

Ortega, Josué; Ziegler, Gabriel; Arribillaga, R. Pablo

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Unimprovable Students and Inequality in School Choice

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Unimprovable Students and Inequality in School Choice

Josué Ortega

Queen's University Belfast

Gabriel Ziegler

The University of Edinburgh

R. Pablo Arribillaga

Universidad Nacional de San Luis

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Unimprovable Students and Inequality in School Choice

JOSUÉ ORTEGA

Department of Economics, Queen's University Belfast

GABRIEL ZIEGLER

School of Economics, The University of Edinburgh

R. PABLO ARRIBILLAGA

Instituto de Matemática Aplicada San Luis, Universidad Nacional de San Luis, and CONICET

The Efficiency-Adjusted Deferred Acceptance (EADA) mechanism addresses the Pareto inefficiency of the celebrated Deferred Acceptance (DA) algorithm by assigning every student to a weakly more preferred school. However, it remains uncertain which and how many students do not see an improvement in their DA placement under EADA. We show that, despite its advantages, EADA does not benefit students assigned to their worst-ranked schools or those who remain unmatched under DA. Additionally, it limits the placement improvement of marginalized students, thereby maintaining school segregation. The placement of worst-off students under EADA can be exceptionally poor, even though significantly more egalitarian allocations are possible. Lastly, we provide a bound on the expected number of unimproved students using a random market approach valid for small markets. Our findings shed light on why EADA fails to mitigate the inequality produced by DA in empirical evaluations.

KEYWORDS. School choice, efficiency-adjusted deferred acceptance.

JEL CLASSIFICATION. C78, D47.

Josué Ortega: j.ortega@qub.ac.uk

Gabriel Ziegler: ziegler@ed.ac.uk

R. Pablo Arribillaga: rarribi@unsl.edu.ar

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1. INTRODUCTION

The student-proposing deferred acceptance (DA) algorithm is used all around the world to assign pupils to schools because it generates a stable assignment that incentivizes students to report their preferences truthfully. However, DA can generate an allocation that is Pareto-inefficient for students and, in some cases, it may assign all students to their worst or second-worst school (Kesten, 2010). DA's inefficiency has also been empirically documented: Abdulkadiroğlu et al. (2009) show that around 2% of students in the New York City high school match, which uses a version of DA, could be assigned to a more desirable school without affecting the placement of other students.

To improve DA's efficiency, Kesten (2010) proposed an influential modification of DA known as efficiency-adjusted deferred acceptance (EADA). EADA first implements DA, then iteratively finds students who make the algorithm inefficient by applying to schools that will eventually reject them, and blocks their application to such schools if students consent. EADA assigns every student to either the same school or a better-ranked school than DA. In fact, EADA produces a Pareto-efficient allocation when all interrupter students consent, which is likely to occur in practice because interrupters never benefit from not consenting.¹ Furthermore, a vast literature has shown that EADA also satisfies a plethora of desirable properties both theoretically and in the lab. For these reasons, the Flemish Ministry of Education has considered using EADA in practice (Cerrone et al., 2024).

Despite EADA's enhanced efficiency, it is unclear how EADA's efficiency improvements are distributed among students. The efficiency gains could be consistently concentrated across high-achievers, thus failing to improve the placement of the worst-off students under DA. In this regard, EADA's Pareto-efficiency does not guarantee a more egalitarian allocation than DA, because it is “*orthogonal to distributive concerns, possibly selecting the most unfair of all distributive systems*” (Moulin, 2003), and therefore constitutes “*a very limited kind of success, which in itself may or may not guarantee much*” (Sen, 1999).

Understanding the distribution of EADA's efficiency improvements is particularly important because DA's original allocation can be extremely unequal (Galichon et al., 2023), and therefore EADA may achieve Pareto-efficiency without improving the placement of those students who need said improvement most. Indeed, we find that this is the case in Propositions 1 and 2: students who are either assigned to their least-preferred school or not assigned at all are always unimprovable by EADA.

Moreover, even when DA's outcome is Pareto-efficient, it may still be highly unequal. In those cases, EADA could preserve an unreasonable amount of inequality, as in Example 1 below.

EXAMPLE 1. Consider a school choice problem with 6 students and schools with preferences and priorities as in Table 1.

¹Throughout the paper, we consider the case that every student consents. This assumption is not restrictive for our purposes because, if fewer students consented to waive their priorities, the inequality documented in EADA would be even worse.

TABLE 1. EADA assigns a student to his worst-off school, even when all students can be assigned to their top two choices.

i_1	i_2	i_3	i_4	i_5	i_6	s_1	s_2	s_3	s_4	s_5	s_6
s_1	s_1	s_2	s_3	s_4	s_5	i_1	i_2	i_3	i_4	i_5	i_6
s_6	s_2	s_1	s_1	s_1	s_1	i_2	i_3	i_4	i_5	i_6	
		s_3	s_2	s_2	s_2	i_3	i_4	i_5	i_6		
			s_4	s_3	s_3	i_4	i_5	i_6			
				s_5	s_4	i_5	i_6				
					s_6	i_6					

The EADA allocation (which coincides with DA, as there are no interrupters) appears in squares. Students i_4 , i_5 and i_6 are allocated to a school in the bottom-half of their preferences, with student s_6 being assigned to his least-preferred school. An alternative Pareto-efficient, Rawlsian allocation, in circles, is remarkably more egalitarian, because all students are assigned to a school among their top two choices. All but one students prefer the alternative Pareto-efficient allocation to the one generated by EADA, and yet EADA yields a much more unequal (and rank-inefficient) allocation.²

Furthermore, the maximum rank is exactly $n/2$ larger than that in the Rawlsian allocation in circles, a measure we call *Rawlsian inequality*. In Proposition 3 we show that a generalization of this example serves as a tight upper bound for EADA's and DA's Rawlsian inequality.

