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Speculating on Higher Order Beliefs

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Speculating on Higher Order Beliefs

Abstract

Higher order beliefs – beliefs about others’ beliefs – may be important for trading behaviour and asset prices, but have received little systematic empirical examination. We study more than twenty years of evidence from the Robert Shiller Investor Confidence surveys, which directly elicit details on investors’ higher order beliefs about the U.S. stock market. We find that investors’ higher order beliefs provide substantial motivations for non-fundamental speculation, e.g., to buy into a stock market perceived to be overvalued. To explore the equilibrium implications, we construct a model of level k thinking that matches the evidence, where some speculative investors mistakenly believe that asset price movements are driven by other, less sophisticated investors. The model reveals that speculators’ higher order beliefs amplify stock market overreaction and excess volatility; these phenomena persist in equilibrium due to investors’ limited strategic reasoning.

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Higher order beliefs – beliefs about others’ beliefs – are important in many equilibrium models in economics and finance. As emphasized by a strand of primarily theoretical literature starting with Keynes (1936), higher order beliefs may be particularly important for understanding investor behavior and financial market fluctuations. This is because investors have strong profit incentives to trade based on predictions of others’ beliefs rather than their own valuations, potentially leading asset prices to deviate substantially from fundamental values.¹ Despite their ubiquity and potential importance, higher order beliefs have received little systematic empirical examination, primarily due to challenges in measurement.

In this paper, we examine survey data on investors’ higher order beliefs, and consider the corresponding asset pricing implications. Our data come from the Robert Shiller Investor Confidence survey, which directly elicits investors’ perceptions of other investors’ beliefs. The survey also reports investors’ return expectations, which embed additional information about higher order beliefs. This is because an investor’s return expectations reflect her forecasts of other investors’ future stock market demand, which, in turn, depend on her forecasts of their beliefs. Guided by the evidence, we construct a theoretical model that illustrates the potential importance of higher order beliefs in driving stock market fluctuations. Below, we summarize our empirical and theoretical findings.

Empirical Results. While previous work has documented episodes of non-fundamental speculation – e.g., buying into risky assets perceived to be overvalued – our headline finding is that for the U.S. stock market, *non-fundamental speculation is the rule, not the exception.*² Higher order beliefs are important for giving rise to non-fundamental speculation.

We examine the Shiller survey, which, for more than 20 years, has asked individual and institutional investors a number of questions regarding their stock market expectations. Particularly relevant for our exploration, the survey asks investors if they perceive other investors to be overly optimistic (or pessimistic) about the U.S. stock market’s prospects, as well as if they perceive the stock market to be over- or under-valued. The majority of survey respondents report that others have mistaken beliefs, with the direction of their responses aligned with their perception of the under- or over-valuation of the stock market.

We find that when investors report that others are overly optimistic, they also report expectations of higher than average returns for the short term (1-to-3 months ahead), before expecting reversion in subsequent periods. A natural, higher order belief-based interpretation is that investors forecast other investors will become even more optimistic in the near term, fueling short horizon returns, before optimism and prices revert. These expectations

¹For example, De Long et al. (1990) note that George Soros “has been generally successful... by betting not on fundamentals but, he claims, on future crowd behavior.” Brunnermeier and Nagel (2004) argue the evidence from the dot-com bubble is “consistent with the view that hedge fund managers were able to predict some of the investor sentiment that was arguably behind the wild fluctuations in valuations of technology stocks.”

²This is speculation in the sense of Keynes (1936) and Harrison and Kreps (1978). We use the prefix ‘non-fundamental’ to distinguish from another common use of the term, where investors buy or sell an asset based on their relative optimism and pessimism about the asset’s fundamental value, e.g., Simsek (2021).

provide investors with strong incentives to buy into a stock market they see as overvalued. Consistent with such speculation, surveyed investors recommend holding positions aligned with their short term expectations, and against their fundamentals-based valuations. Moreover, investors' short term return expectations have strong explanatory power for the stock market positioning of asset managers.

Exploring the source of investors' higher order beliefs, we find that these beliefs and the accompanying non-fundamental speculation emerge in response to macroeconomic news. For example, following positive macroeconomic news, investors report beliefs that others have become increasingly optimistic and markets have become overvalued. At the same time, they report expectations of higher short term returns followed by lower long term returns, which are also consistent with a belief in momentum and reversal. The evidence indicates that investors believe that others overreact to fundamental news upon its arrival and will continue to overreact in subsequent periods. This belief induces investors to engage in non-fundamental speculation.

Evaluating investors' short term return expectations, we find that non-fundamental speculation is unprofitable on average for the investors in our sample. A monthly-rebalanced market timing strategy that takes long and short positions in proportion to the average investor's reported one-month ahead return expectations earns a Sharpe ratio of -0.29. This poor performance aligns with previous work documenting a negative relationship between survey-based measures of return expectations and realized returns (e.g., [Greenwood and Shleifer \(2014\)](#)).

Model. Our empirical results pose a challenge for existing models. Most notably, models without an explicit focus on higher order beliefs cannot simultaneously explain investors' return expectations, valuations, and perceptions of others' relative optimism and pessimism.

To interpret the evidence and understand its implications for equilibrium asset prices, we construct a theoretical model that reveals that higher order beliefs and the accompanying non-fundamental speculation induce asset price overreaction and excess volatility. The model features a riskless asset and a risky asset (the stock market) that pays a publicly observed dividend each period, which is drawn from persistent but unobserved fundamentals. The economy is populated by two types of investors: speculators, whose beliefs match features of the survey data; and arbitrageurs, who jointly behave as a mass of traders with rational expectations. Each investor receives a noisy but unbiased private signal about fundamentals. In the spirit of differences-of-opinion models, each investor believes that others' signals are uninformative conditional on their own. The average investor's belief about fundamentals is consistent with rational expectations, so all excess price movements in the model come from higher order beliefs.

In our baseline specification, speculators (incorrectly) believe that all other investors are non-strategic fundamental traders, i.e., that all other investors trade based on their fundamental valuation of the risky asset and ignore the impact that other investors may have on the

price the following period. Speculators also believe that others overestimate the persistence of the fundamentals process. This specification is sufficient to match the survey evidence on non-fundamental speculation. A belief that the economy is comprised of fundamental investors that overestimate the persistence of fundamentals leads speculators to believe that the risky asset tends to become overvalued whenever fundamentals are strong, due to other investors' over-optimism. However, given that the economy is composed of a combination of speculators and arbitrageurs with correct beliefs on average, the risky asset consistently exhibits less overvaluation than speculators expect. Accordingly, speculators infer that other investors received attenuated signals about fundamentals, and forecast that the risky asset will become even more overvalued in subsequent periods as other investors fully internalize information about fundamentals.³ Based on their forecasts, speculators willingly buy into an asset they perceive is overvalued, in the process *causing* its overvaluation. In subsequent periods, the risky asset price declines, as speculators' forecasts of increasing overvaluation do not manifest.

We next generalize our model to allow speculators to recognize that other investors are strategic, by embedding our baseline specification into a model of level k thinking. We find that unsophisticated equilibrium reasoning is crucial for explaining the evidence. Level k thinking is an idea developed in the experimental literature, where agents believe other agents to be less strategically sophisticated than themselves (see [Crawford, Costa-Gomes and Iriberry \(2013\)](#) for a review). In our setting, we define our baseline specification as a level 1 equilibrium, where speculators believe that all other investors are strategically unsophisticated level 0 thinkers (fundamental traders). A level k equilibrium is defined recursively as one featuring speculators who perceive prices as arising from a level $k - 1$ equilibrium. A higher k captures more sophisticated thinking, in the sense of incorporating more rounds of strategic reasoning. In this setup, speculators' strategic mistake can be summarized as follows: they believe that they are arbitrageurs taking advantage of level $k - 1$ speculators, when in fact, they are level k speculators trading against arbitrageurs that rationally engage in one more level of strategic reasoning.

We find that every level k equilibrium qualitatively matches the empirical evidence. That is, the model matches the patterns in the data as long as there is a seed of a belief that other speculators (believe that other speculators believe that other speculators...) believe that investors overestimate the persistence of fundamentals. Importantly, however, for a given set of model parameters, as we increase strategic sophistication, asset price overreaction and excess volatility fall. In the limit, as speculators' sophistication becomes unbounded ($k \rightarrow \infty$), as assumed in traditional models, the risky asset price converges to the rational expectations value, and non-fundamental speculation disappears. In order to rationalize non-fundamental

³The features that investors attribute to other investors' beliefs – that they overestimate the persistence of fundamentals and persistently update their beliefs in the direction of past news – are the same frictions that can explain forecast errors of macroeconomic forecasters ([Angeletos, Huo and Sastry \(2021\)](#)).

speculation, investors must lack a sophisticated understanding of equilibrium typically assumed in traditional models.

We can understand this point as follows: level k speculators recognize that non-fundamental speculation induces the risky asset to become overvalued whenever fundamentals are positive. They still buy into the market, because their underestimation of other investors' strategic sophistication means that they observe less overvaluation than they expect. They accordingly infer that others received an attenuated signal about fundamentals and that the risky asset will become increasingly overvalued in the near term, as other investors update their beliefs. However, as we increase speculators' strategic sophistication, they increasingly recognize that the current price is driven by speculators buying into the market in anticipation of increasing overvaluation that does not reliably arrive. This recognition leads speculators to decrease their expected returns from non-fundamental speculation, in turn decreasing the degree of overvaluation of the risky asset. As they engage in multiple iterations of such reasoning, speculators fully internalize non-fundamental speculation as the cause of asset price overvaluation and as being unprofitable, and they become arbitrageurs. The risky asset price converges to the rational expectations value. The fact that we observe evidence consistent with non-fundamental speculation indicates the lack of such reasoning by speculators.

In addition to highlighting that higher order beliefs and imperfect strategic sophistication give rise to overreaction and excess volatility, we also use our model to understand the interaction between higher order beliefs and fundamental beliefs. In isolating the role of higher order beliefs, our model presents a straightforward way to understand how higher order beliefs may interact with fundamental belief biases documented in the literature. For example, if investors' fundamental beliefs overreact to news, as suggested by recent work, then higher order beliefs and fundamental beliefs reinforce one another to produce even stronger equilibrium overreaction of asset prices.

Related Literature. Our paper relates to work on higher order beliefs in asset pricing, to which we bring empirical discipline using survey data.⁴ Previous work can be partitioned into two traditions: noisy rational expectations models, where rational investors face frictions that prevent them from observing others' beliefs and fundamentals (Singleton (1987), Allen, Morris and Shin (2006), Bacchetta and van Wincoop (2006, 2008), Makarov and Rytchkov (2012), Kasa, Walker and Whiteman (2014), Cespa and Vives (2015), and Nimark (2017)); and differences-of-opinion models, where investors know and disagree with other investors' valuations (Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003), and Banerjee and Kremer (2010)).

Relative to both, our paper presents empirical evidence on higher order beliefs, and seeks to model higher order beliefs in a manner consistent with the evidence. In doing so, we find

⁴There are two notable exceptions that empirically examine higher order beliefs. Egan, Merkle and Weber (2014) survey private investors and find that beliefs about others' return expectations affect investment decisions. Coibion et al. (2021) survey firm managers in New Zealand, with a focus on testing noisy information models.

relaxations of the underlying assumptions of both types of models to be important. Noisy rational expectations models typically assume full depth of reasoning ($k = \infty$) and suggest that investors only speculate on information that is orthogonal to public information; we find that both are inconsistent with the evidence. Differences-of-opinion models typically assume investors' beliefs to be common knowledge and do not give rise to non-fundamental speculation. In contrast, we find that incorrect higher order beliefs may be important for rationalizing the empirical evidence (see also [Banerjee, Kaniel and Kremer \(2009\)](#) and [Han and Kyle \(2018\)](#) for models featuring incorrect higher order beliefs). Our paper also highlights survey evidence that may be useful for future work – namely direct questions about investors' higher order beliefs, as well as data on the term structure of expected returns, which help pin down investors' beliefs about the dynamics of other investors' beliefs.

Our modeling approach contributes to a small, but growing literature in finance and macroeconomics on level k thinking and errors in strategic reasoning. In finance, previous models study investors neglecting the information content of prices ([Eyster, Rabin and Vayanos \(2019\)](#)) and neglecting that other investors may learn about fundamentals from prices ([Bastianello and Fontanier \(2024, 2022\)](#)), with implications for trading volume and asset price fluctuations. We find that level k thinking helps explain investors' higher order beliefs and return expectations, and can rationalize unprofitable non-fundamental speculation that drives excess volatility.⁵ Our results also comport with a nascent empirical literature in finance documenting investors' neglect of general equilibrium effects in formulating their return expectations ([Andre, Schirmer and Wohlfart \(2023\)](#), [Bybee \(2023\)](#)).

Our paper also relates to a literature on non-fundamental speculation in financial markets, where investors willingly buy into markets they see as overvalued. The literature documents a number of instances of non-fundamental speculation in bubble-like episodes (e.g, [McKay \(1841\)](#), [Kindleberger \(1978\)](#), [Temin and Voth \(2004\)](#), [Brunnermeier and Nagel \(2004\)](#), [Soros \(2015\)](#)) and prominent theoretical work on the topic includes [De Long et al. \(1990\)](#), [Harrison and Kreps \(1978\)](#), [Scheinkman and Xiong \(2003\)](#), [Abreu and Brunnermeier \(2002, 2003\)](#), and [Martin and Papadimitriou \(2022\)](#). Our paper illustrates that non-fundamental speculation is a pervasive feature of the U.S. stock market, and illuminates how higher order beliefs give rise to such speculation. Moreover, while the literature focuses on episodes of non-fundamental speculation where informed investors (e.g., hedge funds) may have profited, we find evidence that non-fundamental speculation is unprofitable for the investors in our sample.

Finally, our paper is related to a literature in finance using survey data to understand market participants' beliefs ([Adam and Nagel \(2022\)](#) provide a survey). A sizeable literature has studied the importance of fundamental and return expectations for investor behavior

⁵Theoretical work on level k thinking in macroeconomics includes [Farhi and Werning \(2019\)](#) and [García-Schmidt and Woodford \(2019\)](#), who argue that level k thinking leads to underreaction of macroeconomic aggregates in response to shocks. [Angeletos and Lian \(2022\)](#) discuss the relationship between macroeconomic models of level k thinking and models of incomplete information.

and stock market returns. Work on return expectations includes [Vissing-Jorgensen \(2003\)](#), [Bacchetta, Mertens and van Wincoop \(2009\)](#), [Greenwood and Shleifer \(2014\)](#), [Amromin and Sharpe \(2014\)](#), [Barberis et al. \(2015, 2018\)](#), [Adam, Marcet and Beutel \(2017\)](#), and [Nagel and Xu \(2022b\)](#), while work on fundamental expectations includes [Chen, Da and Zhao \(2013\)](#), [Bordalo et al. \(2020\)](#), [De La O and Myers \(2021\)](#), and [Nagel and Xu \(2022a\)](#).⁶ Our results bridge together fundamental and return expectations via higher order beliefs. See also [Giglio et al. \(2021\)](#), [Jin and Sui \(2022\)](#), and [McCarthy and Hillenbrand \(2021\)](#) for work relating fundamental and return expectations.

The rest of the paper proceeds as follows. In Section 1, we discuss the survey data and present empirical evidence on investors' short horizon return expectations. In Section 2, we present a theoretical asset pricing model that interprets the empirical evidence and explores its equilibrium implications. Section 3 concludes.

1 Empirical Evidence from Survey Data

We study expectations of U.S. equity market returns reported by retail and institutional investors in the Robert Shiller Investor Confidence survey. Whereas other available investor surveys mostly ask a single or a few questions to investors about their return expectations at a fixed horizon, the Shiller survey is unique in providing a long time-series where investors are simultaneously asked about their higher order beliefs, their stock market valuations, and their return expectations over multiple horizons. This makes it particularly well-suited for studying the questions of interest.

We find that investors often believe that other investors hold incorrect stock market valuations, but find it profitable to speculate in the direction of these incorrect valuations. We also find that investors report a belief that stock markets overreact to news upon its arrival, and report return expectations that are consistent with the stock market exhibiting momentum and reversal. We discuss the ingredients required of higher order beliefs to match the empirical evidence, and the predictions made by existing models, which are able to match some, but not all of the evidence.

1.1 Data Description

The main data used in our empirical analysis come from the Shiller surveys, which are the microdata underlying the Robert Shiller Stock Market Confidence indices. The survey data have been collected continuously since 1989 – semi-annually for a decade, and then monthly

⁶Prior work largely focuses on expectations of returns at a fixed future horizon (e.g., one-year ahead). We focus on the future path of expected returns. In contemporaneous work, [Gandhi, Gormsen and Lazarus \(2023\)](#) also study the term structure of return expectations. Our evidence complements and provides a potential explanation for their finding that investors overestimate the persistence of returns.

by the International Center for Finance at the Yale School of Management since July 2001. [Shiller \(2000\)](#) discusses the survey questions in more detail. For our analysis, we focus on the continuous sample from July 2001 through April 2023.

1.1.1 Survey Respondents

The surveys are conducted by a market survey firm, which mails 500 surveys to high net-worth individual investors, and 500 surveys to institutional investors each month, with a sampling goal of 20 to 50 responses by each of the two types - individual and institutional. For both institutional and individual investors, the investor mailing lists are purchased from Data Axle (previously known as InfoUSA).

The micro data do not provide detailed demographic information on survey respondents (or non-respondents). There is likely to be selection into responding to the survey, as in other work surveying investors. For example, [Giglio et al. \(2021\)](#) find in a survey of Vanguard investors that their survey respondents tend to be older, wealthier, more likely to be male, and tend to trade more often than nonrespondents. The selection criteria for both individual and institutional investors in the Shiller survey, and the data that are available on investor characteristics, indicate that individual investor respondents are likely to have high income and be wealthy, and that institutional respondents manage large portfolios. While likely not fully representative of the investor population, survey respondents are a substantial and important class of investors. In [Appendix C.1](#), we test and find no evidence of systematic business cycle variation in the number of responses to the survey.

For individual investors, the mailing list for the surveys is constructed by sampling households with a household income of greater than \$150,000 per year from the Infogroup Consumer Database. We have no additional demographic information on the respondents.

For institutional investors, the mailing list is constructed by sampling companies from the Infogroup Business Database with the SIC codes 628202 (Investment Management), 628203 (Financial Advisory Services), 628204 (Financing Consultants), and 628205 (Financial Planning Consultants). Survey respondents are asked to provide the ‘Size of the common stock portfolio(s) you make decisions about.’ In the sample, the 25th, 50th, and 75th percentiles of responses are \$1.8 million, \$25 million, and \$100 million. Summing across respondents by month, the 25th, 50th, and 75th percentiles of the sum of responses are \$642 million, \$1.57 billion, and \$9.00 billion.

1.1.2 Questions of Interest

Especially relevant to us, the survey data contain investor responses to a series of questions regarding investors’ beliefs about other investors’ beliefs, their beliefs about stock market valuations, and their expectations of returns over different time horizons. These questions

include (with potential responses in parentheses):

(i) Questions regarding higher order beliefs

- (a) *Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them (True; False; No opinion).*
- (b) *Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them (True; False; No opinion).*

(ii) Questions regarding stock market returns, valuations, and behavior

- (a) *How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?*
- (b) *Stock prices in the United States, when compared with measures of true fundamental value or sensible investment value are (Too low; Too high; About right; Do not know).*
- (c) *Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while (True, False, No opinion; if True, indicate best guess for date of peak).*
- (d) *Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while (True, False, No opinion; if True, indicate best guess for date of bottom).*
- (e) *If the Dow dropped 25% over the next six months, I would guess that the succeeding six months, the Dow would: (Increase (Give percent), Decrease (Give percent), Stay the same, No opinion).*

(iii) Questions regarding drivers of higher order beliefs

- (a) *What do you think is the cause of the trend of stock prices in the United States in the past six months (It properly reflects the fundamentals of the U.S. economy and firms; It is based on speculative thinking among investors or overreaction to current news; Other (write-in); No opinion)?*

Answers to question (i.a) and (i.b) directly provide information regarding investors' higher order beliefs. The questions themselves may submit multiple interpretations, especially given the multi-part nature of the questions, and the fact that respondents may answer "True" to both questions. We present cross-sectional and time-series evidence in this section that when investors respond that they believe many others are overly optimistic and they must be careful not to be influenced by them, they also report beliefs that stock markets are overvalued, and that they expect long term stock market returns to be low. Based on this evidence, investors appear to interpret these questions as asking:

- (i.a) *I believe that many other investors hold overly optimistic stock market valuations.*

(i.b) *I believe that many other investors hold overly pessimistic stock market valuations.*⁷

While there may be some noise associated with assigning this interpretation to the question – it is likely that some respondents may have differing interpretations – the interpretation is consistent with responses to other questions on average and is informative about investors’ views. This is especially the case in light of the long time series of evidence relative to other surveys that may ask similar questions.

Answers to questions (ii.a-e) provide information about investors’ return expectations at different horizons, and the behavior associated with those expectations. These questions provide additional validation for our interpretation of the questions regarding investors’ higher order beliefs. Moreover, the questions on investors’ return expectations at different horizons encode additional information about investors’ higher order beliefs, in particular capturing how investors expect others’ beliefs to evolve over time. Questions (ii.c-e) also link investors’ higher order beliefs and return expectations with potential speculative trading behavior, as we discuss in more detail.

While questions (ii.a) and (ii.e) ask investors to explicitly report numerical values for their expected stock market returns, the other questions are provided in multiple choice format. For empirical analysis, we map answers for most of the questions to numerical values. For questions (i.a-b) and (ii.c-d), we map the responses (True; False; No opinion) to (1, -1, 0), so that higher numbers indicate increasing agreement with the questions. For question (ii.b), we map the responses (Too low; Too high; About right; Do not know) to (-1; 1; 0; 0), so that lower numbers correspond with perceived undervaluation and higher numbers correspond with perceived overvaluation; we find our results are robust to dropping observations where investors report ‘Do not know.’

1.2 Summary Statistics

We begin by summarizing responses in the Shiller survey. Table 1 reports summary statistics over the full sample in response to the relevant questions. For all questions except for questions (ii.a) and (ii.e), the table reports the proportion of respondents in the sample that gave a specific answer in response to that question; for question (ii.a) and (ii.e), the table reports expected returns averaged across survey respondents. The table reports statistics separately for individual and institutional investor respondents; the results are qualitatively and quantitatively similar for the two groups.

Focusing on the first two rows, 59% of individual investors report that they believe many others to be overly optimistic and 62% report that they believe many others to be overly pes-

⁷There is particular ambiguity about the meaning of the second part of the questions – ‘I must be careful not to be influenced by them.’ Based on the evidence that we present in this section comparing investors’ responses across questions, particularly questions (ii.c) and (ii.d), investors seem to indicate that they believe others’ valuations may be overly- optimistic or pessimistic, but nevertheless that others’ valuations still enter into their investment decisions given the influence that others have on short term market returns.

simistic (for institutional investors, the proportions are 52% and 62%). These results indicate that several respondents simultaneously indicate a belief that many others are overly optimistic and overly pessimistic.⁸ They also suggest the presence of substantial disagreement about the stock market; the majority of respondents in the sample report that other investors have incorrect beliefs.

Looking to the third and fourth rows, 35% of individual investors report that they expect the stock market to eventually drop but recommend being overweight, while 35% report that they expect markets to eventually rise, but recommend being underweight (the numbers are 33% and 32% for institutional investors). Given that the questions are logically mutually exclusive, the responses suggest that a substantial majority of investors expect short and long term returns to be differently signed.⁹ This result is notable, because it suggests the presence of non-fundamental speculative motivations on the part of investors, e.g., many investors that believe the stock market to be presently overvalued (and that market returns will be low in the long run) still recommend being overweight stocks due to potential short term profits from doing so.

The fifth row displays investors' return expectations over different horizons as reported on the survey (labeled total) and the sixth row reports return expectations in excess of the corresponding maturity U.S. Treasury bill rate (labeled excess). Individual and institutional investors report small return expectations for the next month, with an expectation of more positive returns for 3- to 6-months ahead. Total return expectations for 12-months ahead are 3.7% on average for individual investors and 4.9% on average for institutional investors; average excess return expectations are 2.1% and 3.5% annualized.¹⁰ Regarding prices vis-a-vis fundamentals, 11% of individual investor respondents indicate that stock valuations are low relative to fundamentals, 37% say they are high, 44% say they are about right, and 8% express no opinion (these numbers are 19%, 30%, 49% and 2% for institutional investors). Regarding recent stock market trends, of individual investor responses, 24% indicate that market movements properly reflect fundamental news, while 52% indicate that the movements reflect speculative thinking and overreaction by other investors; these numbers are 28% and 37% among institutional investors. And lastly, given the hypothetical situation where the stock market drops 25% in the next 6 months, individual investors expect returns in the sub-

⁸The simultaneity can be seen by the fact that the sum of the proportions of investors reporting that others are overly optimistic and pessimistic is greater than 100%. This simultaneity does not affect our analysis, but does suggest that investors generally perceive that others have more extreme beliefs than they do.

⁹By-and-large, survey respondents recognize that the questions are mutually exclusive, though some do not. Only 7.2% of institutional investors and 7.8% of individual responses are *True* to both questions (ii.c) and (ii.d) at the same time. Among individual investors, 71% that respond *True* to (ii.c) response *False* to (ii.d) and 73% that report *True* to (ii.d) report *False* to (ii.c); these numbers are 70% and 72% for institutional investors.

¹⁰The small average expected excess return suggests that investors expect a small equity risk premium; or (not exclusively) that some investors may implicitly subtract the risk-free rate when reporting return expectations. In further analyses, we use expected excess returns, but the results are not sensitive to the choice of using total versus excess returns.

sequent 6 months to be +13.6% on average, and institutional investors expect returns to be +16.9%. This is consistent with investors believing that stock market declines reflect overreaction that will revert in the intermediate term.

1.3 Higher Order Beliefs and Perceived Market Valuations

We next examine responses to questions (i.a) and (i.b), regarding other investors' optimism and pessimism. We provide evidence consistent with respondents generally interpreting the questions to indicate that other investors are *overly* optimistic or *overly* pessimistic in their stock market valuations.

