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Returns to scale:
New evidence from administrative firm-level data

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Non-technical summary

Research Question

In many economic studies, assumptions about the production technology of companies are of central importance. In most cases, it is assumed that firms have constant returns to scale. Against the background of technological change and increasing evidence of pronounced firm heterogeneity, the question arises whether the assumption of constant returns to scale is still justified.

Contribution

We provide new microeconometric evidence on the returns to scale of firms. We use a new, high-quality firm dataset based on administrative firm data from five euro area countries (Belgium, France, Italy, Portugal, Spain), covering almost all sectors of the economy over a recent period of time (2008 to 2018). In contrast, previous studies typically only had access to data for manufacturing, past time periods and the US. Furthermore, we apply recent estimation techniques to estimate the returns to scale parameter. By doing so, we estimate returns to scale for more than 370 industries.

Results

The majority of economic sectors exhibit constant returns to scale. However, a non-trivial share also exhibits decreasing or increasing returns to scale. Especially in manufacturing, the transport sector and the IT sector, there are numerous industries with increasing returns to scale. In service industries, on the other hand, there are relatively often decreasing returns to scale. The results may allow other studies to make more precise assumptions about the returns to scale of companies in the future.

Nichttechnische Zusammenfassung

Fragestellung

In einer Vielzahl ökonomischer Studien sind Annahmen über die Produktionstechnologie von Unternehmen von zentraler Bedeutung. Meist wird dabei unterstellt, dass Unternehmen konstante Skalenerträge aufweisen. Vor dem Hintergrund des technologischen Wandels und vermehrter Evidenz einer hohen Unternehmensheterogenität, stellt sich die Frage, ob die Annahme konstanter Skalenerträge weiterhin gerechtfertigt ist.

Beitrag

Wir liefern neue mikroökonometrische Evidenz zu den Skalenerträgen von Unternehmen. Dabei verwenden wir einen neuen, hochwertigen Firmendatensatz, welcher auf administrativen Firmendaten aus fünf Euroraumländern (Belgien, Frankreich, Italien, Portugal, Spanien) basiert und dabei nahezu alle Wirtschaftszweige sowie einen vergleichsweise aktuellen Zeitraum (2008 bis 2018) abdeckt. Bisherige Studien hatten hingegen typischerweise lediglich Zugriff auf Daten für das Verarbeitende Gewerbe, länger zurückliegende Zeiträume und die USA. Des Weiteren verwenden wir zur Schätzung der Skalenerträge neuste Schätzverfahren.

Ergebnisse

Die Mehrzahl der Wirtschaftszweige weist konstante Skalenerträge auf. Ein nicht unerheblicher Teil weist jedoch auch abnehmende oder zunehmende Skalenerträge auf. Insbesondere im Verarbeitenden Gewerbe, dem Transportsektor sowie dem IT-Sektor finden sich zahlreiche Wirtschaftszweige mit zunehmenden Skalenerträgen. In Dienstleistungsbranchen finden sich hingegen vergleichsweise häufig abnehmende Skalenerträge. Die Ergebnisse erlauben es anderen Studien in Zukunft möglicherweise päzisere Annahmen über die Skalenerträge von Unternehmen zu treffen.

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Returns to Scale:

New Evidence from Administrative Firm-Level Data*

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Abstract

Using a new administrative dataset, we provide fresh micro-level evidence on firms' returns to scale (RTS). We employ a new administrative database, *iBACH*, which contains extensive high-quality annual balance sheet, financial, and demographic information on more than two million non-financial manufacturing, trade and service corporations for five European countries over 2008-2018. Whereas on average, we find sectoral RTS to be close to one (0.98, with a $0.74-1.18$ range), 32 percent of firms exhibit decreasing returns, and 10 percent increasing returns to scale (IRTS). Although the RTS values have remained relatively stable, there is evidence of some tendency for them to increase over time. When we allow for imperfect competition, the RTS range tightens to $0.98 - 1.08$, with a higher share of IRTS industries (15 percent) and essentially zero DRTS cases. Increasing returns are mostly a feature of manufacturing. Finally, we analyze the relationship between different industry characteristics and our RTS estimates.

Keywords: Firm & sectoral production function estimation, imperfect competition, firm characteristics, Gandhi-Navarro-Rivers, *iBACH* database.

JEL Classification: E2, D2, L1.

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The fact that the world seems best described by approximately constant returns at the firm level does not necessarily allow us to reject macroeconomic parables in which increasing returns at a representative firm play a central role in explaining economic fluctuations. Identifying which paradigm provides better macroeconomic insight is an important task for future research.

Basu and Fernald (1997)

1 Introduction

Production functions play a commanding role across several areas of economics. Such functions are key primitives of many economic models and shed light on a variety of phenomena: for example, growth accounting and the degree of factor complementarity in production. Likewise, the degree to which there are constant, increasing, or decreasing returns to scale (hereafter, CRTS, IRTS, DRTS, respectively) is fundamental to the study of growth, technology choice, productivity dispersion, international trade, market power, and industrial policy, to name but a few.

The contribution of our paper is threefold. First, we introduce a new and high-quality administrative firm-level dataset, *iBACH*, covering essentially the entire economy of five major European countries: Belgium, France, Spain, Italy, and Portugal (constituting around 50% of the euro area's GDP). This is an important development because estimates for European economies have previously been hampered by data availability and quality issues. Second, we provide RTS parameters at the 4-, 2-, and 1-digit levels, using both a standard estimation approach (commonly applied in the literature) and one better suited to account for imperfect competition. Third, the paper makes a first-pass at linking firms' RTS values with their specific economic characteristics (such as intangible capital usage, export intensity).

iBACH is a new comprehensive firm-level dataset for selected European countries containing detailed financial and balance sheet information of non-financial corporations. The dataset has been collected from national statistical sources in a coordinated manner under the mandate of the European Committee of Central Balance Sheet Data Offices. Since 2018, this dataset has been disseminated by the Statistics Directorate of the European Central Bank which acts as a hub for the data collection, to its internal users and eurosystem statistical working groups that are in charge of disseminating and promoting the dataset in their countries. To the best of our knowledge, this is the first time the dataset has been used for the estimation of firm-level production relationships. As access to the dataset becomes more widely available among researchers, we expect its size and coverage to grow in the coming years, as well as its usage.

More specifically, using *iBACH* has several advantages:

1. Our analysis can essentially straddle the whole private economy. By contrast, many other studies focus on manufacturing only.[1] Though a key sector in advanced

[1] See for example, the following manufacturing studies: de Loecker (2011), Petrin and Sivadasan (2013), Ackerberg,

economies (and one typically synonymous with increasing returns), manufacturing tends to be modest in terms of turnover and employment share; consequently, an overly narrow sectoral focus may thus paint a distorted picture of RTS and its determinants.[2]

2. The database covers both privately- and publicly held companies. This gives it a considerable advantage over other widely-used dataset (e.g. Compustat, WorldScope etc).

3. Given the quality and consistency aspects embodied in *iBach* (such as common legal definitions), we can be confident of using and combining data from different European countries and sectors.

4. The flexibility and transparency of this data implies that we can additionally vary our unit of analysis: although our main results pertain to the 4-digit level of industrial classification, we can also aggregate and map up to 1-digits.

5. Another departure from the standard literature lies in our geographical coverage. The bulk of studies estimating production functions and assessing returns to scale have been US based (and often limited to manufacturing). *European* studies, by contrast, have been hampered by limited and fragmented stocks of data, as well as data of insufficient quality for cross-sectoral/cross-country purposes.

We find that across the whole economy, sectoral RTS are on average close to one. Notwithstanding, a nontrivial share (42 percent) of industries depart from constant returns. In our baseline specification, which is consistent with most estimation approaches in the literature in assuming perfect competition in product markets, 32 percent of industries exhibit statistically significant DRTS. Overall, RTS values range from 0.74 to 1.18. Similarly, when we allow for *imperfect* competition, still almost one fifth of industries exhibit non-constant returns with the share of DRTS cases shrinking essentially to zero (3 percent) and those with IRTS at 15 percent.

In both approaches (standard and allowing for imperfect competition), some sectors such as *Manufacturing* (NACE C), *Transport & Storage* (H), and *IT* (J) have relatively high RTS estimates (and a large statistically significant share of IRTS-industries), whilst other sectors such as *Accommodation and Food* (I), *Administrative Services* (N) or *Professional Services* (M)

Caves and Frazer (2015), and Gandhi, Navarro and Rivers (2020). Notwithstanding, perhaps reflecting the early and enduring influence of Baumol, Kaldor, and Verdoorn, much of the literature tended to view manufacturing as the source of increasing returns and thus the engine of aggregate growth (see Baumol and Bowen, 1966; Kaldor, 1966).

[2] For instance, Burnside, Eichenbaum and Rebelo (1995) concluded that 'there is virtually no evidence' (p.96) of non-constant returns in the US but limited their study to manufacturing. Burnside, Eichenbaum and Rebelo (1995) also argue that omitted measures of capital utilization can bias RTS estimates. However, Fernald and Wang (2016) have argued, for the US, that there were important shifts from the mid-1980s in which labor productivity turned from pro to counter cyclical – largely reflecting the weakening pro-cyclicality of factor utilization. This may be due to increased economic flexibility (i.e., the expansion of female labor participation, perhaps declining labor power); but also, more likely, the decline of manufacturing (where utilization was traditionally a more important margin of adjustment).

have lower mean RTS values and are much less disposed to increasing returns. Beyond that, we find some evidence that RTS values have exhibited some tendency to drift rightward over time, thinning their lower tail.

Finally, we make a first attempt to analyze the relationship between industry-level RTS and other industry characteristics. Doing so, we find a positive relationship between industries' RTS and their average firm size, their use of intangible capital, trade intensity, or market concentration. Additionally, some evidence points to a negative, though weaker, relationship between TFP growth and RTS.

Organization Section 2 reviews the literature. Section 3 discusses the estimator used (namely, that of Gandhi, Navarro and Rivers, 2020, hereafter **GNR**) and how imperfect competition can be integrated into the estimation. Section 4 discusses the data. Section 5 provides RTS estimates for the 4-, 2-, and 1-digit level, for both the baseline (perfect competition) and imperfect competition estimators. Section 6 makes a first attempt to link industry-level RTS and other industry characteristics. Section 7 concludes. Additional material appears in the appendices.

2 Brief Literature Review

The notion of returns to scale dates back to Marshall's explicit recognition of opposing forces of returns facing firms (Marshall, 1890) and the 1920s and 1930s debates (see, e.g. Special Economic Symposium, Economic Journal, 1930). While Allyn Young's provocative paper at the time (Young, 1928) argued for increasing returns by reviving Adam Smith's division of labor, until the late 1970's, basic theory embraced constant returns with John Hicks and Nicholas Kaldor as major proponents (Hicks, 1936; Kaldor, 1972). The revival of increasing returns (see, Buchanan and Yoon, 2000) was associated with several strands of theory, including monopolistic competition (Dixit and Stiglitz, 1977), international trade (Krugman, 1979; Ethier, 1982), unemployment dynamics (Weitzman, 1982), and growth (Romer, 1987; Growiec, 2022).

Our paper follows in this rich tradition and specifically relates to the vast literature on:

- micro and macro ***production function estimation methods*** (inter alia, Olley and Pakes, 1996; León-Ledesma, McAdam and Willman, 2010; Gandhi, Navarro and Rivers, 2020);
- ***returns to scale*** (e.g., Burnside, Eichenbaum and Rebelo, 1995, Basu and Fernald, 1997, Fernald, Ahmad and Khan, 2019);
- ***imperfect competition*** (de Loecker, Eeckhout and Unger, 2020, Lu, Sugita and Zhu, 2019); and
- ***firm performance and productivity*** (Fernald and Wang, 2016; Syverson, 2017).

Consider first estimation methods. A central concern in the estimation of production functions is the (positive) correlation between firms' input choices and their idiosyncratic productivity shocks. Since such shocks are unobserved by the econometrician, standard least-squares estimates will be biased and inconsistent. This "transmission bias" has given rise to a family of remedial techniques, the most popular being structural control function approaches.[3] These derive moment conditions from assumptions about an optimizing firm (i.e., regarding the timing of input decisions, the evolution of productivity shocks, monotonicity between the shocks and the proxy, production structure) and, thus, correct for that endogeneity (see Ackerberg, Caves and Frazer, 2015, and Gandhi, Navarro and Rivers, 2020 for detailed discussions of these methods).

Within this family, we use the gross output production function estimation method recently proposed by the influential paper of Gandhi, Navarro and Rivers (2020). They show that the standard proxy-variable approach applied to gross output production functions does not in general identify the elasticities of the 'flexible input.[4] Instead, they show, using a two-step nonparametric procedure, how to combine the proxy-variable approach with first-order conditions of cost minimization to estimate the gross output production function, and in turn the mean RTS parameter.[5]

Moreover, we have known at least since Solow (1957), that the analysis of technical change and RTS may be confounded by the presence of imperfect competition.[6] In much of the literature (e.g. Basu and Fernald, 1997; de Loecker and Warzynski, 2012) information on markups and imperfect competition is inferred after production-function estimation. However, here, following the pathway of Gandhi, Navarro and Rivers (2020) and Lu, Sugita and Zhu (2019), we also consider the joint estimation of RTS and the markup in order to account for imperfect competition in the product market.

Next, consider the data sets used. Most of the existing literature focus the analysis on manufacturing including the prominent papers by de Loecker (2011), Petrin and Sivadasan (2013), Ackerberg, Caves and Frazer (2015), and Gandhi, Navarro and Rivers (2020). Though a key sector, manufacturing tends to be modest in terms of turnover and employment share; an overly narrow sectoral focus may thus paint a distorted picture of RTS and its determinants. For instance, Burnside, Eichenbaum and Rebelo (1995) concluded that 'there is virtually no evidence' (p.96) of non constant returns in the US, but limited their study to manufacturing. They also argue that omitted measures of capital utilization can bias RTS estimates. However, Fernald and Wang (2016) have argued, for the US, that there were important shifts from the mid 1980s in which labor productivity turned from pro to counter cyclical – largely reflecting the weakening pro-cyclicality of factor utilization. This may be due to increased economic flexibility (i.e., expansion of female labor participation, perhaps declining labor power); but also, more likely, the decline of manufacturing (where utilization was traditionally a more important margin of adjustment). In contrast to the

[3] See Villacorta (2022) for an interesting application of Bayesian panel techniques to production-function estimation.

[4] For further discussions of potential pitfalls in the value-added production estimation with mark-ups and the likely upward bias in estimates of RTS, see the seminal paper of Basu and Fernald (1997).

[5] GNR also comment that their method is useful for cases without long panels of data without access to firm-specific prices or other external instruments. This largely corresponds to our case.

[6] For an analysis of recent trends in market power in European economies see Cavalleri et al. (2019).

narrow focus in manufacturing, the *iBach* database is significantly more comprehensive covering all sectors for our country sample.

3 Empirical Methodology

We derive returns to scale parameters by summing the output elasticities of firms' inputs obtained by estimating production functions. We estimate the production function at a detailed 4-digit NACE industry level to allow for differences in technologies across industries.[7] In particular, we consider the following general production function:

$$Q_{it} = F(K_{it}, L_{it}, M_{it})e^{\nu_{it}}, \tag{1}$$

which in logs takes the form:

$$q_{it} = f(k_{it}, l_{it}, m_{it}) + \nu_{it}, \tag{2}$$

where Q_{it} denotes the output of firm i in year t, and L_{it}, M_{it}, and K_{it} are, respectively, labor (number of employees), materials (intermediate goods and services), and capital (fixed assets) inputs, and $f(\cdot)$ is a generic production mapping. In what follows, we consider intermediate goods to be the only static and thus fully flexible input factor, while capital and labor are treated as dynamic inputs.[8] Due to the highly regulated labor markets in many European countries, treating labor as a dynamic input factor appears most plausible.

Equations (1) and (2) imply that we are considering a gross output production function. Besides arguments related to value added not being a natural measure of firm output, we prefer a gross output specification because extending our baseline framework by incorporating firm demand in order to allow for imperfect competition in the output market is more plausible.[9] This latter point is important in our context, since, as in most firm-level datasets, we do not observe firm-level prices or produced quantities (Q_{it}) but rather firm revenue (i.e., $P_{it}Q_{it}$). The term ν_{it} constitutes Hicks neutral total factor productivity, which can be decomposed into an ex-ante component (ω_{it}) that is known to the firm when making decisions in period t, and an *iid* ex-post productivity shock ϵ_{it} realized only after decisions

[7] Nomenclature Generale des Activites Economiques dans les Communautes Europeennes (NACE) codes are used in the European industry standard classification system. They are similar in nature to the Standard Industry Classification (SIC) typically used in North American countries.

