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## Evaluating fiscal policy reforms using the fiscal frontier

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# Evaluating Fiscal Policy Reforms using the Fiscal Frontier

*JRC Working Papers on  
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Reforms No 02/2023*

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## **Executive summary**

- We develop a Fiscal Frontier which traces out the most advantageous trade-off between sustaining any level of government debt and social welfare.
- We use the Fiscal Frontier to evaluate a variety of fiscal reforms demonstrating how each proposal compares with alternatives, and how far it lies from the optimum implied by the Frontier.

## ***Policy context***

- Government debt has rapidly grown since 2007 in many advanced economies. In the US, it is today at an historical high since World War II.
- This has led to a renewed interest in the sustainability of public finances and in the nature of desirable fiscal reforms.

## ***Main findings***

- We use the Fiscal Frontier to evaluate the following set of fiscal reforms: (1) dynamic scoring considered in Laffer curve calculations, (2) a gradual reduction in capital taxation proposed by Lucas (1990), and (3) fiscal consolidation strategies akin to those considered by the Congressional Budget Office on the behalf of the US congress.
- Policy reforms underpinning classical Laffer curve calculations are highly inefficient for they fail to vary the level of tax instruments and the composition of the tax mix over time. Therefore, they fail to place the tax burden where it is less distortionary in consideration of how the elasticity of production factors varies over time.
- On the other hand, an appropriately time-phased capital tax reform is found to be very efficient. While the optimal degree of gradualism in capital tax reduction has changed over time, through decades we found the reform to remain very close to the optimum implied by the Frontier.
- Finally, we consider alternative debt reduction targets. These are costly, especially when the required fiscal consolidation is relatively rapid, but the costs of even the worst package of debt reduction measures can be more than offset by simultaneously introducing a capital tax reform.
- While some of the results above mentioned are dependent on the specific model we employ in our analysis, the Fiscal Frontier is a general tool that can be applied to any model (provided that duality applies). Using the Frontier, fiscal reforms can be evaluated and compared along the key dimensions of the policy maker problem, i.e: how much fiscal surplus do they raise and at what welfare cost?

# Evaluating Fiscal Policy Reforms using the Fiscal Frontier\*

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## Abstract

We develop a Fiscal Frontier which traces out the maximum government debt level that can be sustained at a given welfare cost. Through duality, the intertemporal policy mix underpinning the Frontier mirrors standard Ramsey policy and defines an upper limit on the welfare gains that can be achieved by any fiscal reform. The Frontier is then used to evaluate a variety of fiscal reforms: (1) one-off changes in tax instruments considered in Laffer curve calculations, (2) a gradual reduction in capital taxation proposed by Lucas (1990), and (3) fiscal consolidation strategies akin to those considered by the Congressional Budget Office. Conventional Laffer curve calculations significantly under-estimate the sustainable debt of the US. The desirable pace of capital tax abolition has slowed since the 1970s, but the reform remains close to the Frontier. Achieving debt reduction targets considered by the Congressional Budget Office is typically very costly, especially when the fiscal consolidation is large and must be achieved quickly, but a simultaneous capital tax reform can more than offset those costs in all cases we consider.

**Keywords:** Laffer Curve, Optimal Policy, Debt Sustainability, Fiscal Limit, Duality

**JEL Codes:** E62, H30, H60

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# 1 Introduction

The rapid growth in debt levels around the world has led to increased interest in both the sustainability of public finances and the nature of desirable fiscal reforms. We develop a Fiscal Frontier which traces out the maximum sustainable debt level that can be achieved at a given welfare cost. We demonstrate that there is a duality between this problem and a standard Ramsey tax problem. As a result, the surplus maximization problem which underpins the Fiscal Frontier implies a policy mix which mirrors standard Ramsey fiscal policies. Therefore, the Fiscal Frontier shows the most advantageous trade-off between sustaining any level of government debt and social welfare. It can be used as a framework for the evaluation of any conceivable policy reform, demonstrating how a particular proposal compares with alternatives, and how far it lies from the optimum implied by the Frontier.

To illustrate the applicability of this approach we consider three sets of potential reforms. Firstly, we consider the experiment which underpins traditional Laffer curve/Fiscal Limit calculations. Namely, a permanent one-off change in a specific tax instrument. By comparing the traditional Laffer curve experiments with the Fiscal Frontier, we can assess the welfare costs of failing to exploit changes in the policy mix over time. We show that the potential gains from adopting the time-varying multiple-instrument fiscal reforms implied by the Ramsey policy are huge.<sup>1</sup> Secondly, we assess the policy of replacing capital taxation with labor taxation proposed by Lucas (1990), following the insights of Chamley (1986) and Judd (1985). Here we consider the optimal degree of gradualism in adopting Lucas (1990)'s proposal - should we have a short sharp shock or a more gradual adjustment? We find that the fiscal situation at the time the reform is implemented is important. The policy advice that we would have given President Reagan in 1980 is different from the policy advice we would give President Biden today. However, in both cases an appropriately time-phased abolition of capital taxation generates a welfare benefit close to the potential maximum defined

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<sup>1</sup>Since Laffer curve calculations are not intended to mimic optimal policy, this should not be taken as a criticism of this literature.

by the Fiscal Frontier. Thirdly, we consider alternative debt reduction targets similar to those analyzed by the Congressional Budget Office (CBO) on behalf of the US Congress. These are costly, especially when the required fiscal consolidation is both large and/or relatively rapid, but the costs of even the worst package of debt reduction measures we consider can be more than offset by simultaneously introducing a capital tax reform.

The Fiscal Frontier is a reinterpretation of the optimal taxation literature which serves as a benchmark for any fiscal reform or assessment you care to think of including dynamic scoring, fiscal consolidation, Laffer curves or fiscal limits. In doing so, our paper provides a bridge between two largely distinct literatures. Firstly, since the Fiscal Frontier is underpinned by a set of Ramsey fiscal policy problems, it is clearly related to the optimal policy literature (see Chari and Kehoe (1999) for a survey, and Dyrda and Pedroni (2023) for a recent example). As a result, many of the insights from this literature, such as the Chamley (1986) and Judd (1985) result on capital taxation reform in the absence of heterogeneity, will apply to the policies implicit in the frontier. However, it also moves beyond this literature in assessing how those policy recommendations change as inherited debt levels change. This is clearly important given how rapidly government debt levels can change, rising from 63% of GDP immediately prior to the financial crisis in 2007, to 134% in Q2 2020, following the Covid-19 pandemic.<sup>2</sup>

Secondly, since the Frontier can also be interpreted as the maximum discounted surplus that can be generated at a given welfare cost, it can serve as a benchmark against which other policy reforms can be contrasted. Therefore, it also links to any literature offering a quantitative model-based assessment of fiscal policy which is not necessarily fully optimal. This can include, but is not limited to, discussions of fiscal sustainability (for a survey see D’Erasmus *et al.* (2015)), dynamic scoring (Mankiw and Weinzierl (2006) and Leeper and Yang (2008)), fiscal consolidation (Alesina and Passalacqua (2016) and Brinca *et al.* (2021)), time-consistency in fiscal policy (Klein and Rios Rull (2003), Balke and Ravn (2016) and Laczó and Rossi (2020)), Laffer curves (Trabandt and Uhlig (2011), Mendoza *et al.* (2014) and Fève

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<sup>2</sup>See <https://fred.stlouisfed.org/series/GFDEGDQ188S>



*et al.* (2018)), fiscal rules (Schmitt-Grohé and Uribe (2007) and Alfaro and Kanczuk (2017)), fiscal reforms (Auray *et al.* (2016), Fotiou *et al.* (2020) and Malley and Philippopoulos (2023)) and fiscal limits (Bi (2012), Davig *et al.* (2011) and Bi *et al.* (2016)). These analyses can be focused on the revenue generating capabilities of alternative fiscal policies or on their welfare properties. By reducing the modelled economy’s revenue generating capabilities to a single measure of discounted fiscal surplus, and the costs of multiple tax distortions to a single welfare measure, the Fiscal Frontier offers a framework within which to compare and contrast this wide range of results. It should be noted that much of this work will have been focused on exploring economic mechanisms rather than making policy recommendations. Therefore finding that a particular piece of policy analysis generates outcomes which lie well below the Frontier should not be taken as a criticism of that work, but as a spur to explain why the implicit policies perform badly and design alternative reforms which remain feasible, while supporting debt sustainability at a lower welfare cost.

The remainder of the paper is structured as follows. In Section 2, we present the features of our model economy. In Section 3, we describe the fiscal surplus maximization which underpins the construction of the Fiscal Frontier. In Section 4, we discuss the calibration strategy and present a Fiscal Frontier for the US economy. Section 5 then contrasts our Fiscal Frontier with the policy reforms implied by conventional Laffer curve calculations. Section 6 considers a policy reform of gradually eliminating capital taxation inspired by Lucas (1990). Section 7 assesses various approaches to debt reduction similar to those considered by the CBO (2018). Across Sections 5-7 the Fiscal Frontier allows us to rank the reforms considered and assess whether any significant gains to further reform remain. Section 8 concludes.

## 2 The Model Economy

In the following, we outline our model which extends the neoclassical growth model of Trabandt and Uhlig (2011) to include variable capacity utilization of

capital. This is essentially a closed economy version of the model considered in Mendoza *et al.* (2014).<sup>3</sup> The economy features exogenous growth, at rate  $\gamma$ , which is driven by labor-augmenting technological change. Accordingly, all variables (except labor, leisure and the interest rate) are rendered stationary by dividing them by the level of technology.<sup>4</sup> This stationarity-inducing transformation of the model requires discounting the re-scaled utility flows at the rate  $\tilde{\beta} = \beta(1 + \gamma)^{1-\sigma}$  where  $\beta$  is the standard subjective discount factor of time-separable preferences, and adjusting the laws of motion of physical and financial assets so that date  $t+1$  stocks are scaled by the balanced-growth factor  $1 + \gamma$ .

## 2.1 Households

The utility function of the representative household in our economy is

$$\sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, 1 - l_t), \quad (1)$$

where we assume the period utility function is a standard CRRA function in terms of a CES composite good made of consumption,  $c_t$ , and leisure,  $1 - l_t$  given by,

$$U(c_t, 1 - l_t) = \frac{[c_t(1 - l_t)^a]^{1-\sigma}}{1 - \sigma}, \sigma > 1, \text{ and, } a > 0.$$

The household's budget constraint is given by,

$$(1 + \tau_t^c)c_t + x_t + (1 + \gamma)q_t d_{t+1} = (1 - \tau_t^l)w_t l_t + (1 - \tau_t^k)r_t m_t k_t + d_t + e_t, \quad (2)$$

where  $\tau_t^c$ ,  $\tau_t^l$  and  $\tau_t^k$  are proportional tax rates on consumption,  $c_t$ , labor income,  $w_t l_t$ , and capital income,  $r_t m_t k_t$ , respectively. Households also receive a lump-sum transfer from the government,  $e_t$ , which is treated as being ex-

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<sup>3</sup>As our method is quite general, we could easily extend our framework to include, for example, the open economy dimension or other externalities. However, our aim is to demonstrate the application of the concept of the Fiscal Frontier in a simple environment that still implies a non-trivial optimal tax problem.

<sup>4</sup>We could have presented the model in its non-stationary form and then undertaken the transformation of the equilibrium conditions. This is equivalent to undertaking the scaling by technology when setting up the model, as we do.

ogenous and is held at its steady-state value ( $e_t = \bar{e}$ ). Finally, the household saves in the form of government bonds,  $d_{t+1}$ , which are priced at  $q_t$ , as well as physical capital,  $k_{t+1}$ .

