia shocks, back to the household in the form of a lump sum payment. This assumption ensures final aggregate relationships do not include adjustment costs or risk premia shock processes.

where  $\lambda_t$  is the shadow price of consumption/wealth and  $Q_t$  is the shadow price of new capital goods. The household is subject to the labour demand constraint for the  $j$ th variety of labour,

$$\ell_{S,t}(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\eta_t} \ell_{S,t}. \quad (40)$$

which follows from the labour aggregator's cost minimization problem, where the elasticity of substitution,  $\eta_t$ , follows the process,

$$\eta_t = \eta_{t-1}^{\rho_\eta} \eta^{1-\rho_\eta} \exp(\varepsilon_{\eta,t}). \quad (41)$$

Substituting the labour demand function into equation (39) gives,

$$\mathcal{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \dots - \lambda_t \left[ \begin{aligned} & \frac{(C_{S,t}^* - \theta \check{C}_{S,t-1}^*)^{1-\gamma}}{1-\gamma} - \kappa_t \int_0^1 \frac{\left( \frac{W_t(j)}{W_t} \right)^{-\eta_t} \ell_{S,t}^{1+\xi}}{1+\xi} dj \dots \\ & (1 + \tau_{C,t}) C_{S,t} + \frac{P_{I,t}}{P_{C,t}} I_{S,t} + B_{S,t} \dots \\ & \dots + F_{S,t} - \frac{R_{t-1}}{\pi_t} B_{S,t-1} \exp(A_{B,t-1}) \dots \\ & \dots - \frac{R_{t-1}}{\pi_t} F_{S,t-1} \exp(A_{B,t-1}) - \int_0^1 (1 - \tau_{\ell,t}) \frac{W_t(j)^{1-\eta_t} W_t^{\eta_t}}{P_{C,t}} \ell_{S,t} dj \dots \\ & \dots + \int_0^1 \frac{\phi_W}{2} \frac{W_t}{P_{C,t}} \ell_{S,t} \left[ \frac{W_t(j)}{W_{t-1}(j)} - \tilde{\pi}_{W,t} \right]^2 dj \dots \\ & \dots - (1 - \tau_{K,t}) R_{K,t} v_t \bar{K}_{S,t-1} \dots \\ & \dots + \left[ \psi_1 (v_t - 1) + \frac{\psi_2}{2} (v_t - 1)^2 \right] \frac{P_{I,t}}{P_{C,t}} \bar{K}_{S,t-1} \dots \\ & \dots - Z_{S,t} - D_{S,t} - \Phi_{S,t} \dots \\ & \dots + Q_t \left[ \begin{aligned} & \bar{K}_{S,t} - (1 - \delta) \bar{K}_{S,t-1} \dots \\ & \dots - A_{I,t} \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 \right] I_{S,t} \end{aligned} \right] \end{aligned} \right] \right\}. \quad (42)$$

I obtain the first order conditions with respect to consumption, wages, investment, net public debt, physical capital and capacity utilisation:

$$\frac{\partial \mathcal{L}_t}{\partial C_{S,t}} = (C_{S,t}^* - \theta C_{S,t-1}^*)^{-\gamma} - \lambda_t (1 + \tau_{C,t}) = 0, \quad (43)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial W_t(j)} &= \eta_t \kappa_t \frac{\ell_{S,t}(j)^{1+\xi}}{W_t(j)} + \lambda_t (1 - \eta_t) (1 - \tau_{\ell,t}) \left( \frac{W_t(j)}{W_t} \right)^{-\eta_t} \frac{\ell_{S,t}}{P_{C,t}} \dots \\ &\dots - \lambda_t \phi_W \frac{W_t}{P_{C,t} W_{t-1}(j)} \ell_{S,t} \left[ \frac{W_t(j)}{W_{t-1}(j)} - \tilde{\pi}_{W,t} \right] \dots \\ &\dots + E_t \left\{ \beta \lambda_{t+1} \phi_W \frac{W_{t+1} W_{t+1}(j)}{P_{C,t+1} W_t(j)^2} \ell_{S,t+1} \left[ \frac{W_{t+1}(j)}{W_t(j)} - \tilde{\pi}_{W,t+1} \right] \right\} = 0, \end{aligned} \quad (44)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial I_{S,t}} &= -\lambda_t \frac{P_{I,t}}{P_{C,t}} + Q_t \lambda_t A_{I,t} \left( 1 - \frac{\phi_I}{2} \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 - \phi_I \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right) \frac{I_{S,t}}{I_{S,t-1}} \right) \dots \\ &\dots + E_t \left\{ \beta Q_{t+1} \lambda_{t+1} A_{I,t+1} \phi_I \left( \frac{I_{S,t+1}}{I_{S,t}} - 1 \right) \left( \frac{I_{S,t+1}}{I_{S,t}} \right)^2 \right\} = 0, \end{aligned} \quad (45)$$

$$\frac{\partial \mathcal{L}_t}{\partial B_{S,t}(j)} = -\lambda_t + E_t \left\{ \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \exp(A_{B,t}) \right\} = 0, \quad (46)$$

$$\frac{\partial \mathcal{L}_t}{\partial \bar{K}_{S,t}} = -\lambda_t Q_t + E_t \left\{ \beta \lambda_{t+1} \left( (1 - \tau_{K,t+1}) R_{K,t+1} v_{t+1} - \frac{P_{I,t+1}}{P_{C,t+1}} \psi_1 (v_{t+1} - 1) \dots \right) \right\} = 0, \quad (47)$$

$$\frac{\partial \mathcal{L}_t}{\partial v_t} = \lambda_t (1 - \tau_{K,t}) R_{K,t} - \lambda_t \frac{P_{I,t}}{P_{C,t}} \psi_1 - \lambda_t \frac{P_{I,t}}{P_{C,t}} \psi_2 (v_t - 1) = 0. \quad (48)$$

From equation (43), I obtain the marginal utility of consumption,

$$\lambda_t = \frac{(C_{S,t}^* - \theta C_{S,t-1}^*)^{-\gamma}}{1 + \tau_{C,t}}. \quad (49)$$

From equation (44), I obtain the wage Phillips curve,

$$\begin{aligned} \left(\frac{\phi_W}{\eta_t - 1}\right) \frac{W_t}{W_{t-1}(j)} \frac{\ell_{S,t}}{\ell_{S,t}(j)} \left[ \frac{W_t(j)}{W_{t-1}(j)} - \tilde{\pi}_{W,t} \right] = \\ \left(\frac{\eta_t}{\eta_t - 1}\right) \kappa_t \frac{\ell_{S,t}(j)^\xi}{\lambda_t} \frac{P_{C,t}}{W_t(j)} - 1 + \tau_{\ell,t} \dots \\ \dots + E_t \left\{ \left(\frac{\phi_W}{\eta_t - 1}\right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{W_{t+1} W_{t+1}(j)}{\pi_{t+1} W_t(j)^2} \frac{\ell_{S,t+1}}{\ell_{S,t}(j)} \left[ \frac{W_{t+1}(j)}{W_t(j)} - \tilde{\pi}_{W,t+1} \right] \right\}. \end{aligned} \quad (50)$$

In symmetric equilibrium,  $W_t(j) = W_t$  and  $\ell_{S,t}(j) = \ell_{S,t}$ , which means equation (50) can be expressed as,

$$\begin{aligned} \left(\frac{\phi_W}{\eta_t - 1}\right) \pi_{W,t} [\pi_{W,t} - \tilde{\pi}_{W,t}] = \left(\frac{\eta_t}{\eta_t - 1}\right) \kappa_t \left(\frac{\ell_{S,t}^\xi}{\lambda_t \tilde{W}_t}\right) - 1 + \tau_{\ell,t} \dots \\ \dots + E_t \left\{ \left(\frac{\phi_W}{\eta_t - 1}\right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{\tilde{W},t+1}^2}{\pi_{t+1}} \frac{\ell_{S,t+1}}{\ell_{S,t}} [\pi_{W,t+1} - \tilde{\pi}_{W,t+1}] \right\}, \end{aligned} \quad (51)$$

where  $\tilde{W}_t = \frac{W_t}{P_{C,t}}$  is the real wage,  $\pi_{W,t} = \frac{W_t}{W_{t-1}}$  is gross wage inflation, and real wages are related to both price and wage inflation according to,

$$\tilde{W}_t = \frac{\pi_{W,t}}{\pi_t} \tilde{W}_{t-1}. \quad (52)$$

From equation (45), I obtain an expression that equates the marginal benefit of increasing investment with the marginal cost,

$$\begin{aligned} \tilde{P}_{I,t} = Q_t A_{I,t} \left( 1 - \frac{\phi_I}{2} \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 - \phi_I \left( \frac{I_{S,t}}{I_{S,t-1}} - 1 \right) \frac{I_{S,t}}{I_{S,t-1}} \right) \dots \\ \dots + E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} A_{I,t+1} \phi_I \left( \frac{I_{S,t+1}}{I_{S,t}} - 1 \right) \left( \frac{I_{S,t+1}}{I_{S,t}} \right)^2 \right\}, \end{aligned} \quad (53)$$

where  $\tilde{P}_{I,t} = \frac{P_{I,t}}{P_{C,t}}$  is the relative price of investment. From equation (46), I obtain the consumption Euler equation for net government debt,

$$\lambda_t = E_t \left\{ \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \exp(A_{B,t}) \right\}. \quad (54)$$

From equation (47), I obtain an asset pricing equation for Tobin's Q,

$$\lambda_t Q_t = E_t \left\{ \beta \lambda_{t+1} \left( \begin{aligned} &(1 - \tau_{K,t+1}) R_{K,t+1} v_{t+1} \dots \\ &\dots - \tilde{P}_{I,t+1} \left[ \psi_1 (v_{t+1} - 1) + \frac{\psi_2}{2} (v_{t+1} - 1)^2 \right] + Q_{t+1} (1 - \delta) \end{aligned} \right) \right\}. \quad (55)$$

From equation (48), I get an expression that equates the marginal benefit of increasing capacity utilization with the marginal cost,

$$(1 - \tau_{K,t}) R_{K,t} = \tilde{P}_{I,t} [\psi_1 + \psi_2 (v_t - 1)]. \quad (56)$$

## E.2 The Representative Rule-of-Thumb Household (Non-Savers)

The economy also has a representative rule of thumb household. Just like the representative Ricardian household, they supply differentiated labour services, where  $j$  is used to index the type of labour service provided. The rule of thumb household is unable to save or borrow and does not own any assets. As a consequence they must consume all income, including net benefit payments, in the period they are received. I use the subscript  $N$  to define variables specific to non-savers or the rule of thumb household. The rule-of-thumb household is subject to the following period by period budget constraint,

$$(1 + \tau_{C,t}) C_{N,t} = \int_0^1 (1 - \tau_{\ell,t}) \frac{W_t(j)}{P_{C,t}} \ell_{N,t}(j) dj + Z_{N,t}, \quad (57)$$

where  $C_{N,t}$  is consumption by the rule-of-thumb household,  $\ell_{N,t}(j)$  is the  $j$ th variety of labour employed and  $Z_{N,t}$  is transfers to the rule-of-thumb household. In a symmetric equilibrium,

$$(1 + \tau_{C,t}) C_{N,t} = (1 - \tau_{\ell,t}) \tilde{W}_t \ell_{N,t} + Z_{N,t}. \quad (58)$$

Following Leeper et al. (2011), I assume that the rule of thumb household supplies labour according to the same labour supply schedule used by the Ricardian household. In aggregate this implies,

$$\ell_t = \mu \ell_{S,t} + (1 - \mu) \ell_{N,t} = \ell_{N,t} = \ell_{S,t}. \quad (59)$$

As a result, I drop the  $S$  and  $N$  subscripts for labour in equilibrium.

### E.3 Unemployment

Following Galí (2011), unemployment is determined as the difference between the total labour force, those that would like to be employed at the current wage rate, and the number of people currently employed. Taking the real wage  $\tilde{W}_t(j)$  as given, the  $k$ th family member in the  $j$ th Ricardian household is willing to work if,

$$(1 - \tau_{\ell,t}) \tilde{W}_t(j) \geq \kappa_t \frac{k^\xi}{\lambda_t}. \quad (60)$$

The marginal person prepared to supply labour at the given real wage, denoted by  $L_t(j)$  is determined by,

$$(1 - \tau_{\ell,t}) \tilde{W}_t(j) = \kappa_t \frac{L_t(j)^\xi}{\lambda_t}. \quad (61)$$

The aggregate labour force is defined as,

$$L_t = \int_0^1 L_t(j) dj. \quad (62)$$

The aggregate labour force is then determined by,

$$(1 - \tau_{\ell,t}) \tilde{W}_t = \kappa_t \frac{L_t^\xi}{\lambda_t}. \quad (63)$$

It then follows that the unemployment rate is given by,

$$\Upsilon_t = 1 - \frac{\ell_t}{L_t}, \quad (64)$$

where  $\Upsilon_t$  is the unemployment rate.

### E.4 Domestically Produced Tradeable Goods Sector

There is a continuum of tradeable goods producers producing differentiated tradeable intermediate goods. Domestic tradeables producers combine physical capital with labour, government capital and technology to produce intermediate goods. The  $i$ th domestic tradeable goods producer produces tradeable goods according to the Cobb-Douglas production function,

$$Y_t(i) = A_t K_t(i)^\alpha \ell_t(i)^{1-\alpha} K_{G,t-1}^{\alpha_G} - A_t^{\frac{1}{1-\alpha}} \psi_H, \quad (65)$$

where  $Y_t(i)$  is the intermediate good,  $A_t$  is total factor productivity,  $K_t(i)$  is effective capital,  $\ell_t(i)$  is hours worked,  $K_{G,t}$  is government capital and  $\psi_H$  is a fixed cost of production which is scaled by total factor productivity. Total factor productivity is assumed to follow an autoregressive process,

$$A_t = A_{t-1}^{\rho_A} \exp(\varepsilon_{A,t}), \quad (66)$$

and effective capital is defined as,

$$K_t(i) = v_t(i) \bar{K}_{t-1}(i). \quad (67)$$

The  $i$ th intermediate goods producer is subject to the following demand function for their variety of goods,

$$Y_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t, \quad (68)$$

which follows from the perfectly competitive aggregator's cost minimization problem. The  $i$ th tradeable goods producer maximizes the expected net present value of their current and future profits by choosing prices for the variety of good they produce, and allocations of effective capital and labour, subject to a quadratic adjustment cost of changing prices and their production function. The  $i$ th producer's objective function is thus,

$$\Phi_{H,0}(i) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ \begin{array}{l} \exp(A_{P,t}) \frac{P_{H,t}(i)}{P_{C,t}} Y_t(i) - R_{K,t} K_t(i) - \frac{\tilde{W}_t \ell_t(i)}{A_{L,t}} \dots \\ \dots - \frac{\phi_P}{2} \frac{P_{H,t}}{P_{C,t}} Y_t \left[ \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \tilde{\pi}_{H,t} \right]^2 \dots \\ \dots - \Omega_t \left[ Y_t(i) - \left( A_t K_t(i)^\alpha \ell_t(i)^{1-\alpha} K_{G,t-1}^{\alpha_G} - A_t^{\frac{1}{1-\alpha}} \psi_H \right) \right] \end{array} \right] \right\}, \quad (69)$$

where  $A_{P,t}$  is a stochastic subsidy to domestic tradeable goods producers that evolves according to,

$$A_{P,t} = \rho_{AP} A_{P,t-1} + \varepsilon_{AP,t}, \quad (70)$$

$A_{L,t}$  is a labour demand shifter that follows the process,

$$A_{L,t} = A_{L,t-1}^{\rho_{AL}} \exp(\varepsilon_{AL,t}), \quad (71)$$

$\pi_{H,t} = \frac{P_{H,t}}{\tilde{P}_{H,t-1}}$  is domestically produced tradeables inflation, and  $\tilde{\pi}_{H,t}$  is a tradeables inflation index defined as,

$$\tilde{\pi}_{H,t} = \pi_{H,t-1}^{1-\epsilon} \pi_H^{\epsilon}. \quad (72)$$

Substituting the demand function into the firm's problem gives,

$$\Phi_{H,0}(i) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ \begin{array}{l} \exp(A_{P,t}) \frac{P_{H,t}(i)^{1-\epsilon} P_{H,t}^\epsilon Y_t - R_{K,t} K_t(i) \dots \\ \dots - \frac{\tilde{W}_t \ell_t(i)}{A_{L,t}} \dots \\ \dots - \frac{\phi_P P_{H,t}}{2 P_{C,t}} Y_t \left[ \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \tilde{\pi}_{H,t} \right]^2 \dots \\ \dots - \Omega_t \left[ \begin{array}{l} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_t \dots \\ \dots - \left( A_t K_t(i)^\alpha \ell_t(i)^{1-\alpha} K_{G,t-1}^{\alpha_G} - A_t^{\frac{1}{1-\alpha}} \psi_H \right) \end{array} \right] \end{array} \right] \right\}. \quad (73)$$