Closely related to inequality is the concern that DA generates highly segregated allocations, in which marginalized students are condemned to attend schools with predominantly low-achieving peers (Terrier et al., 2021, Kutscher et al., 2023). A natural question is whether EADA's efficiency improvement can help marginalized students to attend more diverse schools. In Propositions 4 and 5, we show that, in a large class of problems in which students can be partitioned into advantaged and marginalized, there is always a marginalized student whose DA placement does not improve under EADA. Under an additional, but empirically reasonable, preference restriction, all unassigned students in EADA are marginalized, and moreover, marginalized students are mostly matched with inferior schools under EADA. Furthermore, under the same assumptions, any fully segregated school in DA, admitting exclusively only advantaged or marginalized students, remains fully segregated under EADA too.

Finally, we adopt a random market approach to study the expected number of under-demanded schools. All students matched to such schools, will not improve their placement in EADA. Considering markets with an equal size of students and schools—as is customary in the literature—in Proposition 6, we provide a small markets result showing that the number of under-demanded schools—equivalently, the number of removed

²We study EADA's rank-efficiency in a separate paper, which requires a different analysis, but builds on the expected number of removed students computed in Proposition 6.

students in the first iteration of EADA—exactly equals the n -th Harmonic number H_n , for any market size n . Thereby, we obtain a lower bound on the number of improvable students. For example, this result implies that in a school choice problem with 100 students and schools, we expect at least 5 students to have the same expected rank as in DA.

2. RELATED LITERATURE

As already mentioned, EADA was proposed by [Kesten \(2010\)](#), who shows that when all interrupters consent, EADA generates an assignment that is Pareto-efficient for students, a property which comes at the expense of losing DA's guarantees of stability and strategy-proofness. Nonetheless, EADA does relatively well on these two fronts, satisfying weak versions of stability ([Trojan et al., 2020](#), [Ehlers and Morrill, 2020](#), [Tang and Zhang, 2021](#), [Reny, 2022](#), [Doğan and Ehlers, 2021](#)) and strategy-proofness ([Trojan and Morrill, 2020](#), [Chen and Möller, 2023](#)). Instead of studying incentive and stability properties, we focus on EADA's inequality, a desideratum that has been rarely studied in the literature on two-sided matching with a few exceptions that we discuss below.

On the theoretical side, [Galichon et al. \(2023\)](#) show that if preferences and priorities are aligned, DA may produce an allocation in which some students attend the school next door while others are forced to travel across the city. [Calsamiglia and Miralles \(2023\)](#) obtain a similar finding: under DA, students from disadvantaged backgrounds experience significant limitations in accessing good schools. [Pittel \(1992\)](#) computes the placement of the worst-off student under DA when there are n schools and students and shows that it is asymptotically equivalent to $\log^2(n)$, which is significantly higher than in other Pareto-efficient mechanisms ([Ortega and Klein, 2023](#)). To our knowledge, EADA's inequality has been unexplored theoretically.

On the empirical side, [Ortega and Klein \(2023\)](#) and [Diebold and Bichler \(2017\)](#) examine the rank distribution generated by EADA using real data. Ortega and Klein compute a counterfactual EADA allocation for the assignment of pupils to secondary schools in Budapest (which currently uses DA). They find that the worst-off matched student in both DA and EADA is assigned to their 13th ranked school, although no student ranks more than 14 schools. Furthermore an alternative Pareto-efficient allocation would assign all students to a school ranked better or equal than their 6th choice. When analysing complete, estimated preferences, both DA and EADA assign some student to their 241st choice out of 244 possible schools, whereas an alternative Pareto-efficient mechanism assigns all students within their 16 most preferred schools.³ Using 19 datasets on course allocation, Diebold and Bichler compare the outcome generated by DA and EADA. They find that the difference between the rank distributions generated by DA and EADA is not statistically different in 7 out of 19 datasets, and furthermore, in each of the 19 datasets, DA and EADA match the same number of students. The authors do not report the placement of the worst-off students.

³EADA's outcome is computed assuming truthful revelation of preferences, an assumption supported by the high ratio of truthful reports under EADA observed in experimental work ([Cerrone et al., 2024](#)).

On the issue of how much segregation emerges under DA, two articles have documented how school segregation changes whenever DA is implemented at a country-level, either replacing a decentralized system, as in Chile in 2016 (Kutscher et al., 2023), or replacing an immediate-acceptance mechanism, as in England in 2007 (Terrier et al., 2021). Both articles paint a bleak picture. In England, switching to DA harmed students with low socioeconomic status, who after the transition attended schools with lower value added and a greater fraction of low-status and low-achieving peers. In Chile, switching to DA increased the probability that students from high socioeconomic status attend private schools. Within-school segregation increased in districts with high levels of pre-existing residential segregation or a high presence of private schools. To our knowledge, there are no articles studying whether school segregation under EADA.

Finally, the placement generated by DA and EADA has also recently been compared in the lab (Cerrone et al., 2024). The authors find that EADA generates a Pareto-efficient allocation more frequently than DA (44% - 80% versus 6% of the cases), but they do not examine EADA's inequality or the distribution of its efficiency improvements.