Gross investment,  $x_t$ , is defined as,

$$x_t = (1 + \gamma)k_{t+1} - [1 - \delta(m_t)] k_t, \quad (3)$$

where the depreciation rate depends on the rate of capital utilization  $m_t$  as follows,

$$\delta(m_t) = \frac{\chi_0 m_t^{\chi_1}}{\chi_1}, \quad \chi_0 > 0 \text{ and } \chi_1 > 1 \quad (4)$$

The household chooses the path of consumption, leisure, government bonds, investment and the rate of capital utilization to maximize utility (1) subject to the budget constraint (2) and the law of motion for capital (3). Its optimization yields the following set of first order conditions.<sup>5</sup> The consumption Euler equation,

$$(1 + \gamma)q_t = \tilde{\beta} \frac{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})}{U'_{c_t}(c_t, 1 - l_t)} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c}, \quad (5)$$

consumption-leisure margin,

$$-\frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{c_t}(c_t, 1 - l_t)} = \frac{1 - \tau_t^l}{1 + \tau_t^c} w_t, \quad (6)$$

gross investment,

$$\begin{aligned} & \frac{U'_{c_t}(c_t, 1 - l_t)}{(1 + \tau_t^c)} [1 + \gamma] \\ = & \tilde{\beta} \frac{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})}{(1 + \tau_{t+1}^c)} \left[ 1 - \delta(m_{t+1}) + (1 - \tau_{t+1}^k) r_{t+1} m_{t+1} \right], \end{aligned} \quad (7)$$

and, finally, the capital utilization condition,

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<sup>5</sup>We use the notation  $f'_{x_i}(\cdot)$  to denote the partial derivative of function  $f(\cdot)$  with respect to argument  $x_i$ .

$$(1 - \tau_t^k)r_t k_t = \delta'_{m_t}(m_t)k_t. \quad (8)$$

## 2.2 Firms

Firms rent labor,  $l_t$ , and capital services,  $v_t$ , from households at a given wage,  $w_t$ , and capital rental rate,  $r_t$ , to maximize profits,

$$\Pi_t = y_t - w_t l_t - r_t v_t,$$

subject to a production function which is assumed to be of the Cobb-Douglas form,

$$y_t = F(v_t, l_t) = v_t^{1-\alpha} l_t^\alpha.$$

The firms' maximization problem gives rise to standard first order conditions

$$F'_{v_t}(v_t, l_t) = r_t, \quad (9)$$

and

$$F'_{l_t}(v_t, l_t) = w_t, \quad (10)$$

while linear homogeneity implies  $y_t = w_t l_t + r_t v_t$ .

## 2.3 Public Sector

The government's budget constraint is given by,

$$b_t - (1 + \gamma)q_t b_{t+1} = s_t, \quad (11)$$

where  $b_t$  is one-period bonds issued by the government, and the primary surplus,  $s_t$ , is defined as,

$$s_t = \tau_t^c c_t + \tau_t^l w_t l_t + \tau_t^k r_t m_t k_t - (g_t + \bar{e}),$$

and government consumption,  $g_t$ , is set to its steady-state value  $g_t = \bar{g}$ .<sup>6</sup> Iterating on this budget constraint and imposing the transversality condition,

$$\lim_{t \rightarrow \infty} \tilde{\beta}^{t+1} U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1}) b_{t+1} = 0$$

we obtain the government's intertemporal budget constraint,

$$b_0 = \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_0}(c_0, 1 - l_0)} s_t. \quad (12)$$

## 2.4 Market Clearing

Market clearing in the goods market requires,

$$F(s_t, l_t) = c_t + g_t + (1 + \gamma)k_{t+1} - [1 - \delta(m_t)] k_t, \quad (13)$$

while capital and bond market clearing imply,

$$m_t k_t = v_t \quad (14)$$

and

$$d_t = b_t. \quad (15)$$

## 2.5 The Competitive Equilibrium

The equilibrium of our model consists of a sequence of prices  $\{w_t, r_t, q_t\}_{t=0}^{\infty}$ , government policy  $\{\tau_t^c, \tau_t^l, \tau_t^k, b_{t+1}\}_{t=0}^{\infty}$  and allocations  $\{c_t, l_t, v_t, x_t, m_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$  such that:

- $\{c_t, l_t, x_t, m_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$  solves the households' problem given prices and government policy;
- $\{v_t, l_t\}_{t=0}^{\infty}$  solves firms' problem given prices;

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<sup>6</sup>Although government expenditures are treated as being exogenous, the fact that tax policy can affect real interest rates provides an additional channel through which the policy maker can influence the discounted value of fiscal surpluses. Therefore, our optimal policy seeks to maximise the discounted fiscal surplus, rather than simply tax revenues.

- The government's budget constraint (11) holds for all  $t \geq 0$ ;
- All markets clear as in (13), (14) and (15).

The definition above implies that for any government policy  $\{\tau_t^c, \tau_t^l, \tau_t^k, b_{t+1}\}_{t=0}^\infty$ , satisfying the government budget constraint (11), we have a different competitive equilibrium. In Section 3, we describe the optimal policy problem that selects the policy corresponding to the government's desired equilibrium. However, before considering such a problem, we need to put some structure on what instruments the government has access to.

The distortionary taxes in our model act on three margins. The first margin is the intratemporal consumption-leisure decision obtained by combining the first order conditions (6) and (10),

$$-\frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{c_t}(c_t, 1 - l_t)} = \frac{1 - \tau_t^l}{1 + \tau_t^c} F'_{l_t}(m_t k_t, l_t). \quad (16)$$

The second margin is the intertemporal investment decision which is obtained by combining equations (7) and (9),

$$\begin{aligned} & \frac{U'_{c_t}(c_t, 1 - l_t)}{(1 + \tau_t^c)} [1 + \gamma] \\ = & \tilde{\beta} \frac{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})}{(1 + \tau_{t+1}^c)} \left[ (1 - \tau_t^k) F'_{s_{t+1}}(m_{t+1} k_{t+1}, l_{t+1}) m_{t+1} + 1 - \delta(m_{t+1}) \right]. \end{aligned} \quad (17)$$

Finally, combining equations (8) and (9) gives rise to the third margin, namely, the capital utilization condition,

$$(1 - \tau_t^k) F'_{s_t}(m_t k_t, l_t) k_t = \delta'_{m_t}(m_t) k_t. \quad (18)$$

The labor tax,  $\tau_t^l$ , distorts the first margin; the consumption tax,  $\tau_t^c$ , distorts the first and second, while the capital tax,  $\tau_t^k$ , affects the latter two. In the case of the intratemporal consumption-leisure decision, if labor income is subsidized (taxed) at the same rate as the policy maker taxes (subsidizes) consumption (i.e.  $-\tau_t^l = \tau_t^c$ ), it will eliminate this distortion.<sup>7</sup> Furthermore,

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<sup>7</sup>Whether the policy maker finds optimal to subsidize labor or consumption depends on which tax base is larger.

if such a tax/subsidy on consumption and labor is set to be constant over time, it will not distort the intertemporal investment decision nor any other margin of this economy. Given that those taxes and subsidies are then applied to different tax bases, the policy maker will be able to raise revenues without any welfare costs. It would, effectively, give the Ramsey policy maker access to a lump-sum tax thereby rendering the policy problem trivial. Since the US economy makes only limited use of consumption taxes, we rule out this possibility by fixing  $\tau_t^c$  at a calibrated value consistent with the data,  $\tau_t^c = \tau^c$ . Therefore, the capital and labor tax rates are the only fiscal instruments available to the optimizing policy maker. Appendix A summarizes the decentralized equilibrium conditions after imposing this assumption.

### 3 The Surplus Maximization Problem

In this section, we consider the details of the surplus maximization problem for our model economy. This problem represents the highest possible discounted value of fiscal surplus that can be generated at a given welfare cost. We then show that this problem can be mapped into the more familiar Ramsey tax problem, given that strong Lagrangian duality holds. Finally, we discuss the main features of the surplus maximizing policy which, given duality, will be in line with the optimal taxation literature.

#### 3.1 The Primal Form

Under the surplus maximization problem, the policy maker chooses the sequences of labor and capital taxes,  $\{\tau_t^l, \tau_t^k\}_{t=0}^\infty$ , so as to maximize the discounted value of the fiscal surplus, subject to our model economy and a given level of welfare loss.<sup>8</sup> This will enable us to trace out a Fiscal Frontier which describes how the maximum sustainable level of government debt varies as we increase the welfare costs of the distortionary taxes underpinning that debt. It is important to note that the objective of this problem allows for the

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<sup>8</sup>This problem is time inconsistent and we assume that government has access to a commitment technology.

endogeneity of the real interest rate, such that the policy maker factors this into the policy problem. This implies that they are not simply maximizing fiscal revenues, but the present value of the fiscal surplus, which is affected by discounting even when government expenditure is treated as being exogenous.

We follow Lucas and Stokey (1983) in writing this problem in the primal form that solves for allocations only. Once allocations have been determined, prices and policy can be recovered from the competitive economy's equilibrium conditions. The surplus maximization problem consists of maximizing the discounted fiscal surplus in (12), which is equivalent to the implementability constraint for the Ramsey problem,

$$b_0 = \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_0}(c_0, 1 - l_0)} (1 + \tau^c) \left[ c_t - \frac{\bar{e}}{1 + \tau^c} + \frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{c_t}(c_t, 1 - l_t)} l_t \right] - B \quad (19)$$

where

$$B \equiv [(1 - \tau_0^k) F'_{m_0 k_0}(m_0 k_0, l_0) m_0 + 1 - \delta(m_0)] k_0$$

that collects the initial level of capital.<sup>9</sup>

Under the primal form of the surplus maximization problem, the policy maker maximizes the discounted value of the primary balance expressed in (19), subject to four constraints. The first is the resource constraint implied by the market clearing conditions in the goods (13) and capital (14) markets, respectively,

$$F(m_t k_t, l_t) - c_t - \bar{g} - (1 + \gamma) k_{t+1} + (1 - \delta(m_t)) k_t = 0 \quad (20)$$

The second constraint is due to the presence of endogenous capacity utilization. It is obtained by combining the intertemporal investment decision (17) and capital utilization condition (18) after leading the latter forward one

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<sup>9</sup>See Appendix B for full derivation of the implementability constraint.



period,

$$\frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})} = \frac{\tilde{\beta}}{1 + \gamma} (\delta'_{m_{t+1}}(m_{t+1})m_{t+1} + 1 - \delta(m_{t+1})) \quad (21)$$

However, by moving the capital utilization condition (18) one-period forward, we omitted this condition at period-0 in the second constraint. Therefore, we reintroduce the period-0 capital utilization condition as a third constraint,

$$(1 - \tau_0^k) F'_{m_0 k_0}(m_0 k_0, l_0) = \delta'_{m_0}(m_0) \quad (22)$$

Finally, the fourth constraint is the welfare constraint which requires the policy maker to achieve a given level of social welfare,  $\bar{W}$ ,

$$W_0 = \sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, 1 - l_t) = \bar{W} \quad (23)$$

The Lagrangian function of the surplus maximization problem is constructed as follows,

$$\begin{aligned} & \max_{\{c_t, l_t, m_t, k_{t+1}, \tau_0^k\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_0}(c_0, 1 - l_0)} \left[ c_t - \frac{\bar{e}}{1 + \tau^c} + \frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{c_t}(c_t, 1 - l_t)} l_t \right] (1 + \tau^c) \right. \\ & + \zeta_t^2 [F(m_t k_t, l_t) - c_t - \bar{g} - (1 + \gamma) k_{t+1} + (1 - \delta(m_t)) k_t] \\ & + \zeta_t^1 \left[ \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})} \frac{1 + \gamma}{\tilde{\beta}} - 1 - \delta'_{m_{t+1}}(m_{t+1})m_{t+1} + \delta(m_{t+1}) \right] \left. \right\} - B \\ & + \zeta^0 [(1 - \tau_0^k) F'_{m_0 k_0}(m_0 k_0, l_0) - \delta'_{m_0}(m_0)] \\ & + \psi \left[ \sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, 1 - l_t) - \bar{W} \right] \end{aligned}$$

where  $\zeta^0$ ,  $\zeta_t^1$ ,  $\zeta_t^2$  and  $\psi$  are the four multipliers attached to the period-0 capital utilization condition, the capital utilization constraint for  $t \geq 1$ , the resource constraint and welfare constraint, respectively.

