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Sovereign Debt Issuance and Selective Default

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October 2023

Abstract

Sovereigns issue debt on both domestic and foreign markets and the two debts are uncorrelated in the data. Sovereigns default mostly selectively. We propose a theory to rationalize these observations. A government chooses the optimal combination of two debts to smooth consumption, which is subject to output shock and volatile tax distortions. In equilibrium, it mostly relies on domestic debt to smooth the tax wedge and on foreign debt to smooth the output shock. Issuing either debt is less costly than raising taxes, but it is subject to default risk due to the government's limited commitment. A quantitative, calibrated model with two shocks and two debts replicates well debt-to-GDP ratios, default frequencies, cyclical properties of emerging economies and behavior of aggregates around default episodes.

Keywords: sovereign debt, selective default, debt composition

JEL Classification: F34, G15, H63

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1 Introduction

Humanity has witnessed sovereign debt crises for hundreds of years. Foreign debt and default have been studied extensively in the literature, but recently the “forgotten history of domestic debt” has become an important research agenda started by [Reinhart and Rogoff \(2011a\)](#). Domestic debt is large and plays a significant role in the history of sovereign defaults. Worldwide, domestic debt has accounted for a large fraction of total public debt in the postwar era. This paper contributes to the literature by establishing new stylized facts about domestic and foreign debt and default and proposing a new theory that can rationalize them.

Using data on 97 economies in the years 1950-2010 we establish two stylized facts: (i) governments issue debt on both domestic and foreign markets and the issuances of two debts are uncorrelated, (ii) defaults are mostly selective on either domestic or foreign debt.

We build a dynamic model, in which domestic and foreign debts and selective defaults arise endogenously. Markets are incomplete, a government issues bonds to domestic and foreign investors, and it cannot commit to future repayments. The government has three means of financing: taxes and two defaultable bonds. Domestic bonds are bought by domestic households and foreign bonds are bought by risk-neutral foreign investors. From the government’s perspective, the two debts are distinguishable, because different investors hold them, and a government cares differently about their welfare. The economy is subject to two exogenous processes: an output shock and a volatile tax wedge. A government weighs the benefits of defaulting and holding on to borrowed resources against a loss of output triggered by default.

Domestic and foreign debts are hardly similar. Foreign debt issuance and repayment involve transferring resources into and out of an economy, which can help to achieve consumption smoothing over the business cycle. Domestic debt cannot achieve this, as its issuance and repayment occur within an economy: domestic borrowing does not bring in additional resources. In the absence of distortions, taxes and domestic debt are perfect substitutes. The tax wedge, however, breaks this Ricardian equivalence and draws a distinction between tax-financed and debt-financed expenditures. The government chooses the optimal combination of two debts to smooth the two exogenous processes. In equilibrium, the government mostly relies on domestic debt to smooth the tax wedge and mostly relies on foreign debt to smooth the output shock.

Defaults on both markets are triggered by a negative output shock but occur subject to different histories of debt stocks accumulations. Foreign default occurs when accumulated stock of foreign debt is high, after a boom followed by a one-off sudden, deep recession. A government repays domestic debt with taxes only when the tax wedge is low. When the tax wedge is high, a government prefers to issue domestic debt rather than collect taxes. Domestic default occurs in a recession after a period of persistently high tax wedge.

We estimate output and tax wedge processes for the postwar Argentina and default penalties parameters are calibrated to match frequencies of defaults. The inclusion of domestic debt brings the average level of the total public debt close to the data. Our model, with one-period bonds delivers 43% total debt-to-GDP, in line with empirical evidence for emerging economies. The model successfully predicts the frequencies of selective defaults, the average debts-to-gdp ratios, as well as several untargeted moments including the low correlation between the two debts and average output paths around domestic and foreign default. It is able to explain several salient features of emerging markets' business cycles such as the countercyclical spreads, excess consumption volatility, countercyclical trade balance, and procyclical fiscal policy.

Literature Review. The canonical sovereign default model of [Eaton and Gersovitz \(1981\)](#) and its quantitative versions of [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#) are built around the four main assumptions: single asset, risk-neutral foreign investor, one shock and exogenous default penalties. The literature has since been tasked with relaxing each of those assumptions, as motivated by empirical regularities that the canonical model found hard to replicate. Our motivation is also empirical: quantitative importance of domestic debt and a pattern of selectivity in sovereign defaults. Thus, our contribution is an extension to the asset structure and investor's heterogeneity. In our framework, foreign investors are risk neutral, but domestic investors are risk-averse in the spirit of [Lizarazo \(2013\)](#), who improves the debt pricing kernel, by allowing for risk-aversion on the foreign investors' side.

In our model taxes are distortionary, which creates incentives for a government to borrow domestically. Thus, the model is closely related to [Pouzo and Presno \(2022\)](#) and [Karanounias \(2017\)](#) where a government defaults to mitigate endogenous tax distortions, albeit in a closed economy setting. Other contributions that study defaultable domestic debt in a closed economy setting include: [Bocola \(2016\)](#), [Coimbra \(2020\)](#), [D'Erasmus and Mendoza](#)

(2016). By introducing two debt instruments, our framework improves the model fit in terms of debt-to-GDP ratios.

In our model, we make the assumption that the process of tax collection is expensive and that this cost fluctuates over time. Two recent studies, [Casalin, Dia and Hallett \(2020\)](#), [Cerniglia, Dia and Hallett \(2021\)](#) also investigate the stability of domestic public debt under a similar assumption. They explore the idea that governments face challenges when attempting to rapidly change tax policies. These studies demonstrate, albeit using a different framework, that ineffective tax collection can lead to the instability of public debt. In a recent contribution, [Izumi \(2020\)](#) shows how a bank run, through fire sale of assets and flight to liquidity, can lower tax base, which can result in a domestic sovereign default. Our model incorporates a similar mechanism, but we focus on examining the impact of external fluctuations in the tax base on both: sovereign default incentives and debt issuance policies.

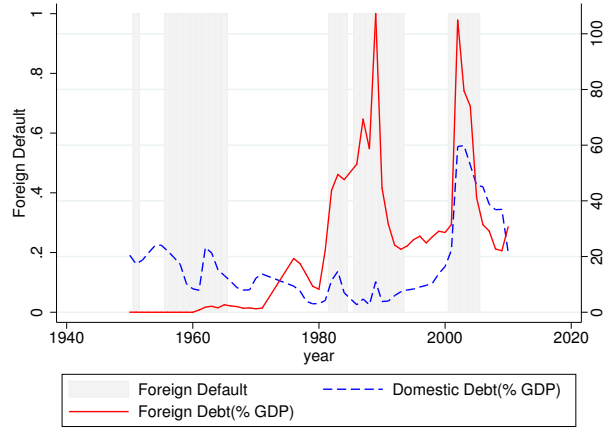
Three recent contributions use alternative frameworks to study, similarly to us, sovereign debt composition allowing for selective default. [Erce and Mallucci \(2018\)](#) study impact of sovereign default on credit and trade in a two bond economy, where domestic debt is held by domestic banks. [Niepelt \(2016\)](#) builds a theoretical model with overlapping generations and elections where a government weights default benefits to taxpayers against costs to creditors. [Vasishtha \(2010\)](#) studies the selective nature of sovereign defaults but in equilibrium, contrary to empirical evidence, foreign default never happens.¹ Several recent contributions study how debt composition affects default incentives under the assumption that a government cannot discriminate between domestic and foreign investors ([Brutti, 2011](#), [D’Erasmus and Mendoza, 2021](#), [Engler and Große Steffen, 2016](#), [Gennaioli, Martin and Rossi, 2014](#), [Guembel and Sussman, 2009](#), [Mengus, 2014](#), [Perez, 2015](#), [Sosa-Padilla, 2018](#)).

2 Stylized Facts

The mechanism of foreign debt issuance and foreign default is well understood. The role that domestic debt plays - less so. Is domestic debt used by governments as a substitute to foreign debt, or are the two complementary? Are governments concerned about total debt issuance, or do they use two instruments to achieve different objectives? Do governments

¹[Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo and Martinez \(2009\)](#) enrich asset structure by introducing long-term debt and [Bigio, Nuno and Passadore \(2022\)](#) by using continuous maturities. [Aguiar, Chatterjee, Cole and Stangebye \(2016\)](#) provide a comprehensive review of sovereign default models.

Figure 1: Domestic and foreign debt as fraction of GDP in Argentina 1950-2010



Notes: Domestic and foreign debt outstanding, GDP series and default dates for Argentina are from [Reinhart and Rogoff \(2011b\)](#). See Online Appendix for details.

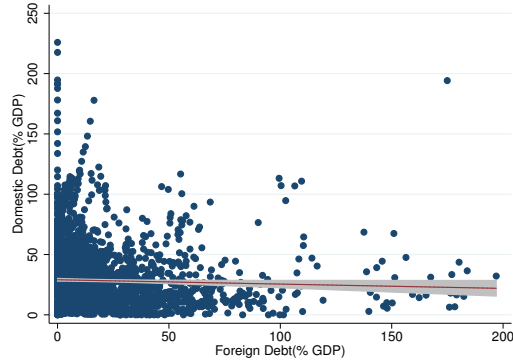
default on domestic debt and if so, do domestic and foreign defaults coincide? In this section, we shed light on these questions demonstrating empirical regularities.

Throughout the paper we use the *economic* definition of domestic and foreign debt (residency of debt holder), as it creates clear differential incentives for a sovereign to default. However, the data on selective defaults and debt compositions usually come in *legal* definition (country’s legal framework applying to an issuance). The literature argues that the two definitions have been historically close to each other.²

1. Governments issue debt on both domestic and foreign markets and two issuances are uncorrelated. Figure 1 plots foreign and domestic debts to GDP for Argentina between 1950 and 2010. Interestingly, in the years immediately following World

² [Reinhart and Rogoff \(2011b\)](#) show that markets for domestic and foreign debt have remained segmented for the most of the sovereign debt history: “*The overwhelming majority of external public debt, debt under the legal jurisdiction of foreign governments, has been denominated in foreign currency and held by foreign residents*”. Recently, literature argues that these markets are still segmented: [Borri and Shakhnov \(2017\)](#) show that there is a gap between local and foreign currency bond spreads beyond what a capital asset pricing model would predict, [Du and Schreger \(2014\)](#) show, that the entry of foreign investors into local currency debt markets is a recent phenomenon: before 2004 only 10% of local currency public debt was held by foreign investors on average. The same pattern emerges in a wider dataset by [Arslanalp and Tsuda \(2014\)](#).

Figure 2: Domestic vs foreign debt in emerging economies



Notes: Correlation=-0.08. When zero-debt observations are excluded, the correlation is equal to -0.03. When country-year fixed effects are used, the correlation is equal to 0.16. Domestic and Foreign debt outstanding and GDP series are from [Panizza \(2008\)](#) and [Reinhart and Rogoff \(2011b\)](#). The dataset covers 97 economies (including developing and developed) in the years 1950-2010 yielding 3245 observations. See Online Appendix for details.

War II foreign, not domestic debt was missing. Other than a period before 1960, Argentina issued both domestic and foreign debts throughout its recent history. The shaded areas represent foreign defaults. Foreign defaults in the case of Argentina were mostly selective, except for the first default (when there was no domestic debt) and the last default (which was total default).

Argentina is plotted here as it is a representative example (and will be later used in the calibration exercise). The fact that governments use both domestic and foreign debt is a cross-country, systematic phenomenon confirmed in Figure 2. It plots domestic debt-to-GDP on the vertical axis against foreign debt-to-GDP in the horizontal axis after excluding default episodes. The vast majority of observations lie outside of the two axes.

The correlation between domestic and foreign debt as fractions of GDP in the full sample is -0.08. The figure includes a plotted correlation line with a 95% confidence interval.

The composition of foreign and domestic debt is important. Empirically it has been shown that, among other things, it determines the size of fiscal multipliers ([Priftis and Zimic \(2018\)](#)), and the size of international spillovers ([Borri and Shakhnov \(2017\)](#)).

