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## Domestic and Foreign Sovereign Debt Stability

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## Domestic and Foreign Sovereign Debt Stability

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#### Abstract

We present a theory of determinants of sovereign debt stability on foreign and domestic markets. Besides the two traditional factors - debt size and output contractions, we highlight the role of the third factor: distortionary tax, which hinders the government's ability to freely raise revenues. We emphasise the impact of tax distortions and output fluctuations on the trade-off between domestic and foreign debt stability. The paper explains why outright defaults in domestic debt are rare, despite its significant share in public debt, and provides insights into optimal debt issuance and taxation strategies.

**Keywords**: sovereign debt, debt stability, selective default, debt composition, distortionary tax

JEL Classification: F34, G15, H63

### 1 Introduction

There is no one-size-fits-all debt-to-GDP threshold that triggers sovereign defaults. Instead, defaults happens under wide spectrum of conditions, from as little as 20% to more than a 100% debt-to-GDP, while other countries surpass these thresholds without facing default.<sup>1</sup> To address this puzzle, we extend a standard theory of default on foreign debt, which derives the default threshold only as a function on foreign debt and an aggregate GDP contraction, by introducing tax distortions and domestic debt. These distortions represent a cost that hinders government's ability to promptly raise tax in order to repay its debt obligation. Domestic debt plays an important role of distortion absorber.

Debt stability studies typically focus on the stability and incetives to default on foreign debt. Domestic debt, however constitutes a substantial portion of overall public debt in both emerging and advanced economies. Despite the growing prevalence of domestic debt, instances of outright default are noticeably rare. This study asks two research questions. First, what factors drive debt stability on domestic and foreign markets. Second, what are the differences between domestic and foreign debt; in their economic role and in their stability.

To answer these questions we construct two distinct models. The first is a simple and a general model of an endowment economy. The economy is subject to output shocks and the government has to cover an exogenous stream of expenditures. To cover expenditures government can either tax or borrow on domestic and foreign markets. Taxation is costly and these costs can vary over time. Taxation costs come in a simple form of an exogenous and time-varying tax distortion. The model admits an analytical solution and we derive closed-form formulas for domestic and foreign debt stability. The results highlight that debt stability on each market depends on the size of respective debt, and the level of output, and the level of tax distortion. To explicitly account for the role of tax distortions for debt stability is a novelty of this paper. The model highlights an important trade-off. Foreign debt can be used to smooth consumption when output is volatile, but is inherently less stable. Domestic debt cannot smooth consumption, as it is only a redistribution within an economy, but its stability threshold is an order of magnitude higher. Domestic debt can play a useful role, as it can smooth volatile tax distortions. To highlight this trade-off, we construct an equilibrium with perfect separation: output shocks drive foreign debt issuance

<sup>&</sup>lt;sup>1</sup>In the years 1970-2000, according to Reinhart, Rogoff and Savastano (2003), tables 3 and 6.

and default, while tax distortion shocks drive domestic debt issuance and default. We show that the equilibrium holds for a wide range of parameters.

The second model is a more specific model of a production economy, with volatile government expenditures. Tax distortions arise endogenously. The model does not admit closed-form solution and we solve it numerically for a plausible set of calibrated parameter values. The model confirm the trade-off between domestic and foreign debt and that domestic debt is inherently more stable. It also shows that the perfect separation is never possible: while productivity shocks mostly drive foreign debt issuance and government expenditures mostly drive domestic debt issuance, the stability of both debts depends on both shocks, and is history dependent. In particular, when domestic debt is high, domestic default is strictly preferred to foreign default, and when the debts are of similar sizes, or foreign debt is higher, foreign default is strictly preferred to domestic default. The analysis further reveals that foreign debt stability remains almost insensitive to the level of domestic debt, and vice versa. The model derives additional important results about optimal debts issuance and optimal taxation in the face of endogenous domestic and foreign debt fragility. Optimal foreign debt issuance is countercyclical, while optimal domestic debt issuance is procyclical. Labour tax is optimally acyclical when debts are stable, but becomes countercyclical when debts are fragile.

Literature Review. Our contribution is in the tradition of the strategic sovereign default framework of Eaton and Gersovitz (1981). In this tradition, which in the recent years has evolved into empirically plausible, quantitative, calibrated general equilibrium models (Aguiar and Gopinath, 2006, Arellano, 2008, Chatterjee and Eyigungor, 2012), foreign debt is used to smooth consumption and its stability depends on output fluctuations and the size of debt, which is driven by past output fluctuations. Reinhart and Rogoff (2011) first noted, that the literature has paid little attention to the stability of domestic debt, although it constitutes a large and growing proportion of total public debt (see also Borri and Shakhnov (2017)), and that outright defaults on domestic debt have also happened. We contribute to the recent studies that add defaultable domestic debt to the sovereign debt stability framework. In our paper, similarly to D'Erasmo and Mendoza (2016, 2021), D'Erasmo, Mendoza and Zhang (2016) the stability of domestic debt takes centre stage. In contrast, in our framework the stability of domestic debt is not driven by distributional incentives, but by tax distortions.

Broner, Martin and Ventura (2010) show that, even in the absence of default penalties, foreign debt can be stable, if foreign creditors can resell their assets to domestic investors on secondary markets, because when tax collection is costless, domestic debt is always safe. We show that costly tax collection critically limits domestic debt stability. As such, our paper is also closely related to the literature on distortionary taxation with default (Karantounias, 2017, Pouzo and Presno, 2022), where a government defaults to mitigate endogenous tax distortions, which drives domestic debt stability, with the mechanism similar to ours, but without including foreign debt.