The above Lagrangian can be re-expressed in a more compact form with the new objective function,  $\tilde{V}(c_t, 1 - l_t, \psi, \zeta_t^1)$ , collecting all terms involving

the utility function,

$$\begin{aligned}\tilde{V}(c_t, 1 - l_t, \psi, \zeta_t^1) &\equiv U(c_t, 1 - l_t) \\ &+ \frac{\zeta_t^1}{\psi} \left[ \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_{t+1}}(c_{t+1}, 1 - l_{t+1})} \frac{1 + \gamma}{\tilde{\beta}} \right] \\ &+ \frac{1}{\psi} \left[ \frac{U'_{c_t}(c_t, 1 - l_t)}{U'_{c_0}(c_0, 1 - l_0)} \left( c_t - \frac{\bar{e}}{1 + \tau^c} \right) + \frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{l_0}(c_0, 1 - l_0)} l_t \right] (1 + \tau^c)\end{aligned}$$

The more compact expression of Lagrangian function is, thus, given by

$$\begin{aligned}\max_{\{c_t, l_t, m_t, k_{t+1}, \tau_0^k\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \psi \left\{ \tilde{V}(c_t, 1 - l_t, \psi, \zeta_t^1) \right. \\ \left. + \frac{\zeta_t^1}{\psi} \left[ -\delta'_{m_{t+1}}(m_{t+1}) m_{t+1} - 1 + \delta(m_{t+1}) \right] \right. \\ \left. + \frac{\zeta_t^2}{\psi} \left[ F(m_t k_t, l_t) - c_t - \bar{g} - (1 + \gamma) k_{t+1} + (1 - \delta(m_t)) k_t \right] \right\} \\ - \psi \mathcal{A}\end{aligned}$$

where

$$\mathcal{A} \equiv \bar{W} + \frac{1}{\psi} B - \frac{\zeta^0}{\psi} \left[ (1 - \tau_0^k) F'_{m_0 k_0}(m_0 k_0, l_0) - \delta'_{m_0}(m_0) \right]$$

collects the period-0 capital utilization condition, the welfare constraint,  $\bar{W}$  and terms in the initial level of capital in  $B$ .

The first order conditions for  $t \geq 0$  are:

$$\{c_t\} : \tilde{V}'_{c_t}(c_{t-1}, 1 - l_{t-1}, \psi, \zeta_{t-1}^1) + \tilde{\beta} \tilde{V}'_{c_t}(c_t, 1 - l_t, \psi, \zeta_t^1) = \tilde{\beta} \frac{\zeta_t^2}{\psi} \quad (24)$$

$$\{l_t\} : \tilde{V}'_{l_t}(c_{t-1}, 1 - l_{t-1}, \psi, \zeta_{t-1}^1) + \tilde{\beta} \tilde{V}'_{l_t}(c_t, 1 - l_t, \psi, \zeta_t^1) = -\tilde{\beta} \frac{\zeta_t^2}{\psi} F'_{l_t}(m_t k_t, l_t) \quad (25)$$

$$\{m_t\} : \frac{\zeta_{t-1}^1}{\psi} \delta''_{m_t}(m_t) m_t = \tilde{\beta} \frac{\zeta_t^2}{\psi} \left[ F'_{m_t k_t}(m_t k_t, l_t) - \delta'_{m_t}(m_t) \right] k_t \quad (26)$$

$$\{k_{t+1}\} : \tilde{\beta} \frac{\zeta_{t+1}^2}{\psi} \left[ F'_{m_{t+1} k_{t+1}}(m_{t+1} k_{t+1}, l_{t+1}) m_{t+1} + 1 - \delta(m_{t+1}) \right] = \frac{\zeta_t^2}{\psi} (1 + \gamma) \quad (27)$$

$$\{c_0\} : \tilde{V}'_{c_0}(c_0, 1 - l_0, \psi, \zeta_0^1) = \frac{\zeta_0^2}{\psi} \quad (28)$$

$$\{l_0\} : \tilde{V}'_{l_0}(c_0, 1 - l_0, \psi, \zeta_0^1) + \frac{\zeta_0^2}{\psi} F'_{l_0}(m_0 k_0, l_0) - \mathcal{A}'_{l_0} = 0 \quad (29)$$

$$\{m_0\} : \frac{\zeta_0^2}{\psi} \left[ F'_{m_0 k_0}(m_0 k_0, l_0) - \delta'_{m_0}(m_0) \right] k_0 - \mathcal{A}'_{m_0} = 0 \quad (30)$$

$$\{\tau_0^k\} : \frac{1}{\psi} F'_{m_0 k_0}(m_0 k_0, l_0) m_0 k_0 - \frac{\zeta_0^0}{\psi} F'_{m_0 k_0}(m_0 k_0, l_0) = 0 \quad (31)$$

The above set of first order conditions (24)-(31) and the four constraints (20)-(23) characterize the solution to the surplus maximization problem.

To save space in the main text, the description of the standard Ramsey problem has been moved to Appendix C. When strong Lagrangian duality holds and the solution to the surplus maximization and the standard Ramsey problem are unique, their first-order conditions are equivalent.<sup>10</sup> We find this to be the case in our model. Therefore, the surplus maximization problem has properties which are familiar from the Ramsey optimal taxation literature.

## 3.2 Duality

In this subsection, we define duality between the surplus maximization problem and standard Ramsey problem, and explore the properties of this dual relationship. It should be noted that we cannot demonstrate analytically that duality holds, but do check for equivalence between the two problems numerically for each computation we undertake. We also rule out other potential solutions to the FOCs which do not satisfy duality.

We begin by representing the surplus maximization and standard Ramsey problems in Table 1. Through the Lagrangian function we show that, when strong Lagrangian duality applies, on the Fiscal Frontier the surplus maximization problem for our economy admits a dual in the form of a welfare maximization problem. We discuss how the properties of duality of the two problems inform the numerical checks we conduct to confirm duality holds

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<sup>10</sup>This is a standard result in convex optimization see, e.g., Boyd and Vandenberghe (2004).

in constructing the Fiscal Frontier.

Table 1: Surplus Maximization and Ramsey

Surplus Maximization	Welfare Maximization
$\max_{\mathbf{x}_t} \sum_{t=0}^{\infty} \tilde{\beta}^t R_{0,t} s_t(\mathbf{x}_t)$	$\max_{\mathbf{x}_t} \sum_{t=0}^{\infty} \tilde{\beta}^t U_t(\mathbf{x}_t)$
Subject to:	Subject to:
$h_t^i(\mathbf{x}_t) = 0, i = 0, 1, 2$	$h_t^i(\mathbf{x}_t) = 0, i = 0, 1, 2$
$\sum_{t=0}^{\infty} \tilde{\beta}^t U_t(\mathbf{x}_t) \geq \bar{W}$	$\sum_{t=0}^{\infty} R_{0,t} s_t(\mathbf{x}_t) \geq \bar{b}$

In the first column of Table 1, we present the surplus maximization problem for our model economy.  $R_{0,t} = \frac{U'_{c_t}(c_t, 1-l_t)}{U'_{c_0}(c_0, 1-l_0)}$  is the discount factor applied to future values of the primary surplus,  $s_t(\mathbf{x}_t)$  in (12), which can be expressed as a function of the endogenous variables,  $\mathbf{x}_t = \{c_t, l_t, m_t, k_{t+1}, \tau_0^k\}$  in the primal form of equation (19). The policy maker maximizes the discounted value of the primary surplus, subject to a constraint on the discounted value of household's utility, and  $h_t^{i=0,1,2}(\mathbf{x}_t) = 0$  that consists of the period-0 capital utilization condition (22), the capital utilization constraint (21), and the resource constraint (20). In the second column of Table 1, we present the welfare maximization problem where the objective and the constraint on household's welfare, under the surplus maximization problem, are exchanged.

To define duality between the two problems presented in Table 1, we begin with the Lagrangian of the surplus maximization problem that can be expressed as the max-min problem,

$$\begin{aligned}
\hat{b}(\bar{W}) &= \max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} \mathcal{L}(\mathbf{x}_t, \zeta_t^i, \psi) \\
&= \max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \left\{ \min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \tilde{\beta}^t [R_{0,t} s_t(\mathbf{x}_t) + \sum_{i=0}^2 \zeta_t^i h_t^i(\mathbf{x}_t) \right. \\
&\quad \left. + \psi \left( \sum_{t=0}^{\infty} \tilde{\beta}^t U_t(\mathbf{x}_t) - \bar{W} \right) \right\}
\end{aligned}$$

where  $\hat{b}(\bar{W})$  denotes the maximum amount of discounted primary surplus attained given the welfare constraint.  $\zeta_t^{i=0,1,2}$  collects Lagrange multipliers for constraints (22), (21) and (20), respectively.  $\psi \geq 0$  is the Lagrange multiplier

of the inequality welfare constraint. The set of feasible solutions  $x_t$  should satisfy the inequality welfare constraint,  $\sum_{t=0}^{\infty} \tilde{\beta}^t U_t(x_t) - \bar{W} \geq 0$ , and render  $\min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} L(\mathbf{x}_t, \zeta_t^i, \psi)$  bounded. Whereas a solution that violates the inequality welfare constraint resulting in  $\min L(\mathbf{x}_t, \zeta_t^i, \psi) \rightarrow -\infty$  cannot be a feasible solution. An optimal solution,  $\hat{x}_t$ , belongs to the feasible set.

Note that when strong Lagrangian duality applies to the model,

$$\max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} \mathcal{L}(\mathbf{x}_t, \zeta_t^i, \psi) = \underbrace{\min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} \max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \mathcal{L}(\mathbf{x}_t, \zeta_t^i, \psi)}_{dual} \quad (32)$$

At this point, we simultaneously add and subtract the level of discounted primary surplus,  $\bar{b}$ , in the dual problem to allow us to re-arrange equation (32) to obtain,

$$\begin{aligned} \hat{b}(\bar{W}) &= \min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} -\psi \bar{W} + \bar{b} \\ &+ \max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \tilde{\beta}^t [\psi U_t(\mathbf{x}_t) + \sum_{i=0}^2 \zeta_t^i h_t^i(\mathbf{x}_t) \\ &+ \left( \tilde{\beta}^t \sum_{t=0}^{\infty} R_{0,t} s_t(\mathbf{x}_t) - \bar{b} \right)] \end{aligned} \quad (33)$$

where the expression in the square brackets is the Lagrangian of the welfare maximization problem presented in the second column of Table 1. To show this, we take  $\psi$  outside the square brackets to obtain,

$$\begin{aligned} \hat{b}(\bar{W}) &= \min_{\{\zeta_t^i, \psi \geq 0\}_{t=0}^{\infty}} -\psi \bar{W} + \bar{b} \\ &+ \psi \max_{\{\mathbf{x}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \tilde{\beta}^t \left[ U_t(\mathbf{x}_t) + \sum_{i=0}^2 \frac{\zeta_t^i}{\psi} h_t^i(\mathbf{x}_t) \right. \\ &\left. + \frac{1}{\psi} \left( \sum_{t=0}^{\infty} \tilde{\beta}^t R_{0,t} s_t(\mathbf{x}_t) - \bar{b} \right) \right] \end{aligned} \quad (34)$$

We denote  $\hat{W}(\bar{b})$  the maximum level of welfare attained under the Ramsey problem given the budget constraint,  $\bar{b}$ .  $\hat{\zeta}_t^{i=0,1,2}$  and  $\hat{\psi}$  are the optimized multipliers in equation (34). Equation (34) can then be simplified to

$$\hat{b}(\bar{W}) - \bar{b} = \hat{\psi} [\hat{W}(\bar{b}) - \bar{W}] \quad (35)$$

Whenever  $\hat{\psi} > 0$ , equation (35) implies duality between the surplus maximization and the welfare maximization problems. Specifically, if we set the government debt in the welfare maximization problem equal to the the maximum level of the discounted primary surplus generated by the surplus maximization problem, i.e.  $\hat{b}(\bar{W}) = \bar{b}$ , then the level of welfare attained in the former problem is equal to the welfare constraint imposed in the latter, i.e.  $\hat{W}(\bar{b}) = \bar{W}$ , and vice versa. In other words, when the constraints of the surplus maximization and of the welfare maximization problems are set consistently, the optimal combination of ‘welfare loss’ - ‘sustainable debt’ the two problems return is identical. This means that, when strong Lagrangian duality applies and  $\hat{\psi} > 0$ , duality between the surplus maximization and the Ramsey problems hold. Importantly, note that condition (35) conversely implies condition (32); i.e when  $\hat{\psi} > 0$ , strong Lagrangian duality is a necessary and sufficient condition for duality between the welfare and the surplus maximization problems to hold. Therefore, to numerically establish strong Lagrangian duality holds on the Fiscal Frontier, it is sufficient to check condition (35) is verified and  $\hat{\psi} > 0$ . Our numerical check confirms this is the case.