2. Defaults are mostly selective. The data on defaults come from the updated database accompanying [Reinhart and Rogoff \(2011b\)](#) and cover up to 130 countries for the years 1800-2014. As the dates of the domestic and foreign debt crises sometimes overlap, there

are many ways to calculate the final number of events. We focus on the postwar period. When a government, in a given year, defaulted *de jure* on the both domestic and foreign debt we label this event as *total default*. There are 13 such instances (19 if we consider a +/- 2 year window). When a government, in a given year, defaulted *only* on its foreign debt, we label it as *foreign default*. There are 161 such instances. Similarly, when a government, in a given year, defaulted *only* on its domestic debt, we label this event as *domestic default*. There are 36 such instances.

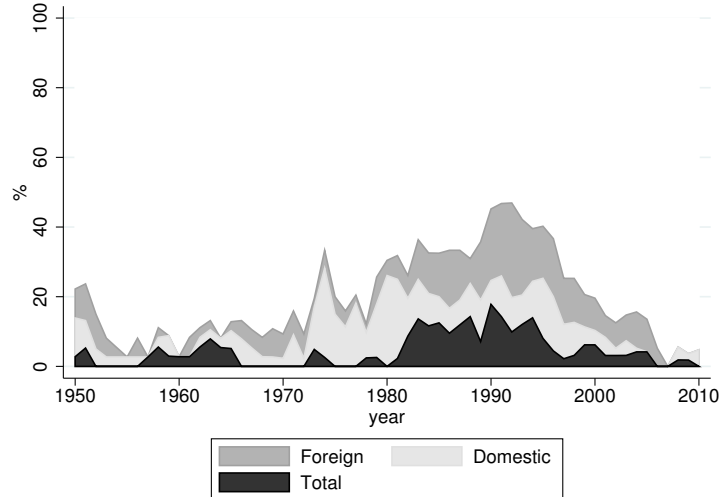
We also recognize that hyperinflation is a *de facto* way to default on domestic debt. Inflation crises help to reduce the burden of local currency denominated debt. This debt was often issued domestically and sold to domestic residents. This is consistent with our theoretical framework, that is a real model of *de facto* defaults. There are 193 instances of domestic default.³

Figure 3 shows the same fact from another perspective. It plots, in any given year, the fraction of independent countries that are experiencing one of the three types of default: selective foreign, selective domestic or total. None of the three defaults is negligible. The figure shows that sovereign selective default is a systematic phenomenon that calls for a unified theory of domestic and foreign debt and selective defaults.

How can a government default on foreign investors while repaying domestic investors or *vice versa*? Among the tools that governments use to discriminate against types of bondholders, the most popular are capital controls, exchange controls and freezes on deposits. In 1990 Brazil defaulted on its domestic debt but kept servicing its foreign debt. All foreign exchange transactions were directed through the central bank. In 1998 Russia defaulted on both foreign and domestic debt, imposing capital and exchange rate controls. Russia kept servicing debts to foreign investors and domestic households, so it effectively defaulted only on domestic debt held by firms. Argentina's 2001 default is often considered as a model calibration case of foreign default, although in fact, it was a total default. Firstly, all resident-held bonds, denominated in both domestic and foreign currency, were converted to the government-guaranteed loans, which were all later converted to pesos at below-market

³Outright domestic defaults are rare, which is consistent with a long-held view that, after all, governments can service these obligations by printing money (dramatically so in cases like Germany in 1923 and Yugoslavia in 1993-94). We find it reasonable to identify the hyperinflation as domestic default. The definitions of foreign and domestic *de jure* defaults are taken from [Reinhart and Rogoff \(2011b\)](#). The list of episodes with dates is available in Online Appendix.

Figure 3: Fraction of countries in different types of default



Notes: The fractions are defined in percentages of the total number of countries for which data are available. Dates of domestic and foreign defaults are [Reinhart and Rogoff \(2011b\)](#). The dataset covers 101 economies (including developing and developed) in the years 1950-2010 yielding 3319 non-missing observations. See Online Appendix for details.

exchange rate. Secondly, 60% of the debt defaulted on in December 2001 was held by Argentinians.⁴

3 The Model

Let time be indexed by $t = 0, 1, 2, \dots$. The economy has an exogenous stochastic stream of income $y_t \in \mathbb{Y}$, which is a Markov process. In each period t the government covers a fixed exogenous stream of expenditures g and decides either to repay or default on outstanding foreign and domestic debts. When the government chooses to default, the economy suffers from output penalties and is excluded from borrowing on the market where the default happened for a random number of periods. We allow the expected exclusion durations and

⁴Recent examples of what could be considered pure foreign default include: Bolivia in 1989 (most of domestic debt was repurchased a year before default), Pakistan in 1999 (which stopped payments on outstanding obligations to creditors in the UK, Europe and the US and put a freeze on foreign currency deposits mostly owned by non-residents) and Cyprus in 2013 (a freeze and a partial expropriation of deposits exceeding €100,000, which were mostly owned by non-residents).

output costs to differ between the two markets. When the government chooses to repay to either type of investors, it issues new bonds on the respective market.

3.1 Households

Households are identical and risk averse. Their instantaneous utility is given by the CRRA function over consumption:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}. \quad (1)$$

Households save using domestically issued government bonds b_d . They face a budget constraint, which is dependent on the government's decision to default on their savings (δ_d).⁵

When the government repays both debts, households' budget constraint reads:

$$c^i = y^i - T(1 + \tau) + (1 - \delta_d)(b_d - q_d^i b'_d), \quad i \in \{r, fd, dd, td\} \quad (2)$$

where b_d is the amount of domestic debt owed and repaid by the government to households, b'_d is the new issuance of government domestic debt, q_d^i is the domestic bond's discount price, which depends on the government default decisions. Output y^i and consumption c^i also depend on the government repayment/default decision. T is the amount of taxes raised by the government.⁶ τ is an exogenous distortion from taxation: a wedge between benefit of taxation for the government and the cost of taxation for the agents. This specification with an exogenous tax wedge is flexible enough to accommodate many models in which tax distortions arise endogenously.⁷

3.2 Recursive equilibrium

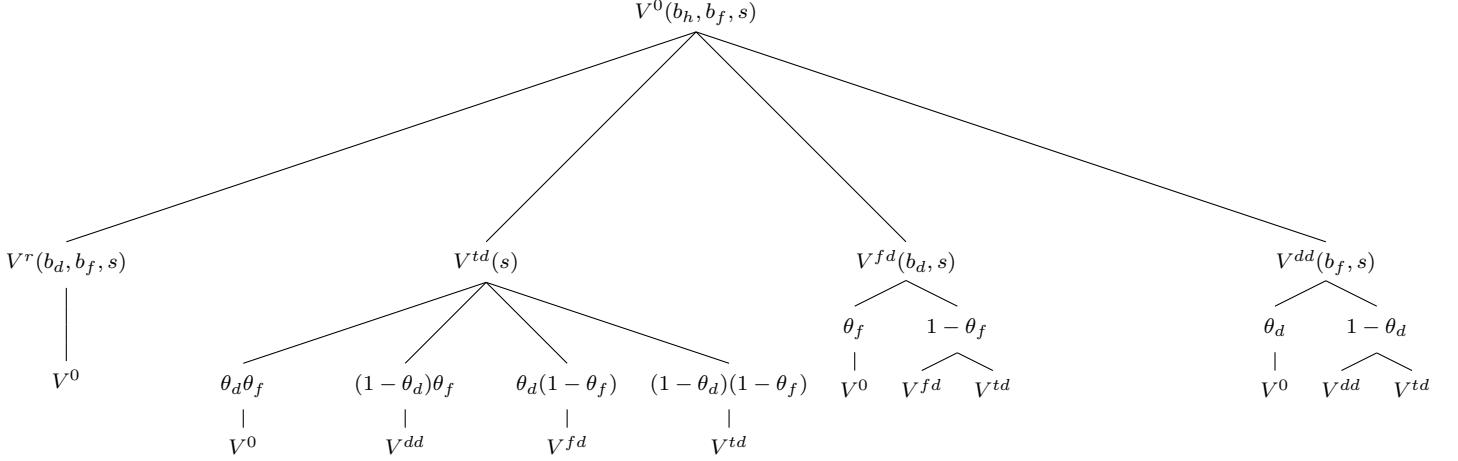
We define a recursive equilibrium in which domestic households, foreign investors and the government act sequentially and the government acts with discretion. The aggregate state of the economy is given by two endogenous debts (b_d, b_f) and two exogenous processes for

⁵In the foreign default literature a government and a representative household are the same agent. We draw a clear distinction between the two. Households decide on savings (domestic debt demand) taking taxes as given. The government internalizes the impact of distortionary taxes and default decisions on households' consumption and decides on domestic debt issuance (debt supply) to smooth the distortions.

⁶Whenever taxes are negative, the household budget constraint is $c^r = y - T(1 - \tau) + b_d - q_d^r b'_d$, so that rebates are also distortionary.

⁷In a [separate note](#) accompanying this paper we formally derive the equivalence between models with endogenous and exogenous tax distortions.

Figure 4: Government decision tree



Notes: When both markets are open (V^0), the government can decide to repay both debts (V^r), default on both debts (V^{td}), repay only domestic debt (V^{fd}) or repay only foreign debt (V^{dd}). Subsequent possible choices are depicted on the lower branches of the decision tree.

income and the tax wedge $s = (y, \tau)$. Every period, the government decides whether to repay its two outstanding debts, default on domestic debt, default on foreign debt or default on both:

$$V^0(b_d, b_f, s) = \max\{V^r(b_d, b_f, s), V^{dd}(b_f, s), V^{fd}(b_d, s), V^{td}(s)\} \quad (3)$$

The government's default decisions are summarized by default indicators δ_j^i assuming value 1 in the case of default, where subscript j stands for the defaulted debt: foreign and domestic ($j = f, d$) and superscript i stands for the current state: repayment, foreign default and domestic default ($i = r, fd, dd$). It is sufficient to define two default indicators for repayment periods: δ_f^r, δ_d^r (repayment decision is taken with both equal to zero and total default decision is taken with both equal to one) and one for each of the two selective default periods: $\delta_d^{fd}, \delta_f^{dd}$. After default, the government is excluded from the market and faces probabilities of returning to borrowing: θ_f to foreign and θ_d to domestic market. The government's choices are presented graphically in Figure 4, where tree branches correspond from the left to the right to a repayment, total, selective foreign and selective domestic defaults.