Two recent studies (Casalin, Dia and Hallett, 2020, Cerniglia, Dia and Hallett, 2021) also investigate the stability of domestic public debt under a similar assumption, that governments face challenges when attempting to rapidly change tax policies. They demonstrate, albeit using a different framework, that ineffective tax collection can lead to the instability of public debt. Eijffinger, Kobielarz and Uras (2015) study debt fragility in monetary unions, Camous and Cooper (2019) study the role of monetary policy on debt fragility, and Camous and Gimber (2018) study the role of fiscal policy on debt fragility. In our model, similarly to the latter, optimal tax becomes procyclical when debt becomes risky.

In our framework, domestic debt is strategically employed to smooth distortions from taxation, in the tradition of seminal works. The theory of tax smoothing is due to Barro (1979) in a partial equilibrium model where debt is not state-contingent and risk-free, spending needs are exogenous, and taxes have convex costs. Followingthis seminal paper, tax smoothing has become an important benchmark in the literature on optimal taxation. In the general equilibrium approach of Aiyagari, Marcet, Sargent and Seppälä (2002), the Ramsey solution for taxes and government debt resemble the results stated in Barro (1979).

Several recent contributions study how debt composition affects default incentives under the assumption that a government cannot discriminate between domestic and foreign investors (Brutti, 2011, Coimbra, 2020, Engler and Große Steffen, 2016, Gennaioli, Martin and Rossi, 2014, Guembel and Sussman, 2009, Sosa-Padilla, 2018), while Borri and Shakhnov (2017)cover the case of partially segmented markets. We show that when domestic and foreign markets are segmented, domestic debt has almost no impact on foreign debt stability, and vice versa.

### 2 Exogenous tax distortions

In this section, we propose a general setup to study foreign and domestic debt sustainability. The economy is an endowment economy with three actors: domestic households, foreign investors and a benevolent government. Households derive utility from consumption. The economy is subject to two shocks: an output shock and an exogenous tax distortion shock. Every period the government must cover an exogenous stream of expenditures  $g_t$ . The government acts on behalf of households; it collects lump-sum taxes and borrows on international and domestic markets to maximize households' lifetime utility. Taxes are lump-sum, but collecting taxes comes at a cost to the economy. Raising an amount T of taxes by the government induces a loss of  $T(1+\tau)$  resources to agents: an exogenous tax distortion that can vary over time. This is a key element that breaks Ricardian equivalence in the endowment economy and creates a trade-off between taxes and domestic debt. Both domestic and foreign debt contracts are non-contingent and the government can discretely default on either. We study this general setup, in which taxes are lump-sum but exert an exogenous, time-varying distortion on the economy, as it admits an analytical solution and allows us to derive closed-form debt stability thresholds. In the next section, we present a specific setup, in which tax distortions arise endogenously.

The government decides on domestic and foreign bond holdings  $(b_t^f, b_t^d)$ , and domestic and foreign default  $(\delta_t^f, \delta_t^d)$  to maximize the lifetime utility from consumption  $(c_t)$  subject to households' budget constraint (2), the government budget constraint (3), foreign investors' pricing schedule of the foreign bond (4), and domestic investors' pricing schedule of the domestic bond (5):

$$\max_{c_t, b_t^f, b_t^d, \delta_t^f, \delta_t^d} \sum_{t=1}^N \beta^{t-1} E_1[u(c_t)]$$
(1)

s.t. 
$$y_t = c_t + T_t(1+\tau_t) + q_t^d b_t^d - (1-\delta_t^d) b_{t-1}^d,$$
 (2)

$$T_t + q_t^f b_t^f = g_t + (1 - \delta_t^f) b_{t-1}^f,$$
(3)

$$q_t^f = \frac{E_t \left( 1 - \delta_{t+1}^f \right)}{1 + r},\tag{4}$$

$$q_t^d = \frac{\beta E_t \left[ \left( 1 - \delta_{t+1}^d \right) u_c(c_{t+1}) \right]}{u_c(c_t)},\tag{5}$$

$$b_0^f = b_0^d = 0, \ b_N^f = b_N^d = 0, \tag{6}$$

where  $y_t$  is an exogenous endowment,  $T_t$  is a lump-sum tax,  $\tau_t$  is an exogenous tax distortion,

 $b_t^f$  denotes foreign bond holdings and  $q_t^f$  is the foreign bond discount price,  $q_t^d$  represents domestic bond holdings and  $q_t^d$  is the domestic bond discount price, and  $\delta_t^f$  and  $\delta_t^d$  designate government's decisions to repay ( $\delta = 0$ ) or default ( $\delta = 1$ ) on foreign and domestic debt, respectively.

The  $\tau$  variable captures the fact that tax collection is costly. This cost is expressed in terms of how far the distorted allocation is compared to the case with costless, lumpsum taxes. Tax distortions depend on several factors: the levels of the marginal tax rates, the progressivity of the tax code, targeted and non-targeted tax reliefs, tax concessions, deductions and credits, administrative costs and the size of the grey economy, to name just a few. Although big reforms to a tax code or the level of marginal tax rates happen infrequently, in a dynamic economic system, most of the other factors change every period. All those changes can be summarized by the changes in the  $\tau$  variable. Foreign investors are risk-neutral and have deep pockets. They borrow on international markets at a risk-free rate r and lend funds to the government at a discount price  $q_f$  to break even in expectation (4). Domestic debt is the only saving device for domestic households: the discount price of domestic debt is derived from their Euler equation in the case of domestic debt repayment (5).