I obtain the first order conditions with respect to prices, effective capital and labour,

$$\begin{aligned} \frac{\partial \Phi_{H,t}(i)}{\partial P_{H,t}(i)} &= (1-\epsilon) \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} \frac{Y_t}{P_{C,t}} \exp(A_{P,t}) + \epsilon \Omega_t P_{H,t}(i)^{-\epsilon-1} P_{H,t}^\epsilon Y_t \dots \\ &\quad \dots - \phi_P \frac{P_{H,t}}{P_{H,t-1}(i)} \frac{Y_t}{P_{C,t}} \left[ \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \tilde{\pi}_{H,t} \right] \dots \\ &\quad \dots + E_t \left\{ \phi_P \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{H,t+1} P_{H,t+1}(i)}{P_{H,t}(i)^2} \frac{Y_{t+1}}{P_{C,t+1}} \left[ \frac{P_{H,t+1}(i)}{P_{H,t}(i)} - \tilde{\pi}_{H,t+1} \right] \right\} = 0, \end{aligned} \quad (74)$$

$$\frac{\partial \Phi_{H,t}(i)}{\partial K_t(i)} = -R_{K,t} + \Omega_t \alpha A_t K_t(i)^{\alpha-1} \ell_t(i)^{1-\alpha} K_{G,t-1}^{\alpha_G} = 0, \quad (75)$$

$$\frac{\partial \Phi_{H,t}(i)}{\partial \ell_t(i)} = -\frac{\tilde{W}_t}{A_{L,t}} + \Omega_t (1-\alpha) A_t K_t(i)^\alpha \ell_t(i)^{-\alpha} K_{G,t-1}^{\alpha_G} = 0. \quad (76)$$

From equation (74), I obtain the tradeables Phillips curve,

$$\begin{aligned} \left( \frac{\phi_P}{\epsilon-1} \right) \frac{P_{H,t}}{P_{H,t-1}(i)} \frac{Y_t}{Y_t(i)} \left[ \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \tilde{\pi}_{H,t} \right] &= \left( \frac{\epsilon}{\epsilon-1} \right) \frac{P_{C,t}}{P_{H,t}(i)} \Omega_t - \exp(A_{P,t}) \dots \\ &\quad \dots + E_t \left\{ \left( \frac{\phi_P}{\epsilon-1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{H,t+1} P_{H,t+1}(i)}{\pi_{t+1} P_{H,t}(i)^2} \frac{Y_{t+1}}{Y_t(i)} \left[ \frac{P_{H,t+1}(i)}{P_{H,t}(i)} - \tilde{\pi}_{H,t+1} \right] \right\}. \end{aligned} \quad (77)$$

In symmetric equilibrium  $P_{H,t}(i) = P_{H,t}$  and  $Y_t(i) = Y_t$ , which means equation (77) can be expressed as,

$$\begin{aligned} \left( \frac{\phi_P}{\epsilon-1} \right) \pi_{H,t} [\pi_{H,t} - \tilde{\pi}_{H,t}] &= \left( \frac{\epsilon}{\epsilon-1} \right) \frac{\Omega_t}{\tilde{P}_{H,t}} - \exp(A_{P,t}) \dots \\ &\quad \dots + E_t \left\{ \left( \frac{\phi_P}{\epsilon-1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{H,t+1}^2 Y_{t+1}}{\pi_{t+1} Y_t} [\pi_{H,t+1} - \tilde{\pi}_{H,t+1}] \right\}, \end{aligned} \quad (78)$$

where  $\tilde{P}_{H,t} = \frac{P_{H,t}}{P_{C,t}}$ , which together with the definition of domestically produced tradeables inflation implies,

$$\tilde{P}_{H,t} = \frac{\pi_{H,t}}{\pi_t} \tilde{P}_{H,t-1} \quad (79)$$

From equation (75), I obtain the firm's demand function for physical capital,

$$R_{K,t} = \Omega_t \alpha A_t K_t(i)^{\alpha-1} \ell_t(i)^{1-\alpha} K_{G,t-1}^{\alpha_G}. \quad (80)$$

From equation (76), I obtain the firm's demand function for labour,

$$\tilde{W}_t = A_{L,t} \Omega_t (1-\alpha) A_t K_t(i)^\alpha \ell_t(i)^{-\alpha} K_{G,t-1}^{\alpha_G}. \quad (81)$$

## E.5 Importers

There is a continuum of importers, that buy imported goods at the docks, differentiate them and sell them locally. The  $i$ th importer maximises their expected discounted stream of profits by choosing prices for their variety of good subject to a quadratic adjustment cost of changing prices. The expected net present value for the  $i$ th importer's current and future profits is defined as follows,

$$\Phi_{F,0}(i) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ \begin{array}{l} \exp(A_{T,t}) \frac{P_{F,t}(i)}{P_{C,t}} M_t(i) - \frac{S_t P_{F,t}^*}{P_{C,t}} M_t(i) - \frac{S_t P_{F,t}^*}{P_{C,t}} A_t^{\frac{1}{1-\alpha}} \psi_M \dots \\ \dots - \frac{\phi_F P_{F,t}}{2 P_{C,t}} M_t \left[ \frac{P_{F,t}(i)}{P_{F,t-1}(i)} - \tilde{\pi}_{F,t} \right]^2 \end{array} \right] \right\}, \quad (82)$$

where

$$A_{T,t} = \rho_{A_T} A_{T,t-1} + \varepsilon_{A_T,t}, \quad (83)$$

is a stochastic subsidy to importers,  $M_t(i)$  is the  $i$ th firm's supply of imports,  $P_{F,t}^*$  is the price of imports at the docks denominated in foreign currency,  $S_t$  is the nominal exchange rate (the local currency price for 1 unit of foreign currency),  $P_{F,t}(i)$  is the import price in local currency set by the  $i$ th importer,  $\psi_M$  is a fixed cost of production which is scaled by technology,  $P_{F,t}$  is the aggregate import price in local currency,  $\pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}}$  is gross import price inflation,  $\tilde{\pi}_{F,t}$  is an imported inflation index that is defined as,

$$\tilde{\pi}_{F,t} = \pi_{F,t-1}^{\lambda_F} \pi_{F,t}^{1-\lambda_F}. \quad (84)$$

Imports are the sum of the imported aggregate demand components:  $M_t = C_{F,t} + I_{F,t} + G_{F,t} + G_{FI,t}$ , where the  $F$  subscript indicates these are imports. Importers are also subject to the following demand constraint for their variety of imported good,

$$M_t(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} M_t. \quad (85)$$

Substituting the demand curve into their profit function gives,

$$\Phi_{F,0}(i) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ \exp(A_{T,t}) \frac{P_{F,t}(i)^{1-\varepsilon} P_{F,t}^\varepsilon}{P_{C,t}} M_t - \frac{S_t P_{F,t}^*}{P_{C,t}} \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} M_t \dots \right] \right. \\ \left. \dots - \frac{S_t P_{F,t}^*}{P_{C,t}} A_t^{\frac{1}{1-\alpha}} \psi_M - \frac{\phi_F}{2} \frac{P_{F,t}}{P_{C,t}} M_t \left[ \frac{P_{F,t}(i)}{P_{F,t-1}(i)} - \tilde{\pi}_{F,t} \right]^2 \right\}. \quad (86)$$

I obtain the first order condition with respect to prices,

$$\frac{\partial \Phi_{F,t}(i)}{\partial P_{F,t}(i)} = (1-\varepsilon) \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} \frac{M_t}{P_{C,t}} \exp(A_{T,t}) + \varepsilon \frac{S_t P_{F,t}^*}{P_{C,t}} P_{F,t}(i)^{-\varepsilon-1} P_{F,t}^\varepsilon M_t \dots \\ \dots - \phi_F \frac{P_{F,t}}{P_{F,t-1}(i)} \frac{M_t}{P_{C,t}} \left[ \frac{P_{F,t}(i)}{P_{F,t-1}(i)} - \tilde{\pi}_{F,t} \right] \dots \\ \dots + E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \phi_F \frac{P_{F,t+1}}{P_{C,t+1}} \frac{P_{F,t+1}(i)}{P_{F,t}(i)^2} M_{t+1} \left[ \frac{P_{F,t+1}(i)}{P_{F,t}(i)} - \tilde{\pi}_{F,t+1} \right] \right\} = 0. \quad (87)$$

This implies,

$$\left( \frac{\phi_F}{\varepsilon-1} \right) \frac{P_{F,t}}{P_{F,t-1}(i)} \frac{M_t}{M_t(i)} \left[ \frac{P_{F,t}(i)}{P_{F,t-1}(i)} - \tilde{\pi}_{F,t} \right] = \left( \frac{\varepsilon}{\varepsilon-1} \right) \frac{S_t P_{F,t}^*}{P_{C,t}} \frac{P_{C,t}}{P_{F,t}(i)} - \exp(A_{T,t}) \dots \\ \dots + E_t \left\{ \left( \frac{\phi_F}{\varepsilon-1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{F,t+1} P_{F,t+1}(i)}{\pi_{t+1} P_{F,t}(i)^2} \frac{M_{t+1}}{M_t(i)} \left[ \frac{P_{F,t+1}(i)}{P_{F,t}(i)} - \tilde{\pi}_{F,t+1} \right] \right\}. \quad (88)$$

In symmetric equilibrium  $P_{F,t}(i) = P_{F,t}$  and  $M_t(i) = M_t$ , so that equation (88) becomes,

$$(1-\varepsilon) \exp(A_{T,t}) + \varepsilon \frac{S_t P_{F,t}^*}{P_{F,t}} - \phi_F \pi_{F,t} [\pi_{F,t} - \tilde{\pi}_{F,t}] + E_t \left\{ \phi_F \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{F,t+1}^2}{\pi_t^2} \frac{M_{t+1}}{M_t} [\pi_{F,t+1} - \tilde{\pi}_{F,t+1}] \right\}, \quad (89)$$

where  $\tilde{P}_{F,t} = \frac{P_{F,t}}{P_{C,t}}$  is the relative import price in domestic currency. This can be rearranged to obtain,

$$\left( \frac{\phi_F}{\varepsilon-1} \right) \pi_{F,t} [\pi_{F,t} - \tilde{\pi}_{F,t}] = \left( \frac{\varepsilon}{\varepsilon-1} \right) \frac{S_t P_{F,t}^*}{P_{F,t}} - \exp(A_{T,t}) \dots \\ \dots + E_t \left\{ \left( \frac{\phi_F}{\varepsilon-1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{F,t+1}^2}{\pi_{t+1}} \frac{M_{t+1}}{M_t} [\pi_{F,t+1} - \tilde{\pi}_{F,t+1}] \right\}. \quad (90)$$

$$\left( \frac{\phi_F}{\varepsilon-1} \right) \pi_{F,t} [\pi_{F,t} - \tilde{\pi}_{F,t}] = \left( \frac{\varepsilon}{\varepsilon-1} \right) \frac{Q_t}{\tilde{P}_{F,t}} - \exp(A_{T,t}) \dots \\ \dots + E_t \left\{ \left( \frac{\phi_F}{\varepsilon-1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{F,t+1}^2}{\pi_{t+1}} \frac{M_{t+1}}{M_t} [\pi_{F,t+1} - \tilde{\pi}_{F,t+1}] \right\}, \quad (91)$$

where  $Q_t = \frac{S_t P_t^*}{P_{C,t}} = \frac{S_t P_{F,t}^*}{P_{C,t}}$  is the real exchange rate.<sup>2</sup> The definition of the relative import price,  $\tilde{P}_{F,t}$ , implies,

$$\tilde{P}_{F,t} = \frac{\pi_{F,t}}{\pi_t} \tilde{P}_{F,t-1}. \quad (92)$$

The definition of the real exchange rate,  $Q_t$ , implies,

$$Q_t = \frac{S_t}{S_{t-1}} \frac{\pi_t^*}{\pi_t} Q_{t-1}. \quad (93)$$

<sup>2</sup>Note that I assume  $P_t^* = P_{F,t}^*$ , so that imports priced in foreign currency are equal to the foreign aggregate price level. This assumption follows from the small open economy assumption, where the domestic economy is infinitesimally small relative to the foreign economy and domestically produced inputs into foreign final goods production are negligible.



## E.6 Final Goods Producers/Aggregators

The final consumption good is produced by combining domestically produced consumption goods with imported consumption goods according to the constant elasticity of substitution (CES) aggregator function,

$$C_t = \left[ (1 - \nu_C)^{\frac{1}{\mu_C}} C_{H,t}^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} C_{F,t}^{1 - \frac{1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}. \quad (94)$$

Aggregation is carried out by a perfectly competitive aggregator firm. The aggregator chooses allocations of foreign and domestically produced consumption goods to minimize their costs as follows,

$$\mathcal{L}_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + P_{C,t} \left[ C_t - \left[ (1 - \nu_C)^{\frac{1}{\mu_C}} C_{H,t}^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} C_{F,t}^{1 - \frac{1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}} \right]. \quad (95)$$

I obtain the first order condition with respect to domestically produced consumption goods,

$$\frac{\partial \mathcal{L}_t}{\partial C_{H,t}} = P_{H,t} - P_{C,t} C_t^{\frac{1}{\mu_C}} (1 - \nu_C)^{\frac{1}{\mu_C}} C_{H,t}^{-\frac{1}{\mu_C}} = 0, \quad (96)$$

and the first order condition with respect to foreign produced consumption goods,

$$\frac{\partial \mathcal{L}_t}{\partial C_{F,t}} = P_{F,t} - P_{C,t} C_t^{\frac{1}{\mu_C}} \nu_C^{\frac{1}{\mu_C}} C_{F,t}^{-\frac{1}{\mu_C}} = 0. \quad (97)$$

From equation (96), the demand function for home produced consumption goods is given by,

$$C_{H,t} = (1 - \nu_C) \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} C_t. \quad (98)$$

From equation (97), the demand function for foreign produced consumption goods is given by,

$$C_{F,t} = \nu_C \left( \frac{P_{F,t}}{P_{C,t}} \right)^{-\mu_C} C_t. \quad (99)$$

Plugging equations (98) and (99) back into equation (94) gives,

$$C_t = \left[ (1 - \nu_C)^{\frac{1}{\mu_C}} \left( (1 - \nu_C) \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} C_t \right)^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} \left( \nu_C \left( \frac{P_{F,t}}{P_{C,t}} \right)^{-\mu_C} C_t \right)^{1 - \frac{1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}, \quad (100)$$

$$C_t = C_t \left[ (1 - \nu_C)^{\frac{1}{\mu_C}} \left( (1 - \nu_C) \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} \left( \nu_C \left( \frac{P_{F,t}}{P_{C,t}} \right)^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}, \quad (101)$$

$$1 = (1 - \nu_C)^{\frac{1}{\mu_C}} \left( (1 - \nu_C) \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} \left( \nu_C \left( \frac{P_{F,t}}{P_{C,t}} \right)^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}}, \quad (102)$$

$$P_{C,t}^{1 - \mu_C} = (1 - \nu_C)^{\frac{1}{\mu_C}} \left( (1 - \nu_C) P_{H,t}^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} \left( \nu_C P_{F,t}^{-\mu_C} \right)^{1 - \frac{1}{\mu_C}}, \quad (103)$$

which after rearranging and cancelling terms leads to an expression for the consumption price index,

$$P_{C,t} = \left[ (1 - \nu_C) P_{H,t}^{1 - \mu_C} + \nu_C P_{F,t}^{1 - \mu_C} \right]^{\frac{1}{1 - \mu_C}}. \quad (104)$$

The final investment goods are produced according to,

$$I_t = \left[ (1 - \nu_I)^{\frac{1}{\mu_I}} I_{H,t}^{1 - \frac{1}{\mu_I}} + \nu_I^{\frac{1}{\mu_I}} I_{F,t}^{1 - \frac{1}{\mu_I}} \right]^{\frac{\mu_I}{\mu_I - 1}}. \quad (105)$$