We compute a perceived *Overvaluation* variable by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). Higher values of *Overvaluation* correspond with higher stock market prices relative to fundamentals. We regress *Overvaluation* on two measures of higher order beliefs: *Higher Order Optimism* and *HO Pessimism*. We construct these variables by mapping the responses to questions (i.a) and (i.b.), (True; False; No Opinion), to the values (1; -1; 0). The *HO Optimism* variable is increasing in agreement with the statement that other investors are overly optimistic, while the *HO Pessimism* variable is increasing in agreement with the statement that other investors are overly pessimistic.

Table 2 reports the regression results. Columns 1 to 3, on which we focus in our discussion, report regression results pooling together individual and institutional investor responses. Columns 1 and 2 report results from regressions using survey response level observations. With month fixed effects (column 2), the regressions capture cross-sectional comparisons, for example, whether an investor that believes others are more optimistic also is more likely to believe the stock market is overvalued. We also run the regressions as time-series regressions (column 3), by using cross-sectional monthly averages of the variables as the observations in our regressions. The time-series regressions capture whether, for example, in time periods where investors believe others to be more optimistic, they are also more likely to believe that markets are overvalued, consistent with what we would expect. Variables in the regressions are standardized to have zero mean and unit standard deviation, so that the coefficients can broadly be interpreted as correlations.

Panel A reports results where *HO Optimism* is the independent variable, Panel B reports results where *HO Pessimism* is the independent variable, and Panel C reports results where the independent variable is *HO Belief*, which we define as $HO\ Optimism - HO\ Pessimism$. We find consistent evidence of a strong relationship in the expected direction for each of the regression specifications. We focus our discussion on Panel C, where the measure incorporates information from their responses about others' optimism and pessimism. The coefficients are 0.28 in the response-level regression with no fixed effects, 0.23 in the response-level regression with time fixed effects, and 0.59 in the regression using monthly averages as observations, with R^2

	Individual				Institutional			
	True	False	No Opin.		True	False	No Opin.	
<i>(i.a) Others overly optimistic about stocks</i>	59%	28%	13%		52%	32%	16%	
<i>(i.b) Others overly pessimistic about stocks</i>	62%	23%	15%		62%	23%	16%	
<i>(ii.c) Expect eventual drop but overweight</i>	35%	54%	11%		33%	56%	12%	
<i>(ii.d) Expect eventual rise but underweight</i>	35%	52%	12%		32%	55%	13%	
	1M	3M	6M	12M	1M	3M	6M	12M
<i>(ii.a) Percent expected return (total)</i>	-0.2%	0.5%	1.4%	3.7%	0.1%	0.6%	2.0%	4.9%
<i>Percent expected return (excess)</i>	-0.3%	0.2%	0.7%	2.1%	0.0%	0.3%	1.4%	3.5%
	Low	High	Right	No Op.	Low	High	Right	No Op.
<i>(ii.b) Stock prices vs. fundamental value are...</i>	11%	37%	44%	8%	19%	30%	49%	2%
	Value	Overreac.	Other	No Op.	Value	Overreac.	Other	No Op.
<i>(iii.a) Cause of 6-month market trend</i>	24%	52%	22%	2%	28%	37%	34%	1%
<i>(ii.e) Expected 6M return after 25% drop</i>			13.6%				16.9%	
<i>Total Number of Responses</i>			6688				5591	

TABLE 1: SHILLER SURVEY SUMMARY STATISTICS

Note: The table reports summary statistics of survey responses to the Shiller surveys, reporting statistics separately for individual and institutional investor respondents. For questions (i.a,b), (ii.b,c,d) and (iii.a), the table reports the proportion of survey respondents in the sample that gave a particular answer in response to a given question. For questions (ii.a) and (ii.e), the table reports the average h -month ahead expected return reported by respondents. For (ii.a), we return expectations both as provided on the survey (labeled total), and also in excess of the h -month U.S. Treasury Bill rate (labeled excess). Questions and potential responses are presented in abbreviated form in the table; details on the questions and responses are provided in Section 1.1.

values of 0.08, 0.17, and 0.34.

For the rest of our empirical analysis in the main text, we focus on the *HO Belief* measure. In each of the panels in Table 2, columns 4-6 report regression results using the sample of individual investors and columns 7-9 report regression results using the sample of institutional investors. The results are qualitatively the same across the individual and institutional investor samples, with some slight quantitative variation. For brevity, we report results in the pooled sample for the rest of the paper and provide additional breakdowns of the results for the individual versus institutional investor samples in the appendix. We note that our main conclusions are largely similar for individual and institutional investors.

The results indicate a strong relationship between the *HO Belief* and the *Overvaluation* measures; when investors report that they think other investors are more optimistic, they are substantially more likely to report that the stock market is overvalued. To better understand investors' valuations and higher order beliefs, Figure 1 plots the quarterly averages of investors' higher order beliefs and valuations over time.

Focusing on *HO Belief* in the top panel of the figure, we see that it exhibits intuitive peaks and troughs related to the broader macroeconomy. We provide context for this variation by looking to survey respondents' open-ended responses on the survey.¹¹ For example, *HO Belief* exhibits a trough in late 2002 and early 2003 (with respondents discussing the Iraq war and geopolitical uncertainty); in 2008-2009, corresponding with the Great Financial Crisis; in late 2011, corresponding with U.S. and European sovereign debt concerns; in Q1 2016, coinciding with concerns about oil prices and slowing Chinese Growth; and in Q2 2022 (with respondents discussing supply chain issues/inflation, and the Ukraine war). Meanwhile, *HO Belief* exhibits peaks in Q2 2007 (in advance of the October 2007 local peak in the Dow Jones Industrial Average); in Q2 2013 (with respondents discussing the impact of quantitative easing); and more generally, is elevated from 2013 through 2021, coinciding with a period of low interest rates and strong stock market performance.

The bottom panel of the figure plots *Overvaluation*, which exhibits related, though not identical, variation to *HO Belief*. Most notably, while *Overvaluation* tends to rise and fall at similar periods as *HO Belief*, it has been more persistently elevated in the period of low interest rates and rising valuations following the Great Financial Crisis. In Section 1.6, we more systematically analyze the dynamics of *HO Belief* and *Overvaluation*, and find that they and investors' return expectations exhibit systematic co-variation with macroeconomic news, rising in periods of good macroeconomic news and falling in periods of poor macroeconomic news.

¹¹These open-ended responses come from people that mark 'Other' in response to question (iii.a) above, regarding the driver of stock prices, as well as a "General Comments" section of the survey. A nascent literature in economics and finance has begun systematically analyzing open-ended survey responses, e.g., see [Haaland et al. \(2024\)](#).

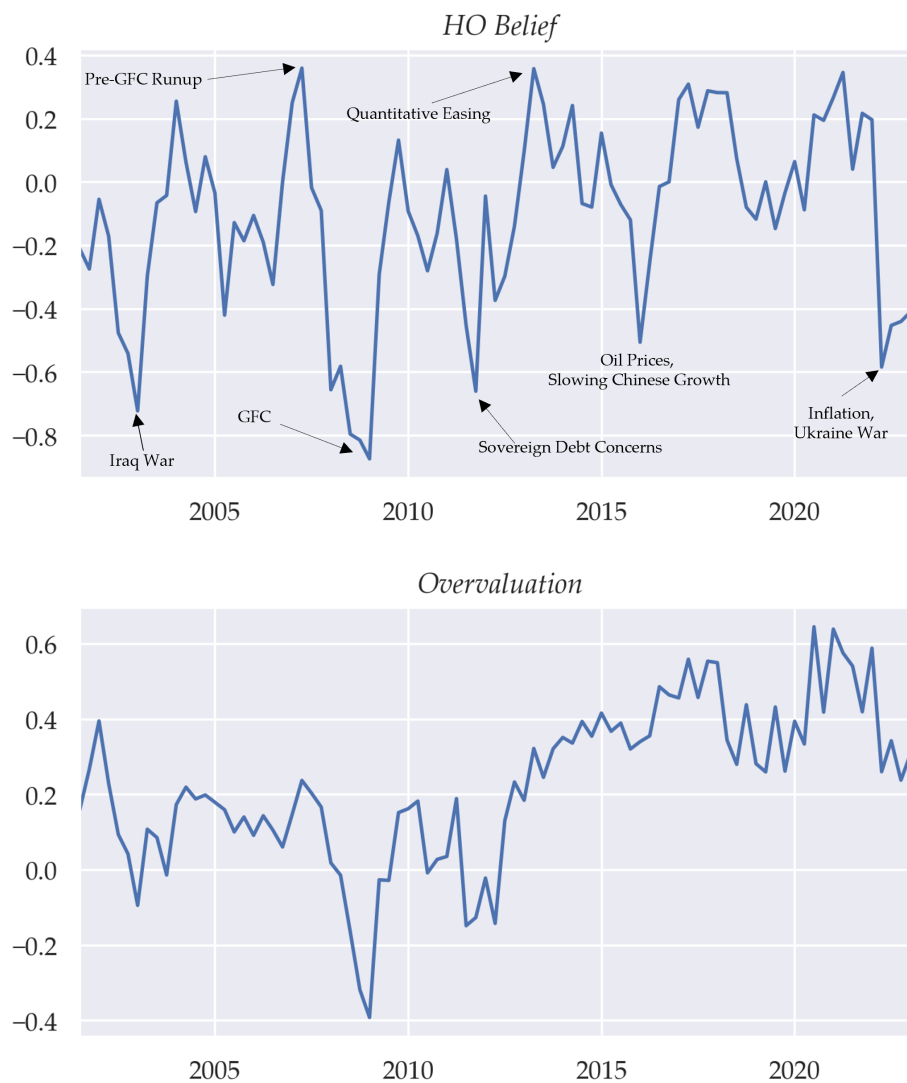


FIGURE 1: HIGHER ORDER BELIEFS AND PERCEIVED VALUATIONS

Note: The figure plots the time-series of quarterly averages of a *Higher Order Belief* measure and a perceived *Overvaluation* measure. The *Overvaluation* measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals, (Too low; Too high; About right; Do not know), to the values (-1; 1; 0; 0). The *HO Optimism* and *HO Pessimism* measures are constructed by mapping the responses to questions (i.a) and (i.b) regarding other investors' optimism and pessimism, (True; False; No Opinion), to the values (1; -1; 0).

Panel A: $x=HO$ Optimism									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>HO Optimism</i>	0.25 (0.02)	0.20 (0.01)	0.61 (0.10)	0.19 (0.02)	0.14 (0.01)	0.54 (0.10)	0.31 (0.02)	0.27 (0.02)	0.49 (0.09)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.06	0.16	0.37	0.04	0.15	0.30	0.10	0.22	0.24
Panel B: $x=HO$ Pessimism									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>HO Pessimism</i>	-0.11 (0.01)	-0.09 (0.01)	-0.28 (0.12)	-0.10 (0.02)	-0.08 (0.02)	-0.13 (0.10)	-0.14 (0.02)	-0.11 (0.01)	-0.20 (0.10)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.01	0.13	0.08	0.01	0.14	0.02	0.02	0.16	0.04
Panel C: $x=HO$ Belief= HO Optimism - HO Pessimism									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>HO Belief</i>	0.28 (0.02)	0.23 (0.01)	0.59 (0.11)	0.22 (0.02)	0.17 (0.01)	0.48 (0.10)	0.34 (0.02)	0.29 (0.02)	0.52 (0.11)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.08	0.17	0.34	0.05	0.16	0.23	0.12	0.23	0.28

TABLE 2: HIGHER ORDER BELIEFS AND PERCEIVED VALUATIONS

Note: The table reports results from regressions of a perceived *Overvaluation* measure constructed from the Shiller surveys on *Higher Order Belief* variables constructed from the surveys. The *Overvaluation* measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals, (Too low; Too high; About right; Do not know), to the values (-1; 1; 0; 0). The *HO Optimism* and *HO Pessimism* measures are constructed by mapping the responses to questions (i.a) and (i.b) regarding other investors' optimism and pessimism, (True; False; No Opinion), to the values (1; -1; 0). Columns 1-3 pool together observations across the individual and institutional investor samples, and columns 4-6 and 7-9 separately report results for the two samples. The unit of observation for Columns 3, 6, and 9 is the monthly cross-sectional average of the variables; Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observations for columns 1, 2, 4, 5, 7, and 8 are survey responses; Driscoll-Kraay standard errors (12 lags) for coefficients are reported in parentheses.

1.4 *HO Belief* and Return Expectations

We next turn to studying the relationship between perceptions of other investors' optimism and return expectations of different horizons. Responses to questions (i.a) and (i.b) capture investors' beliefs about other investors' *present* beliefs. Return expectations capture how stock prices are expected to evolve in the future, embedding beliefs about other investors' *future* stock market demand and beliefs. That is, return expectations capture an additional dimension of higher order beliefs that is important for understanding investor behavior.

We regress investors' reported excess return expectations of different horizons (multiplied by 100) on the *HO Belief* variable. Panel A of Table 3 reports the regression results.

The first four columns report regression results for time-series regressions, where the unit of observation is the cross-sectional average of survey responses in a given month. The coefficient on *HO Belief* for 1-month return expectations is 1.61, indicating that a unit increase in the *HO Belief* variable corresponds with a 1.61% higher expected return for the following month. The coefficients decline with horizon; the coefficients on 3-, 6-, and 12-month return expectations are 0.69, -0.12, and -2.07. Appendix Table C.4 verifies that these results hold for both *HO Optimism* and *HO Pessimism* as independent variables.

The results reveal an interesting, hump-shaped pattern of cumulative return expectations corresponding with *HO Belief*. On average, in periods where investors report a belief that other investors are overly optimistic (and that stock markets are overvalued), they expect stock markets to rise over the next month – they expect stock market demand to rise. Though other factors may contribute, a natural, higher order belief-based interpretation is that investors expect others to become even more optimistic in the short term, fueling the increasing demand.

These expectations provide motivations for non-fundamental speculative trade, whereby investors may choose to take long (or overweight) positions in the stock market even when they perceive it to be overvalued, because they perceive the market may continue to rise before valuations correct and returns are lower.

The last four columns in Panel A of Table 3 report regression results for cross-sectional regressions, which include month fixed-effects, and where the unit of observation is a survey response. The coefficients for 1-, 3-, 6-, and 12-month ahead returns in the cross-sectional regressions are 0.05, -0.30, -1.01, and -1.76. The results indicate that in cross-sectional comparisons, an investor that holds a stronger belief that others are overly optimistic does not necessarily believe that short term returns will be higher than an investor that holds a weaker belief that others are optimistic, though they do expect worse long term market performance.

We provide additional validation for the time-series pattern of investors' return expectations by using investors' responses to questions (ii.c) and (ii.d), which ask investors whether they expect the stock market to reach a peak (trough) in the short run though they expect it to decline (rise) in the long run. We construct *Short-Term Peak* and *ST Trough* variables by

mapping responses to questions (ii.c) and (ii.d), (True; False; No Opinion), to the values (1, -1, 0). We run time-series regressions of *ST Peak* and *ST Trough* on *HO Belief* and *Overvaluation*, with cross-sectional monthly averages as the unit of observation.

Panel B of Table 3 reports the regression results. The first two columns report results where the independent variable is *HO Belief*. The coefficient on *HO Belief* is 0.41 for *ST Peak* (R^2 of 0.22) and -0.28 for *ST Trough* (R^2 of 0.16). These results provide important validation of the relationship between investors' beliefs regarding others' optimism and their belief that markets will continue to rise before eventually declining, using a qualitative elicitation method. This evidence also helps assuage potential concerns with respondents struggling in providing quantitative responses.

Affirmative responses to (ii.c) and (ii.d), used to construct the *ST Peak* and *ST Trough* measures, also indicate investors' recommendations to be overweight stocks even though they expect an eventual decline in stocks, or underweight despite expecting an eventual rise in stocks. The relationship between *HO Belief*, *ST Peak*, and *ST Trough* provide further evidence in support of non-fundamental speculation induced by higher order beliefs.

The last two columns in Panel B of Table 3 reports results where the independent variable in the regression is *Overvaluation*, capturing investors' beliefs that the market is overvalued. We observe a similarly strong relationship between *ST Peak* and *Overvaluation* (coefficient of 0.65, R^2 of 0.26), though a weaker relationship between *ST Trough* and *Overvaluation* (coefficient of -0.06, R^2 of 0.00).

1.5 Non-Fundamental Speculation

The evidence in this section indicates that investors have an incentive to engage in non-fundamental speculation, e.g., to 'ride the bubble' and buy into a stock market they perceived as overvalued due to the expectation of short term positive returns. Indeed, survey respondents' affirmative responses to questions (ii.c) and (ii.d), advising taking positions in the stock market that are opposite their long term stock market return expectations, provide support for non-fundamental speculation motives. An important question is whether the expectations data actually capture investors' trading behavior, and in particular, whether investors speculate based on their short term return expectations.

We provide additional evidence of non-fundamental speculation using data on futures positions. We find that the trading behavior of buy-side investors (asset managers, hedge funds, etc.) tracks short horizon return expectations reported in the Shiller survey. Investors increase their market exposure corresponding with higher short term return expectations, and reduce their market exposure corresponding with lower short term expectations.

We obtain weekly data on the positions of investors in Dow Jones Industrial Average (DJIA) and S&P500 equity index futures from the Traders in Financial Futures report from the Commodity Futures Trading Commission. The data have been published weekly since

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	1.59 (0.31)	0.66 (0.43)	-0.14 (0.58)	-2.07 (0.91)	0.05 (0.04)	-0.30 (0.06)	-1.01 (0.09)	-1.76 (0.12)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.14	0.02	0.00	0.05	0.00	0.00	0.02	0.04

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>		<i>ST Trough</i>	
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.41 (0.07)	-0.28 (0.04)		
<i>Overvaluation</i>			0.65 (0.12)	-0.06 (0.15)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.22	0.16	0.26	0.00

TABLE 3: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS

Note: Panel A of the table reports results from regressions of cumulative return expectations on the *Higher Order Belief* variable constructed from the Shiller surveys, pooling together observations across individual and institutional investors. Each column, labeled $\mathbb{E}_t(R_{t,t+k})$ represents cumulative return expectations in month t for returns from month t to month $t+k$. The unit of observation in the first four columns is the monthly cross-sectional average of survey responses. Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observation for the last four columns are individual survey responses. Driscoll-Kraay standard errors (12 lags) of coefficients are reported in parentheses. Panel B of the table reports regressions of *Short Term Peak* and *ST Trough* on the *HO Belief* and *Overvaluation* measures. The variable *ST Peak* is constructed from question (ii.c), which asks whether investors expect markets to eventually fall but reach a peak in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The variable *ST Trough* is constructed from question (ii.d), which asks whether investors expect markets to eventually rise but reach a trough in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The units of observation in the regressions are monthly cross-sectional averages of the variables. Newey-West standard errors (12 lags) of coefficients are reported in parentheses.

2010, and have been backfilled to 2006. The report presents the number of long and short contracts held in aggregate by investors classified into one of four categories based on self-reported business purposes: futures dealers, levered funds (i.e., hedge funds), institutional asset managers, and other. [Hazelkorn, Moskowitz and Vasudevan \(2023\)](#) find that positions in futures contracts from the report capture demand for equity market exposure that is reflected in equity market valuations.¹² Futures positions reflect the equity exposures taken on by funds in aggregate, which may reflect a combination of individual investors' expected returns and demand for equity market exposure, as well as the return expectations of fund managers. That is, both individual and institutional investors' expectations may be important determinants of funds' futures positioning. We analyze both DJIA and S&P 500 futures positions, since the Shiller survey asks investors about their expectations for the DJIA, but the S&P 500 is the more commonly tracked equity market index.

Building on the previous literature, we construct *Net Positioning* as the number of short contracts minus the number of long contracts held by dealers, normalized by open interest. Futures contracts are in zero net supply, and dealers meet the futures demand of other investor types, so *Net Positioning* is a measure of the net long demand for equity market exposure by buy-side investors ([Hazelkorn, Moskowitz and Vasudevan \(2023\)](#)).¹³ We standardize *Net Positioning* to have zero mean and unit standard deviation. We run contemporaneous regressions of the level of *Net Positioning* on the cross-sectional average of investors' return expectations in the same period as the independent variable. We also run the regression in changes, regressing the quarterly change in net short minus long contracts held by dealers (normalized by lagged open interest) on changes in the cross-sectional average of return expectations; the variables are also standardized to have zero mean and unit standard deviation.

Table 4 reports the regression results. Panel A reports the results from the levels regressions. For the first four columns, where the dependent variables correspond with DJIA futures positions, the coefficient on 1-month return expectations is 0.40 (standard error of 0.10, R^2 of 0.24) in the univariate regression, and 0.27 (standard error of 0.08) in the multivariate regression. The univariate coefficient on 3-month return expectations is 0.35 (standard error of 0.09, R^2 of 0.20) and the multivariate coefficient is 0.23 (standard error of 0.10). The coefficients for 6-month return expectations are 0.18 (standard error of 0.10) and -0.02 (standard error of 0.15), while the coefficients for 12-month return expectations are 0.02 (standard error of 0.09) and -0.03 (standard error of 0.07). When the dependent variable corresponds with S&P 500 futures positions, the regression results are qualitatively and quantitatively simi-

¹²Other papers, for example [Brunnermeier, Nagel and Pedersen \(2008\)](#) and [Moskowitz, Ooi and Pedersen \(2012\)](#), use futures positioning data from the Commitment of Traders Report, a similar report also published by the Commodity Futures Trading Commission that groups traders into more coarse categories. These papers similarly find that the futures positions capture futures demand that is relevant for asset price behavior.

¹³Dealers generally do not take on equity market exposure, but hedge their futures positions by trading in the spot market, as discussed in [Hazelkorn, Moskowitz and Vasudevan \(2023\)](#).

lar. The evidence indicates that return expectations over the next 1-3 months are strongly related to investors' futures positions, while return expectations over the next 6-12 months are weakly related to investors' futures positions.

Panel B reports the results from regressions of changes in futures positions on changes in investors' expectations. These regressions are more restrictive in that they capture the extent to which return expectations and investors' positions change contemporaneously in the same quarter.¹⁴ The conclusions drawn from the regressions in changes are largely the same, however. For example, the coefficients on 1-month return expectations are 0.29 (univariate; standard error of 0.05) and 0.30 (multivariate; standard error of 0.06) for DJIA futures, whereas the coefficients on 12-month return expectations are 0.05 (univariate; standard error of 0.08) and 0.01 (multivariate; standard error of 0.08).

To further explore the relationship between futures positions and investors' stated higher order beliefs, we regress *Net Positioning* on the *HO Belief* and *Overvaluation* variables. All variables are standardized to have zero mean and unit standard deviation, so that the coefficients can be interpreted as correlations. As before, we run the regression both in changes and in levels. Consistent with the previous regressions, we find evidence of non-fundamental speculation, with investors buying into the stock market when they perceive others to be overly optimistic. This evidenced is strengthened when controlling for investors' valuations.

Table 5 reports the results from the regressions. Panel A reports results for the levels regressions. Focusing on DJIA futures, the coefficient on *HO Belief* is 0.37 in the univariate regression (standard error of 0.14; R^2 of 0.15) and 0.53 in the multivariate regression (standard error of 0.18). The coefficient on *Overvaluation* is 0.11 in the univariate regression (standard error of 0.16) and -0.23 (standard error of -0.23). The evidence indicates that a belief that others are overly optimistic tends to correspond with long equity market positioning. Panel B reports results from the regression in changes. The univariate and multivariate coefficients for *HO Belief* are 0.20 (standard error of 0.08; R^2 of 0.04) and 0.23 (standard error of 0.11). The univariate and multivariate coefficients for *Overvaluation* are 0.05 (standard error of 0.10; R^2 of 0.00) and -0.06 (standard error of 0.12). The last three columns of the table report results for the S&P 500, which are qualitatively similar, though with some quantitative differences.

Taking the results together, one interpretation is that the survey data reflect buy side investors' expectations, which are accordingly reflected in their positions. Under this interpretation, the results indicate that investors' short term return expectations lead them to engage in non-fundamental speculation, and they increase their positions because of perceived short term profits. While our evidence is consistent with such an interpretation, we are also cautious, in that we cannot link the identity of survey respondents with their trades.

¹⁴ To see this point, note that an observation of a time-series $\{y_t\}$ at time t , y_t , can be written as $y_t = y_0 + \sum_{i=0}^{t-1} (y_{t-i} - y_{t-i-1})$. For a similarly defined time-series $\{x_t\}$, the correlation between y_t and x_t in levels does not just reflect the contemporaneous correlation of changes in the y_t and x_t , but also their cross-autocorrelations.

Panel A: Levels Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.40 (0.10)				0.27 (0.08)	0.33 (0.12)				0.25 (0.09)
$\mathbb{E}_t(R_{t,t+3})$		0.35 (0.09)			0.23 (0.10)		0.29 (0.11)			-0.06 (0.13)
$\mathbb{E}_t(R_{t,t+6})$			0.18 (0.10)		-0.02 (0.15)			0.29 (0.08)		0.18 (0.14)
$\mathbb{E}_t(R_{t,t+12})$				0.02 (0.09)	-0.03 (0.07)				0.14 (0.06)	0.06 (0.06)
R^2	0.24	0.20	0.08	0.00	0.28	0.16	0.14	0.22	0.13	0.27
N	69	69	69	69	69	69	69	69	69	69

Panel B: Changes Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.29 (0.05)				0.30 (0.06)	0.27 (0.07)				0.20 (0.09)
$\mathbb{E}_t(R_{t,t+3})$		0.08 (0.09)			-0.05 (0.12)		0.25 (0.08)			0.13 (0.10)
$\mathbb{E}_t(R_{t,t+6})$			0.09 (0.13)		0.01 (0.16)			0.21 (0.12)		0.03 (0.14)
$\mathbb{E}_t(R_{t,t+12})$				0.05 (0.08)	0.01 (0.08)				0.10 (0.09)	0.02 (0.10)
R^2	0.18	0.01	0.02	0.01	0.18	0.15	0.11	0.09	0.03	0.20
N	68	68	68	68	68	68	68	68	68	68

TABLE 4: RETURN EXPECTATIONS AND INVESTOR FUTURES POSITIONS

Note: The table reports results from regressions of investors' futures positions on return expectations. Our measure of futures positions is *Net Positioning*, defined as the number of short minus long futures contracts held by futures dealers in aggregate, normalized by open interest. Data are from the Traders in Financial Futures report. Return expectations are the average return expectations in a given period from the Shiller survey. Observations are quarterly levels in Panel A ("Level Regressions"). In Panel B ("Changes Regressions"), observations are quarterly changes in return expectations and the change in short minus long futures contracts held by dealers, normalized by lagged open interest. The first four columns in the table report results where futures positions are those of dealers in Dow Jones Industrial Average (DJIA) futures. The last four columns report results where futures positions are those of dealers in S&P 500 futures. Newey-West standard errors (4 lags) of coefficients are reported in parentheses.