If the government decides to default, the economy suffers a proportional output penalty. Specifically, output is reduced to the  $\gamma^i < 1$  fraction of what it would have been in the case of repayment:

$$y_{t} = \begin{cases} y_{t}^{r}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (0, 0) \\ \gamma^{f} y_{t}^{r}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (1, 0) \\ \gamma^{d} y_{t}^{r}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (0, 1) \\ \gamma^{f} \gamma^{d} y_{t}^{r}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (1, 1) \end{cases}$$
(7)

where  $\gamma_t^f < 1$  and  $\gamma_t^d < 1$  are output penalties in cases of foreign and domestic default, respectively. The presence of output penalties is motivated by the empirical findings of Borensztein and Panizza (2008).

If the debts are repaid with taxes, the government imposes distortions on the economy. The main driving forces of the government's optimal policies are two trade-offs. The first is the trade-off between a transfer of resources away from the economy as foreign debt repayment versus a loss of resources due to a foreign default penalty. The second is between imposing distortions on the economy from tax collection versus imposing a loss of resources from a domestic default penalty. Unlike in models where Ricardian equivalence holds, the timing of taxes matters here.

The economy lasts for two periods (N = 2). In the second period, there is no debt issuance and the government only decides whether to repay or default on domestic and foreign markets. In the first period, there are no outstanding debt obligations from previous periods, and the government only decides on domestic and foreign debt issuance. This setup allows for a separation of default and issuance decisions. We solve the model by backward induction: first, we solve for default decisions in the second period taking debt issuance from the first period as given. This allows us to study debt stability in domestic and foreign markets. Second, we solve for the debt issuance policies in the first period taking debt stability in the second period as given. This allows us to study the full equilibrium of the model.

#### 2.1 Debt stability

Given debt issuance decisions  $b^d$  and  $b^f$  from the first period (time subscripts are suppressed for convenience), the government takes default decisions, in the second period, that maximize households' instant utility from consumption. The only source of income for the government is taxation. In the second period, four scenarios may arise: repayment, foreign default, domestic default and total default. Substituting the government's repayment decisions ( $\delta^d \in \{0,1\}, \delta^f \in \{0,1\}$ ) and default penalties (7) into households' second-period budget constraint (2) and the government's second-period budget constraint (3), household consumption levels in each of the four scenarios are given by the following equations:

$$c_2^r = y_2^r - b^f (1 + \tau_2) - b^d \tau_2, \tag{8}$$

$$c_2^{dd} = \gamma^d y_2^r - b^f (1 + \tau_2), \tag{9}$$

$$c_2^{fd} = \gamma^f y_2^r - b^d \tau_2, \tag{10}$$

$$c_2^{td} = \gamma^d \gamma^f y_2^r, \tag{11}$$

where consumption superscripts r, dd, fd and td stand for repayment, domestic default, foreign default and total default, respectively. **Foreign debt stability.** When deciding whether to default on foreign investors, the government compares household consumption under repayment and foreign default. Foreign debt is repaid whenever:

$$\frac{b^f}{y_2^r} \le \frac{1 - \gamma^f}{1 + \tau_2},\tag{12}$$

where the left-hand side is the foreign debt-to-GDP ratio and the right-hand side is a threshold defined by the parameters of the model. Importantly, when all parameters are fixed, foreign debt is always safe (or zero), and only when at least one parameter is stochastic, debt is fragile. This result is explained and proved in Proposition 1.

**Proposition 1.** If taxation is costly then the government's optimal policy on the international market is characterized by the foreign default threshold (12). Whenever foreign debt is below this threshold, it is riskless and is always repaid. Whenever it is above the threshold, it will always be defaulted on and therefore can never be issued. If either output or tax distortions are stochastic, the foreign default threshold is also stochastic, hence debt can be risky, and default can arise in equilibrium.

*Proof.* The first part follows directly from comparing (8) and (10). For the second part, suppose that the future output  $y_2$  and the tax wedge  $\tau_2$  are known in period one. Any debt  $b_f$  exceeding  $\frac{y_2(1-\gamma_f)}{1+\tau_2}$  will be defaulted on with certainty in period two, therefore its discount price in period one is zero. The government is only able to take out loans  $b_f \leq \frac{y_2(1-\gamma_f)}{1+\tau_2}$  which are repaid with certainty. Foreign default cannot arise in equilibrium. For  $b_f$  to be in the default area with a positive probability, at least one parameter has to be stochastic.  $\Box$ 

Condition (12) provides a closed-form formula for foreign debt stability threshold in an environment with exogenous tax distortion. Debt stability depends on four factors:

- The level of output  $y_2$ : the higher the output, the higher the foreign debt stability threshold. With high output, proportional output loss from default outweighs potential gains from keeping borrowed resources at home.
- The size of output penalty γ<sup>f</sup>: the lower the parameter, the higher the penalty (the more output is lost in default), and the higher the foreign debt stability threshold. With a high output penalty, losses from default outweigh potential gains. Output penalty works as a (limited) commitment substitute.

- The level of tax distortion τ: the higher are distortions from taxation, the lower the foreign debt stability threshold. With high tax distortions, raising tax revenue for debt repayment exerts high deadweight losses on the economy. This is an important and often overlooked channel in the literature on debt stability.
- The size of the debt itself  $b^f$ : the higher the debt, the less stable it becomes. This is a traditional debt stability channel. Importantly, the size of foreign debt is an endogenous object: if the debt issuance is low, the debt is always stable, if it is too high, then it is always defaulted on and will never be issued. The next subsection constructs an equilibrium in which foreign debt issued is fragile: it is repaid when output is high and defaulted when output is low.

**Domestic debt stability.** Similarly, we derive the domestic debt stability threshold. Domestic debt is repaid whenever:

$$\frac{b^d}{y_2^r} \le \frac{1 - \gamma^d}{\tau_2},\tag{13}$$

where the left-hand side is the domestic debt-to-GDP ratio and the right-hand side is a threshold defined by the parameters of the model. Importantly, the denominator on the right-hand side of inequality (13) is of a different magnitude than that in (12). This is because the repayment of foreign debt is a transfer of resources out of the economy, while the repayment of domestic debt is only a redistribution of resources within the economy. This redistribution is costly, and these costs are captured by the parameter  $\tau_2$ . Inequality (13) leads to two propositions.