3. MODEL

Following Abdulkadiroğlu and Sönmez (2003), a school choice problem P consists of a finite set of students I and a finite set of schools S . Each student i has a strict preference \succ_i over the schools. Each school s has a quota of available seats q_s and a strict priority over the students \triangleright_i , determined by local educational regulations. We use $m = |I|$ and $n = |S|$ to denote the number of students and schools, respectively. To allow for unassigned students, we assume the existence of a null school (denoted s_\emptyset), which has unlimited capacity. Let $\mathcal{P}_{m,n}$ denote the set of all school choice problems with m students and n schools.

For a given school choice problem P , a *matching* μ is a mapping from I to S such that no school is matched to more students than its quota. We denote by μ_i the school to which student i is assigned and by μ_s^{-1} the set of students assigned to school s . With this notation, a matching needs to satisfy $|\mu_s^{-1}| \leq q_s$ for every $s \in S$. Furthermore, we call every student i with $\mu_i = s_\emptyset$ *unassigned*.

The function $\text{rk}_i : S \rightarrow \{1, \dots, n\}$ specifies the rank of school s according to the preference profile \succ_i of student i , i.e.

$$\text{rk}_i(s) = |\{s' \in S : s \succ_i s'\}| + 1,$$

so that the most desirable option gets a rank of 1, whereas the least desirable gets a rank of n . With some abuse of notation, we use the same rank function to specify the students' rank per the priority profile of schools, i.e. $\text{rk}_s(i) = |\{j \in I : i \triangleright_s j\}| + 1$.

A matching μ *weakly Pareto-dominates* matching ν if, for every student $i \in I$, $\text{rk}_i(\mu_i) \leq \text{rk}_i(\nu_i)$. A matching μ *Pareto dominates* matching ν if μ weakly dominates ν and there exists a student $j \in I$ with $\text{rk}_j(\mu_j) < \text{rk}_j(\nu_j)$. A matching is *Pareto-dominated* if there exists a matching that Pareto-dominates it and is *Pareto-efficient* if it is not Pareto-dominated.

Student i *desires* school s in matching μ if $\text{rk}_i(s) < \text{rk}_i(\mu_i)$. We say that student j *violates* student i 's priority at school s in matching μ if i desires s , $\mu_j = s$, and $\text{rk}_s(i) < \text{rk}_s(j)$. A matching μ is *non-wasteful* if every school s that is desired by some student in μ satisfies $|\mu_s^{-1}| = q_s$. A matching μ is *stable* if it is non-wasteful and no student's priority at any school is violated in μ .

3.1 Mechanisms

A *mechanism* associates a matching to every school choice problem. We are mainly interested in the following two mechanisms.

3.1.1 *DA*. The *student-proposing Deferred Acceptance (DA) mechanism* (Gale and Shapley, 1962) works as follows:

Round 1: Every student applies to her most preferred school. Every school tentatively accepts the best students according to its priority, up to its capacity, and rejects the rest.

Round k : Every student rejected in the previous round applies to her next best school. Among both new applicants and previously accepted students, every school accepts the best students according to its priority, up to its capacity, and rejects the rest.

The procedure stops when there is a round without any rejection. We use $DA(P)$ to denote the (unique) resulting matching generated by DA in school choice problem P . $DA_i(P)$ denotes the school to which student i is assigned. $DA_s^{-1}(P)$ denotes the set of students assigned to school s in $DA(P)$.

3.1.2 *EADA*. The *Efficiency-Adjusted Deferred Acceptance (EADA) mechanism* (Kesten, 2010) iteratively executes the DA algorithm, by identifying interrupters that cause DA's inefficiency and then modifying students' preferences after each execution. We describe an alternative version of it that is easier to analyse for our purposes, proposed by Tang and Yu (2014). In this implementation, under-demanded schools are removed at each iteration of DA, and instead of changing students preferences we reduce the number of schools and students after each DA execution.

DEFINITION 1. A school s is *under-demanded* if, for every student i , $\text{rk}_i[DA_i(P)] \leq \text{rk}_i(s)$.

That is, an under-demanded school is one that no student strictly prefers to the one he is assigned under DA. Equivalently, an under-demanded school is one that does not reject any student during DA's execution.

If all students consent, the EADA algorithm is implemented as follows:

Round 1: Run DA and find all under-demanded schools.

Round 2: Permanently assign under-demanded schools to the students they are matched with.

Round 3: Eliminate under-demanded schools and associated students.

Round 4: If there are no schools remaining, terminate the algorithm and implement the allocation obtained. Otherwise, go to round 1 (on the simplified school choice problem without under-demanded schools and associated students).

Similar to before, we use $EADA(P)$ to denote the matching generated by EADA in school choice problem P , and $EADA_i(P)$ to represent the school to which student i is assigned. $EADA_s^{-1}(P)$ denotes the set of students assigned to s in $EADA(P)$.

3.2 Relationship between Mechanisms.

EADA always generates an allocation that is weakly preferred by every student to that obtained in DA.

LEMMA 1 (Kesten 2010). *For all school choice problems P and all $i \in I$, $\text{rk}_i[EADA_i(P)] \leq \text{rk}_i[DA_i(P)]$.*

Even though all students weakly prefer EADA over DA, there is always at least one student who does not benefit from EADA's improved efficiency.