Cost minimisation by the final goods producer leads to the following demand equations for domestically produced investment,

$$I_{H,t} = (1 - \nu_I) \left( \frac{P_{H,t}}{P_{I,t}} \right)^{-\mu_I} I_t, \quad (106)$$

and foreign produced investment goods,

$$I_{F,t} = \nu_I \left( \frac{P_{F,t}}{P_{I,t}} \right)^{-\mu_I} I_t. \quad (107)$$

Plugging the demand functions, equations (106) and (107), into the final investment goods aggregator function, equation (105), gives the investment goods price index,

$$P_{I,t} = \left[ (1 - \nu_I) P_{H,t}^{1 - \mu_I} + \nu_I P_{F,t}^{1 - \mu_I} \right]^{\frac{1}{1 - \mu_I}}. \quad (108)$$

The aggregator function for final government goods is given by,

$$G_t = \left[ (1 - \nu_G)^{\frac{1}{\mu_G}} G_{H,t}^{1-\frac{1}{\mu_G}} + \nu_G^{\frac{1}{\mu_G}} G_{F,t}^{1-\frac{1}{\mu_G}} \right]^{\frac{\mu_G}{\mu_G-1}}. \quad (109)$$

Cost minimization leads to the demand function for home produced government consumption goods,

$$G_{H,t} = (1 - \nu_G) \left( \frac{P_{H,t}}{P_{G,t}} \right)^{-\mu_G} G_t, \quad (110)$$

and the demand function for foreign produced government consumption goods,

$$G_{F,t} = \nu_G \left( \frac{P_{F,t}}{P_{G,t}} \right)^{-\mu_G} G_t. \quad (111)$$

Plugging equations (110) and (111) into equation (109) gives the government consumption price index,

$$P_{G,t} = \left[ (1 - \nu_G) P_{H,t}^{1-\mu_G} + \nu_G P_{F,t}^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}. \quad (112)$$

The aggregator function for government investment good is given by,

$$G_{I,t} = \left[ (1 - \nu_{G_I})^{\frac{1}{\mu_{G_I}}} G_{HI,t}^{1-\frac{1}{\mu_{G_I}}} + \nu_{G_I}^{\frac{1}{\mu_{G_I}}} G_{FI,t}^{1-\frac{1}{\mu_{G_I}}} \right]^{\frac{\mu_{G_I}}{\mu_{G_I}-1}}. \quad (113)$$

Cost minimization by the government investment goods aggregator leads to the following demand functions for home produced investment goods,

$$G_{HI,t} = (1 - \nu_{G_I}) \left( \frac{P_{H,t}}{P_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t}, \quad (114)$$

and foreign produced investment goods,

$$G_{FI,t} = \nu_{G_I} \left( \frac{P_{F,t}}{P_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t}. \quad (115)$$

Substituting equations (114) and (115) back into the aggregation function, equation (113), gives the government investment deflator,

$$P_{G_I,t} = \left[ (1 - \nu_{G_I}) P_{H,t}^{1-\mu_{G_I}} + \nu_{G_I} P_{F,t}^{1-\mu_{G_I}} \right]^{\frac{1}{1-\mu_{G_I}}}. \quad (116)$$

Dividing equations (104), (108), (112), and (116) by  $P_{C,t}$  and rearranging gives,

$$\tilde{P}_{H,t} = \left[ (1 - \nu_C) + \nu_C T_t^{1-\mu_C} \right]^{\frac{1}{\mu_C-1}}, \quad (117)$$

$$\frac{\tilde{P}_{I,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_I) + \nu_I T_t^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}}, \quad (118)$$

$$\frac{\tilde{P}_{G,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_G) + \nu_G T_t^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}, \quad (119)$$

$$\frac{\tilde{P}_{G_I,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_{G_I}) + \nu_{G_I} T_t^{1-\mu_{G_I}} \right]^{\frac{1}{1-\mu_{G_I}}}, \quad (120)$$

where  $\tilde{P}_{H,t} = \frac{P_{H,t}}{P_{C,t}}$ ,  $\tilde{P}_{I,t} = \frac{P_{I,t}}{P_{C,t}}$ ,  $\tilde{P}_{G,t} = \frac{P_{G,t}}{P_{C,t}}$  and  $\tilde{P}_{G_I,t} = \frac{P_{G_I,t}}{P_{C,t}}$  are relative prices, and  $T_t$  is the terms of trade defined as,

$$T_t = \frac{P_{F,t}}{P_{H,t}} = \frac{\tilde{P}_{F,t}}{\tilde{P}_{H,t}}. \quad (121)$$

## E.7 Export Demand

The foreign final goods producer produces foreign final output by combining domestically produced export goods,  $M_t^*$ , with foreign produced tradeable goods,  $V_t^*$ , according to the following CES technology,

$$Y_t^* = \left[ (\nu^*)^{\frac{1}{\mu^*}} (M_t^* \Theta_t^*)^{1-\frac{1}{\mu^*}} + (1 - \nu^*)^{\frac{1}{\mu^*}} (V_t^*)^{\frac{\mu^*}{\mu^*-1}} \right]^{\frac{\mu^*}{\mu^*-1}}, \quad (122)$$

where  $\Theta_t^* = 1 - \frac{\phi_M^*}{2} \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right)^2$  is an adjustment cost on changing imports (exports from the domestic economy) relative to their level in the previous period. The accent over  $M_{t-1}^*$  indicates this is an aggregate variable, which means the foreign tradeable goods producer does not take into

account how their actions affect this variable when optimising. The foreign final goods producer minimizes costs by choosing allocations of imported tradeable goods and domestically produced tradeable goods,

$$\mathcal{L}_t^* = \frac{P_{H,t}^* M_t^*}{A_{M^*,t}} + P_{F,t}^* V_t^* + P_t^* \left[ Y_t^* - \left[ (\nu^*)^{\frac{1}{\mu^*}} (M_t^* \Theta_t^*)^{1-\frac{1}{\mu^*}} + (1-\nu^*)^{\frac{1}{\mu^*}} (V_t^*)^{\frac{\mu^*}{\mu^*-1}} \right]^{\frac{\mu^*}{\mu^*-1}} \right], \quad (123)$$

where  $A_{M_t^*}$  is an export demand shock that follows the process,

$$A_{M^*,t} = A_{M^*,t-1}^{\rho_{A_{M^*}}} \exp(\varepsilon_{A_{M^*,t}}). \quad (124)$$

I obtain the first order condition with respect to imports,

$$\frac{\partial \mathcal{L}_t^*}{\partial M_t^*} = \frac{P_{H,t}^*}{A_{M^*,t}} - P_t^* \left( \frac{\mu^*}{\mu^*-1} \right) (Y_t^*)^{\frac{1}{\mu^*}} \left( 1 - \frac{1}{\mu^*} \right) (\nu^*)^{\frac{1}{\mu^*}} (M_t^* \Theta_t^*)^{-\frac{1}{\mu^*}} \dots \dots \times \left[ \Theta_t^* - \phi_M^* \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right) \left( \frac{M_t^*}{M_{t-1}^*} \right) \right] = 0. \quad (125)$$

Rearranging gives,

$$\frac{P_{H,t}^*}{P_t^*} = A_{M^*,t} (\nu^*)^{\frac{1}{\mu^*}} \left( \frac{Y_t^*}{M_t^* \Theta_t^*} \right)^{\frac{1}{\mu^*}} \left[ \Theta_t^* - \phi_M^* \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right) \left( \frac{M_t^*}{M_{t-1}^*} \right) \right]. \quad (126)$$

Substituting in the relative price of domestically produced tradeable goods and the real exchange rate gives an expression for export goods demand,

$$\frac{\tilde{P}_{H,t}}{Q_t} = A_{M^*,t} (\nu^*)^{\frac{1}{\mu^*}} \left( \frac{Y_t^*}{M_t^* \Theta_t^*} \right)^{\frac{1}{\mu^*}} \left[ \Theta_t^* - \phi_M^* \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right) \left( \frac{M_t^*}{M_{t-1}^*} \right) \right], \quad (127)$$

where  $\frac{P_{H,t}^*}{P_t^*} = \frac{P_{H,t}}{S_t P_t^*} = \frac{P_{C,t}}{S_t P_t^*} \frac{P_{H,t}}{P_{C,t}} = \frac{\tilde{P}_{H,t}}{Q_t}$ .

## E.8 Government

The government's period by period budget constraint in real terms is given by,

$$B_t + \tau_{K,t} R_{K,t} v_t \bar{K}_{t-1} + \tau_{\ell,t} \tilde{W}_t \ell_t + \tau_{C,t} C_t = \frac{R_{t-1}}{\pi_t} B_{t-1} + \tilde{P}_{G,t} G_t + \tilde{P}_{G_I,t} G_{I,t} + Z_t + A_{SFA,t} \quad (128)$$

where  $A_{SFA,t}$  is an autocorrelated shock process,

$$A_{SFA,t} = \rho_{SFA} A_{SFA,t-1} + \varepsilon_{SFA,t}. \quad (129)$$

Tax revenue received from consumption is defined as,

$$T_{C,t} = \tau_{C,t} C_t. \quad (130)$$

Tax revenue from labour income is defined as,

$$T_{\ell,t} = \tau_{\ell,t} \tilde{W}_t \ell_t. \quad (131)$$

Tax revenue from capital income is defined as,

$$T_{K,t} = \tau_{K,t} R_{K,t} v_t \bar{K}_{t-1}, \quad (132)$$

where  $K_t = v_t \bar{K}_{t-1}$  is effective capital. Consumption tax rates are set according to the simple rule,

$$\tau_{C,t} = \tau_C \exp(u_{C,t}). \quad (133)$$

Tax rates on labour and capital are set according to the rules,

$$\tau_{\ell,t} = \tau_\ell \left( \frac{Y_t}{Y} \right)^{\varphi_\ell} \left( \frac{B_{t-1}}{B} \right)^{\gamma_\ell} \exp(u_{\ell,t}), \quad (134)$$

$$\tau_{K,t} = \tau_K \left( \frac{Y_t}{Y} \right)^{\varphi_K} \left( \frac{B_{t-1}}{B} \right)^{\gamma_K} \exp(u_{K,t}). \quad (135)$$

Government consumption, government investment, transfers to the Ricardian household ( $S$ ) and transfers to the rule-of-thumb household ( $N$ ) are set according to the rules,

$$G_t = G \left( \frac{Y_t}{Y} \right)^{-\varphi_G} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_G} \exp(u_{G,t}), \quad (136)$$

$$G_{I,t} = G_I \left( \frac{Y_t}{Y} \right)^{-\varphi_{G_I}} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_{G_I}} \exp(u_{G_I,t}), \quad (137)$$

$$Z_{S,t} = Z_S \left( \frac{Y_t}{Y} \right)^{-\varphi Z_S} \left( \frac{B_{t-1}}{B} \right)^{-\gamma Z_S} \exp(u_{Z_S,t}), \quad (138)$$

$$Z_{N,t} = Z_N \left( \frac{Y_t}{Y} \right)^{-\varphi Z_N} \left( \frac{B_{t-1}}{B} \right)^{-\gamma Z_N} \exp(u_{Z_N,t}), \quad (139)$$

where  $u_{C,t}$ ,  $u_{K,t}$ ,  $u_{\ell,t}$ ,  $u_{Z_S,t}$ ,  $u_{Z_N,t}$ ,  $u_{G,t}$  and  $u_{G_I,t}$  are shock processes that evolve according to,

$$u_{C,t} = \rho_{u_C} u_{C,t-1} + \varepsilon_{u_C,t}, \quad (140)$$

$$u_{K,t} = \rho_{u_K} u_{K,t-1} + \varepsilon_{u_K,t}, \quad (141)$$

$$u_{\ell,t} = \rho_{u_\ell} u_{\ell,t-1} + \varepsilon_{u_\ell,t}, \quad (142)$$

$$u_{Z_S,t} = \rho_{u_{Z_S}} u_{Z_S,t-1} + \varepsilon_{u_{Z_S},t}, \quad (143)$$

$$u_{Z_N,t} = \rho_{u_{Z_N}} u_{Z_N,t-1} + \varepsilon_{u_{Z_N},t}, \quad (144)$$

$$u_{G,t} = \rho_{u_G} u_{G,t-1} + \varepsilon_{u_G,t}, \quad (145)$$

$$u_{G_I,t} = \rho_{u_{G_I}} u_{G_I,t-1} + \varepsilon_{u_{G_I},t}. \quad (146)$$

The public capital stock evolves according to,

$$K_{G,t} = (1 - \delta_G) K_{G,t-1} + G_{I,t}. \quad (147)$$

The central bank sets short term interest rates according to,

$$R_t = \varrho (\mathbf{Z}_{1,t} R_t^* + (1 - \mathbf{Z}_{1,t}) R_{ELB}) + (1 - \varrho) (\mathbf{Z}_{2,t} R + (1 - \mathbf{Z}_{2,t}) R_t^*), \quad (148)$$

where  $\varrho$  is a parameter that takes the value 1 when the lower bound is set endogenously and 0 when an interest rate peg is set,  $R_t^*$  is the shadow interest rate,  $R_{ELB}$  is the effective lower bound,  $R$  is the steady state gross nominal interest rate and  $\mathbf{Z}_{1,t}$  and  $\mathbf{Z}_{2,t}$  are indicator functions.  $\mathbf{Z}_{2,t}$  is exogenously determined taking the value 1 in the periods the interest rate is pegged at  $R$  and 0 in all other periods.  $\mathbf{Z}_{1,t}$  is endogenously determined, set according to,

$$\mathbf{Z}_{1,t} = \begin{cases} 0 & \text{if } R_t^* \leq R_{ELB} \\ 1 & \text{if } R_t^* > R_{ELB} \end{cases}, \quad (149)$$

where the shadow interest rate is set according to either,

$$R_t^* = (R_{t-1}^*)^{\rho_R} \left( R \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t}), \quad (150)$$

or,

$$R_t^* = (R_{t-1})^{\rho_R} \left( R \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t}), \quad (151)$$

depending on which form of smoothing is assumed in the shadow rate rule, and  $\varepsilon_{R,t}$  is a monetary policy shock.<sup>3</sup>

## E.9 Market Clearing, Aggregation and Equilibrium

The domestically produced tradeable good can be consumed or invested by households, consumed or invested by government, or exported,

$$Y_t = C_{H,t} + I_{H,t} + G_{H,t} + G_{HI,t} + M_t^*. \quad (152)$$

Substituting in the demand functions from equations (98), (106), (110) and (114) gives,

$$Y_t = (1 - \nu_C) \tilde{P}_{H,t}^{-\mu_C} C_t + (1 - \nu_I) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t \dots \\ \dots + (1 - \nu_G) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t + (1 - \nu_{G_I}) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t} + M_t^*. \quad (153)$$

Imported goods can be consumed or invested by households, or consumed or invested by government, so that,

$$M_t = C_{F,t} + I_{F,t} + G_{F,t} + G_{FI,t}. \quad (154)$$

Substituting in the demand functions from equations (99), (107), (111) and (115) gives,

$$M_t = \nu_C \tilde{P}_{F,t}^{-\mu_C} C_t + \nu_I \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t + \nu_G \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t + \nu_{G_I} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t}. \quad (155)$$

<sup>3</sup>The model is linearised for estimation purposes. Linearisation ignores the lower bound constraint on interest rates. This implies that  $\varrho = 0$  and  $\mathbf{Z}_{2,t} = 0$  during estimation, for all time periods.

I impose a modified uncovered interest rate parity condition on the model,<sup>4</sup>

$$R_t = E_t \left\{ \frac{S_{t+1}}{S_t} R_t^* \exp \left( -\phi_S \left( \frac{F_t}{\tilde{P}_{H,t} Y_t} - \frac{F}{\tilde{P}_H Y} \right) + A_{S,t} \right) \right\}, \quad (156)$$

where  $R_t^*$  is the foreign interest rate,  $F_t$  is net foreign assets denominated in domestic currency and  $A_{S,t}$  is an international risk premium shock that follows the process,

$$A_{S,t} = \rho_{A_S} A_{S,t} + \varepsilon_{A_S,t}. \quad (157)$$

With a bit of algebra, it can be shown that,

$$F_t = \frac{R_{t-1}}{\pi_t} F_{t-1} + P_{H,t} M_t^* - \mathcal{Q}_t \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right), \quad (158)$$

where  $P_{H,t} M_t^* - \mathcal{Q}_t \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right)$  is net exports.