When  $\hat{\psi} = 0$ , duality between welfare maximization and surplus maximization problems need not hold any longer. An example would be the case we ruled out in the introduction to this section; namely, where tax revenues can be raised at no social cost. A simple example can be obtained by allowing the planner to set both labor income and consumption taxation, without further restrictions. As noted in Section 2.5, this effectively endows the policy maker with a lump sum tax instrument. In such a model, the solution of the Ramsey and of the surplus maximization problem will return two different optimal combinations of welfare and sustainable debt, even when the constraints in the two problems are set consistently. Indeed, even though the level of welfare will be the same under both policies, the level of sustainable debt will differ. Specifically, under the Ramsey problem it will correspond

to the one required by the government budget constraint, whereas it will be unbounded under the surplus maximizing policy.

Similarly, we discard numerical solutions where  $\hat{\psi} < 0$  such that the Lagrange Multiplier no longer has the interpretation that relaxing either the budget constraint or welfare constraint improves the policy maker's objective function, when maximising social welfare or discounted fiscal surplus, respectively. This implies we are ruling out situations where the surplus maximisation problem imposes a level of welfare which is lower than would be chosen by the Ramsey planner for any feasible level of government debt.

### 3.3 Features of the Optimal Policy

Because of duality the solution to our surplus maximization problem has elements of the Ramsey optimal taxation literature. In this subsection, we focus on two features of our Fiscal Frontier. First, the famous Chamley-Judd result (see Chamley (1986) and Judd (1985)) applies to our model. In the short-run the capital tax rate is positive as the policy maker exploits the (quasi) lump-sum nature of the tax on the initial capital. However, in the long-run the capital tax approaches zero as the policy maker attempts to raise revenues through the least distorting instrument which is the labor income tax. Second, with endogenous capacity utilization the tax on the initial stock of capital is bounded. This is in contrast to Chamley-Judd. This is because endogenous capacity utilization makes the capital base elastic in the short-run, limiting the extent to which the Ramsey planner can exploit this margin. We now turn to demonstrate these results.

#### Long run capital tax of zero

To illustrate the application of the Chamley-Judd result to our model, we compare the steady-state solution of the Ramsey first order condition for capital (27),

$$\tilde{\beta} [F'_{mk}(mk, l)m + 1 - \delta(m)] = 1 + \gamma,$$

with the intertemporal investment decision (17) under the competitive equilibrium.

$$\tilde{\beta} [(1 - \tau^k) F'_{mk}(mk, l)m + 1 - \delta(m)] = 1 + \gamma,$$

Since the surplus maximizing allocation is a competitive equilibrium, it implies that the capital tax,  $\tau^k$ , is zero in the long-run.

Therefore, the policies underpinning the Fiscal Frontier will ensure that the tax rate on capital is driven to zero in the long-run.

### Taxation of initial capital

In our model with endogenous capacity utilization, the first order condition with respect to the period-0 capital tax,  $\tau_0^k$ , in (31),

$$F'_{m_0k_0}(m_0k_0, l_0)m_0k_0 - \zeta^0 F'_{m_0k_0}(m_0k_0, l_0) = 0$$

offers an insight into why the initial capital tax is bounded. In particular, the term,  $\zeta^0 F'_{m_0k_0}(m_0k_0, l_0)$ , appears in the above first order condition because endogenous capital utilization introduces a distortionary component to the period-0 capital tax. Indeed, as the government increases the capital tax, households will reduce capacity utilization in the same period. Therefore, when setting initial capital taxation, the Ramsey planner will need to balance the benefits associated with lower future distortions with the counteracting short-run costs associated with reduced capacity utilization. This is in contrast to the corresponding condition in Chamley-Judd with an exogenous fixed utilization rate,

$$F'_{k_0}(k_0, l_0) k_0 > 0,$$

where the period-0 stock of capital,  $k_0$ , is given and the capital tax rate is effectively a lump sum tax. Therefore, under Chamley-Judd without endogenous capital utilization, it is optimal to set the capital tax rate as high as possible to increase current revenues without any worsening of welfare through tax distortions.



## 4 The Fiscal Frontier

### 4.1 Calibration

Our baseline calibration for the US economy tracks closely Trabandt and Uhlig (2011) and D’Erasmus *et al.* (2015).<sup>11</sup> We report in Table 2 our calibration and discuss below the approach we follow.

Beginning with technology parameters, the labor share of production,  $\alpha$ , is set to 0.61, a value in line with Trabandt and Uhlig (2011) and D’Erasmus *et al.* (2015). Similarly, following D’Erasmus *et al.* (2015), we set the quarterly rate of labor augmenting technological change,  $\gamma = 0.0038$ . Then, the depreciation function in (4) requires setting two parameters,  $\chi_0$  and  $\chi_1$ . To calibrate  $\chi_0$ , we use the steady-state of the capital utilization constraint (21) which implies that  $\chi_0 \bar{m}^{\chi_1} = \frac{1+\gamma-\tilde{\beta}}{\beta} + \bar{\delta}$ , and we normalize the long-run capacity utilization rate to  $\bar{m} = 1$ . Therefore,  $\chi_0$  is set to 0.026 to match the long-run depreciation rate  $\bar{\delta} = 0.0163$ , a value in line with D’Erasmus *et al.* (2015) and Trabandt and Uhlig (2011). Given the values of  $\bar{m}$ ,  $\bar{\delta}$  and  $\chi_0$ , from the evaluation of the depreciation function (4) in steady-state, it results  $\chi_1 = 1.59$ .

For preference parameters,  $\sigma$ , is set to 2 to deliver the commonly used intertemporal elasticity of substitution of 0.5. Furthermore, following D’Erasmus *et al.* (2015), we set the leisure utility parameter,  $a$ , to 2.675, and the households’ discount factor,  $\beta$ , to 0.998, so that  $\tilde{\beta} = \beta(1 + \gamma)^{1-\sigma} = 0.994$ .

Fiscal variables include tax rates, government expenditures, transfers and debt. Although in our analysis labor and capital tax rates are solutions of the optimal policy problem, the initial equilibrium of our model is parametrized on the basis of the fiscal regime prior to 2008, in line with D’Erasmus *et al.* (2015). Resulting fiscal aggregates are also largely in line with those from Trabandt and Uhlig (2011). Tax rates are set as follows:  $\tau^c = 0.04$ ,  $\tau^l = 0.27$  and  $\tau^k = 0.37$ . Government expenditures is set to be 16% of GDP in line with the OECD definition ‘general government consumption expenditure as

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<sup>11</sup>Although the D’Erasmus *et al.* (2015) calibration applies to an open economy, the assumption of a steady-state trade balance of zero ensures that it remains applicable to our model.

a percentage of GDP'. In addition, public debt-to-GDP ratio,  $\frac{d}{4y}$ , is calibrated to 76% to reflect the US debt at that time. Finally, government transfers are determined as a residual of government budget constraint in (11) such that<sup>12</sup>

$$\frac{e}{y} = \frac{Rev}{y} - \frac{g}{y} - \frac{d}{y} (1 - \tilde{\beta}) = 0.1574,$$

where  $Rev \equiv \tau^c c + \tau^l wl + \tau^k rmk$ .

Table 2: Calibration US economy

Parameter	Description	Value	Calibration strategy
Technology			
$\alpha$	labor income share	0.61	D'Erasmus <i>et al.</i> (2015)
$\gamma$	growth rate	0.0038	D'Erasmus <i>et al.</i> (2015)
$\bar{m}$	capacity utilization	1	steady-state normalization
$\bar{\delta}$	depreciation rate	0.0163	D'Erasmus <i>et al.</i> (2015)
$\chi_0$	$\delta(m)$ coefficient	0.026	set to yield $\bar{\delta} = 0.0163$
$\chi_1$	$\delta(m)$ exponent	1.59	set to yield $\bar{m} = 1$
Preferences			
$\tilde{\beta}$	discount factor	0.994	D'Erasmus <i>et al.</i> (2015)
$\sigma$	risk aversion	2.000	standard RBC value
$a$	labor supply elasticity	2.675	D'Erasmus <i>et al.</i> (2015)
Fiscal Policy			
$\tau^c$	consumption tax	0.04	D'Erasmus <i>et al.</i> (2015)
$\tau^l$	labor tax	0.27	D'Erasmus <i>et al.</i> (2015)
$\tau^k$	capital tax	0.37	D'Erasmus <i>et al.</i> (2015)
$\frac{d}{4y}$	govt debt in GDP	0.76	D'Erasmus <i>et al.</i> (2015)
$\frac{g}{y}$	govt consumption in GDP	0.16	D'Erasmus <i>et al.</i> (2015)
$\frac{e}{y}$	govt transfer in GDP	0.1574	govt budget in SS

Note: our calibration of the US economy at quarterly frequency tracks closely Trabandt and Uhlig (2011) and D'Erasmus *et al.* (2015).

## 4.2 Constructing the Fiscal Frontier

We construct our Fiscal Frontier by iteratively solving the surplus maximization problem presented in Section 3, conditional on different levels of social

<sup>12</sup>Note that the consumption Euler equation in steady state implies that  $\tilde{\beta} = (1 + \gamma)q$ .

welfare.<sup>13</sup> As discussed in Section 3.2, we rule out solutions where  $\hat{\psi} < 0$ . This ensures that duality between our surplus maximization problem and the conventional Ramsey problem holds for every point on our Fiscal Frontier. Compared to ad-hoc policy reforms, the surplus maximizing policy underpinning our Fiscal Frontier allows for an optimal construction of tax plans which account for discounting, expectations and the dynamics of production factor elasticities.

To make our welfare measure meaningful, we convert it to consumption equivalents relative to the welfare implied by the initial calibrated steady-state of the decentralized equilibrium in Table 2.<sup>14</sup> These welfare costs capture the combined distortions implied by the fiscal mix optimally implemented by the policy maker relative to the calibrated policy of the initial steady-state. We then plot each level of sustainable debt (over GDP) against the implied welfare loss measured in consumption equivalent units.<sup>15</sup> This gives rise to our Fiscal Frontier in Figure 1. This curve represents the welfare costs of sustaining any amount of government debt when fiscal policy is carried out optimally. It shows the key trade-offs facing the policy maker: how much debt can be sustained and at what social cost?