LEMMA 2. *For all school choice problem P , $EADA_i(P) = DA_i(P)$ for some $i \in I$.*

PROOF. In the first iteration of EADA, there is always one school s^* who is matched to some student and did not reject any pupils (the last school to receive a student application during DA's execution is one such school). Let i denote an arbitrary student matched to s^* under DA. Student i will be removed in the first iteration of EADA, together with $s^* = DA_i(P)$. Therefore, $EADA_i(P) = s^*$. \square

Moreover, whenever DA's outcome is Pareto-efficient, DA and EADA generate the same placement for every student. This equivalence holds even if there exist interrupters in the execution of DA, provided that the outcome of DA is Pareto-efficient.⁴

LEMMA 3. *If $DA(P)$ is Pareto-efficient, then $DA(P) = EADA(P)$.*

PROOF. Suppose $DA(P)$ is Pareto-efficient but $DA(P) \neq EADA(P)$. The latter, together with Lemma 1, implies that there exists a student $i \in I$ with $\text{rk}_i[EADA_i(P)] < \text{rk}_i[DA_i(P)]$, and $\text{rk}_j[EADA_j(P)] \leq \text{rk}_j[DA_j(P)]$ for all students $j \in I$. Thus, $EADA(P)$ Pareto dominates $DA(P)$ —a contradiction to $DA(P)$ being Pareto-efficient. \square

Because DA is non-wasteful, every Pareto-improvement over DA involves students swapping their places at the schools they have been assigned under DA. A consequence is that each school is assigned the same number of students under DA and EADA.

LEMMA 4 (Tang and Yu 2014). *For any school choice problem P and any school $s \in S$, $|DA_s^{-1}(P)| = |EADA_s^{-1}(P)|$.*

⁴If there are no interrupters then the two mechanisms are equivalent trivially.

3.3 Unimprovable Students

The reason why the alternative algorithm by [Tang and Yu \(2014\)](#) always produces the same matching as Kesten's original EADA algorithm is because of the role that unimprovable students play in both algorithms. We define this key concept below.

DEFINITION 2. A student $i \in I$ is *unimprovable* at matching $DA(P)$ if for every matching μ that weakly Pareto-dominates $DA(P)$, we have $\mu_i = DA_i(P)$.

Unimprovable students do not necessarily need to be in an adverse situation. For example, every student who is assigned to their most preferred school in DA is unimprovable. Nonetheless, if unimprovable students tend to be those with a poor placement, then EADA will consistently fail to improve those students who need the efficiency improvement the most.

We denote by $\mathcal{U}(P)$ the set of unimprovable students in school choice problem P . Similarly, let $\mathcal{R}^1(P)$ denote the students who are removed in the first iteration of the EADA procedure, i.e. those matched to under-demanded schools in DA. The sets $\mathcal{U}(P)$ and $\mathcal{R}^1(P)$ are related: any student who is removed in the first iteration of EADA is unimprovable.

LEMMA 5 ([Tang and Yu 2014](#)). *For any school choice problem P , $\mathcal{R}^1(P) \subseteq \mathcal{U}(P)$.*

4. WHICH STUDENTS ARE CONSISTENTLY UNIMPROVABLE?

Although EADA produces allocations that are at least as good as those generated by the DA algorithm for every student, in empirical studies it appears to fall short in improving the placements of worst-off students as mentioned in Section 2. Therefore, even though EADA may improve the worst-off placement under DA, this is not guaranteed to happen. In fact, we establish next that whenever a student is assigned to his worst ranked school in DA (as in Example 1), EADA will always assigns him to the same school.

PROPOSITION 1. *For any school choice problem P with n schools, if there is a student $i \in I$ with $\text{rk}_i[DA_i(P)] = n$, then i is unimprovable, and therefore $\text{rk}_i[EADA_i(P)] = n$.*

PROOF. Consider student i that has $\text{rk}_i[DA_i(P)] = n$ within a school choice problem P with n schools. That is, i is assigned to his worst ranked school under DA, which means they applied to all other schools during the execution of DA. Thus, all schools besides $DA_i(P)$ are *not* under-demanded, because they all rejected student i and an under-demanded school always exists.⁵ Therefore, in the first iteration of EADA we remove $DA_i(P)$ and the students matched to it (*viz.* i). By Lemma 5, i is an unimprovable student, and is also matched to $DA_i(P)$ under EADA. \square

⁵Alternatively, suppose that $DA_i(P)$ was not under-demanded. Then some student was rejected from it at some point in DA's execution. But then, before student i applied to his worst choice $DA_i(P)$, all schools had received a proposal and thus DA must have stopped, a contradiction.

In the spirit of [Rawls \(1971\)](#), students assigned to their worst-ranked school are particularly important for a mechanism designer when using the welfare of a worst-off agent as the welfare measure for the whole population. In a similar vein, we may argue that students who are not assigned to a school are particularly important from a fairness point of view, and an effective mechanism should place students to schools whenever possible. However, it is generally not possible to rank standard school choice mechanisms based on the number of unassigned students. For some school choice problems, some other well-know mechanisms like Top Trading Cycles and Random Serial Dictatorship assign more students to schools than DA and EADA, but sometimes also less for some other school choice problems.⁶ Nonetheless, we demonstrate that DA and EADA always match the same number of students to schools next.