If the government decides to repay it solves the following problem:

$$V^r(b_d, b_f, s) = \max_{b'_d, b'_f} \left\{ u(c^r) + \beta \mathbb{E} [V^0(b'_d, b'_f, s')] \right\} \quad (4)$$

subject to four constraints: the households' budget constraint (2), the foreign bond price schedule, the domestic bond price schedule and the government budget constraint. The foreign bond price schedule is given by:

$$q_f^r(b'_d, b'_f, s) = \frac{\mathbb{E} \left[1 - \delta_f^r(b'_d, b'_f, s') \right]}{1 + r} \quad (5)$$

where q_f^r is the discount price of a government bond issued with foreign investors b'_f , who are risk-neutral, deep-pocket and have access to international risk free rate r , and δ_f^r is the indicator of the government decision to default on foreign debt. A risk-free foreign investor assumption is employed by the vast majority of the literature. Unlike foreign, we assume that domestic investors are risk-averse.⁸ The risk-averse price of domestic debt is dynamic and depends on six states: two exogenous shocks, two debts issued and two debts outstanding (as they all affect marginal utility of consumption). Unlike in models facing risk-neutral marginal investor, the fluctuations in both current consumption and expected consumption affect debt discount prices. The domestic bond price schedule reads:

$$\begin{aligned} q_d^r(b_d, b_f, b'_d, b'_f, s) &= \\ &= \beta \frac{\mathbb{E} \left[(1 - \delta_f^{r'}) (1 - \delta_d^{r'}) u_c(c^{r'}) + \delta_f^{r'} (1 - \delta_d^{r'}) u_c(c^{fd'}) \right]}{u_c(c^r)}, \end{aligned} \quad (6)$$

where future expected utility consists of two parts: one related to repayment ($\delta_f^r = \delta_d^r = 0$) and one to foreign default ($\delta_f^r = 1$). Only in these two cases domestic debt is being serviced so it is a valid savings vehicle that enables an intertemporal consumption smoothing. In (6) state space notation for default indicators and consumption levels has been suppressed to facilitate legibility. The last constraint is the government budget constraint:

$$T + q_d^r b'_d + q_f^r b'_f = g + b_d + b_f. \quad (7)$$

Second, if the government defaults on foreign debt (but services its domestic obligations) the economy suffers an output cost, with the probability $1 - \theta_f$ is excluded from the foreign debt market (and remains only on domestic one) and the government can still decide to also

⁸This is a necessary condition for a selective sovereign default model, and has some important implications: the pricing kernel for domestic debt is a higher dimensional object than the pricing kernel for foreign debt. The government internalizes the impact of its current and future decisions on current prices. Technically, this will complicate the solution algorithm, as one would need to keep track of the whole (also off-equilibrium) repayment-default schedule at every iteration.

default on domestic debt (yielding total default). The government's problem is:

$$V^{fd}(b_d, s) = \max_{b'_d} \left\{ u(c^{fd}) + \beta \mathbb{E} \left[\theta_f V^0(0, b'_d, s') + (1 - \theta_f) \max\{V^{fd}(b'_d, s'), V^{td}(s')\} \right] \right\} \quad (8)$$

subject to households' budget constraint (2), households' first-order condition

$$q_d^{fd}(b_d, b'_d, s) = \beta \frac{\mathbb{E} \left[\theta_f (1 - \delta_d^{r'}) u_c(c^{r'}(., 0, .)) + (1 - \theta_f) (1 - \delta_d^{fd'}) u_c(c^{fd'}) \right]}{u_c(c^{fd})}, \quad (9)$$

where expected future marginal utility consists of two parts: one related to a readmission to foreign markets and one related to foreign default, where δ_d^{fd} is the probability of government going from foreign into total default. In (9) some arguments have been suppressed to facilitate legibility. The last is the government budget constraint:

$$T + q_d^{fd} b'_d = g + b_d. \quad (10)$$

Third, if the government decides to default selectively on domestic debt it remains active on the foreign market, comes back to domestic borrowing with the probability θ_d , suffers the domestic output penalty and can still default on foreign debt:

$$V^{dd}(b_f, s) = \max_{b'_f} \left\{ u(c^{dd}) + \beta \mathbb{E} \left[\theta_d V^0(b'_f, 0, s') + (1 - \theta_d) \max\{V^{dd}(b'_f, s'), V^{td}(s')\} \right] \right\} \quad (11)$$

subject to households' budget constraint (2), the foreign bond price schedule

$$q_f^{dd}(b'_f, s) = \theta_d \frac{\mathbb{E} \left[1 - \delta_f^r(b'_f, 0, s') \right]}{1 + r} + (1 - \theta_d) \frac{\mathbb{E} \left[1 - \delta_f^{dd}(b'_f, s') \right]}{1 + r}, \quad (12)$$

which is a probabilities-weighted sum of a price of foreign debt in repayment and foreign debt in domestic default, with δ_d^{fd} being the probability of the government going from foreign default into total default. The last is the government budget constraint:

$$T + q_f^{dd} b'_f = g + b_f. \quad (13)$$

Fourth, if the government decides to pursue total default, the economy suffers the output penalties for both domestic and foreign default, and the government comes back to the international and domestic markets with probabilities θ_f and θ_d respectively. The government's problem is summarized by:

$$V^{td}(s) = u(c^{td}) + \beta \mathbb{E} \left[\theta_f \theta_d V^0(0, 0, s) + \theta_f (1 - \theta_d) V^{dd}(0, s') + (1 - \theta_f) \theta_d V^{fd}(0, s') + (1 - \theta_f) (1 - \theta_d) V^{td}(s') \right] \quad (14)$$

subject to households' budget constraint (2) and the government budget constraint

$$T = g. \quad (15)$$

Let $B = \{b_d, b_f\}$ define endogenous and $s = \{y, \tau\}$ define exogenous states in the economy.

Definition 1. *The recursive competitive equilibrium is defined as four sets of default schedules $\{\delta_d^r(B, s), \delta_f^r(B, s), \delta_d^{fd}(b_d, s), \delta_f^{dd}(b_f, s)\}$, four sets of debt discount prices $\{q_f^r(B', s), q_d^r(B, B', s), q_f^{dd}(b_f, s), q_d^{fd}(b_d, s)\}$, four sets of government debt issuance policies $\{b_f'(B, s), b_d'(B, s), b_f^{dd}(b_f, s), b_d^{fd}(b_d, s)\}$ and four sets of consumptions $\{c^r, c^{dd}, c^{fd}, c^{td}\}$ conditional of exogenous processes $\{y, \tau\}$ such that:*

- 1) *Taking prices $\{q_f^r, q_d^r, q_f^{fd}, q_d^{dd}\}$ and government debt issuances $\{b_f', b_d', b_f^{dd}, b_d^{fd}\}$ as given, households' consumptions $\{c^r, c^{dd}, c^{fd}, c^{td}\}$ satisfy the households' budget constraints.*
- 2) *Taking prices $\{q_d^r, q_f^r, q_d^{fd}, q_f^{dd}\}$ as given, government's default schedules $\{\delta_f^r, \delta_d^r, \delta_d^{fd}, \delta_f^{dd}\}$ and debt issuance policies $\{b_d', b_f', b_f^{dd}, b_d^{fd}\}$ solve the government's optimization problems.*
- 3) *Given foreign default schedules $\{\delta_f^r, \delta_f^{dd}\}$ foreign debt prices $\{q_f^r, q_f^{dd}\}$ satisfy foreign investors expected zero profits.*
- 4) *Given domestic default schedules $\{\delta_f^r, \delta_f^{dd}\}$, households' consumptions $\{c^r, c^{fd}\}$ and future expected default schedules, debt issuances and households' consumptions, domestic debt prices $\{q_d^r, q_d^{fd}\}$ satisfy households first order conditions.*

Default schedules and value functions are related in the following way:

$$\begin{aligned} V^r &\geq \max \{V^{fd}, V^{dd}, V^{td}\} \iff \delta_f^r = \delta_d^r = 0 \\ V^{fd} &> \max \{V^r, V^{dd}, V^{td}\} \iff \delta_f^r = 1, \delta_d^r = 0 \\ V^{dd} &> \max \{V^r, V^{fd}, V^{td}\} \iff \delta_f^r = 0, \delta_d^r = 1 \\ V^{td} &> \max \{V^r, V^{fd}, V^{dd}\} \iff \delta_f^r = \delta_d^r = 1 \\ V^{fd} &\geq V^{td} \iff \delta_d^{fd} = 0 & V^{fd} < V^{td} &\iff \delta_d^{fd} = 1 \\ V^{dd} &\geq V^{td} \iff \delta_f^{dd} = 0 & V^{dd} < V^{td} &\iff \delta_f^{dd} = 1 \end{aligned}$$

4 Numerical Analysis

4.1 Calibration

To solve the model numerically, we assume functional forms for the exogenous processes and assign parameter values. The data availability dictates the choice of a frequency for

Table 1: Calibration

Parameter	Value	Source/Moment
Risk aversion	$\sigma = 2$	Standard in the literature
Discount factor	$\beta = 0.825$	Standard in the literature
Risk-free interest rate	$r = 0.017$	5-year US bond yearly yield
Government expenditure	$g = 0.13$	Argentina 1993–2001
Persistence of output	$\rho_y = 0.897$	Argentina 1950–2011
Std. dev. of output	$\epsilon_y = 0.053$	Argentina 1950–2011
Mean tax wedge	$\bar{\tau} = 0.025$	Harberger (1964)
Persistence of tax wedge	$\rho_\tau = 0.738$	Argentina 1980–2002
Std. dev. of tax wedge	$\epsilon_\tau = 0.175$	Argentina 1980–2002
Re-entry to foreign market	$\theta_f = 0.5$	Foreign Debt-to-GDP & f-default frequency
Output cost after f-default	$\gamma_f = 0.905$	Foreign Debt-to-GDP & f-default frequency
Re-entry to domestic market	$\theta_d = 0.5$	Domestic Debt-to-GDP & d-default frequency
Output cost after d-default	$\gamma_d = 0.955$	Domestic Debt-to-GDP & d-default frequency

the model. As default frequencies and debt-to-GDP ratios, are best thought of at annual frequency we set up the model annually. Table 1 presents the numerical values assigned to all the model parameters. We set the risk aversion coefficient σ equal to 2 and the risk free interest rate r to 1.7% yearly, which are the standard values in the literature. The discount factor β we set to 0.825, which is a yearly equivalent of the quarterly $\beta = 0.953$ found in [Arellano \(2008\)](#). The level of government expenditure is set to 13% of GDP, the average Argentinian government expenditure for 1993-2013 ([CEPAL \(2016\)](#)). Without a loss of generality we normalize the mean output to 1 and assume that output evolves according to an $AR(1)$ stochastic process in logs:

$$\log(y_t) = \rho_y \log(y_{t-1}) + u_t \quad u_t \sim \mathcal{N}(0, \epsilon_y^2). \quad (16)$$

Using the yearly output data series for Argentina from Penn Tables ([Feenstra, Inklaar and Timmer \(2015\)](#)), after taking out a linear trend and taking logs, the estimate for ρ_y is 0.897 and for ϵ_y is 0.053. Upon default the government suffers an output cost, that is assumed to be asymmetric as in [Arellano \(2008\)](#):

$$y_t^i = \min\{y_t, \gamma_i y\} \quad i = \{fd, dd, td\}, \quad (17)$$

where y^i is output in either domestic or foreign or total default, y is the mean of the output

process and γ_i takes the separate values for domestic and foreign default and a multiple of two for total default. The cost function implies that default is more costly with high output realizations.

In the model the tax wedge evolves according to an $AR(1)$ in logs around the mean $\bar{\tau}$:

$$\tau_t = \bar{\tau} \tilde{\tau}_t \tag{18}$$

$$\log(\tilde{\tau}_t) = \rho_\tau \log(\tilde{\tau}_{t-1}) + v_t \quad v_t \sim \mathcal{N}(0, \epsilon_\tau^2). \tag{19}$$

As there is no readily available measure of the tax wedge in the data, we identify the movements in the tax wedge via the movements in the tax base. The two are inversely related (Feldstein (1999) formula). Using the yearly data for the corporate taxes in Argentina (ICTD 2014 and WTD 2015) the estimate of ρ_τ is 0.738 and of ϵ_τ is 0.175.⁹ For the mean tax wedge $\bar{\tau}$ we take a conservative stand and parametrize it at 0.025, which is the lowest estimate of the static deadweight loss from taxation that we have found in the literature Harberger (1964). Additionally, we assume that the two processes are uncorrelated. This will allow the model isolate the endogenous spillover effects from the exogenous interdependence, and is consistent with the data (empirically, the correlation is equal to -0.05).

Finally, we are left with the four default-specific parameters: the exclusion probabilities θ_d, θ_f and the output penalties γ_d, γ_f . We calibrate those four parameters jointly to match two default frequencies and two debt-to-GDP ratios. In the postwar period Argentina had five foreign *de jure* and three domestic *de jure* defaults. Twice these defaults started in the same year, yielding a total default frequency of 2.8% over the period of seventy years (1945-2014). Selective foreign default happened three times, yielding a frequency of 4.3%. Finally, combining one *de jure* domestic default and seven episodes of hyperinflation, the frequency of selective domestic default is 11.4%. In this same period the average domestic debt-to-GDP ratio in Argentina was 16.93% while foreign debt-to-GDP was 23.5% (Reinhart and Rogoff (2011b)).

We calibrate default penalties to $\gamma_d = 0.955$ and $\gamma_f = 0.905$ and the exclusion parameters to $\theta_d = 0.5$ and $\theta_f = 0.5$. Our calibrated exclusion parameters are within the range of parameters typically used in the literature.¹⁰ The probabilities of returning to financial

⁹If instead we use individual income tax (value added tax) the estimates are 0.66 (0.61) for the persistence and 0.43 (0.08) for the standard deviation.