Finally, from Figure 1 some further insights can be obtained. For example, at zero welfare cost, our Fiscal Frontier implies a sustainable debt-to-GDP ratio in the US of 263% as opposed to the 76% supported by the initial calibrated tax policies in Table 2. That means implementing an optimal tax policy can generate an additional 187% of GDP in discounted fiscal surplus at no welfare cost. In addition, moving along the Frontier gives us a sense of the trade-offs facing policy makers. The highest sustainable debt-to-GDP ratio is 408%, with the associated tax distortions being equivalent to a welfare loss of 17.67% of steady-state consumption.

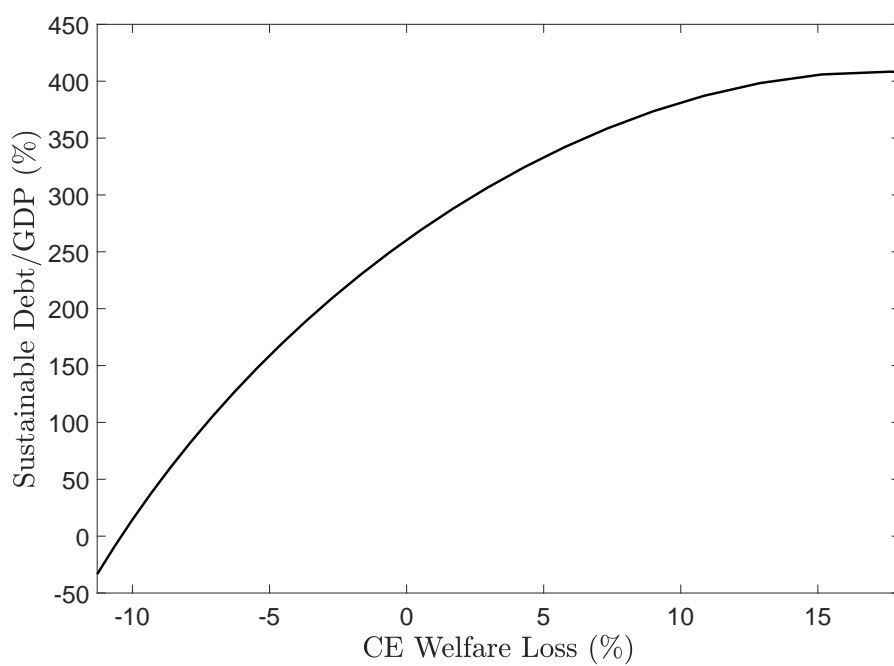
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<sup>13</sup>We solve the model using Dynare 4.6. The deterministic setup together with rational expectations imply perfect foresight. Therefore, we use the Dynare's perfect foresight facility based non-linear solution techniques, see Adjemian *et al.* (2011).

<sup>14</sup>See Appendix D for the computation of consumption equivalent units of welfare.

<sup>15</sup>Our notion of sustainable debt corresponds to the initial level of debt the government must generate fiscal backing for in the form of future surpluses. The policies implemented to achieve this may generate higher or lower levels of debt during and after the transition to the ultimate steady-state.

Figure 1: The Fiscal Frontier



Note: each point on the Fiscal Frontier represents the solution to the surplus maximization problem detailed in Section 3 for a different level of social welfare. Duality ensures it is equivalent to the Ramsey problem of maximizing social welfare subject to the government's intertemporal budget constraint. Welfare is measured in consumption equivalent units as described in Appendix D.

Having constructed the Fiscal Frontier for our model economy, we now turn to explore what determines its position and shape.

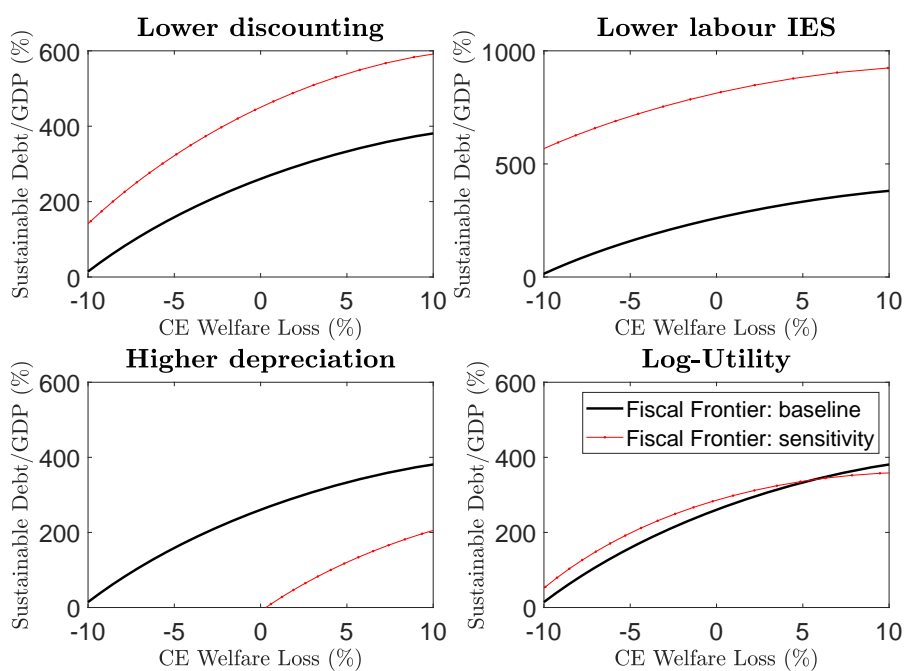
### 4.3 Determinants of the Fiscal Frontier

To gain further insights into the properties of the Fiscal Frontier, we undertake an analysis of the key determinants of the shape and position of the curve by varying the households' discount factor, their elasticity of labor supply, the intertemporal elasticity of substitution and the rate of capital depreciation. Specifically, the four panels of Figure 2 compare the Fiscal Frontier under our baseline calibration against alternative parameterizations of selected parameters including a lower discount factor ( $\tilde{\beta} = 0.9954$ , implying a reduction in the steady-state interest rate of 1%), decreasing the elasticity of labor supply ( $a = 2$ ), increasing the long-run depreciation rate ( $\bar{\delta} = 0.02$ ) and reducing the intertemporal elasticity of substitution ( $\sigma = 1$ ), respectively.

The first row of Figure 2 shows that an increase in the household's discount factor or a decrease in the elasticity of labor supply substantially increases the policy maker's ability to sustain debt by raising the discounted value of future fiscal surpluses. In particular, the higher discount factor encourages households to accumulate more capital, which increases the size of the capital tax base during transition, while the lower interest rate enables the policy maker to service a given level of debt at a lower welfare cost. Similarly, the lower elasticity of labor supply allows the policy maker to exploit the labor income tax base more effectively. Conversely, panel 3 increases the long-run depreciation rate,  $\bar{\delta}$ , equal to 0.02, *cet. par.*, which discourages capital accumulation. This reduces the capital tax base during the transition and the size of the labor income tax base in both transition and steady-state.

Finally, the two Fiscal Frontiers plotted in panel 4 have different curvatures. Under log utility ( $\sigma = 1$ ), there is less curvature of the utility function compared with the benchmark case of ( $\sigma = 2$ ). The reduction in curvature means that losses in consumption count for less and gains matter relatively more. As a result, when the economy is relatively efficient, the policy maker

Figure 2: The determinant of the Fiscal Frontier



Note: each subplot compares the Fiscal Frontier under our baseline calibration against alternative parametrization of selected parameters including: the discount factor ( $\tilde{\beta} = 0.9954$ ), the intertemporal elasticity of labour supply ( $a = 2$ ), the long run depreciation rate ( $\tilde{\delta} = 0.02$ ) and the intertemporal elasticity of substitution ( $\sigma = 1$ ), respectively.

would want to exploit the transition to a greater extent when ( $\sigma = 1$ ) and promise a better tomorrow in doing so. When the economy is highly inefficient the transition has already been exploited significantly so that further attempts to do so worsen the eventual steady-state, relative to the situation when ( $\sigma = 2$ ) which is costly and can reverse the ranking of the Fiscal Frontiers for the two values of  $\sigma$ .

For the remainder of the paper we illustrate how it can be used as a guide to policy by undertaking the three policy exercises discussed in the Introduction. Specifically, (1) a comparison of the Fiscal Frontier with conventional Laffer curves; (2) an exploration of the optimal pace of capital tax reform; and (3) an analysis of debt reduction packages similar to those undertaken by the CBO on behalf of the US Congress. This will reveal that standard Laffer curves significantly underestimate the level of sustainable public debt and, equivalently, overestimate the welfare costs of sustaining a given level of debt. Relatedly, ad-hoc debt reduction measures can have huge welfare costs. Moreover, using insights from the design of optimal policies underpinning the Fiscal Frontier, gradual capital taxation reform either on its own, or as part of a wider debt reduction package, can achieve debt targets in a timely manner while simultaneously improving social welfare. Such policies, although simple, can come close to achieving the welfare gains experienced under full Ramsey optimization.

## 5 Reforms underpinning Conventional Laffer Curves

Our first policy exercise applies the Fiscal Frontier to the analysis of the policy reforms underpinning conventional Laffer curves. In particular, we assess to what extent these policy reforms based on a one-off tax hike, of either labor or capital tax rates leave potential revenue or welfare gains unexploited. In doing so, we address two issues. The first one is quantitative: how much additional fiscal revenues can be raised when the government is pursuing an optimal tax policy rather than adopting constant tax rates as assumed in conventional Laffer curve calculations? The second issue concerns policy design: how should an optimal tax policy be carried out both in the new

steady-state and during the transition to that steady-state?

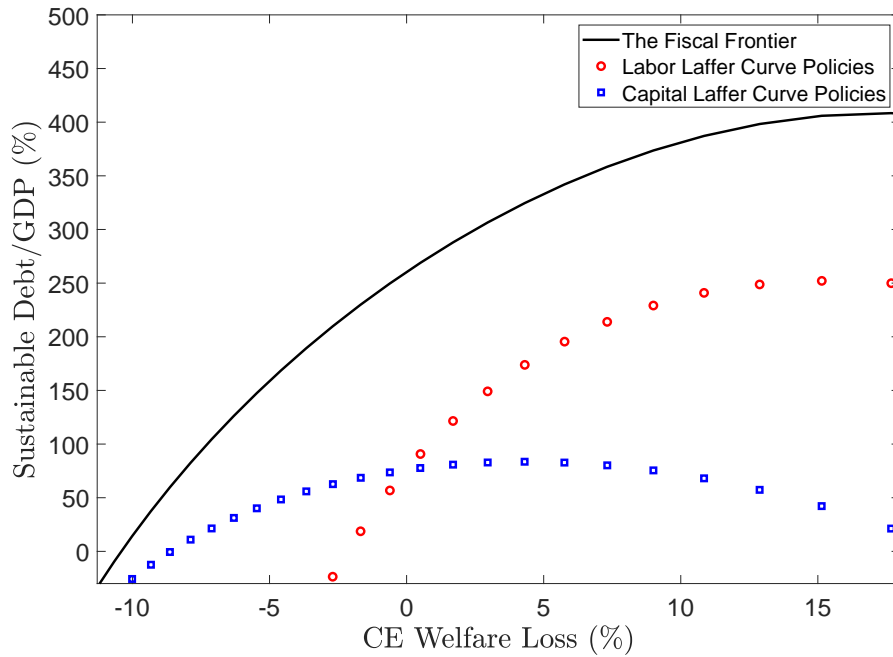
To simulate the policy reforms underpinning the conventional labor and capital Laffer curves, we begin with our decentralized economy in its calibrated steady-state defined in Table 2. We then permanently change either tax rate to a value from 0 to 100%, while keeping the other fixed. Through the equilibrium conditions, we can solve the transitional dynamics from the initial steady-state to the new steady-state underpinned by the tax change. We can then measure the discounted fiscal surplus and social welfare associated with such a policy: this will constitute a single point on the Laffer curve for that tax. For example, progressively varying the capital rate of taxation from 0 to 100%, and repeating these calculations, we can trace out the points underpinning the capital Laffer curve. Those underpinning the labor Laffer curve are derived analogously. It is important to stress that these calculations include the impact on both the discounted surplus and social welfare of transitioning from the initial to the new steady-state. We plot these points against our Fiscal Frontier in Figure 3.