PROPOSITION 2. *For any school choice problem P , if a student i is unmatched in DA, i.e. if $\text{DA}_i(P) = s_\emptyset$, then i is unimprovable, and therefore $\text{EADA}_i(P) = s_\emptyset$.*

PROOF. Let i be an unmatched student in $\text{DA}(P)$, where P is an arbitrary school choice problem. That is $\text{DA}_i(P) = s_\emptyset$ and note that s_\emptyset is under-demanded, because s_\emptyset has unlimited capacity and therefore does not reject anyone. Thus, we remove school s_\emptyset and i in the first iteration of EADA, and—as in the proof of Proposition 1—the conclusion follows. \square

Proposition 2 sheds light on why, in all 19 datasets in [Diebold and Bichler \(2017\)](#), as well as in the Hungarian data of [Ortega and Klein \(2023\)](#), DA and EADA match the same number of students. The Diebold and Bichler dataset shows that most algorithms are incomparable on the basis of number of students matched, highlighting the strength of Proposition 2, despite its simple proof.

Note that Lemma 4 does not imply Proposition 2, because Lemma 4 only establishes that the number of unassigned students is equal in DA and EADA, whereas Proposition 2 establishes that the identity of all unassigned students in DA and EADA is the same. Moreover, Proposition 1 does not imply Proposition 2 either, because the latter does not assume that being unmatched is worse than being assigned to any school.

Proposition 2 is hinted at in [Tang and Yu \(2014\)](#) and can be obtained as a corollary of a general equivalence between a large class of mechanisms in a more general allocation model, obtained by [Alva and Manjunath \(2019\)](#). We nonetheless present it here because its simple proof allows us to clearly observe the relationship between DA and EADA, and this relationship will be useful for our results in the next sections.

5. RAWLSIAN INEQUALITY

In the previous section, we argued that the placement of the worst-off student is a natural measure of the inequality generated by a school choice mechanism. Now, we will

⁶Note that we consider non-stable mechanisms like the EADA. For stable mechanisms, the *rural hospital theorem* applies: any two stable mechanisms always match the same number of students ([McVitie and Wilson, 1970](#), [Roth, 1986](#)).

examine inequality from another perspective by considering the ratio between the maximum ranks achieved by two different mechanisms.

The *Rawlsian mechanism*, denoted by $\text{RW}(P)$, selects a matching that minimizes the maximum rank for every school choice problem P (Demeulemeester and Pereyra, 2022, Afacan and Dur, 2024, Kuvalekar and Romero-Medina, 2024). The matching selected by $\text{RW}(P)$ is generally not unique, but this multiplicity and how it is resolved is of no consequence for our purposes.

We define the *Rawlsian inequality* (RI) of a mechanism M for a school choice problem with n schools as the following ratio:⁷

$$\text{RI}(M; n) := \sup_{m \in \mathbb{N}, P \in \mathcal{P}_{m,n}} \left[\frac{\max_{i \in I} \text{rk}_i[M_i(P)]}{\max_{i \in I} \text{rk}_i[\text{RW}_i(P)]} \right] \quad (1)$$

Armed with this definition, we can state EADA's Rawlsian inequality, showing that EADA may assign the worst-off student to a school $\frac{n}{2}$ times worse than that of the Rawlsian mechanism.

PROPOSITION 3. *For every $m \in \mathbb{N}$,*

$$\text{RI}(\text{EADA}; n) = \text{RI}(\text{DA}; n) = \frac{n}{2},$$

and the supremum is attained.

PROOF. The school choice problem in Example 1 shows that the $\text{RI}(\text{EADA}; n) = \text{RI}(\text{DA}; n) = n/2$ ratio can be achieved when $n = m = 6$, and it can be generalized for any other $n = m$ values with the following construction. Let students' preferences be as follows:

- for i_1 : $\text{rk}_{i_1}(s_1) = 1$ and $\text{rk}_{i_1}(s_n) = 2$
- for any other i_k : $\text{rk}_{i_k}(s_{k-1}) = 1$, $\text{rk}_{i_k}(s_k) = k$ and, for any s_j with $2 \leq j \leq k$, $\text{rk}_{i_k}(s_j) = j + 1$.

Additionally, schools' priorities are so that for any s_k and any two students i_ℓ, i_f with $k \leq \ell < f \leq n$, $\text{rk}_{s_k}(i_\ell) < \text{rk}_{s_k}(i_f)$.⁸

In DA's allocation, student i_k is matched with s_k , for all $1 \leq k \leq n$, because s_1 is the best school for student i_1 and vice versa, and by stability of DA they must be matched together. Among the remaining schools and students, i_2 and s_2 rank each other as best, and must be matched, and so on. However, DA's allocation is Pareto-efficient and thus coincides with EADA, cf. Lemma 3. In both matchings, $\text{rk}_{i_n}[\text{DA}_{i_n}(P)] = \text{rk}_{i_n}[\text{EADA}_{i_n}(P)] =$

⁷This concept is related to the *price of anarchy* and *price of stability* concepts in algorithmic game theory, which establish the efficiency losses that arise in equilibrium (Koutsoupas and Papadimitriou, 1999, Anshelevich et al., 2008). A similar concept was proposed by Boudreau and Knoblauch (2013) to measure the efficiency loss of stable matching mechanisms.

⁸Although we have not specified preferences and priorities fully, the given partial order will suffice for our purposes because any unspecified preferences or priorities will be irrelevant.

n , even though the alternative Pareto-efficient matching $\mu_{i_j} = s_{j-1}$ (modulus n) is such that $\max_{i \in I} \text{rk}_i(\mu_i) = 2$, thus achieving the $n/2$ ratio.