The remaining aggregate variables,  $C_t$ ,  $Z_t$ ,  $I_t$ ,  $\bar{K}_t$ ,  $K_t$ ,  $B_t$ ,  $F_t$  and  $D_t$  are defined as follows,

$$C_t = \mu C_{S,t} + (1 - \mu) C_{N,t}, \quad (159)$$

$$Z_t = \mu Z_{S,t} + (1 - \mu) Z_{N,t}, \quad (160)$$

$$I_t = \mu I_{S,t}, \quad (161)$$

$$\bar{K}_t = \mu \bar{K}_{S,t}, \quad (162)$$

$$K_t = \mu K_{S,t}, \quad (163)$$

$$B_t = \mu B_{S,t}, \quad (164)$$

$$F_t = \mu F_{S,t}, \quad (165)$$

$$D_t = \mu D_{S,t}, \quad (166)$$

where,

$$D_t = \tilde{P}_{H,t} Y_t - R_{K,t} K_t - \tilde{W} \ell_t + \tilde{P}_{F,t} M_t - \mathcal{Q}_t \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right), \quad (167)$$

is the combined profit of the domestic intermediate producers and the imported good producers.<sup>5</sup> For reporting purposes I define the real interest rate as follows,

$$\mathfrak{R}_t = E_t \left\{ \frac{R_t}{\pi_{t+1}} \right\}. \quad (168)$$

## E.10 Foreign Economy

I model the foreign variables as autoregressive processes,

$$Y_t^* = (Y^*)^{1-\rho_{Y^*}} (Y_{t-1}^*)^{\rho_{Y^*}} \exp(\varepsilon_{Y^*,t}), \quad (169)$$

$$R_t^* = (R^*)^{1-\rho_{R^*}} (R_{t-1}^*)^{\rho_{R^*}} \exp(\varepsilon_{R^*,t}), \quad (170)$$

$$\pi_t^* = (\pi^*)^{1-\rho_{\pi^*}} (\pi_{t-1}^*)^{\rho_{\pi^*}} \exp(\varepsilon_{\pi^*,t}), \quad (171)$$

where  $Y_t^*$  is foreign GDP,  $R_t^*$  is the gross nominal foreign interest rate and  $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$  is the gross foreign inflation rate.

<sup>4</sup>Ordinarily the UIP condition would fall out of the household's first order conditions with respect to net foreign debt. However, because net foreign debt is priced in domestic currency, the UIP condition should fall out of the foreign household's optimisation problem. In this case I don't find the foreign economy's first order conditions as I assume the foreign variables follow autoregressive processes, so I impose the UIP condition on the model.

Alternatively, specifying interest costs on net foreign debt as,

$$\frac{R_{t-1}^*}{\pi_t} \frac{S_t}{S_{t-1}} F_{S,t-1} \exp(A_{B,t-1}) \exp \left( -\phi_S \left( \frac{F_{t-1}}{\tilde{P}_{H,t-1} Y_{t-1}} - \frac{F}{\tilde{P}_H Y} \right) + A_{S,t-1} \right),$$

instead of,

$$\frac{R_{t-1}}{\pi_t} F_{S,t-1} \exp(A_{B,t-1}),$$

in the same way as Algozhina (2012) does would allow the derivation of the UIP condition from the household's first order conditions.

<sup>5</sup>I assume that price adjustment costs are rebated back to the Ricardian household.

## E.11 Measurement Equations

For estimation purposes, I augment the model with the following measurement equations,

$$\hat{Y}_t = 100 \times \log \left( \frac{Y_t}{\bar{Y}} \right), \quad (172)$$

$$\hat{R}_t = 100 \times (R_t - R), \quad (173)$$

$$\hat{\pi}_t = 100 \times (\pi_t - \pi), \quad (174)$$

$$\hat{\pi}_{W,t} = 100 \times (\pi_{W,t} - \pi_W), \quad (175)$$

$$\hat{\Upsilon}_t = 100 \times (\Upsilon_t - \Upsilon), \quad (176)$$

$$\hat{I}_t = 100 \times \log \left( \frac{I_t}{\bar{I}} \right), \quad (177)$$

$$\hat{C}_t = 100 \times \log \left( \frac{C_t}{\bar{C}} \right), \quad (178)$$

$$\hat{M}_t^* = 100 \times \log \left( \frac{M_t^*}{\bar{M}^*} \right), \quad (179)$$

$$\hat{G}_t = 100 \times \log \left( \frac{G_t}{\bar{G}} \right), \quad (180)$$

$$\hat{G}_{I,t} = 100 \times \log \left( \frac{G_{I,t}}{\bar{G}_I} \right), \quad (181)$$

$$\hat{Z}_t = 100 \times \log \left( \frac{Z_t}{\bar{Z}} \right), \quad (182)$$

$$\hat{T}_{C,t} = 100 \times \log \left( \frac{T_{C,t}}{\bar{T}_C} \right), \quad (183)$$

$$\hat{T}_{\ell,t} = 100 \times \log \left( \frac{T_{\ell,t}}{\bar{T}_\ell} \right), \quad (184)$$

$$\hat{T}_{K,t} = 100 \times \log \left( \frac{T_{K,t}}{\bar{T}_K} \right), \quad (185)$$

$$\frac{\widehat{B}}{\bar{Y}_t} = 100 \times \left( \frac{B_t}{4 \times \tilde{P}_{H,t} Y_t} - \frac{B}{4 \times \tilde{P}_H \bar{Y}} \right), \quad (186)$$

$$\frac{\widehat{S}_t}{\bar{S}_{t-1}} = 100 \times \left( \frac{S_t}{\bar{S}_{t-1}} - 1 \right), \quad (187)$$

$$\hat{Y}_t^* = 100 \times \log \left( \frac{Y_t^*}{\bar{Y}^*} \right), \quad (188)$$

$$\hat{R}_t^* = 100 \times (R_t^* - R^*), \quad (189)$$

$$\hat{\pi}_t^* = 100 \times (\pi_t^* - \pi^*). \quad (190)$$

## E.12 Model Equations

The 93 model variables:  $\tau_{C,t}$ ,  $\tau_{K,t}$ ,  $\tau_{\ell,t}$ ,  $Z_t$ ,  $Z_{N,t}$ ,  $Z_{S,t}$ ,  $Y_t$ ,  $C_{S,t}$ ,  $C_{N,t}$ ,  $C_t$ ,  $I_t$ ,  $G_t$ ,  $\ell_t$ ,  $L_t$ ,  $\Upsilon_t$ ,  $\tilde{P}_{I,t}$ ,  $\tilde{P}_{H,t}$ ,  $\tilde{P}_{G,t}$ ,  $T_t$ ,  $\frac{S_t}{\bar{S}_{t-1}}$ ,  $B_t$ ,  $F_t$ ,  $R_t$ ,  $\pi_t$ ,  $\pi_{H,t}$ ,  $\tilde{\pi}_{H,t}$ ,  $Q_t$ ,  $R_{K,t}$ ,  $\tilde{W}_t$ ,  $\pi_{W,t}$ ,  $\tilde{\pi}_{W,t}$ ,  $v_t$ ,  $\bar{K}_t$ ,  $K_t$ ,  $A_t$ ,  $D_t$ ,  $\lambda_t$ ,  $\Omega_t$ ,  $Y_t^*$ ,  $R_t^*$ ,  $\pi_t^*$ ,  $u_{C,t}$ ,  $u_{K,t}$ ,  $u_{\ell,t}$ ,  $u_{Z_{N,t}}$ ,  $u_{Z_{S,t}}$ ,  $u_{G,t}$ ,  $K_{G,t}$ ,  $G_{I,t}$ ,  $\tilde{P}_{G,t}$ ,  $u_{G_{I,t}}$ ,  $\pi_{F,t}$ ,  $Q_t$ ,  $M_t$ ,  $\tilde{P}_{F,t}$ ,  $\tilde{\pi}_{F,t}$ ,  $C_{S,t}^*$ ,  $A_{I,t}$ ,  $\kappa_t$ ,  $A_{B,t}$ ,  $A_{S,t}$ ,  $A_{T,t}$ ,  $A_{P,t}$ ,  $A_{SFA,t}$ ,  $A_{X,t}$ ,  $A_{L,t}$ ,  $\eta_t$ ,  $\mathfrak{R}_t$ ,  $M_t^*$ ,  $R_t^*$ ,  $\tilde{\epsilon}_{AB,t}$ ,  $T_{C,t}$ ,  $T_{K,t}$ ,  $T_{\ell,t}$ ,  $\hat{Y}_t$ ,  $\hat{R}_t$ ,  $\hat{\pi}_t$ ,  $\hat{\pi}_{W,t}$ ,  $\hat{\Upsilon}_t$ ,  $\hat{I}_t$ ,  $\hat{C}_t$ ,  $\hat{M}_t^*$ ,  $\hat{G}_t$ ,  $\hat{G}_{I,t}$ ,  $\hat{Z}_t$ ,  $\hat{T}_{C,t}$ ,  $\hat{T}_{\ell,t}$ ,  $\hat{T}_{K,t}$ ,  $\frac{\hat{B}}{\bar{Y}_t}$ ,  $\frac{\hat{S}_t}{\bar{S}_{t-1}}$ ,  $\hat{Y}_t^*$ ,  $\hat{R}_t^*$ , and  $\hat{\pi}_t^*$ , are determined by the 93 equations: (27) – (32), (41), (49), (51) – (56), (58), (63) – (67), (70) – (72), (78) – (81), (83), (84), (90), (92), (93), (117) – (121), (124), (127) – (148), (150), (151), (153), (155) – (157), (159), (160), (167) – (190), as follows,

$$\begin{aligned} \mu(1 + \tau_{C,t})C_{S,t} + \tilde{P}_{I,t}I_t + B_t + F_t &= \frac{R_{t-1}}{\pi_t}B_{t-1} \dots \\ &\dots + \frac{R_{t-1}}{\pi_t}F_{t-1} + \mu(1 - \tau_{\ell,t})\tilde{W}_t\ell_t \dots \\ &\dots + (1 - \tau_{K,t})R_{K,t}v_t\bar{K}_{t-1} + \mu Z_{S,t} + D_t, \end{aligned} \quad (191)$$

$$D_t = \tilde{P}_{H,t}Y_t - R_{K,t}K_t - \tilde{W}_t\ell_t + \tilde{P}_{F,t}M_t - \mathcal{Q}_t \left( M_t + A_t^{\frac{1}{1-\alpha}}\psi_M \right), \quad (192)$$

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + A_{I,t} \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad (193)$$

$$K_{G,t} = (1 - \delta_G) K_{G,t-1} + G_{I,t}, \quad (194)$$

$$\lambda_t = \frac{(C_{S,t}^* - \theta C_{S,t-1}^*)^{-\gamma}}{1 + \tau_{C,t}}, \quad (195)$$

$$C_{S,t}^* = C_{S,t} + \varphi_G G_t + \varphi_{G_I} G_{I,t}, \quad (196)$$

$$\begin{aligned} \left( \frac{\phi_W}{\eta_t - 1} \right) \pi_{W,t} [\pi_{W,t} - \tilde{\pi}_{W,t}] &= \left( \frac{\eta_t}{\eta_t - 1} \right) \kappa_t \left( \frac{\ell_t^\xi}{\lambda_t \tilde{W}_t} \right) - 1 + \tau_{\ell,t} \dots \\ &\dots + E_t \left\{ \left( \frac{\phi_W}{\eta_t - 1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{W,t+1}^2}{\pi_{t+1}} \frac{\ell_{t+1}}{\ell_t} [\pi_{W,t+1} - \tilde{\pi}_{W,t+1}] \right\}, \end{aligned} \quad (197)$$

$$(1 - \tau_{\ell,t}) \tilde{W}_t = \kappa_t \frac{L_t^\xi}{\lambda_t}, \quad (198)$$

$$\Upsilon_t = 1 - \frac{\ell_t}{L_t}, \quad (199)$$

$$\begin{aligned} \tilde{P}_{I,t} &= Q_t A_{I,t} \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) \dots \\ &\dots + E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} A_{I,t+1} \phi_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\}, \end{aligned} \quad (200)$$

$$\lambda_t = E_t \left\{ \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \exp(A_{B,t}) \right\}, \quad (201)$$

$$R_t = E_t \left\{ \frac{S_{t+1}}{S_t} R_t^* \exp \left( -\phi_S \left( \frac{F_t}{\tilde{P}_{H,t} Y_t} - \frac{F}{\tilde{P}_H Y} \right) + A_{S,t} \right) \right\}, \quad (202)$$

$$\lambda_t Q_t = E_t \left\{ \beta \lambda_{t+1} \left( \begin{aligned} &(1 - \tau_{K,t+1}) R_{K,t+1} v_{t+1} \dots \\ &\dots - \tilde{P}_{I,t+1} \left[ \psi_1 (v_{t+1} - 1) + \frac{\psi_2}{2} (v_{t+1} - 1)^2 \right] + Q_{t+1} (1 - \delta) \end{aligned} \right) \right\}, \quad (203)$$

$$(1 - \tau_{K,t}) R_{K,t} = \tilde{P}_{I,t} [\psi_1 + \psi_2 (v_t - 1)], \quad (204)$$

$$(1 + \tau_{C,t}) C_{N,t} = (1 - \tau_{\ell,t}) \tilde{W}_t \ell_t + Z_{N,t}, \quad (205)$$

$$C_t = \mu C_{S,t} + (1 - \mu) C_{N,t}, \quad (206)$$

$$Z_t = \mu Z_{S,t} + (1 - \mu) Z_{N,t}, \quad (207)$$

$$Y_t = A_t K_t^\alpha \ell_t^{1-\alpha} K_{G,t-1}^{\alpha_G} - A_t^{\frac{1}{1-\alpha}} \psi_H, \quad (208)$$

$$R_{K,t} = \Omega_t \alpha A_t K_t^{\alpha-1} \ell_t^{1-\alpha} K_{G,t-1}^{\alpha_G}, \quad (209)$$

$$\tilde{W}_t = A_{L,t} \Omega_t (1 - \alpha) A_t K_t^\alpha \ell_t^{-\alpha} K_{G,t-1}^{\alpha_G}, \quad (210)$$

$$\begin{aligned} \left( \frac{\phi_P}{\varepsilon - 1} \right) \pi_{H,t} [\pi_{H,t} - \tilde{\pi}_{H,t}] &= \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{\Omega_t}{\tilde{P}_{H,t}} - \exp(A_{P,t}) \dots \\ &\dots + E_t \left\{ \left( \frac{\phi_P}{\varepsilon - 1} \right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{H,t+1}^2}{\pi_{t+1}} \frac{Y_{t+1}}{Y_t} [\pi_{H,t+1} - \tilde{\pi}_{H,t+1}] \right\}, \end{aligned} \quad (211)$$

$$\left(\frac{\phi_F}{\varepsilon-1}\right) \pi_{F,t} [\pi_{F,t} - \tilde{\pi}_{F,t}] = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{Q_t}{\tilde{P}_{F,t}} - \exp(A_{T,t}) \dots$$

$$\dots + E_t \left\{ \left(\frac{\phi_F}{\varepsilon-1}\right) \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{F,t+1}^2}{\pi_{t+1}} \frac{M_{t+1}}{M_t} [\pi_{F,t+1} - \tilde{\pi}_{F,t+1}] \right\}, \quad (212)$$

$$\tilde{P}_{H,t} = \left[ (1 - \nu_C) + \nu_C T_t^{1-\mu_C} \right]^{\frac{1}{\mu_C-1}}, \quad (213)$$

$$\frac{\tilde{P}_{I,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_I) + \nu_I T_t^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}}, \quad (214)$$

$$\frac{\tilde{P}_{G,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_G) + \nu_G T_t^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}, \quad (215)$$

$$\frac{\tilde{P}_{G_I,t}}{\tilde{P}_{H,t}} = \left[ (1 - \nu_{G_I}) + \nu_{G_I} T_t^{1-\mu_{G_I}} \right]^{\frac{1}{1-\mu_{G_I}}}, \quad (216)$$

$$Y_t = (1 - \nu_C) \tilde{P}_{H,t}^{-\mu_C} C_t + (1 - \nu_I) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t \dots$$

$$\dots + (1 - \nu_G) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t + (1 - \nu_{G_I}) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t} + M_t^*, \quad (217)$$