¹⁰We have also studied the model under alternative calibrations. Argentina has output and tax distortion process of a typical total defaulter. The non-defaulters have less volatile output and more persistent tax

markets after a default vary greatly in quantitative studies: [Aguiar and Gopinath \(2006\)](#) set it to 0.1 yearly, while [Arellano \(2008\)](#) sets it to 0.282 quarterly, which is equivalent to 0.73 yearly.¹¹

4.2 Policy Functions

In this section we analyze default and debt issuance policies in the calibrated model. We describe the algorithm for solving the model in Online Appendix. Throughout the analysis we find, that the policies for foreign debt and default respond strongly to the changes in output and little (or not at all) to changes in the tax wedge. However, policies for domestic debt and default respond strongly to both output and the tax wedge. Despite the fact that markets are segmented, we also document spillovers from one market to the other.¹²

Default sets. Figure 5 plots the repayment-default policies for foreign debt in the top two panels and domestic debt in the bottom two panels. The level of debt is plotted horizontally on each graph and output (left panel) and the tax wedge (right panel) are plotted vertically. The tax wedge is plotted inversely, so that top values, similarly to output, represent good times (low distortions). White area stands for a repayment and black area stands for a default.

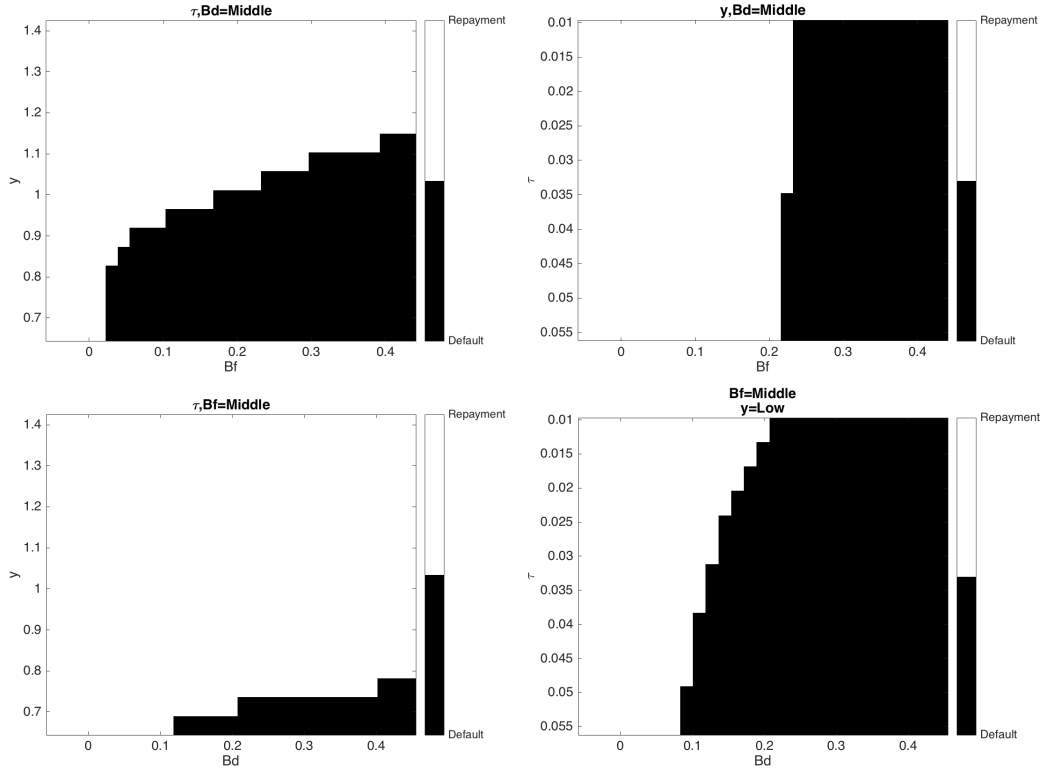
Foreign default becomes more likely as the level of foreign debt goes up and as the output goes down. For the high levels of output any level of foreign debt is safe. There is no variability of the foreign default decision with respect to the tax wedge. Foreign default set is decreasing in y , therefore foreign interest rate is decreasing in y . Foreign default happens when output suddenly drops.

Default on each debt becomes more likely when output drops. The empirical work provides strong evidence that domestic and foreign defaults are associated with sizable and wedge process. Calibrating the model to a non-defaulter substantially reduces the probability of foreign default from 4.6% to 1.9%. In order to completely eliminate foreign default however, we also need to increase foreign default output costs from 0.955 to 0.905. The similar story holds for the domestic default, but the cost adjustment is even more important. The detailed results of the model under alternative calibrations are available upon request.

¹¹See Chapter 13 in [Uribe and Schmitt-Grohé \(2017\)](#) for discussion on calibrating sovereign default models.

¹²In what follows we will use some generic terms for the levels of variables. High, Middle and Low translate to highest, middle and lowest points on the grids: $y(High) = 1.4$, $y(Middle) = 1$, $y(Low) = 0.7$, $\tau(High) = 0.0544$, $\tau(Middle) = 0.025$, $\tau(Low) = 0.0115$, $b(High) = 0.433$, $b(Middle) = 0.183$, $b(Low) = -0.05$.

Figure 5: Default sets for foreign debt (top) and domestic debt (bottom) in output (left) and tax wedge (right)

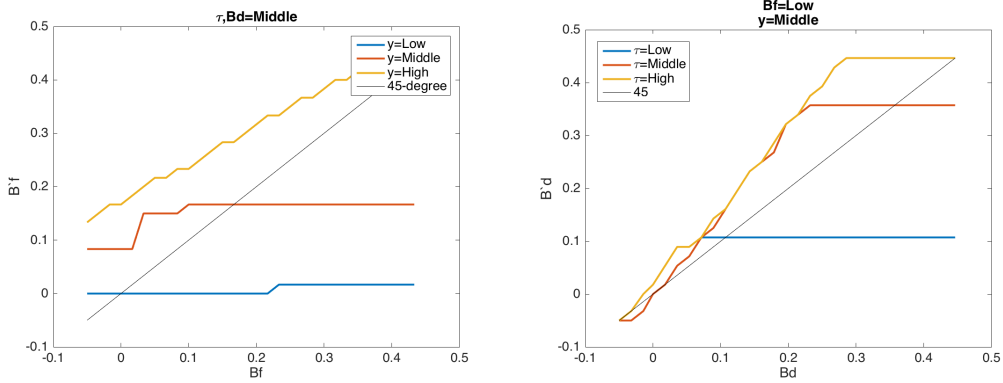


protracted contractions in output (Reinhart and Rogoff, 2011a, Schmitt-Grohé and Uribe, 2017).¹³

Domestic default is not only driven by tax wedge, but (due to the specification of the default costs) it also depends on output fluctuations. Domestic default becomes more likely as the tax wedge goes up. Domestic default happens only with low output. Middle and high output render domestic debt virtually safe. In Online Appendix we show how foreign and domestic default policies change with respect to other variables.

¹³In the model this is reinforced by the specification of default costs. We employ the commonly used Arellano (2008)'s specification of the asymmetric costs for the both types of default. This specification is intuitive, as it discourages default in good states of nature, i.e., when output is high. Since it discourages default, it helps to increase the amount of debt sustainable in equilibrium.

Figure 6: Foreign (left) and domestic (right) debt issuance policies after repayment.



Debt issuance. Figure 6 plots the debt issuance policies for foreign debt (left panel) and for domestic debt (right panel) in repayment periods. The economy accumulates foreign debt when output is high due to a countercyclical in the interest rate. For middle and low output there are well defined debt limits above which the discount price of new debt is zero. Debt issuance for middle and low y flattens out above those limits. An economy with default risk engages in a procyclical foreign borrowing policy. Similar results for foreign debt are found across quantitative models of sovereign default. Foreign debt issuance is almost exclusively driven by output fluctuations. Foreign default occurs in a recession when foreign debt is high.

Domestic debt issuance is however qualitatively different from the foreign debt issuance and almost exclusively driven by the tax wedge. When the tax wedge is low (blue line), the government prefers to finance its expenditures via taxation, because raising taxes comes at the lowest cost for the economy. Therefore, debt issuance is low, regardless of the domestic debt outstanding (only a small amount of debt, which is always safe, is being rolled over). When the tax wedge goes up (orange and yellow lines) the government employs a “gambling for redemption” policy. The government increases its domestic debt position - the issuance lines lie above the 45-degree line. The government finds it optimal to pile up domestic debt in a hope that it will be repayed with taxes, when the low-tax wedge day comes. The tax wedge is thus instrumental to the build-up of domestic debt.

Figure 13 in Online Appendix plots four extensions of the left panel of Figure 6 when τ and b_d are changing and Figure 14 plots four extensions of the right panel when y and b_f are

Table 2: Stylized Facts: Model v Data

ID	Moment	Data	Model	Model w/out dom. debt
(1a)	Avg. Foreign Debt-to-GDP	23.5%	21.3%	19.2%
(1b)	Avg. Domestic Debt-to-GDP	16.9%	22.9%	x
(2a)	Foreign Default frequency	4.6%	4.6%	7.2%
(2b)	Domestic Default frequency	11.4%	11.5%	x
(2c)	Total Default frequency	2.8%	0.3%	x
(3)	Correlation (Foreign Debt, Domestic Debt)	-0.03	-0.27	x

Notes: Data moments (1)-(5) are based on annual series 1950-2013. Corresponding model moments are obtained from calibrated model at an annual frequency. Data sources are introduced and data transformations are explained in Online Appendix.

changing. The foreign debt policy is almost entirely driven by fluctuations in output and does not respond to changes in the tax wedge and domestic debt outstanding. Domestic debt issuance policy depends on the fluctuations in the tax wedge, output (due to the decreasing interest rate) and foreign debt outstanding. The last is the refinancing operation: more domestic debt must be raised when more foreign debt comes due.

5 Simulation Results

5.1 Stylized Facts

The model is replicating the stylized facts laid out as the motivation for this paper. Table 2 compares the debt-to-GDP ratios, the default frequencies and the debt correlations obtained from the data versus respective figures obtained from the model. We also show, wherever possible, the corresponding moments obtained from a model with the same calibration but without domestic debt. The most striking finding is that the model is easily capable of simultaneously delivering observed high debt-to-GDP ratios (1a and 1b) and low default frequencies (2a and 2b). It is a well documented fact that the standard model without domestic debt fails at this exercise (see discussion in [Chatterjee and Eyigungor \(2012\)](#)). Because of the shape of the default area in the debt-output space, high foreign debt goes hand in hand with high foreign default frequency. This is also visible in our exercise: when domestic debt channel is shut down, high foreign debt of 19.2% corresponds with high foreign

Table 3: Cyclical Properties: Model v Data

ID	Moment	Data	Model	Model w/out dom. debt
(1)	Corr(Foreign spread, Output)	-0.71	-0.65	-0.75
(2)	Std(Consumption)/Std(Output)	1.41	1.23	1.02
(3)	Corr(Net Exports/Output, Output)	-0.80	-0.45	-0.48
(4)	Corr(Primary Balance/Output, Output)	-0.26	-0.21	-0.48

Notes: Data moments are based on quarterly series 1991Q1-2013Q4. Corresponding model moments are obtained from the calibrated model at an annual frequency. Data sources are introduced and data transformations are explained in Online Appendix.

default frequency of 7.2%.

Including defaultable domestic debt breaks this strong interdependence. It changes the government decision problem by increasing the menu of options available to the government, thus making foreign default relatively less attractive. Instead of entirely relying on foreign debt as a sources of income, the government can raise domestic debt. As foreign default is less attractive, higher levels of foreign debt can be sustained with a lower foreign default probability, *ceteris paribus*.

Selective default frequencies closely mirror those seen in the data. However, the model underestimates the total default frequency (2c). This is a necessary consequence of the assumption that the two exogenous processes are uncorrelated.