Panel A: Levels Regressions						
	DJIA Futures			S&P 500 Futures		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>HO Belief</i>	0.37 (0.14)		0.53 (0.18)	0.11 (0.20)		0.62 (0.16)
<i>Overvaluation</i>		0.11 (0.16)	-0.23 (0.21)		-0.34 (0.12)	-0.74 (0.17)
R^2	0.15	0.01	0.18	0.01	0.14	0.36
N	69	69	69	69	69	69

Panel B: Changes Regressions						
	DJIA Futures			S&P 500 Futures		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>HO Belief</i>	0.20 (0.08)		0.23 (0.11)	0.16 (0.09)		0.17 (0.10)
<i>Overvaluation</i>		0.05 (0.10)	-0.06 (0.12)		0.05 (0.12)	-0.03 (0.13)
R^2	0.04	0.00	0.04	0.03	0.00	0.03
N	68	68	68	68	68	68

TABLE 5: HIGHER ORDER BELIEFS, VALUATIONS, AND INVESTOR FUTURES POSITIONS

Note: The table reports results from regressions of *Net Positioning* on *HO Belief* and *Overvaluation*. Observations are quarterly levels in Panel A (“Level Regressions”). In Panel B (“Changes Regressions”), observations are quarterly changes in the independent variables and the change in short minus long futures contracts held by dealers, normalized by lagged open interest. The first four columns in the table report results where futures positions are those of dealers in Dow Jones Industrial Average (DJIA) futures. The last four columns report results where futures positions are those of dealers in S&P 500 futures. Newey-West standard errors (4 lags) of coefficients are reported in parentheses.

1.6 What Drives Higher Order Beliefs and Return Expectations?

Given the observed time-series relationship between investors' reported higher order beliefs and return expectations, we next seek to better understand the causes and drivers of these beliefs. We find macroeconomic news to be a key driver, with positive macroeconomic news increasing *HO Belief* and short term return expectations while decreasing long term return expectations.

We use two measures of macroeconomic news in our analysis. The first is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators.¹⁵ The second is quarterly AR(1) innovations in discussion of recessions in the Wall Street Journal, from [Bybee et al. \(2021\)](#).¹⁶ The first measure corresponds with positive macroeconomic news, while the second corresponds with negative macroeconomic news.

We regress quarterly changes in the cross-sectional averages of *HO Belief*, *Overvaluation*, and return expectations of different horizons on the measures of macroeconomic news. We standardize the independent variables, and changes in *HO Belief* and *Overvaluation*, to have zero mean and unit standard deviation. The coefficients for return expectations can be interpreted as the change in expected returns (in percentage points) corresponding with a one standard deviation innovation to the independent variable, and the coefficients for the other dependent variables can be interpreted as correlation coefficients.¹⁷

Figure 2 plots the regression coefficients. With innovations to leading economic indicators as the independent variable, the coefficient on 1-month return expectations is 0.49, indicating that a one standard deviation innovation corresponds with a 49 basis point higher return expectation for the next month. The coefficients for 3-, 6-, and 12-month return expectations are 0.06, -0.28, and -0.56, indicating that investors lower their return expectations for the next year contemporaneous with the arrival of positive macroeconomic news. Innovations to the leading economic indicators index are 0.45 correlated with changes in *HO Belief* and *Overvaluation*, indicating that substantial variation in investors' reports that others are overly optimistic and that markets are overvalued occurs in quarters with positive macroeconomic news. With innovations to recession attention as the independent variable, the coefficients on 1-, 3-, 6-, and 12-month return expectations are -0.72, -0.18, -0.03, and 0.30, indicating expectations of strong negative short term performance that will revert in the future. Innovations to recession attention are -0.40 and -0.24 correlated to *HO Belief* and *Overvaluation*.

¹⁵The data are provided as an index – we construct innovations in the percent change in the index. We lag observations by one month to ensure that innovations are in investors' information sets. We also report results in the appendix using coincident business cycle indicators, and find similar results.

¹⁶[Bybee et al. \(2021\)](#) find that discussion of recessions has substantive predictive power for macroeconomic outcomes. [Bybee, Kelly and Su \(2023\)](#) find that the recession topic has substantial power to explain risk premia as an asset pricing factor. We use an updated series from the authors, which contains data through January 2021.

¹⁷In the appendix, we also report results running the regressions in levels rather than changes, which are similar

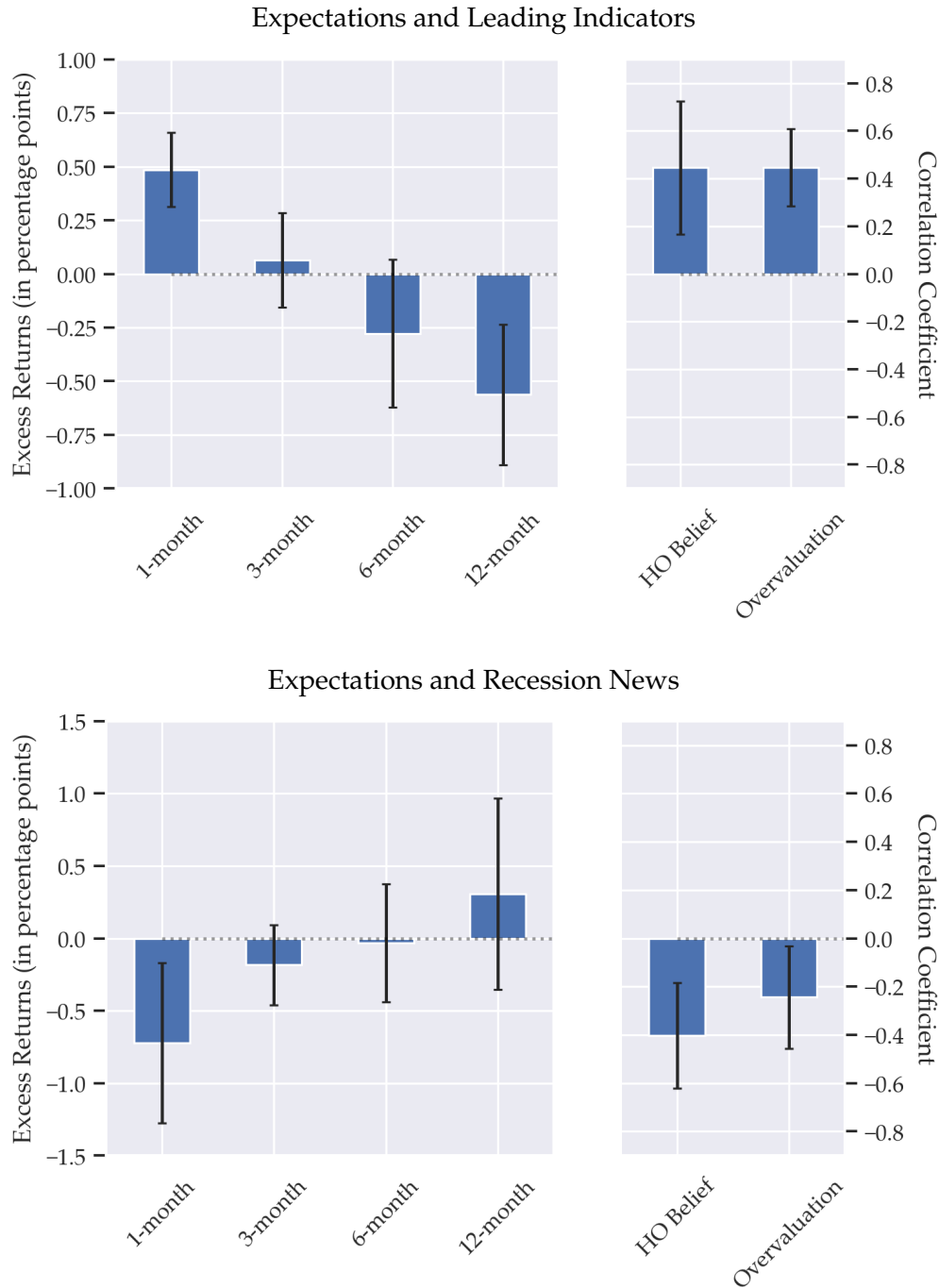


FIGURE 2: MACROECONOMIC NEWS, RETURN EXPECTATIONS, AND HIGHER ORDER BELIEFS

Note: The figure plots coefficients from contemporaneous regressions of changes in quarterly average 1-,3-,6-, and 12-month excess return expectations, *HO Belief*, and *Overvaluation* on measures of macroeconomic news. The measure of macroeconomic news in the first panel is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators. The measure of macroeconomic news in the second panel is AR(1) innovations in attention paid to recession news in the Wall Street Journal from [Bybee et al. \(2021\)](#). The independent variables, *HO Belief* and *Overvaluation*, are scaled to have zero mean and unit standard deviation, and return expectations are multiplied by 100. Standard errors are Newey-West standard errors (4 lags). The figure also plots plus and minus two standard errors for the estimated coefficients.

We can interpret the evidence as follows: in quarters with positive macroeconomic news, the stock market tends to appreciate. The contemporaneous quarterly return associated with a one standard deviation shock to leading indicators is 1.68%. Investors perceive that in the month following positive macroeconomic news, returns will be 49 basis points higher, but that in the subsequent 11 months, this short term return will entirely revert, and further, that returns will be lower by nearly a quarter of the contemporaneous response to the news. That is, the evidence is consistent with the 1.68% return reflecting overreaction to news, with investors having a sense that the initial reaction is about 30% larger than justified by fundamentals.

The evidence from these time-series regressions and the accompanying interpretation are further augmented by responses in the survey that do not rely on regressions from a single time-series. In particular, the evidence is consistent with investors' direct responses when asked about the drivers of stock markets in question (iii.a). Unconditionally, 52% of individual investors and 37% of institutional investors answer that the cause of the 6-month market trend is overreaction and speculative thinking by other investors.¹⁸ Additionally, the interpretation of investors' perception of overreaction is also consistent with investors' direct responses to a hypothetical situation in question (ii.e) – when asked how they expect the market to perform following a 25% drop in the next 6 months, investors report expectations of reversals of 13.6% to 16.9%.

1.7 Relationship to Evidence on Extrapolation

Prior work, most notably [Greenwood and Shleifer \(2014\)](#), finds evidence that investors exhibit extrapolative return expectations; they expect high returns following periods of positive market performance and low returns following periods of poor market performance. Given our evidence relating investors' expectations with macroeconomic news, we may naturally expect a similar relationship to hold in our setting. We examine the relationship between past returns and expectations in the Shiller survey, and discuss it in the context of [Greenwood and Shleifer \(2014\)](#).

Panel A of Table 6 reports the correlations of investors' return expectations, *HO Belief*, and *Overvaluation* with trailing 12-month returns. Short-term return expectations are positively correlated with trailing 12-month returns (correlation of 0.37 for 1-month return expectations; and 0.28 for 3-month return expectations), while the correlation of 6-month ahead returns is insignificantly positive (0.07), and that of 12-month ahead return expectations is negative (-0.18). *HO Belief* is 0.59 correlated with trailing 12-month returns, and *Overvaluation* is 0.42

to the results reported in the main text.

¹⁸The true proportions are a bit higher; several respondents select 'other,' and choose to fill in custom responses that indicate a view that stock prices are driven by others' overreaction or speculation acting in conjunction with additional forces, such as monetary policy.

correlated with 12-month trailing returns. As reported in the appendix, we find similar evidence when we separately break down the results for individual and institutional investors.

The correlations in Panel A are consistent with investors' 1- to 3-month return expectations being extrapolative and 12-month return expectations being somewhat contrarian. Studying investors' stated bullishness or bearishness about subsequent 12-month returns from the Gallup survey, Chief Financial Officers' (CFOs') expectations of the returns of the U.S. stock market over the next 12 months, individual investors' bullishness or bearishness about subsequent 6-month returns from the American Association of Individual Investors (AAII) Investor Sentiment Survey, and the bullishness or bearishness of various financial newsletters' forecasts of 'near term' stock market returns as surveyed by "Investors Intelligence," [Greenwood and Shleifer \(2014\)](#) find consistent evidence of extrapolative return expectations. The extrapolative 1- to 3-month return expectations we document are broadly consistent with the findings of [Greenwood and Shleifer \(2014\)](#), though the contrarian 12-month ahead return expectations differ from their results found in other surveys.¹⁹

A potential explanation for this difference has to do with differences in survey design.²⁰ The Shiller survey is unique from other surveys of stock market expectations, in that it asks investors about their return expectations at multiple horizons. It is plausible that investors have a general sense that the stock market may increase over the short horizon (the period relevant for their portfolio choice) and decrease over a longer horizon, but they do not formulate precise frequency-specific forecasts in their minds. When asked about returns over multiple horizons, as in the Shiller survey, respondents may report numerical return expectations at different horizons consistent with their belief in high short term returns to be followed by lower longer term returns. But when asked only about returns at the 6- to 12-month ahead horizon, as in other surveys, investors may simply report the short horizon return expectations used to form their portfolios. That is, the omission of questions about expectations at multiple horizons may lead a survey respondent to report their beliefs differently.

Such a difference is consistent with psychological evidence on framing effects (e.g., see [Zauberman et al. \(2010\)](#) and [Read, Frederick and Scholten \(2013\)](#)).²¹ Additionally, we discuss two pieces of evidence that are consistent with this line of reasoning. First, Panel B of

¹⁹[Greenwood and Shleifer \(2014\)](#) also analyze the Yale ICF 1-year confidence index for individual investors, which is the proportion of individual investor respondents to the Shiller survey that report strictly positive 12-month ahead return expectations. In a sample that runs through 2011, [Greenwood and Shleifer \(2014\)](#) find a relationship close to zero between the confidence index and trailing 12-month returns (t -statistic of 0.18).

²⁰Another potential reason for the difference is that the sample of Shiller survey respondents may systematically differ from those in other surveys. The investors in other surveys are often individual investors, whereas the Shiller survey respondents are selected to either be wealthy investors or institutional investors. While this selection may play a role, it cannot explain why the CFO respondents in the Graham and Harvey survey, who are financially sophisticated, also exhibit extrapolative expectations.

²¹Also see [Hartzmark and Sussman \(2024\)](#), who argue that differences in how questions are framed may significantly influence reported beliefs about return expectations. Their focus is on designing surveys to accurately capture beliefs about distributions.

Table 6 displays the correlations of expectations measures from the Shiller survey with the monthly proportion of investors that report being bullish minus the proportion that report being bearish about 6-month stock market returns in the AAI survey.²² We find that the AAI responses are highly correlated with short-horizon return expectations in the Shiller survey (correlation of 0.53 with 1-month ahead return expectations), with the correlations declining with forecast horizon (correlations of 0.48, 0.42, and 0.23 with 3-, 6-, and 12-month ahead return expectations). These correlations suggest that the AAI responses are particularly well aligned with Shiller survey respondents' short-horizon return expectations, and less so with their 12-month ahead return expectations. Second, in a previous version of this paper, analyzing multi-period forecasts of exchange rates by financial institutions from FX4casts, we also find that respondents report expectations of high 1- to 3-month ahead returns followed by low 6- to 12-months ahead returns for developed market currencies versus the USD that experienced interest rate increases and positive excess returns in the previous quarter. We reproduce this analysis in the appendix. The consistency of the patterns of more extrapolative short term return expectations and more contrarian longer term return expectations suggest that these might be general features of investors' expectations.

1.8 Return Expectations and Realizations

Our evidence suggests that investors' decisions are driven by their short horizon return expectations. Table 7 illustrates the performance associated with investors' 1-month return expectations. The first row displays coefficients from regressions of realized returns on 1-month return expectations; coefficients are negative for the pooled, individual, and institutional sample. The coefficient in the pooled regression is statistically significant at the 10% level. The table displays the Sharpe ratio of a market-timing strategy that takes long and short positions in the market in proportion to the average return expectation of investors in the survey. The Sharpe ratios are -0.29, -0.32, and -0.15. While the sample is somewhat limited in length, the evidence suggests that investors' short horizon return expectations are often wrong and that short term speculation is unprofitable. Moreover, the evidence is in line with results found in survey data of longer time samples that investors' return expectations tend to be negatively predictive of future returns.

1.9 Summarizing the Evidence and Implications for Theory

We can summarize the evidence presented in this section as follows:

- (i) **(Non-fundamental speculation).** In time periods when investors perceive others to be overly optimistic and markets to be overvalued, they forecast short term returns to be

²²The AAI data are weekly; we follow Greenwood and Shleifer (2014) and aggregate them to be monthly. Greenwood and Shleifer (2014) report a high correlation between the AAI survey and the other surveys they examine.

Panel A: Expectations and Trailing Returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
$R_{t-12,t}$	0.37 (0.07)	0.28 (0.08)	0.07 (0.12)	-0.18 (0.13)	0.59 (0.10)	0.42 (0.15)
R^2	0.14	0.08	0.00	0.03	0.34	0.18
N	259	259	259	259	259	259

Panel B: Shiller and AAI Survey Expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
AAII	0.53 (0.08)	0.48 (0.07)	0.42 (0.10)	0.23 (0.12)	0.32 (0.10)	0.02 (0.13)
R^2	0.28	0.23	0.18	0.05	0.10	0.00
N	259	259	259	259	259	259

TABLE 6: EXPECTATIONS AND TRAILING RETURNS

Note: Panel A of the table displays time-series correlation coefficients between measures of expectations from the Shiller survey, averaged across investors in a given month, with the trailing 12-month excess returns of the Dow Jones Industrial Average. Panel B of the table reports time-series correlation coefficients of the same measures of expectations from the Shiller Survey with the the proportion of investors that reporting being bullish minus the proportion of investors that report being bearish about the future 6-month returns of the U.S. stock market from the American Association of Individual Investors (AAII) Investor Sentiment Survey. The AAI survey data are aggregated from the weekly to monthly frequency by averaging across observations within a month. Newey-West standard errors (12 lags) are reported in parentheses.

	Pooled	Individual	Institutional
Coefficient	-0.32 (0.19)	-0.26 (0.16)	-0.15 (0.17)
Mkt Timing Sharpe	-0.29	-0.32	-0.15

TABLE 7: RETURN EXPECTATIONS AND REALIZED RETURNS

Note: The top row of the table displays coefficients from a regression of 1-month realized returns on investors' return expectations for the same period. Newey-West standard errors (12 lags) of coefficients are reported in parentheses. The last row of the table reports the Sharpe ratio of a market timing strategy that takes long and short positions in the stock market in proportion to the average respondent's return expectations for the next month.

high and long term returns to be low. Speculators seek to ‘ride-the-bubble’ and buy into an overvalued stock market.

- (ii) **(Overreaction)**. Investors perceive that other investors become overly optimistic, and stock markets become overvalued with the arrival of fundamental news. That is, the stock market displays initial overreaction to fundamental news.
- (iii) **(Time-series momentum and reversal)**. Investors forecast that the stock market exhibits momentum and reversal in response to fundamental news. In periods following positive fundamental news, investors forecast positive returns in the short term and negative returns multiple periods into the future.

The results provide systematic evidence from survey data that for the U.S. stock market, non-fundamental speculation is the rule not the exception, where previous work documents such behavior in particular episodes (e.g., [Brunnermeier and Nagel \(2004\)](#), [Temin and Voth \(2004\)](#)). Investors believe in patterns such as overreaction-driven momentum and reversal, but choose to buy into the stock market when they perceive it to be driven up by overly optimistic investors, because they see short term profits from doing so. Additionally, in episodes previously highlighted where informed investors profited from their non-fundamental speculation, our evidence suggests that non-fundamental speculation is unprofitable for the investors studied.

Below, we discuss mechanisms presented in the literature, and how they may help explain the results. A belief that other investors make systematic errors in forecasting fundamentals is sufficient to explain the results. Two other mechanisms highlighted in the literature – higher order uncertainty, and beliefs that other investors may form their return expectations on the basis of past price changes – may also help explain the results.

Higher Order Uncertainty. Higher order uncertainty – uncertainty about whether other investors agree with one’s beliefs – is a form of higher order beliefs that has received particular attention in the literature. In the presence of higher order uncertainty, investors with short investment horizons may not trade fully towards their beliefs, because of uncertainty regarding whether prices will reflect their beliefs in the near future. In one strand of literature, higher order uncertainty leads asset prices to underreact to fundamental news upon its arrival, and monotonically drift towards fundamental values in subsequent periods. This can happen even when the average belief is an unbiased estimate of fundamentals ([Allen, Morris and Shin \(2006\)](#), [Banerjee, Kaniel and Kremer \(2009\)](#)).²³ However, such uncertainty does not capture the patterns of expectations we document, namely that asset prices initially overreact

²³In [Allen, Morris and Shin \(2006\)](#), noisy signals about fundamentals lead to underreaction of prices to fundamental news, though [Banerjee, Kaniel and Kremer \(2009\)](#) note that if investors learn from prices, this effect may disappear. [Banerjee, Kaniel and Kremer \(2009\)](#) argue that price drift can reliably occur when investors agree-to-disagree about fundamental values, but have uncertainty about other investors’ higher order beliefs. [Banerjee, Kaniel and Kremer \(2009\)](#) also note that in noisy rational expectations models, speculation must be driven by information orthogonal to public information; we document speculation in response to public information.

to news, and that they continue to overreact before reversing.

A form of higher order uncertainty that generates synchronization risk can help explain our results. In [Abreu and Brunnermeier \(2002, 2003\)](#), arbitrageurs become aware of mispricing sequentially, and have higher order uncertainty about other arbitrageurs' awareness of the mispricing. With short investment horizons and the need for coordination to correct mispricings, arbitrageurs may engage in non-fundamental speculation and hold long positions in a stock market they see as overvalued.

In and of itself, however, this form of uncertainty is not sufficient to explain the patterns we observe, and requires mispricing to arise and persist from an exogenous set of investors. In the model that we present, the source of mispricing is endogenized as the investors engaged in non-fundamental speculation; such an assumption is also consistent with the poor performance associated with non-fundamental speculation.

Self-fulfilling expectations. Another form of higher order beliefs that can produce asset price fluctuations comes in the form of self-fulfilling beliefs (e.g., [Khorrami and Mendo \(2021\)](#), [Gârleanu and Panageas \(2021\)](#) and [Zentefis \(2022\)](#)). Models of self-fulfilling expectations feature multiple equilibria, with fluctuations driven by investors' coordination on a particular equilibrium. Here, we find that investors systematically report that other investors have mistaken beliefs and report return expectations that are negatively correlated with realized returns, features that are not present in models focused on self-fulfilling expectations. While self-fulfilling expectations may play a role more generally (and do, in fact, appear under some parametrizations of our model), they are insufficient to explain the results for our sample of investors.

Return Extrapolation. Recent work emphasizes that return extrapolation – investors forming their expected returns based on past returns – may explain investor behavior. Our empirical results provide some evidence consistent with return extrapolation: investors' short term return expectations are correlated with past returns. However, without additional assumptions, return extrapolation cannot speak to investors' multi-period return expectations and perceptions of others' beliefs.

Interestingly, beliefs that other investors return extrapolate can lead to non-fundamental speculation. In an economy populated by non-speculative fundamental traders, return extrapolators, and rational speculators, the rational speculators can push prices to overreact to fundamental news due to a recognition that such speculation may trigger future excitement by return extrapolators ([De Long et al. \(1990\)](#)).²⁴ While the rational speculators in the model of [De Long et al. \(1990\)](#) would report return expectations and valuations consistent with the survey responses we observe, we would not expect them to report that they increasingly find others to be overly optimistic contemporaneous with positive news shocks (as in the data).

²⁴If fundamental traders and return extrapolators are the only investors in the economy, and there is no third type, as in [Barberis et al. \(2018\)](#), the fundamental traders would believe that stock markets exhibit initial underreaction rather than believing in initial overreaction (as the investors in the Shiller surveys do).

The speculators in [De Long et al. \(1990\)](#) expect excess optimism to arise from return extrapolators in the periods following news. Moreover, non-fundamental speculation is profitable for these investors. Beliefs in other investors' return extrapolation may help explain the patterns, but existing models do not fully match the evidence.

Errors in Forecasting Fundamentals. A growing literature has suggested that investors may make systematic mistakes in forecasting asset price fundamentals ([Chen, Da and Zhao \(2013\)](#), [Bordalo et al. \(2020\)](#), [De La O and Myers \(2021\)](#), [Nagel and Xu \(2022a\)](#)). In the absence of assumptions about higher order beliefs, such theories can make vastly different predictions about return expectations. For example, if investors with mistaken fundamental beliefs believe all other investors share their beliefs, they always expect constant returns in the absence of time-varying risk premia, inconsistent with our evidence.

However, a belief in other investors making errors in forecasting fundamentals can explain the evidence. In the model we present in the next section, investors believe that other investors persistently update their beliefs in the direction of past news, and overestimate the persistence of fundamentals. These belief updating biases are effectively identical to the ones in [Angeletos, Huo and Sastry \(2021\)](#), who show that these frictions help match macroeconomic forecasters' expectations.²⁵ As we further discuss in an extension of the model, however, this is not the only explanation consistent with the evidence, which can also be rationalized, for example, by investors believing that other investors believe that *other* investors make fundamental belief mistakes.