$$\frac{\tilde{P}_{H,t}}{Q_t} = A_{M^*,t} (\nu^*)^{\frac{1}{\mu^*}} \left( \frac{Y_t^*}{M_t^*} \right)^{\frac{1}{\mu^*}} \left( 1 - \frac{\phi_M^*}{2} \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right)^2 \right)^{\frac{-1}{\mu^*}} \left[ \begin{array}{l} 1 - \frac{\phi_M^*}{2} \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right)^2 \dots \\ \dots - \phi_M^* \left( \frac{M_t^*}{M_{t-1}^*} - 1 \right) \left( \frac{M_t^*}{M_{t-1}^*} \right) \end{array} \right], \quad (218)$$

$$M_t = \nu_C \tilde{P}_{F,t}^{-\mu_C} C_t + \nu_I \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t + \nu_G \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t + \nu_{G_I} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t}, \quad (219)$$

$$K_t = v_t \bar{K}_{t-1}, \quad (220)$$

$$\tilde{P}_{H,t} = \frac{\pi_{H,t}}{\pi_t} \tilde{P}_{H,t-1}, \quad (221)$$

$$\tilde{P}_{F,t} = \frac{\pi_{F,t}}{\pi_t} \tilde{P}_{F,t-1}, \quad (222)$$

$$\tilde{W}_t = \frac{\pi_{W,t}}{\pi_t} \tilde{W}_{t-1}, \quad (223)$$

$$Q_t = \frac{S_t}{S_{t-1}} \frac{\pi_t^*}{\pi_t} Q_{t-1}, \quad (224)$$

$$T_t = \frac{\tilde{P}_{F,t}}{\tilde{P}_{H,t}}, \quad (225)$$

$$\tilde{\pi}_{F,t} = \pi_{F,t-1}^{\iota_F} \pi_F^{1-\iota_F}, \quad (226)$$

$$\tilde{\pi}_{H,t} = \pi_{H,t-1}^{\iota_H} \pi_H^{1-\iota_H}, \quad (227)$$

$$\tilde{\pi}_{W,t} = \pi_{W,t-1}^{\iota_W} \pi_W^{1-\iota_W}, \quad (228)$$

$$B_t + \tau_{K,t} R_{K,t} v_t \bar{K}_{t-1} + \tau_{\ell,t} \tilde{W}_t \ell_t + \tau_{C,t} C_t = \frac{R_{t-1}}{\pi_t} B_{t-1} + \tilde{P}_{G,t} G_t + \tilde{P}_{G_I,t} G_{I,t} + Z_t + A_{SFA,t}, \quad (229)$$

$$R_t = \varrho (\mathbf{Z}_{1,t} R_t^* + (1 - \mathbf{Z}_{1,t}) R_{ELB}) + (1 - \varrho) (\mathbf{Z}_{2,t} R + (1 - \mathbf{Z}_{2,t}) R_t^*), \quad (230)$$



$$R_t^* = (R_{t-1}^*)^{\rho_R} \left( R \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t})$$

or

$$R_t^* = R_{t-1}^{\rho_{R^*}} \left( R \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1-\rho_{R^*}} \exp(\varepsilon_{R,t}) \quad (231)$$

$$\tau_{C,t} = \tau_C \exp(u_{C,t}), \quad (232)$$

$$\tau_{\ell,t} = \tau_\ell \left( \frac{Y_t}{Y} \right)^{\varphi_\ell} \left( \frac{B_{t-1}}{B} \right)^{\gamma_\ell} \exp(u_{\ell,t}), \quad (233)$$

$$\tau_{K,t} = \tau_K \left( \frac{Y_t}{Y} \right)^{\varphi_K} \left( \frac{B_{t-1}}{B} \right)^{\gamma_K} \exp(u_{K,t}), \quad (234)$$

$$G_t = G \left( \frac{Y_t}{Y} \right)^{-\varphi_G} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_G} \exp(u_{G,t}), \quad (235)$$

$$G_{I,t} = G_I \left( \frac{Y_t}{Y} \right)^{-\varphi_{G_I}} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_{G_I}} \exp(u_{G_I,t}), \quad (236)$$

$$Z_{S,t} = Z_S \left( \frac{Y_t}{Y} \right)^{-\varphi_{Z_S}} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_{Z_S}} \exp(u_{Z_S,t}), \quad (237)$$

$$Z_{N,t} = Z_N \left( \frac{Y_t}{Y} \right)^{-\varphi_{Z_N}} \left( \frac{B_{t-1}}{B} \right)^{-\gamma_{Z_N}} \exp(u_{Z_N,t}), \quad (238)$$

$$\mathfrak{R}_t = E_t \left\{ \frac{R_t}{\pi_{t+1}} \right\}, \quad (239)$$

$$u_{C,t} = \rho_{u_C} u_{C,t-1} + \varepsilon_{u_C,t}, \quad (240)$$

$$u_{K,t} = \rho_{u_K} u_{K,t-1} + \varepsilon_{u_K,t}, \quad (241)$$

$$u_{\ell,t} = \rho_{u_\ell} u_{\ell,t-1} + \varepsilon_{u_\ell,t}, \quad (242)$$

$$u_{Z_N,t} = \rho_{u_{Z_N}} u_{Z_N,t-1} + \varepsilon_{u_{Z_N},t}, \quad (243)$$

$$u_{Z_S,t} = \rho_{u_{Z_S}} u_{Z_S,t-1} + \varepsilon_{u_{Z_S},t}, \quad (244)$$

$$u_{G,t} = \rho_{u_G} u_{G,t-1} + \varepsilon_{u_G,t}, \quad (245)$$

$$u_{G_I,t} = \rho_{u_{G_I}} u_{G_I,t-1} + \varepsilon_{u_{G_I},t}, \quad (246)$$

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \quad (247)$$

$$Y_t^* = (Y^*)^{1-\rho_{Y^*}} (Y_{t-1}^*)^{\rho_{Y^*}} \exp(\varepsilon_{Y^*,t}), \quad (248)$$

$$R_t^* = (R^*)^{1-\rho_{R^*}} (R_{t-1}^*)^{\rho_{R^*}} \exp(\varepsilon_{R^*,t}), \quad (249)$$

$$\pi_t^* = (\pi^*)^{1-\rho_{\pi^*}} (\pi_{t-1}^*)^{\rho_{\pi^*}} \exp(\varepsilon_{\pi^*,t}), \quad (250)$$

$$A_{I,t} = A_{I,t-1}^{\rho_{A_I}} \exp(\varepsilon_{A_I,t}), \quad (251)$$

$$\kappa_t = \kappa_{t-1}^{\rho_\kappa} \kappa^{1-\rho_\kappa} \exp(\varepsilon_{\kappa,t}), \quad (252)$$

$$A_{B,t} = \rho_{A_B} A_{B,t-1} + \tilde{\varepsilon}_{A_B,t}, \quad (253)$$

$$A_{S,t} = \rho_{A_S} A_{S,t-1} + \varepsilon_{A_S,t}, \quad (254)$$

$$A_{T,t} = \rho_{A_T} A_{T,t-1} + \varepsilon_{A_T,t}, \quad (255)$$

$$A_{P,t} = \rho_{AP} A_{P,t-1} + \varepsilon_{AP,t}, \quad (256)$$

$$A_{SFA,t} = \rho_{SFA} A_{SFA,t-1} + \varepsilon_{SFA,t}, \quad (257)$$

$$A_{M^*,t} = A_{M^*,t-1}^{\rho_{AM^*}} \exp(\varepsilon_{AM^*,t}), \quad (258)$$

$$A_{L,t} = A_{L,t-1}^{\rho_{AL}} \exp(\varepsilon_{AL,t}), \quad (259)$$

$$\eta_t = \eta_{t-1}^{\rho_\eta} \eta^{1-\rho_\eta} \exp(\varepsilon_{\eta,t}), \quad (260)$$

$$\mathbf{X}_t (R_t - \mathbb{R}_t) + (1 - \mathbf{X}_t) (\tilde{\varepsilon}_{AB,t} - \varepsilon_{AB,t}) = 0, \quad (261)$$

$$T_{C,t} = \tau_{C,t} C_t, \quad (262)$$

$$T_{\ell,t} = \tau_{\ell,t} \tilde{W}_t \ell_t, \quad (263)$$

$$T_{K,t} = \tau_{K,t} R_{K,t} v_t \bar{K}_{t-1}, \quad (264)$$

$$\hat{Y}_t = 100 \times \log \left( \frac{Y_t}{Y} \right), \quad (265)$$

$$\hat{R}_t = 100 \times (R_t - R), \quad (266)$$

$$\hat{\pi}_t = 100 \times (\pi_t - \pi), \quad (267)$$

$$\hat{\pi}_{W,t} = 100 \times (\pi_{W,t} - \pi_W), \quad (268)$$

$$\hat{\Upsilon}_t = 100 \times (\Upsilon_t - \Upsilon), \quad (269)$$

$$\hat{I}_t = 100 \times \log \left( \frac{I_t}{I} \right), \quad (270)$$

$$\hat{C}_t = 100 \times \log \left( \frac{C_t}{C} \right), \quad (271)$$

$$\hat{M}_t^* = 100 \times \log \left( \frac{M_t^*}{M^*} \right), \quad (272)$$

$$\hat{G}_t = 100 \times \log \left( \frac{G_t}{G} \right), \quad (273)$$

$$\hat{G}_{I,t} = 100 \times \log \left( \frac{G_{I,t}}{G_I} \right), \quad (274)$$

$$\hat{Z}_t = 100 \times \log \left( \frac{Z_t}{Z} \right), \quad (275)$$

$$\hat{T}_{C,t} = 100 \times \log \left( \frac{T_{C,t}}{T_C} \right), \quad (276)$$

$$\hat{T}_{\ell,t} = 100 \times \log \left( \frac{T_{\ell,t}}{T_\ell} \right), \quad (277)$$

$$\hat{T}_{K,t} = 100 \times \log \left( \frac{T_{K,t}}{T_K} \right), \quad (278)$$

$$\frac{\widehat{B}_t}{Y_t} = 100 \times \left( \frac{B_t}{4 \times \tilde{P}_{H,t} Y_t} - \frac{B}{4 \times \tilde{P}_H Y} \right), \quad (279)$$

$$\frac{\widehat{S}_t}{S_{t-1}} = 100 \times \left( \frac{S_t}{S_{t-1}} - 1 \right), \quad (280)$$

$$\hat{Y}_t^* = 100 \times \log \left( \frac{Y_t^*}{Y^*} \right), \quad (281)$$

$$\hat{R}_t^* = 100 \times (R_t^* - R^*), \quad (282)$$

$$\hat{\pi}_t^* = 100 \times (\pi_t^* - \pi^*). \quad (283)$$

## F Steady State

I describe two approaches for solving the steady state model. The first approach is a short hand method that results in a pencil and paper solution of the steady state model. This approach can be used to solve the steady state when estimating the model and when steady state fiscal policy remains unchanged. When permanent changes are made to fiscal policy, the second more complicated approach must be used. This approach involves solving a system of non-linear equations using numerical methods.

Using the first method, I fix the steady state employment level and the real exchange rate, which allows me to find a pencil and paper expression for output. As a consequence I need to solve for the value of labour preferences,  $\kappa_t$ , that is consistent with the level employment is fixed at. With the level of employment fixed, an analytical expression for labour preferences exists. This method is useful for calibrating a baseline version of the model because the level of employment and fiscal ratios can easily be chosen to match calibration targets in the data.

When investigating counterfactual fiscal policy, the level of labour and GDP, along with all the other endogenous variables, need to adjust to changes in fiscal policy. This means holding the value of labour preferences constant and solving for the levels of employment and the real exchange rate that are consistent with the new fiscal policy. This is equivalent to solving for the level of employment and the real exchange rate consistent with the new policy. There is no pencil and paper solution with this approach, which leads to the solution of a system of two (non-linear) equations in two unknowns.

### F.1 Solving the Steady State by Fixing the Employment Level

I start by setting all the shock processes to the steady state values implied by their respective processes. Using equations (251), (240) - (247) and (253) - (259), I set the exogenous shock processes as follows,

$$A_t = A_{I,t} = A_{M^*,t} = A_{L,t} = 1, \quad (284)$$

$$A_{B,t} = A_{S,t} = A_{T,t} = A_{P,t} = A_{SFA,t} = u_{C,t} = u_{L,t} = u_{K,t} = u_{G,t} = u_{G_I,t} = u_{Z_S,t} = u_{Z_N,t} = 0. \quad (285)$$

I set the endogenous shock  $\tilde{\varepsilon}_{A_{B,t}}$  equal to 0 in the steady state,

$$\tilde{\varepsilon}_{A_{B,t}} = 0. \quad (286)$$

I set the tax rates equal to their calibrated steady state levels,

$$\tau_{C,t} = \tau_C, \quad (287)$$

$$\tau_{K,t} = \tau_K, \quad (288)$$

$$\tau_{\ell,t} = \tau_{\ell}. \quad (289)$$

The inflation and inflation indices are all set equal to the steady state inflation rate,

$$\pi_t = \pi_{H,t} = \tilde{\pi}_{H,t} = \pi_{W,t} = \tilde{\pi}_{W,t} = \pi_t^* = \tilde{\pi}_{F,t} = \tilde{\pi}_{F,t} = \pi. \quad (290)$$

I set the steady state real exchange rate to ensure that  $\tilde{P}_{F,t} = 1$ ,

$$Q_t = \tilde{P}_{F,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right). \quad (291)$$

Using equation (212) in steady state,

$$\tilde{P}_{F,t} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) Q_t = 1. \quad (292)$$

Using equation (213), I find an expression for  $\tilde{P}_{H,t}$  as a function of  $\tilde{P}_{F,t}$ ,

$$\tilde{P}_{H,t} = \left[ (1 - \nu_C) + \nu_C T_t^{1-\mu_C} \right]^{\frac{1}{\mu_C-1}}, \quad (293)$$

$$1 = \left[ (1 - \nu_C) \tilde{P}_{H,t}^{1-\mu_C} + \nu_C \tilde{P}_{F,t}^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}, \quad (294)$$

$$\tilde{P}_{H,t} = \left[ \frac{1 - \nu_C \tilde{P}_{F,t}^{1-\mu_C}}{1 - \nu_C} \right]^{\frac{1}{1-\mu_C}}. \quad (295)$$

With expressions for  $\tilde{P}_{H,t}$  and  $\tilde{P}_{F,t}$ , I can determine the terms of trade,

$$T_t = \frac{\tilde{P}_{F,t}}{\tilde{P}_{H,t}}. \quad (296)$$

The other relative prices can be determined using equations (214), (215) and (216), combined with the values for  $\tilde{P}_{H,t}$  and  $T_t$ ,

$$\tilde{P}_{I,t} = \tilde{P}_{H,t} \left[ (1 - \nu_I) + \nu_I T_t^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}}, \quad (297)$$

$$\tilde{P}_{G,t} = \tilde{P}_{H,t} \left[ (1 - \nu_G) + \nu_G T_t^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}, \quad (298)$$

$$\tilde{P}_{G_I,t} = \tilde{P}_{H,t} \left[ (1 - \nu_{G_I}) + \nu_{G_I} T_t^{1-\mu_{G_I}} \right]^{\frac{1}{1-\mu_{G_I}}}, \quad (299)$$

In steady state, the nominal exchange rate does not grow,

$$\frac{S_t}{S_t} = 1. \quad (300)$$

In steady state, equation (200) implies,

$$Q_t = \tilde{P}_{I,t}. \quad (301)$$

In steady state, equation (201) implies

$$R_t = \frac{\pi_t}{\beta}. \quad (302)$$

I assume that the steady state foreign interest rate is equal to the steady state domestic interest rate,

$$R_t^* = R_t. \quad (303)$$

The shadow interest rate is equal to the nominal interest rate in steady state,

$$R_t^* = R_t. \quad (304)$$

The real interest rate is determine by,

$$\mathfrak{R}_t = \frac{R_t}{\pi_t}. \quad (305)$$

I set the steady state capacity utilization rate to 1,

$$v_t = 1. \quad (306)$$

From equation (211), I obtain the steady state real marginal cost,

$$\Omega_t = \tilde{P}_{H,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right). \quad (307)$$

The fixed cost in the tradeables sector,  $\psi_H$ , is chosen to ensure that profits in the domestic tradeables sector equal 0 in equilibrium,

$$\tilde{P}_{H,t} Y_t - \Omega_t \left( Y_t + A_t^{\frac{1}{1-\alpha}} \psi_H \right) = 0. \quad (308)$$