[8] Considering capital and labor as dynamic inputs implies that firms chooses variable inputs to minimize cost, given the levels of capital and labor that was set in the previous period. Reasons for this are e.g. adjustment costs, time to build.

[9] As noted by Basu and Fernald (1997), and more recently by Rivers (2018), specifying a gross output production is preferable, not the least since value added is not a natural measure of output (i.e., output can only be produced with intermediates). Moreover, parameters obtained from a value-added production function do not correspond to those of a gross output function if returns to scale are not constant and the assumption of perfect competition is violated. Besides, a value added production function does not allow for substitution between intermediates and capital and labor, and its extension to imperfect competition by modeling firm demand is less obvious, since consumer demand typically relates to a firm's gross output instead of its value added.

in period t have been made.[10] The presence of ω_{it} implies that a firm's input choice is potentially endogenous since firm managers observe it when making input decisions.

We apply the recent approach of GNR to estimating production functions in order to address this endogeneity concern. GNR observe that the widely applied proxy approach of production function estimation cannot be easily extended from the value-added to the gross-output case. In particular, the proxy approach generally fails to identify the coefficients of static and flexible inputs such as intermediate goods when applied to a gross output production function.[11] The GNR approach, however, allows doing so. Next, we summarize GNR's general estimation framework in the context of perfectly competitive output markets and briefly discuss the econometric implementation.[12]

3.1 Identification

The first step of GNR's identification strategy rests on a non-parametric link between the production function (2) and the first-order condition of firms' profit maximization problem. Similar to the proxy approach, GNR also relies on the assumption that ω_{it} follows a first-order Markov process. Assuming (for the moment) also a perfectly-competitive output market and that firms are price takers in the input market, a firm's maximization problem with respect material inputs (M_{it}) is given by:

$$\max_{M_{it}} P_{it}Q_{it} - P_t^M M_{it} = \max_{M_{it}} P_{it}\mathbb{E}\big[F(K_{it}, L_{it}, M_{it})e^{\nu_{it}} \mid \mathcal{I}_{it}\big] - P_t^M M_{it}, \tag{3}$$

where P_t^M are material input prices, $\mathbb{E}$ is the expectations operator and $\mathcal{I}_{it}$ is the information set available to the firm when making the input decisions.[13] The first-order condition is then,

$$P_{it}\frac{\partial}{\partial M_{it}}\Big(F(K_{it}, L_{it}, M_{it}\Big)e^{\omega_{it}}\mathcal{E} - P_t^M = 0, \tag{4}$$

where $\mathcal{E} = \mathbb{E}[e^{\epsilon_{it}} \mid \mathcal{I}_{it}] = \mathbb{E}[e^{\epsilon_{it}}]$.[14]

The non-parametric link between the production function (2) and the first-order condi-

[10] We thus do not consider the role of factor-augmenting technological change. See Raval (2023) for an analysis of the role of labor-augmenting technological change in the context of the production approach to estimating markups.

[11] This is because in the proxy framework, there is usually nothing that varies with intermediate inputs independently of productivity and the other inputs. This identification problem could be addressed with external instruments that shift intermediate input demand without directly affecting firm productivity and the other inputs (e.g., input prices). Moreover, under certain assumptions, using investment as a proxy for productivity would allow the identification of the material coefficient; however, this requires limiting the estimation sample to firms with positive investment.

[12] Estimation details of this approach as well as a detailed description of how this approach can be extended to the case of imperfect competition are in Appendix B.

[13] Note that $\omega_{it} \in \mathcal{I}_{it}$, while $\epsilon_{it} \notin \mathcal{I}_{it}$.

[14] Note that one can normalize $\mathbb{E}[\epsilon_{it} \mid \mathcal{I}_{it}] = \mathbb{E}[\epsilon_{it}] = 0$ without loss of generality. However, as GNR point out, since the latter is in units of log output, the expectation of the ex-post shock in units of the level of output becomes a free parameter denoted here by $\mathcal{E}$.

tion (4) is exploited by taking logs of (4) and differencing with (2), which gives:

$$
\begin{aligned}
s_{it} &= \ln \mathcal{E} + \ln \left(\frac{\partial}{\partial m_{it}} f(k_{it}, l_{it}, m_{it}) \right) - \epsilon_{it} \\
&= \ln \mathcal{E} + \ln D(k_{it}, l_{it}, m_{it}) - \epsilon_{it},
\end{aligned} \tag{5}
$$

where $s_{it} = \ln(P_t^M M_{it}/P_{it} Q_{it})$ is the log of the share of material inputs in firm revenue, and $D(\,\cdot\,)$ is the output elasticity of intermediate goods.[15]

GNR show that regressing s_{it} on the vector of firm inputs yields an estimate of the output elasticity of intermediate goods and ϵ_{it}. The information derived from the share regression can then be used to recover the rest of the production function. In particular, the material input elasticity defines a partial differential equation that can be integrated up to the part of the production function f related to m:

$$
\int \frac{\partial}{\partial m_{it}} f(k_{it}, l_{it}, m_{it}) dm_{it} + \mathcal{C}(k_{it}, l_{it}), \tag{6}
$$

where $\mathcal{C}(k_{it}, l_{it})$ refers to the constant of integration. Subtracting this term from the production function (2), we obtain the estimation equation for the second stage:

$$
\mathcal{Q}_{it} \equiv q_{it} - \epsilon_{it} \int \frac{\partial}{\partial m_{it}} f(k_{it}, l_{it}, m_{it}) dm_{it} = -\mathcal{C}(k_{it}, l_{it}) + \omega_{it}, \tag{7}
$$

where $\mathcal{Q}_{it}$ constitutes an observable random variable derived from data and estimation of the share equation. Similar to the proxy approach, the second stage then relies on the law of motion of productivity in order to exploit moments used to estimate the output elasticities of the dynamic inputs k_{it} and l_{it}.

Estimation Framework We follow GNR and estimate non-parametrically the model outlined above. In particular, we use a 2nd-degree polynomial series for the share equation and the integration constant. In order to obtain the intermediate goods output elasticity, we estimate equation (5) by non-linear least squares using:

$$
\begin{aligned}
s_{it} = \ln(&\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 \\
&+ \gamma_{kl} k l_{it} + \gamma_{km} k m_{it} + \gamma_{lm} l m_{it}) + \epsilon_{it}.
\end{aligned} \tag{8}
$$

The constant of integration, in turn, is estimated as:

$$
\mathcal{C}(k_{it}, l_{it}) = \kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} k l_{it}. \tag{9}
$$

[15] Strictly speaking, we obtain the elasticity as $e^{\ln D(\cdot)}$.

The estimated production function is therefore given by:

$$\begin{aligned} f(k_{it}, l_{it}, m_{it}) = m_{it}\Big(&\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \frac{\gamma_m}{2} m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 \\ &+ \frac{\gamma_{mm}}{3} m_{it}^2 + \gamma_{kl} k l_{it} + \frac{\gamma_{km}}{2} k m_{it} + \frac{\gamma_{lm}}{2} l m_{it}^2\Big) \\ &- \kappa_k k_{it} - \kappa_l l_{it} - \kappa_{kk} k_{it}^2 - \kappa_{ll} l_{it}^2 - \kappa_{kl} k l_{it} + \nu_{it}. \end{aligned} \tag{10}$$

The output elasticities for materials, labor, and capital are respectively obtained as:

$$\begin{aligned} \theta_{it}^M &= \gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} k l_{it} + \gamma_{km} k m_{it} + \gamma_{lm} l m_{it}, \\ \theta_{it}^L &= m_{it}\left(\gamma_l + 2\gamma_{ll} l_{it} + \gamma_{kl} k_{it} + \frac{\gamma_{lm}}{2} m_{it}\right) \quad + \kappa_l + 2\kappa_{ll} l_{it} + \kappa_{kl} k_{it}, \\ \theta_{it}^K &= m_{it}\left(\gamma_k + 2\gamma_{kk} k_{it} + \gamma_{kl} l_{it} + \frac{\gamma_{km}}{2} m_{it}\right) + \kappa_k + 2\kappa_{kk} k_{it} + \kappa_{kl} l_{it}. \end{aligned} \tag{11}$$

Firms' returns to scale are then computed as $\theta = \theta_{it}^M + \theta_{it}^L + \theta_{it}^K$ and averaged across all firms belonging to a specific 4-digit NACE industry. Estimation details are presented in greater detail in Appendix B.

The Imperfect Competition Case The existence of increasing returns has long been related to imperfect competition, and thus markups and positive profits. In estimating the production functions, markups and returns to scale are hard to identify separately when only revenue data is available. Consider a firm that has CRTS but also has some degree of market power: if that firm doubles its inputs, it will produce twice as much; but its revenue will not increase by the same rate because of the downward-sloping demand curve it faces for its products. Accordingly, market power might appear as DRTS.

An Example Assume price P equals some markup $mu \geq 1$ over marginal cost $C'(Q)$ in the production of output Q:

$$P = mu \times C'(Q) \tag{12}$$

$$\Rightarrow P\frac{Q}{C} = \frac{mu}{\theta} \tag{13}$$

with returns to scale given by $\theta = \frac{C}{C'(Q)Q} > 0$. Under increasing returns, we would expect that average costs exceed marginal costs, $C/Q > C'(Q)$.

Moreover, equation (13) makes clear that under perfect competition, increasing returns imply negative profits, $PQ - C < 0$. Under standard conditions, this combination can only pertain if the firm is subsidized. Accordingly, an environment of perfect competition (or at least 'many' firms) would instead suggest the presence of either constant or decreasing returns.

If there is imperfect competition, by contrast, the size of profits would depend on the relationship between θ and mu. To illustrate, in the estimation context, if positive profits are observed and we incorrectly assumed perfect competition, this would bias down the returns to scale value. Likewise, if zero or 'small' profits are observed and we over-estimated (or overvalued) the markup, this would bias upward the returns to scale value. Accordingly, it makes sense that returns to scale and the markup are estimated jointly.

Accounting for Unobserved Prices To this end, we follow Klette and Griliches (1996) and de Loecker (2011) and account for unobserved firm-level prices by introducing a constant elasticity of substitution (CES) demand system in the model:

$$\frac{P_{it}}{P_t} = \left(\frac{Q_{it}}{Q_t}\right)^{\frac{1}{\sigma_t}} e^{\chi_{it}}, \tag{14}$$

where P_t is the industry price index , Q_t is an index of industrial output which features in the framework as an aggregate demand shifter, χ_{it} is a (ex-post) firm-specific demand shock, and σ_t is the elasticity of demand. Firms are assumed to produce horizontally-differentiated products within a given industry and to be active in a monopolistically-competitive environment such that they charge a constant (expected) markup over marginal costs $1/(\sigma_t + 1)$. Following the case of perfect competition, we can derive the equivalent share equation to (5):

$$s_{it} = (\Gamma_t + \mu) + \ln D^{\mu}(k_{it}, l_{it}, m_{it}) + \ln \tilde{\mathcal{E}} - \tilde{\epsilon}_{it}, \tag{15}$$

where $\Gamma_t = \ln\left(\frac{1}{\sigma_t} + 1\right)$ denotes expected industry-level markups, $\tilde{\mathcal{E}} = \mathbb{E}[e^{\tilde{\epsilon}_{it}}]$, and μ is an additional constant. Hence, this equation now also includes an expression for markups and the constant μ. Note that $\phi_t = \Gamma_t + \mu$ above can be approximated by year fixed effects and that markups are $\left(\frac{1}{\sigma_t} + 1\right) = e_t^{\phi} e^{-\mu}$.

The final estimated production function in the imperfect competition case becomes:

$$\begin{aligned} &f(k_{it}, l_{it}, m_{it}) = \\ &e^{\mu} m_{it}(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \frac{\gamma_m}{2} m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \frac{\gamma_{mm}}{3} m_{it}^2 + \gamma_{kl} k l_{it} + \frac{\gamma_{km}}{2} k m_{it} + \frac{\gamma_{lm}}{2} l m_{it}) \\ &+ e_t^{\phi} e^{-\mu}(\kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} k l_{it}) \\ &- (e_t^{\phi} e^{-\mu} - 1) q_t + \tilde{\omega}_{it} + \chi_{it} + \tilde{\epsilon}_{it}. \end{aligned} \tag{16}$$

Analogously, the output elasticities are calculated as:

$$\theta_{it}^M = e^{\mu}(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} k l_{it} + \gamma_{km} k m_{it} + \gamma_{lm} l m_{it}^2)$$

$$\theta_{it}^L = e^{\mu} m_{it}(\gamma_l + 2\gamma_{ll} l_{it} + \gamma_{kl} k_{it} + \frac{\gamma_{lm}}{2} m_{it}) \quad + e_t^{\phi} e^{-\mu}(\kappa_l + 2\kappa_{ll} l_{it} + \kappa_{kl} k_{it})$$

$$\theta_{it}^K = e^{\mu} m_{it}(\gamma_k + 2\gamma_{kk} k_{it} + \gamma_{kl} l_{it} + \frac{\gamma_{km}}{2} m_{it}) + e_t^{\phi} e^{-\mu}(\kappa_k + 2\kappa_{kk} k_{it} + \kappa_{kl} l_{it}). \tag{17}$$

4 Data

We use the new *iBACH* database, which is a new, comprehensive firm-level dataset for selected European countries containing detailed financial and balance sheet information of non-financial corporations.[16]

Data Collection and Coverage The dataset has been collected from national statistical sources in a coordinated manner under the mandate of the European Committee of Central Balance Sheet Data Offices. Since 2018, this dataset has been disseminated by the Statistics Directorate of the European Central Bank which acts as a hub for the data collection, to its internal users and eurosystem statistical working groups who are in charge of disseminating and promoting the dataset in their countries. To the best of our knowledge, ours is the first time the dataset has been used for the estimation of firm-level production relationships.

The dataset contains yearly balance sheet, financial and demographic variables of non-financial corporations for five Euro Area countries, namely Belgium, Spain, Italy, France and Portugal (see Table 1). These countries were chosen on the basis of current full availability; other country datasets are likely to become available as the *iBach* project continues.

Currently, the database covers more than 300 four-digit manufacturing and service sectors (Rev. 2) for the period from 2000 to 2018 (for most countries).[17] The dataset covers either the full or at least large shares of the population of companies in participating European countries and is thus representative of the underlying firm population - both in terms of sectoral coverage as well as in terms of size structure (see Tables 2 and 3). Accounting and reporting standards are *fully harmonized* across legal entities and firm/unit definitions, and thus across countries and years. Data quality is checked extensively by applying a set of accounting validation rules that are iterated to convergence. Compared to competing datasets, such as Orbis, the quality, internal consistency, comparability and the timeliness is appreciably higher. Nonetheless, it is worth noting that there is significant variability in coverage across countries: for instance, while for Italy, Portugal and Belgium we have complete or near complete coverage of the population of firms, for Spain coverage is at 41% and for France it is 27%.

[16] For further information on *iBach*, see "The Bank for the Accounts of Companies Harmonized (BACH) database", Statistics Paper Series , No. 11 and further documentation at https://www.bach.banque-france.fr/?lang=en.

[17] Slovakia is also covered in *iBACH*, but due to its limited data dimension and availability of the required variables in the empirical analysis, it was not used in this study.

Table 1: *iBACH* data sources

Country	Source	Type of source	Type of information	Notes
Belgium	Nationale Bank van België	Administrative data	Accounting data collected at individual level	Almost full coverage of all corporate sizes and activity sectors due to access to mandatory Official Accounts of the companies
Spain	Banco de España / Mercantile Registries	Statistical survey & Administrative data	Accounting data collected at individual level	
France	Banque de France / Ministry of Finance	Administrative data & Tax returns	Fiscal data collected at individual level	Sample with an over-representation of the manufacturing sector. Criteria for sample selection: Sales >= EUR 750K AND commercial firms or state-owned firms subject to commercial law and firms with 12 months of activity and subject to corporation tax (iBACH)
Italy	Cerved/Banca d'Italia	Multiple sources	Accounting data collected at individual level	Bias towards total population of limited companies
Portugal	IES (Ministry of Justice, Ministry of Finance, Statistics Portugal and Banco de Portugal)	Administrative data	Accounting data collected at individual level	Full coverage of non-financial corporations from 2006 onward. A statistical survey was used for collecting data before 2006, biased toward large companies

Notes: This table is sourced from DG-Statistics at the ECB and through the eurosystem central banks.

Information Available in *iBACH* The variables available in *iBACH* are divided into four groups: demographics, assets, liabilities, and income statements, providing detailed information about firms' balance sheets and profit and loss statements. Moreover, the demographic variables provide information about the number of employees, industry affiliation, location, and legal form of the firms, and, furthermore, potentially enable (string) merges with other datasets based on company names (see Table A.1 in Appendix A).