**Proposition 2.** If taxation is costless and domestic default induces small positive costs to the economy, then any level of domestic debt is repaid.

This is the result of Broner et al. (2010), where taxes are assumed to be lump sum and default on domestic agents induces redistribution costs, which are endogenously derived (here captured by the parameter  $\gamma^d$ ). This result has powerful consequences. For example, if any level of debt is sustainable on the domestic market, then if secondary debt markets are efficient, any level of foreign debt is also sustainable in repayment equilibrium. Foreign debt can always be repaid even without exogenous default penalties, and a sufficient solution to the default problem is to improve the efficiency of secondary debt markets.

Proposition 2 shows that the assumption of lump-sum taxes is key to deriving the Broner et al. (2010)'s result. Without this assumption, there is a finite limit to the amount of domestic debt that can be sustained in the repayment equilibrium.

**Proposition 3.** If taxation is costly then the government's optimal policy on the domestic market is characterized by the domestic default threshold (13). Whenever domestic debt is below this threshold, it is riskless and is always repaid. Whenever it is above the threshold, it will always be defaulted on and therefore can never be issued. If either output or tax distortions are stochastic, the domestic default threshold is also stochastic, hence debt can be risky, and default can arise in equilibrium.

*Proof.* The first part follows directly from comparing (8) and (9). The proof of the second part is analogous to the proof of Proposition 1.

Condition (13) provides a closed-form formula for domestic debt stability threshold in an environment with exogenous tax distortion. Similarly to the foreign case, domestic debt stability depends on four factors: output level, default penalty, tax distortion, and the level of domestic debt itself. Importantly, domestic debt stability depends differently on tax distortion than foreign debt stability. For domestic debt, tax distortions play a much larger role than for foreign debt, as can be seen by comparing the denominators on the righthand side of conditions (12) and (13). With the same output and similar level of default penalties, more domestic debt is sustainable in repayment. This is because repayment of foreign debt constitutes a transfer of resources out of the economy, and efficiency losses from taxation are of second-order importance. In the case of no tax distortions  $\tau = 0$ , the foreign debt stability threshold is still well defined. A repayment of domestic debt is a redistribution within the economy, and a (potentially small) deadweight loss from taxation is the only source of resource loss when repaying the debt. In the case of no tax distortions, the domestic debt stability threshold approaches infinity and any level of domestic debt is stable.

Inequalities (12) and (13) completely characterize the government's policies in the second period. Notice that whenever both inequalities are reversed, it is also the case that  $c_{td} > c_r$ , which is consistent with the definition of total default being simultaneous default on both domestic and foreign debts outstanding. **Debt fragility.** For exposition purposes, we construct an equilibrium with fragile debt: the realizations of output and tax distortion shocks can lead to either repayments or defaults on domestic and foreign markets. According to debt stability conditions (12) and (13) both domestic and foreign stability depends on both output and tax distortion. For simplicity, we assume that each process admits two outcomes:

$$y_t^r = \begin{cases} y_H & \text{with prob. } \pi_y \\ y_L & \text{with prob. } 1 - \pi_y \end{cases}, \quad \tau_t = \begin{cases} \tau_L & \text{with prob. } \pi_\tau \\ & \\ \tau_H & \text{with prob. } 1 - \pi_\tau \end{cases}, \quad (14)$$

where subscript H stands for high and L for low. To construct an equilibrium we use guess and verify procedure. First, we guess that the four possible realizations of  $(y, \tau)$  stochastic processes are mapped into four equilibrium outcomes, one to one: repayment occurs when output is high and tax distortion is low; domestic default occurs when tax distortion is high (regardless of output); and foreign default occurs when output is low (regardless of tax distortion), hence total default occurs when output is low and tax distortion is high.<sup>2</sup> In the next subsection we verify our guess, showing that for a wide set of parameter values, the first-period debt issuances are consistent with the second-period equilibrium outcomes described here.

For this, we assume that government expenditure only occurs in the second period ( $g_1 = 0$ ). This gives the government an incentive to borrow on foreign markets in the first period, to smooth consumption. We also assume that tax distortion is high in the first period. This gives the government an incentive to postpone taxation, in the hope that the tax distortion will be low in the second period, and borrow on domestic markets in the first period, to smooth tax distortion. For graphical illustration, we need to impose specific numerical parameters on the model, which we show in Table 2 in Appendix 5.2.

In Figure 1 the exogenous tax distortion  $\tau$  is on the horizontal axis, while the vertical axis represents the domestic and foreign debt-to-GDP ratios in the second period. A negative shock to output is shown as an increase along the y-axis, and a negative tax distortion

$$\frac{y_L(1-\gamma_f)}{1+\tau_L} < b_f \le \frac{y_H(1-\gamma_f)}{1+\tau_H},$$
(15)

$$\frac{y_H(1-\gamma_d)}{\tau_H} < b_h \le \frac{y_L(1-\gamma_d)}{\tau_L},\tag{16}$$

<sup>&</sup>lt;sup>2</sup>Mathematically these conditions can be summarized by four inequalities that follow from substituting the realizations of y and  $\tau$  into (12) and (13):

#### Figure 1: Domestic and foreign debt fragility



Notes: The blue solid line shows the foreign debt stability threshold, red dashed line shows the domestic debt stability threshold. A-D represent four different scenarios. Circles stand for debt repayment, and crosses stand for debt default (blue for foreign, red for domestic). Source: Authors' elaboration.