[Martin and Papadimitriou \(2022\)](#) propose a model where investors have differences-of-opinion about the (fixed) fundamental value of an asset. Investors that are correct in hindsight become wealthier, and so the belief of the representative agent becomes more optimistic following good news and more pessimistic following bad news. Investors internalize the role of this shifting 'sentiment,' and may engage in non-fundamental speculation. Investors' beliefs regarding the shifting relative wealth of optimists and pessimists in response to news may also help explain the facts we document.

Investor Sentiment. More broadly, a voluminous body of work studies *investor sentiment*, defined in a literature review of the topic by [Baker and Wurgler \(2007\)](#) as "a belief about future cash flows and investment risks that is not justified by the facts." Investor sentiment may encompass both errors in forecasting fundamentals and errors in forecasting returns.²⁶ Work on investor sentiment, while related to our results, cannot directly speak to the reason that investors buy into overvalued markets – is it because they are overly optimistic about the asset's fundamentals, or because of their forecasts of other investors' future behavior? In our model, we discuss the roles that each of these might play and their potential interactions, which can help decompose the drivers of sentiment.

²⁵[Valente, Vasudevan and Wu \(2021\)](#) use a model based on [Angeletos, Huo and Sastry \(2021\)](#), to match survey data on investors' interest rate expectations in order to understand puzzles in foreign exchange markets.

²⁶[Baker and Wurgler \(2007\)](#) cite examples of both in their literature review.

Other Models. The previously discussed forms of higher order beliefs – higher order uncertainty and beliefs about other investors’ fundamental and return expectations – may help explain the results. We briefly discuss other common models in the literature.

Expectations of time-varying returns are commonly attributed to time-varying risk premia (Campbell and Cochrane (1999), Bansal and Yaron (2004), Gabaix (2008), Wachter (2013)). Such models typically assume a representative agent with correct beliefs, and accordingly struggle to confront evidence on beliefs that other investors hold incorrect expectations, and that the stock market is mispriced. They also generally predict that risk premia and return expectations decrease contemporaneously with positive news (which makes agents less risk-averse), whereas we find evidence that investors’ short term return expectations and equity market exposures increase contemporaneously with positive news.²⁷ Moreover, these models predict that investors’ return expectations should be positively related to future returns, which we do not find to be the case.

A common explanation for non-fundamental speculation is that investors may face short sale costs and constraints (Harrison and Kreps (1978), Scheinkman and Xiong (2003), Duffie, Garleanu and Pedersen (2002)). The presence of short sale costs leads pessimistic investors to avoid taking short positions, and asset prices reflect the valuations of optimistic investors, plus the added benefit they receive from the possibility of reselling to more optimistic investors that arrive in the future. In our setting – the aggregate U.S. stock market – short sale constraints are unlikely to be binding for most investors.²⁸ Moreover, short sale constraints alone cannot explain the cyclical patterns of return expectations we document. Short sale constraints likely contribute to our results, but may not be the primary driver, and are not a sufficient explanation.

2 A Model of Non-Fundamental Speculation

In this section, we present a stylized asset pricing model. The model provides a framework for interpreting and understanding the empirical results, clarifying the relationship between return expectations and higher order beliefs, and illustrating the impact that investors’ higher order beliefs have on equilibrium asset prices.

The model features two types of investors: speculators and arbitrageurs. Investors trade a risky asset that pays a dividend each period that is drawn from fundamentals that follow a persistent but unobserved process. The average investor’s beliefs about fundamentals match rational expectations. Accordingly, all excess asset price volatility in the model arises from higher order beliefs.

²⁷Though, with specific parametrizations of preferences and the cash flow process, models of time-varying risk premia may be able to generate hump-shaped patterns of return expectations in response to fundamental news.

²⁸It is essentially costless to short the stock market via futures (Hazelkorn, Moskowitz and Vasudevan (2023)).

Speculators, who can be thought to reflect the survey respondents we study, are strategically naive – they underestimate the strategic sophistication of other investors. This underestimation means that speculators misattribute the source of asset price fluctuations and make systematic mistakes in predicting asset price movements. Arbitrageurs, in contrast, are strategically sophisticated and correctly understand other investors’ beliefs and behavior.

We capture the empirical evidence by endowing speculators with a belief that other investors overestimate the persistence of fundamentals. This belief, in turn, means that speculators believe that when fundamentals are positive, other investors tend to hold overly optimistic valuations and the risky asset tends to be overvalued. However, because speculators do not correctly understand other investors’ behavior, when fundamentals are positive, the risky asset is always less overvalued than speculators expect it to be. Speculators accordingly consistently misinfer that other investors must have received attenuated signals about the risky asset’s fundamentals. In turn, when fundamentals are positive, speculators forecast other investors will become even more optimistic in the future as they update their beliefs, and that the risky asset will appreciate. Speculators buy into the risky asset they know to be overvalued, in the process causing its overvaluation. Hence, speculators’ higher order beliefs and the accompanying non-fundamental speculation amplify excess volatility. Additionally, speculators’ misunderstanding of equilibrium means that their non-fundamental speculation is unprofitable on average.

In an extension of our model to a more general form of level k thinking, we find that the effects of speculators’ higher order beliefs are dampened as their strategic sophistication increases and they treat other investors as more strategically sophisticated. In the limit, speculators’ behavior matches that of the arbitrageurs, and the price of the asset matches its rational expectations fundamental value. Accordingly lack of strategic sophistication may play an important role in giving rise to non-fundamental speculation.

As an expositional note, for convenience, we discuss the risky asset becoming overvalued and investors having positive return expectations corresponding with positive fundamentals. However, the model is symmetric, in the sense that it also produces undervaluation and lower than average return expectations corresponding with negative fundamentals, consistent with the survey evidence.

2.1 Baseline Model Setup

We begin by laying out the model environment and describing investors’ beliefs.

2.1.1 Model Environment

There is a risky asset (the stock market) and a riskless asset. The payoff of the riskless asset is normalized to zero. The risky asset pays a dividend D_t each period, where D_t evolves

according to the process

$$\begin{aligned} D_t &= \mathbf{d}_t + v_t, \text{ where } v_t \sim N(0, \sigma_v^2), \text{ and} \\ \mathbf{d}_t &= \rho \mathbf{d}_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma_\epsilon^2). \end{aligned} \tag{1}$$

The term \mathbf{d}_t captures the persistent component of dividends, which we refer to as the asset's fundamentals, while v_t captures a transitory component of dividends. While dividends are observed each period, the underlying fundamentals are never revealed to investors. The riskless asset is in perfectly elastic supply and the risky asset is in zero net supply.²⁹

The model follows an overlapping generations structure. Each period, a unit mass of individually infinitesimal investors is born, indexed by $i \in [0, 1]$. Investors born in period t make an investment decision in that period. In period $t + 1$, they liquidate their investments, consume the proceeds, and pass their beliefs onto the newly born investor i . The assumption of overlapping generations is common in work on higher order beliefs (e.g., [Allen, Morris and Shin \(2006\)](#)), and serves to accentuate the importance of short term price movements for traders. All investors have exponential utility, with risk-aversion parameter $\gamma > 0$.

In period t , in addition to observing the publicly announced dividend, D_t , each investor i also receives a private signal,

$$\begin{aligned} s_t^i &= s_t + \phi_t^i, \text{ where} \\ s_t &= \mathbf{d}_t + \eta_t, \\ \eta_t &\sim N(0, \sigma_\eta^2), \text{ and } \phi_t^i \sim N(0, \sigma_\phi^2). \end{aligned}$$

Each investor's private signal contains a common component that is informative about fundamentals, s_t , as well as idiosyncratic noise, ϕ_t^i . We later provide additional structure on how investors treat these signals in forming their higher order beliefs.

There are two types of investors: a mass $\theta \in (0, 1)$ of strategically naive speculators (whom we refer to as speculators, for short), indexed by $i \in [0, \theta]$; and a mass $(1 - \theta)$ of strategically sophisticated arbitrageurs, with mass $(1 - \theta)$, indexed by $i \in [\theta, 1]$. Investors of each type share beliefs about the parameters governing the economy with other investors of the same type.

Both investor types are identical in how they receive and process information about the risky asset's fundamentals, but differ in their understanding of the equilibrium behavior of other investors. Speculators are our primary focus, and have higher order beliefs that

²⁹The assumption of zero net supply is not critical but simplifies and focuses our analysis. In the appendix we outline a version of our baseline specification where the risky asset is in positive and fixed supply. As is standard, the level of supply determines the risk premium on the risky asset. Interestingly, in our setting, positive supply also induces a persistent sign bias in speculators' return expectations. This bias can be positive (i.e., speculators overestimate the average expected returns on the risky asset) or negative, depending on the risk aversion of investors in the economy.

align with those of the survey respondents we study. They do not correctly understand the structure of equilibrium, and particularly, other investors' behavior, in a manner that we further detail. Their misunderstanding helps match the empirical evidence we document. Arbitrageurs correctly understand other investors' behavior; the average arbitrageur's beliefs match rational expectations.³⁰

Each investor i 's demand is given by

$$Q_t^i = \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})}, \quad (2)$$

where $\mathbb{E}_t^i(\cdot)$ and $\mathbb{V}_t^i(\cdot)$ are the subjective expectations and variance operators respectively, and P_t is the price of the risky asset in period t , determined by the market clearing condition

$$0 = \int_0^1 Q_t^i di.$$

2.1.2 Investors' Beliefs

We next turn to specifying how investors form their beliefs. All investors are Bayesian in forming their beliefs about fundamentals, \mathbf{d}_t . Using their beliefs about the dividend process and their observations of past dividends, they form their expectations of \mathbf{d}_t by Kalman filtering. We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. This means that investors' Kalman gain – the weight they place on new information that arrives in period t versus their prior in forming their fundamental beliefs – is constant each period.

Before presenting the exact formulation of investors' belief updating, we make an assumption about how investors process their own and other investors' signals.

Assumption 1 (Differences-of-opinion) *The noise term in investor i 's private signal, ϕ_t^i , is an idiosyncratic interpretation that i imputes to the informative component of s_t . Investors treat other investors' signals as being uninformative about fundamentals conditional on their own private signals. When updating their beliefs about \mathbf{d}_t , investor i treats their private signal s_t^i as if it has variance σ_η^2 .*

The assumption that investors treat others' signals as uninformative follows in the spirit of 'differences-of-opinion' models (e.g., [Harris and Raviv \(1993\)](#), [Kandel and Pearson \(1995\)](#), [Banerjee and Kremer \(2010\)](#)).³¹ Given Assumption 1, investor i perceives that the average

³⁰For example, they may be large institutional asset managers that make equity allocations based on capital market assumptions of equity returns that appear close to correct on average (as studied in [Dahlquist and Ibert \(2024\)](#)).

³¹Differences-of-opinion means that our model also matches the cross-sectional evidence on investors' return expectations, and helps rationalize the substantial belief heterogeneity in return expectations found in other work (e.g., [Giglio et al. \(2021\)](#)), though these are not our main focus. Microfoundations for differences of opinion may

signal received by other investors, $s_t^{-i} \equiv \mathbb{E}_t^i \int s_t^j dj$, is a biased signal about fundamentals.³² With this assumption in hand, Lemma 1 outlines how investors' fundamental beliefs evolve.

Lemma 1 (Fundamental Beliefs) *Investor i 's beliefs about fundamentals, \mathbf{d}_t , evolve according to the updating process*

$$\mathbf{d}_t^i \equiv \mathbb{E}_t^i(\mathbf{d}_t) = (1 - \kappa_1 - \kappa_2)\rho\mathbf{d}_{t-1}^i + \kappa_1 D_t + \kappa_2 s_t^i, \quad (3)$$

where

$$\begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} = \Sigma H^T (H \Sigma H^T + R)^{-1}, H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}, \text{ and} \\ \Sigma = \rho^2 \Sigma - \rho^2 \Sigma H^T (H \Sigma H^T + R)^{-1} H \Sigma + \sigma_\epsilon^2.$$

Proof. All proofs are contained in Appendix A. □

Investors update their beliefs in response to new information based on the signal-to-noise ratio of D_t and s_t . When these signals are informative about dividends, investors give them additional weight (higher κ_1 and κ_2), whereas they rely more on their priors when these signals are less informative. For notational convenience, we denote investor i 's beliefs about fundamentals as $\mathbf{d}_t^i = \mathbb{E}_t^i(\mathbf{d}_t)$.

We next make two assumptions about speculators' higher order beliefs that form the core frictions in our baseline specification.

Assumption 2 (Second Order Beliefs) *Speculators believe that all other investors misperceive the persistence of fundamentals. That is, their second order belief is that others perceive the persistence of fundamentals, ρ , as $\hat{\rho}$.*

Assumption 3 (Additional Higher Order Beliefs) *Speculators believe that all other investors trade as if the price in period $t + 1$ will reflect their buy-and-hold valuation for the risky asset. That is, speculators believe that other investors are non-strategic.*

Jointly, assumptions 2 and 3 characterize speculators' higher order beliefs. Assumption 2 pins down how speculators believe that other investors update their beliefs. In particular, their second order belief is that the average fundamental belief in the economy evolves according to

$$\mathbf{d}_{t,2}^i \equiv \mathbb{E}_t^i \int \mathbf{d}_t^j = (1 - (\hat{\kappa}_1 + \hat{\kappa}_2))\hat{\rho}\mathbf{d}_{t-1,2}^i + \hat{\kappa}_1 D_t + \hat{\kappa}_2 s_t^{-i}, \quad (4)$$

include overconfidence (Odean (1998), Daniel and Hirshleifer (2015)), or motivated reasoning (Banerjee, Davis and Gondhi (2021)).

³²The treatment of s_t^i as having variance σ_η^2 , rather than $\sigma_\eta^2 + \sigma_\phi^2$, means that the average fundamental belief is an unbiased signal of \mathbf{d}_t . In the alternative case, the average fundamental belief underreacts to news, as in models of noisy rational expectations (Woodford (2001) and Sims (2003)). This would not meaningfully affect our results, but we shut down this channel to clearly isolate the role of higher order beliefs in driving asset price fluctuations.

where $[\hat{\kappa}_1 \quad \hat{\kappa}_2]^T$ are speculators' second order beliefs about Kalman gains, constructed using the expression in Lemma 1. These differ from speculators' own Kalman gains, because speculators' belief that others misperceive the persistence of the fundamentals process means that they also believe that other investors differ in their speed of learning. Assumption 3 specifies all higher than second order beliefs, with speculators treating others as non-strategic. This assumption can be thought to coincide with a form of level k thinking – where investors believe that other investors are less strategically sophisticated than they are. In an extension of the model, we relax assumptions 2 and 3 to capture a more general form of level k thinking.

2.1.3 Equilibrium

We next define equilibrium, and then derive expressions for the risky asset price in equilibrium.

Definition 2.1 (Equilibrium) *An equilibrium in period t is a combination of a price, P_t , and beliefs, such that*

- (i) *Investor i 's demand, Q_t^i , maximizes their subjective expected utility;*
- (ii) *Markets clear ($\int_0^1 Q_t^i di = 0$); and*
- (iii) *Investors' (potentially incorrect) beliefs about fundamentals and higher order beliefs are consistent with the price they observe.*

Given its definition, we derive equilibrium in two steps:

- (i) We derive speculators' perceived pricing function for the risky asset, given their higher order beliefs.
- (ii) We derive the true pricing function for the risky asset, subject to the restriction that speculators' beliefs equate the true price and the perceived price implied by (i).

Lemma 2 (Speculators' Perceived Pricing Function) *Speculators perceive the period t price as*

$$P_t = B_0 \mathbf{d}_{t,2}, \quad (5)$$

where $B_0 \equiv \frac{\hat{\rho}}{1-\hat{\rho}}$, and $\mathbf{d}_{t,2}$ is the second order belief shared by all speculators about the average investors' belief about fundamentals, i.e., $\mathbf{d}_{t,2} \equiv \mathbf{d}_{t,2}^i = \mathbb{E}_t^i \int \mathbf{d}_{t,2}^j, \forall i \in [0, \theta]$.

Speculators all share the same second order beliefs. Given that they believe that all other investors perceive the persistence of fundamentals as $\hat{\rho}$ and are non-strategic investors, to rationalize the price they observe, they must believe that the average investor's belief about fundamentals is $\mathbf{d}_{t,2}$.

Given how speculators perceive prices, we can derive the true pricing function for the economy.

Lemma 3 (Equilibrium Pricing Function) *The linear equilibrium pricing rule for the economy is given by*

$$P_t = B_1 \mathbf{d}_t^s,$$

where $B_1 > 0$ is the solution to a cubic equation, and $\mathbf{d}_t^s \equiv \int_0^1 \mathbf{d}_t^i$ is the average of investors' first order beliefs.

We briefly outline the proof of Lemma 3. The total risky asset demand from each investor type is proportional to the ratio of the average expected return to the average perceived variance of that investor type. Speculators forecast expected returns using the pricing equation in Lemma 2. They use prices to infer the signal that they believe other investors received, so that the price under their perceived pricing function matches the price they observe; in turn, this requires that $\mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t^s$.

Given their strategic sophistication, arbitrageurs' forecasted expected returns reflect the true equilibrium pricing function in Lemma 3. As is a common feature of models featuring short-lived investors trading a long-lived asset, arbitrageurs' beliefs about volatility can become self-fulfilling.³³ The equilibrium pricing coefficient is the solution to a cubic equation; accordingly, there are potentially three values for the pricing coefficient, B_1 , that satisfy the market clearing condition. Any real valued root of the cubic equation that B_1 must satisfy corresponds with a potential equilibrium, with higher price coefficients corresponding with higher asset price volatility. We discuss this multiplicity in more detail when discussing how the model matches the survey evidence.

2.2 Matching the Survey Evidence and Implications for Asset Prices

We next turn to describing how the model can match the evidence on investors' expectations. We then proceed to exploring the equilibrium asset pricing implications of the model.

2.2.1 Matching the Evidence

In the context of the model, we can summarize the conditions required to match the empirical evidence:

- (i) **(Perceived overreaction and overvaluation)**: on average, when fundamentals are positive, the risky asset price exceeds the average speculator's valuation (their expected sum of future dividends). That is, $\mathbf{d}_t > 0 \implies P_t > \sum_{h=1}^{\infty} \rho^h \mathbf{d}_t^s = \frac{\rho}{1-\rho} \mathbf{d}_t^s$.
- (ii) **(Perceived time-series momentum and reversal)**: on average, following a positive fundamental innovation, speculators perceive that the risky asset return will exhibit pos-

³³See Spiegel (1998), Bacchetta and van Wincoop (2006), Banerjee (2011), Greenwood and Vayanos (2014), and Albagli (2015) for examples and discussions. We note that while other similar models commonly select a particular equilibrium to study, equilibrium selection does not affect the qualitative conclusions that we draw.

itive short-term returns and negative long term returns. That is, given an innovation $\epsilon_t > 0$, $\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) > 0$ (momentum); and $\lim_{h \rightarrow \infty} \mathbb{E}_t^s P_{t+h} + \sum_{j=1}^h \rho^j \mathbf{d}_t^s - P_t < 0$ (reversal), where \mathbb{E}_t^s is the expectation of the average speculator.

- (iii) **(Non-fundamental speculation)**: the average speculator buys into the risky asset when they perceive other investors as overly optimistic and the risky asset as overvalued.

Based on these conditions, we can derive the parameter values under which the model can match the survey evidence:

Proposition 1 (Matching the Survey Evidence) *If, and only if, $\hat{\rho} > \rho$ (speculators perceive that other investors overestimate the persistence of fundamentals), the average speculator*

- (i) *perceives that the risky asset is overvalued when fundamentals are positive;*
- (ii) *perceives that the risky asset exhibits time-series momentum and reversal;*
- (iii) *and engages in non-fundamental speculation.*

In equilibrium, speculators expect other investors to overreact and overvalue the risky asset when fundamentals are positive, as they believe that other investors are fundamental investors that overestimate the persistence of fundamentals. Since the economy is composed of speculators and arbitrageurs with correct beliefs on average, the equilibrium price does not reflect as much overvaluation as speculators expect given their belief about other investors' behavior. Hence, speculators infer that other investors must have received attenuated signals that led them to not fully internalize the level of fundamentals ($\mathbf{d}_{t,2} < \mathbf{d}_t^s$ when $\mathbf{d}_t > 0$). This leads speculators to hold positive return expectations in spite of viewing the market as overvalued, as they forecast that other investors will buy the risky asset in the future when they fully incorporate past information about fundamentals. Speculators willingly buy into a risky asset they perceive is overvalued because they forecast that it will become even more overvalued, before subsequently reverting. Note that speculators' recognition of the risky asset's overvaluation and their positive expected returns co-exist in equilibrium because speculators' positive expected returns are the cause of overvaluation. When fundamentals are positive, the average speculator takes a long position in the risky asset, while the average arbitrageur takes a short position to meet speculators' long demand.

Remark 1 (Mapping the Model to the Data) *The HO Belief variable can be thought to map to the proportion of speculators that perceive the average investor to be overly optimistic and the risky asset to be overvalued. In the model, this can be expressed as $\frac{1}{\theta} \int_0^\theta \mathbb{1}_{\text{if } P_t > \frac{\rho}{1-\rho} \mathbf{d}_t^s}$, where $\mathbb{1}$ is a 0/1 indicator variable. As we show in the appendix, this expression is strictly increasing in \mathbf{d}_t . Hence, consistent with our empirical evidence, in the model, for stronger fundamentals (higher \mathbf{d}_t), we also observe higher values of HO Belief and speculators' short-horizon return expectations. Accordingly, when fundamentals are more positive, more speculators recognize the asset price is overvalued, and see others as being overly optimistic (HO Belief increases); and more speculators also have positive short-horizon*

return expectations. This relationship between fundamentals, return expectations, and HO Belief means that the model matches the empirical evidence in Sections 1.4 and 1.6. Putting these together, the proportion of speculators engaged in non-fundamental speculation increases with fundamentals, \mathbf{d}_t .

Remark 2 (Equilibrium Multiplicity) *When speculators believe that other investors overestimate the persistence of fundamentals, they always believe that the risky asset is overvalued corresponding with positive fundamentals, and they always believe in time-series momentum and reversal. When there are multiple equilibria, the difference across equilibria is in the quantitative degree of overvaluation, momentum, and reversal that speculators perceive. All qualitative insights of the model we discuss remain the same. Moreover, multiplicity does not arise generically and is only present under specific parameter restrictions on the proportion of speculators, θ , their higher order beliefs about the persistence of fundamentals, $\hat{\rho}$, and the noise in signals about fundamentals. For example, we also show in the appendix that as a necessary but not sufficient condition for multiplicity, we must have that $\hat{\rho} > 2/3 + \frac{\rho}{3}$, i.e., speculators must believe that other investors sufficiently overestimate the persistence of fundamentals. For example, if $\rho = 0.6$, there can only be multiple equilibria when $\hat{\rho} > 0.86$.*

Remark 3 (Perceived Belief Mistakes) *Investors' higher order beliefs about others' belief mistakes in the model – that the average investor's fundamental expectations tend to underreact to shocks ($\mathbf{d}_{t,2} < \mathbf{d}_t^s$ following a positive shock), and that other investors overestimate persistence, $\hat{\rho} > \rho$ – are consistent with the frictions that can explain the dynamics of forecast errors of macroeconomic fundamentals made by professional forecasters (Angeletos, Huo and Sastry (2021); Reis (2020)).*

2.2.2 Equilibrium Asset Pricing Implications

Having matched the survey evidence, we next turn to explore the equilibrium asset pricing implications.

Result 1 (Overreaction and Reversal in Equilibrium) *Whenever speculators engage in non-fundamental speculation, given positive fundamentals in period t , the risky asset is overvalued, and has negative objective expected returns in subsequent periods.*

When fundamentals are positive, the risky asset is overvalued in period t , in spite of the fact the average investor's belief about fundamentals is consistent with rational expectations, due to speculators' higher order beliefs. Note this also corresponds with asset prices overreacting to news since fundamentals are, on average, positive following good news and negative following bad news. The logic of this result holds more generally than the particular setup where speculators believe all other investors to be fundamental investors. We further highlight this point when extending our model to more general level k thinking.

Following overvaluation, asset prices experience a gradual reversal, corresponding with speculators revising their beliefs about the average investor's valuation, $\frac{\hat{\rho}}{1-\hat{\rho}} \mathbf{d}_{t,2}$. Speculators'



FIGURE 3: NON-FUNDAMENTAL SPECULATION, OVERREACTION, AND REVERSAL

Note: The figure plots the asset price in period t given $\mathbf{d}_t^s = \mathbf{d}_t = 1$. The blue line represents speculators' cumulative return expectations from period t to $t+h$. The red line represents the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \hat{\rho}, \sigma_\epsilon^2, \sigma_v^2, \sigma_\eta^2, \sigma_\phi^2) = (0.5, 0.6, 0.7, 1, 1.13, 1.13, 1.13)$. The choice of σ_v^2 and σ_η^2 is made to set the Kalman gains to be $\kappa_1 = \kappa_2 = 0.2$.

initial excitement – that other investors would overreact even more, leading to short term profits – turns out to be incorrect, resulting in negative forecast errors of returns.

Figure 3 summarizes Proposition 1 and Result 1. The figure plots the price in period t given $\mathbf{d}_t^s = \mathbf{d}_t = 1$ in the unique equilibrium for a set of chosen parameters. The asset price exceeds the average speculator's valuation in period t . The figure displays the period t cumulative return expectations of the average speculator in blue. Though the asset price exceeds their valuation, speculators expect to earn even more positive returns in period $t+1$, though they expect the cumulative returns to revert to their buy-and-hold valuations in the long run. This pattern matches the analogous survey evidence. The figure plots the average realized returns in red. In the periods following the shock, the cumulative returns are negative, as asset prices revert.

We make two additional remarks to help interpret these results.

Remark 4 (Overreaction and Perceived Overreaction) *That asset prices overreact to news despite speculators' beliefs that other investors overreact is unique from the logic in static models (e.g., Bastianello and Fontanier (2024)). In a static model, when investors believe that other investors overreact to news, they trade in the opposite direction of news, leading equilibrium asset prices to underreact. Here, in a dynamic model, speculators' forecasts of continued overreaction makes them seek to "ride*

the bubble” in spite of their belief in overreaction. In turn, their behavior gives rise to overreaction in equilibrium.