The first thing to note is that the policy reform underpinning the capital Laffer curve does not support the efficient generation of tax revenues. For capital tax Laffer curves in particular, in context of neo-classical models such as ours, the capital tax is highly distortionary, typically: a policy based on increasing capital taxation is condemned to be relatively ineffective. Therefore, we focus on the comparison between the Fiscal Frontier and the points associated with the labor Laffer curve reforms. The distance between these points and the Fiscal Frontier is striking. At their peak, the sustainable government debt-to-GDP implied by the Fiscal Frontier is more than 156% of GDP higher than that generated by a constant tax rate policy of a one-off increase in the labor tax rate. In addition, differences in debt sustainability between Labor Laffer Curve policy reforms and the Fiscal Frontier appear to be of broadly similar magnitude, measuring about 150% of GDP, across all welfare costs. By inverting this argument it can be noted that, in sustaining the same level of debt, optimal tax policy typically offers welfare gains of about 11% in constant consumption equivalent units. In a sense this isn't surprising since the policy changes considered in Laffer curve cal-



culations aren't designed to be efficient and/or optimal. Nevertheless, we conclude that an optimal tax policy has strong quantitative implications for debt sustainability, tax revenues and welfare gains.

Figure 3: Laffer Curve Policy Reforms against the Fiscal Frontier



Note: the black line reproduces the Fiscal Frontier from Figure 1. The red dots hold the capital tax rate at its calibrated value, while permanently shifting labor taxes from their calibrated value to a tax rate between 0 and 100%. The blue squares do the same, but hold labor tax rates at their calibrated value while varying capital tax rates.

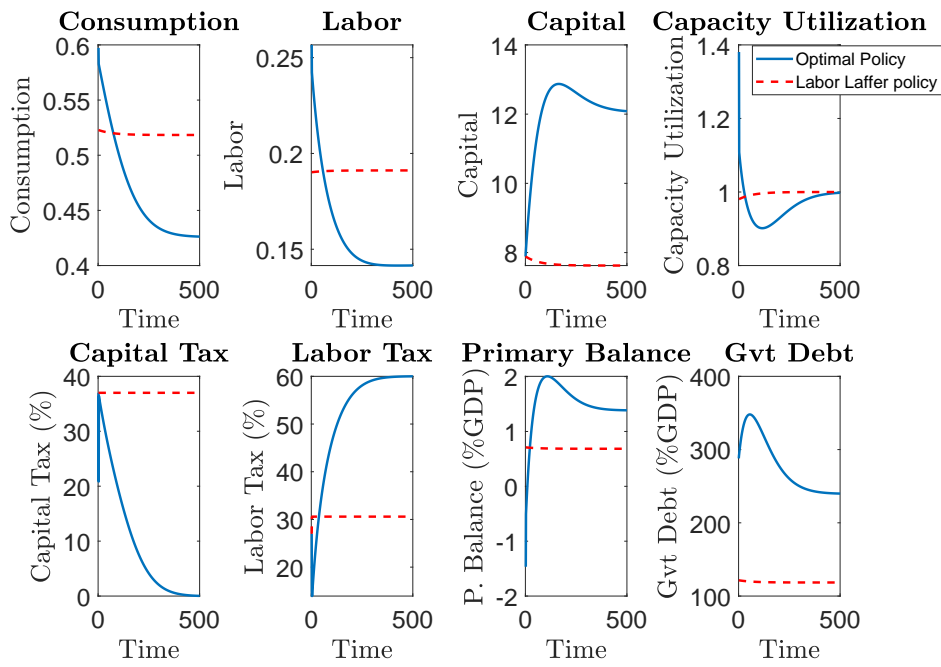
We now turn to analyze how the fiscal policy underpinning our Fiscal Frontier curve achieves its benefits relative to the constant tax rate policy implied by the conventional labor Laffer curve. In Figure 4, we show for both policies the transitional dynamics associated with a particular welfare cost of 1.50% of steady-state consumption. We note that with the constant tax rate policy there is very limited variation in the endogenous variables during the transition. In contrast, under the optimal tax policy, there is an initial front-loading of capital taxation which is coupled with a complementary cut

in the labor tax rate. The capital tax rate then declines until converging to zero in the steady-state, while the labor income tax rate rises consistently until achieving a relatively high long-run value. Given the commitment to abolish capital taxation in the long-run and the relatively low labor taxes during the initial stages of transition, capital keeps accumulating despite it being taxed at a positive rate.

The core feature of the optimal tax policy is the commitment to cut capital taxation in the long-run and the shift towards labor income taxation. This enables the policy maker to tax capital during the transition without discouraging its accumulation. This dynamic switch in tax policy during the transition drives the extremely large fiscal/welfare gains. This can be further appreciated in Figure 5 where the highest revenues (about 40% of GDP) are raised when both capital and labor taxes are at ‘intermediate rates’ (e.g:  $\tau^l = 0.40$  and  $\tau^k = 0.20$ ) and the quantity of capital has reached its maximum (see the corresponding time period in Figure 4). Therefore, we conclude that the striking gains in tax revenue generation are, in part, due to using one tax instrument to complement another and, in part, due to the variation in these instruments over time.

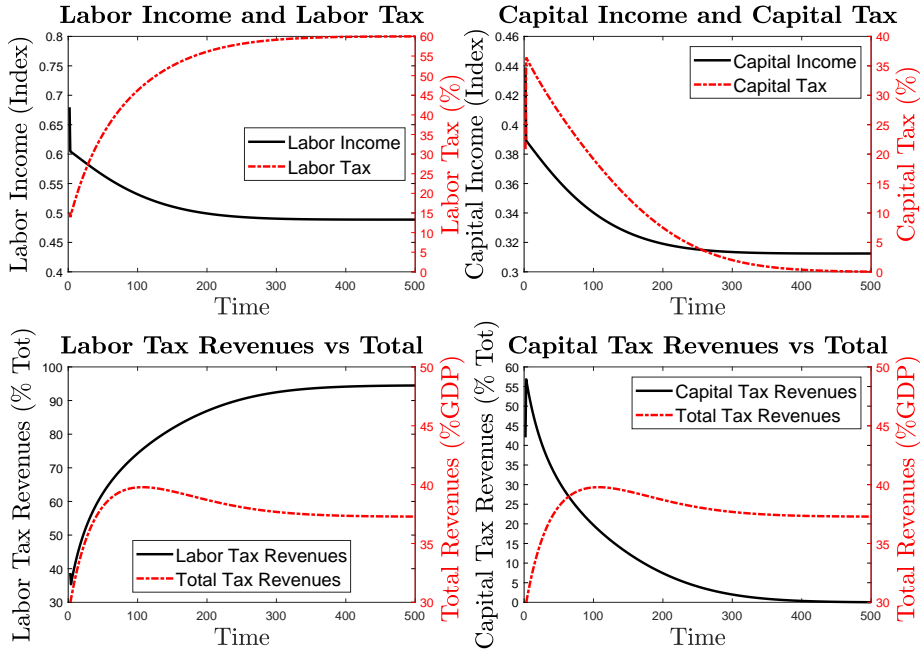
The simple tax reform underpinning the Laffer curve was not designed to mimic optimal policy, but to gain insight into the economic costs of tax distortions. We now turn to consider whether other simple reforms can be designed to achieve some of the benefits of optimal policy.

Figure 4: Transitional dynamics for CE=1.50%



Note: this plots the transition for two different policies which share the same welfare cost of 1.5% of calibrated steady-state consumption. The equivalence in welfare costs is achieved by setting the initial value of debt appropriately. The dashed red line starts from a slightly higher than calibrated level of debt from which a permanent increase in the labor tax rate to 30% is stabilizing. The blue line is the Ramsey policy given the economy inherits a particularly high level of debt.

Figure 5: Optimal fiscal mix (CE=1.50%)



Note: the Figure gives further detail on the fiscal implications of the Ramsey policy plotted in Figure 4.

## 6 Replacing capital taxation

We know from economic theory that, in models such as ours, capital taxation is usually more distortionary than labor taxation, prompting Lucas (1990) to call for the replacement of capital taxation with labor taxation. We consider that proposal here. However, rather than simply replacing one tax with the other in a single fiscal reform, we allow for a gradualist approach which captures the incremental nature of tax reforms seen in the data. We therefore allow the capital tax to be replaced by the labor income tax linearly over a period of  $n$  years. However, since such calls for tax reform are perennial, we produce this exercise conditional on the fiscal and macroeconomic conditions

applying in each decade from the 1970s to the present day. We assess how the optimal degree of gradualism has changed over time in response to changing economic conditions. We also show that the Frontier against which we compare these reforms has also shifted over time, which offers further insight into the determinants of the Frontier.

## 6.1 Calibrating the US economy across decades, 1970 - 2020

We compute time series for the US economy for effective tax rates, the growth rate in real GDP per capita as well as ratios to GDP of government debt, government consumption, private capital and private investment. Using these fiscal and macroeconomic data, we can calibrate our baseline model in each decade, as shown in Table 3. In total, we consider five decades: 1971-1980, 1981-1990, 1991-2000, 2001-2010 and 2011-2020.

Again our calibration strategy tracks closely Trabandt and Uhlig (2011) and D’Erasmus *et al.* (2015). Accordingly, technology parameters including the quarterly rate of labor-augmenting technological change,  $\gamma$ , and the depreciation function parameters,  $\bar{\delta}$ ,  $\chi_0$  and  $\chi_1$ , are calculated as averages over each decade. Fiscal policy parameters,  $\tau^c$ ,  $\tau^l$ ,  $\tau^k$ ,  $\frac{d}{4y}$  and  $\frac{g}{y}$ , are calibrated using fiscal data at the end point of each decade (e.g. 1980 for the decade of 1971-1980). The transfer-to-GDP ratio,  $\frac{e}{y}$ , is derived as a residual from the government budget constraint. We assume other parameters, such as preferences, remain unchanged and therefore are the same as in Table 2.

Debt has risen substantially since the 1970s. It began to rise in the 1981-1990 decade due to Reagan’s tax cuts and defence spending increases; it then stabilized as a result of the Clinton budget, but rose sharply during the last two decades due to the financial crisis and the COVID-19 pandemic. On the other hand, economic growth has slowed in the last two decades. The effective labor and consumption tax rates have been relatively stable over time, while the effective capital tax rate has tended to fall. Capital taxation was at its peak in 1980 at 40%, due to significant increase in capital gains tax rates in the 1969 and 1976 Tax Reform Acts. The following rounds of

reforms, including tax rate reductions in 1981, the Taxpayer Relief Act of 1997, the Economic Growth and Tax Relief Reconciliation Act of 2001 and the Jobs and Growth Tax Relief Reconciliation Act of 2003, have led to a considerable reduction in the effective capital tax to around 32% (see Romer and Romer (2010)).