shock is shown as an increase along the x-axis. The red (dotted) line plots the domestic debt stability threshold (13), while the blue (solid) line plots the foreign debt stability threshold (12). Circles show the allocations for which debt is repaid, while crosses show defaults. Colours represent the respective debt types (red for domestic, blue for foreign). The starting point is shown in (A), which shows domestic and foreign debt inherited from the first period. When the output is high and tax distortion is low (as in the first period), both debts fall below the default thresholds and therefore both are repaid. When the output and the tax distortion are low (B), both debt-to-GDP ratios shoot up. Since the foreign debt stability threshold is relatively flat in the tax distortion dimension, foreign debt-to-GDP (blue cross) goes above its stability threshold and is therefore defaulted on. However, since the domestic debt stability is highly concave in the tax distortion dimension, domestic debt (red circle) is still repaid, as it falls below its threshold. When output and tax distortion are high (C), the situation is the reverse of (B). The difference between domestic and foreign

debt defaults derives from the different shapes of their respective stability thresholds (13) and (12). In particular, when the tax distortion parameter  $\tau$  is low, the domestic debt stability threshold is highly volatile in the tax distortion, while foreign debt is almost flat in the tax distortion, which makes it more dependent on the output level. Finally, when output is low and the tax distortion is high, both debts are above their stability thresholds and are therefore defaulted on.

#### 2.2 Debt issuance

In this section, we verify the existence of the equilibrium proposed in the previous subsection, by solving for the first-period debt issuance decisions that are consistent with the secondperiod default decisions. We assume a constant relative risk aversion (CRRA) instantaneous utility function for domestic agents:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

This section aims to establish the equilibrium existence, in which foreign default is driven by the output shock and domestic default is driven by the tax distortion. The solution algorithm is provided in Appendix 5.1. Debt issuances must obey the first-order conditions given by:

$$(b_d:) (\tau_L - \tau_1)q_d = \tau_1 b_d \frac{\partial q_d}{\partial b_d}, (17)$$

$$(b_{f}:) \quad u'(c_{1})\left((1+\tau_{1})q_{f}+\tau_{1}b_{d}\frac{\partial q_{d}}{\partial b_{d}}\right) = \beta\left(1+\tau_{L}\right)\left(\pi_{y}\pi_{\tau}u'(c^{r})+\pi_{y}\left(1-\pi_{\tau}\right)u'(c^{dd})\right).$$
(18)

Table 2 in Appendix 5.2 presents the values we assign to each parameter that guarantee the existence of an equilibrium with the four outcomes. The set is non-empty. For each parameter of interest, we show the range in which the equilibrium with four outcomes exists. The ranges are wide and within plausible parameter values. Finally, for the firstperiod issuance, we perform comparative statics exercises, where we vary each parameter at a time and show how changes to parameters affect total debt issuance and the composition of debt between domestic and foreign investors. Comparative statics reveal that this general model environment can account for the two empirically observed facts shown in Andritzky (2012). First, that the share of foreign investors is negatively correlated with the interest rate (top graph in Figure 7); and second, that the share of domestic investors is positively correlated with the total public debt of the economy (middle and bottom graphs in Figure 7).

### 3 Endogenous tax distortions

In the second model, we operationalize the idea that tax distortions can be volatile over the business cycle. We build a model where tax distortions arise endogenously. Households derive utility from consumption and leisure, and supply labour elastically. The government has to cover volatile expenditures  $g_t$  each period. To cover expenditures, the government can raise distortionary labour taxes, or borrow on domestic and/or foreign markets. Labour taxes are discretionary and liable to change each period, which creates an endogenous tax distortion in the economy. The rest of the model is the same as in the previous section. In particular, the government can discretionarily default on either domestic or foreign debt.

We solve the model with a primal approach: the planner chooses allocations, taking households' first-order conditions as constraints. In period t, the planner chooses an allocation  $\{c_t, n_t, b_t^f, b_t^d, \delta_t^f, \delta_t^d\}$  to maximize the household's expected lifetime utility such that the implementability condition in (20) (derived from household's intratemporal substitution optimal condition and their budget constraint), along with the economy's resource constraint in (21), and the domestic and foreign bond price schedules in (22) and (23) are satisfied. The condition in (24) ensures that households' initial and final debt levels are null.

$$\max_{c_t, n_t, b_t^f, b_t^d, \delta_t^f, \delta_t^d} \quad \sum_{t=1}^N \beta^{t-1} E_1 \big[ u(c_t, n_t) \big], \tag{19}$$

s.t. 
$$c_t = -\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} n_t + (1 - \alpha) \gamma_t^i A_t n_t^\alpha - q_t^d b_t^d + (1 - \delta_t^d) b_{t-1}^d$$
 (20)

$$\gamma_t^i A_t n_t^{\alpha} = c_t + g_t - q_t^f b_t^f + (1 - \delta_t^f) b_{t-1}^f, \tag{21}$$

$$q_t^d = \frac{\beta E_t \lfloor (1 - \delta_{t+1}^a) u_c(c_{t+1}, n_{t+1}) \rfloor}{u_c(c_t, n_t)},$$
(22)

$$q_t^f = \frac{E_t \left[ 1 - \delta_{t+1}^f \right]}{1+r},$$
(23)

$$b_0^f = b_0^d = 0, \ b_T^f = b_T^d = 0, \tag{24}$$

where  $\gamma_t^i$  is a proportional TFP penalty, similar to the one introduced in (7):

$$\gamma_{t}^{i} = \begin{cases} 1, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (0, 0) \\ \gamma^{f}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (1, 0) \\ \gamma^{d}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (0, 1) \\ \gamma^{f} \gamma^{d}, & \text{if } (\delta_{t}^{f}, \delta_{t}^{d}) = (1, 1) \end{cases}$$
(25)