Remark 5 (Interaction with Fundamental Beliefs) *Result 1 isolates the impact of higher order beliefs on equilibrium asset prices by assuming that the average belief about fundamentals \mathbf{d}_t^s matches rational expectations. Lemma 3 indicates the market clearing price can be written as a linear function of \mathbf{d}_t^s , and it follows in a straightforward way that the average speculator’s return expectations can be written as a linear function of \mathbf{d}_t^s . These facts suggest that return expectations (and investor demand) can be understood as driven by speculators’ higher order beliefs conditional on their fundamental expectations.*

More generally, the behavior of equilibrium asset prices relies on the interaction of fundamental and higher order beliefs. Our focus is on higher order beliefs, so in our model, $\mathbf{d}_t^s = \mathbf{d}_t$, i.e., the average investor’s belief about fundamentals matches rational expectations. However, empirical work using analysts’ expectations suggests that \mathbf{d}_t^s may depart from rational expectations, though there is disagreement about the exact nature of the departure (Chen, Da and Zhao (2013); Bordalo et al. (2020); De La O and Myers (2021); McCarthy and Hillenbrand (2021)). If \mathbf{d}_t^s sluggishly responds to dividend or earnings shocks, as suggested in models of sticky or noisy information (Mankiw and Reis (2002), Bouchaud et al. (2019)), then asset prices may display momentum in addition to overreaction and reversal. If fundamental beliefs overreact to news, as suggested in Bordalo et al. (2021), then asset price overreaction may be even stronger. Additionally, investors’ higher order beliefs may influence their fundamental beliefs, for example leading investors to misinfer fundamental information from prices based on their misunderstanding of the behavior of other investors (e.g., see Bastianello and Fontanier (2024)); this may introduce further excess volatility into asset prices. Finally, we highlight that though empirical work often separately considers return expectations and fundamental expectations (e.g., McCarthy and Hillenbrand (2021); De La O and Myers (2021)), our analysis suggests the two are tightly linked, as empirically documented by (Giglio et al. (2021)), with higher order beliefs playing an important role.

Remark 6 (Long Run Survival) *In the main specification, speculators lose money each period on average, leading to the question as to whether we may expect non-fundamental speculation to survive in the long run. While our setup of short-lived investors that have exponential utility is not fit to formally study this question, we discuss two potential reasons we may expect non-fundamental speculation to survive. The first is that the entry of new investors into the market may mean that even if speculators lose money on average, we may still expect speculation to persist if new speculators consistently enter the market; this implicitly occurs in the overlapping generations setup of our model. We may also expect new or inexperienced investors to be non-fundamental speculators (Greenwood and Nagel (2009)). Second, in an extension with risk premium in Appendix B, we find that if risk aversion is sufficiently high, speculators may have overly optimistic return expectations on average. Under some conditions (outside the scope of our model), such optimism can help with wealth accumulation when the asset’s risk premium is positive, as noted by Borovička (2020) in a detailed analysis of long-run*

survival of investors with heterogeneous beliefs and Duffie-Epstein-Zin preferences.

2.3 Level k Thinking

We next generalize our model by embedding our baseline specification into a more general form of level k thinking, where speculators recognize that other investors are strategic, but underestimate others' strategic sophistication. This allows our model to more closely match the survey question on whether the cause of the trend in stock prices is 'speculative thinking among other investors.' Whereas speculators in the baseline specification only form second order beliefs and believe that the risky asset may be overvalued because other investors have overly optimistic fundamental valuations, in the extension, speculators may believe that overreaction and overvaluation are driven by higher order belief driven-speculation by other investors.

In our more general specification, the main conclusion – that higher order beliefs can explain non-fundamental speculation and induce excess volatility – still follows through under the same parameter restriction ($\hat{\rho} > \rho$). However, as we increase speculators' depth of reasoning (how sophisticated they treat other speculators to be), the risky asset price converges towards the rational expectations fundamental value. Hence, the survival of non-fundamental speculation as an equilibrium outcome indicates limited strategic reasoning by investors.

Definition 2.2 (Level k Equilibrium) *A level 1 equilibrium is the equilibrium solved in the previous section, where the speculators are defined as level 1 speculators. For $k > 1$, a level k equilibrium features level k speculators who believe that the equilibrium structure is a level $k - 1$ equilibrium, i.e., they believe that all speculators are level $k - 1$ speculators.*

Given the definition of a level k equilibrium, for $k > 1$, a level k speculator can be thought to exhibit the following form of bounded rationality: they think of themselves as a strategically sophisticated arbitrageur, when in fact, they are mistaken in treating other speculators as less sophisticated than they are. A level k speculator correctly recognizes that a mass θ of investors are speculators, but incorrectly believes that the other speculators are level $k - 1$ thinkers. Similarly, level k speculators recognize the presence of a mass $(1 - \theta)$ of strategically sophisticated arbitrageurs; however, they mistakenly believe that those arbitrageurs are just as strategically sophisticated as they are, not recognizing that the arbitrageurs rationally engage in one additional level of reasoning.

Note that for $k > 1$, in a level k equilibrium, speculators believe that all investors' fundamental belief updating follows the same structure as their own (Lemma 1), but recognize that other investors have private interpretations of news that lead them to disagree about fundamental values. Each speculator i believes that the average fundamental belief in the economy

evolves according to.

$$\mathbf{d}_{t,2}^i = \mathbb{E}_t^i \int \mathbf{d}_t^i = (1 - \kappa_1 - \kappa_2)\rho \mathbf{d}_{t-1,2}^i + \kappa_1 D_t + \kappa_2 s_t^{-i}. \quad (6)$$

As before, speculators each seek to infer what other investors' signals are, which they use to reconcile their perceived pricing function with the true price they observe. Given the definition of a level k equilibrium, speculators' second order beliefs are that all investors perceive the persistence of the fundamental process as ρ , but they believe that level 1 speculators believe that all other investors perceive persistence as $\hat{\rho}$.

Proposition 2 (Level k Pricing Rule and Equilibrium Beliefs) *In the level k equilibrium, the linear equilibrium pricing function can be expressed as*

$$P_t = B_k \mathbf{d}_t^s, \text{ where}$$

for $k > 1$, B_k is the positive root of a cubic equation that is a function of B_{k-1} and deep parameters of the model, and B_1 is defined as in Lemma 3.

Proposition 2 provides a recursive representation for the asset price in the level k equilibrium which can always be expressed in terms of the level $k - 1$ equilibrium pricing coefficient. Although they are not explicitly provided, in the level k equilibrium, speculators form k higher order beliefs, e.g., level 3 speculators form beliefs about level 2 speculators' beliefs about level 1 speculators' beliefs and second order beliefs about the average fundamental belief. Each of these higher order beliefs is exactly pinned down by the fact that speculators' (higher order) beliefs have to be consistent with the prices they observe, and their beliefs about other investors' beliefs about equilibrium. Given the level k equilibrium's representation, we next turn to studying the impact of strategic sophistication.

Result 2 (Overreaction and Reversal in a Level k Equilibrium) *In the level k equilibrium, if, and only if, $\hat{\rho} > \rho$:*

- (i) *the average speculator perceives that the risky asset is overvalued when fundamentals are positive;*
- (ii) *the average speculator perceives that the risky asset exhibits time-series momentum and reversal;*
- (iii) *the average speculator engages in non-fundamental speculation;*
- (iv) *given positive fundamentals in period t , the risky asset is overvalued, and has negative objective expected returns in subsequent periods.*

Result 2 clarifies that the results derived in the case of level 1 speculators extend to cases where speculators have higher depth of reasoning, and understand that not all investors are fundamental traders. Asset price overreaction and reversals persist for the exact same parameter values, $\hat{\rho} > \rho$, regardless of the level of investors' strategic sophistication. That is,

the results obtain as long as there is a seed of a belief that other speculators (believe that other speculators believe that other speculators...) believe that investors overestimate the persistence of fundamentals.³⁴

We can understand the result as follows: a level 2 speculator understands that level 1 speculators' risky asset demand tends to lead the risky asset to become overvalued when fundamentals are positive. However, in a level 2 equilibrium, given that all speculators are level 2 speculators, the risky asset exhibits less overvaluation than speculators expect given their fundamental beliefs and their perception of equilibrium, so speculators infer that other investors received attenuated signals, and will buy into the asset more aggressively in the next period. The level 2 speculators accordingly still have positive return expectations and buy into the overvalued risky asset. Prices revert when speculators' second order beliefs are revealed to be incorrect, and other investors do not revise their fundamental beliefs upwards in the subsequent period. The same logic holds for each level k of reasoning.

While the results qualitatively remain the same when we increase depth of reasoning, the equilibrium behavior of the model is not identical.

Result 3 (Equilibrium with Sophisticated Speculators)

- (i) For a given $\hat{\rho} > \rho$, asset price overreaction is lower for higher levels of strategic sophistication (higher k).
- (ii) In the limit, as speculators have infinite depth of reasoning ($k \rightarrow \infty$),
 - (a) The asset price converges to its rational expectations fundamental value—that is, $\lim_{k \rightarrow \infty} B_k = \frac{\rho}{1-\rho}$;
 - (b) Non-fundamental speculation disappears; speculators become arbitrageurs.

Result 3 indicates that as we increase speculators' depth of reasoning, prices overreact less, and converge to the rational expectations fundamental value in the limit. For each level k of reasoning, the equilibrium overreaction is attenuated relative to the corresponding level $k - 1$ equilibrium that speculators believe holds. This attenuation is due to level k speculators' understanding of level $k - 1$ speculators' overreaction.

Iterating ad infinitum, non-fundamental speculation does not survive as an equilibrium outcome with sophisticated ($k = \infty$) speculators, who are able to correctly extract the average fundamental belief of other investors because they understand the structure of equilibrium. Sophisticated speculators behave just like the sophisticated arbitrageurs in the model.

The convergence of prices to the rational expectations fundamental value with higher levels of reasoning can be rapid. Figure 4 plots the price in period t given $\mathbf{d}_t^s = \mathbf{d}_t = 1$, for different levels of k . Overvaluation exponentially decays as we increase speculators' strategic

³⁴For a given level K , there are potentially 3^K equilibria. This is because the level k equilibrium is defined recursively, and since for each level $k \leq K$, there are potentially three equilibria; and level K speculators must select an equilibrium for each level k . As long as $\hat{\rho} > \rho$, however, we obtain Result 2, regardless of the equilibrium chosen.

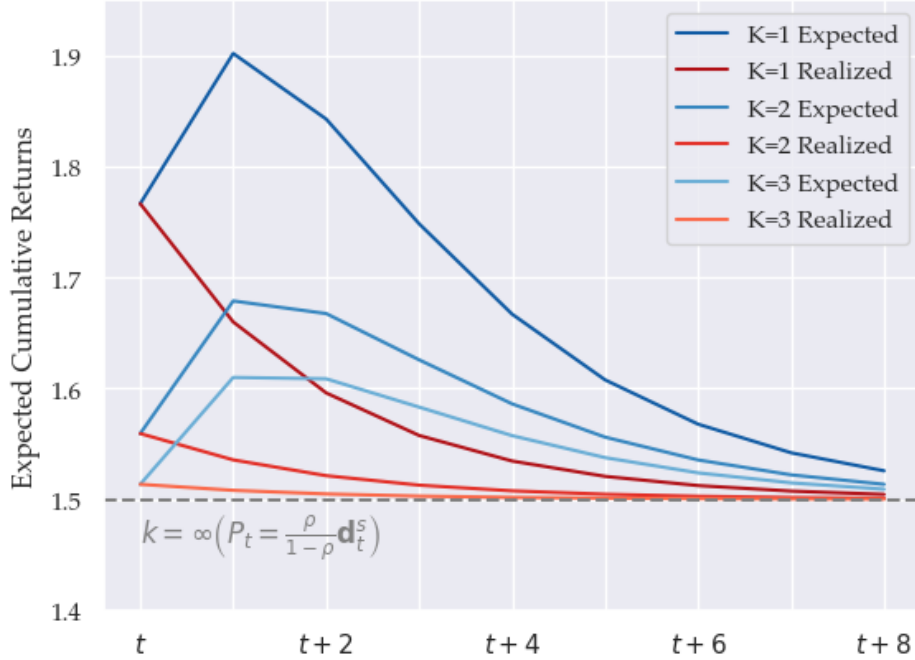


FIGURE 4: CUMULATIVE RETURN EXPECTATIONS AND REALIZED RETURNS FOR DIFFERENT k

Note: The figure plots the asset price in period t given $\mathbf{d}_t^s = \mathbf{d}_t = 1$, for different levels of strategic sophistication, k . The blue lines represent speculators' cumulative return expectations from period t to $t+h$. The red lines represent the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \hat{\rho}, \sigma_\epsilon^2, \sigma_v^2, \sigma_\eta^2, \sigma_\phi^2) = (0.5, 0.6, 0.7, 1, 1.13, 1.13, 1.13)$. The choice of σ_v^2 and σ_η^2 is made to set the Kalman gains to be $\kappa_1 = \kappa_2 = 0.2$.

sophistication. The figure displays the period t cumulative return expectations of the average speculator in blue for different values of k . The declining overvaluation can be understood by the fact that the risky asset price declines with k , though speculators still have positive return expectations. The figure plots the average realized returns in red. Consistent with overvaluation being considerably smaller, reversals are considerably less sharp as we increase speculators' strategic sophistication.

Result 3 highlights the importance of imperfect depth of reasoning for non-fundamental speculation, overreaction, and reversals as equilibrium outcomes. These phenomena each decline substantially with even limited amounts of higher order reasoning. If speculators engaged in higher order reasoning, then speculators should bet on returns moving in the direction of their perceived valuation, as arbitrageurs do.

Remark 7 (Models of Rational Non-Fundamental Speculation) *Our conclusion that non-fundamental speculation does not survive as we increase depth of reasoning differs from the conclusions drawn by models where rational investors may find it profitable to engage in non-fundamental speculation, such as De Long et al. (1990) and Abreu and Brunnermeier (2002, 2003). There, the underlying cause of mispricing and overvaluation is the behavior of non-strategic investors. Here, we endogenize the*

source of mispricing as coming from speculators that engage in non-fundamental speculation; and when these speculators become sophisticated, non-fundamental speculation disappears. That is, we can trace the differences in conclusions drawn to a focus on different types of investors.

3 Conclusion

We study investors' higher order beliefs, using survey data from the Robert Shiller Investor Confidence surveys. While previous work has documented instances of non-fundamental speculation – investors taking positions in a risky asset in a direction that conflicts with their fundamental views – we find that such speculation is the norm for the U.S. stock market. The majority of investors in the Shiller surveys, who represent an important class of investors, report that other investors have mistaken beliefs, but nevertheless report positive return expectations from speculating in the direction of these mistaken beliefs. In addition, investors report that they believe that stock markets overreact and exhibit momentum and reversal in response to news. Investors' non-fundamental speculation is unprofitable, however; investors' short term return expectation tend to perform poorly in predicting subsequent market returns.

To explore the equilibrium implications of the empirical evidence, we construct a theoretical model that can match the survey evidence, where investors believe that the patterns in prices are driven by other, less sophisticated investors. We find that investors' higher order beliefs amplify stock market overreaction and drive excess volatility. Moreover, we find that for higher order belief-driven non-fundamental speculation to survive in equilibrium, investors must not engage in the types of sophisticated strategic reasoning typically assumed in traditional models.

Our paper also provides direction for future work. The types of higher order beliefs we study may be at play in other asset markets. For example, in a previous version of this paper, we examined foreign exchange markets, and find similar patterns of investors' return expectations that we document here. Additionally, quantitative work may be helpful for better understanding the extent to which stock market fluctuations can be attributed to speculation. We also do not provide micro-foundations for investors' higher order beliefs (and our approach to modeling them is admittedly 'backwards-engineered'). But exploring the source of investors' higher order beliefs, and how they interact with fundamental beliefs, may prove fruitful.

References

- Abreu, Dilip, and Markus K Brunnermeier.** 2002. "Synchronization risk and delayed arbitrage." *Journal of Financial Economics*, 66(2-3): 341–360. [5](#), [30](#), [46](#)
- Abreu, Dilip, and Markus K Brunnermeier.** 2003. "Bubbles and crashes." *Econometrica*, 71(1): 173–204. [5](#), [30](#), [46](#)
- Adam, Klaus, Albert Marcet, and Johannes Beutel.** 2017. "Stock price booms and expected capital gains." *American Economic Review*, 107(8): 2352–2408. [6](#)
- Adam, Klaus, and Stefan Nagel.** 2022. "Expectations Data in Asset Pricing." National Bureau of Economic Research. [5](#)
- Albagli, Elias.** 2015. "Investment horizons and asset prices under asymmetric information." *Journal of Economic Theory*, 158: 787–837. [38](#)
- Allen, Franklin, Stephen Morris, and Hyun Song Shin.** 2006. "Beauty contests and iterated expectations in asset markets." *Review of Financial Studies*, 19(3): 719–752. [4](#), [29](#), [34](#)
- Amromin, Gene, and Steven A Sharpe.** 2014. "From the horse's mouth: Economic conditions and investor expectations of risk and return." *Management Science*, 60(4): 845–866. [6](#)
- Andre, Peter, Philipp Schirmer, and Johannes Wohlfart.** 2023. "Mental models of the stock market." [5](#)
- Angeletos, George-Marios, and Chen Lian.** 2022. "Dampening General Equilibrium: Incomplete Information and Bounded Rationality." National Bureau of Economic Research. [5](#)
- Angeletos, George-Marios, Zhen Huo, and Karthik A Sastry.** 2021. "Imperfect macroeconomic expectations: Evidence and theory." *NBER Macroeconomics Annual*, 35(1): 1–86. [3](#), [31](#), [40](#)
- Bacchetta, Philippe, and Eric van Wincoop.** 2006. "Can information heterogeneity explain the exchange rate determination puzzle?" *American Economic Review*, 96(3): 552–576. [4](#), [38](#)
- Bacchetta, Philippe, and Eric van Wincoop.** 2008. "Higher order expectations in asset pricing." *Journal of Money, Credit and Banking*, 40(5): 837–866. [4](#)
- Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop.** 2009. "Predictability in financial markets: What do survey expectations tell us?" *Journal of International Money and Finance*, 28(3): 406–426. [6](#)
- Baker, Malcolm, and Jeffrey Wurgler.** 2007. "Investor sentiment in the stock market." *Journal of Economic Perspectives*, 21(2): 129–151. [31](#)
- Banerjee, Snehal.** 2011. "Learning from prices and the dispersion in beliefs." *Review of Financial Studies*, 24(9): 3025–3068. [38](#)
- Banerjee, Snehal, and Ilan Kremer.** 2010. "Disagreement and learning: Dynamic patterns of trade." *Journal of Finance*, 65(4): 1269–1302. [4](#), [35](#)

- Banerjee, Snehal, Jesse Davis, and Naveen Gondhi.** 2021. "Choosing to disagree: Endogenous dismissiveness and overconfidence in financial markets." *Journal of Finance, Forthcoming*. 36
- Banerjee, Snehal, Ron Kaniel, and Ilan Kremer.** 2009. "Price drift as an outcome of differences in higher-order beliefs." *Review of Financial Studies*, 22(9): 3707–3734. 5, 29
- Bansal, Ravi, and Amir Yaron.** 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance*, 59(4): 1481–1509. 32
- Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer.** 2015. "X-CAPM: An extrapolative capital asset pricing model." *Journal of Financial Economics*, 115(1): 1–24. 6
- Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer.** 2018. "Extrapolation and bubbles." *Journal of Financial Economics*, 129(2): 203–227. 6, 30
- Bastianello, Francesca, and Paul Fontanier.** 2022. "Partial Equilibrium Thinking, Extrapolation, and Bubbles." 5
- Bastianello, Francesca, and Paul Fontanier.** 2024. "Expectations and Learning from Prices." 5, 41, 42
- Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer.** 2020. "Expectations of fundamentals and stock market puzzles." National Bureau of Economic Research. 6, 31, 42
- Bordalo, Pedro, Nicola Gennaioli, Spencer Yongwook Kwon, and Andrei Shleifer.** 2021. "Diagnostic bubbles." *Journal of Financial Economics*, 141(3): 1060–1077. 42
- Borovička, Jaroslav.** 2020. "Survival and long-run dynamics with heterogeneous beliefs under recursive preferences." *Journal of Political Economy*, 128(1): 206–251. 42
- Bouchaud, Jean-Philippe, Philipp Krueger, Augustin Landier, and David Thesmar.** 2019. "Sticky expectations and the profitability anomaly." *Journal of Finance*, 74(2): 639–674. 42
- Brunnermeier, Markus K, and Stefan Nagel.** 2004. "Hedge Funds and the Technology Bubble." *Journal of Finance*, 59(5): 2013–2040. 1, 5, 29
- Brunnermeier, Markus K., Stefan Nagel, and Lasse H. Pedersen.** 2008. "Carry trades and currency crashes." *NBER Macroeconomics Annual*, 23(1): 313–348. 19
- Bybee, J Leland.** 2023. "The Ghost in the Machine: Generating Beliefs with Large Language Models." Yale University Working Paper. 5
- Bybee, Leland, Bryan Kelly, and Yinan Su.** 2023. "Narrative Asset Pricing: Interpretable Systematic Risk Factors from News Text." *The Review of Financial Studies*. 23
- Bybee, Leland, Bryan T Kelly, Asaf Manela, and Dacheng Xiu.** 2021. "Business news and business cycles." National Bureau of Economic Research. 23, 24
- Campbell, John Y, and John H Cochrane.** 1999. "By force of habit: A consumption-based explanation of aggregate stock market behavior." *Journal of Political Economy*, 107(2): 205–251. 32

- Cespa, Giovanni, and Xavier Vives.** 2015. "The beauty contest and short-term trading." *Journal of Finance*, 70(5): 2099–2154. [4](#)
- Chen, Long, Zhi Da, and Xinlei Zhao.** 2013. "What drives stock price movements?" *Review of Financial Studies*, 26(4): 841–876. [6](#), [31](#), [42](#)
- Coibion, Olivier, Yuriy Gorodnichenko, Saten Kumar, and Jane Ryngaert.** 2021. "Do You Know that I Know that You Know... Higher-Order Beliefs in Survey Data." *Quarterly Journal of Economics*, 136(3): 1387–1446. [4](#)
- Crawford, Vincent P, Miguel A Costa-Gomes, and Nagore Iriberrri.** 2013. "Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications." *Journal of Economic Literature*, 51(1): 5–62. [3](#)
- Dahlquist, Magnus, and Markus Ibert.** 2024. "Equity return expectations and portfolios: Evidence from large asset managers." *The Review of Financial Studies*, 37(6): 1887–1928. [35](#)
- Daniel, Kent, and David Hirshleifer.** 2015. "Overconfident investors, predictable returns, and excessive trading." *Journal of Economic Perspectives*, 29(4): 61–88. [36](#)
- De La O, Ricardo, and Sean Myers.** 2021. "Subjective cash flow and discount rate expectations." *Journal of Finance*, 76(3): 1339–1387. [6](#), [31](#), [42](#)
- De Long, J Bradford, Andrei Shleifer, Lawrence H Summers, and Robert J Waldmann.** 1990. "Positive feedback investment strategies and destabilizing rational speculation." *Journal of Finance*, 45(2): 379–395. [1](#), [5](#), [30](#), [31](#), [46](#)
- Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen.** 2002. "Securities lending, shorting, and pricing." *Journal of Financial Economics*, 66(2-3): 307–339. [32](#)
- Egan, Daniel, Christoph Merkle, and Martin Weber.** 2014. "Second-order beliefs and the individual investor." *Journal of Economic Behavior & Organization*, 107: 652–666. [4](#)
- Eyster, Erik, Matthew Rabin, and Dimitri Vayanos.** 2019. "Financial markets where traders neglect the informational content of prices." *Journal of Finance*, 74(1): 371–399. [5](#)
- Farhi, Emmanuel, and Iván Werning.** 2019. "Monetary policy, bounded rationality, and incomplete markets." *American Economic Review*, 109(11): 3887–3928. [5](#)
- Gabaix, Xavier.** 2008. "Variable rare disasters: A tractable theory of ten puzzles in macro-finance." *American Economic Review*, 98(2): 64–67. [32](#)
- Gandhi, Mihir, Niels Joachim Gormsen, and Eben Lazarus.** 2023. "Excess Persistence in Return Expectations." [6](#)
- García-Schmidt, Mariana, and Michael Woodford.** 2019. "Are low interest rates deflationary? A paradox of perfect-foresight analysis." *American Economic Review*, 109(1): 86–120. [5](#)
- Gârleanu, Nicolae, and Stavros Panageas.** 2021. "What to expect when everyone is expecting: Self-fulfilling expectations and asset-pricing puzzles." *Journal of Financial Economics*, 140(1): 54–73. [30](#)

- Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus.** 2021. "Five facts about beliefs and portfolios." *American Economic Review*, 111(5): 1481–1522. [6](#), [7](#), [35](#), [42](#)
- Greenwood, Robin, and Andrei Shleifer.** 2014. "Expectations of returns and expected returns." *Review of Financial Studies*, 27(3): 714–746. [2](#), [6](#), [25](#), [26](#), [27](#)
- Greenwood, Robin, and Dimitri Vayanos.** 2014. "Bond supply and excess bond returns." *Review of Financial Studies*, 27(3): 663–713. [38](#)
- Greenwood, Robin, and Stefan Nagel.** 2009. "Inexperienced investors and bubbles." *Journal of Financial Economics*, 93(2): 239–258. [42](#)
- Haaland, Ingar K, Christopher Roth, Stefanie Stantcheva, and Johannes Wohlfart.** 2024. "Measuring what is top of mind." National Bureau of Economic Research. [13](#)
- Hamilton, James Douglas.** 2020. *Time series analysis*. Princeton university press. [54](#)
- Han, Jungsuk, and Albert S Kyle.** 2018. "Speculative equilibrium with differences in higher-order beliefs." *Management Science*, 64(9): 4317–4332. [5](#)
- Harris, Milton, and Artur Raviv.** 1993. "Differences of opinion make a horse race." *Review of Financial Studies*, 6(3): 473–506. [4](#), [35](#)
- Harrison, J Michael, and David M Kreps.** 1978. "Speculative investor behavior in a stock market with heterogeneous expectations." *Quarterly Journal of Economics*, 92(2): 323–336. [1](#), [4](#), [5](#), [32](#)
- Hartzmark, Samuel M, and Abigail B Sussman.** 2024. "Eliciting Expectations." [26](#)
- Hazelkorn, Todd M, Tobias J Moskowitz, and Kaushik Vasudevan.** 2023. "Beyond Basis Basics: Liquidity Demand and Deviations from the Law of One Price." *Journal of Finance*, 78(1): 301–345. [19](#), [32](#)
- Jin, Lawrence J, and Pengfei Sui.** 2022. "Asset pricing with return extrapolation." *Journal of Financial Economics*, 145(2): 273–295. [6](#)
- Kandel, Eugene, and Neil D Pearson.** 1995. "Differential interpretation of public signals and trade in speculative markets." *Journal of Political Economy*, 103(4): 831–872. [4](#), [35](#)
- Kasa, Kenneth, Todd B Walker, and Charles H Whiteman.** 2014. "Heterogeneous beliefs and tests of present value models." *Review of Economic Studies*, 81(3): 1137–1163. [4](#)
- Keynes, John Maynard.** 1936. *The General Theory of Interest, Employment, and Money*. London:Macmillan. [1](#)
- Khorrami, Paymon, and Fernando Mendo.** 2021. "Rational sentiments and financial frictions." [30](#)
- Kindleberger, Charles Poor.** 1978. *Manias, panics, and crashes: A history of financial crises*. Basic Books, New York. [5](#)
- Makarov, Igor, and Oleg Rytchkov.** 2012. "Forecasting the forecasts of others: Implications for asset pricing." *Journal of Economic Theory*, 147(3): 941–966. [4](#)

- Mankiw, N Gregory, and Ricardo Reis.** 2002. "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve." *Quarterly Journal of Economics*, 117(4): 1295–1328. [42](#)
- Martin, Ian WR, and Dimitris Papadimitriou.** 2022. "Sentiment and speculation in a market with heterogeneous beliefs." *American Economic Review*, 112(8): 2465–2517. [5](#), [31](#)
- McCarthy, Odhrain, and Sebastian Hillenbrand.** 2021. "Heterogeneous Investors and Stock Market Fluctuations." [6](#), [42](#)
- McKay, Charles.** 1841. *Delusions of The Madness of Crowds*. Harriman House Classics. [5](#)
- Moskowitz, Tobias J, Yao Hua Ooi, and Lasse Heje Pedersen.** 2012. "Time series momentum." *Journal of Financial Economics*, 104(2): 228–250. [19](#)
- Nagel, Stefan, and Zhengyang Xu.** 2022a. "Asset pricing with fading memory." *Review of Financial Studies*, 35(5): 2190–2245. [6](#), [31](#)
- Nagel, Stefan, and Zhengyang Xu.** 2022b. "Dynamics of subjective risk premia." National Bureau of Economic Research. [6](#)
- Nimark, Kristoffer.** 2017. "Dynamic higher order expectations." [4](#)
- Odean, Terrance.** 1998. "Volume, volatility, price, and profit when all traders are above average." *Journal of Finance*, 53(6): 1887–1934. [36](#)
- Read, Daniel, Shane Frederick, and Marc Scholten.** 2013. "DRIFT: an analysis of outcome framing in intertemporal choice." *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 39(2): 573. [26](#)
- Reis, Ricardo.** 2020. "Imperfect Macroeconomic Expectations: Yes, But, We Disagree." *NBER Macroeconomics Annual*, 35(1): 99–111. [40](#)
- Scheinkman, Jose A, and Wei Xiong.** 2003. "Overconfidence and speculative bubbles." *Journal of Political Economy*, 111(6): 1183–1220. [4](#), [5](#), [32](#)
- Schmidt-Engelbertz, Paul, and Kaushik Vasudevan.** 2023. "Asset Pricing with Higher Order Beliefs."
- Shiller, Robert J.** 2000. "Measuring bubble expectations and investor confidence." *The Journal of Psychology and Financial Markets*, 1(1): 49–60. [7](#)
- Sims, Christopher A.** 2003. "Implications of rational inattention." *Journal of Monetary Economics*, 50(3): 665–690. [36](#)
- Simsek, Alp.** 2021. "The macroeconomics of financial speculation." *Annual Review of Economics*, 13: 335–369. [1](#)
- Singleton, Kenneth J.** 1987. "Asset prices in a time series model with disparately informed, competitive traders." In *New Approaches to Monetary Economics*, ed. W. A. Barnett and Kenneth J. Singleton, 249–72. Cambridge University Press. [4](#)
- Soros, George.** 2015. *The alchemy of finance*. John Wiley & Sons. [5](#)

- Spiegel, Matthew.** 1998. "Stock price volatility in a multiple security overlapping generations model." *Review of Financial Studies*, 11(2): 419–447. [38](#)
- Temin, Peter, and Hans-Joachim Voth.** 2004. "Riding the south sea bubble." *American Economic Review*, 94(5): 1654–1668. [5](#), [29](#)
- Valente, Joao Paulo, Kaushik Vasudevan, and Tianhao Wu.** 2021. "The Role of Beliefs in Asset Prices: Evidence from Exchange Rates." [31](#)
- Vissing-Jorgensen, Annette.** 2003. "Perspectives on behavioral finance: Does "irrationality" disappear with wealth? Evidence from expectations and actions." *NBER Macroeconomics Annual*, 18: 139–194. [6](#)
- Wachter, Jessica A.** 2013. "Can time-varying risk of rare disasters explain aggregate stock market volatility?" *Journal of Finance*, 68(3): 987–1035. [32](#)
- Woodford, Michael.** 2001. "Imperfect common knowledge and the effects of monetary policy." *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, 25. [36](#)
- Zauberman, Gal, Jonathan Levav, Kristin Diehl, and Rajesh Bhargave.** 2010. "1995 feels so close yet so far: The effect of event markers on subjective feelings of elapsed time." *Psychological Science*, 21(1): 133–139. [26](#)
- Zentefis, Alexander K.** 2022. "Self-fulfilling asset prices." *Review of Asset Pricing Studies*, 12(4): 886–917. [30](#)

A Proofs

Proof of Lemma 1

Proof. Given the linear-Gaussian environment, belief updating follows the standard Kalman filter recursion method derived in [Hamilton \(2020\)](#). \square

Proof of Lemma 2

Proof. Given their higher order beliefs, speculator i believes each other investor j believes (1) that the persistence of fundamentals is $\hat{\rho}$ and (2) that investors in period $t + 1$ will share their beliefs, i.e., $\mathbb{E}_t^i(\mathbb{E}_t^j(P_{t+1})) = \frac{\hat{\rho}}{1-\hat{\rho}} \mathbf{d}_t^j$.

Hence, each speculator perceives that each other investor j 's demand for the risky asset can be expressed as

$$Q_t^j = \frac{\mathbb{E}_t^j(P_{t+1} + D_{t+1}) - P_t}{\gamma \mathbb{V}_t^j(P_{t+1} + D_{t+1})}.$$

Applying the market clearing condition, $\int Q_t^j = 0$, re-writing in terms of P_t , investor i perceives that the equilibrium price is given by $P_t = \frac{\hat{\rho}}{1-\hat{\rho}} \mathbf{d}_{t,2}^i$. Since the market clearing price does not vary across investors, and $\frac{\hat{\rho}}{1-\hat{\rho}}$ is a constant, it follows that $\mathbf{d}_{t,2}^i$ must also be equal across speculators. \square

Proof of Lemma 3

Proof. For notational simplicity, we define $\kappa = \kappa_1 + \kappa_2$, $\hat{\kappa} = \hat{\kappa}_1 + \hat{\kappa}_2$. We also note that throughout the proofs, we freely make use of the substitution $\frac{1}{\theta} \int_0^\theta \mathbf{d}_t^i = \frac{1}{1-\theta} \int_0^1 \mathbf{d}_t^i = \mathbf{d}_t^s = \mathbf{d}_t$, i.e., the average speculator and the average arbitrageur have the same beliefs about fundamentals, which are equal to the true level of the unobserved fundamental.

Derivation of B_1 : We conjecture that the pricing formula is of the form $P_t = B_1 \mathbf{d}_t^s$. Then note that, by the definition of equilibrium, in particular that speculators' perceived price must coincide with the true price, we have that $\mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t^s$.

To forecast the price in period $t + 1$, speculator i forecasts the average belief in period $t + 1$, based on their forecast of \mathbf{d}_{t+1} and s_{t+1} :

$$\begin{aligned} \mathbb{E}_t^i(\mathbf{d}_{t+1,2}) &= \mathbb{E}_t^i \int \mathbf{d}_{t+1}^j = (1 - (\hat{\kappa}_1 + \hat{\kappa}_2)) \hat{\rho} \mathbf{d}_{t,2} + \hat{\kappa}_1 \underbrace{\mathbb{E}_t(D_{t+1})}_{=\rho \mathbf{d}_t^i} + \hat{\kappa}_2 \underbrace{\mathbb{E}_t(s_{t+1}^{-i})}_{=\rho \mathbf{d}_t^i} \\ &= (1 - \hat{\kappa}) \hat{\rho} \mathbf{d}_{t,2} + \hat{\kappa} \rho \mathbf{d}_t^i. \end{aligned}$$

Speculator i 's expected period $t + 1$ payoff is

$$\begin{aligned}\mathbb{E}_t^i(D_{t+1} + P_{t+1}) &= \underbrace{\rho \mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{\mathbb{E}_t^i(B_0 \mathbf{d}_{t+1,2})}_{=\mathbb{E}_t^i(P_{t+1})} \\ &= (1 + \hat{\kappa} B_0) \rho \mathbf{d}_t^i + (1 - \hat{\kappa}) \hat{\rho} B_0 \mathbf{d}_{t,2}.\end{aligned}$$

Speculator i 's subjective perceived variance of the period $t + 1$ dividend is

$$\mathbb{V}_t^j(D_{t+1}) = \sigma_\epsilon^2 + \sigma_v^2,$$

and his subjective perceived variance of the period $t + 1$ price is

$$\begin{aligned}\mathbb{V}_t^j(P_{t+1}) &= \mathbb{V}_t^j\left(\frac{\hat{\rho}}{1 - \hat{\rho}} \mathbf{d}_{t+1}^j\right) \\ &= B_0^2 \mathbb{V}_t^j((1 - \hat{\kappa}) \hat{\rho} \mathbf{d}_{t,2} + \hat{\kappa}_1 D_{t+1} + \hat{\kappa}_2 S_{t+1}^-) \\ &= B_0^2 \left(\hat{\kappa}_1^2 (\sigma_\epsilon^2 + \sigma_v^2) + \hat{\kappa}_2^2 (\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2 \hat{\kappa}_1 \hat{\kappa}_2 \sigma_\epsilon^2 \right).\end{aligned}$$

Hence, his perceived variance of the period $t + 1$ payoff is given by

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + \underbrace{2 \mathbf{C}(P_{t+1}, D_{t+1})}_{=B_0(\hat{\kappa}\sigma_\epsilon^2 + \hat{\kappa}_1\sigma_v^2)} \\ &= (1 + \hat{\kappa} B_0)^2 \sigma_\epsilon^2 + (1 + \hat{\kappa}_1 B_0)^2 \sigma_v^2 + B_0^2 \hat{\kappa}_2^2 (\sigma_\eta^2 + \sigma_\phi^2).\end{aligned}$$

We note that this perceived variance does not depend on the coefficient of interest, B_1 , and is shared across all speculators. So, for notational simplicity, we denote it as a constant $A_S \equiv \mathbb{V}_t^i(P_{t+1} + D_{t+1})$. Speculator demand is then given by

$$\begin{aligned}\int_0^\theta Q_t^i &= \int_0^\theta \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})} \\ &= \frac{\theta}{\gamma} \frac{(1 + \hat{\kappa} B_0) \rho \mathbf{d}_t^i + (1 - \hat{\kappa}) \hat{\rho} B_0 \mathbf{d}_{t,2} - P_t}{A_S},\end{aligned}$$

where the second line comes from substituting $\int_0^\theta \mathbf{d}_t^i = \theta \mathbf{d}_t^s$. Substituting $\mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t^s$, this becomes

$$\int_0^\theta Q_t^i = \frac{\theta}{\gamma} \frac{((1 + \hat{\kappa} B_0) \rho + (1 - \hat{\kappa}) \hat{\rho} B_1) \mathbf{d}_t^s - P_t}{A_S}.$$

Turning to the arbitrageurs, they know that the form of the pricing rule is $B_1 \mathbf{d}_t^s$. Moreover, they know that other investors correctly perceive the parameters governing the risky asset's

fundamentals. Hence, arbitrageur i 's expected period $t + 1$ payoff is

$$\mathbb{E}_t^i(P_{t+1} + D_{t+1}) = \rho \mathbf{d}_t^i + B_1((1 - \kappa)\rho \mathbf{d}_t^s + \kappa \rho \mathbf{d}_t^i).$$

Arbitrageur i 's perceived variance of dividends and the next period's price are

$$\begin{aligned}\mathbb{V}_t^i(D_{t+1}) &= \sigma_\varepsilon^2 + \sigma_v^2, \text{ and} \\ \mathbb{V}_t^i(P_{t+1}) &= B_1^2(\kappa_1(\sigma_\varepsilon^2 + \sigma_v^2) + \kappa_2(\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\varepsilon^2).\end{aligned}$$

Hence, arbitrageur i 's perceived variance of the period $t + 1$ payoff is

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbf{C}(P_{t+1}, D_{t+1})}_{=B_1(\kappa\sigma_\varepsilon^2 + \kappa_1\sigma_v^2)} \\ &= (1 + \kappa B_1)^2 \sigma_\varepsilon^2 + (1 + \kappa_1 B_1)^2 \sigma_v^2 + B_1^2 \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2).\end{aligned}$$

This perceived variance is quadratic in B_1 . For notational simplicity, we define A_0 , A_1 , and A_2 as the quadratic equation coefficients, i.e.,

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2 \sigma_v^2 + \kappa^2 \sigma_\varepsilon^2 + \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2))}_{A_2} B_1^2 + \underbrace{2(\kappa_1 \sigma_v^2 + \kappa \sigma_\varepsilon^2)}_{A_1} B_1 + \underbrace{\sigma_v^2 + \sigma_\varepsilon^2}_{A_0},$$

and note that $A_0, A_1, A_2 > 0$.

Arbitrageur demand is then given by

$$\begin{aligned}\int_\theta^1 Q_t^i &= \int_\theta^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})} \\ &= \frac{(1 - \theta)}{\gamma} \frac{\rho(1 + B_1) \mathbf{d}_t^s - P_t}{A_2 B_1^2 + A_1 B_1 + A_0},\end{aligned}$$

where the second line comes from the fact that $\int_\theta^1 \mathbf{d}_t^i = (1 - \theta) \mathbf{d}_t^s$. Imposing market clearing ($\int Q_t^i = \int_0^\theta Q_t^i + \int_\theta^1 Q_t^i = 0$), and solving for P_t , we get that

$$P_t = \frac{\theta(A_0 + A_1 B_1 + A_2 B_1^2)(\rho(1 + \hat{\kappa} B_0) + \hat{\rho}(1 - \hat{\kappa}) B_1) + (1 - \theta) A_S \rho(1 + B_1)}{(1 - \theta) A_S + \theta(A_0 + A_1 B_1 + A_2 B_1^2)} \mathbf{d}_t^s.$$

Matching coefficients, we have that

$$B_1 = \frac{\theta(A_0 + A_1 B_1 + A_2 B_1^2)(\rho(1 + \hat{\kappa} B_0) + \hat{\rho}(1 - \hat{\kappa}) B_1) + (1 - \theta) A_S \rho(1 + B_1)}{(1 - \theta) A_S + \theta(A_0 + A_1 B_1 + A_2 B_1^2)}.$$

Multiplying both sides by $(1 - \theta) A_S + \theta(A_0 + A_1 B_1 + A_2 B_1^2)$, subtracting the resulting left

hand side from both sides, and simplifying, we get that B_1 is the solution to the cubic equation

$$\begin{aligned}
0 = & \theta(1 - (1 - \hat{\kappa})\hat{\rho})A_2B_1^3 - \frac{\theta(1 - (1 - \hat{\kappa})\hat{\rho})(\rho A_2 - (1 - \hat{\rho})A_1)}{1 - \hat{\rho}}B_1^2 \\
& + \left(A_S(1 - \theta)(1 - \rho) + \theta(1 - (1 - \hat{\kappa})\hat{\rho})A_0 - \theta\rho \left(\frac{1 - (1 - \hat{\kappa})\hat{\rho}}{1 - \hat{\rho}} \right) A_1 \right) B_1 \quad (\text{A.1}) \\
& - \rho \left(A_S(1 - \theta) + \theta \left(\frac{1 - (1 - \hat{\kappa})\hat{\rho}}{1 - \hat{\rho}} \right) A_0 \right).
\end{aligned}$$

Positivity of B_1 : We prove by contradiction. Assume that $B_1 \leq 0$. Assume without loss of generality that $\mathbf{d}_t > 0$.

We denote the average speculator's expected return and perceived variance as

$$\begin{aligned}
ER_t^S &= \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t), \text{ and} \\
V_t^S &= \frac{1}{\theta} \int_0^\theta \mathbb{V}_t^i(P_{t+1} + D_{t+1} - P_t),
\end{aligned}$$

and the average arbitrageur's expected return and perceived variance as

$$\begin{aligned}
ER_t^A &= \frac{1}{1 - \theta} \int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t), \text{ and} \\
V_t^A &= \frac{1}{1 - \theta} \int_\theta^1 \mathbb{V}_t^i(P_{t+1} + D_{t+1} - P_t).
\end{aligned}$$

The market clearing condition implies that

$$0 = \frac{\theta ER_t^S V_t^A + (1 - \theta) ER_t^A V_t^S}{V_t^A V_t^S}.$$

Since θ , $(1 - \theta)$, V_t^S , and V_t^A are all positive, for the market to clear, we must have that $\text{Sign}(ER_t^S) = -\text{Sign}(ER_t^A)$, i.e., the average speculator and arbitrageur must have opposite sign expected returns.

The objective expected return of the risky asset is positive, i.e.,

$$\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = \underbrace{\rho \mathbf{d}_t}_{>0} + \underbrace{B_1}_{\leq 0} \underbrace{(\rho \mathbf{d}_t - \mathbf{d}_t)}_{\leq 0} > 0.$$

Because arbitrageurs have correct fundamental beliefs on average ($\frac{1}{1-\theta} \int \mathbf{d}_t^i = \mathbf{d}_t$), and because they know the form of the true pricing formula, the average arbitrageur's expected return is correct. It follows that the average speculator must have negative expected returns.

Next note that $\mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t < 0$, since $B_0 > 0$, $B_1 < 0$, and $\mathbf{d}_t > 0$. The average speculator's expected return is

$$ER_t^S = \underbrace{\rho \mathbf{d}_t^s}_{>0} + B_0 \underbrace{((1 - \hat{\kappa}) \hat{\rho} \mathbf{d}_{t,2} + \hat{\kappa} \rho \mathbf{d}_t - \mathbf{d}_{t,2})}_{>0} > 0.$$

But this contradicts that the average speculator's expected return is negative. \square

Proof of Proposition 1

Proof. Without loss of generality, we assume that $\mathbf{d}_t > 0$. We first show Lemma A.1.

Lemma A.1 $B_1 > \frac{\rho}{1-\rho}$ if and only if $\hat{\rho} > \rho$.

Proof. For the if direction, assume that $\hat{\rho} > \rho$, and assume by contradiction that $B_1 \leq \frac{\rho}{1-\rho}$. Then the objective one-period ahead expected return of the risky asset is positive:

$$\begin{aligned} \mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) &= \rho \mathbf{d}_t + B_1 \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1}^s - \mathbf{d}_t^s)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\ &\geq \rho \mathbf{d}_t - \frac{\rho}{1-\rho} (1-\rho) \mathbf{d}_t \\ &= 0. \end{aligned}$$

Because the average arbitrageur has correct beliefs and knows the form of the equilibrium pricing rule, they also must have a positive one-period ahead expected return. Hence, by the same argument in the proof of Lemma 3, the average speculator must have a negative one-period ahead expected return.

Next, considering speculators' perception of the economy, their second order belief must satisfy $\mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t$. Making use of the substitution that $\mathbf{d}_t^s = \frac{1}{\theta} \int_0^\theta \mathbf{d}_t^i = \mathbf{d}_t$, we can write the average speculator's expected returns as

$$\begin{aligned} \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t &= \rho \mathbf{d}_t + B_0 ((1 - \hat{\kappa}) \hat{\rho} \mathbf{d}_{t,2} + \hat{\kappa} \rho \mathbf{d}_t - \mathbf{d}_{t,2}) \\ &= (\rho - (1 - \hat{\rho}) B_1 + \hat{\kappa} (\rho B_0 - \hat{\rho} B_1)) \mathbf{d}_t \text{ (substituting } \mathbf{d}_{t,2} = \frac{B_1}{B_0} \mathbf{d}_t) \end{aligned} \tag{A.2}$$

$$\begin{aligned}
&\geq \frac{\rho((1-\hat{\kappa})\hat{\rho} + (1-\rho)\hat{\kappa}B_0 - \rho)}{1-\rho} \mathbf{d}_t \text{ (substituting } B_1 \leq \frac{\rho}{1-\rho}\text{)} \\
&> \frac{\rho(1-\hat{\kappa})(\hat{\rho}-\rho)}{1-\rho} \mathbf{d}_t \text{ since } B_0 > \frac{\rho}{1-\rho} \iff \hat{\rho} > \rho \\
&> 0 \text{ since } \hat{\rho} > \rho.
\end{aligned}$$

But this is a contradiction since the average speculator's expected return must be negative. Hence we must have that $B_1 > \frac{\rho}{1-\rho}$.

For the only if direction, assume that $B_1 > \frac{\rho}{1-\rho}$. Then the objective one-period ahead expected return is negative:

$$\begin{aligned}
\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) &= \rho \mathbf{d}_t + B_1 \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1} - \mathbf{d}_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
&< \rho \mathbf{d}_t - \frac{\rho}{1-\rho} (1-\rho) \mathbf{d}_t \quad (\text{A.3}) \\
&= 0.
\end{aligned}$$

Hence, by similar argument as before, the average speculator's one-period ahead expected return is positive. This means that

$$\begin{aligned}
0 &< \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t \\
&= (\rho - (1-\hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1)) \\
&< \frac{\rho(\hat{\rho}-\rho)(1-(1-\hat{\kappa})\hat{\rho})}{(1-\rho)(1-\hat{\rho})} \mathbf{d}_t \text{ (substituting } B_0 = \frac{\hat{\rho}}{1-\hat{\rho}} \text{ and } B_1 \leq \frac{\rho}{1-\rho}\text{)},
\end{aligned}$$

which implies that $\hat{\rho} > \rho$. □

Additionally, we note that the objective expected returns of the risky asset are negative if and only if $B_1 > \frac{\rho}{1-\rho}$, which can be seen by the fact that

$$\begin{aligned}
0 &> \mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = \rho \mathbf{d}_t + B_1 \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1} - \mathbf{d}_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
&\iff B_1 > \frac{\rho}{1-\rho}.
\end{aligned}$$

With Lemma A.1 and the auxiliary claim in hand, we turn to the proof of the main claim.

For claim (i), we can see the average speculator's belief about fundamentals is correct, i.e., $\frac{1}{\theta} \int_0^\theta \mathbf{d}_t^i = \mathbf{d}_t$. Because they also know that the true persistence of the fundamental process is ρ , the average speculator perceives that the fundamental (buy-and-hold) value of the asset is

$\frac{\rho}{1-\rho} \mathbf{d}_t^s$. Using Lemma A.1, this valuation is less than the price, $B_1 \mathbf{d}_t^s$, if and only if $\hat{\rho} > \rho$, hence proving claim (i).

For time-series momentum in claim (ii), using the auxiliary claim, because the arbitrageur has correct beliefs on average and knows the form of the equilibrium pricing rule, they expect negative returns for the risky asset one period ahead if and only if $\hat{\rho} > \rho$. Then, by similar argument as before regarding the opposing signs of the average speculator's and arbitrageur's return expectations, the average speculator must have positive expected returns if and only if $\hat{\rho} > \rho$. Since this is true for $\mathbf{d}_t > 0$, positive expected returns are sufficient for perceived time-series momentum since $\mathbf{d}_t > 0$ on average following positive news. Perceived long-term reversals follow immediately from perceived overvaluation in (i), since the average speculator perceives the long-term buy-and-hold return of the risky asset to be negative.

For non-fundamental speculation in (iii), this follows immediately from perceived overvaluation in (i) and positive return expectations in (ii). □

Proof of Mapping the Model to the Data (Remark 1)

Proof. We can observe that $\frac{1}{\theta} \int_0^\theta \mathbb{1}_{P_t > \frac{\rho}{1-\rho} \mathbf{d}_t^i} = \mathbb{P}(B_1 \mathbf{d}_t > \frac{\rho}{1-\rho} \mathbf{d}_t^i)$, where \mathbb{P} is the probability operator. Note that given the linear Gaussian structure of the model, and the fact that the average speculator's belief about fundamentals matches rational expectations, each speculator's fundamental belief can be written as $\mathbf{d}_t^i = \mathbf{d}_t + \zeta_i$, where $\zeta_i \sim N(0, \sigma_\zeta^2)$ for some variance σ_ζ^2 that is pinned down in the model by the variance of the signals observed by speculators. Hence, we can observe that $\mathbb{P}(B_1 \mathbf{d}_t > \frac{\rho}{1-\rho} \mathbf{d}_t^i) = \mathbb{P}((B_1 - \frac{\rho}{1-\rho}) \mathbf{d}_t > \zeta_i)$. Since $B_1 > \frac{\rho}{1-\rho}$ when $\hat{\rho} > \rho$, and ζ_i is a random variable with mean zero and fixed variance, this probability is strictly increasing in \mathbf{d}_t .