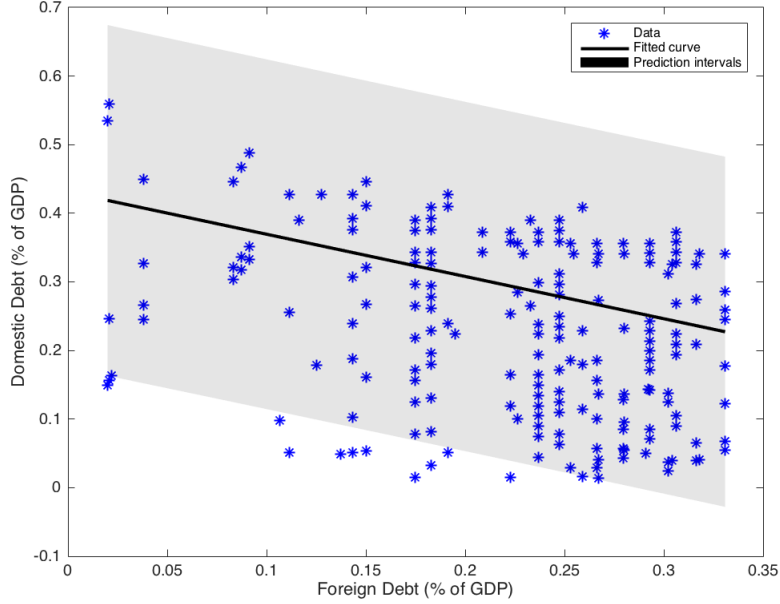
Finally, the model predicts that the correlation between the two debts is -0.27 and statistically not significantly different from zero. Figure 7 plots values of foreign debt-to-GDP ratios on the horizontal axis versus domestic debt-to-GDP ratios obtained from the simulated model on the vertical axis . The simulated data exclude default episodes and zero debt issuance episodes.¹⁴

5.2 Cyclical Properties

In the second step we test our model on the four stylized facts of the emerging economies by Neumeyer and Perri (2005). These are the standard data tests for small open economy models. Table 3 compares the moments from the data with those obtained from our model

¹⁴After including zero debt issuance episodes the correlation is -0.38 and statistically not significantly different from zero. The theoretical predictions are based on the model calibrated to Argentinean data.

Figure 7: Domestic vs foreign debt in the model



Correlation=-0.27

and obtained in the same model without domestic debt.

i) Countercyclical spread on foreign debt. As the default area decreases in output the government's commitment problem is less severe and the interest rate goes down. The correlation (1) is of the same sign and magnitude as in the data (-0.71 vs -0.65).

ii) Excess consumption volatility is the well known feature of emerging economies (2). In the model consumption is not smoothed relative to output, because sovereign risk makes prices of debts volatile. The fact that the two processes are uncorrelated is helpful in obtaining a number than is closer to the data than in the model without domestic debt.

iii) Countercyclical trade balance and *iv) procyclical fiscal policy.* A procyclical fiscal policy is defined as a positive response of primary balance-to-GDP to output gap (countercyclical primary balance). In the standard model, with only foreign debt, the primary balance and the trade balance are identical and equal to the net foreign debt position. In recessions the government faces higher spreads due to higher default risk and relies more on taxation to finance expenditures. In expansions the cost of credit is lower so there is an

Table 4: Novel Features: Model v Data

ID	Moment	Data	Model
(1)	Average Domestic Spread	10.61	7.18
(2)	Std(Domestic Spread)	10.09	25.62
(3)	Average Foreign Spread	6.83	3.51
(4)	Std(Foreign Spread)	4.59	3.69
(5)	Corr(Domestic Spread, Output)	-0.35	-0.77
(6)	Std(Net Exports-to-Output, Output)	0.34	0.41
(7)	Std(Primary Balance-to-Output, Output)	0.26	1.38

Notes: Data moments are based on quarterly series 1991Q1-2013Q4. Corresponding model moments are obtained from calibrated model at an annual frequency. Data sources are introduced and data transformations are explained in Online Appendix.

increase in foreign borrowing. Thus, the fiscal policy is procyclical and the trade/primary balance is countercyclical. In our model the trade balance (3) is negatively correlated with output, as in the data (-0.8 vs -0.45).

Our model allows to draw a clear distinction between the two. The change in the primary balance equals to the sum of changes in both foreign and domestic debts. The correlation between the primary balance and output (4) is closely matched (-0.26 vs -0.21). The primary balance is less correlated with output than trade balance, because domestic debt issuance b'_d is less sensitive to output than b'_f .¹⁵

5.3 Novel features

The novel features of our model, the tax wedge and domestic debt, allow us to look at the data in previously unexplored dimensions. Table 4 explores the features of the model related to domestic variables. We show the abilities and limitations of the model in matching those features with the data.

¹⁵Our model comes closer to accounting for the cyclical properties than the literature up to date. In [Arellano \(2008\)](#), the cyclicalities of the trade balance and spreads are about 33 percent of those in the data, in [Chatterjee and Eyigungor \(2012\)](#) they are 50 percent and 82 percent respectively and in our model they are 56 percent and 91 percent respectively. Our model captures 81 percent of the primary balance cyclicity. The comparison model used here is one without domestic debt and without tax wedge. A model without domestic debt but with the tax wedge is quantitatively very similar to the comparison model presented in this section. Detailed results are available upon request.

Domestic spread has not yet been looked at in either business cycle or sovereign default literature. In simulated series *i) average domestic spread* (1) is reasonably close to the data (10.61 in the data vs 7.18 in the model), but the *ii) standard deviation of domestic spread* (2) is twice higher than in the data. This is because the interest rate on domestic debt is high on average, due to the risk aversion of domestic households and the fact that we use an inverse of the discount factor as domestic risk-free rate¹⁶. On the other hand, the average foreign spread is twice lower (3) and less volatile (4) than in the data. These discrepancies follow from the assumptions: of a risk-neutrality of foreign and a risk-aversion of domestic investors. On the domestic market, introducing second, safe asset would bring down both the level and the volatility of domestic spread (albeit at an expense of lower domestic debt levels and lower default probabilities). On the foreign market, [Lizarazo \(2013\)](#) shows that introducing risk aversion on the side of foreign investors helps to bring foreign spreads to the empirically observable levels.

iii) Domestic spread is countercyclical (5) (similarly to foreign) and the model replicates this fact qualitatively (-0.35 vs -0.77). Domestic spread in the model is mostly driven by the changes in consumption, which are affected by output and foreign debt. As foreign debt is procyclical, both work in the same direction. This is why domestic spread is strongly countercyclical, much more than in the data.

The volatility of the trade balance relative to output (6) closely mimics the data (0.34 vs 0.41), however the model overshoots the *iv) relative volatility of the primary balance* (7) (0.26 vs 1.38). The reason for this is the high sensitivity of domestic debt to the tax wedge. As soon as the tax wedge hits the lower limit, the government optimally chooses to repay all domestic debt (see [Figure 6](#)).

5.4 Default Episodes

Using our model we can compare the behavior of aggregate variables around different default episodes. [Table 5](#) reports the average levels of debt, debt prices, output, tax wedge and consumption prior to and at the time of a default episode.

First, the output drop is on average more severe during foreign default. This is partly

¹⁶ $1/(1+r)$ is foreign discounting and β is domestic discounting. Hence, we calculate domestic and foreign default risk premium subtracting different rates. If we had subtracted the same rate, we would have taken the point of view of foreign investors, who evaluate possibility of entering into domestic market.

Table 5: Default Episodes

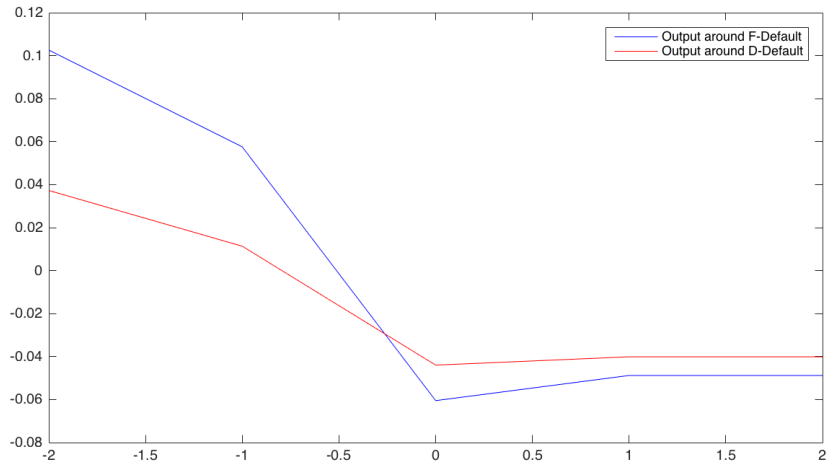
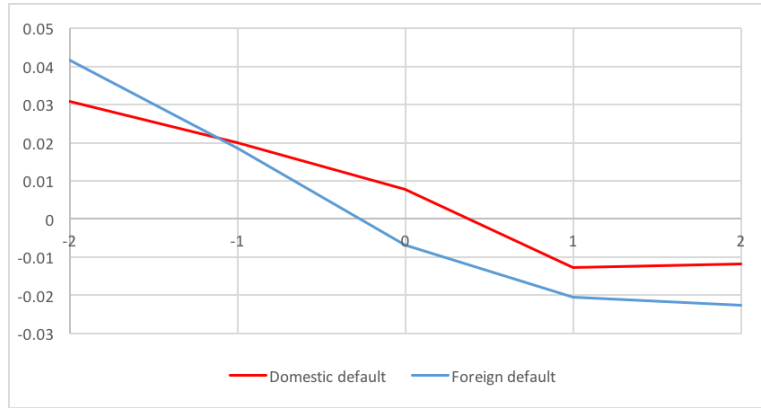
	Average	Prior to F-def.	At F-def.	Prior to D-def.	At D-def.
Output	1.01	1.05	0.93	0.99	0.95
Tax Wedge	0.0258	0.0255	0.0256	0.0258	0.0259
Consumption	0.96	0.90	0.76	0.83	0.85
Foreign Debt-to-GDP	16.70%	18.99%	0%	14.35%	19.85%
Domestic Debt-to-GDP	21.40%	26.99%	35.57%	30.25%	0%
Discount Price of F-Debt	0.95	0.91	0.98	0.87	0.27
Discount Price of D-Debt	0.78	0.71	0.08	0.55	0.72

a reflection of an assumption that output cost of foreign default is higher than that of domestic. However, an endogenous prediction of the model is that the output drop upon foreign default is more sudden. In the model, one year prior to default output is still above the trend. The paths of output around different default episodes are qualitatively similar to those in the data, as shown in Figure 8. The top panel plots empirical output deviations from the trend (in the HP-filtered data) around domestic and foreign default, while the bottom panel plots the respective lines obtained in the model. Both in the data and in the model output is higher prior to foreign default than prior to domestic default. At the time of foreign default the drop in output is larger and it is more pronounced and longer-lasting than during and after domestic default.

Second, quantitatively it turns out, that the tax wedge does not play a triggering role for neither of defaults, even though according to the static model it could qualitatively play such a role. Instead, it is instrumental in a build-up of domestic debt before domestic default. Third, output drop and consumption drop are larger during foreign than during domestic default.

Fourth, both debts are above their respective averages in the period prior to default. This is a reflection of a fact that riskiness of debt is an increasing function of its size as shown in Figure 5. Finally, even though the two markets are segmented and the stochastic processes are uncorrelated, the model delivers endogenous spillovers between the two debt markets. The discount price of foreign (domestic) debt goes down upon domestic (foreign) default. Foreign debt becomes more risky when domestic default happens for two reasons. First, as output drops foreign default becomes more likely. Second, government loses the ability of repaying foreign debt with domestic debt and must rely on costly taxation. Upon foreign

Figure 8: Average output path around foreign and domestic default in the data (top) and model (bottom)



default the price of domestic default drops sharply, due to risk-averse pricing of domestic bond. When output drops sharply, domestic citizens must be compensated heavily by the drop in the price in order to give up an extra unit of consumption and to save in domestic bond.