The economy lasts for two periods (N = 2). We solve the model using backward induction. In the second period, the social planner takes the debt levels as given and solves consumption and labour allocations as well as default decisions, subject to the secondperiod implementability and resource constraints. Second-period optimal default decisions yield equilibrium foreign bond discount price. In the first period, the planner takes the foreign debt price as given and solves for the optimal consumption and labour allocations and domestic and foreign bond issuances, subject to the first-period implementability and resource constraints, and the domestic bond price schedule (households' Euler equation). Optimal labour tax is then derived from optimal allocations using the following formula

$$(1 - \tau_t) = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} \frac{1}{w_t},$$
(26)

where  $w_t$  is the wage equal to the marginal product of labour  $w_t = \alpha A_t n_t^{\alpha-1}$ . We solve the model numerically using the benchmark parameters in Table 1. The utility function is given by:

$$u(c,n) = \frac{\left[c^{\eta}(1-n)^{1-\eta}\right]^{1-\sigma}}{1-\sigma}.$$
(27)

Under the benchmark specification (A = 1, g = 0.063) and with no debts outstanding, the reference output is  $\bar{y} = 0.422$ . For simplicity, we assume equal default penalties for the two debts. In the previous section, we have constructed an equilibrium with fragile debt, using a simplified stochastic structure, where output and tax distortion processes assumed two values, and showed that this equilibrium is supported in a wide range of parameter values. Here, we take a different approach. We solve for an equilibrium using a single set of plausible parameters specified in Table 1, and we show outcomes for a wide range of realizations of stochastic processes: output and government expenditures. We assume that the TFP and the government expenditure values are *i.i.d.*, randomly drawn from normal

Parameter	Description		
$\gamma^d$	Output penalty of domestic default	0.95	
$\gamma^f$	Output penalty of foreign default	0.95	
$\sigma$	Relative risk aversion	4	
$\eta$	Consumption-leisure substitution	0.377	
$\alpha$	Labor share of output	0.63	
eta	Discount factor	0.9	
r	Interest rate	0.05	
$\mu_A$	Mean TFP	1	
$\sigma_A$	TFP standard deviation	0.01	
$\mu_g$	Mean government expenditure	0.15	
$\sigma_{g}$	Government expenditure standard deviation	0.05	

Table 1: Benchmark parameters

distributions given, respectively, by  $A_t \sim N(\mu_A, \sigma_A)$  and  $g_t/\bar{y} \sim tN(\mu_g, \sigma_g, -\infty, 0)$ . We assess how default and debt issuance decisions vary in response to variations in the TFP and government expenditure in each period. We vary output and government expenditures below and above its benchmark value A = 1 and we vary government expenditures below and above its benchmark value expressed in percentages of the benchmark output,  $g/\bar{y} = 0.15$ .

#### 3.1 Debt stability

Figure 2 presents the stability graphs for domestic and foreign debt. In each graph, potential levels of foreign debt-to-GDP are plotted on the vertical axis, and potential levels of domestic debt-to-GDP are plotted on the horizontal axis. The top left graph plots debt stability in the benchmark specification ( $A_2 = 1, g_2/\bar{y} = 0.15$ ), the top right graph plots debt stability with high government expenditures ( $g_2/\bar{y} = 0.25$ ), and bottom graph plots debt stability with low TFP ( $A_2 = 0.9$ ), both shocks are of the same magnitude (10% of the benchmark GDP).

The first result that is evident in all three graphs is that the debt size matters: foreign default happens for high levels of foreign debt and domestic default happens for high levels of domestic debt. When both debts are high, total default is the preferred option. A lower



Figure 2: Foreign and domestic default sets.

Notes: Graphs present default and repayment regions for different levels of domestic (horizontal axis) and foreign debt (vertical axis) in the second period. The top left graph presents results for the benchmark case, when  $A_2 = 1$  and  $g_2/\bar{y} = 0.15$ , top right graph presents results when government expenditures are high  $(g_2/\bar{y} = 0.25)$ , bottom left graph presents results when TFP is low  $(A_2 = 0.9)$ .

level of TFP (bottom graph) decreases the stability of both debts: foreign and domestic default areas increase and defaults are more likely for lower levels of both debts (default on foreign debt is 7% more likely and on domestic debt is 9% more likely than in the benchmark case). Most interestingly, a higher level of government expenditure (right graph) decreases the stability of domestic debt, and has only a small effect on the stability of foreign debt:

when compared to the benchmark case (top-left graph), the number of events in which domestic default occurs is 16% higher when government expenditures are high (top-right graph), whereas the number of events in which foreign default occurs increases by only 4%.

The results about debt stability in the specific model with endogenous tax distortions echo the general results that we have derived in the previous section (debt stability equations 12 and 13). Debt stability on both markets depends on the size of each debt, on the output level (approximated here by the TFP), and the level of tax distortions (here driven by both, a TFP shock and a government expenditure shock). However, in the specific model with endogenous tax distortions, domestic debt stability mostly depends on the size of debt and the level of government expenditures, and foreign debt stability mostly depends on its size and the level of TFP. Unlike in the previous model, where we derived general formulas of domestic and foreign debt stability and constructed a specific equilibrium in which output shock affects foreign debt stability and tax distortions affect domestic debt stability, the result of a weak separation arises endogenously in a model with endogenous tax distortions.