Additionally, observe that in equilibrium, from Equation (A.2), the average speculator's one-period ahead return expectation is given by

$$\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) = (\rho - (1 - \hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho} B_1)) \mathbf{d}_t^s,$$

which is similarly strictly increasing in \mathbf{d}_t^s , since $(\rho - (1 - \hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho} B_1)) > 0$, per Lemma A.1. □

Proof of Necessary Condition for Multiplicity (Remark 2)

Proof. From the proof of Proposition 1, we know that all equilibria have to satisfy $B_1 > \frac{\rho}{1-\rho}$. Hence, it is useful to perform a change of variables, $B_1 = \beta_1 + \frac{\rho}{1-\rho}$, and re-write the cubic equation that B_1 must satisfy (Equation A.1) as a function of β_1 . Doing so and simplifying, any

equilibrium must coincide with a real and positive root of the equation

$$0 = z_3\beta_1^3 + z_2\beta_1^2 + z_1\beta_1 + z_0,$$

where

$$\begin{aligned} z_3 &\equiv \theta(1 - (1 - \hat{\kappa})\hat{\rho})A_2, \\ z_2 &\equiv \frac{\theta(1 - (1 - \hat{\kappa})\hat{\rho})((1 - \rho)(1 - \hat{\rho})A_1 + \rho(2 + \rho - 3\hat{\rho})A_2)}{(1 - \rho)(1 - \hat{\rho})}, \\ z_1 &\equiv ((1 - \rho)^2(1 - \hat{\rho}))^{-1} (\theta(1 - \rho)^2(1 - \hat{\rho})(1 - (1 - \hat{\kappa})\hat{\rho})A_0 \\ &\quad - \theta(1 - \rho)\rho(1 + \rho - 2\hat{\rho})(1 - (1 - \hat{\kappa})\hat{\rho})A_1 \\ &\quad - \theta\rho^2(3\hat{\rho} - 1 - 2\rho)(1 - (1 - \hat{\kappa})\hat{\rho})A_2 + (1 - \theta)(1 - \rho)^3(1 - \hat{\rho})A_3), \text{ and} \\ z_0 &\equiv \frac{\theta\rho(\rho - \hat{\rho})(1 - (1 - \hat{\kappa})\hat{\rho})((1 - \rho)^2A_0 + \rho((1 - \rho)A_1 + \rho A_2))}{(1 - \rho)^3(1 - \hat{\rho})}. \end{aligned}$$

Descartes' Rule of Signs states that the upper bound for the number of real and positive roots of a polynomial is equal to the number of sign changes in the coefficients (e.g., a sign change is if z_3 and z_2 have different signs or if z_2 and z_1 have different signs). Assuming that $\hat{\rho} > \rho$, as in Proposition 1, it is necessarily the case that $z_3 > 0$ and $z_0 < 0$. Hence, for there to be more than one sign change in the coefficients (and more than one equilibrium), a necessary condition is that $z_2 < 0$, which in turn requires that $3\hat{\rho} > 2 + \rho$, which reduces to $\hat{\rho} > 2/3 + 1/3\rho$.

An additional necessary condition is that $z_1 > 0$. This can be seen as placing additional joint restrictions on the noise in signals observed by investors, the proportion of speculators in the economy, and higher order beliefs about persistence.

We note that these conditions are not sufficient. In particular, it may be the case that there is only one real root, even if there are three sign changes in the coefficients. Additionally, inspecting z_2 , for it to be negative, we also need that $\rho(3\hat{\rho} - 2 - \rho)A_2 > (1 - \rho)(1 - \hat{\rho})A_1$, i.e., there are additional restrictions on A_1 and A_2 , which depend upon the noise in public and private signals relative to fundamentals, in order for z_2 to be negative.

□

Proof of Result 1

Proof. All claims follow immediately from the proof of Proposition 1. Perceived overvaluation and long term reversal follow from the fact that the average speculator has correct fundamental beliefs on average, and correctly recognizes that the risky asset is overvalued.

Short term reversal is also proven in the proof of Proposition 1, since arbitrageurs have correct one-period ahead return expectations and expected negative returns whenever $\mathbf{d}_t > 0$, which is true on average following positive news. \square

Proof of Proposition 2

Proof. The proof follows by induction. For the level 1 equilibrium, the claim follows immediately from Lemma 3.

For $k > 1$, assume that in the level k equilibrium, speculators perceive the period t price as governed by the level $k - 1$ pricing function, i.e., speculator i perceives that the equilibrium price is given by

$$P_t = \mathbb{E}_t^i \left(B_{k-1} \int_0^1 \mathbf{d}_t^j \right) = B_{k-1} \mathbf{d}_{t,2},$$

where the second order belief about fundamentals, $\mathbf{d}_{t,2}$, is equal across all speculators because B_{k-1} is a constant and P_t is the same across all investors. We conjecture that the true pricing formula is of the form $B_k \mathbf{d}_t^s$. Note that in equilibrium, for speculators' beliefs to be consistent with the price they observe, we must have that $\mathbf{d}_{t,2} = \frac{B_k}{B_{k-1}} \mathbf{d}_t^s$.

The average speculator's forecasted expected period $t + 1$ payoff is then given by

$$\begin{aligned} \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) &= \rho \mathbf{d}_t^s + B_{k-1}((1 - \kappa)\rho \mathbf{d}_{t,2} + \kappa \rho \mathbf{d}_t^s) \\ &= (1 + \kappa B_{k-1} + (1 - \kappa)B_k)\rho \mathbf{d}_t^s. \end{aligned} \quad (\text{A.4})$$

Each speculator i 's forecast variance of the period $t + 1$ dividend and price are given by

$$\begin{aligned} \mathbb{V}_t^i(D_{t+1}) &= \sigma_\epsilon^2 + \sigma_v^2 \\ \mathbb{V}_t^i(P_{t+1}) &= B_{k-1}^2(\kappa_1^2(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2^2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2), \end{aligned}$$

and speculator i 's forecast variance of the period $t + 1$ payoff is

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = (1 + \kappa B_{k-1})^2 \sigma_\epsilon^2 + (1 + \kappa_1 B_{k-1})^2 \sigma_v^2 + B_{k-1}^2 \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2).$$

Defining, A_2 , A_1 , and A_0 as in Lemma 3, note that this forecast variance can be written as

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2 \sigma_v^2 + \kappa^2 \sigma_\epsilon^2 + \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2))}_{A_2=} B_{k-1}^2 + \underbrace{2(\kappa_1 \sigma_v^2 \kappa + \kappa \sigma_\epsilon^2)}_{A_1=} B_{k-1} + \underbrace{\sigma_v^2 + \sigma_\epsilon^2}_{A_0=}.$$

Hence, the total speculator demand can be written as

$$\int_0^\theta Q_t^i = \int_0^\theta \frac{\theta}{\gamma} \frac{(1 + \kappa B_{k-1} + (1 - \kappa) B_k) \rho \mathbf{d}_t^s - P_t}{A_2 B_{k-1}^2 + A_1 B_{k-1} + A_0}. \quad (\text{A.5})$$

The average arbitrageur knows that the form of the period t price is $B_k \mathbf{d}_t$. Accordingly, their forecasted expected period $t + 1$ payoff is

$$\begin{aligned} \frac{1}{1 - \theta} \int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1}) &= \rho \mathbf{d}_t^s + B_k \rho \mathbf{d}_t^s \\ &= (1 + B_k) \rho \mathbf{d}_t^s. \end{aligned} \quad (\text{A.6})$$

Each arbitrageur i 's forecast variance of the period $t + 1$ payoff is

$$\begin{aligned} \mathbb{V}_t^i(D_{t+1}) &= \sigma_\epsilon^2 + \sigma_v^2 \\ \mathbb{V}_t^i(P_{t+1}) &= B_k^2 (\kappa_1^2 (\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2^2 (\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1 \kappa_2 \sigma_\epsilon^2), \end{aligned}$$

and arbitrageur i 's forecast variance of the period $t + 1$ payoff is

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = (1 + \kappa B_k)^2 \sigma_\epsilon^2 + (1 + \kappa_1 B_k)^2 \sigma_v^2 + B_k^2 \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2),$$

which can be re-written as

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2 \sigma_v^2 + \kappa^2 \sigma_\epsilon^2 + \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2))}_{A_2=} B_k^2 + \underbrace{2(\kappa_1 \sigma_v^2 \kappa + \kappa \sigma_\epsilon^2)}_{A_1=} B_k + \underbrace{\sigma_v^2 + \sigma_\epsilon^2}_{A_0=}.$$

Then, the total arbitrageur demand can be written as

$$\int_\theta^1 Q_t^i = \int_\theta^1 \frac{1 - \theta}{\gamma} \frac{(1 + B_k) \rho \mathbf{d}_t^s - P_t}{A_2 B_k^2 + A_1 B_k + A_0}.$$

Imposing the market clearing condition, $\int Q_t^i = 0$, re-writing in terms of P_t , and matching coefficients, we get that B_k is the solution to the cubic equation

$$0 = y_3 B_k^3 + y_2 B_k^2 + y_1 B_k + y_0,$$

where

$$\begin{aligned} y_3 &\equiv \theta(1 - (1 - \kappa)\rho)A_2, \\ y_2 &\equiv \theta((1 - (1 - \kappa)\rho)A_1 - \rho A_2(1 + \kappa B_{k-1})), \end{aligned}$$

$y_1 \equiv (1 - (1 - \theta\kappa)\rho)A_0 + (1 - \theta)(1 - \rho)A_2B_{k-1}^2 + A_1((1 - \rho - \theta(1 - (1 - \kappa)\rho))B_{k-1} - \theta\rho)$, and
 $y_0 \equiv -A_0\rho(1 + \theta\kappa B_{k-1}) - (1 - \theta)\rho(A_1B_{k-1} + A_2B_{k-1}^2)$.

The proof of positivity is identical to the proof of positivity in Lemma 3, replacing B_0 and B_1 with B_{k-1} and B_k . \square

Proof of Result 2

Proof. The proof is nearly identical to the proofs of Proposition 1 and Result 1, replacing B_0 and B_1 with B_{k-1} and B_k . Note that $\hat{\rho} > \rho \iff B_0 > \frac{\rho}{1-\rho}$. Hence, the proof of Proposition 1 implies we have that $B_0 > \frac{\rho}{1-\rho} \iff B_1 > \frac{\rho}{1-\rho}$. By a nearly identical argument, $B_{k-1} > \frac{\rho}{1-\rho} \iff B_k > \frac{\rho}{1-\rho}, \forall k > 1$. In turn, this corresponds with overreaction, short- and long-term reversals, and non-fundamental speculation if $\hat{\rho} > \rho$. \square

Proof of Result 3

Proof. First, we show that $B_k < B_{k-1}$. Assume that $\hat{\rho} > \rho$, and assume without loss of generality that $\mathbf{d}_t^s > 0$.

From Result 2, we know that $B_k > \frac{\rho}{1-\rho}$, so the objective one-period ahead expected return of the risky asset is negative:

$$\begin{aligned} \mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) &= \rho \mathbf{d}_t + B_k(\rho \mathbf{d}_t - \mathbf{d}_t) \\ &< \rho \mathbf{d}_t - \frac{\rho}{1-\rho}(1-\rho)\mathbf{d}_t \\ &= 0. \end{aligned}$$

Because the average arbitrageur has correct expectations on average, the average arbitrageur has negative expected returns, and accordingly, the average speculator has positive expected returns. This, in turn, holds if and only if $\frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) > \frac{1}{1-\theta} \int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1})$. Using Equations (A.4) and (A.6), we can observe that

$$\begin{aligned} \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) &> \frac{1}{1-\theta} \int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1}) \\ &\iff (1 + \kappa B_{k-1} + (1 - \kappa)B_k)\rho \mathbf{d}_t^s > (1 + B_k)\rho \mathbf{d}_t^s \\ &\iff B_{k-1} > B_k. \end{aligned}$$

It immediately follows that the price in the level k equilibrium is lower than the price in the level $k - 1$ equilibrium, i.e., there is less overvaluation when fundamentals are positive.

Moreover, this holds for each K , so we have a sequence, $B_1, B_2, \dots, B_k, \dots$ such that $B_k < B_{k-1}$, where $B_k > \frac{\rho}{1-\rho}, \forall k$, i.e., we have a monotonically decreasing sequence that is bounded

below. By the monotone convergence theorem, $\lim_{k \rightarrow \infty} B_k \rightarrow \bar{B}$ for some value \bar{B} .

To solve for \bar{B} , consider the limit as $k \rightarrow \infty$, where $\bar{B} = B_{k-1} = B_k$, i.e., arbitrageurs and speculators conjecture the same pricing rule. Then, we can summarize the market clearing condition as

$$\begin{aligned} 0 &= \int_0^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)}{\mathbb{V}_t^i(P_{t+1} + D_{t+1})} \\ &= \int_0^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t) \\ &= \rho \mathbf{d}_t^s + \bar{B}(\rho \mathbf{d}_t^s - \mathbf{d}_t^s). \end{aligned}$$

Solving for \bar{B} , we get that $\bar{B} = \frac{\rho}{1-\rho}$. Hence, $\lim_{k \rightarrow \infty} B_k = \frac{\rho}{1-\rho}$.

□

B Model with Positive Supply

Here, we briefly outline a version of the baseline specification with positive asset supply to illustrate the impact of asset supply. The model is identical to the structure of our baseline specification, with two modifications. First, the risky asset is in fixed supply $Q > 0$. Second, rather than normalizing the payoff of the riskless asset to zero, we assume that it pays a gross return of $(1 + r)$.

With these modifications in hand, we can then turn to deriving the equilibrium in the model.

Lemma B.1 (Speculators' Perceived Pricing Function) *Speculators perceive the period t price as*

$$P_t = C_0 Q + B_0 \mathbf{d}_{t,2},$$

where

$$B_0 \equiv \frac{\hat{\rho}}{1 + r - \hat{\rho}},$$

$$C_0 \equiv -\gamma \frac{(1 + r - (1 - \hat{\kappa})\hat{\rho})^2 \sigma_\epsilon^2 + (1 + r - \hat{\rho}(1 - \hat{\kappa}_1))^2 \sigma_v^2 + \hat{\rho}^2 \sigma_\eta^2 \hat{\kappa}_2^2}{r(1 + r - \hat{\rho})^2},$$

and $\mathbf{d}_{t,2}$ is the second order belief shared by all speculators about the average investors' belief about fundamentals, i.e., $\mathbf{d}_{t,2} = \mathbf{d}_{t,2}^i, \forall i \in [0, \theta)$.

Proof. Proofs presented at the end of the section. □

Note that there are two differences between the expressions in Lemma B.1 and the corresponding expression in the main text. First, the constant $C_0 Q < 0$ reflects a risk premium associated with the fixed supply of the risky asset. Second, B_0 includes $1 + r - \hat{\rho}$ rather than $1 - \hat{\rho}$ in the denominator, to reflect the return on the riskless asset.

Next, we turn to deriving the pricing equation for the true equilibrium price.

Lemma B.2 (Equilibrium Pricing Function) *The linear equilibrium pricing rule for the economy is given by*

$$P_t = C_1 Q + B_1 \mathbf{d}_t^s,$$

where $B_1 > 0$ is the solution to a cubic equation

$$0 = \theta(1 + r - (1 - \hat{\kappa})\hat{\rho})A_2 B_1^3$$

$$+ \left(\theta(1 + r - (1 - \hat{\kappa})\hat{\rho})A_1 - \frac{\theta\rho(1 - (1 - \hat{\kappa})\hat{\rho})A_2}{1 - \hat{\rho}} \right) B_1^2$$

$$+ \left(\theta(1 + r - (1 - \hat{\kappa})\hat{\rho})A_0 - \frac{\theta\rho(1 - (1 - \hat{\kappa})\hat{\rho})A_1}{1 - \hat{\rho}} + (1 - \theta)(1 + r - \rho)A_S \right) B_1,$$

$$-\frac{\theta\rho(1 - (1 - \hat{\kappa})\hat{\rho})A_0}{1 - \hat{\rho}} - (1 - \theta)\rho A_S,$$

$$C_1 \equiv \frac{(A_0 + B_1(A_1 + A_2B_1))(-\gamma A_S + \theta(1 - (1 - \hat{\kappa})\hat{\rho})C_0)}{\theta(1 + r - (1 - \hat{\kappa})\hat{\rho})A_0 + (1 - \theta)rA_S + \theta(1 + r - (1 - \hat{\kappa})\hat{\rho})B_1(A_1 + A_2B_1)},$$

$A_0, A_1, A_2, A_3,$ and $A_S > 0$ are functions of deep parameters of the model, and $\mathbf{d}_t^s \equiv \int_0^1 \mathbf{d}_t^i$ is the average of investors' first order beliefs.

The expression for B_1 is nearly identical to the analogous expression in the main text (identical setting the risk free return $r = 0$). Notably, however, the coefficient on risky asset supply, C_1 , which captures the risk premium, is different than the risk premium coefficient perceived by speculators, C_0 . For sufficiently high risk aversion, γ , we can have $C_1 < C_0$ and for risk aversion closer to zero, we have $C_1 > C_0$. This misperception of the risk premium term leads to a bias in speculators' return expectations; when risk aversion is high ($C_0 > C_1$), speculators overestimate expected returns on average, and they underestimate expected returns when risk aversion is low ($C_1 > C_0$).³⁵ This bias arises in part because of the second order belief about fundamentals that speculators extract from prices, $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_0}Q + \frac{B_1}{B_0}\mathbf{d}_t^s$. For example, when $C_1 > C_0$, speculators extract an upward-biased belief about the average investor's belief about fundamentals; this lowers how they forecast others will revise their beliefs, and accordingly their return expectations. And additionally, when $C_1 > C_0$, speculators also have a downward-biased belief about the risk premium.

The bias component is not the main focus of our empirical analysis, though we do find some empirical evidence that speculators' return expectations may be biased. Similar to the main analysis, the cyclicity of speculators' beliefs that the market is overvalued and the cyclicity of their return expectations are determined by $\hat{\rho}$. By similar arguments as the main proofs, both return expectations and the proportion of investors seeing the market as overvalued are procyclical when $\hat{\rho} > \rho$. Namely, objective expected returns (those held by the average arbitrageur) are countercyclical when $B_1 > \frac{\rho}{1+r-\rho}$, and accordingly, the average speculator's expected returns are procyclical in this case. And $B_1 > \frac{\rho}{1+r-\rho}$ when $\hat{\rho} > \rho$.

B.1 Proofs

Proof of Lemma B.1

Proof. Each speculator conjectures the form of the pricing rule is $C_0Q + B_0\mathbf{d}_{t,2}$. Speculators believe that each other investor j trades as if all investors born in period $t + 1$ will share their

³⁵The bias in expected returns from perceiving the risk premium coefficient as C_0 can be written as $Q(C_0 - C_1)(1 - (1 - \hat{\kappa})\hat{\rho})$, which is the difference in the unconditional expected return forecasted by the average speculator and average arbitrageur.

beliefs. They believe the average investor's expected payoff in period $t + 1$ is

$$\begin{aligned}\int \mathbb{E}_t^i(P_{t+1} + D_{t+1}) &= C_0Q + (B_0 + 1) \int \mathbb{E}_t^i(\mathbf{d}_{t+1}) \\ &= C_0Q + (1 + B_0)\hat{\rho}\mathbf{d}_{t,2}.\end{aligned}$$

Each investor j 's forecast variance of the period $t + 1$ payoff can be written as

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + \underbrace{2\mathbf{C}(P_{t+1}, D_{t+1})}_{=B_0(\hat{\kappa}\sigma_\epsilon^2 + \hat{\kappa}_1\sigma_v^2)} \\ &= (1 + \hat{\kappa}B_0)^2\sigma_\epsilon^2 + (1 + \hat{\kappa}_1B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2\sigma_\eta^2.\end{aligned}$$

Solving that

$$\int \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - (1 + r)P_t}{\gamma\mathbb{V}_t^i(P_{t+1} + D_{t+1})} = Q,$$

and matching coefficients, we get the following system of equations:

$$\begin{aligned}B_0 &= \frac{\hat{\rho}(1 + B_0)}{1 + r}, \text{ and} \\ C_0 &= \frac{C_0 - \gamma((1 + \hat{\kappa}B_0)^2\sigma_\epsilon^2 + \sigma_v^2(1 + B_0\hat{\kappa}_1)^2 + B_0^2\sigma_\eta^2\hat{\kappa}_2^2)}{1 + r}.\end{aligned}$$

Solving the system of equations yields the expressions in the lemma. \square

Proof of Lemma B.2

Proof. For notational simplicity, we define $\kappa = \kappa_1 + \kappa_2$, $\hat{\kappa} = \hat{\kappa}_1 + \hat{\kappa}_2$.

We conjecture that the pricing formula is of the form $P_t = C_1Q + B_1\mathbf{d}_t^s$. Note that in equilibrium, we must have that $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_0}Q + \frac{B_1}{B_0}\mathbf{d}_t^s$, since speculators' second order beliefs at the equilibrium must equal their perceived price to the prevailing equilibrium price.

To forecast the price in period $t + 1$, speculator i forecasts the average belief in period $t + 1$, based on their forecast of \mathbf{d}_{t+1} and s_{t+1} :

$$\begin{aligned}\mathbb{E}_t^i(\mathbf{d}_{t+1,2}) &= \mathbb{E}_t^i \int \mathbf{d}_{t+1}^j = (1 - (\hat{\kappa}_1 + \hat{\kappa}_2))\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 \underbrace{\mathbb{E}_t(D_{t+1})}_{=\rho\mathbf{d}_t^i} + \hat{\kappa}_2 \underbrace{\mathbb{E}_t(s_{t+1}^{-i})}_{=\rho\mathbf{d}_t^i} \\ &= (1 - \hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}\rho\mathbf{d}_t^i.\end{aligned}$$

Speculator i 's expected period $t + 1$ payoff is

$$\mathbb{E}_t^i(D_{t+1} + P_{t+1}) = \underbrace{\rho\mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{C_0Q + \mathbb{E}_t^i(B_0\mathbf{d}_{t+1,2})}_{=\mathbb{E}_t^i(P_{t+1})}$$

$$=C_0Q + (1 + \hat{\kappa}B_0)\rho\mathbf{d}_t^i + (1 - \hat{\kappa})\hat{\rho}B_0\mathbf{d}_{t,2}.$$

Speculator i 's subjective perceived variance of the period $t + 1$ dividend is

$$\mathbb{V}_t^i(D_{t+1}) = \sigma_\varepsilon^2 + \sigma_v^2,$$

and his subjective perceived variance of the period $t + 1$ price is

$$\begin{aligned}\mathbb{V}_t^j(P_{t+1}) &= \mathbb{V}_t^j(B_0\mathbf{d}_{t+1}^j) \\ &= B_0^2\mathbb{V}_t^j((1 - \hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1D_{t+1} + \hat{\kappa}_2s_{t+1}^{-i}) \\ &= B_0^2\left(\hat{\kappa}_1^2(\sigma_\varepsilon^2 + \sigma_v^2) + \hat{\kappa}_2^2(\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\hat{\kappa}_1\hat{\kappa}_2\sigma_\varepsilon^2\right).\end{aligned}$$

Hence, his perceived variance of the period $t + 1$ payoff is given by

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + \underbrace{2\mathbf{C}(P_{t+1}, D_{t+1})}_{=B_0(\hat{\kappa}\sigma_\varepsilon^2 + \hat{\kappa}_1\sigma_v^2)} \\ &= (1 + \hat{\kappa}B_0)^2\sigma_\varepsilon^2 + (1 + \hat{\kappa}_1B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2(\sigma_\eta^2 + \sigma_\phi^2).\end{aligned}$$

As in the baseline specification, this variance does not depend on the coefficients of interest, B_1 and C_1 , and we write it as $A_S \equiv \mathbb{V}_t^i(P_{t+1} + D_{t+1})$. Speculator demand is then given by

$$\begin{aligned}\int_0^\theta Q_t^i &= \int_0^\theta \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - (1 + r)P_t}{\gamma\mathbb{V}_t^i(P_{t+1} + D_{t+1})} \\ &= \frac{\theta}{\gamma} \frac{C_0Q + (1 + \hat{\kappa}B_0)\rho\mathbf{d}_t^s + (1 - \hat{\kappa})\hat{\rho}B_0\mathbf{d}_{t,2} - (1 + r)P_t}{A_S}.\end{aligned}$$

Substituting $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_0}Q + \frac{B_1}{B_0}\mathbf{d}_t^s$, this becomes

$$\int_0^\theta Q_t^i = \frac{\theta}{\gamma} \frac{QC_0 + (1 + \hat{\kappa}B_0)\rho\mathbf{d}_t^s + (1 - \hat{\kappa})\hat{\rho}(Q(C_1 - C_0) + B_1\mathbf{d}_t^s) - (1 + r)P_t}{A_S}.$$

Turning to the arbitrageurs, they know that the form of the pricing rule is $QC_1 + B_1\mathbf{d}_t^s$. Moreover, they know that other investors correctly perceive the parameters governing the risky asset's fundamentals. Hence, arbitrageur i 's expected period $t + 1$ payoff is

$$\mathbb{E}_t^i(P_{t+1} + D_{t+1}) = \rho\mathbf{d}_t^i + B_1((1 - \kappa)\rho\mathbf{d}_t^s + \kappa\rho\mathbf{d}_t^i) + C_1Q.$$

Arbitrageur i 's perceived variance of dividends and the next period's price are

$$\mathbb{V}_t^i(D_{t+1}) = \sigma_\varepsilon^2 + \sigma_v^2, \text{ and}$$

$$\mathbb{V}_t^i(P_{t+1}) = B_1^2(\kappa_1(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2).$$

Hence, arbitrageur i 's perceived variance of the period $t + 1$ payoff is

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + \underbrace{2\mathbf{C}(P_{t+1}, D_{t+1})}_{=B_1(\kappa\sigma_\epsilon^2 + \kappa_1\sigma_v^2)} \\ &= (1 + \kappa B_1)^2 \sigma_\epsilon^2 + (1 + \kappa_1 B_1)^2 \sigma_v^2 + B_1^2 \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2).\end{aligned}$$

This perceived variance is quadratic in B_1 . For notational simplicity, we define A_0 , A_1 , and A_2 as the quadratic equation coefficients, i.e.,

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2 \sigma_v^2 + \kappa^2 \sigma_\epsilon^2 + \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2))}_{A_2 \equiv} B_1^2 + \underbrace{2(\kappa_1 \sigma_v^2 \kappa + \kappa \sigma_\epsilon^2)}_{A_1 \equiv} B_1 + \underbrace{\sigma_v^2 + \sigma_\epsilon^2}_{A_0 \equiv},$$

and note that $A_0, A_1, A_2 > 0$.