As described in Section 3, we estimate a gross output production function. We use firms' turnover, which is reported in *iBACH* (net of sales and excise taxes) as a measure of gross output (see Table 4). Intermediate inputs are computed as the sum of material and services inputs, that are typically reported separately in the dataset.

Moreover, the capital stock is approximated based on the sum of the book values of tangible and intangible fixed assets. These variables are deflated with industry- and country-specific deflators for gross output, intermediate goods, and gross fixed capital formation, obtained from the OECD. Finally, we measure firms' labor input using the number of employees. In additional analysis presented later (see Section 6), we will make use of additional information present in *iBACH* to compute variables used to investigate relationships between industry characteristics and estimated RTS parameters.

Table 2: *iBACH* coverage

Country	Period	Coverage ratio 2017 (in %)			Notes
		Number of Corporations	Turnover	Employment	
Belgium	2000-2018	97.6		98.3	No break in time series
Spain	2008-2018	41.0		58.8	2010: Inclusion of unemployment corporations and exclusion of holdings and head offices classified by ESA 2010 as financial corporations; 2016: Inclusion of professional civil partnerships
France	2003-2018	27.3	81.9	78.8	2013: Reduction of the number of firms legally obliged to report (drop of the alternative size criteria Bank debt > 380 thousand Euro)
Italy	2003-2018	100.0	100.0	100.0	No break in time series
Portugal	2003-2018	100.0	100.0	100.0	2006: Full coverage of NFC population; sample until 2005 was biased for large firms; 2010: Change in the Portuguese GAAP; some variables may present time series breaks, particularly financial liabilities structure and cost

Notes: The coverage rate is determined by comparing the sample of corporations recorded in the database with the population of BIC-BRN corporations. Only non-financial corporation's (excluding sole proprietors) are covered by each country. The coverage rates are based on the aggregate Zc - Total NACE (without K642 and M701).

Table 3: *iBACH* coverage of size classes

Size class	Definition	Belgium		Spain		France		Italy		Portugal	
		Sample	Popl.	Sample	Popl.	Sample	Popl.	Sample	Popl.	Sample	Popl.
All sizes		100%	100%	100%	NA	100%	100%	100%	100%	100%	100%
Small	Turnover < 10 mil.€	37%	37%	45%	NA	30%	37%	23%	23%	37%	37%
Medium	10 mil.€< turn.< 50 mil.€	22%	22%	12%	NA	20%	18%	19%	19%	20%	20%
SME	Turnover < 50 mil.€	58%	58%	57%	NA	50%	55%	42%	42%	57%	57%
Large	Turnover > 50 mil.€	42%	42%	43%	NA	50%	45%	58%	58%	43%	43%

Notes: This table reports the size structure of firms in each country. NA denotes Not Available.

Sample We focus on observations from the period 2008 to 2018, that are available for all countries under consideration. Moreover, we use the following broad sectors and their subsectors: *Manufacturing* (NACE **C**), *Construction* (**F**); *Wholesale & Retail Trade* **G**); *Transportation & Storage* (**H**); *Accommodation & Food Service Activities* (**I**); *Information and Communication* (**J**), *Professional, Scientific and Technical Activities* (**M**) as well as *Administrative and Support Service Activities* (**N**). *Agriculture, Financial Sector, Real Estate, Utilities*, and publicly-dominated

TABLE 4: List of key variables

Variable	Description	*iBACH* name	Deflator
go	Gross output, measured as turnover net of VAT and excise taxes	$i0100$	*dfl_PRDP*
im	Materials inputs, measured as cost of materials and consumables	$i0500$	
is	Services inputs, measured as external supplies and services	$i0600$	
i	Intermediate inputs, computed as $im + is$	$i0500 + i0600$	*dfl_INTP*
ki	Intangible fixed assets	$a1100$	
kt	Tangible fixed assets	$a1200$	
k	Fixed assets, computed as $ki + kt$	$a1100 + a1200$	*dfl_GFCP*
l	Number of employees	*DNUMBEREMPL*	

Notes: dfl_PRDP is a gross output deflator, *dfl_INTP* a intermediate inputs deflator, *dfl_VALP* a value added deflator, and *dfl_GFCP* an investment deflator. Deflators vary across sectors roughly equivalent to 2-digit NACE sectors) and countries and are obtained from OECD.

sectors such as *Education* and *Health Services* are excluded in part due to data availability (e.g. as in financial services), and in part because estimating production functions for some sectors (e.g. education and health care) appears particularly challenging.

Cleaning the data Given that small companies typically exhibit very different production technologies compared to larger-sized firms, we exclude micro firms with less than 10 employees. In addition, since the French data only contain firms with an annual turnover of more than €750,000, to increase comparability across countries, we apply this condition to firms in all countries. In addition, firms of certain legal forms, such as sole proprietorships, public authorities, or nonprofit organizations, are not included to ensure common legal frameworks across countries.

Finally, we clean the dataset from obvious outliers. In particular, observations with values of certain ratio variables that deviate from the country-industry median by more than three standard deviations are discarded. These ratios are computed as firms' employees, capital stock, labor costs, and intermediate input costs scaled by firms' turnover. Furthermore, observations which are missing values in key variables (such as go, k, l or i) are naturally omitted. In the last cleaning effort, we only kept those four-digit sectors that exhibit at least 200 observations (across all countries). Having done so, this leaves us with a sample of around 400,000 firms per year (see Table 5).

5 Estimation

Our empirical analysis is based on two estimators: the GNR estimator which we use as our baseline and which assumes perfect competition in output markets, and one that accounts for imperfect competition. Arguably, this last estimator is the more useful and realistic of

Table 5: Size of the estimation sample by country

Year	Sample	Annual number of observations					
		Belgium	Spain	France	Italy	Portugal	Total
	All firms	217,871	788,895	197,662	521,998	327,296	2,053,722
2008	No missing	24,993	449,092	184,225	373,773	211,502	1,243,585
	No missing & Empl. > 9	10,142	110,885	115,403	122,236	42,373	401,039
	All firms	326,100	691,574	222,984	591,826	369,862	2,202,346
2018	No missing	16,368	359,493	207,133	433,642	201,711	1,218,347
	No missing & Empl. > 9	9,604	81,670	122,579	137,107	40,514	391,474

Notes: The 'no missing'-sample consists of all firms which have available values for the following key variables: go, l, k, and i. The 'non-missing & Empl. > 9'-sample represents our estimation sample.

the two.

In either case, we consider industry-level RTS parameter estimates to be reliable only if they fulfill certain criteria; namely, that their production function coefficients θ_h with $h = (l, k, m)$, individually lie in the unit interval, and the implied returns to scale parameter, $\theta = \theta_l + \theta_k + \theta_m$, lies within a $\frac{1}{3} - 3$ range.[18] For the baseline GNR estimator, these criteria are mostly met.

Baseline Results Based on these firm-level RTS estimates, we compute for each of the 373 4-digit industries average RTS values and report these point estimates in Figures 1 - 2 along with their NACE identifier, from sub-sectors in Manufacturing **(C)** to Administrative and Support Service Activities **(N)**.

Applying a *bootstrap* procedure, we denote sectoral estimates as not significantly different from unity (CRTS) at the 5% level of significance as the symbol ○, and estimates significantly below or above unity (DRTS or IRTS), as the ■ or ▲, respectively. We do the same for the Imperfect Competition estimator in Figures 6-7.

Notwithstanding a spread of $0.74 - 1.18$,[19] average sectoral RTS estimates are close to 1 (with a mean and a median equal to 0.98, see Figure 3, Panel A). This is also the case when computing gross-output weighted average RTS values for each 4-digit industry. In the weighted case, the average RTS point estimate is numerically slightly above unity (1.03) with a wider support, suggesting a long tail of large firms with IRTS of up to 2, alongside a somewhat shorter tail of low values below 0.5. For more detail, Table C.1 and Figure C.1 tabulate and show histograms for the baseline case and the imperfect competition case by Total and 1-digit sectors for the weighted and unweighted cases.

[18] Recalling Section 3.1, although IRTS would be counter-intuitive under perfect competition, we chose not to restrict the range. This is akin to assessing the robustness and plausibility of the estimator.

[19] Specifically, 0.74: *(7830(N) Other human resources provision activities*) and 1.18: *(7733(N) Rental & leasing of office machinery)*.

In line with this, Panel B, shows that the majority of industries are characterized by constant returns (58 percent).[20] That said, our findings also point to a significant sectoral heterogeneity and a non-trivial share of industries which do not operate under constant returns: 32 percent of 4-digit industries exhibit statistically significant decreasing returns to scale and 10 percent by increasing returns.

Manufacturing industries have, in general, higher RTS than service industries. For example, 17 percent of 4-digit manufacturing sub-industries exhibit IRTS, compared to only 4 percent for service sub-industries. Within services, industries like *Transportation & Storage* (with a mean RTS value of 1), or *IT* (mean, 0.98), for instance, have on average relatively high RTS, while *Accommodation & Food* (mean, 0.92) or *Professional Services* (mean, 0.94) and *Administrative Services* (mean, 0.91) exhibit lower RTS.

The results for decreasing returns are quite plausible. When production is characterized by DRTS, it is generally difficult for a large firm to compete with smaller firms and so there are likely to be many firms in the market. Our results show that some sectors are dominated by decreasing returns such as the aforementioned *Accommodation & Food* (I), *Professional Services* (M), and *Administrative Services* (N). Examples of particularly low values can be found in 5621*I* (event catering), 5911*J* (film and video production) 7022*M* (consulting) and 7830*N* (employment placing agencies). Such sub-sectors indeed tend to be dominated by small and locally-servicing firms (Eurostat, 2008). Panel C shows that even within those 1-digit sectors, RTS point estimates vary widely.

Finally, Figure 4 presents RTS at the 2-digit level (i.e. for 38 sectors). Variability of the estimates and their associated ranges are more vividly displayed compared to the more aggregated 1-digit level. It is interesting to note that in several of the 38 sectors, there are industries with RTS above one. Examples include *Manufacture of machinery & equipment* (C28), *Warehousing & Support Activities for Transportation* (H52), *Publishing Activities* (J58), *Advertising & Market Research* (M73). It has been argued that these sub-sectors are associated with technological advancements, especially in the broadband space, revolutionizing their business (see, e.g. Arthur (1994)).

[20] The sectors exhibiting DRTS include: *Construction*, *Accommodation & Food*, Professional *Services*, and *Administrative Services*.

Figure 1: RTS by 4-digit industry (Part I), Baseline case

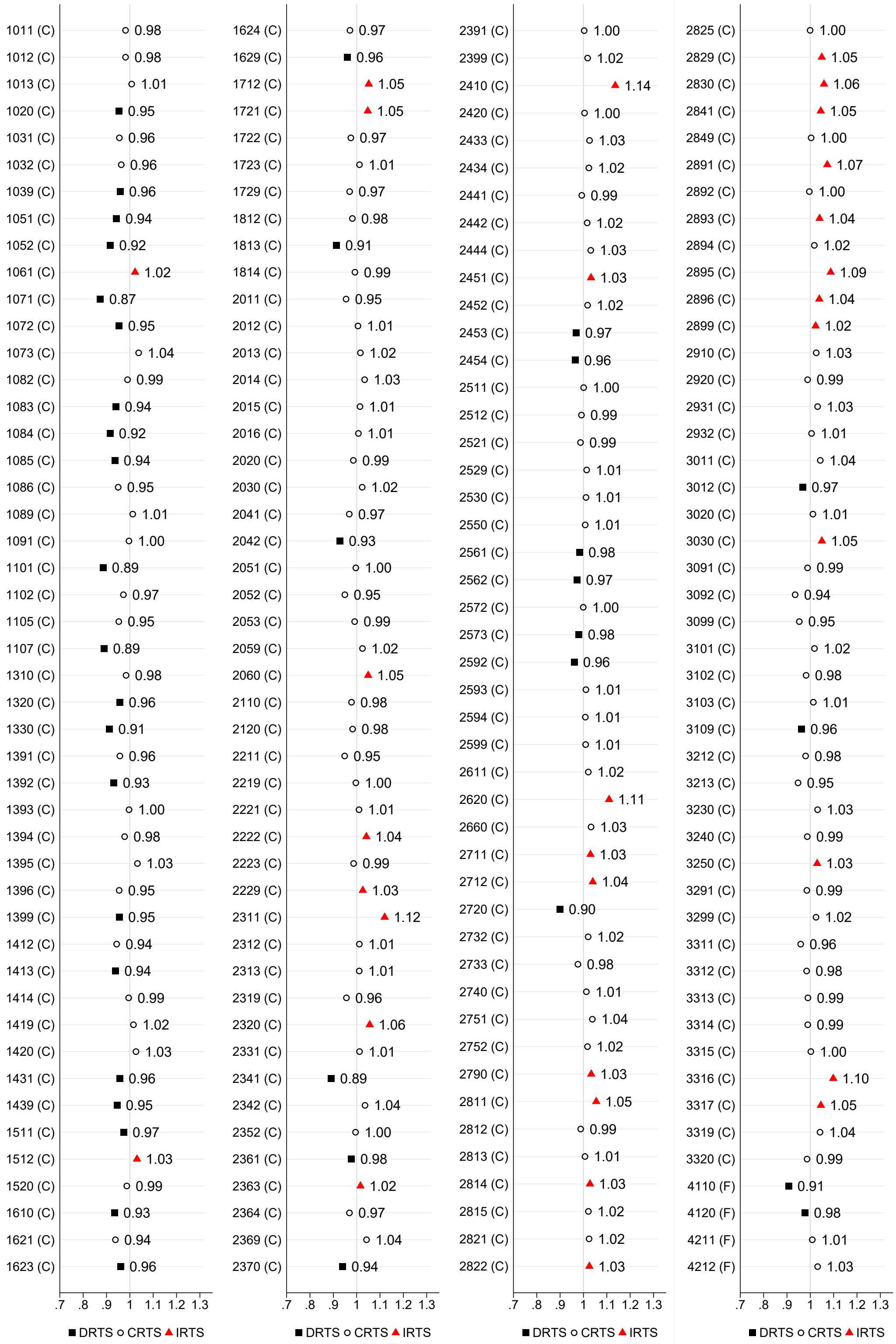
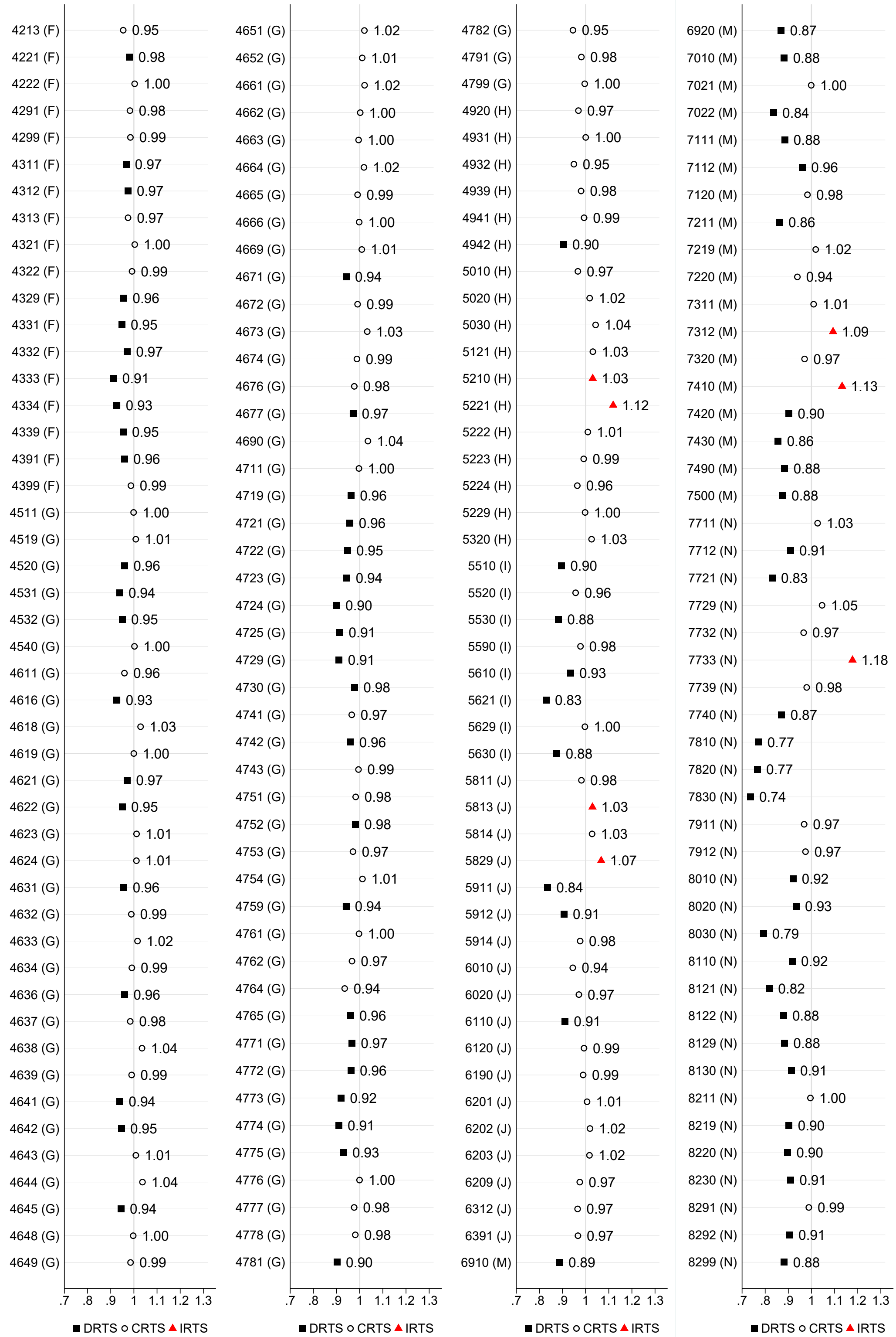