Using equation (308),  $\psi_H$  can be written as a function of  $Y_t$ . This expression can then be used to substitute for the fixed cost of production in subsequent expressions, which is necessary to solve the steady state model. Substituting  $\tilde{P}_{H,t} \left( \frac{\varepsilon-1}{\varepsilon} \right)$  for  $\Omega_t$  in equation (308) and rearranging for  $\psi_H$ ,

$$\tilde{P}_{H,t} Y_t - \tilde{P}_{H,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( Y_t + A_t^{\frac{1}{1-\alpha}} \psi_H \right) = 0, \quad (309)$$

$$Y_t - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( Y_t + A_t^{\frac{1}{1-\alpha}} \psi_H \right) = 0, \quad (310)$$

$$Y_t - \left( \frac{\varepsilon - 1}{\varepsilon} \right) Y_t = \left( \frac{\varepsilon - 1}{\varepsilon} \right) A_t^{\frac{1}{1-\alpha}} \psi_H, \quad (311)$$

$$\psi_H = \left( \frac{1}{\varepsilon - 1} \right) \frac{Y_t}{A_t^{\frac{1}{1-\alpha}}}. \quad (312)$$

From equation (209), the demand for capital is given by,

$$R_{K,t} = \Omega_t \alpha A_t K_t^{\alpha-1} \ell_t^{1-\alpha} K_{G,t}^{\alpha_G}, \quad (313)$$

This can be rewritten as,

$$R_{K,t} = \Omega_t \alpha \left( \frac{Y_t + A_t^{\frac{1}{1-\alpha}} \psi_H}{K_t} \right), \quad (314)$$

Using equation (312),

$$R_{K,t} = \Omega_t \alpha \left( \frac{Y_t + \left( \frac{1}{\varepsilon-1} \right) Y_t}{K_t} \right), \quad (315)$$

$$R_{K,t} = \Omega_t \alpha \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{Y_t}{K_t}. \quad (316)$$

Substituting equation (307) into equation (316),

$$R_{K,t} = \tilde{P}_{H,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right) \alpha \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{Y_t}{K_t}. \quad (317)$$

Cancelling terms gives,

$$R_{K,t} = \tilde{P}_{H,t} \alpha \frac{Y_t}{K_t}, \quad (318)$$

From equation (203),

$$\lambda_t Q_t = \beta \lambda_t \left( (1 - \tau_{K,t}) R_{K,t} v_t + Q_t (1 - \delta) \right). \quad (319)$$

From equation (319), I obtain an expression for the steady state output capital ratio,

$$\frac{Y}{K} = \frac{Q_t (1 - \beta (1 - \delta))}{\tilde{P}_{H,t} \alpha \beta (1 - \tau_{K,t})}. \quad (320)$$

I use equation (318) to obtain the steady state rental/dividend rate,

$$R_{K,t} = \alpha \tilde{P}_{H,t} \frac{Y}{K}, \quad (321)$$

which I use to find the capacity utilization parameter,

$$\psi_1 = R_{K,t} (1 - \tau_{K,t}), \quad (322)$$

which leads to,

$$\psi_2 = \chi \psi_1. \quad (323)$$

I fix the number of employed workers,

$$\ell_t = \ell, \quad (324)$$

and find an expression for the steady state output level, starting from equation (208),

$$Y_t = A_t K_t^\alpha \ell_t^{1-\alpha} K_{G,t}^{\alpha_G} - A_t^{\frac{1}{1-\alpha}} \psi_H. \quad (325)$$

I substitute equation (312) for the fixed cost of production in equation (208) and rearrange for  $Y_t$ ,

$$Y_t = A_t K_t^\alpha \ell_t^{1-\alpha} K_{G,t}^{\alpha_G} - \left( \frac{1}{\varepsilon - 1} \right) Y_t, \quad (326)$$

$$Y_t \left( \frac{\varepsilon}{\varepsilon - 1} \right) = A_t K_t^\alpha \ell_t^{1-\alpha} K_{G,t}^{\alpha_G}. \quad (327)$$

I rewrite the production function in terms of  $\frac{K}{Y}$ , which has already been determined, and  $\frac{K_G}{Y}$ ,

$$Y_t^{1-\alpha-\alpha_G} \left( \frac{\varepsilon}{\varepsilon - 1} \right) = A_t \left( \frac{K}{Y} \right)^\alpha \ell_t^{1-\alpha} \left( \frac{K_G}{Y} \right)^{\alpha_G}, \quad (328)$$

$$Y_t = \left[ \left( \frac{\varepsilon - 1}{\varepsilon} \right) A_t \left( \frac{K}{Y} \right)^\alpha \left( \left( \frac{G_I}{Y} \right) \delta_G^{-1} \right)^{\alpha_G} \ell_t^{1-\alpha} \right]^{\frac{1}{1-\alpha-\alpha_G}}, \quad (329)$$

where I make use of  $K_{G,t} = \frac{G_{I,t}}{\delta_G}$  which can be obtained from equation (194) in steady state and I assume  $\frac{G_I}{Y}$  is a parameter that I set when calibrating the model.<sup>6</sup> With  $Y_t$  determined, I can solve for the remaining model variables. Using equation (210) and the steady state value of  $Y_t$  I can find the steady state real wage,

$$\tilde{W}_t = (1 - \alpha) \tilde{P}_{H,t} \frac{Y_t}{\ell_t}. \quad (330)$$

Steady state government consumption is obtained according to,

$$G_t = \left( \frac{G}{Y} \right) Y_t, \quad (331)$$

where  $\left( \frac{G}{Y} \right)$  is a parameter set when calibrating the model.<sup>7</sup> Steady state government investment is determined according to,

$$G_{I,t} = \left( \frac{G_I}{Y} \right) Y_t, \quad (332)$$

<sup>6</sup>Alternatively, I could have set the steady state value of  $G_{I,t}$  rather than the ratio of  $G_{I,t}$  to GDP, in which case equation (329) becomes,

$$Y_t = \left[ \left( \frac{\varepsilon - 1}{\varepsilon} \right) A_t \left( \frac{K}{Y} \right)^\alpha \left( G_{I,t} \delta_G^{-1} \right)^{\alpha_G} \right]^{\frac{1}{1-\alpha}} \ell_t.$$

<sup>7</sup>Alternatively I could have set the level of government spending directly. However, setting the ratio makes it easier to calibrate the model as this is one of the targets that is matched.

where  $\left(\frac{G_t}{Y}\right)$  is a parameter set when calibrating the model.<sup>8</sup> The steady state capital stock is found using equation (320),

$$K_t = \left(\frac{K}{Y}\right) Y_t. \quad (333)$$

I use equation (67) to determine the steady state level of the capital stock,

$$\bar{K}_t = \frac{K_t}{v_t}. \quad (334)$$

Using equation (193) in steady state, I obtain the value of investment,

$$I_t = \delta \bar{K}_t. \quad (335)$$

Net foreign debt is determined by,

$$F_t = \left(\frac{F}{\tilde{P}_H Y}\right) \tilde{P}_{H,t} Y_t, \quad (336)$$

where  $\left(\frac{F}{\tilde{P}_H Y}\right)$  is a parameter set when calibrating the model. Combining equations (191), (205), (229) and (206) and rearranging for  $C_t$  gives,

$$C_t = \tilde{P}_{H,t} Y_t - \tilde{P}_{G,t} G_t - \tilde{P}_{G_I,t} G_{I,t} - \tilde{P}_{I,t} I_t - \left(1 - \frac{R_t}{\pi_t}\right) F_t. \quad (337)$$

From equation (194) in steady state, I obtain,

$$K_{G,t} = \frac{G_I}{\delta_G}. \quad (338)$$

Net government debt is obtained according to,

$$B_t = \left(\frac{B}{\tilde{P}_H Y}\right) Y_t, \quad (339)$$

where  $\left(\frac{B}{\tilde{P}_H Y}\right)$  is a parameter set in the calibration of the model. I obtain the steady state level of transfers from equation (229),<sup>9</sup>

$$Z_t = B_t \left(1 - \frac{R_t}{\pi_t}\right) - G_t - G_{I,t} - \tau_{S,t} W_t \ell_t + \tau_{K,t} R_{K,t} K_t + \tau_{\ell,t} \tilde{W}_t \ell_t + \tau_{C,t} C_t. \quad (340)$$

I set transfers payments for the Ricardian household,

$$Z_{S,t} = Z_S, \quad (341)$$

where  $Z_S$  is a parameter set to calibrate the model. Using equation (207), I find the steady state level of transfers paid to the rule of thumb household,

$$Z_{N,t} = \frac{Z_t - \mu Z_{S,t}}{1 - \mu}. \quad (342)$$

I use equation (205) to find the steady state level of consumption by the rule of thumb household,

$$C_{N,t} = \frac{(1 - \tau_{\ell,t}) \tilde{W}_t \ell_t + Z_{N,t}}{1 + \tau_{C,t}}. \quad (343)$$

I use equation (206) to determine the steady state level of consumption by the Ricardian household,

$$C_{S,t} = \frac{C_t - (1 - \mu) C_{N,t}}{\mu}. \quad (344)$$

From equation (219), I obtain an expression for the steady state level of imports,

$$M_t = \nu_C \tilde{P}_{F,t}^{-\mu_C} C_t + \nu_I \left(\frac{\tilde{P}_{F,t}}{\tilde{P}_{I,t}}\right)^{-\mu_I} I_t + \nu_G \left(\frac{\tilde{P}_{F,t}}{\tilde{P}_{G,t}}\right)^{-\mu_G} G_t + \nu_{G_I} \left(\frac{\tilde{P}_{F,t}}{\tilde{P}_{G_I,t}}\right)^{-\mu_{G_I}} G_{I,t}. \quad (345)$$

Next, I find an expression for the fixed cost in the importer's production function. I choose  $\psi_M$  so that profits are zero in steady state,

$$\tilde{P}_{F,t} M_t - Q_t \left(M_t + A_t^{\frac{1}{1-\alpha}} \psi_M\right) = 0, \quad (346)$$

<sup>8</sup>As mentioned in an earlier footnote, I could set the value of  $G_{I,t}$  directly, but setting the ratio makes it easier to calibrate the model.

<sup>9</sup>I am assuming here that transfers payments are the residual in steady state, chosen to match the desired debt to GDP ratio and the other fiscal policy choices. It is possible to make other fiscal variables the residual or balancing item in steady state, although this would change around some of the ordering for the steady state solution.

$$\tilde{P}_{F,t}M_t - \tilde{P}_{F,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right) = 0, \quad (347)$$

$$M_t - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right) = 0, \quad (348)$$

$$\frac{1}{\varepsilon} M_t = \left( \frac{\varepsilon - 1}{\varepsilon} \right) A_t^{\frac{1}{1-\alpha}} \psi_M, \quad (349)$$

$$\psi_M = \left( \frac{1}{\varepsilon - 1} \right) \frac{M_t}{A_t^{\frac{1}{1-\alpha}}}. \quad (350)$$

From equation (192), I get an expression for the steady state profit levels,

$$D_t = \tilde{P}_{H,t}Y_t - R_{K,t}K_t - \tilde{W}_t\ell_t + \tilde{P}_{F,t}M_t - \mathcal{Q}_t \left( M_t + A_t^{\frac{1}{1-\alpha}} \psi_M \right). \quad (351)$$

I use equation (196) to obtain the consumption index,

$$C_{S,t}^* = C_{S,t} + \varphi_G G_t + \varphi_{G_I} G_{I,t}, \quad (352)$$

which allows a steady state expression for the shadow price of consumption to be found,

$$\lambda_t = \frac{(C_{S,t}^* (1 - \theta))^{-\gamma}}{1 + \tau_{C,t}}. \quad (353)$$

The steady state unemployment rate is only a function of the inverse of the Frisch labour supply elasticity and the elasticity of substitution between differentiated labour. I set the steady state unemployment rate and the labour supply elasticity and find the elasticity between differentiated varieties of labour.

$$\Upsilon_t = \Upsilon, \quad (354)$$

In steady state, equations (197) and (198) imply

$$\left( \frac{\eta_t}{\eta_t - 1} \right) \kappa_t \frac{\ell_t^\xi}{\lambda_t} = \kappa_t \frac{L_t^\xi}{\lambda_t} \quad (355)$$

Rearrange for  $\eta_t$ ,

$$\left( \frac{\eta_t}{\eta_t - 1} \right) \ell_t^\xi = L_t^\xi, \quad (356)$$

$$\left( \frac{\eta_t}{\eta_t - 1} \right) \ell_t^\xi = L_t^\xi, \quad (357)$$

$$(1 - \Upsilon_t)^\xi = 1 - \frac{1}{\eta_t}, \quad (358)$$

$$1 - (1 - \Upsilon_t)^\xi = \frac{1}{\eta_t}, \quad (359)$$

$$\eta_t = \frac{1}{1 - (1 - \Upsilon_t)^\xi}. \quad (360)$$

Using equation (197), I find an expression for the value of labour preferences,

$$\kappa_t = \left( \frac{\eta_t - 1}{\eta_t} \right) (1 - \tau_{\ell,t}) \left( \frac{\lambda_t \tilde{W}_t}{\ell_t^\xi} \right). \quad (361)$$

I rearrange equation (217) to find exports,  $M_t^*$ ,

$$M_t^* = Y_t - \left( \begin{array}{c} (1 - \nu_C) \tilde{P}_{H,t}^{-\mu_C} C_t + (1 - \nu_I) \left( \frac{\tilde{P}_{H,t}}{P_{I,t}} \right)^{-\mu_I} I_t \dots \\ \dots + (1 - \nu_G) \left( \frac{\tilde{P}_{H,t}}{P_{G,t}} \right)^{-\mu_G} G_t + (1 - \nu_{G_I}) \left( \frac{\tilde{P}_{H,t}}{P_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t} \end{array} \right). \quad (362)$$

I use equation (218) to find  $Y_t^*$ ,

$$Y_t^* = \frac{M_t^*}{A_{M^*,t}^{\mu^*} \nu^* \left( \frac{\tilde{P}_{H,t}}{\mathcal{Q}_t} \right)^{-\mu^*}}. \quad (363)$$

Total tax revenue is determined by equations (262), (263) and (264),

$$T_{C,t} = \tau_{C,t} C_t, \quad (364)$$

$$T_{\ell,t} = \tau_{\ell,t} \tilde{W}_t \ell_t, \quad (365)$$

$$T_{K,t} = \tau_{K,t} R_{K,t} v_t \bar{K}_t. \quad (366)$$

The measurement variables are all gaps and are equal to zero in steady state,

$$\begin{aligned} \hat{Y}_t = \hat{R}_t = \hat{\pi}_t = \hat{\pi}_{W,t} = \hat{\Upsilon}_t = \hat{I}_t = \hat{C}_t = \hat{M}_t^* = \hat{G}_t = \hat{G}_{I,t} = \hat{Z}_t = \dots \\ \dots \hat{T}_{C,t} = \hat{T}_{\ell,t} = \hat{T}_{K,t} = \frac{\widehat{B}_t}{\widehat{Y}_t} = \frac{\widehat{S}_t}{\widehat{S}_{t-1}} = \hat{Y}_t^* = \hat{R}_t^* = \hat{\pi}_t^* = 0. \end{aligned} \quad (367)$$

## F.2 Solving the Steady State With Fixed Labour Preferences

Using the second approach for solving the steady state, I fix the value for labour preferences,  $\kappa_t$ , and I solve for the steady state levels of GDP and the Terms of trade, which also means solving the level of employment and the real exchange rate, as well as all the other endogenous variables.

I start by setting the steady state values for the shock processes, the tax rates and the inflation rates using equations (284) through (290) and I set the nominal and real interest rates using equations (302) through (305).