In this section we showed that the model with two debts and two shocks delivers a substantial improvement in replicating business cycle statistics of an emerging economy and, in spite of the fact that the markets are segmented on the demand side, model generates spillovers from one market to the other.

6 Conclusions

We develop a model of sovereign debt issuance on the international and domestic markets, and of selective defaults. Our model shows, that although foreign debt is more valuable and can in principle be used to smooth both output and taxation shocks, the government would still use domestic debt to smooth the domestic tax wedge shock. In a world with two shocks, foreign and domestic debts are issued, and selective defaults arise endogenously. The model is capable of matching debt-to-GDP ratios, default frequencies and low correlation between two debt instruments. It quantitatively accounts for the cyclical properties of the emerging economies and qualitatively replicates behavior of aggregate variables around different defaults.

The model is a first step to study debt issuance and default decisions of government with multiple investors and multiple debt instruments. There is an interesting dynamic to study, when a government can discriminate between investors when issuing and repaying debt, but has no control over what happens with bonds in the meantime. That is, when bonds can be re-traded on the secondary markets in the spirit of [Broner, Martin and Ventura \(2010\)](#), but with costly taxation in the spirit of our paper.

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7 Online Appendix for “Sovereign Debt Issuance and Selective Default”

7.1 Definitions and Default Episodes

We follow the definitions of [Reinhart and Rogoff \(2011b\)](#) for default episodes. Their dataset contains several types of crisis including three dummy indicators: for a country being in foreign default, for a country being in domestic default and for a country being in an instance of hyperinflation. Default is defined as failure to meet a principal or interest payment on the due date. Inflation above 20% per annum is classified as hyperinflation. We further define the beginning of foreign default as an instance when the dummy indicator switches from zero to one. We combine domestic default and hyperinflation to define the beginning of domestic default. As [Reinhart and Rogoff \(2011b\)](#) point out “Domestic public debt is issued under home legal jurisdiction. In most countries, over most of their history, it has been denominated in the local currency and held mainly by residents.” Consequently, the instance of inflation is essentially partial default on domestic debt. We define the beginning of domestic default as when either the dummy indicator for domestic default or the dummy indicator for hyperinflation switches from zero to one. The total default is defined as a situation when both domestic and foreign defaults happen simultaneously. All instances of foreign and domestic defaults according to our definition are listed in the table below.

Table 6: Default Episodes

	Foreign	Domestic	Domestic <i>de jure</i>
Albania	1991		
Algeria	1991	1991	
Angola	1985	1964, 1967, 1974, 1976, 1991, 1992	1976, 1992
Antigua Barbuda	1996	1998	1998
Argentina	1951, 1956, 1982, 2001, 2013	1957, 1962, 1971, 1982, 1989, 2001, 2002, 2008, 2010	1982, 1989, 2001
Australia		1951, 1956, 1975	
Bangladesh	1974		
Belize	2006, 2012		
Bolivia	1980, 1986	1950, 1973, 1979, 1982, 1991	1982
Bosnia Herzegovina	1992		
Brazil	1961, 1983	1954, 1959, 1961, 1974, 1986, 1990	1986, 1990
Bulgaria	1990		
Burkina Faso	1983		
Cameroon	1985	2004	2004
Cape Verde	1981		
Central African Rep	1981, 1983	1971, 1994	
Chile	1961, 1963, 1965, 1972, 1983	1951, 1953, 1958, 1962, 1967, 1983, 1985, 1990	
China		1994	
Colombia		1950, 1963, 1973, 1979, 1985, 187	
Congo (Brazzav- ille)	1983		

Congo (Kinshasa)	1976	1979	1979
Cook Islands	1995		
Costa Rica	1962, 1981	1974, 1981, 1988, 1991, 1995	
Cote D'Ivoire	1983, 2000	1958, 1977, 1979, 1994	
Croatia	1992	1993	1993
Cuba	1960, 1982		
Czechoslovakia	1959	1959	1959
Domenican Republic	1982, 2005	1980, 1981, 1984, 1988, 2003	1981
Dominica	2003	2003	2003
Ecuador	1982, 1999, 2008	1973, 1983, 1999, 2008	1999, 2008
Egypt	1984	1980, 1986, 1989, 1992	
Ethiopia	1991		
Gabon	1978, 1986, 1999	1999	1999
Gambia	1986		
Ghana	1966, 1987	1965, 1973, 1975, 1979, 1982, 1986, 1993, 2000, 2003	1979, 1982
Greece	2012	1957, 1980, 1983, 1985, 1990, 2012	2012
Grenada	2004, 2012	2005	2005
Guatemala	1986, 1989	1974, 1986, 1990	
Guinea	1986, 1991		
Guinea-Bissau	1983		
Guyana	1979, 1982		
Haiti	1952, 1965, 1982		
Honduras	1981	1990, 1994	
Hungary		1951, 1990, 1995	
India	1958, 1969	1973	
Indonesia	1966, 1999, 2002	1950, 1955, 1957, 1960, 1973, 1979, 1998	
Iran	1978		
Iraq	1987		
Italy		1974, 1980	
Jamaica	1970, 1978, 1981, 1987, 2010	2010	2010
Japan		1974	
Jordan	1989		
Kenya	1994, 2000	1984, 1992	
Korea		1952, 1961, 1964, 1974, 1980	
Kuwait		1990	1990
Liberia	1963, 1968, 1981	1989	1989
Macedonia	1992		
Madagascar	1981	2002	2002
Malawi	1982, 1988		
Malaysia		1950	
Mali	1967, 1988, 1992, 2012	2012	2012
Mauritania	1992		
Mauritius		1974, 1980	
Mexico	1982	1973, 1976, 1979, 1982, 1990, 1995	1982
Moldova	1998, 2002		

Mongolia		1997	1997
Morocco	1983, 1986	1951	
Mozambique	1980, 1983	1980	1980
Myanmar	1997	1966, 1973, 1984, 1987, 1988, 1997, 2001, 2006	1984, 1987
Nauru	2002		
Nicaragua	1979	1973, 1979, 2003, 2008	2003, 2008
Niger	1982		
Nigeria	1987, 2004	1975, 1977, 1981, 1983, 1988, 1992	
North Korea	1974		
Pakistan	1972, 1981, 1998		
Panama	1983	1988	1988
Paraguay	1968, 1986, 2003	1974, 1979, 1984, 1994	
Peru	1969, 1976, 1978, 1980, 1983	1975, 1985	1985
Philippines	1983	1971, 1974, 1984	
Poland	1981	1953, 1981, 1987	
Portugal		1974, 1982	
Romania	1981, 1986		
Russia	1991	1993, 1998	1998
Rwanda		1995	1995
Sao Tome Principe	1987	1981, 1985, 1990, 1993	1981
Senegal	1981, 1990, 1992		
Serbia and Montenegro	2003		
Seychelles	2000, 2008		
Sierra Leone	1983, 1986	1995	1995
Singapore		1950, 1973	
Slovenia	1992		
Solomon Islands	1998	1995	1995
South Africa	1985, 1989, 1993		
Spain		1977	
Sri Lanka	1979, 1981	1996	1996
St Kitts Nevis	2011		
Sudan	1979	1991	1991
Suriname		2001	2001
Tanzania	1984		
Thailand		1974	
Togo	1979, 1982, 1988, 1991		
Trinidad Tobago	1988	1974	
Tunisia	1956, 1958, 1963, 1979		
Turkey	1956, 1958, 1963, 1978, 1982	1958, 1977	
Uganda	1980		
Ukraine	1998	1998	1998
United Kingdom		1975	
Uruguay	1965, 1983, 1987, 1990, 2003	1959, 1963, 1971, 1983	
Venezuela	1983, 1990, 1995, 2004	1980, 1987, 1995, 2002, 2008	1995

Vietnam	1985	1975	1975
Yemen	1985		
Yugoslavia	1965, 1983		
Zambia	1983	1985	
Zimbabwe	1965, 2000	1983, 1991, 1994, 1997, 2006	2006

7.2 Solution Algorithm

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1. Discretize two exogenous AR(1) processes for y and τ (Tauchen with 3 std. devs.).
2. Create a grid for b_d and b_f ¹⁷.
3. Guess price schedules q_f^0 and q_d^0 .
 - (a) Calculate consumption in autarky c^{aut} and the value of permanent autarky V^{aut} . Guess four value functions $V^{0,0}$, $V^{0,fd}$, $V^{0,dd}$ and $V^{0,td}$ using V^{aut} .
 - (b) Calculate b'_d and b'_f in repayment given prices and $V^{0,0}$ as continuation value.
 - (c) Calculate value of repayment V^r given optimal policies and continuation value.
 - (d) Repeat (b) and (c) for foreign and domestic default to obtain $V^{1,fd}$ and $V^{1,dd}$.
 - (e) Calculate the value of total default $V^{1,td}$ given $V^{1,fd}$ and $V^{1,dd}$ and $V^{0,0}$.
 - (f) Derive default policies comparing four value functions at each grid point.
 - (g) Derive the new value function $V^{1,0}$ as the maximum of the four value functions.
 - (h) Substitute $V^{0,0} := V^{1,0}$.
 - (i) Repeat (b)-(h) until convergence in value function.
4. Given optimal default policies calculate q_f^1 and q_d^1 using pricing rules (9) and (10).
5. Update prices $q_f^0 = \alpha^f q_f^1 + (1 - \alpha^f) q_f^0$ and $q_d^0 = \alpha^d q_d^1 + (1 - \alpha^d) q_d^1$.
6. Repeat until convergence in prices.

After prices converge, we use the policy functions to simulate the model economy 10000 times over 200 periods. We burn the first 10 observations to avoid any dependence on the starting conditions. We also discard the first 10 periods after reentry following each default. We do this because the model economy reenters capital markets without any debt, while countries always emerge from each default/restructuring episode with a positive level of debt. By ignoring the first 10 years following reentry, we ignore years with counter-factually low debt levels. To calculate all moments (other than default frequencies) we use only data for repayment periods.

¹⁷Given that domestic discount price is a 6-dimensional object, we quickly run into the curse of dimensionality. We spread the maximum size of array over the six dimensions so that results are not affected by the grid size in any dimension. More than 10 points for the exogenous states is enough. We use 17 points for y , 13 points for τ , 29 for b_d and 31 for b_f .

7.3 Data for Simulations

Data sources. To calculate the cyclical component of output y , we take linearly detrended seasonally adjusted quarterly real GDP of Argentina. We calculate the spread on foreign debt as the difference between the JPMorgan Emerging Market Bond Index (EMBI) for Argentina and 30-days T-bills. As we do not directly observe the spread of domestic debt, we proxy it by the money market rate in the local currency from the IFS. The correlation between foreign spread and money market rate in \$ is 0.71, while the correlation between foreign spread and money market rate in pesos is only 0.52, therefore we see it as a reasonable approximation. We use seasonally adjusted by x13 procedure quarterly series of GDP, import and export and government expenditure to calculate trade balance as the difference between export and import ($NX/y = (X - M)/y$). The primary balance is calculated as the difference between tax revenue and government expenditure $PB/y = (T - G)/y$. However, tax revenues are not available at quarterly frequency, so we interpolate them using annual data. Also, the quarterly government expenditure is available only from 1993Q1. Therefore, our primary balance series are shorter. Government expenditure, imports and exports, as well as consumption data come from [CEPAL \(2016\)](#).

In the model, we follow the literature and calculate net export as the change in net foreign asset (NFA) position: $NX = (b'_f q_f - b_f)$. The government budget constraint defines primary balance as: $PB = T - G = -(b'_f q_f - b_f) - (b'_d q_d - b_d)$.

Transformations. Flow variables, which are expressed in relative terms are independent of frequency. The ratio of stocks over flows, such as foreign and domestic debt-to-GDP, when transformed from quarterly to yearly need to be divided by four. The similar procedure applies to the exclusions. We need to take a square root to the power of four for the remaining variables, such as discount factor $\beta_a = \beta_q^4$ and annual spreads $\frac{1}{q_a} - R_f = \left(\frac{1}{q_q}\right)^{1/4} - R_f$. To calculate spreads we invert the discount prices and subtract the risk free rate. We use R_f to calculate spreads on foreign debt and we use inverse of the discount factor $1/\beta - 1$ for the domestic debt.