Arbitrageur demand is then given by

$$\begin{aligned}\int_\theta^1 Q_t^i &= \int_\theta^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - (1+r)P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})} \\ &= \frac{(1-\theta)C_1 Q + \rho(1+B_1)\mathbf{d}_t^s - (1+r)P_t}{\gamma(A_2 B_1^2 + A_1 B_1 + A_0)}.\end{aligned}$$

Imposing market clearing ($\int_0^\theta Q_t^i = \int_\theta^1 Q_t^i = Q$), and solving for P_t , we get that

$$\begin{aligned}P_t &= \frac{(1-\theta)\rho A_S(1+B_1) + \theta A_0(\rho + \hat{\rho}B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1)) + \theta B_1(A_1 + A_2 B_1)(\rho + \hat{\rho}B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1))}{(1+r)(\theta A_0 + (1-\theta)A_S + \theta B_1(A_1 + A_2 B_1))} \mathbf{d}_t^s \\ &+ \frac{\frac{(1-\theta)C_1 - \gamma(A_0 + A_1 B_1 + A_2 B_1^2)}{A_2 B_1^2 + A_1 B_1 + A_0} + \frac{\theta(1-(1-\hat{\kappa})\hat{\rho})C_0 + \theta(1-\hat{\kappa})\hat{\rho}C_1}{A_S}}{(1+r)\left(\frac{\theta}{A_S} + \frac{1-\theta}{A_0 + B_1(A_1 + A_2 B_1)}\right)} Q.\end{aligned}$$

Matching coefficients, and through algebraic manipulation, we get the expressions provided in the lemma. \square

C Additional Empirical Analyses

In this section, we present additional empirical analyses. We first analyze if the number of survey responses in the Shiller survey displays any business cycle variation, and find no evidence that it does. The rest of the section presents tables and figures that replicate the main results for different subsets of the data (e.g., individual versus institutional investors).

C.1 Survey Responses in the Shiller Survey

We analyze if there is any business cycle frequency variation in responses to the Shiller survey. We regress the quarterly change in the log number of survey responses to the survey each quarter on S&P 500 returns, and quarterly innovations in the Conference Board Coincident indicators index (labeled 'Macro').³⁶ The independent variables are standardized to have zero mean and unit standard deviation. Table C.1 reports the results, and Newey-West standard errors (4 lags) are reported in parentheses. There is little evidence to indicate systematic business cycle variation in survey response counts.

	All	Retail	Inst
Returns	1.52 (4.12)	0.44 (2.57)	1.07 (2.21)
Macro	-0.61 (2.99)	-2.58 (1.59)	1.97 (1.54)

TABLE C.1: RESPONSE COUNTS AND BUSINESS CYCLE VARIATION

³⁶Unfortunately, we do not observe the number of questionnaires that were sent out each quarter, so we use changes in total responses to proxy for response rates.

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	1.46 (0.53)	0.83 (0.57)	0.17 (0.63)	-1.02 (0.72)	0.21 (0.06)	-0.04 (0.08)	-0.58 (0.10)	-1.13 (0.13)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	258	258	258	258	5651	5651	5651	5651
<i>R</i> ²	0.09	0.02	0.00	0.01	0.00	0.00	0.01	0.02

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>		<i>ST Trough</i>	
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.28 (0.04)	-0.25 (0.05)		
<i>Overvaluation</i>			0.54 (0.07)	-0.02 (0.17)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.13	0.10	0.21	0.00

TABLE C.2: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS (INDIVIDUAL INVESTORS)

Note: This table replicates Table 3 for the individual investor subset of our sample.

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	0.14 (0.17)	-0.35 (0.32)	-1.18 (0.46)	-2.35 (0.76)	-0.12 (0.05)	-0.61 (0.08)	-1.51 (0.13)	-2.47 (0.18)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	258	258	258	258	4486	4486	4486	4486
<i>R</i> ²	0.00	0.01	0.04	0.07	0.00	0.01	0.05	0.07

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>		<i>ST Trough</i>	
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.28 (0.07)	-0.10 (0.07)		
<i>Overvaluation</i>			0.61 (0.14)	0.04 (0.13)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.08	0.02	0.16	0.00

TABLE C.3: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS (INSTITUTIONAL INVESTORS)

Note: This table replicates Table 3 for the institutional investor subset of our sample.

Panel A: Term Structure of Expected Cumulative Returns and Higher Order Optimism								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Optimism</i>	1.93 (0.44)	1.04 (0.51)	0.02 (0.67)	-2.00 (1.25)	0.06 (0.05)	-0.38 (0.08)	-1.02 (0.17)	-1.78 (0.15)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.11	0.02	0.00	0.02	0.00	0.00	0.01	0.02
Panel B: Term Structure of Expected Cumulative Returns and Higher Order Pessimism								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Pessimism</i>	-1.93 (0.63)	-0.41 (0.71)	0.48 (1.05)	3.32 (1.46)	-0.03 (0.05)	0.14 (0.07)	0.78 (0.10)	1.32 (0.14)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.07	0.00	0.00	0.04	0.00	0.00	0.01	0.01

TABLE C.4: HIGHER ORDER OPTIMISM, PESSIMISM, AND RETURN EXPECTATIONS

Note: This table replicates Table 3, separately breaking down the results for *HO Optimism* and *HO Pessimism*.

Panel A: Levels Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.17 (0.08)				0.16 (0.08)	0.33 (0.11)				0.30 (0.07)
$\mathbb{E}_t(R_{t,t+3})$		0.06 (0.10)			0.14 (0.08)		0.22 (0.10)			-0.18 (0.11)
$\mathbb{E}_t(R_{t,t+6})$			-0.08 (0.11)		-0.10 (0.15)			0.24 (0.08)		0.22 (0.13)
$\mathbb{E}_t(R_{t,t+12})$				-0.12 (0.07)	-0.10 (0.09)				0.11 (0.05)	0.02 (0.07)
R^2	0.04	0.01	0.02	0.08	0.14	0.16	0.08	0.15	0.08	0.23
N	69	69	69	69	69	69	69	69	69	69

Panel B: Changes Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.08 (0.10)				0.06 (0.09)	0.33 (0.08)				0.27 (0.08)
$\mathbb{E}_t(R_{t,t+3})$		0.08 (0.09)			-0.05 (0.08)		0.25 (0.07)			0.13 (0.14)
$\mathbb{E}_t(R_{t,t+6})$			0.01 (0.11)		-0.13 (0.17)			0.21 (0.09)		0.12 (0.15)
$\mathbb{E}_t(R_{t,t+12})$				0.00 (0.06)	0.02 (0.09)				0.07 (0.06)	-0.05 (0.08)
R^2	0.01	0.02	0.00	0.04	0.23	0.10	0.10	0.10	0.01	0.26
N	68	68	68	68	68	68	68	68	68	68

TABLE C.5: RETURN EXPECTATIONS AND ASSET MANAGER FUTURES POSITIONS

Note: This table replicates Table 4, using the positioning of asset managers rather than dealers as the dependent variable.

Panel A: Levels Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.26 (0.09)				0.22 (0.08)	0.13 (0.09)				-0.03 (0.11)
$\mathbb{E}_t(R_{t,t+3})$		0.18 (0.08)			0.14 (0.09)		0.27 (0.10)			0.17 (0.14)
$\mathbb{E}_t(R_{t,t+6})$			0.08 (0.07)		-0.08 (0.11)			0.23 (0.09)		0.09 (0.23)
$\mathbb{E}_t(R_{t,t+12})$				-0.01 (0.07)	-0.01 (0.06)				0.12 (0.07)	0.04 (0.13)
R^2	0.10	0.06	0.01	0.00	0.11	0.03	0.12	0.14	0.08	0.15
N	69	69	69	69	69	69	69	69	69	69

Panel B: Changes Regressions										
	DJIA Futures					S&P 500 Futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.19 (0.09)				0.21 (0.05)	0.05 (0.08)				0.00 (0.08)
$\mathbb{E}_t(R_{t,t+3})$		0.00 (0.09)			-0.14 (0.10)		0.16 (0.11)			0.23 (0.14)
$\mathbb{E}_t(R_{t,t+6})$			0.09 (0.10)		0.04 (0.11)			0.06 (0.10)		-0.16 (0.12)
$\mathbb{E}_t(R_{t,t+12})$				0.09 (0.06)	0.07 (0.08)				0.07 (0.09)	0.10 (0.09)
R^2	0.08	0.00	0.02	0.03	0.11	0.00	0.04	0.0	0.02	0.07
N	68	68	68	68	68	68	68	68	68	68

TABLE C.6: RETURN EXPECTATIONS AND HEDGE FUND FUTURES POSITIONS

Note: This table replicates Table 4, using the positioning of leverage funds (hedge funds) rather than dealers as the dependent variable.

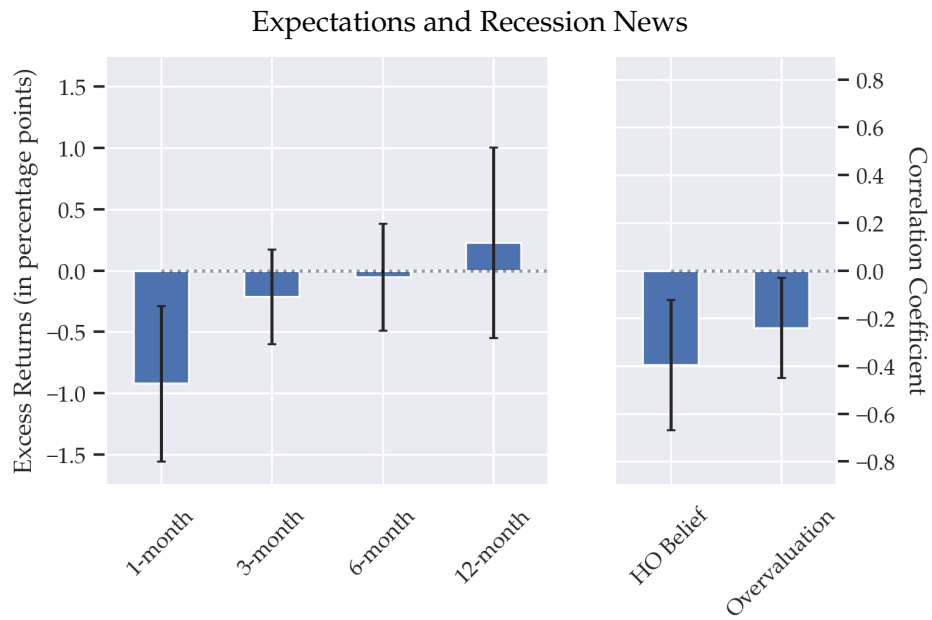
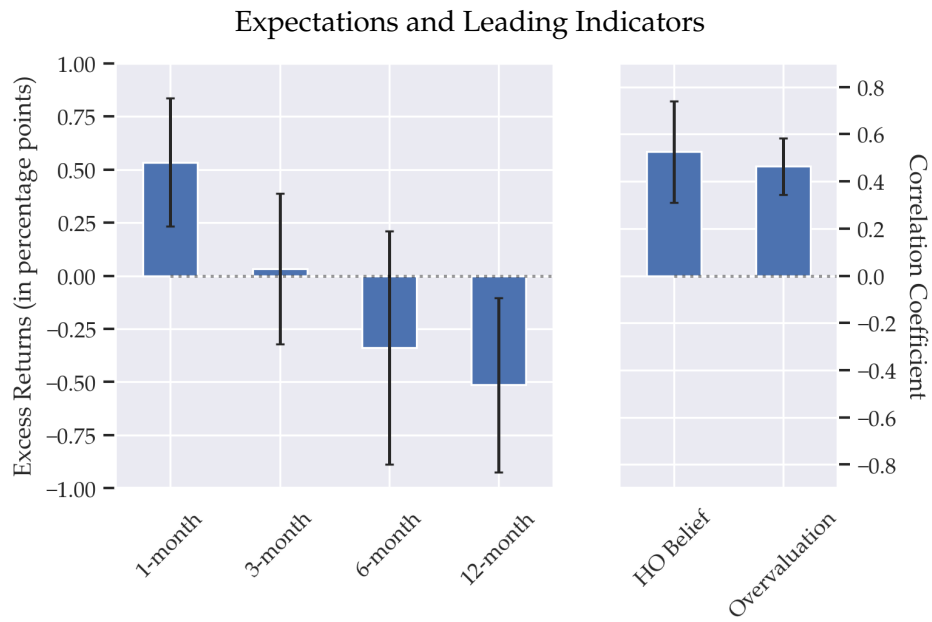


FIGURE C.1: MACROECONOMIC NEWS AND EXPECTATIONS (INDIVIDUAL INVESTORS)

Note: The figure replicates Figure 2 for the individual investor subset of our sample.

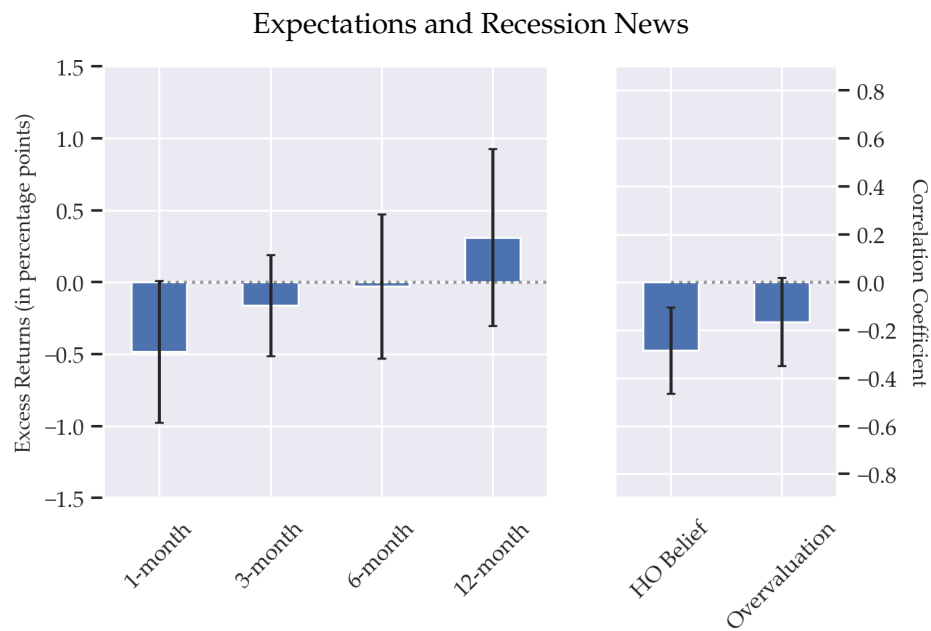
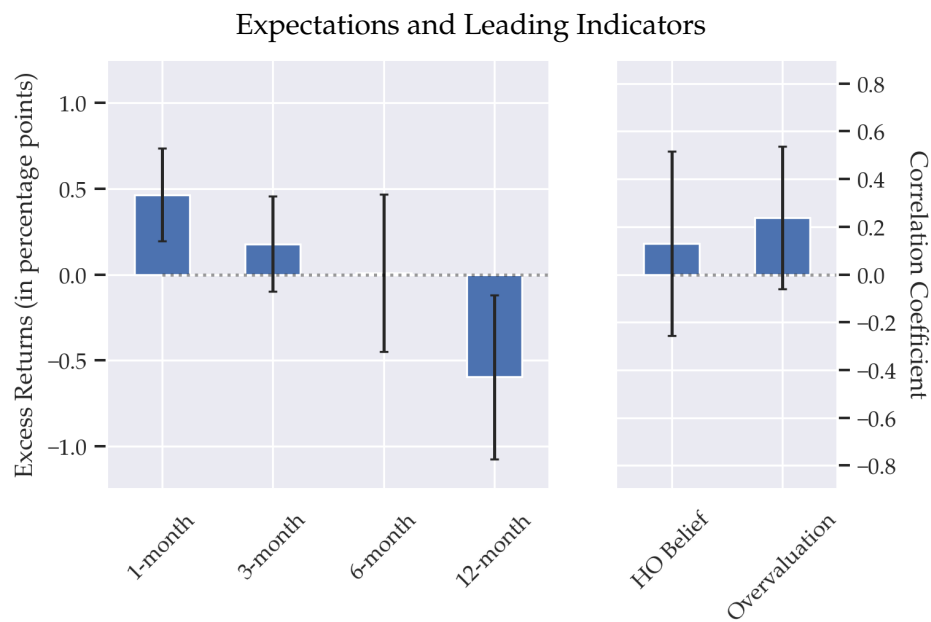


FIGURE C.2: MACROECONOMIC NEWS AND EXPECTATIONS (INSTITUTIONAL INVESTORS)

Note: The figure replicates Figure 2 for the institutional investor subset of our sample.

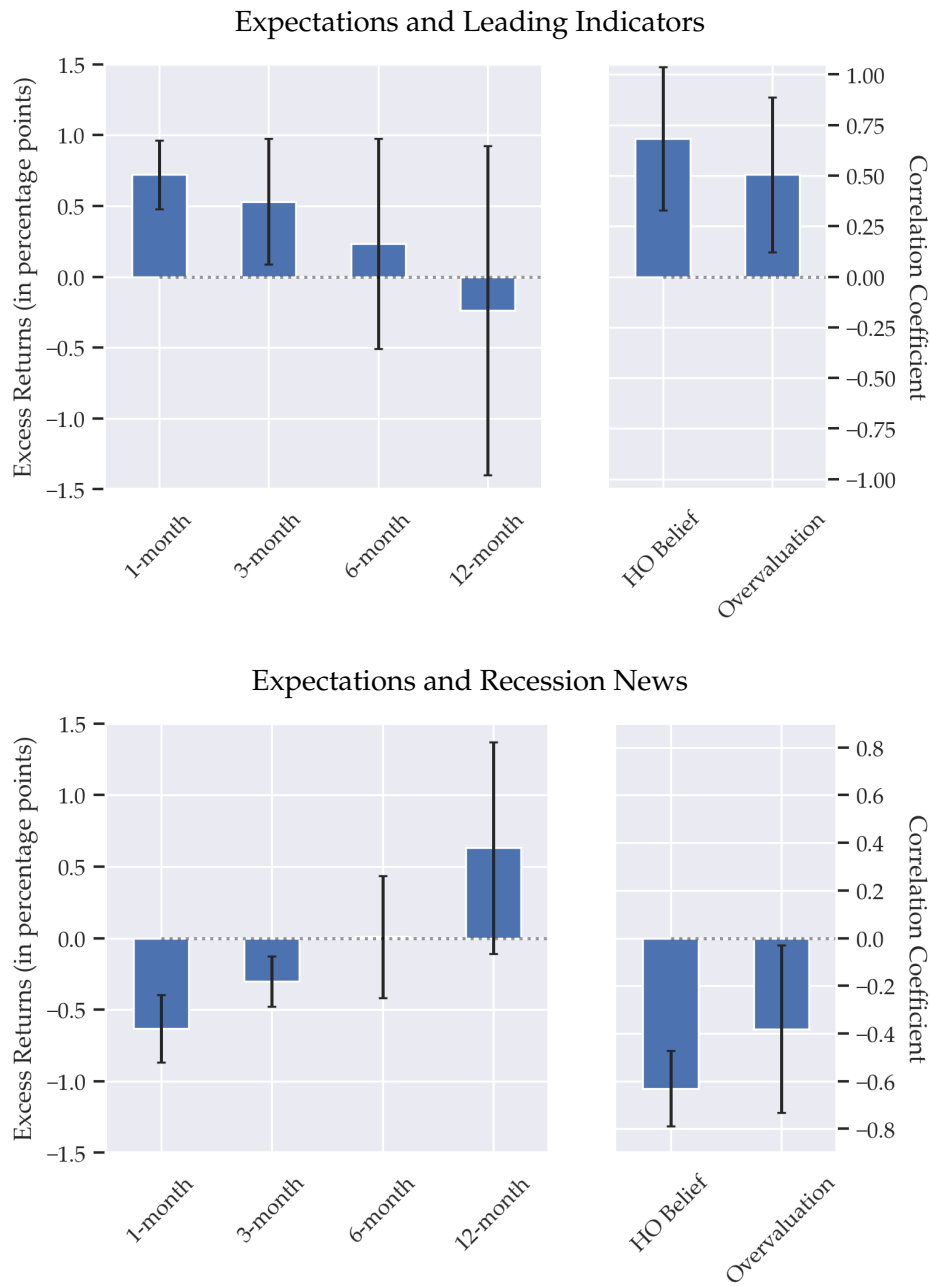


FIGURE C.3: MACROECONOMIC NEWS AND EXPECTATIONS (LEVELS)

Note: The figure replicates Figure 2 using the levels of the dependent and independent variables, rather than changes in return expectations and innovations to the dependent variable.

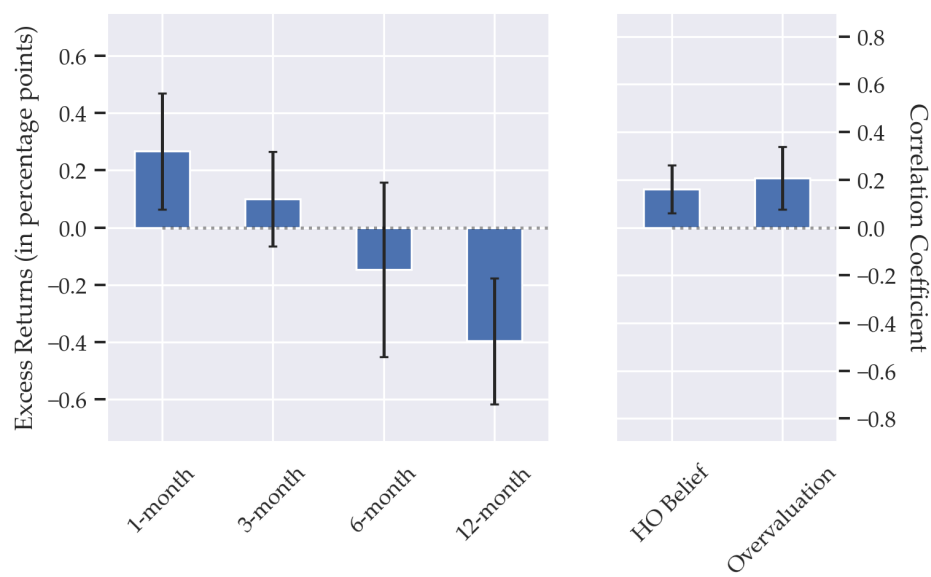


FIGURE C.4: COINCIDENT INDICATORS AND EXPECTATIONS

Note: The figure replicates the top panel of Figure 2, using innovations to the Coincident Macroeconomic Indicators index from the Conference Board.

Panel A: Expectations and Trailing Returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
$R_{t-12,t}$	0.39 (0.07)	0.31 (0.07)	0.17 (0.09)	-0.06 (0.10)	0.52 (0.08)	0.37 (0.12)
R^2	0.15	0.10	0.03	0.00	0.27	0.14
N	259	259	259	259	259	259

Panel B: Shiller and AAI Survey Expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
AAII	0.47 (0.08)	0.45 (0.07)	0.44 (0.08)	0.30 (0.11)	0.23 (0.09)	-0.04 (0.10)
R^2	0.22	0.20	0.20	0.09	0.05	0.00
N	259	259	259	259	259	259

TABLE C.7: EXPECTATIONS AND TRAILING RETURNS (INDIVIDUAL INVESTORS)

Note: This table replicates Table 6 for the subset of individual investors in our sample.

Panel A: Expectations and Trailing Returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
$R_{t-12,t}$	0.20 (0.07)	0.13 (0.10)	-0.04 (0.11)	-0.24 (0.11)	0.45 (0.10)	0.35 (0.14)
R^2	0.04	0.02	0.00	0.06	0.20	0.12
N	258	258	258	259	259	259

Panel B: Shiller and AAI Survey Expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	<i>HO Belief</i>	<i>Overvaluation</i>
AAII	0.38 (0.07)	0.31 (0.07)	0.20 (0.09)	0.08 (0.10)	0.28 (0.08)	0.05 (0.11)
R^2	0.15	0.10	0.04	0.01	0.08	0.00
N	258	258	258	259	259	259

TABLE C.8: EXPECTATIONS AND TRAILING RETURNS (INSTITUTIONAL INVESTORS)

Note: This table replicates Table 6 for the subset of institutional investors in our sample.

	$\mathbb{E}_t^s(r_{t,t+3})$	$\mathbb{E}_t^s(r_{t+3,t+6})$	$\mathbb{E}_t^s(r_{t+6,t+12})$
Interest Rate Innovations	0.63 (0.18)	-0.02 (0.14)	-0.51 (0.21)
Trailing 3-Month Return	0.39 (0.17)	0.14 (0.13)	-0.13 (0.19)

TABLE C.9: CURRENCY MARKET EXPECTED RETURNS IN RESPONSE TO NEWS

Note: The table reports regression results from regressions of consensus return expectations over different horizons on past news using data on currency market expectations. $\mathbb{E}_t^s(r_{t+h,t+h+k})$ represents the consensus k -month return expectation for h months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation change in the independent variable. The first row corresponds with regressions where the news measure is AR(1) innovations to interest rate differentials, and the second row corresponds with regressions where the independent variable is trailing 3-month returns. The table reports the average coefficient across countries. Standard errors are HAC-panel standard errors and are reported in parentheses. The return expectations data are from FX4casts, which provides the average forecast of 3-, 6-, and 12-month ahead exchange rate forecasts from a number of large financial institutions that actively participate in foreign exchange markets across the world. The sample begins in August 1986 and ends in December 2019, and contains monthly observations of forecasts for developed market G11 currencies versus the USD.