Table 3: Calibrating the US economy 1970 - 2020

	1971-1980	1981-1990	1991-2000	2001-2010	2011-2020
<b>Technology:</b>					
$\gamma$	0.0052	0.0058	0.0054	0.0021	0.0024
$\bar{\delta}$	0.012	0.013	0.013	0.017	0.016
$\chi_0$	0.023	0.024	0.025	0.025	0.024
$\chi_1$	1.931	1.939	1.853	1.474	1.541
<b>Investment:</b>					
x/y	0.18	0.18	0.17	0.17	0.17
k/4y	2.55	2.43	2.27	2.26	2.31
<b>Fiscal policy:</b>					
$\tau^c$	0.05	0.04	0.05	0.04	0.04
$\tau^l$	0.25	0.27	0.29	0.26	0.27
$\tau^k$	0.40	0.38	0.38	0.31	0.32
d/4y	0.30	0.58	0.58	0.97	1.42
g/y	0.16	0.16	0.14	0.17	0.15
e/y	0.17	0.16	0.20	0.12	0.14

Note: We use data from the OECD database (Available at: [stats.oecd.org](http://stats.oecd.org)) and the BEA NIPA database (Available at: [www.bea.gov](http://www.bea.gov));  $\tau^c$ ,  $\tau^l$  and  $\tau^k$  are calculated following Mendoza *et al.* (1994).

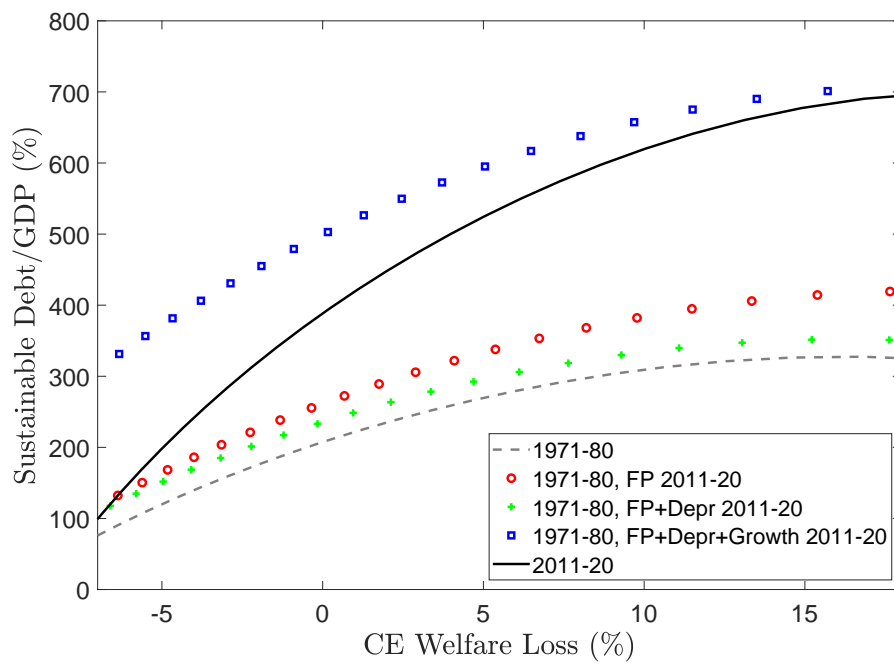
We can see how the changing fiscal and economic circumstances have impacted the Fiscal Frontier by plotting the Frontier based on the data from 1971-1980 (the grey-dashed line) and 2011-2020 (the black-solid line), respectively, in Figure 6. The Frontier has shifted up over time, although more so towards the maximum discounted fiscal surplus that can be generated. It is possible to explain this shift, by decomposing the impact of the various

changes detailed in Table 3. We begin from the grey-dashed line which represents the 1971-1980's Fiscal Frontier for the calibration implied by the first column of Table 3. The first step in moving from this Frontier to that for the 2011-2020 period is to change the calibrated fiscal policy to be consistent with the final column of Table 3. The combined impact of a slightly lower consumption tax rate and lower levels of government consumption and transfers serve to improve the primary budget, *cet. par.*, allowing the Frontier to shift upwards to the curve implied by the red circles. The rate of depreciation has also increased over time, which offsets, although only partially, the improvement due to fiscal changes for the reasons detailed in section 4.3. This resultant frontier is denoted by the green dots.

Furthermore, incorporating the decline in the growth rate over this time period leads to a substantial upward shift in the Frontier, as shown by the blue squares. That a reduction in growth should increase the sustainability of government debt is slightly counter-intuitive. However, in our model the steady-state real interest rate is given by,  $1 + r = \beta^{-1}(1 + \gamma)^\sigma$ . Since the calibrated value of the intertemporal elasticity of substitution is less than one,  $1/\sigma = 0.5$ , the real interest rates falls by more than the growth rate implying a significant upward shift in the Frontier. Note that had interest rates fallen due to a decrease in households' rate of time preference the upward shift in the Frontier would have been even greater, as discussed in Section 4.3. Finally, allowing for the fact that the capital stock is lower in 2011-2020 than in 1971-1980, implies that the final Frontier (black solid line) is lower at significantly negative levels of welfare loss as exploiting the pre-existing capital stock, while promising to eliminate capital taxation is a key element in generating fiscal surplus without incurring a significant welfare cost.

We now turn to consider how the changing economic situation affects the desirability of capital tax reform. How do our policy recommendations change over time?

Figure 6: Decomposing changes in the US Fiscal Frontier, 1971 - 2020





## 6.2 Simulating the capital tax replacement

To simulate the policy reform of replacing capital taxation with labor taxation, we begin with our decentralized economy at its calibrated steady-state for each decade in Table 3. Therefore, for a given decade economic conditions (in the form of the capital output ratio, growth rate and parameterization of capital utilization costs) and fiscal variables (initial tax rates, government expenditure and the debt-to-GDP ratio) will differ in line with the data.

In the initial steady-state, we impose the following fiscal rules for capital and labor taxes:

$$\tau_t^i = \rho \tau_{t-1}^i + (1 - \rho) \bar{\tau}^i, i \in [l, k] \quad (36)$$

where  $\rho$  captures the degree of gradualism in converging to the new long-run mean  $\bar{\tau}^i$ . As far as capital taxes are involved (i.e.  $i = k$ ), the long-run mean is set to zero (i.e.  $\bar{\tau}^k = 0$ ). On the other hand, to ensure fiscal solvency, the long-run mean for labor taxes (i.e.  $\bar{\tau}^l$ ) is set such that the path of government debt stabilizes in the long-run. It is important to note that the long-run debt-to-GDP ratio is likely to differ from its initial value as a result, being higher or lower depending on the tax revenues generated during the transition. We then use the equilibrium conditions to solve the transitional dynamics from the initial steady-state to the new steady-state underpinned by the tax change for each decade.

We measure the social welfare associated with any degree of gradualism  $\rho$ , in each decade. For a more straightforward interpretation of the results, we transform  $\rho$  into a measure of the capital tax rate half-life (in years), i.e. half-life =  $\frac{\log 0.5}{4 \log \rho}$ . This is the length of time it takes to halve the capital tax rate. We then vary the degree of gradualism in replacing the capital tax with a labor tax. This results in different levels of welfare. The half-life (in years) that attains the highest level of welfare defines the optimal degree of gradualism. This should be read as the maximum amount of welfare attainable under the capital tax replacement reform given the economic and fiscal conditions at the time of the reform.

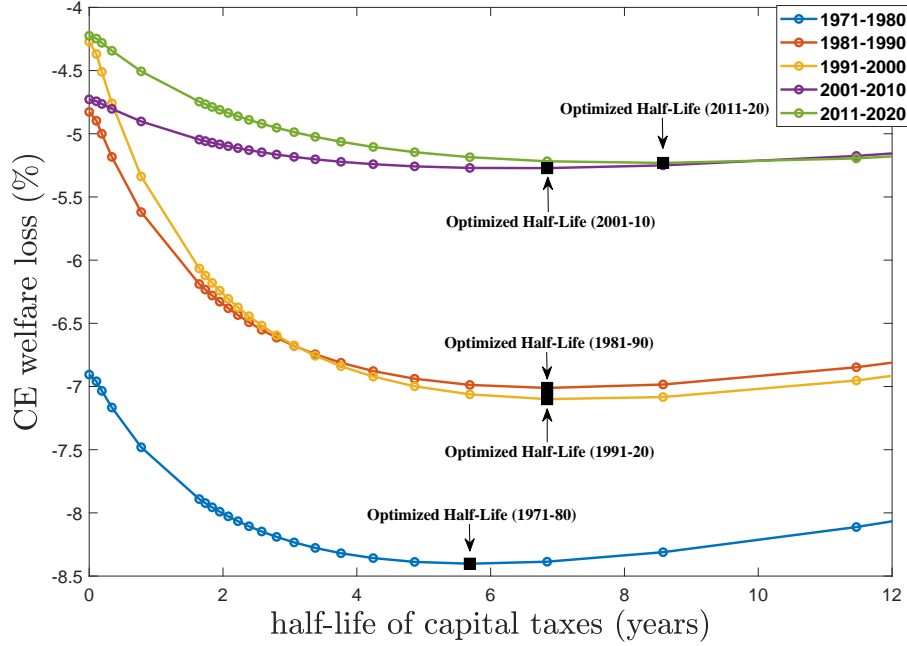
Figure 7 plots welfare losses against the half-life measure for each decade.<sup>16</sup> The first thing to note is that the welfare gains attainable through replacing capital taxation have reduced over time, nearly halving from the initial decade in our analysis (1971-1980) to the final one (2011-2020). Over the same period, the optimal pace of capital taxation reduction has slowed from a half life of 5.7 years ( $\rho = 0.97$ ) to one of 8.6 years ( $\rho = 0.98$ ). Broadly speaking, in decades where debt levels are low and capital taxation high, the gains from a rapid capital reform are greatest. As a result the desirability of capital tax cuts today, while still substantial, is significantly lower than at the time of the Reagan tax cuts of 1980. Across all decades, no gradualism yields the lowest welfare gains compared to reforms where gradualism is allowed.

Finally, when contrasting the capital tax reform with the optimized half-life against the Fiscal Frontier it can be appreciated how close this straightforward reform comes to the optimum. The case for the decade 2011-2020 is presented in Figure 8, with different potential initial levels of debt. Regardless of the initial level of debt, an implementation of the simple Lucas capital tax reform comes close to the Fiscal Frontier indicating just how important this aspect of the optimal policy is. The only desirable change in the implementation of the policy is that the reduction in capital taxation is slower the higher the initial level of debt to be sustained. The policy maker would like to exploit the transition to the ultimate steady-state, which is free of capital taxation, for longer when debt is higher. As a result the optimized half-life of capital taxation is 8.6 years when the debt-to-GDP ratio is 100% and 17.2 years ( $\rho = 0.99$ ) when the debt ratio rises to 300%. Importantly, this is not a feature of this decade only. As we show in Table 4, the welfare gains from the capital tax reform with optimized half-life are very similar to the ones attained by the Ramsey policy in each decade.

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<sup>16</sup>Note that since welfare losses are negative, this implies the reduction in capital taxation represents a significant welfare improvement.