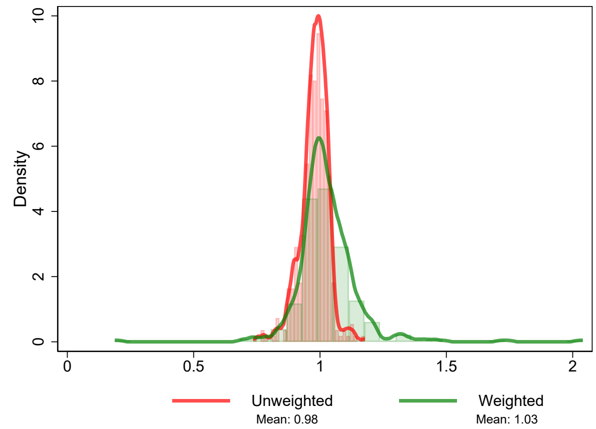
FIGURE 2: RTS by 4-digit industry (Part II), Baseline case



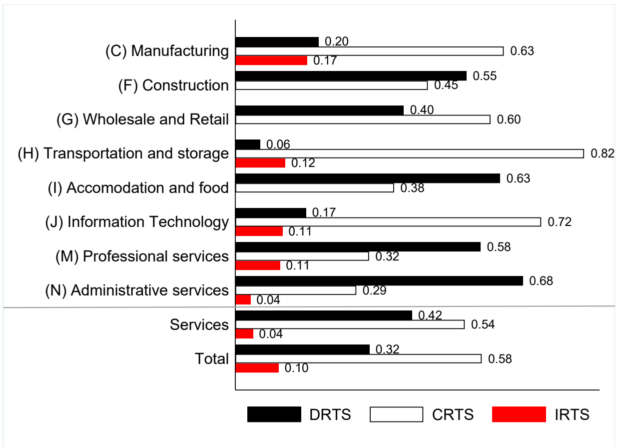
Notes: This figure shows the RTS estimate using the baseline estimator (assuming perfect competition in output markets) for each 4-digit industry. Industries that exhibit RTS significantly below one (at 5% significance level) are depicted as black squares, those that exhibit RTS significantly above one are depicted as red triangles, whereas those whose RTS differ not significantly from one are depicted as hollow circles.

Figure 3: RTS Summary Panel: Baseline Estimator

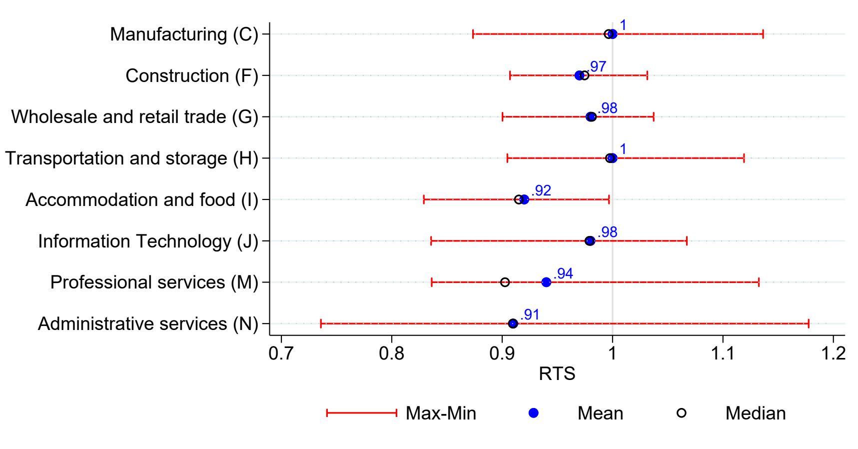
(a) Total RTS: Unweighted & Weighted



(b) RTS Shares Within 1-Digit Sectors



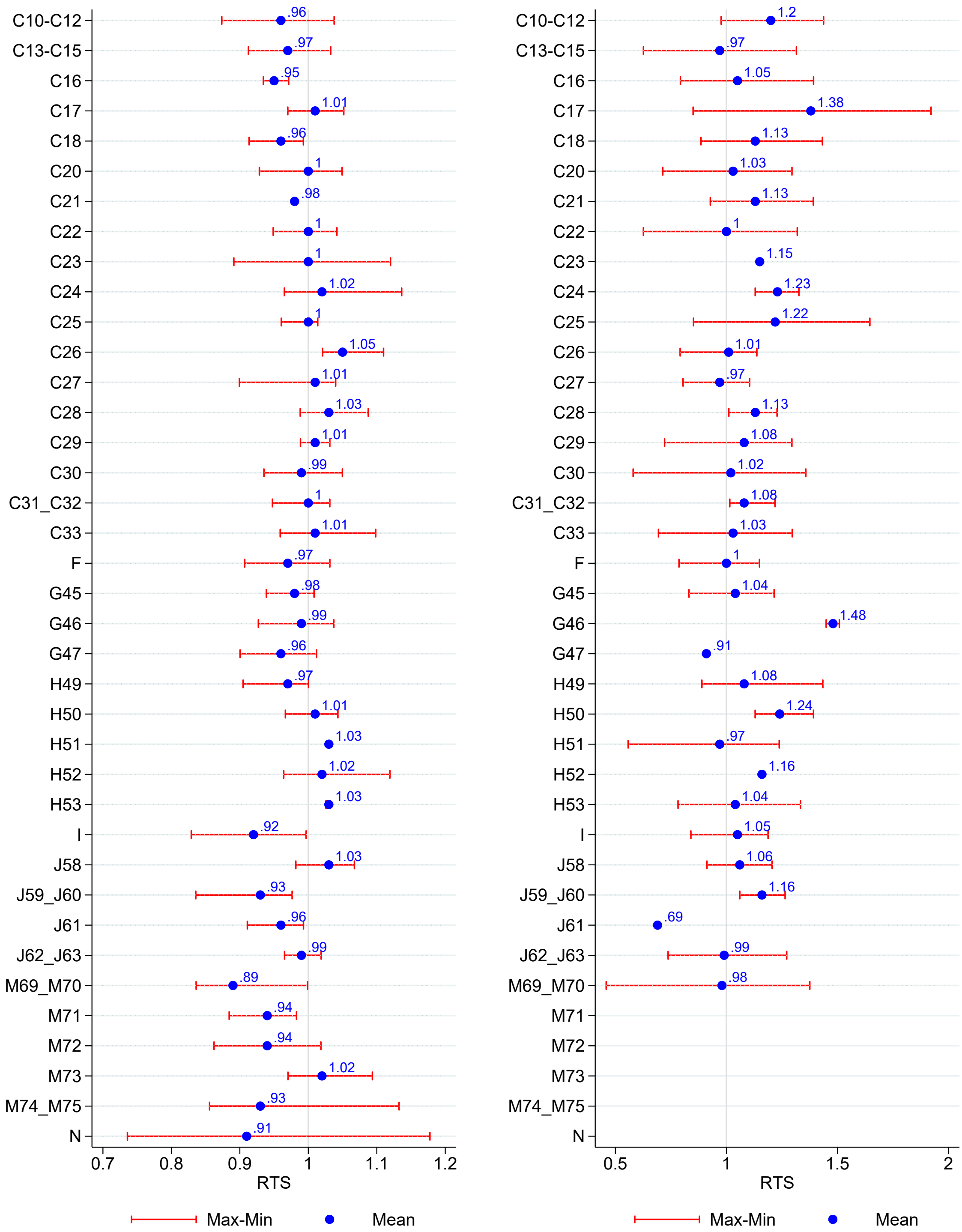
(c) Distribution of 4-digit industries
with 1-digit Aggregation



Notes: Panel (A) shows the histogram and kernel density estimates of the total-economy RTS in unweighted and weighted form (where the weight is by firm gross output). Panel (B) shows the share of 4-digit industries that exhibit RTS that are at 5% significance levels below (DRTS), above (IRTS) or not significantly different from unity (CRTS), falling within each 1-digit sector. Standard errors/p-values for the industry-specific RTS estimates are based on a bootstrap. The services aggregate comprises sectors G to N. Panel (C) illustrates the RTS distribution of 4-digit industries within 1-digit sectors.

Figure 4: Distribution of 4 digit industries within 2-digit sectors

(a) Baseline (b) Imperfect Competition



Notes: This figure illustrates the distribution of RTS estimates of 4-digit industries within 2-digit Sectors. Unweighted means (circles) and max-min ranges are shown. We suppress the medians for legibility.

One sub-sector that stands out is *Manufacture of Computer, Electronic & Optical Products* (division 26) which not only exhibits a mean estimate for RTS of 1.05 but also has the entire range of its respective industries operating under IRTS. While it is not the scope of this paper to examine these specific sectors and industries more granularly, it is our hope that our data can provide enough detail for such a future investigation and shed light on the characteristic of particular industries and firms that exhibit IRTS. We do so at a high level aggregation below in Section 6.

Although only a subset of results is characterized by increasing returns, that is nonetheless puzzling since, as noted, increasing returns under perfect competition imply the firm requires subsidies to survive. This does not seem a plausible or attractive result. An alternative explanation may be that the increasing returns detected are somehow *external* to the firm (i.e., result from industry-wide or agglomeration phenomena such as a common pool of skilled labor, technology set etc).[21] Other more general explanations include misspecification: measurement error in the data, or that the restrictive model assumptions (perfect competition; materials being the flexible factor) are biasing the estimator. Indeed, given that market power is a commonplace assumption in macroeconomic models as well as theory, it makes sense to supplement our estimators with that in mind. This is our next step, after briefly examining changes over time.

Another caveat of our analysis is the fact that we cannot isolate fixed cost in our data. However, the existence of fixed costs could affect our estimation results. For instance, if a firm produces with a constant returns technology but faces fixed costs at the same time, it would face increasing returns to scale (from a cost perspective). Thus our results have to be interpreted as describing the RTS properties of firms' production technology.

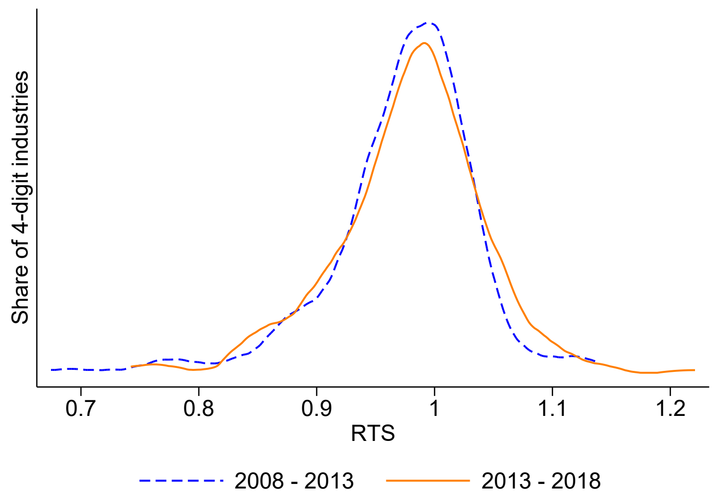
Changes Over Time Finally, Panels A and B in Figure 5 assess whether the RTS values have changed over time. There is much emerging scientific and anecdotal evidence pointing to a shift of power from small to medium size firms to large and "superstar" firms. In his book *Profit Paradox*, Eeckhout (2021) notes that over the past forty years, a handful of companies have profited due to technological advancements and liberal market reforms. Along increasing market concentration, RTS could have been rising in particular firms and sectors.

To this end, we split our sample into two sub-periods, 2008-2013 and 2013-2018, and estimate accordingly. Overall, it seems that the parameters have remained quite stable over the sample period. There is, though, some tendency for RTS estimates to drift rightward over time, thinning their lower tail. This is corroborated by the sectoral picture. While manufacturing (which has the highest RTS estimate and IRTS share) has stood still, *Construction* (F), *Transportation & Storage* (H), *Accommodation & Food* (I), *Professional* (M) and *Administrative Services* (N) values have increased slightly on average.

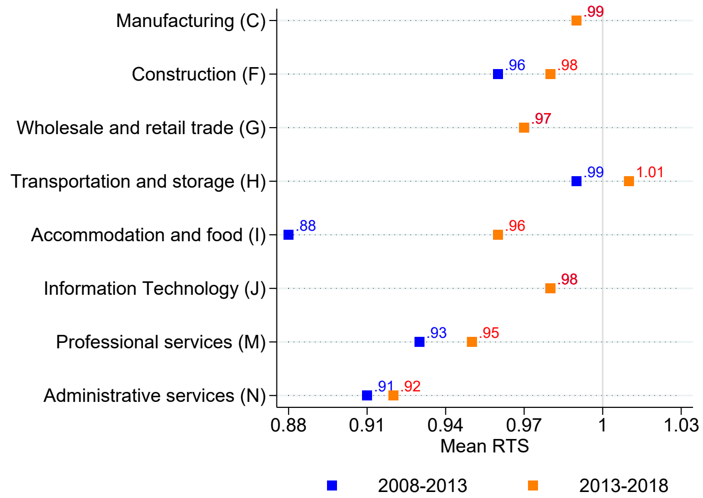
[21] These mechanisms are discussed in Caballero and Lyons (1992) and Basu and Fernald (1995). Indeed, we found IRTS cases for the baseline estimator concentrated in the manufacturing sub-sectors, which is the sector discussed by Caballero and Lyons (1992).

Figure 5: RTS Summary Panel: Baseline Case Over Split Time Periods

(A) RTS Changes Over Time



(B) RTS Changes Over Time
(Within 1-Digit Industries)



Notes: Panel (A) illustrates the total RTS distribution across all 373 4-digit industries for two periods (2008-2013 and 2013-2018). Panel (B) shows, for those samples, mean RTS of 4-digit industries falling within each 1-digit sector.

Accounting For Unobserved Prices As mentioned in Section 3, by imposing some additional assumptions on the empirical baseline model, it is possible to obtain parameter estimates consistent with an environment characterized by imperfect competition in the context of the GNR estimator (we dub this the GNR-IC estimator).[22]

This modified estimator, though, is occasionally prone to extreme values which we try to correct for by imposing an additional condition: namely that any RTS parameter, which is estimated to not differ from 1 according to our bootstrap procedure, should not exceed (fall short of) the median RTS parameter of the industries in the group with RTS parameters that are significantly above 1 (below 1). These additional conditions change the number of 4-digit industries for which we can estimate the RTS parameters from 373, for the baseline estimator, to 219.[23]

Figure 8 presents the equivalent charts to those in Figure 3, but now adjusted to account for imperfect competition. Panel A shows that, based on this estimator, the range of RTS parameters increases, with significantly more industries featuring returns that lie outside the $0.9 - 1.1$ interval. This is an outcome one might expect. Recall relationship (13): assuming $P/C'(Q) > 1$ and $\theta < 1$, would imply positive profits. Thus, conditional on an observed positive profit, if we incorrectly assumed near perfect competition $P/C'(Q) \gtrsim 1$ (i.e, a markup barely above one) compared to a true case of marked imperfect competition, $P/C'(Q) \gg 1$, there would be downward bias in the θ value.

Moreover, the mean RTS increases by 10% from 0.98, to 1.08.[24] Consistently, Figure 8, Panel B reveals that in the case of the IC estimator, we find fewer industries that exhibit DRTS but more industries with IRTS than in the baseline specification. Overall, industries are now uniformly dominated by CRTS, with a more substantial share of statistically significant IRTS shares. Nonetheless, we find 15 percent of industries exhibit statistically significant IRTS, and 3 percent exhibit DRTS.

[22] See also Flynn, Traina and Gandhi (2019).