Another important result of this model is the quantitative difference between the domestic and foreign debt stability thresholds. As is visible in the top left graph, domestic debt is more stable, i.e. higher level of domestic debt-to-GDP than foreign debt-to-GDP can be sustained in repayment. This echoes the result we have derived in the previous, more general model, and documented in the discussion under equation (13). The result is the most pronounced when the default costs are similar, which is the case in the current parametrization (see Table 1). In this simple two-period model, where all debts are due after one period, the correct empirical counterpart of the model debt-to-output variables  $b^f/\bar{y}$  and  $b^d/\bar{y}$  are the debt service-to-GDP ratios (rather than debt-to-GDP ratios). Importantly, even though the model is simple, the quantitative results of debt stability thresholds are within plausible range. In the model, the foreign debt (service)-to-GDP stability threshold is around 3-4%, and the domestic debt (service)-to-GDP stability threshold is around 8-9%. Empirically, debt service-to-GDP in emerging economies (which typically hold little domestic debt) is on average 2.1%, and in advanced economies (which typically hold more domestic debt) is 11.8%, and quantitative models of sovereign default usually target values around 2% (Paczos and Froemel, 2022).

Figure 3 plots debt stability in different dimensions. Here government expenditures are plotted on the horizontal axis and TFP levels are plotted on the vertical axis. The





Notes: Graphs present default and repayment regions for fixed levels of domestic and foreign debt  $(b^f/\bar{y} = b^d/\bar{y} = 0.037)$  on the left and  $(b^f/\bar{y}, b^d/\bar{y}) = (0.016, 0.037)$  on the right) and varying levels of government expenditure (horizontal axis) and TFP (vertical axis) in the second period.

left panel plots default areas when domestic and foreign debt are similar in size, the right panel plots the default areas when domestic debt is much larger than foreign debt. The situation depicted in Figure 3 is an example of a more general result, that domestic and foreign debt stability are history-dependent. In particular, given domestic and foreign debt levels in period two, either domestic or foreign default dominates the other. In other words, unlike in the constructed equilibrium of the general model of Section 3, it is not possible that in period two, one shock would trigger foreign default and another shock would trigger domestic default. This is a direct consequence of the previous result, that domestic debt is more stable than foreign debt. When this is the case, and the two debts are similar in size (or foreign debt is larger than domestic), then any shock that reduces the availability of resource in period two triggers foreign default first. If the shock is large in size it would trigger total default. On the other hand, domestic default can only happen, when foreign debt is small in size (Figure 3, right panel). A size of foreign and domestic debt is an endogenous object, it is driven by the shocks in the first period. Therefore, since debt stability depends on debt size, and these depend on the history of shocks, we show that debt stability is history-dependent.



Figure 4: Domestic (left) and foreign (right) debt discount prices.

Notes: Graphs plot domestic (left) and foreign (right) debt discount prices. Discount prices are on the vertical axes, and TFP levels are on the horizontal axes. Lines represent different government expenditure levels.

Next, given debt stability in the second period, and ensuing probabilities of default, we turn to the equilibrium analysis. In the next subsection, we present the equilibrium debt discount prices, the optimal sizes of domestic and foreign debt issued, and the optimal tax policy in both periods.

#### 3.2 Debt issuance

Figure 4 presents the discount prices for domestic and foreign debt. Both discount prices are procyclical: they are high when TFP is high and low when TFP is low. A procyclical discount price of a foreign bond is driven by the probability of default, which is decreasing in today's TFP (see eq. 21) and is a standard result in the literature. The discount price of a domestic bond is also procyclical, but the underlying mechanism is somewhat different. A domestic bond is priced by risk-averse households, who take into their pricing kernel differences in consumption today and tomorrow. When TFP is low, households expect it to be somewhat higher tomorrow (due to the *i.i.d.* nature of the TFP shock). The differences in expected consumption drive the risk-averse price of the bond down (see eq. 22). Owing to consumption smoothing, the price of a domestic bond is also smooth in the TFP level.



Figure 5: Domestic (left) and foreign (right) optimal debt policies.

Notes: The graphs are contour plots for the optimal debt issuances in the first period. Government expenditures are plotted on the horizontal axis, and TFP levels are plotted on the vertical axis. Light colours represent low debt levels, and dark colours represent high debt levels.

Figure 5 presents the optimal policies for debt issuance in the first period. Government expenditures are plotted on the horizontal, and TFP levels are plotted on the vertical axes. The first striking result is that foreign debt is countercyclical (debt issuance falls when TFP increases, moving up along the vertical axis in the right panel), but domestic debt issuance is procyclical (debt issuance increases when TFP increases). This demonstrates how different instruments domestic and foreign debts are. Foreign debt allows the government to bring in additional resources to the economy from outside and hence is a valid instrument for consumption smoothing. Domestic debt is only an intertemporal redistribution of resources within the economy: it can help to smooth tax distortion, but it cannot achieve the consumption smoothing objective when total resources available (driven by TFP) vary over time.

There are two competing mechanisms in each market. The first is consumption smoothing: households prefer to save (borrow) in the domestic bond when TFP is high (low), and the government prefers to save (borrow) in the foreign bond when TFP is high (low). This is a countercyclical motive for domestic and foreign debt issuance. The second is the price mechanism: the discount price of each debt is high when TFP is high, so issuing debt is cheaper when TFP is high. This is a procyclical motive for domestic and foreign debt issuance. As we have discussed in the previous paragraph, the countercyclical consumption smoothing motive is much stronger with foreign debt. Also, the procyclical price mechanism is much stronger with domestic debt (see Figure 4). Then, the consumption smoothing motive dominates on the market for foreign debt, and foreign debt issuance becomes countercyclical, whereas the price mechanism dominates on the domestic market, and domestic debt issuance becomes countercyclical.