Figure 7: Degree of gradualism across decades



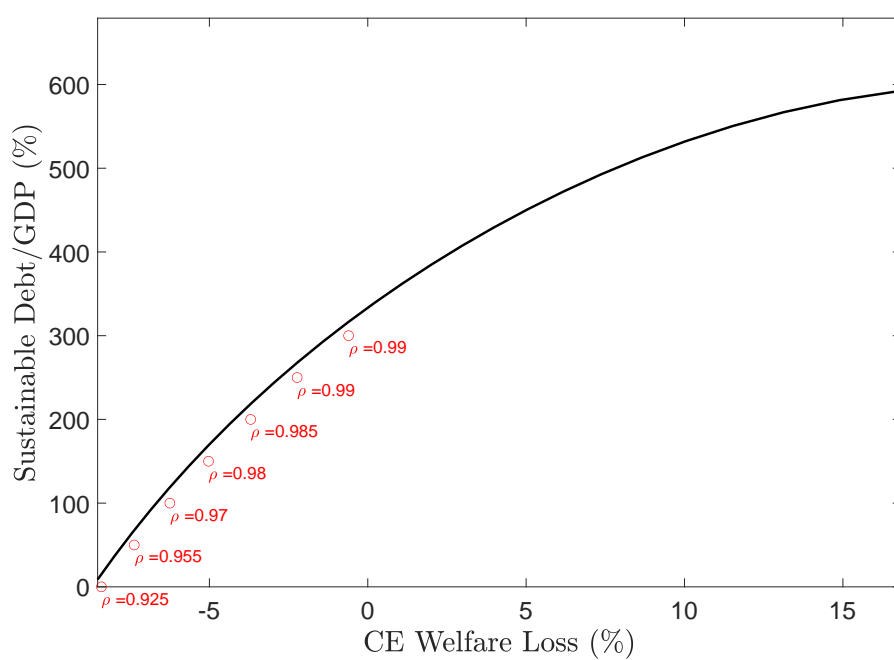
Note: starting from the decade-specific calibration in Table 3, the figure plots the welfare losses associated with eliminating capital tax rates in line with equation (36) under different degrees of gradualism. The lowest point of each line represents the optimal speed of fiscal reform measured as the half-life of the capital tax rate.

Table 4: Comparing Lucas optimized policy and Ramsey

	1971-1980	1981-1990	1991-2000	2001-2010	2011-2020
Ramsey % CE	-8.83	-7.50	-7.78	-5.63	-5.70
Lucas % CE	-8.40	-7.01	-7.10	-5.27	-5.23
Half-life (years)	5.7	6.8	6.8	6.8	8.6

Note: The table reports the welfare losses under the Ramsey policy vs the simple Lucas capital tax reform using the calibration for each decade detailed in Table 3. The final row reports the welfare maximizing half-life of capital taxes.

Figure 8: Lucas optimized policy within the Fiscal Frontier for 2010-2020



Note: the red dots represent the welfare outcome associated with the capital tax reform under alternative initial level of government debts for 2011 - 2020. In each case the fiscal rule is optimized and the associated  $\rho$  is reported. These reforms are then compared against the Fiscal Frontier constructed using the same calibration from the ultimate column of Table 3.

## 7 Debt reduction policies

Our third and final exercise explores the relative efficacy of alternative debt reduction strategies explored by the CBO (2018) on behalf of the US Congress. Starting from the current deficit, the CBO examines alternative long-run debt targets and time horizons over which the deficit is adjusted to achieve those targets. We conduct a similar analysis, but allow for the impact of changing distortionary tax instruments on economic incentives (the CBO abstracts from this), as well as computing the welfare costs associated with each policy. We consider labor taxation to be the instrument being used to achieve the required debt target.

To be specific, we consider our economy starting from debt levels of 120%, 140%, 160% and 180%, respectively. In the spirit of the CBO (2018), we consider debt reduction scenarios with government debt decreasing to 100% of GDP. Again following CBO (2018), we consider time horizons of 20, 25 and 30 years over which to achieve this debt target.

In order to achieve the debt target, we maintain capital taxation at the initial calibrated level and let labor taxation be determined by the following debt target rule,

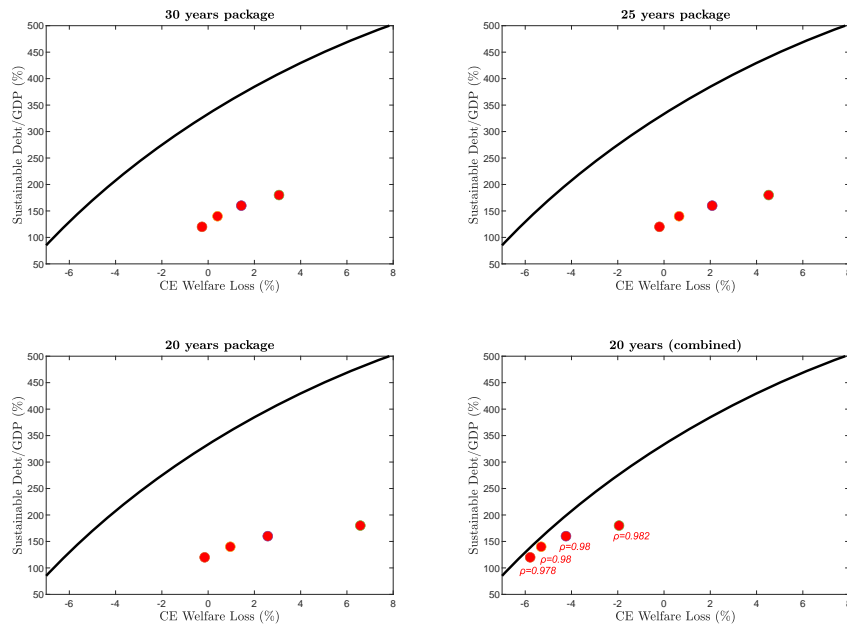
$$\frac{\tau_t^l}{\tau^l} = \left( \frac{\tau_{t-1}^l}{\tau^l} \right)^\psi * \left( \frac{d_t}{d} \right)^{(1-\psi)\mu} \quad (37)$$

where,  $\mu$  is set to achieve the debt target by the desired time horizon in each scenario. The parameter,  $\psi$ , that controls the degree of inertia in fiscal policy, is set to 0.995 in all scenarios as this prevents any significant overshooting of the debt target beyond the target horizon.

Results are presented in Figure 9. They show that a lower debt target of 100% comes at a welfare cost which progressively rises as both the speed of adjustment and the size of the fiscal consolidation increases. Conditioning on the speed of adjustment, we locate the policies associated with different degrees of fiscal correction within the Fiscal Frontier. This shows that they lie well inside the Frontier, increasingly so as the required quantity of fiscal adjustment is increased. Such a pattern is to be expected as adopting a

policy which reduces debt rather than, simply, maintaining it is extremely costly as it requires greater variation in labor tax rates. This goes against the tax smoothing arguments of Barro (1979) inherent in the optimal policy underpinning the Frontier. Therefore as we increase the debt reduction required to be achieved within the time frame the welfare costs rise at an increasing rate.

Figure 9: Achieving CBO debt targets under different time horizons and initial debt levels



Note: in each subplot the circles represent the welfare costs of reducing debt to 100% of GDP from alternative initial debt levels of 120%, 140%, 160% and 180%. Each quadrant depicts a different time horizon over which the target is achieved. The time horizons considered are 20, 25 and 30 years. The debt reduction is achieved through increasing labor taxation in line with fiscal rule in equation (37). The right bottom quadrant combines this approach to fiscal consolidation with a simultaneous gradual reduction in capital taxation to zero, with an optimized half-life of the capital tax rate ranging from 7.8 years ( $\rho = 0.978$ ) to 9.5 years ( $\rho = 0.982$ ). The debt target is achieved after 20 years under this combined policy.

Our analysis so far suggests that a capital tax replacement reform has

large welfare benefits which bring the economy close to the Fiscal Frontier, while conventional debt reduction strategies lie far from the Frontier and are hugely costly, particularly when they are large and done quickly. In light of this we try to mitigate the costs of fiscal consolidation by combining them with a simultaneous capital tax reform. To do so, we consider the case of a 20 year debt reduction strategy which was shown to be the most costly of such policies. We then combine that fiscal consolidation with a capital tax reform. Specifically, using the calibration for the 2011-2020 decade in combination with different initial values of debt, we allow the capital tax rate to be reduced in line with the optimized half-life that implies, while at the same time allowing labor taxes to follow the fiscal rule in equation (37) in order to achieve a debt target of 100% of GDP within 20 years. We find that the capital tax reform is so beneficial that it implies a welfare improvement despite the additional increase in labor taxation needed to reduce debt at the same time as compensating for the elimination of capital taxation. The net gains are, however, reduced as we increase the size of required fiscal consolidation and even the combined package of capital reform and fiscal consolidation moves away from the Fiscal Frontier.

In summary, the need to reduce debt implies a significant increase in labor income taxation which acts as a drag on the economy; however, when combined with a gradual reduction in capital taxation, the negative effect of this reform is more than offset. Indeed, such a combined policy package implies welfare gains which come very close to the optimum provided the size of the fiscal consolidation remains relatively modest.

## 8 Conclusions

In the context of a standard neo-Classical growth model, we explore policies which maximize the discounted value of fiscal surplus given the welfare costs of the implied tax distortions and which, through duality, are equivalent to conventional Ramsey policies. Such policies trace out a Fiscal Frontier in sustainable debt-welfare space which shows the most advantageous trade-off between fiscal sustainability and welfare. Any conceivable fiscal policy

reform must lie within this Frontier. For a specific proposed reform, the horizontal distance to the Frontier measures how much additional welfare could be generated if policy was optimal, but constrained to sustain the same level of debt. While the vertical distance from the Frontier measures how much additional debt could be sustained without any additional welfare cost.

We then use this Fiscal Frontier to assess the efficacy of three alternative sets of fiscal reform. Firstly, the permanent one-off adjustments to individual fiscal instruments underlying conventional Laffer curve calculations are shown to lie significantly within the Fiscal Frontier. The welfare cost of sustaining a particular debt level is far lower and/or the maximum debt that can be sustained at a given welfare cost is higher than implied by Laffer curves once we take account of the ability to simultaneously vary tax instruments intertemporally. For the US economy the maximum sustainable debt-to-GDP ratio for a given welfare cost, is typically around 150% of GDP higher than that implied by the corresponding Laffer curve experiment. To some extent, this is to be expected since the Laffer curve reform was never intended to be optimal, although the magnitude of the difference is striking.

Secondly, since the Ramsey policy supports the Chamley-Judd result that in the long-run capital taxation should be eliminated, we consider the policy proposal of Lucas (1990) to gradually eliminate such taxes, while simultaneously raising labor taxes to ensure long-run fiscal sustainability. Adopting an optimal pace of fiscal reform implies that, for the US economy, such a policy is very close to the Fiscal Frontier, despite the fact that the latter involves a full-blown non-linear intertemporal policy optimization. This shows that the Fiscal Frontier is not an abstract benchmark which simple policy reform can never hope to approach. This exercise also suggests that the need for capital tax reform was more pressing in 1980 than it is today, as government debt has risen, but capital tax rates have been reduced.

Thirdly, we replicate and extend the analysis undertaken by the CBO (2018) on behalf of the US Congress exploring the implications of debt reduction strategies needed to achieve a debt target of 100% of GDP over different time horizons and different initial debt positions. We find these policies to be



costly, especially when the required debt reduction is relatively rapid and/or large. Since our second exercise revealed the value of capital tax cuts, we then examined the extent to which such a reform could mitigate the costs of debt reduction. We find the costs of even the least desirable package of debt reduction measures can be more than offset by simultaneously introducing a capital tax reform, although its ability to do so is reduced as the scale of the require fiscal consolidation is increased.

The reliance on multiple instruments and time-variation in the setting of those instruments, make Ramsey policy too complex to be implemented as a real-world tax reform. However, by constructing a Fiscal Frontier which defines the best possible outcomes a policy reform can hope for, this can serve as a useful benchmark for policy evaluation. In a series of exercises, we placed a variety of straight-forward reforms of the kind often considered in the literature within the Fiscal Frontier. In doing so, we could clearly see which reforms were most effective in reducing the welfare costs of sustaining debt, and whether these preferred reforms enjoyed any scope for further improvement. The ability to compare different potential reforms in a common framework also suggested ways of combining reforms to mitigate welfare costs and/or enhance debt sustainability. While some of the results in our analysis are likely to be dependent on the model specification we adopted, the Fiscal Frontier is a general tool that can be applied to any model (provided that duality applies). Future research will seek to apply this approach to richer environments, as well as considering a wider range of policy proposals.

## **Disclaimer**

The views expressed in this article are the sole responsibility of the authors and in no way represent the ones of the European Commission.

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