[23] For completeness the first four moments of the four total-economy series (i.e., the baseline and imperfect competition estimators and both variants weighted by gross output, *GO*), the inter-quartile range, and probability values for the Shapiro-Wilks normality test, are as follows:

Estimator	Obs	Mean	Median	Std. Dev	Skew	Kurtosis	IQR	$\sim \mathcal{N}$
GNR	373	0.979	0.984	0.056	−0.669	5.378	0.059	[0.000]
GNR(GO)	372	1.027	1.014	0.124	1.446	22.511	0.105	[0.000]
GNR-IC	219	1.075	1.065	0.214	0.267	3.891	0.256	[0.194]
GNR-IC(GO)	219	1.035	1.037	0.284	0.177	4.731	0.333	[0.003]

[24] Consistent with previous results, we observe that when output-weighted, the RTS distribution widens in a similar fashion to that of 2B (weighted) although the comparison with the GNR-IC *un*weighted case is far less dramatic than with the GNR.

Figure 6: RTS by 4-digit industry (Part I), Imperfect Competition case

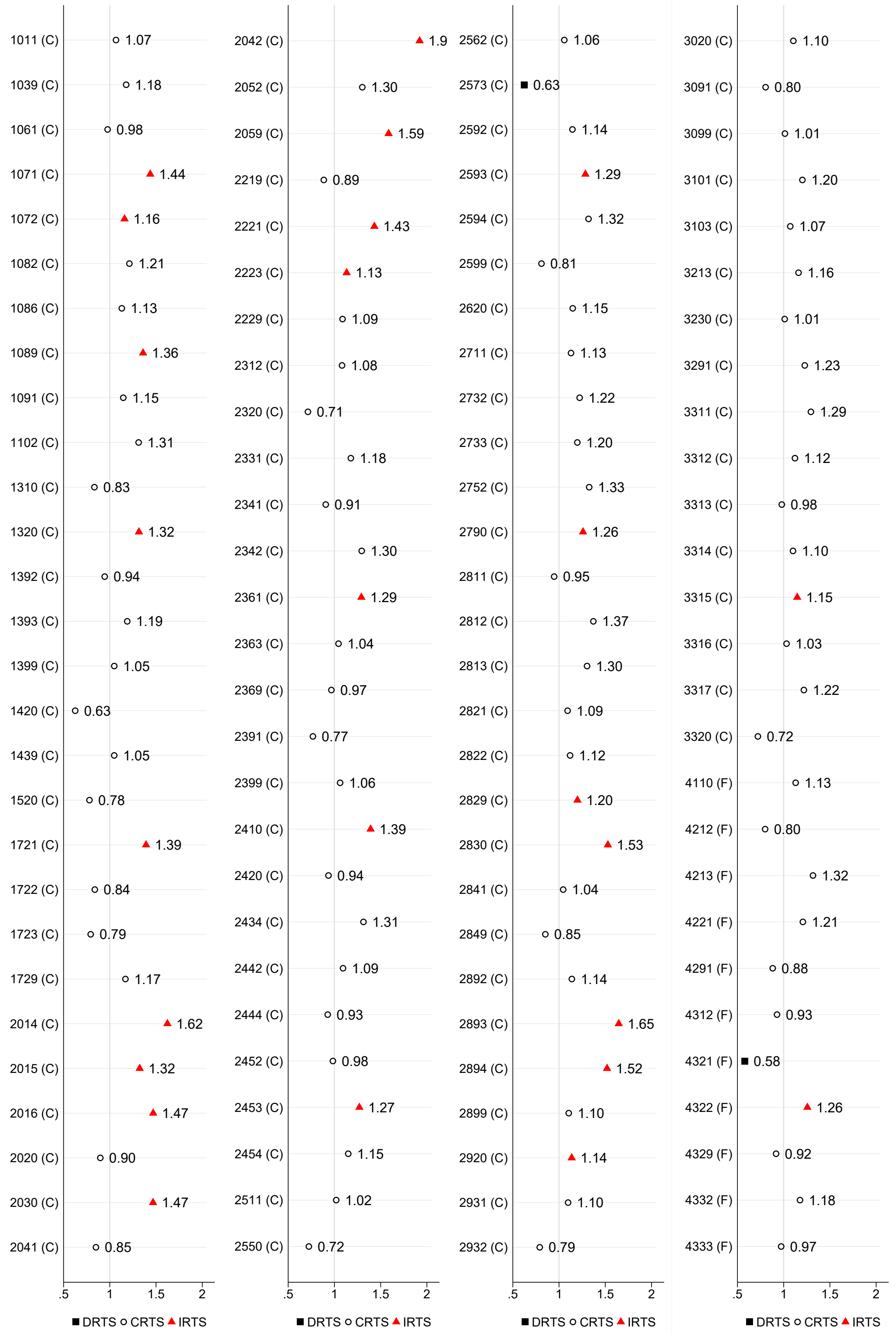
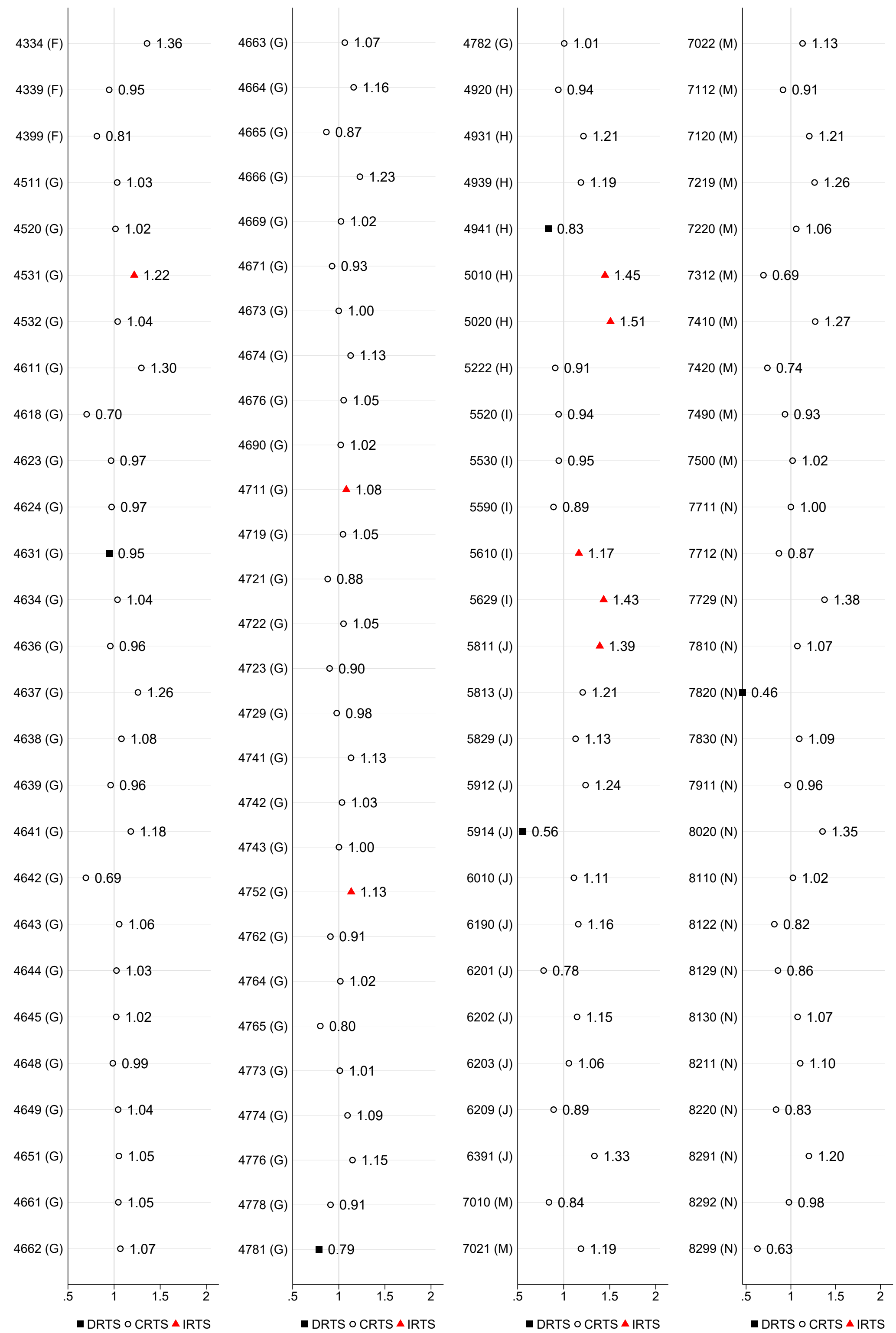


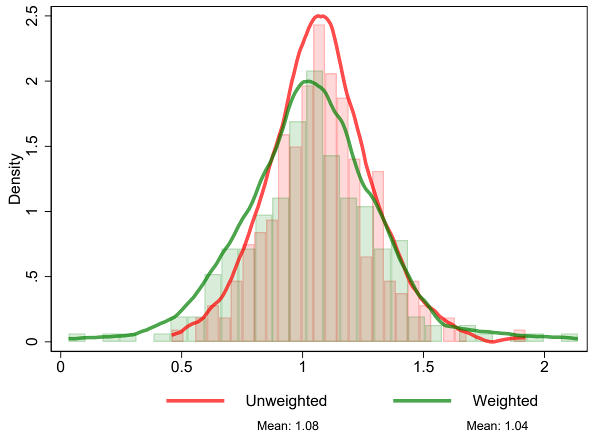
Figure 7: RTS by 4-digit industry (Part II), Imperfect Competition case



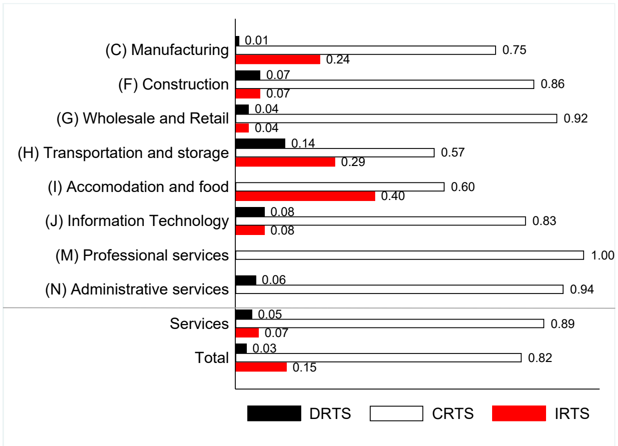
Notes: This figure shows the RTS estimates for each 4-digit industry. Industries that exhibit RTS significantly below one (at 5% significance level) are depicted as black squares, those that exhibit RTS significantly above one are depicted as red triangles, whereas those whose RTS differs not significantly from one are depicted as hollow circles.

Figure 8: RTS Summary Panel: Imperfect Competition Case

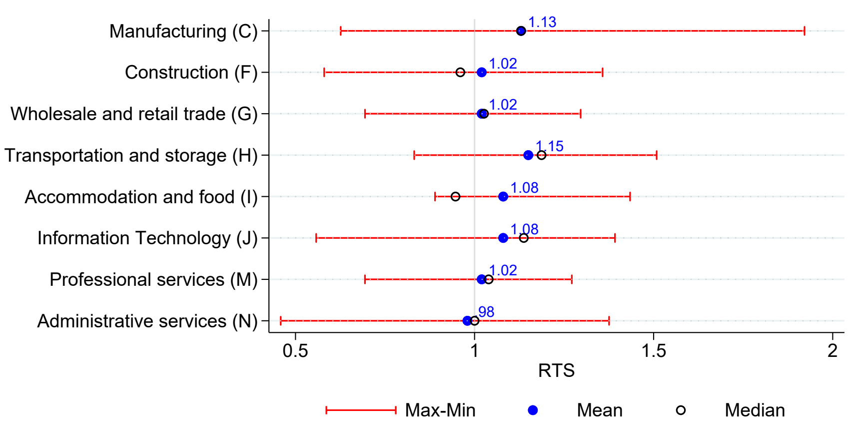
(a) Total RTS: Unweighted & Weighted



(b) RTS By 1-Digit Sectors (At 5%-Significance)



(c) Distribution of 4-digit Industries at 1-digit Aggregation



Notes: Panel (A) shows the histogram and kernel density estimates of the total-economy RTS in unweighted and weighted form (where the weight is by firm gross output). Panel (B) shows the share of 4-digit industries that exhibit RTS that are at 5% significance levels below (DRTS), above one (IRTS) or not significantly different from one (CRTS), falling within each 1-digit sector. Standard errors/p-values for the industry-specific RTS estimates are based on a bootstrap. The services aggregate comprises sectors G to N.

5.1 Rank Correlation Between the Different RTS Estimators

A natural question to ask is how similar are the distributions of the two RTS estimators (GNR and GNR-IC). This applies not just to the first and second moments, but also whether their rank order is preserved. If that ordering is similar then we can essentially be as satisfied with the ranking from one series as with the other, implying that, in this particular dimension, controlling for imperfect competition has limited value added.

Indeed, at first glance, considering the estimators, (comparing, say, the 1- and 2-digit results) suggests some commonalities. For example across both measures, *Manufacturing* and *Information Technology* are dominated by constant returns with a similar share (0.63 vs 0.75, and 0.72 vs. 0.83, respectively). Moreover, *Manufacturing* and *Transport & Storage* remain the two sectors with the highest average RTS values, while *Administrative Services* in both specifications exhibit the lowest RTS values. However, when we come to the 2-digit results, there are also some stark differences: for example both industries C17 and G46 predict marked IRTS under GNR-IC, whilst the baseline estimator selects constant returns. We examine this more formally below, at the 4-digit level.

Several approaches exist to measure the *distance* between two ranked lists. Standard tests (see Table 6) reveal a low but statistically significant positive correlation between the two series. But these measures merely provide a single summary measure and do not distinguish between agreement in the top versus towards the bottom of the RTS values.

Accordingly, in the adjacent figure (Figure 9) we plot the new sequential rank agreement measure (*sra*) of Ekstrøm, Gerds and Jensen (2018). This provides a more meaningful measure of correlation across the range of rank orderings of two series. The lower the sra value, the greater the rank agreement. Here, we observe that at the very top ranks of each measure, there is little agreement in the ranking. That suggests that industries under one estimator that are ranked with high RTS values, do not rank as high when controlling for imperfect competition. However, as we go through lower values this correlation across ranks improves.

Test	Value
Pearson	0.1971 [0.0030]
Spearman's ρ	0.1864 [0.0060]
Kendall's τ_b	0.1230 [0.0070]

TABLE 6: Rank Correlation Test

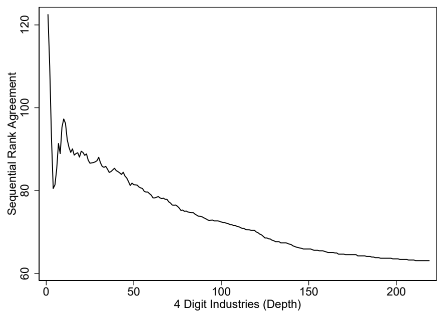


FIGURE 9: Sequential Rank Agreement Measure

Note, that this comparative analysis comes with caveats. For example, to make the comparison means we have to restrict ourselves for the 227 industries for which there are RTS estimates. This truncation involves some loss of power for the comparison. Nonetheless, if we consider discarding issues of competition in the estimator, we risk making specification errors in both the mean RTS values found and their qualitative ranking across the industries.