A pencil and paper solution for the entire set of steady state model equations does not exist when this method is used, which means I need to solve a system of equations. This can be drastically simplified to a system of two equations in two unknowns,  $Y_t$  and  $T_t$ , as follows,

$$\epsilon_{1,t} = \left( \frac{\eta_t}{\eta_t - 1} \right) \kappa_t \frac{\ell_t^\xi}{\lambda_t \tilde{W}_t} - 1 + \tau_{\ell,t}, \quad (368)$$

$$\epsilon_{2,t} = Y_t - \begin{pmatrix} (1 - \nu_C) \tilde{P}_{H,t}^{-\mu_C} C_t + (1 - \nu_I) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t \dots \\ \dots + (1 - \nu_G) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t \dots \\ \dots + (1 - \nu_{G_I}) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t} + M_t^* \end{pmatrix}, \quad (369)$$

where equations (368) and (369) follow from equations (361) and (217) respectively. Note that  $T_t$  does not appear directly in equations (368) and (369) but is important for determining a number of variables that appear in these equations as illustrated by the equations below. I find values of output  $Y_t$  and the terms of trade  $T_t$  so that  $\epsilon_{1,t} = 0$  and  $\epsilon_{2,t} = 0$ , using a root finding algorithm. I set the following variables in these equations according to,

$$\kappa_t = \kappa, \quad (370)$$

$$\tau_{\ell,t} = \tau_\ell, \quad (371)$$

$$\tau_{K,t} = \tau_K, \quad (372)$$

$$\tau_{C,t} = \tau_C, \quad (373)$$

$$Y_t^* = Y^*, \quad (374)$$

$$\Upsilon_t = \Upsilon. \quad (375)$$

I either set the levels or the ratios of government consumption, government investment and transfers,

$$G_t = G, \quad \text{or} \quad G_t = \left( \frac{G}{Y} \right) Y_t, \quad (376)$$

$$G_{I,t} = G_I, \quad \text{or} \quad G_{I,t} = \left( \frac{G_I}{Y} \right) Y_t. \quad (377)$$

$$Z_t = Z, \quad \text{or} \quad Z_t = \left( \frac{Z}{Y} \right) Y_t, \quad (378)$$

depending whether the government targets the level or ratio of spending. The other variables in the system of two equations are determined by equations (360), (295), (297), (298), (299), (296), (291), (301), (307), (320), (338), (327), (334), (335), (210), (337), (343), (344), (353) and (218) as follows,

$$\eta_t = \frac{1}{1 - (1 - \Upsilon_t)^\xi}, \quad (379)$$

$$\tilde{P}_{H,t} = \left[ (1 - \nu_C) + \nu_C T_t^{1-\mu_C} \right]^{\frac{1}{\mu_C-1}}, \quad (380)$$

$$\tilde{P}_{I,t} = \tilde{P}_{H,t} \left[ (1 - \nu_I) + \nu_I T_t^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}}, \quad (381)$$

$$\tilde{P}_{G,t} = \tilde{P}_{H,t} \left[ (1 - \nu_G) + \nu_G T_t^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}, \quad (382)$$

$$\tilde{P}_{G_I,t} = \tilde{P}_{H,t} \left[ (1 - \nu_{G_I}) + \nu_{G_I} T_t^{1-\mu_{G_I}} \right]^{\frac{1}{1-\mu_{G_I}}}, \quad (383)$$

$$\tilde{P}_{F,t} = T_t \tilde{P}_{H,t}, \quad (384)$$

$$Q_t = \tilde{P}_{F,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right), \quad (385)$$

$$Q_t = \tilde{P}_{I,t}, \quad (386)$$



$$\Omega_t = \tilde{P}_{H,t} \left( \frac{\varepsilon - 1}{\varepsilon} \right), \quad (387)$$

$$\frac{Y}{\bar{K}} = \frac{Q_t - Q_t \beta (1 - \delta)}{\tilde{P}_{H,t} \alpha \beta (1 - \tau_{K,t})}, \quad (388)$$

$$K_{G,t} = \delta_G^{-1} G_{I,t}, \quad (389)$$

$$\frac{Y}{\bar{\ell}} = \left( \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( \frac{K}{\bar{Y}} \right)^\alpha K_{G,t}^{\alpha_G} \right)^{\frac{1}{1-\alpha}}, \quad (390)$$

$$K_t = \left( \frac{K}{\bar{Y}} \right) Y_t, \quad (391)$$

$$\bar{K}_t = \frac{K_t}{v_t}, \quad (392)$$

$$I_t = \delta \bar{K}_t, \quad (393)$$

$$\ell_t = \left( \frac{Y}{\bar{\ell}} \right)^{-1} Y_t, \quad (394)$$

$$\tilde{W}_t = (1 - \alpha) \Omega_t K_t^\alpha \ell_t^{-\alpha} K_{G,t}^{\alpha_G}, \quad (395)$$

$$F_t = \left( \frac{F}{\bar{Y}} \right) \tilde{P}_{H,t} Y_t, \quad (396)$$

$$C_t = \tilde{P}_{H,t} Y_t - \tilde{P}_{G,t} G_t - \tilde{P}_{G_I,t} G_{I,t} - \tilde{P}_{I,t} I_t - F_t \left( 1 - \frac{R_t}{\pi_t} \right), \quad (397)$$

$$C_{N,t} = \frac{(1 - \tau_{\ell,t}) \tilde{W}_t \ell_t + Z_{N,t}}{1 + \tau_{C,t}}, \quad (398)$$

$$C_{S,t} = \frac{C_t - (1 - \mu) C_{N,t}}{\mu}, \quad (399)$$

$$C_t^* = C_{S,t} + \phi_G G_t + \phi_{G_I} G_{I,t}, \quad (400)$$

$$\lambda_t = \frac{(C_t^* (1 - \theta))^{-\gamma}}{1 + \tau_{C,t}}, \quad (401)$$

$$M_t^* = A_{M^*,t}^{\mu^*} \nu^* \left( \frac{\tilde{P}_{H,t}}{Q_t} \right)^{-\mu^*} Y_t^*. \quad (402)$$

When values of  $T_t$  and  $Y_t$  have been determined such that the residuals of equations (368) and (369) are sufficiently small, I can use equations (380), (384), (381), (382) and (383) to determine  $\tilde{P}_{H,t}$ ,  $\tilde{P}_{F,t}$ ,  $\tilde{P}_{I,t}$ ,  $\tilde{P}_{G,t}$  and  $\tilde{P}_{G_I,t}$ . The real exchange rate,  $Q_t$ , is determined using equation (385), real marginal costs are determined by equation (387), Tobin's Q is determined by equation (386). Government consumption and government investment are determined by equations (376) and (378) and the public capital stock is determined by equation (389). The private capital stock is determined by dividing output by  $\frac{K}{\bar{Y}}$  from equation (388), while private investment is determined by equation (393). The number employed is determined by equation (394), while real wages and the real rental/dividend rate are determined by equations (395) and (318). Exports are determined by equation (402) and aggregate consumption is determined by rearranging equation (217) to obtain,

$$C_t = \frac{Y_t - \left( (1 - \nu_I) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{I,t}} \right)^{-\mu_I} I_t + (1 - \nu_G) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G,t}} \right)^{-\mu_G} G_t + (1 - \nu_{G_I}) \left( \frac{\tilde{P}_{H,t}}{\tilde{P}_{G_I,t}} \right)^{-\mu_{G_I}} G_{I,t} + M_t^* \right)}{(1 - \nu_C) \tilde{P}_{H,t}^{-\mu_C}}. \quad (403)$$

Transfers to the Ricardian household are set according to equation (341) while transfers to rule of thumb consumers are determined by equation (342). Consumption by the rule of thumb household is determined by equation (343) and consumption by the Ricardian household is determined by equation (344).  $C_{S,t}^*$  is determined by equation (352) while the marginal utility of consumption  $\lambda_t$  is determined by equation (353). Imports are determined by equation (219) and net foreign debt is determined by equation (396). Foreign output is determined by equation (374) and net government debt is determined by,

$$B_t = \left( \tau_K v_1 \bar{K}_t + \tau_{\ell,t} \tilde{W}_t \ell_t + \tau_{C,t} C_t - Z_t - \tilde{P}_{G,t} G_t - \tilde{P}_{G_I,t} G_{I,t} \right) \left( \frac{R_t}{\pi_t} - 1 \right)^{-1}. \quad (404)$$

Note that net government debt is the residual or balancing item in steady state. Other fiscal variables could be made the residual, although this would change the ordering of the solution. Consumption tax revenue, labour tax revenue and capital income tax revenue are determined by equations (364), (365), and (366). The steady states for the measurement variables are determined by equation (367).

## G Prior Predictive Analysis Parameter Comparisons

I carry out prior predictive analysis in Section 6 of this paper under a number of different monetary policy assumptions. In nearly all cases, the estimated government consumption bands fall outside the 99% prior bands. To understand which parameters are causing some of the estimated multipliers to be higher or lower than the 99% prior bands, I compare the distribution of the estimated parameters that generate draws that fall outside the prior bands with those that always fall within the prior bands. Figures 16 through 19 plot the distributions of the estimated parameters that fall within the 99% prior bands for the government consumption multipliers (red) against the distributions of the estimated parameters that fall outside the 99% prior bands for at least 1 quarter (blue).