The second (horizontal) dimension in Figure 5 is the government expenditure. Both domestic and foreign debts increase when government expenditure goes up. This is intuitive but is driven by somewhat different mechanisms in each market. On the foreign market, higher demand for debt when government expenditures are high derives from the resource constraint (21). Absent foreign borrowing, when expenditures go up, the government would have to either default on foreign debt (which also happens to some extent, see Figure 2, but is mitigated by the default costs), or induce households to work more and consume less, with higher taxes (which is costly in utility terms via endogenous tax distortions). On the domestic market, however, the resource constraint does not apply - the government cannot relax the resource constraint with domestic borrowing. An increase in domestic debt issuance is driven by the implementability constraint (20). With domestic debt, the government can induce households to postpone their consumption (via their intertemporal condition by offering debt at attractive prices) without affecting their optimal labour-leisure choice, and so without distorting equilibrium efficiency (avoiding utility loss from an increased labour supply).

The debt policies on both markets induce certain implicit optimal tax policies, which we now turn to discuss. Figure 6 presents the optimal tax policy (on the vertical axis) as the function of the TFP level (on the horizontal axis). The left graph presents the optimal tax policies in the first period, while the right graph presents the optimal tax policies in the second period. There are two effects at play. First, the planner wants to smooth the tax distortion in the first period by issuing domestic debt, hence the lower variance of tax rates in the first period, relative to the second. Moreover, the planner smoothes consumption by issuing foreign debt, thus the tax level is independent of TFP in the first period. In the second period, however, there no is debt issuance, so there is no space for smoothing - apart from default.



Figure 6: Tax rate in the first period (left) and in the second period (right).

Notes: Graphs plot optimal tax policies in the first (left) and second (right) periods. Tax rates are on the vertical axes, and TFP levels are on the horizontal axes. Lines represent different government expenditure levels.

This concludes the analysis of the model with endogenous tax distortions. We summarize the findings and conclude the study in the next, final section.

### 4 Conclusions

In this paper we have proposed a joint theory of sovereign debt issuance and stability on domestic and foreign markets. Foreign debt is useful for consumption smoothing when output is volatile, but is inherently less stable than domestic debt. Domestic debt cannot smooth output fluctuations, but is useful for smoothing tax distortions, which can arise due to fluctuations in government expenditures. Optimally, foreign debt is countercyclical, while domestic debt is procyclical. Taxes are optimally acyclical, and when debt becomes fragile, taxes are optimally countercyclical. Domestic and foreign defaults are triggered either by suddenly low output, or suddently high government expenditures, or both. Foreign debt is more likely to be defaulted on, as its default threshold is lower, ceteris paribus, than for domestic debt. Only when shocks from the past lead to higher domestic debt build up, domestic default is the preferred option. Debt stability depends on debt size (which depends on past shocks), output fluctuations, and tax distortions.

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## 5 Appendix for "Domestic and Foreign Sovereign Debt Stability"

#### 5.1 Exogenous Tax Distortions - Solution Algorithm

We solve for the government's optimal domestic and foreign debt policies in the first period following these steps:

- 1. Assuming that (15) and (16) are satisfied in the second period, we write the government's problem as (28).
- 2. The solution to the problem is then a set of two first-order conditions (17) and (18) and pricing rules (29) and (30).
- 3. We pick a set of parameters and solve (28) numerically.
- 4. We confirm that the resulting policy functions  $b_f$ ,  $b_d$  and equilibrium prices  $q_f$ ,  $q_d$  satisfy conditions (15)–(16), and therefore that expectations in (28) are consistent in equilibrium. If not, we update parameters.
- 5. We vary one parameter at a time within a range where (15)–(16) are satisfied to derive comparative statics.

The government chooses debt issuances  $b_d$  and  $b_f$  to maximize the lifetime utility of domestic agents:

$$\max_{\{b_d, b_f\}} u(c_1) + \beta \mathbb{E} \left[ u(c_2) \right], \tag{28}$$

where

$$c_{1} = y_{1} + \tau_{1}q_{d}b_{d} - (1 + \tau_{1})(g - q_{f}b_{f}),$$

$$c_{2} = \begin{cases} (8) \text{ with prob. } \Pi_{y}\Pi_{\tau} \\ (9) \text{ with prob. } \Pi_{y}(1 - \Pi_{\tau}) \\ (10) \text{ with prob. } (1 - \Pi_{y})\Pi_{\tau} \\ (11) \text{ with prob. } (1 - \Pi_{y})(1 - \Pi_{\tau}), \end{cases}$$

subject to price schedules derived from foreign investors' zero-profit condition and domestic households' first-order condition:

$$q_f = \frac{\Pi_y}{1+r},\tag{29}$$

$$q_d = \beta \frac{\Pi_y \Pi_\tau u'(c^r) + (1 - \Pi_y) \Pi_\tau u'(c^{fd})}{u'(c_1)}.$$
(30)

## 5.2 Exogenous Tax Distortions - Parametrization

Parameter	Value	Range	Description
y	1		Output today / High output tomorrow
$\sigma$	1		Risk aversion
$\pi_y$	0.72	[0.5 , 1]	Probability of high output
$\pi_{ au}$	0.8	[0.5 , 1]	Probability of low tax distortion
g	0.7	[0.5 , 0.8]	Gov. expenditure
$y_L$	0.5	$[0.1 \ 0.7]$	Low output
$ au_1$	0.1	[0, 0.2]	Tax distortions today
$ au_{H}$	0.15	[0.1 , 0.2]	Tax distortions tomorrow (high)
$ au_L$	0.05	[0, 0.15]	Tax distortions tomorrow (low)
$\delta_{f}$	0.65	[0.42 , 0.87]	Output cost of Foreign default
$\delta_h$	0.05	[0, 0.15]	Output cost of Home default
r	0.00		Risk-free interest rate

Table	2:	Parametrization

### 5.3 Exogenous Tax Distortions - Debt Issuance Graphs

Figure 7: Comparative statics in the Model with Exogenous Tax Distorions