6 Increasing Returns and Firm Characteristics

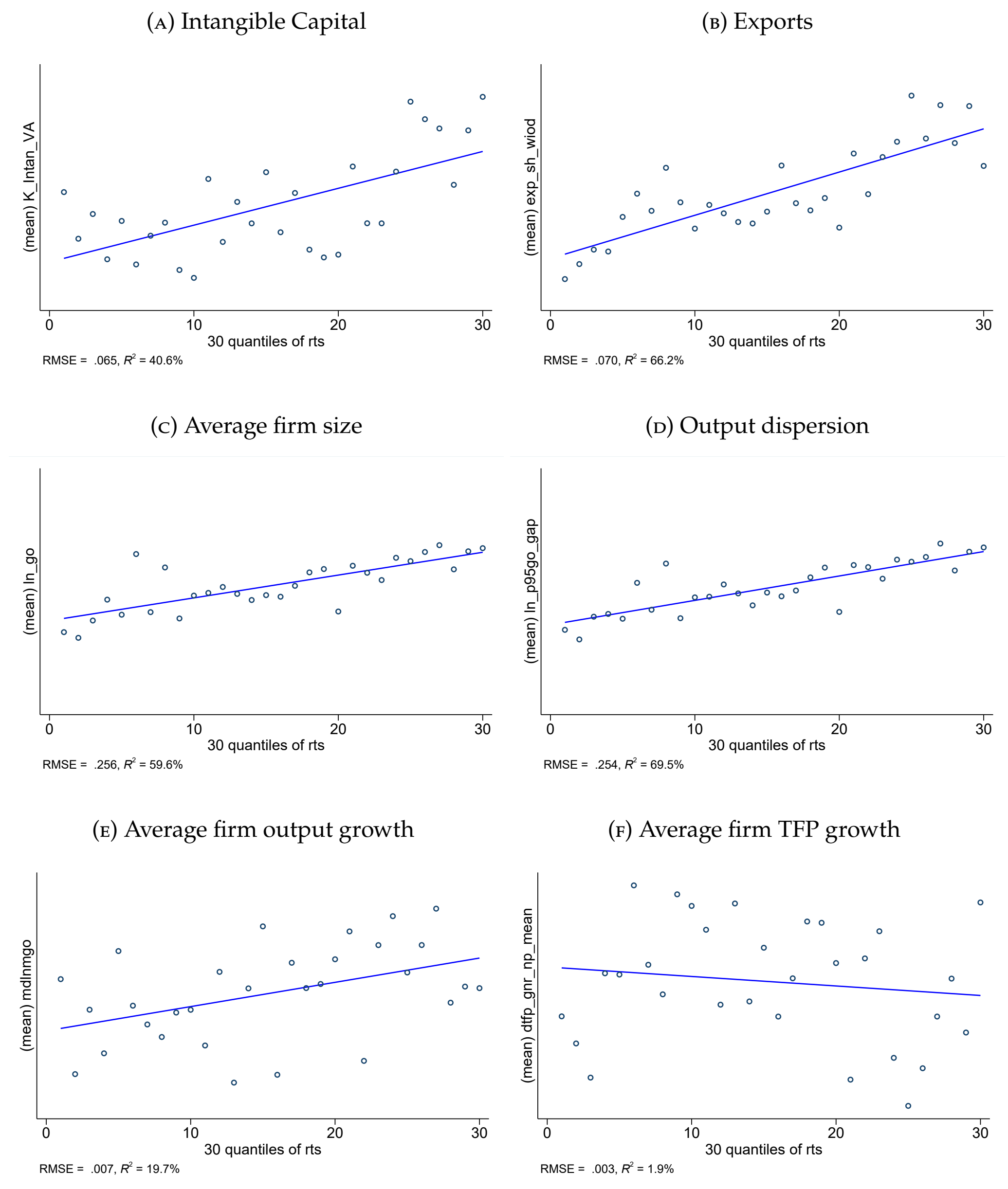
Next, we make a first pass at analyzing the relationship between industry characteristics and our RTS estimates. While there is no pretense of inferring watertight causal links, it is informative nonetheless to see whether broad commonalities can be found on relationships that resonate with the relevant literature. To this end, we plot our RTS parameters (in 30 quantiles) against potential determinants and variables of interest at the industry level, like firm size, export intensity, TFP dynamics etc. We do this for the baseline and imperfect competition estimator (respectively, Figure 10 and 11). In several cases, a fairly tight linear relationship can be detected. Generally speaking, the relationships are stronger for the baseline estimator (as judged by the diagnostics under the charts).[25]

We begin by investigating the relationship with indicators of **intangible capital**. Intangible capital is often associated with a high degree of scalability due to its nature of high fixed and low marginal costs (e.g. Crouzet et al., 2022). As a result, we might expect to find a positive relationship between a sector's use of intangible capital and its RTS. We use *EU-KLEMS* data to compute the ratio of intangible capital to value added roughly at the 2-digit industry-level. The intangible capital stock measure is computed as the sum of software and databases, R&D, and other intellectual property products. As Panel A of both figures shows, we indeed find a strong positive relationship, suggesting that industries with higher RTS make more intense use of intangible capital to produce their output.

Similarly, access to foreign markets may allow firms to reap the benefits of increasing returns to scale. Indeed, international trade models in the spirit of Krugman (1979) and Melitz (2003) typically feature firms with increasing returns. In order to assess the relationship between our RTS estimates and industries' **exports**, we compute the share of exports in gross output roughly at the 2-digit industry-level using data from the World Input Output Database (WIOD). We again find the expected positive relationship, as displayed in Figure 10 Panel (B). Hence, RTS are higher in more export-oriented sectors.

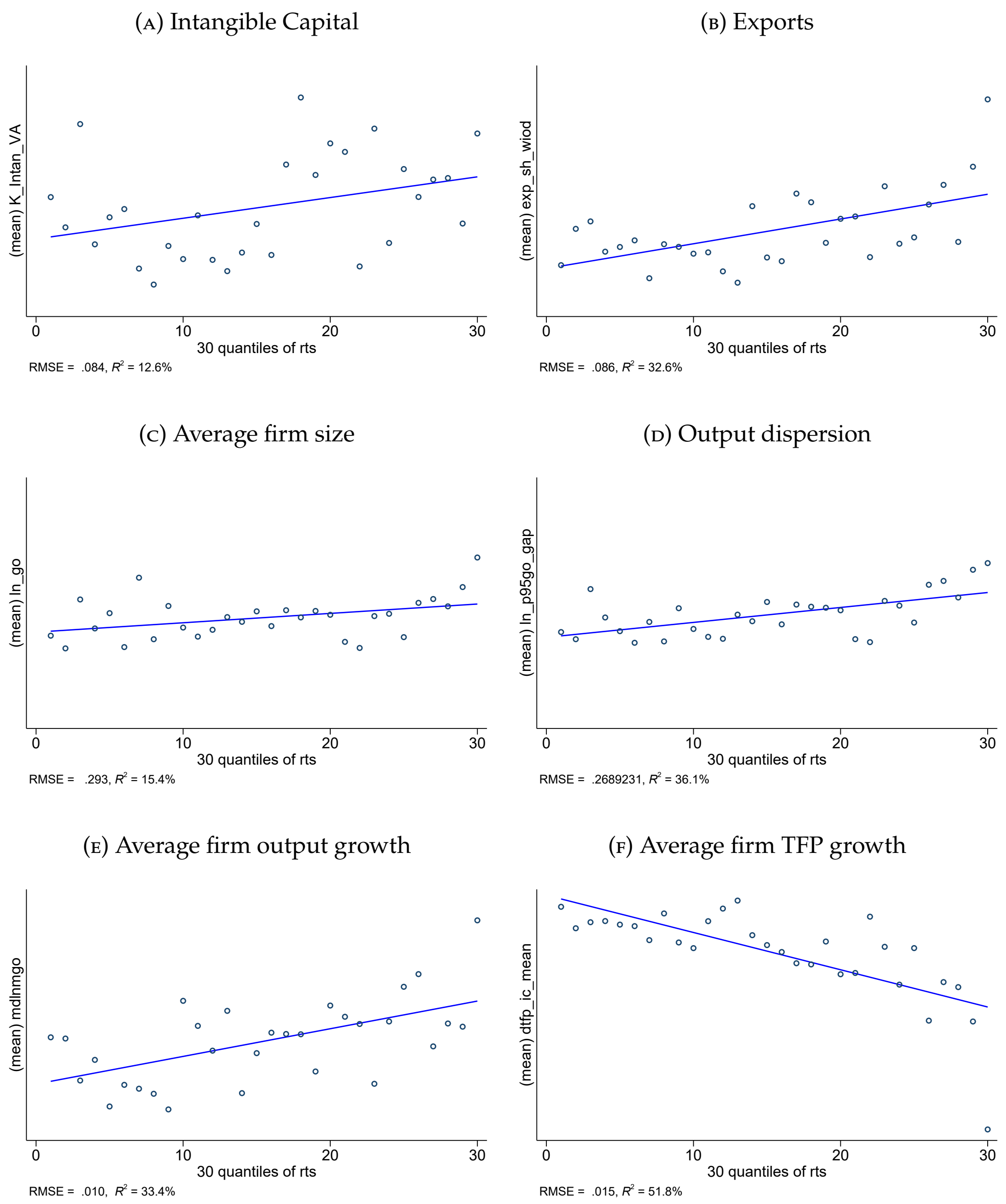
[25] Note, the regression parameter estimates are suppressed for compactness but are available on request.

Figure 10: Returns to scale and Industry Characteristics (Baseline Case)



Notes: Plots of estimated RTS parameters against industry characteristics. RTS estimates are grouped into 30 bins of equal size. Intangible capital in Chart (A) measured as the sum of software and databases, R&D, and other intellectual property products over value added, using EU-KLEMS data. Exports in Chart (B) are measured as exports over gross output, using the WIOD. Average firm size in Chart (C) refers to mean firm sales by industry according to *iBACH*. Output dispersion in Chart (D) refers to the gap in sales between the firm at the 95^{th} and the firm at the 5^{th} percentile of the industry sales distribution according to *iBACH*. Average firm output growth in Chart (E) refers to mean firm sales growth according to *iBACH*. Average firm TFP growth is in Chart (F).

Figure 11: Returns to scale and Industry Characteristics (Imperfect Competition Case)



Notes: See notes to Figure 10.

One might also expect that RTS are positively associated with **average firm size**. Generally, a firm's average-total-cost curve will continually decline when production exhibits IRTS. This will tend to reduce competition (though by no means strictly, e.g., Koutsoyiannis, 2017) and increase market power. Panels (C) show that industries' **average firm sales**, as a measure of average firm size, are indeed positively related to our RTS estimates. Moreover, we find a positive relationship between the RTS parameters and the **output dispersion** between the largest and smallest firms in an industry in Panel (D).[26] This positive correlation between the industries' sales dispersion and the RTS parameters implies that mostly the largest firms in an industry drive the positive relationship between average firm size and RTS parameters. This also suggests that higher RTS parameters are found in more concentrated markets. This may raise concerns that industries featuring IRTS are actually less dynamic sectors; for instance, if returns to scale enable incumbent firms to deter competitors. If this was the case, one would expect these sectors to display rather sluggish growth. However, Figure 10 Panel (E) suggests the contrary, since it shows a positive relationship between an industry's **average firm output growth** and the RTS parameters. Besides, the relationship with **average firm TFP growth** tends to be weak, as shown in Panel (F).

The findings are qualitatively similar when accounting for imperfect competition (see Figure 11). The notable difference is in the relationship between average firm TFP growth and RTS between baseline and IC estimates – with IC estimates showing a clear negative relationship (Panel F).

7 Conclusions

This paper takes a fresh look at estimates of returns to scale (RTS): a deep production parameter of fundamental economic importance. Its value and nature touches on issues related to the sources of growth, inequality, trade patterns, and market power, to mention but a few.

Thus far, much of the literature has restricted itself to examining the US economy and the manufacturing sector. Part of that narrow focus reflected the lack of high-quality, comparable data for other comparable economies. We use a new administrative dataset, *iBach*, that contains high quality and most comparable micro data currently available, covering 5 countries constituting around 50% of the euro area's GDP. We apply recently developed non-parametric production estimation techniques, including *joint* estimation of RTS and the degree of imperfect competition.

Our main findings are the following:

– While on average, we find sectoral RTS to be close to one (0.98) and the majority of 4-digit industries (58 percent) to exhibit constant returns to scale, a non-trivial share

[26] The largest firms refer here to those at the 95$^{\text{th}}$ percentile of a sector's sales distribution, while the smallest firms are those at the 5$^{\text{th}}$ percentile.

of industries is not characterized by CRTS. In our baseline specification, where we assume perfect competition in output markets, 32 percent and 10 percent of 4-digit industries respectively exhibit decreasing and increasing returns to scale. Sectoral RTS estimates range from 0.74 to 1.18.

– When allowing for imperfect competition, the RTS range tightens to $0.98 - 1.08$, the share of DRTS sectors drops to near zero, while the share of industries with IRTS rises to 15 percent. Reflecting this sectoral heterogeneity, some broad sectors like *Manufacturing*, *Transport*, and *IT* have quite high median RTS estimates (and a large statistical significant share of IRTS firms). Others, such as *Professional Services* have lower average RTS values and are much less disposed to increasing returns in general.

– These results could constitute a useful benchmark for other researchers, investigating disaggregate empirical and policy questions. In particular, we hope that the granular estimates on RTS at the 4-digit level, and subsequent aggregations to 1-digit level, could be of use in the recent macro and trade literature using models with multi-sector production functions (e.g., Baqaee and Farhi, 2019).

– Moreover, although the RTS values have remained relatively stable, there is some tendency for them to drift rightward over time, thinning their lower tail. This speaks to a large contemporary literature that has stressed increases in market power in many advanced economies (e.g., Duval et al., 2023; Cavalleri et al., 2019; Ferrando et al., 2023; de Loecker, Eeckhout and Unger, 2020). In such cases, the market power need not be associated with increasing markups but an increasing share of sales derived from technological advantages.

– A widely noted puzzle in the literature, moreover, is the prevalence of sectors and industries with *decreasing* returns; the puzzle being to understand how industries can operate under decreasing returns. This puzzle essentially evaporates when we control for imperfect competition.

– Using the richness of our dataset, we examine the correlations between industry characteristics and our RTS estimates. We find that industries with higher RTS are larger in size (sales) and exist in more concentrated markets, are associated with more intense use of intangible capital, and are export oriented. We also obtain a negative though weaker relationship between TFP growth and RTS.

Hence while an aggregate constant returns takeaway is not obviously incompatible with the data, its acceptance does suppress a substantial degree of heterogeneity and detail – detail which may in different contexts matter. For example, in issues of business-cycle modeling it may be important to differentiate between durable and non durable consumption; this would speak to the importance of differentiating between parts of the economy susceptible to increasing returns. In models of endogenous growth or multiple equilibria, or network effects, the prevalence and drivers of increasing returns are fundamental. Likewise, in analyses of public policy and industrial policy, a sectoral knowledge of the dispersion of RTS estimates can be highly insightful. More recently, in micro-to-macro multi-sector models, sectoral RTS are crucial for calibration and policy analysis.

Future Research The dataset used constitutes an important landmark in micro data for European countries. Future research could drill down to the individual country and individual industry level. This may lead to additional insights into cross sectoral and cross country differences in TFP growth and the sources of growth; network effects and productivity spillovers; model calibrations; as well as the trade offs in modeling production at different levels of aggregation. Future work could also try to understand the extent to which differences between estimated RTS at the 4-digit, 2-digit, and 1-digit level can be put down to reallocation effects and inefficiencies in the spirit of Baqaee, Farhi and Sangani (forthcoming), where it is shown that micro- and aggregate returns to scale can be sensitive to such factors.

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A Variables and Summary Statistics

This section presents variables used in the empirical analysis and summary statistics. Table A.1 presents the subset of *iBACH* variables used in our empirical analysis. It also demonstrates the richness and detail of the source dataset.

Core dataset Since the focus of this paper is on understanding cross-sectoral heterogeneity in firms' RTS parameters based on the firm-level dataset used to estimate the production function, we construct a sector-level dataset. It contains sector-specific moments of distribution of firm characteristics within 4-digit sectors (see Table A.2).

Additional datasets To study how differences in sectoral RTS parameters relate to certain other characteristics of those sectors' firms, we exploit additional data sources. In particular the following variables are considered:

EU KLEMS As an alternative to the information on intangible fixed assets given in *iBACH*, we exploit information on intangible capital (which comprises software, R&D, and other intellectual property product assets) from the EU KLEMS database, see Stehrer et al. (2019). It provides information for the 27 EU countries, the UK, the US and Japan, for around 30 one-to-two digit sectors, and over the period 1995–2017 (depending on country). Specifically, we construct We construct the following six variables: 1) Intangible capital share: $INTANG/CAP$; 2) Intangible capital share - alternative measure: $(INTANG - R\&D)/CAP$; 3) R&D share: $R\&D$ share: $R\&D/CAP$; 4) Intangible capital intensity: $INTANG/VA$; 5) Intangible capital intensity: $(INTANG - R\&D)/VA$; 6) R&D intensity: $R\&D/VA$.

World Input-Output Database The World Input-Output Database (WIOD) presents international input-output tables which allow to compute shares of a country's gross exports and gross imports over total gross output (Timmer et al., 2015). In particular, we use this database to compute a country's mean export and import shares over the period 2008-2014.