Figure 16: Normal Times: Inside vs Outside Parameter Distributions



Figure 17: Counterfactual Government Consumption Multipliers



Figure 18: Counterfactual Government Consumption Multipliers



Figure 19: Counterfactual Government Consumption Multipliers



## H Monte Carlo Filtering Results

Figure 20: Normal Times Impact

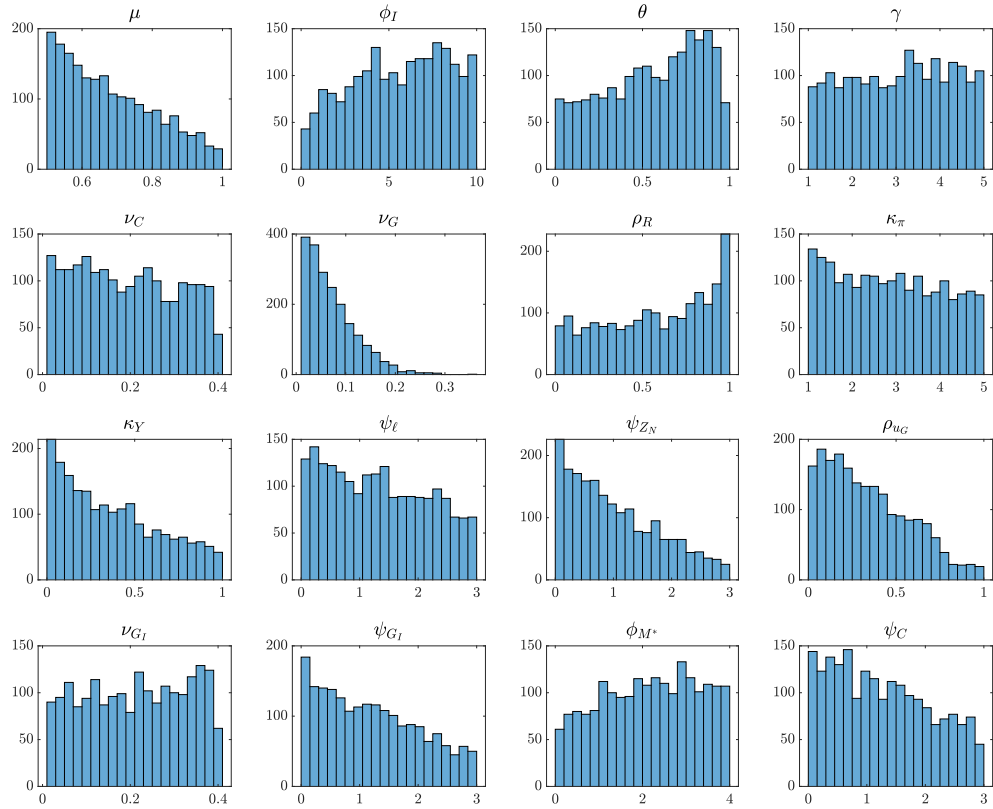


Figure 21: Normal Times 2 Years

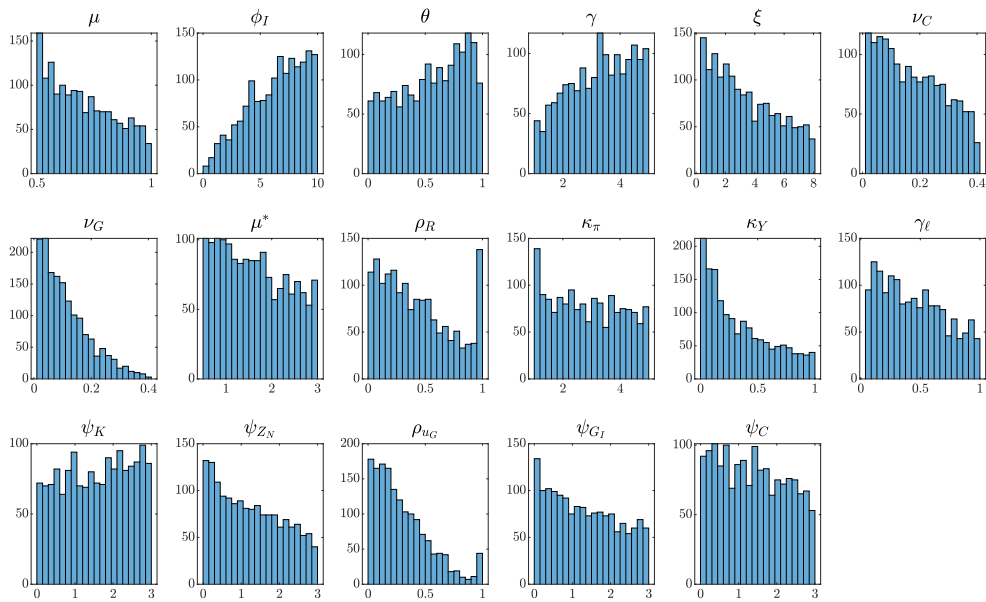


Figure 22: ELB Lagged Shadow Rate Impact

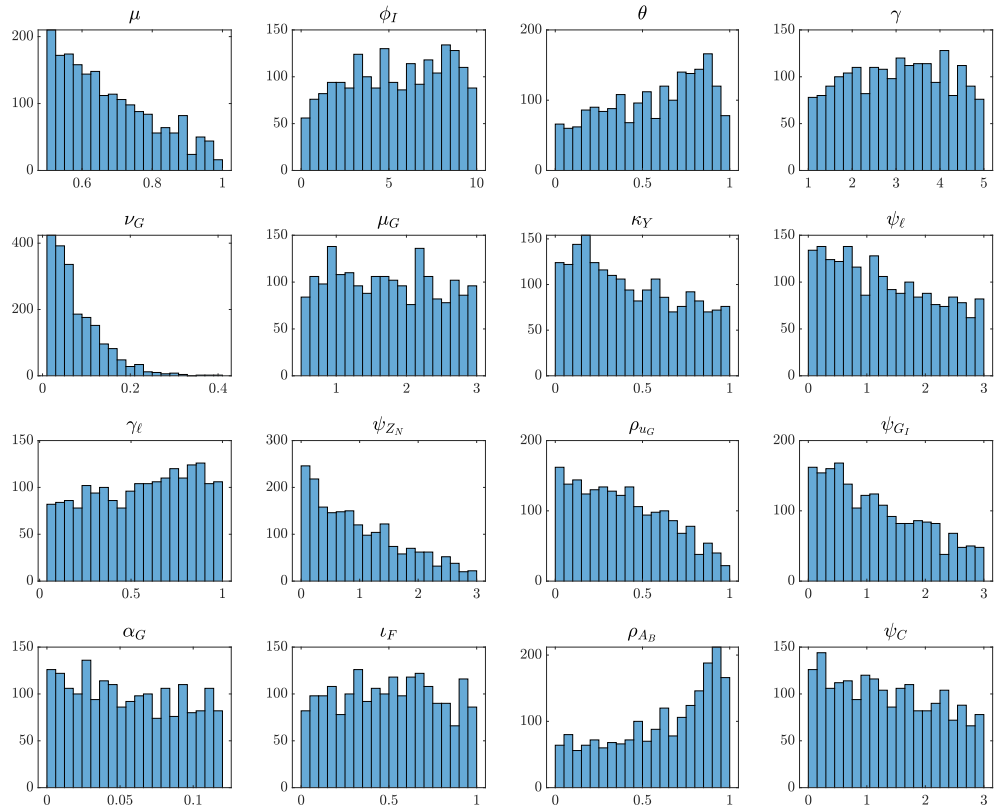


Figure 23: ELB Lagged Shadow Rate Two Years

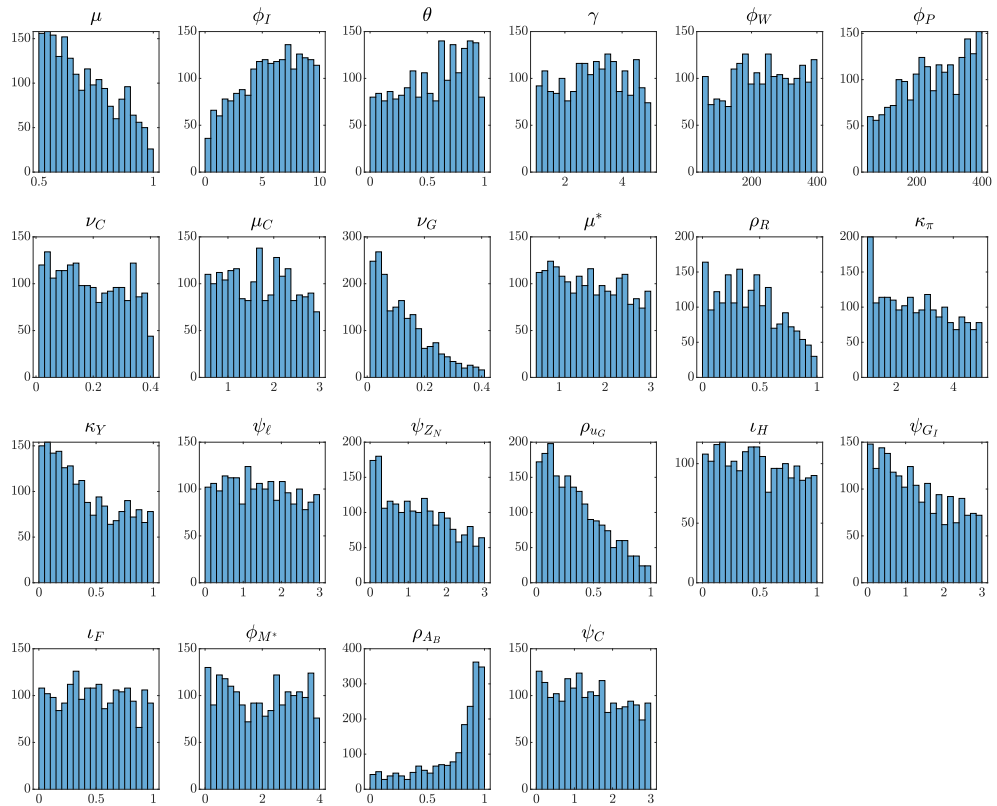




Figure 24: ELB Lagged Actual Rate Impact

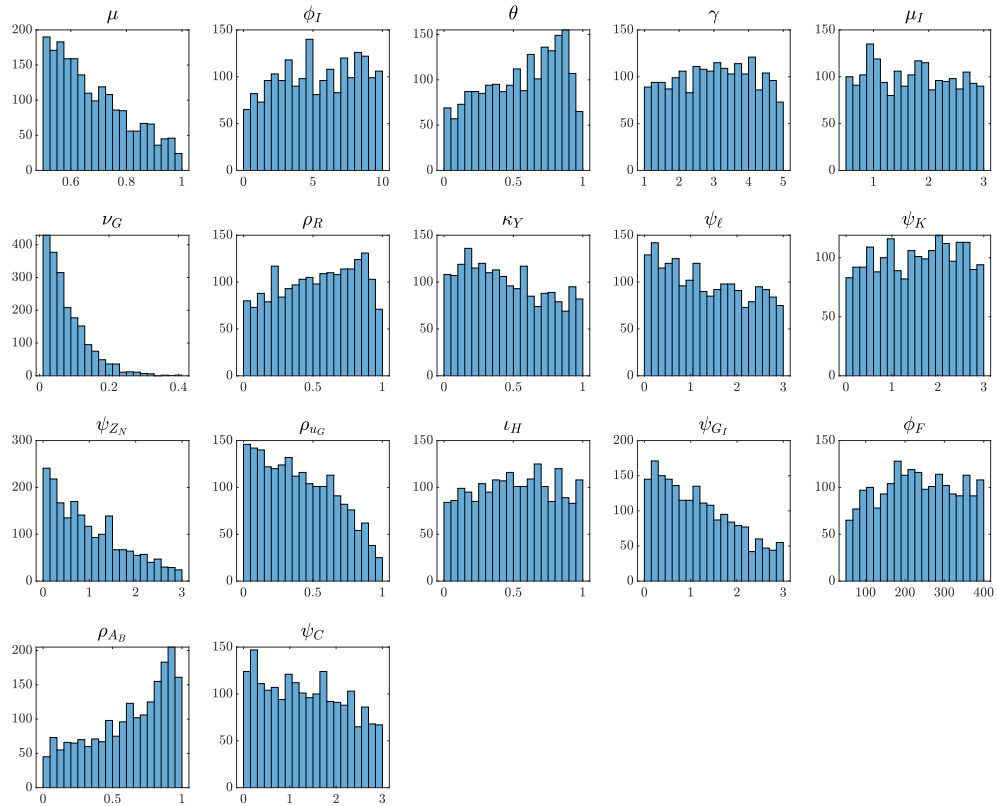


Figure 25: ELB Lagged Actual Rate Two Years

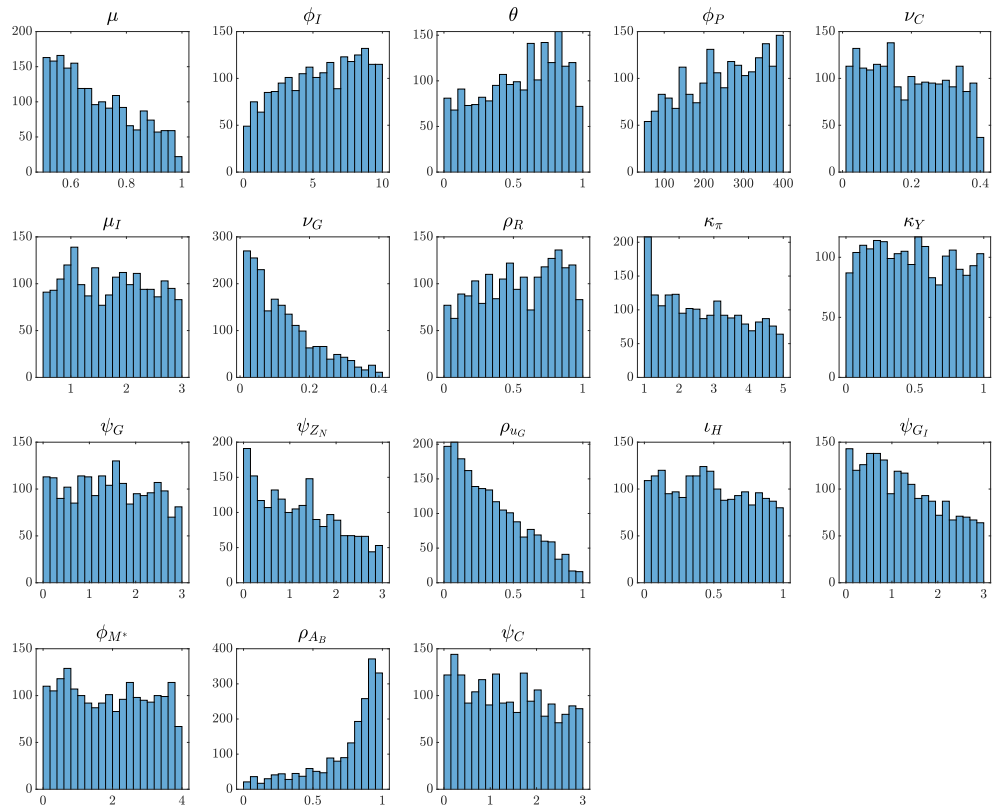


Figure 26: Four Quarter Interest Rate Peg Impact

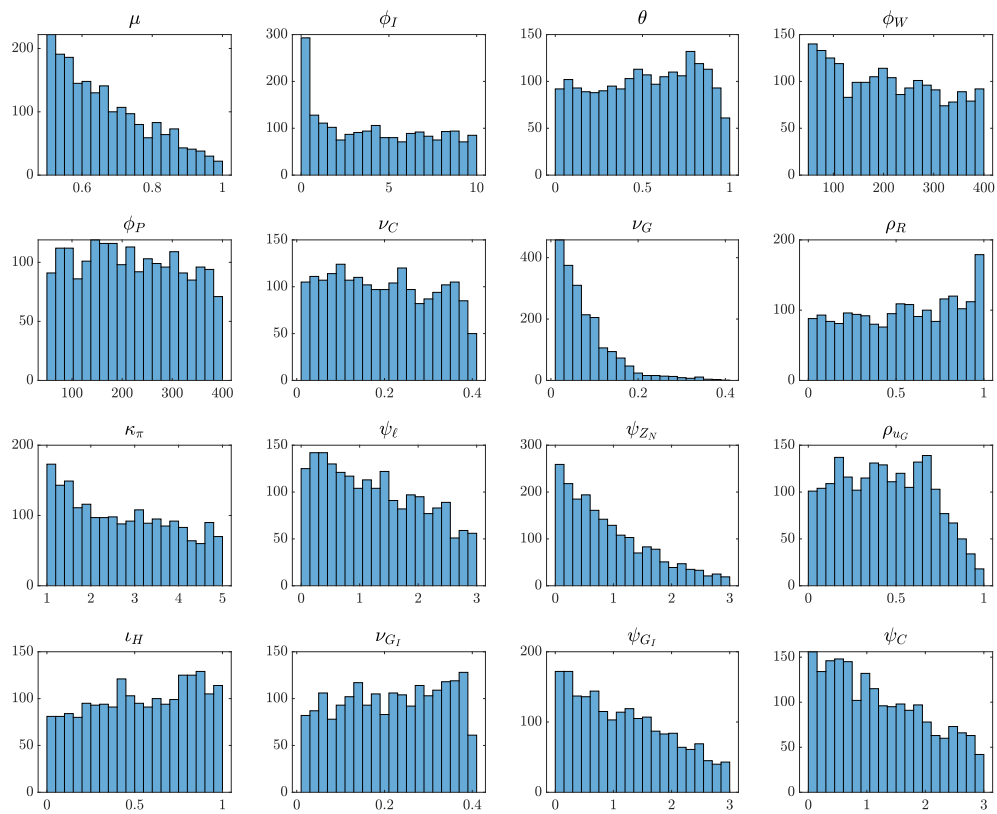


Figure 27: Four Quarter Interest Rate Peg Two Years

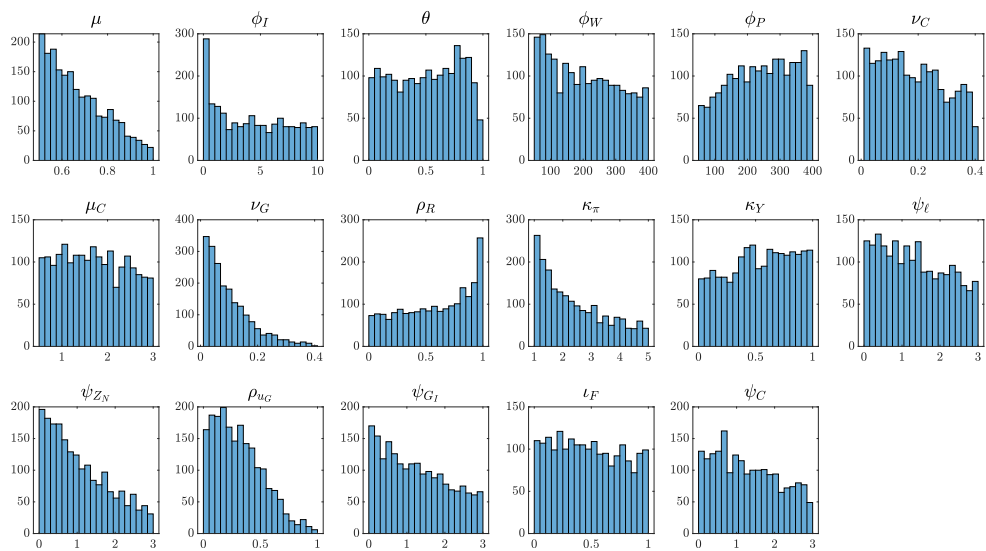


Figure 28: Eight Quarter Interest Rate Peg Impact

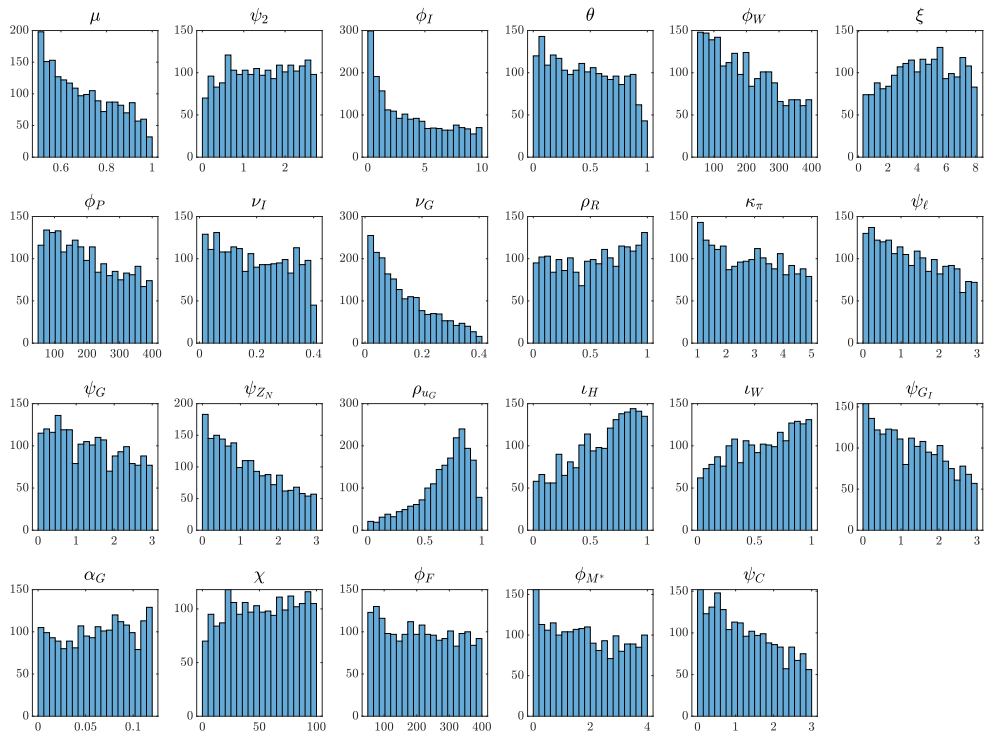
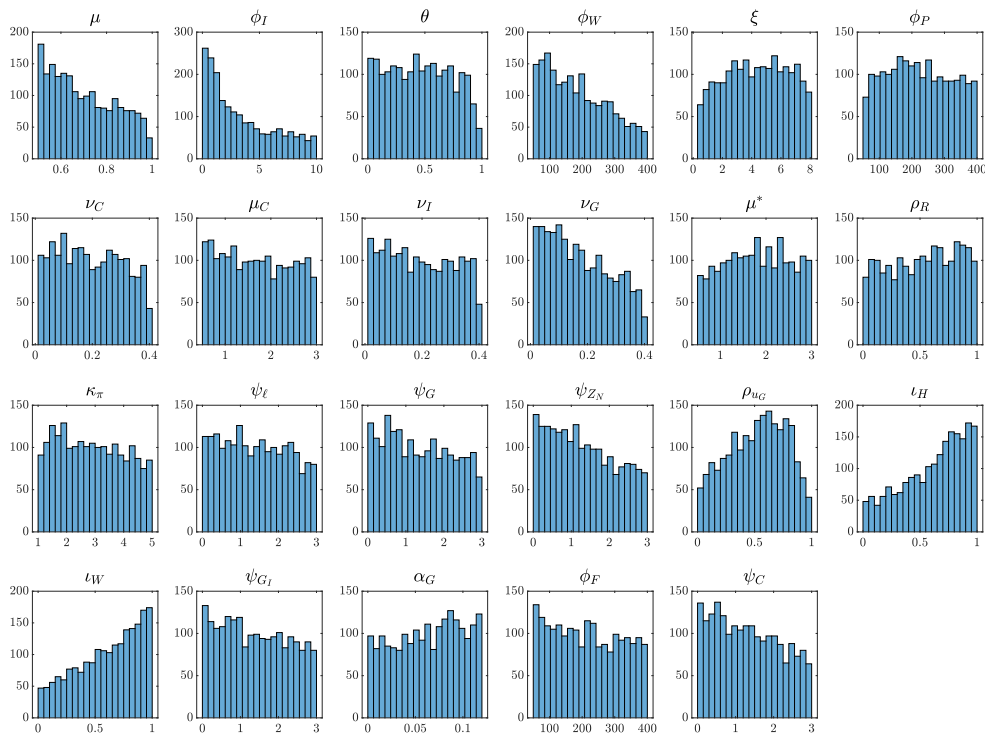


Figure 29: Eight Quarter Interest Rate Peg Two Years



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