Finally, we show that $n/2$ is the highest ratio possible for any school choice problem P with n schools. The maximum rank for a student is n , so the numerator cannot exceed n . The denominator can only be smaller than 2 if $\max_{i \in I} \text{rk}_i[\text{RW}_i(P)] = 1$. However, if such an allocation were possible, DA would stop at the first proposal round, assigning every student to their top school. In this case, we would have $\text{RI}(\text{EADA}; n) = \text{RI}(\text{DA}; n) = \frac{1}{1} < \frac{n}{2}$. \square

6. UNIMPROVABLE, MARGINALIZED, AND SEGREGATED

Now we turn to the question of how EADA may improve (or not) the placement of marginalized students in society. Marginalized students can be thought of as those from low-income households, immigrants, ethnic, racial, or religious minorities, students with disabilities, and others facing systemic disadvantages. With this in mind, we are also interested in whether EADA alters the segregation in school compositions. Several studies have shown that DA generates high levels of school segregation both theoretically and practically (Calsamiglia et al., 2020, Terrier et al., 2021, Escobar and Huerta, 2022, Calsamiglia and Miralles, 2023). Therefore, understanding whether EADA reduces the segregation created by DA provides important underpinning for its real-life implementation.⁹

In this section, we partition the set of students and schools into two non-empty groups. Some students will be *advantaged* and some will be *marginalized*. A marginalized student has a lower priority than every advantaged student at every school. The set of advantaged and marginalized students are denoted by $Y \subsetneq I$ and $Z \subsetneq I$, respectively, with $Y \cup Z = I$ and $Y \cap Z = \emptyset$.

In this setup, we obtain the following result.

PROPOSITION 4. *There is at least one marginalized student whose DA assignment does not improve under EADA, i.e. there exists $i \in Z$ such that $\text{DA}_i(P) = \text{EADA}_i(P)$.*

PROOF. Denote by S_Z the set of schools who are assigned at least one marginalized student under DA, i.e. $S_Z := \{s \in S : \text{DA}_s(P)^{-1} \cap Z \neq \emptyset\}$. We will prove that there exist a school $s \in S_Z$ that is under-demanded, and thus the placement of the students matched to it in DA (including at least one marginalized student) do not change in EADA.

By contradiction, assume that all schools in S_Z are not under-demanded, which means that all such schools reject some applicant during DA's execution. Let t be the last round of DA's execution in which a school in S_Z receives a student proposal (call s one of such schools). Let $O^t(s)$ denote the set of student proposals that school s receives at time t , and q_s^{t-1} the number of students accepted by school s at time $t-1$. We consider two cases:

⁹Irrespective of the mechanism used, school choice is known to generate more segregation than neighbourhood assignment (Söderström and Uusitalo, 2010, Böhlmark et al., 2016, Ukanwa et al., 2022, Greaves, 2024).

- Case 1: $|O^t(s)| + q_s^{t-1} \leq q_s$. Then school s does not reject any student at time t or at any other time $t' < t$. But then s is under-demanded, a contradiction.
- Case 2: $|O^t(s)| + q_s^{t-1} > q_s$. Then school s rejects at least one student at time t . We know that school s eventually accepts and is matched with some marginalized student, *viz.* i_z . Therefore, by the definition of marginalized students, the student who was rejected at time t was himself marginalized, and must apply to a school in S_Z at a round $t' > t$, contradicting our definition of t .

□

For our next results, we will also partition the set of schools into *adequate* and *inferior* institutions, so that every student prefers any adequate school over every inferior one. The set of adequate and inferior schools is denoted by $A \subsetneq S$ and $B \subsetneq S$, respectively, with $A \cup B = S$ and $A \cap B = \emptyset$. We impose that every student prefers being assigned to a school than remaining unassigned, which implies that $s_\emptyset \in B$.

Let $Q = \sum_{s \in S \setminus \{s_\emptyset\}} q_s$ denote the sum of (finite) school capacities. We will assume that there are more students than available seats at schools excluding s_\emptyset , i.e. $m > Q$, and that the set of marginalized students is large enough to avoid trivial problems, i.e. $|Z| \geq m - Q$.

The class of school choice problems satisfying the aforementioned restrictions are said to be *bottom-aligned*. This class is denoted by \mathcal{P}^{BA} .

Our measure of school segregation is simple:¹⁰ We say that a school s is *fully segregated* under matching μ if it only accepts either advantaged or marginalized students, i.e. if $\mu_s^{-1} \subseteq Y$ or $\mu_s^{-1} \subseteq Z$.

Now we are ready to state the main result on this section:

PROPOSITION 5. *If $P \in \mathcal{P}^{\text{BA}}$, then*

1. *all unassigned students in DA and EADA are marginalized students.*
2. *under both DA and EADA, either 2.1) every student matched to an inferior school is marginalized, or 2.2) every marginalized student is assigned to an inferior school (or both).*
3. *any marginalized student assigned to an inferior school in DA is also matched to an inferior school in EADA.*
4. *any school that is fully segregated under DA is also fully segregated under EADA.*

PROOF. i) By stability of DA and the fact that being unassigned is the worst option for all students, it is never the case that a marginalized student is matched in DA but an advantaged one is not. But because only $m - Q$ students are unassigned, and there are at least $m - Q$ marginalized students, every unmatched student under DA must be marginalized. Proposition 2 extends this result to EADA.