Table A.1: Selected *iBACH* variables

Code	Name	Description
Demographic variables:		
DID	ID	Firm identification number Country-specific
DNAME	Name	The denomination of the firm as recorded in the national registers
DADDRESS1	Address	The address of the firm
DREGIO	Localization	Localization of the firm (or the headquarter of the firm), at NUTS-3 level
DCOUNTRY	Country	The 2-letter ISO code for the country of the firm
DYEAR	Year	Period of reference for the accounting exercise
DLEGAL	Legal form	Classification of the firm's legal form, according to the Bureau van Dijks framework: 1: Public limited companies; 2: Private limited companies; 3: Partnerships; 4: Sole proprietorships; 5: Public authorities; 6: Non-profit organizations; 7: Branches; 8: Foreign companies; 9: Others
DSECTOR	Sector of activity	Principal economic activity of the firm, according to the NACE rev. 2 classification at 4-digit level
DYINCORP	Year of incorportation	Year of incorporation in the register or the database used to fill in the financial statements
DCEASE	Status of activity	Firm's status of activity: 1: Still exist; 2: Has been merged with other firms; 3: Has been liquidated
DNUMBEREMPL	Number of employees	Number of employees for the reference period (the year), with regards to the firm's payroll
DFOREIGN	Foreign owned firm	Ultimate controlling institutional unit (UCI) is a non-resident: 1 – Yes; 0 – No
DRIAD	RIAD code	Register of Institutions and Affiliates Database (RIAD) code
DISIN	ISIN	International Securities Identification Numbers associated to the firm
Income statement:		
i0100	Net turnover	Includes sales of goods and services net of returns, deductions and rebates. Sales are net of VAT and Excise taxes
i0200	Variation in stocks of finished goods and work in progress	Includes change in inventories of production recognized in the income statement
i0300	Capitalised production	Includes costs capitalized by the entity recognized as income in the period
i0400	Other income	Includes other income not identified in previous items (I0100, I0200 and I0300)
i0410	Of which: Operating subsidies and supplementary operating income	Details of other income relating to operating subsidies and supplementary operating income
i0500	Cost of goods sold, materials and consumables	Includes cost of materials and consumables used and the cost of goods sold in the period.
i0600	External supplies and services	Includes expenses with external supplies and services in the period
i0700	Staff costs	Includes expenses with the staff recognized in the period
i0800	Other expenses	Includes other expenses not identified in previous items (I0500, I0600 and I0700)
i0810	Of which: Operating taxes and other operating charges	Details of other expenses relating to operating taxes and other operating charges
i0900	Depreciation and amortization on intangible and tangible fixed assets	Includes depreciation and amortization of assets included in the items A1100 and A1200 recognized in the period

Continued on next page

Code	Name	Description
it100	Total income	I0100+I0200+I0300+I0400
IT200	Total expenses	I0500+I0600+I0700+I0800+I0900+I1000+I1100
IT300	Net profit or loss for the period	IT100-IT200
Assets:		
a1000	Fixed assets	A1100+A1200+A1300
a1100	Intangible fixed assets	Includes brands, patents, copyrights, licenses, etc., even if such assets are held under finance lease contracts. This item also includes the Goodwill recognized separately
a1200	Tangible fixed assets	Includes Lands, buildings, machineries, administration and transport equipments, etc., even if such assets are held under finance lease contracts. This item also includes the bearer's biological assets and investment properties
a2000	Inventories	Includes raw materials and consumables, goods, work in progress and finished products, as well as consumable biological assets
a3000	Trade receivables	Includes credit granted to customers for sales or services net of advances received (except for payments received on account of orders, included in L5)
a4000	Other receivables	Includes other accounts receivable (except trade receivables), as well as non-current assets held for sale (net of any associated liabilities)
a5000	Deferred assets	Includes deferred tax assets and expenses to be recognized in future periods
a6000	Other financial assets, current	Includes financial assets held for trading and derivatives
a7000	Cash and bank	Includes the amount available in cash, demand deposits and other deposits in financial institutions
a0000	TOTAL BALANCE SHEET	A1000+A2000+A3000+A4000+A5000+A6000+A7000

Notes: This table shows the definition and computation of various *iBach* variables used in the empirical analysis. We do not include liability statements, which are available but are unused in our analysis.

Table A.2: List of variables used in the empirical analysis

Variable	Description
$ln(sales)$	Log. of industry-specific average of go
$\Delta ln(sales)$	Δ of log. of industry-specific average of go
$ln(va)$	Log. of industry-specific average of va
$\Delta ln(va)$	Δ of log. of industry-specific average of va
$ln(empl)$	Log. of industry-specific average of l
$\Delta ln(empl)$	Δ of log. of industry-specific average of l
$ln(assets)$	Log. of industry-specific average of k
$\Delta ln(assets)$	Δ of log. of industry-specific average of k
$sales\ share\ top\ 10$	Average go-share of 10 firms with highest go within 4-digit sector and year
$profit\ share\ top\ 10$	Average $ebit$-share of 10 firms with highest $ebit$ within 4-digit sector and year
$ln(sales\ p95 - p05\ gap)$	Average log. of gap between 95th and 5th percentile of go-distribution within 4-digit sector and year
$ln(profit\ p95 - p05\ gap)$	Average log. of gap between 95th and 5th percentile of $ebit$-distribution within 4-digit sector and year
$\Delta lprod$	Δ of log. of industry-specific average of lp^{GO}
$\Delta lprod\ p95 - p05\ gap$	Average gap between 95th and 5th percentile of $\Delta \ln lp^{GO}$ within 4-digit sector and year
ΔTFP	Δ of log. of industry-specific average of TFP
$\Delta TFP\ (wgt)$	Δ of log. of industry-specific X-weighted average of TFP
$\Delta TFP\ p95 - p05\ gap$	Average gap between 95th and 5th percentile of $\Delta \ln TFP$ within 4-digit sector and year
$profit\ share$	Average industry-specific share of $ebit$ in go
$profit\ share\ p95 - p05gap$	Average gap between 95th and 5th percentile of profit share distribution within 4-digit sector and year
$lerner\ index$	Average industry-specific $lerner$ index
$lerner\ p95 - p05\ gap$	Average gap between 95th and 5th percentile of $lerner$ distribution within 4-digit sector and year

Table A.3: NACE industry codes (1- and 2-digits)

Code	Description
C	**MANUFACTURING**
C10	Manufacture of food products
C11	Manufacture of beverages
C12	Manufacture of tobacco products
C13	Manufacture of textiles
C14	Manufacture of wearing apparel
C15	Manufacture of leather & related products
C16	Manufacture of wood & of products of wood & cork, except furniture
C17	Manufacture of paper & paper products
C18	Printing & reproduction of recorded media
C19	Manufacture of coke & refined petroleum products
C20	Manufacture of chemicals & chemical products
C21	Manufacture of basic pharmaceutical products & pharmaceutical preparations
C22	Manufacture of rubber & plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery & equipment
C26	Manufacture of computer, electronic & optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery & equipment n.e.c.
C29	Manufacture of motor vehicles, trailers & semi-trailers
C30	Manufacture of other transport equipment
C31	Manufacture of furniture
C32	Other manufacturing
C33	Repair & installation of machinery & equipment
F	**CONSTRUCTION**
F41	Construction of buildings
F42	Civil engineering
F43	Specialised construction activities
G	**WHOLESALE & RETAIL TRADE; REPAIR OF MOTOR VEHICLES & MOTORCYCLES**
G45	Wholesale & retail trade & repair of motor vehicles & motorcycles
G46	Wholesale trade, except of motor vehicles & motorcycles
G47	Retail trade, except of motor vehicles & motorcycles

Table A.4: NACE industries (1- and 2-digits) cont.

H	**TRANSPORTING AND STORAGE**
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing & support activities for transportation
H53	Postal & courier activities
I	**ACCOMMODATION & FOOD SERVICE ACTIVITIES**
I55	Accommodation
I56	Food & beverage service activities
J	**INFORMATION & COMMUNICATION**
J58	Publishing activities
J59	Motion picture, video & television programme production, sound recording & music publishing
J60	Programming & broadcasting activities
J61	Telecommunications
J62	Computer programming, consultancy & related activities
J63	Information service activities
K	**FINANCIAL & INSURANCE ACTIVITIES**
K64	Financial service activities, except insurance & pension funding
K65	Insurance, reinsurance & pension funding, except compulsory social security
K66	Activities auxiliary to financial services & insurance activities
L	**Real estate activities**
L68	Real estate activities
M	**PROFESSIONAL, SCIENTIFIC & TECHNICAL ACTIVITIES**
M69	Legal & accounting activities
M70	Activities of head offices; management consultancy activities
M71	Architectural & engineering activities; technical testing & analysis
M72	Scientific research & development
M73	Advertising & market research
M74	Other professional, scientific & technical activities
M75	Veterinary activities
N	**ADMINISTRATIVE & SUPPORT SERVICE ACTIVITIES**
N77	Rental & leasing activities
N78	Employment activities
N79	Travel agency, tour operator & other reservation service & related activities
N80	Security & investigation activities
N81	Services to buildings & landscape activities
N82	Office administrative, office support & other business support activities

B Estimation Framework

This section first presents the econometric implementation of the baseline GNR estimator. Then is lays out how this estimation approach can be extended to account for unobserved prices and thus for imperfect competition.

B.1 Baseline Estimator

The first stage of the GNR methodology entails estimating the following equation by non-linear least squares:

$$s_{it} = \ln\Big(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} kl_{it} + \gamma_{km} km_{it} + \gamma_{lm} lm_{it}\Big) - \epsilon_{it}, \tag{B.1}$$

where γ is a vector of parameters to be estimated. Based on the residuals, we can construct $\mathcal{E} = \mathbb{E}[e^{\epsilon_{it}}]$. Since $D(k_{it}, l_{it}, m_{it}; \gamma) = \partial f(k_{it}, l_{it}, m_{it})/\partial m_{it}$, its integration gives:

$$f(k_{it}, l_{it}, m_{it}) = \int D(k_{it}, l_{it}, m_{it} dm_{it} + \mathcal{C}(k_{it}, l_{it}), \tag{B.2}$$

where $\mathcal{C}(k_{it}, l_{it})$ is the constant of integration. From equation (B.1) we obtain:

$$\int D(k_{it}, l_{it}, m_{it}; \gamma) dm_{it} = m_{it}\Big(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \frac{\gamma_m}{2} m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \frac{\gamma_{mm}}{3} m_{it}^2 + \gamma_{kl} kl_{it} + \frac{\gamma_{km}}{2} km_{it} + \frac{\gamma_{lm}}{2} lm_{it}\Big) \tag{B.3}$$

We have thus all the ingredients to compute:

$$\mathcal{Q}_{it} \equiv q_{it} - \int D(k_{it}, l_{it}, m_{it}) dm_{it} - \epsilon_{it}, \tag{B.4}$$

which constitutes an observable random variable derived from data and outcomes from estimating equation (B.1). Importantly, $\mathcal{Q}_{it}$ is the dependent variable of the second stage of the estimation approach, which requires estimating:

$$\mathcal{Q}_{it} = e^{-\mu}(\kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} kl_{it}) + \omega_{it}. \tag{B.5}$$

As mentioned before, ω_{it} is assumed to follow a first order Markov process such that

$$\omega_{it} = g(\omega_{it-1}) + \xi_{it}, \tag{B.6}$$

where $g(\cdot)$ is a third order polynomial function in period $t-1$ and ξ_{it} is an unanticipated *iid* innovation. The second stage is then estimated by GMM. In particular, for a given value of κ, we can regress $\omega_{it}(\kappa)$ on $\omega_{it-1}(\kappa)$ to obtain $\xi_{it}(\kappa)$. Following GNR, we assume that $\Upsilon_{it} = k_{it}, l_{it}, k_{it}^2, l_{it}^2, kl_{it}$ are predetermined at time t and thus orthogonal to ξ_{it}. We therefore have five moment conditions $\mathbb{E}(\xi_{it}(\kappa)\Upsilon_{it}) = 0$ and five κ parameters to be estimated by GMM.

B.2 Accounting for Unobserved Prices in the GNR Framework

We follow Klette and Griliches (1996) and de Loecker (2011) to account for unobserved firm-level prices by introducing a constant elasticity of substitution (CES) demand system in the model:

$$\frac{P_{it}}{P_t} = \left(\frac{Q_{it}}{Q_t}\right)^{\frac{1}{\sigma_t}} e^{\chi_{it}}, \tag{B.7}$$

where P_t is the industry price index, Q_t is an index of industrial output which features in the framework as an aggregate demand shifter, χ_{it} is a (ex-post) firm-specific demand shock, and σ_t is the elasticity of demand. Firms are assumed to produce horizontally differentiated products within a given industry and to be active in a monopolistically competitive environment such that they charge a constant (expected) markup over marginal costs $1/(\sigma_t + 1)$.

Introducing the CES demand system allows us to obtain an expression for observed output; namely, revenue as:

$$R_{it} = Q_{it}P_{it} = Q_{it}^{\left(\frac{1}{\sigma_t}+1\right)} Q_t^{-1/\sigma_t} P_t (e^{\chi_{it}})^{-1/\sigma_t}, \tag{B.8}$$

so that the model to be estimated becomes:

$$r_{it} - p_t = \left(\frac{1}{\sigma_t} + 1\right) f(, k_{it}, l_{it}, m_{it}) - \frac{1}{\sigma_t} q_t + \tilde{\omega}_{it} + \chi_{it} + \tilde{\epsilon_{it}}, \tag{B.9}$$

where $\tilde{\omega}_{it} = \left(\frac{1}{\sigma_t} + 1\right)\omega_{it}$ and $\tilde{\epsilon}_{it} = \left(\frac{1}{\sigma_t} + 1\right)\epsilon_{it}$. In their online appendix, GNR lay out a conceptual framework for estimating (B.9) and see Lu, Sugita and Zhu (2019) for a recent corresponding empirical application. Re-arranging the first-order condition corresponding to (B.9) yields the following share equation:

$$s_{it} = (\Gamma_t + \mu) + \ln D^{\mu}(k_{it}, l_{it}, m_{it}) + \ln \tilde{\mathcal{E}} - \tilde{\epsilon}_{it}, \tag{B.10}$$

where $\Gamma_t = \ln\left(\frac{1}{\sigma_t} + 1\right)$ denotes expected industry-level markups, $\tilde{\mathcal{E}} = E[e^{\tilde{\epsilon}_{it}}]$, and μ is an additional constant. Hence, this equation is generally similar to before, while it now also includes an expression for markups and the constant μ. Note that $\phi_t = \Gamma_t + \mu$ in (B.10) can be approximated by year fixed effects and that markups $\left(\frac{1}{\sigma_t} + 1\right) = e_t^{\phi} e^{-\mu}$ such that the second stage estimation equation can be written as:

$$\begin{aligned} \mathcal{R}_{it} &\equiv r_{it} - e^{\phi_t} \int D^{\mu}(k_{it}, l_{it}, m_{it}) dm_{it} - \tilde{\epsilon}_{it} \\ &= -(e_t^{\phi} e^{-\mu})\mathcal{C}(k_{it}, l_{it}) - (e_t^{\phi} e^{-\mu} - 1)q_t + (e_t^{\phi} e^{-\mu})\omega_{it} + \chi_{it}, \end{aligned} \tag{B.11}$$

where $D^{\mu}(k_{it}, l_{it}, m_{it}) = e^{-\mu} \frac{\partial f(k_{it}, l_{it}, m_{it})}{\partial m_{it}}$. Equation (B.11) makes it clear that the constant μ is identified based on variation in the demand shifter q_t.