7.4 Default Sets - Additional Graphs

Figure 9: Default sets for foreign debt in output

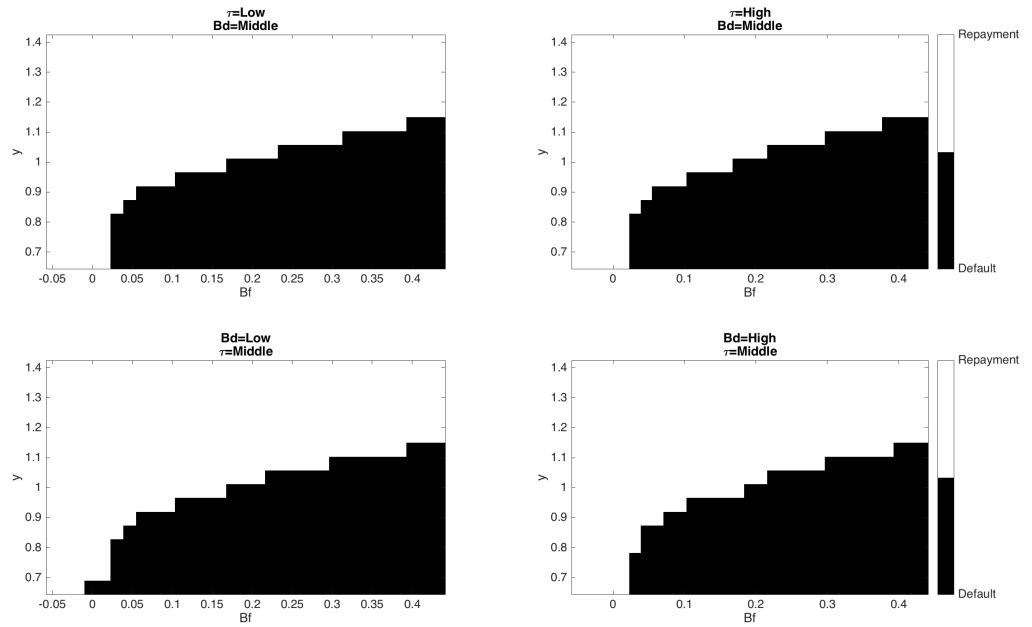
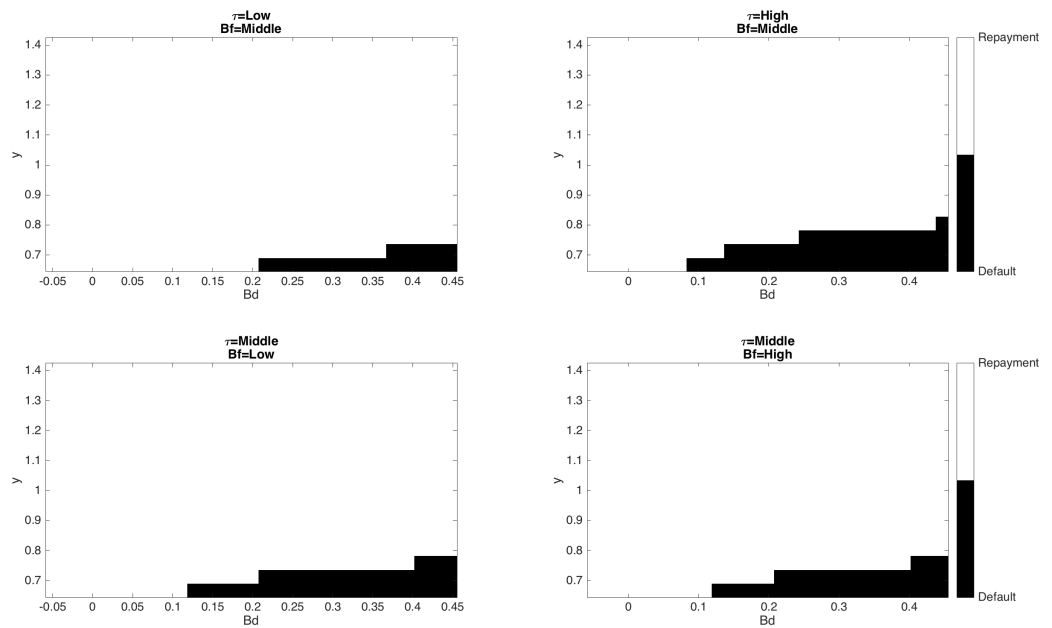


Figure 10: Default sets for domestic debt in output



7.5 Debt Policies - Additional Graphs

Figure 11: Foreign debt policies after repayment

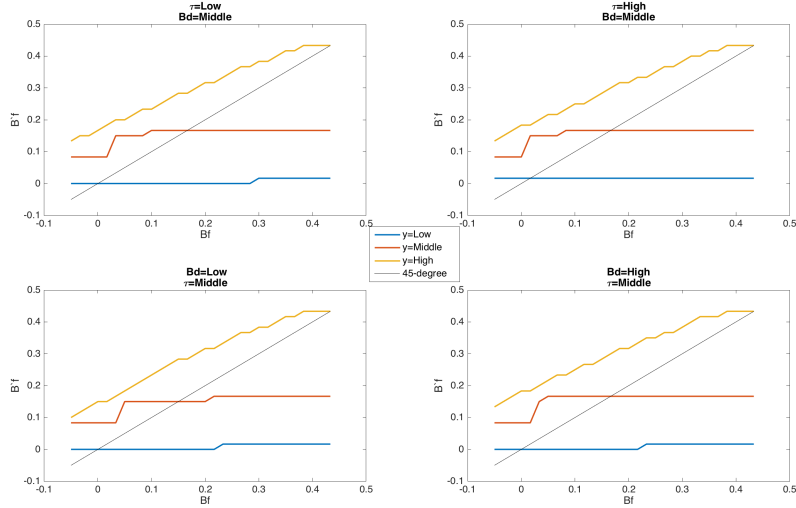
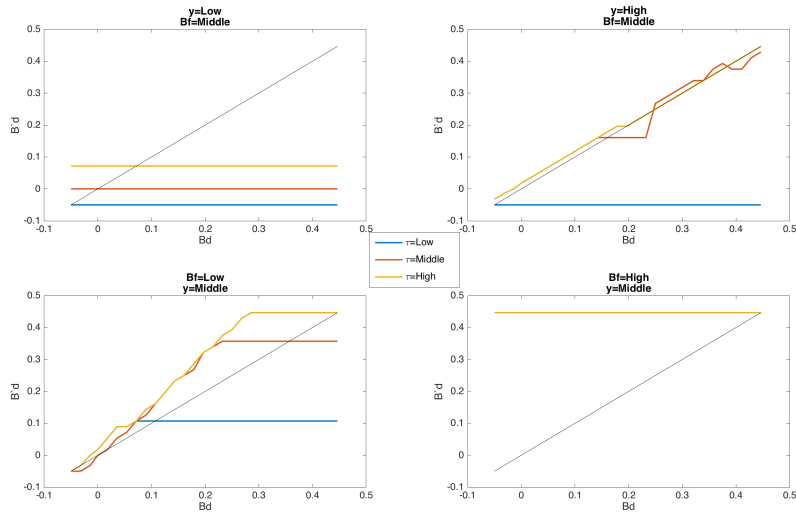


Figure 12: Domestic debt policies after repayment



7.6 Equilibrium Discount Prices

Figure 13: Equilibrium discount prices for foreign debt

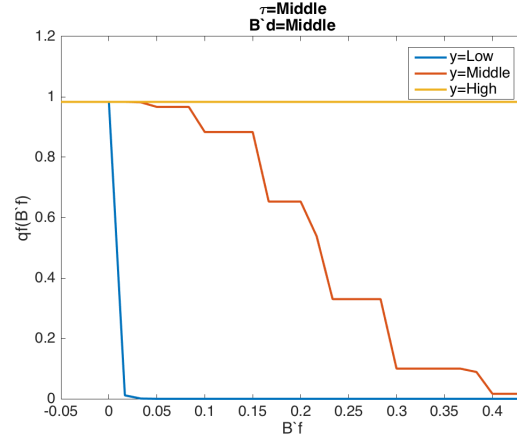


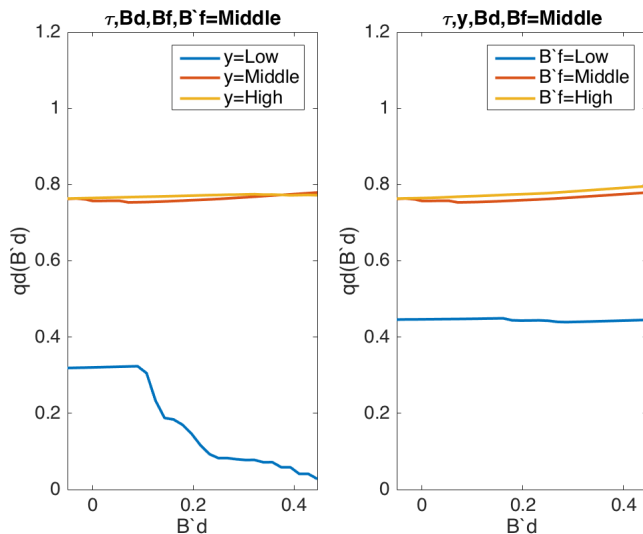
Figure 13 plots the equilibrium discount prices for the newly issued foreign debt. As foreign investors are risk-neutral the price only reflects the default probability. For high level of output any newly issued debt is safe, therefore the price is constant and equal to $\frac{1}{1+r}$. For the middle and low output the discount price goes down in the level of the newly issued debt, as the probability of default goes up. In particular, for the lowest level of output almost any debt will surely be defaulted, therefore the price quickly hits zero. For the middle values of output the risky borrowing area is the biggest. This is the standard result in the sovereign default quantitative studies.

However, domestic investors are risk-averse. Therefore the price of domestic debt reflects not only the default probability but also the current and expected consumptions (recall q_d is a six-dimensional object). In Figure 6 we have found that the tax wedge has a small effect on the domestic default probability, compared to the effect of output on the foreign default probability. Tax wedge has also only a second order effect on consumption - see budget constraints (6a)-(6d). The price of domestic debt is therefore mostly influenced by the two key determinants of consumption: output and foreign debt issuance.

In the left panel of Figure 14 we plot how the price q_d changes in debt b_d controlling for output. With high (yellow line) and middle output (orange line) the probability of default next period is zero. Given that output process is persistent, the expected future consumption is close to today's consumption. Hence, the price of debt is almost equal to the discount factor¹⁸. Foreign default

¹⁸Expected consumption tomorrow is slightly higher than today, due to governments borrowing policies that we will discuss next. This makes the safe price of domestic debt slightly below the discount factor.

Figure 14: Equilibrium discount prices for domestic debt



would result in output penalties, that would decrease consumption tomorrow. With low output however, the probability of domestic default increases sharply with the amount of domestic debt outstanding b_d . This drives the price of domestic debt q_d down to zero. With middle foreign debt, high domestic debt and low output, no new domestic debt can be issued.