¹⁰This measure is inspired by Schelling's model where segregation is measured by the number of agents who have no neighbours of different types (Schelling, 1971). The literature has also used other, more complex measures of segregation such as the Duncan index (Escobar and Huerta, 2022). See also Calsamiglia et al. (2021) for a measure of segregation based on differences in reported preferences (due to different risk attitudes) instead of differences in outcomes.

ii) Suppose 2.1 does not hold. Then some advantaged student i_n is matched to an inferior school s_b . Suppose additionally that 2.2 also does not hold. Then some marginalized student i_z is assigned to an adequate school s_a . Combining the negations to statements 2.1 and 2.2 yields that an advantaged student i_n is assigned to an inferior school s_b , even though he would prefer to be matched to some adequate school s_a , that instead accepted a marginalized student i_z over him. Such matching is clearly not stable, so DA could not have generated it.

Now we will show that EADA cannot generate the aforementioned matching either. We know that $EADA_{i_z}(P) \in A$, but from the previous point we also know that $DA_{i_z}(P) \in B$ by stability of DA. Therefore, i_z improved his DA placement by participating in a trading cycle in which some other student i' is assigned to the inferior school $DA_{i_z}(P)$. In other words, by Lemma 4, some student who was getting assigned to a good school in DA is now assigned to an inferior school in EADA, violating the Pareto improvement of EADA over DA, a contradiction.

iii) By contradiction, suppose that a marginalized student i_z is assigned to an inferior school under DA *and* to an adequate school under EADA. Because of Lemma 4, a trading cycle occurred through which a student $i_y \neq i_z$ who was assigned to an adequate school under DA is now assigned to an inferior school in EADA. But then we have a contradiction of EADA Pareto-dominating DA, concluding the proof.

iv) Consider the case that a school is fully segregated in DA, which means that either it is matched only with marginalized students (case 1) or only with advantaged students (case 2). We will prove that then it is also segregated under EADA using the previous two cases.

In case 1, there is a fully segregated school s that accepts only marginalized students under DA. By the definition of marginalized students, no advantaged student i_y applied to s during DA's execution, as otherwise i_y would have been accepted. Combining this finding with the Pareto dominance of EADA over DA, we find that for every advantaged student $i_y \in Y$, $rk_{i_y}[EADA_{i_y}(P)] \leq rk_{i_y}[DA_{i_y}(P)] < s$, and therefore s is not matched to any advantaged student in EADA, remaining fully segregated.

In case 2, some school s_1 accepts only advantaged students under DA. By contradiction, if s_1 is not fully segregated under EADA, then there is a marginalized student i_z assigned to s_1 under EADA but not under DA, and an advantaged student i_y who is assigned to s_1 under DA, but not under EADA, where he gets assigned to a more desired school s_0 .

Student i_z must have been rejected by school s_1 in DA's execution, and this rejection occurs because an interrupter student i^* applied to s_0 at round t , causing i_y to be rejected from s_0 and applying to s_1 , leading to i_z been rejected at school s_1 at round $t' > t$ (which means i^* must be an advantaged student because he has higher priority at some school than i_y). But interrupters' placement is the same in DA and EADA (Proposition 3 in Kesten, 2010), so somebody (possibly s_z) must apply later at school s_0 , leading to i^* being rejected from s_0 at time $t'' > t'$.

Let us denote an application from a student i to school s as $i \rightarrow s$, and a rejection of student i by school s as $s \rightarrow i$. Therefore, in DA's execution there is the following chain

of applications and rejections:

$$i^* \rightarrow s_0 \succ i_y \rightarrow s_1 \succ i_z \rightarrow s_2 \succ \dots \rightarrow s_0 \succ i^*$$

But if i_z caused some student i_1 to be rejected, such student is marginalized, and any student that is rejected by i_1 's applications is marginalized too, until we eventually find that i^* must be also marginalized, a contradiction. \square

The assumption that students and schools agree on which potential matches are good and bad has been imposed previously in the literature studying segregation in school choice. For example, [Calsamiglia and Miralles \(2023\)](#) consider a model with three schools and impose that one of them is universally perceived as worst. They discuss empirical evidence showing almost unanimous consensus among parents on which schools are inferior, supporting their assumption ([He, 2015](#), [Agarwal and Somaini, 2018](#), [Calsamiglia et al., 2020](#)). Similarly, [Escobar and Huerta \(2022\)](#) consider a school choice model in which the set of students is partitioned into regular and minority pupils, and the set of schools into popular (tier 1) and unpopular (tier 2) colleges. Both papers share similar conclusions to our Proposition 5, albeit focusing only on DA, instead of EADA, and under different model specifications.

7. THE EXPECTED NUMBER OF UNDER-DEMANDED SCHOOLS

We have shown that there is always at least one student who does not improve his DA placement with EADA (Lemma 2), but often we find that there is more than one such student in school choice problems. In particular, every student assigned to an under-demanded school will not improve his placement with EADA (Lemma 5). In this section, we will address the question of how many schools are under-demanded *on average* which then provides a lower bound on the number of students who do not improve their DA placement under EADA on average.

For this, we consider a *random school choice problem* where strict preferences and priorities are drawn independently and uniformly at random. Random matching problems have been useful to understand the average efficiency of several matching mechanisms ([Wilson, 1972](#), [Knuth, 1976](#), [Pittel, 1989](#), [Liu and Pycia, 2016](#), [Che and Tercieux, 2018, 2019](#), [Pycia, 2019](#), [Nikzad, 2022](#), [Ortega and Klein, 2023](#)), the expected number of stable matchings ([Pittel, 1992](#), [Immorlica and Mahdian, 2005](#)), the potential gains from strategic behaviour ([Lee, 2016](#)) and the effect of adding one more agent to a matching problem ([Ashlagi et al., 2017](#)). In this section, we will use this framework assuming (as the majority of the aforementioned literature) that the number of students and schools is equal and given by n , i.e that each school has a quota of 1, and there is no null school for students to remain unmatched.