B.3 Econometric Implementation

We follow GNR and others in estimating the model outlined above non-parametrically. In particular, we are using a polynomial series of degree 2 for the share equation and the integration constant. Hence, in order to obtain the intermediate goods output elasticity, we estimate equation (B.10) by non-linear least squares using:

$$s_{it} = \phi_t + \ln\big(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} kl_{it} + \gamma_{km} km_{it} + \gamma_{lm} lm_{it}\big) - \tilde{\epsilon}_{it}, \tag{B.12}$$

where ϕ_t are year fixed effects. The constant of integration is estimated as:

$$\mathcal{C}(k_{it}, l_{it}) = \kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} kl_{it}, \tag{B.13}$$

such that the second stage of the estimation procedure becomes:

$$\mathcal{R}_{it} = -e_t^{\phi} e^{-\mu} (\kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} kl_{it}) - (e_t^{\phi} e^{-\mu} - 1) q_t + \tilde{\omega}_{it}^*, \tag{B.14}$$

where $\tilde{\omega}_{it}^* = \tilde{\omega}_{it} + \chi_{it}$ and q_t is measured as in Klette and Griliches (1996) and de Loecker (2011) by the revenue weighted average of log deflated revenues of firms present in the sample in periods t and $t-1$: $q_{it} = \sum_{i=1}^{N} \left(\frac{R_{it}}{\sum_{i=1}^{N} R_{it}} \right) r_{it}$. Equation (B.14) is estimated using GMM techniques exploiting moment conditions derived from assumptions on the law of motion of $\tilde{\omega}_{it}^*$. Details on this estimation step are described in the subsection below. The estimated production function is thus given by:

$$\begin{aligned} f(k_{it}, l_{it}, m_{it}) = {} & \\ e^{\mu} m_{it} (\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \frac{\gamma_m}{2} m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \frac{\gamma_{mm}}{3} m_{it}^2 + \gamma_{kl} kl_{it} + \frac{\gamma_{km}}{2} km_{it} + \frac{\gamma_{lm}}{2} lm_{it}) & \\ + e_t^{\phi} e^{-\mu} (\kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk} k_{it}^2 + \kappa_{ll} l_{it}^2 + \kappa_{kl} kl_{it}) & \\ - (e_t^{\phi} e^{-\mu} - 1) q_t + \tilde{\omega}_{it} + \chi_{it} + \tilde{\epsilon}_{it}, & \end{aligned} \tag{B.15}$$

and the output elasticities for materials, labor, and capital, are respectively computed as:

$$\begin{aligned} \theta_{it}^M &= e^{\mu} (\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} kl_{it} + \gamma_{km} km_{it} + \gamma_{lm} lm_{it}^2), \\ \theta_{it}^L &= e^{\mu} m_{it} \left(\gamma_l + 2\gamma_{ll} l_{it} + \gamma_{kl} k_{it} + \frac{\gamma_{lm}}{2} m_{it} \right) + \left(e_t^{\phi} e^{-\mu} \right) (\kappa_l + 2\kappa_{ll} l_{it} + \kappa_{kl} k_{it}), \\ \theta_{it}^K &= e^{\mu} m_{it} \left(\gamma_k + 2\gamma_{kk} k_{it} + \gamma_{kl} l_{it} + \frac{\gamma_{km}}{2} m_{it} \right) + \left(e_t^{\phi} e^{-\mu} \right) (\kappa_k + 2\kappa_{kk} k_{it} + \kappa_{kl} l_{it}). \end{aligned} \tag{B.16}$$

Returns to scale are then obtained from $\theta_{it}^M + \theta_{it}^L + \theta_{it}^K$ and averaged across all firms belonging to a specific 4-digit NACE industry.

B.4 Estimation Details for GNR-IC

The first stage of the GNR methodology extended to account for imperfect competition entails estimating the following equation by non-linear least squares:

$$s_{it} = \phi_t + \ln(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \gamma_m m_{it} + \gamma_{kk} k_{it}^2 + \gamma_{ll} l_{it}^2 + \gamma_{mm} m_{it}^2 + \gamma_{kl} kl_{it} + \gamma_{km} km_{it} + \gamma_{lm} lm_{it}) - \tilde{\epsilon}_{it}, \tag{B.17}$$

where ϕ_t are year fixed effects, γ is a vector of parameters to be estimated, and $\tilde{\epsilon}_{it}$ is an error term. Based on the residuals, we can construct $\tilde{\mathcal{E}} = E[e^{\tilde{\epsilon}_{it}}]$. $\left(\frac{1}{\sigma_t} + 1 \right) = e_t^{\phi} e^{-\mu}$, implying that we need to keep track of the constant μ in the following, which can be estimated in the second step of the estimation framework. Since $D^{\mu}(k_{it}, l_{it}, m_{it}; \gamma) = e^{-\mu} \frac{\partial f(k_{it}, l_{it}, m_{it})}{\partial m_{it}}$, its integration gives:

$$e^{-\mu}f(k_{it},l_{it},m_{it}) = \int D^{\mu}(k_{it},l_{it},m_{it}dm_{it} + e^{-\mu}\mathcal{C}(k_{it},l_{it}), \tag{B.18}$$

where $\mathcal{C}(k_{it},l_{it})$ is the constant of integration. From equation (B.17) we obtain:

$$\int D^{\mu}(k_{it},l_{it},m_{it};\gamma)dm_{it} = m_{it}(\gamma_0 + \gamma_k k_{it} + \gamma_l l_{it} + \frac{\gamma_m}{2}m_{it} + \gamma_{kk}k_{it}^2 + \gamma_{ll}l_{it}^2 + \frac{\gamma_{mm}}{3}m_{it}^2 + \gamma_{kl}kl_{it} + \frac{\gamma_{km}}{2}km_{it} + \frac{\gamma_{lm}}{2}lm_{it}). \tag{B.19}$$

We have thus all the ingredients to compute:

$$\mathcal{R}_{it} \equiv r_{it} - e^{\phi_t}\int D^{\mu}(k_{it},l_{it},m_{it})dm_{it} - \tilde{\epsilon}_{it}, \tag{B.20}$$

which constitutes an observable random variable derived from data and outcomes from estimating equation (B.17). Importantly, $\mathcal{R}_{it}$ is the dependent variable of the second stage of the estimation approach, which requires estimating:

$$\mathcal{R}_{it} = e_t^{\phi}e^{-\mu}(\kappa_k k_{it} + \kappa_l l_{it} + \kappa_{kk}k_{it}^2 + \kappa_{ll}l_{it}^2 + \kappa_{kl}kl_{it}) - (e_t^{\phi}e^{-\mu} - 1)q_t + \tilde{\omega}_{it}^*. \tag{B.21}$$

We do so by first re-parametrizing equation (B.21) in line with the suggestions by Lu, Sugita and Zhu (2019). In particular, defining $\rho = [e^{\mu}]^{-1}$; $\nu = \rho\kappa$; $\tilde{\mathcal{R}}_{it} = \mathcal{R}_{it} - q_t$; $\tilde{q}_t = e_t^{\phi}q_t$; and $\tilde{\nu} = e^{\phi}\nu$, equation (B.21) can be expressed as:

$$\tilde{\mathcal{R}}_{it} = \tilde{\nu}_k k_{it} + \tilde{\nu}_l l_{it} + \tilde{\nu}_{kk}k_{it}^2 + \tilde{\nu}_{ll}l_{it}^2 + \tilde{\nu}_{kl}kl_{it} - \rho q_t + \tilde{\omega}_{it}^*. \tag{B.22}$$

$\tilde{\omega}_{it}^*$ is assumed to follow a first order Markov process[1] such that:

$$\tilde{\omega}_{it}^* = g(\tilde{\omega}_{it-1}^*) + \xi_{it}, \tag{B.23}$$

where $g(\cdot)$ is a third order polynomial function in period $t-1$, and ξ_{it} is an unanticipated *iid* innovation. The second stage in then estimated by GMM. In particular, for a given value of (ρ,ν), we can regress $\tilde{\omega}_{it}^*(\rho,\nu)$ on $\tilde{\omega}_{it-1}^*(\rho,\nu)$ in order to obtain $\xi_{it}(\rho,\nu)$. Following GNR and de Loecker (2011), we assume that $\Upsilon_{it} = k_{it}, l_{it}, k_{it}^2, l_{it}^2, kl_{it}, q_{t-1}$ are pre-determined at time t and thus orthogonal to ξ_{it}. We therefore have six moment conditions $\mathbb{E}(\xi_{it}(\rho,\nu)\Upsilon_{it}) = 0$ and six parameters (ρ,ν) to be estimated by GMM.

We pick the initial value of (ρ_0,ν_0) by considering nine different initial values of ρ_0 {0.1, 0.2, . . . , 0.9} and choosing the value that minimizes the objective function. We then obtain ν_0 by regression $\tilde{\mathcal{R}}_{it}$ on the second order polynomial of (k_{it},l_{it}) and q_{it} and multiplying the estimated coefficients by ρ_0. Once we have (ρ_0,ν_0), we commence the GMM estimation as outlined before.

Conditional on convergence, we thus obtain $\hat{\kappa} = \hat{\nu}\left[e^{\hat{\phi}_t}\hat{\rho}\right]^{-1}$ and $e^{\hat{\mu}} = [\hat{\rho}]^{-1}$, which in conjunction with estimates of $\hat{\gamma}$ from equation (B.17) allows us to compute the output elasticities as shown in the main paper. Note though that we retain the estimated output elasticities to compute returns to scale only if the average industry j output elasticity of capital, labor, and materials $(\theta^K,\theta^L,\theta^M)_j$ lies between 0 and 1, the implied returns to scale $(\theta_j^K + \theta_j^L + \theta_j^M)$ are between $\frac{1}{2}$ and 3, and the estimated average markup is between $\frac{1}{3}$ and 2.

[1] We thus assume that the combination of productivity ω_{it} and the demand shock χ_{it} is Markovian. Alternatively, one could assume that χ_{it} is *iid* and rely on the more standard assumption that only ω_{it} is Markovian (see, e.g., de Loecker, 2011).

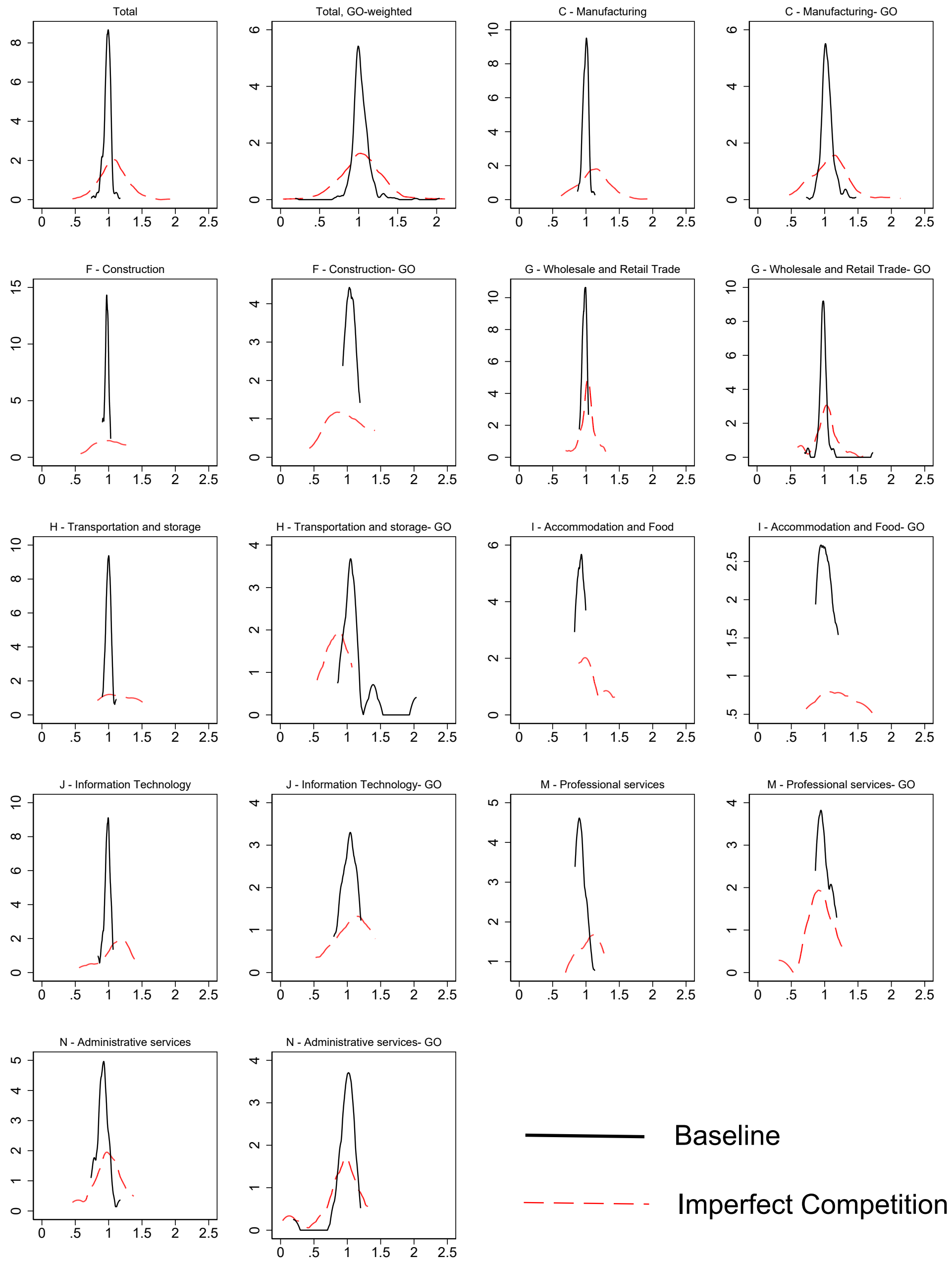
C Additional Empirical Results

This section include additional plots and tables. Specifically,

1. Figure C.1 reports the full set of results for the total economy and for the 4-digit industries expressed in 1-digit sectors using both unweighted and weighted RTS values. For the latter we consider gross-output weights.

2. Tables C.1-C.2 tabulate these data and also show RTS weighted by costs as well as by gross output.

FIGURE C.1: RTS Distributions: Baseline and Imperfect Competition
4-digit industries by 1-digit sector (unweighted and weighted)



Notes: This figure plots, first for the total and then for the 1-digit sectors the Baseline and Imperfect Competition Densities first unweighted and then weighted by Gross Output (GO).

TABLE C.1: Data companion to Figure C.1

	Mean	St. Dev.	Median
Total			
Baseline	0.979	0.056	0.984
Baseline (Cost weighted)	1.029	0.122	1.014
Baseline (Gross Output weighted)	1.027	0.124	1.014
Imperfect Competition	1.075	0.214	1.065
Imperfect Competition (Cost weighted)	1.036	0.280	1.038
Imperfect Competition (Gross Output weighted)	1.035	0.284	1.037
C – Manufacturing			
Baseline	0.995	0.043	0.996
Baseline (Cost weighted)	1.043	0.099	1.028
Baseline (Gross Output weighted)	1.042	0.098	1.029
Imperfect Competition	1.128	0.233	1.130
Imperfect Competition (Cost weighted)	1.086	0.301	1.084
Imperfect Competition (Gross Output weighted)	1.086	0.298	1.087
F – Construction			
Baseline	0.971	0.030	0.975
Baseline (Cost weighted)	1.053	0.078	1.056
Baseline (Gross Output weighted)	1.051	0.075	1.054
Imperfect Competition	1.021	0.225	0.960
Imperfect Competition (Cost weighted)	0.988	0.291	0.929
Imperfect Competition (Gross Output weighted)	0.989	0.289	0.927
G – Wholesale and Retail Trade			
Baseline	0.976	0.034	0.981
Baseline (Cost weighted)	0.991	0.108	0.985
Baseline (Gross Output weighted)	0.990	0.108	0.985
Imperfect Competition	1.020	0.121	1.026
Imperfect Competition (Cost weighted)	1.025	0.195	1.038
Imperfect Competition (Gross Output weighted)	1.027	0.199	1.035

Notes: This table shows the first two moments of the data from Figure C.1. In addition it shows RTS weighted by costs as well as by gross output, although the numbers are quite similar.

TABLE C.2: Data companion to Figure C.1: [Table C.1 Continued]

	Mean	St. Dev.	Median
H – Transport and Storage			
Baseline	1.000	0.047	0.998
Baseline (Cost weighted)	1.138	0.287	1.085
Baseline (Gross Output weighted)	1.130	0.274	1.085
Imperfect Competition	1.149	0.267	1.187
Imperfect Competition (Cost weighted)	0.829	0.175	0.850
Imperfect Competition (Gross Output weighted)	0.832	0.175	0.871
I- Accommodation & Food			
Baseline	0.919	0.057	0.915
Baseline (Cost weighted)	1.029	0.120	0.990
Baseline (Gross Output weighted)	1.024	0.118	0.988
Imperfect Competition	1.076	0.226	0.947
Imperfect Competition (Cost weighted)	1.207	0.409	1.221
Imperfect Competition (Gross Output weighted)	1.201	0.408	1.205
J – Information Technology			
Baseline	0.977	0.054	0.979
Baseline (Cost weighted)	1.020	0.110	1.019
Baseline (Gross Output weighted)	1.019	0.111	1.018
Imperfect Competition	1.083	0.236	1.137
Imperfect Competition (Cost weighted)	1.061	0.283	1.095
Imperfect Competition (Gross Output weighted)	1.061	0.284	1.095
M – Professional Services			
Baseline	0.939	0.083	0.903
Baseline (Cost weighted)	0.994	0.099	0.955
Baseline (Gross Output weighted)	0.994	0.099	0.955
Imperfect Competition	1.021	0.199	1.039
Imperfect Competition (Cost weighted)	0.914	0.248	0.919
Imperfect Competition (Gross Output weighted)	0.912	0.247	0.921
N – Administrative Services			
Baseline	0.913	0.094	0.910
Baseline (Cost weighted)	0.986	0.133	1.015
Baseline (Gross Output weighted)	0.975	0.179	1.014
Imperfect Competition	0.982	0.233	1.000
Imperfect Competition (Cost weighted)	0.924	0.318	0.961
Imperfect Competition (Gross Output weighted)	0.906	0.361	0.958

Notes: See notes to Table C.1.