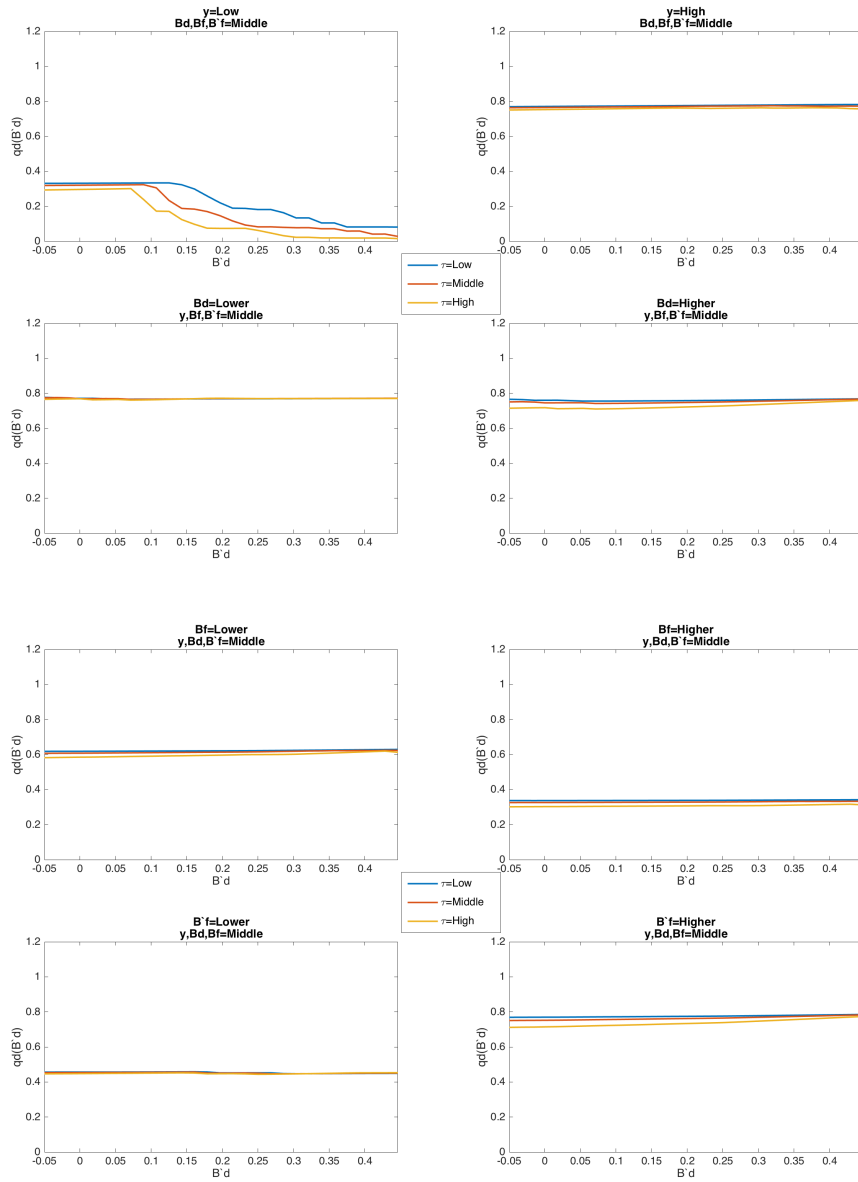
This discussion already previews a single numerical difficulty our model will face in the simulations. The calibrated discount factor of 0.825 implies a domestic risk-free rate of 21%. This will give rise of a counterfactually high average interest rate on domestic debt. In our calibration we decided to employ the standard value used in the literature to facilitate comparisons. This value is calibrated in such a way, that it gives rise to realistic levels of foreign debt, because households are less patient than international markets (foreign risk-free rate is 1.7%). As a result, our novel framework implies a sharp trade-off between the average foreign debt level and the average price of domestic debt. This link arises endogenously, as by assumption markets are separated and stochastic processes uncorrelated.

In the right panel of Figure 14 we plot how the price q_d changes in debt b_d controlling for foreign debt. The level of foreign debt outstanding is middle. With middle output the risk of domestic default is nil, therefore the lines are virtually flat in b_d . With both high and middle issuance of foreign debt (orange and yellow lines) foreign debt is rolled-over and the risk of foreign default is almost constant. Therefore the price of domestic debt is almost equal to the discount factor.

Low issuance of foreign debt however (blue line), means that foreign debt is being paid off. This decreases consumption today relative to tomorrow and drives the price of domestic debt down.

Figure 15 plots equilibrium discount prices for domestic debt controlling for foreign debt outstanding and the tax wedge. We conclude that foreign debt outstanding has exactly the opposite effect to the new foreign debt issuance and that the tax wedge τ has a small effect on the price of domestic debt.

Figure 15: Equilibrium discount prices for domestic debt



8 A note on the equivalence between endogenous and exogenous tax distortions

In this note we establish that a static model with an exogenous tax wedge can deliver the same allocations in terms of consumption and utility as different classes of models with endogenous distortions. Since we are interested in establishing general allocation equivalence, we abstract from debt considerations in this section (we later introduce debts into a two-period setup)

I) Exogenous tax wedge model

The economy is subject to two shocks, y and τ :

$$\begin{aligned} \max_c \quad & \sum_t \beta^t (u(c_t) - \zeta_t) \\ \text{s.t.} \quad & c_t = y_t - T_t(1 + \tau_t) \\ & g_t = T_t, \end{aligned} \tag{20}$$

where ζ_t is a utility shifter that does not affect real allocations, but is useful to show equivalence in utility levels. Without a loss of generality we abstract from the debt considerations. Consumption is determined by the resource constraint and, hence, the combination of two shocks:

$$c_t = y_t - g_t(1 + \tau_t) \tag{21}$$

II) Labor income tax model

Consider a production economy with separable utility from consumption and leisure. Government collects labor income tax with a marginal rate of t^l . Households maximize utility subject to households' and government budget constraints:

$$\begin{aligned} \max_{c,n} \quad & \sum_t \beta^t (u(c_t) - v(n_t)) \\ \text{s.t.} \quad & c_t = w_t n_t (1 - t_t^l) \\ & g_t = w_t n_t t_t^l. \end{aligned} \tag{22}$$

As decisions are static, time subscripts can be dropped. The firms' sector is standard: firms produce with constant returns to scale using only labor $y = Af(n)$. The equilibrium is given by c^l and n^l that satisfy Euler equation and the resource constraint:

$$\begin{aligned} u'(c^l) &= \frac{v'(n^l)}{Af'(n^l)(1 - t^l)} \\ c^l &= Af'(n^l)n^l - g. \end{aligned} \tag{23}$$

The first order Taylor expansion around an undistorted solution ($t^l = 0$) yields the following solution for labor:

$$n^l = n^u + \frac{dn}{dt^l} t^l \tag{24}$$

where n^u is labor in undistorted allocation and

$$\begin{aligned}\frac{dn}{dt^l}\Big|_{t^l=0} &= \left((1-t^l)\frac{u''}{u'}A(f'+nf'') + (1-t^l)\frac{f''}{f'} - \frac{v''}{u' Af'} \right)^{-1}\Big|_{t^l=0} \\ &= \left(\frac{u''(c^u)}{u'(c^u)}A(f'(n^u) + n^u f''(n^u)) + \frac{f''(n^u)}{f'(n^u)} - \frac{v''(n^u)}{v'(n^u)} \right)^{-1},\end{aligned}\quad (25)$$

where c^u is consumption in undistorted allocation. The resource constraint takes the following form:

$$c^l = Af'(n^l)n^l - g = Af'(n^l)n^l = c^u + (f'(n^u) + f''(n^u)n^u)A\frac{dn}{dt^l}t^l. \quad (26)$$

III) Consumption tax model

Consider the previous model with consumption tax instead of labor income tax:

$$\begin{aligned}\max_{c,n} & u(c) - v(n) \\ \text{s.t.} & c(1+t^c) = wn \\ & g = ct^c.\end{aligned}\quad (27)$$

The firms' sector is the same as before. The equilibrium is given by c^c and n^c that satisfy Euler equation and the resource constraint:

$$\begin{aligned}u'(c^c) &= \frac{v'(n^c)(1+t^c)}{Af'(n^c)} \\ c^c &= Af'(n^c)n^c - g,\end{aligned}\quad (28)$$

We again use the first order Taylor expansion with respect to t^n around the undistorted solution:

$$n^c = n^u + \frac{dn}{dt^c}t^c \quad (29)$$

where

$$\begin{aligned}\frac{dn}{dt^c}\Big|_{t^c=0} &= \left(\frac{u''}{u'}A(f'+nf'') + \frac{f''}{f'} - (1+t^c)\frac{v''}{u' Af'} \right)^{-1}\Big|_{t^c=0} \\ &= \left(\frac{u''(c^u)}{u'(c^u)}A(f'(n^u) + n^u f''(n^u)) + \frac{f''(n^u)}{f'(n^u)} - \frac{v''(n^u)}{v'(n^u)} \right)^{-1}.\end{aligned}\quad (30)$$

Comparing the expressions (30) and (25), we observe that they are the same. Hence, if $t^c = t^n$ the solutions of two problems are identical up to the first order approximation. Resource constraint is therefore equivalent to (26).

IV) Capital income tax model

The model with capital income tax is slightly different form the models above. Consider a production economy with capital accumulation and no labor. Government collects capital income tax

t^k :

$$\begin{aligned} \max_{c_t, k_{t+1}} \quad & \sum_t \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + k_{t+1} = k_t(1 + (1 - t_t^k)r_t) \\ & g_t = r_t k_t t_t^k, \end{aligned} \tag{31}$$

The firms' sector is standard, only now firms produce with capital only: $y_t = A_t f(k_t)$. The equilibrium is given by c_t^k and k_{t+1} satisfying:

$$\begin{aligned} \frac{u'(c_t^k)}{u'(c_{t+1}^k)} &= \beta(1 + (1 - t_t^k)r_t) \\ c_t^k + k_{t+1} &= k_t(1 + (1 - t_t^k)r_t) \\ r_t &= A_t f'(k_t) \\ g_t &= r_t k_t t_t^k. \end{aligned} \tag{32}$$

The steady state becomes:

$$\begin{aligned} 1 &= \beta(1 + (1 - \tau^k)A f'(k)) \\ c^k &= k A f'(k) - g \\ g &= A f'(k) k t^k, \end{aligned} \tag{33}$$

We further use the first order Taylor expansion with respect to t^k around the undistorted solution:

$$k^d = k^u + \left. \frac{dk}{dt^k} \right|_{t^k=0} t^k, \tag{34}$$

where:

$$\left. \frac{dk}{dt^k} \right|_{t^k=0} = \left. \frac{f'(k^u)}{f''(k^u)(1 - t^k)} \right|_{t^k=0} = \frac{f'(k^u)}{f''(k^u)}, \tag{35}$$

$$c^k = y^u - g - g \left(\left| \frac{f'(k^u)}{k^u f''(k^u)} \right| - 1 \right). \tag{36}$$

V) Equivalence

By selecting proper y and τ in (21), the prototype model generates the same allocations as any of the three subsequent models with an endogenous deadweight loss from taxation:

$$\begin{aligned} c = c^l = c^c \Rightarrow \quad & y = A n^u f'(n^u) \quad \& \quad \tau = \left(\left| \frac{f''(n^u)}{f'(n^u)} \right| - \frac{1}{n^u} \right) \left. \frac{dn}{dt} \right|_{t=0} \\ c = c^k \Rightarrow \quad & y = A k^u f'(k^u) \quad \& \quad \tau = \left(\left| \frac{f'(k^u)}{k^u f''(k^u)} \right| - 1 \right) \end{aligned}$$

With a proper choice of ζ , the prototype model can also deliver the same utility allocations:

$$u(c) - \zeta = u(c^l) - v(n^l) = u(c^c) - v(n^c) \Rightarrow \quad \zeta = v(n^l) = v(n^c)$$

VI) Labor income tax model with domestic and foreign debt

The introduction of an exogenous foreign debt into a two-period model is straightforward, one

needs to replace g_t by $g_t + b_t^f$. The foreign debt threshold remains unchanged. The same is true for domestic debt. For example, in the case of labor tax:

$$c^l = y^u - g - t^l A (f'(n^u) + n^u f''(n^u)) \left| \frac{dn}{dt} \right|_{t^l=0} + b^d \quad (37)$$

$$g + b^f + b^d = t^l A f'(n^u) n^u \quad (38)$$

Combining the two, we obtain

$$c^l = y^u - b^f \left(1 + \left(\left| \frac{f''(n^u)}{f'(n^u)} \right| - \frac{1}{n^u} \right) \right) - b^d \left(\left| \frac{f''(n^u)}{f'(n^u)} \right| - \frac{1}{n^u} \right) \quad (39)$$

which is equivalent to the expression for consumption in the case of repayment in our baseline model.

An equivalence in the class of infinite-horizon dynamic general equilibrium models can only be approximated using simulations, this task we leave for the future research.