We use $\mathbb{E}(|R_n^1|)$ to denote the expected number of students removed in the first round of EADA in a random school choice problem of size n . In this setting, by the definition of EADA, $\mathbb{E}(|R_n^1|)$ equals the number of under-demanded schools. Proposition 6 shows that the expected number of under-demanded schools and removed students in the first stage of EADA equals the n -th Harmonic number.

PROPOSITION 6. *In a random school choice problem of size n , $\mathbb{E}(|R_n^1|) = H_n := \sum_{i=1}^n \frac{1}{i}$.*

PROOF. A key result for the analysis of random matching markets is that DA can alternatively be implemented by an algorithm in which each student applies to one school in each step, proposed by [McVitie and Wilson \(1971\)](#). Whenever a student is rejected, he goes back to the queue of students who need to apply to a school. McVitie and Wilson show that, for whichever way of queueing unassigned and rejected students, this sequential algorithm generates the same matching as DA. The McVitie-Wilson algorithm (called the *fundamental algorithm* by [Knuth \(1976\)](#)) has a close resemblance to the so called *coupon collector problem*, in which n coupons are drawn with replacement n over a number of rounds until all coupons have appeared at least once. The analogy between both problems is that coupon drawing is equivalent to a random applications to schools, and both algorithms end once all coupons have been collected, i.e. once all schools have received at least one application.¹¹

We use the equivalence between the McVitie-Wilson algorithm and the coupon collector problem to bound the number of unimproved students in EADA as follows. The number of *singleton* coupons that appear exactly once in the coupon collector problem is equivalent to the number of under-demanded schools, as this is the number of schools who received exactly one application during the execution of the McVitie-Wilson algorithm. The last school to receive a proposal (resp. the last coupon to appear) is always under-demanded (resp. a singleton), but there might be some more. [Myers and Wilf \(2006\)](#) derive the joint probability distribution that there are s singleton coupons and the problem stops after r draws, and use it to compute the expected number of singleton coupons, which equals H_n . It follows that the expected number of under-demanded schools removed in the first round exactly equals H_n . \square

As an illustration of what Proposition 6 implies, consider a situation with 100 students and schools, then, in expectation, approximately 5 students (and schools) will be removed in the first iteration of the EADA algorithm. In simulations, described in Figure 1, we observe that the average number of schools removed in the first iteration of EADA in random school choice problems closely resembles our theoretical prediction.

Furthermore, it is worth highlighting that the number of students removed in the first stage of EADA is a lower bound on the number of unimprovable students. Some students may be removed in later stages of EADA and yet are unimprovable, e.g. those who are assigned to their most preferred school. This, together with Lemma 5, gives a natural corollary: namely that the expected number of unimprovable students in a random school problem of size n is at least as large as the n -th Harmonic number. Another interesting feature of Proposition 6 is that it applies to any positive integer n , and in particular to small markets too, unlike the vast majority of the results on random matching markets, which establish asymptotic results. To our knowledge, the only other small

¹¹The coupon collector problem actually takes a bit longer to terminate, because one coupon may appear more than n times before the problem terminates. This is impossible in the McVitie-Wilson algorithm, as a school can never receive more than n applications. Nonetheless, this error margin is small ([Knuth, 1976](#)).

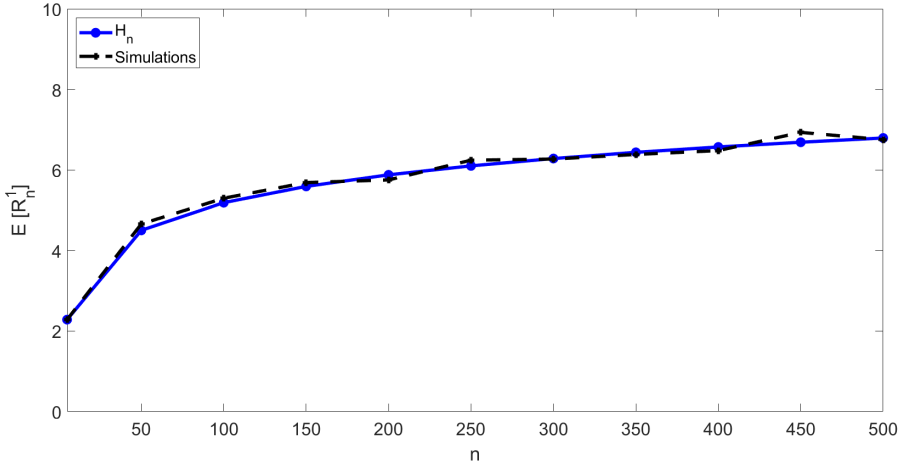


FIGURE 1. Average number of students removed in EADA's first iteration (average over 500 random school choice problems for each n).

markets result is the equivalence amongst strategy-proof and Pareto-efficient mechanisms obtained by [Pycia \(2019\)](#).

8. CONCLUSION

The Efficiency-Adjusted Deferred Acceptance (EADA) algorithm is one of the most influential mechanisms in market design literature, and for valid reasons. This paper does not criticize EADA for failing to achieve what it was not built for. Instead, our paper emphasizes that EADA's efficiency gains over DA are not evenly distributed across students, and do not benefit particular groups of pupils who are poorly placed under both mechanisms. Depending on the specific context, these limitations can have significant policy implications.

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