

Ács, Zoltán J.; Sanders, Mark

Working Paper

Intellectual property rights and the knowledge spillover theory of entrepreneurship

Jena Economic Research Papers, No. 2008,069

Provided in Cooperation with:

Max Planck Institute of Economics

Suggested Citation: Ács, Zoltán J.; Sanders, Mark (2008) : Intellectual property rights and the knowledge spillover theory of entrepreneurship, Jena Economic Research Papers, No. 2008,069, Friedrich Schiller University Jena and Max Planck Institute of Economics, Jena

This Version is available at:

<https://hdl.handle.net/10419/31742>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



JENA ECONOMIC RESEARCH PAPERS



2008 – 069

Intellectual Property Rights and the Knowledge Spillover Theory of Entrepreneurship

by

**Zoltan J. Acs
Mark Sanders**

www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

Friedrich Schiller University Jena
Carl-Zeiss-Str. 3
D-07743 Jena
www.uni-jena.de

Max Planck Institute of Economics
Kahlaische Str. 10
D-07745 Jena
www.econ.mpg.de

© by the author.

Intellectual Property Rights and the Knowledge Spillover Theory of Entrepreneurship

September 2008

Zoltan J. Acs

George Mason University & Max Planck Institute of Economics

Mark Sanders

Utrecht School of Economics & Max Planck Institute of Economics

Abstract

We develop a model in which stronger protection of intellectual property rights has an inverted U-shaped effect on innovation. Intellectual property rights protection allows the incumbent firms to capture part of the rents of commercial exploration that would otherwise accrue to the entrepreneurs. Stronger patent protection will increase the incentive to do R&D and generate new knowledge. This has a positive impact on entrepreneurship and innovation. However, after some point, further strengthening patent protection will reduce the returns to entrepreneurship sufficiently to reduce overall economic growth.

JEL-classification: J24, L26, M13, O3

Keywords: Intellectual Property Rights; Endogenous Growth; Entrepreneurship; Incentives; Knowledge Spillovers; Rents.

Acknowledgement: We are grateful to Daron Acemoglu, Josh Lerner, David Audretsch, Adriaan van Zon, Claire Economidou, Sameeksha Desai and participants at the 2007 ECEI conference in Utrecht, the Netherlands and 2008 IECER conference in Regensburg, Germany for useful comments and discussion on earlier drafts. Any remaining mistakes remain the responsibility of the authors.

I. Introduction

Reforms in the US patent system over the past few decades have caused an explosion in patent applications and grants (Gallini (2002), Jaffe and Lerner (2004)). These reforms were aimed at strengthening the position of patent holders and they were successful in increasing the productivity of research measured in patents. But it has also been argued that the quality and importance of these patents has decreased and that the patent boom have not generated the economic growth that might have been expected (Jaffe and Lerner (2004)). This has provoked a debate on the theoretical and empirical justifications for strengthening patent protection among policy makers and academics.

The debate on patents is not new. In fact, for as long as patents have existed, scholars have debated the optimal length, strength and breadth of protection. A strong rationale for more protection has been formalized in endogenous, innovation driven growth models such as those put forth by Romer (1986, 1990), Aghion and Howitt (1992) and Grossman and Helpman (1991). In these models, knowledge creation drives economic growth in the long run. Consequently, intellectual property rights (IPR) protection is considered a key institution that allows inventors to market their inventions and thereby recover their costs. The logic in these models implies that stronger protection stimulates investment in knowledge creation and consequently, higher growth.¹

The empirical growth literature indeed strongly supports the notion that institutions in general (Barro (1996), Sala-I-Martin (1996) and Acemoglu et al. (2001)) and IPR-protection in particular (Branstetter et al. (2006) and Allred and Park (2007)), contribute to growth performance. But this same literature does not support the premise that more and stronger protection is always better. Instead, evidence of an inverted-U-shaped relationship is growing

¹ Stronger protection will increase the return as well as reduce the risk on investments in R&D.

(Gould and Gruben (1996)) and some theoretical arguments for such a relationship have already been proposed.

For example, Nordhaus (1969) pointed out that static efficiency losses need to be traded-off against dynamic innovation gains. And several other mechanisms have been suggested in what one might label the *patent-literature*.² This literature, however, is less formal than modern growth theory and relies largely on partial equilibrium modeling techniques. This makes it difficult to evaluate the importance of these mechanisms for overall economic growth and innovation. Analyzing the trade-offs in the context of general equilibrium, endogenous innovation driven growth models is a recent research trajectory aimed at connecting these two literatures, and the area of focus in this paper.

Nordhaus' arguments, for example, have been formalized in general equilibrium innovation driven growth models by Kwan and Lai (2003) and Iwaisako and Futagami (2003). Both papers showed that static losses can be weighed against dynamic gains and thus, an optimum level of protection exists. Horii and Iwaisako (2007) and Furukawa (2007) focus on the reduced growth potential in an economy with more monopolized sectors. These models show that IPR-protection can indeed be too much of a good thing.

Still, these contributions remain strongly committed to the assumption that patent protection is required to provide economic incentives for innovation. They weigh static efficiency costs (that increase in the level of protection) against dynamic benefits of innovation (that remain constant or increase at a decreasing rate in the level of protection) to find an optimum.

² There exists, for example, a large industrial organization literature on the strategic use of patents and the implications for optimal patent policy design. In particular issues such as disclosure in sequential innovation processes, fragmented innovation processes and cumulative or cooperative research projects have been addressed. Examples of papers in this literature include Gilbert and Shapiro (1990), Gallini (1992), Scotchmer (1991, 1996), Gallini and Scotchmer (2001), Green and Scotchmer (1995), Maurer and Scotchmer (2002), Bessen and Mashkin (2006) and Kullti and Takalo (2008). Gallini (2002) gives a good overview. We thank Josh Lerner for bringing this literature to our attention.

We present a model of innovation driven endogenous growth in which more protection can also reduce the rate of innovation in equilibrium³ following the narrative developed in Jaffe (2000), Gallini (2002) and Jaffe and Lerner (2004), who discuss the patent system in the United States. They argue that stricter enforcement and easier establishment of intellectual property has turned the highly successful US patent system into an impediment to innovation. Their main argument is that stricter enforcement by the central appeals court has tilted the system towards the interests of patent holders, while the fees-based financing of the patent and trademark office made patent evaluators directly dependent on the number of patents granted. Thus, patents became easier to obtain and easier to enforce.

The implications of these reforms were unforeseen, unintended and largely undesirable. For example, large incumbent firms and individual inventors seek patents on all potentially valuable knowledge, even if they have no intention of ever commercializing the knowledge. Rather, they aim to capture rents once an idea is commercialized by another party and generates profits. Large corporations have even set up specialized patent enforcement departments which quickly became profit centers in their own right.⁴ The threat of patent infringements suits and rent-seeking inventors stifled small firm competition and strongly reduced incentives to commercialize and exploit knowledge that was not 100% home made and fully protected.

These outcomes cannot be understood in the context of existing modern general equilibrium endogenous growth models. Commercialization in these models is, after all, trivial. We argue that this illustrates a rather fundamental flaw in these models. Innovation driven endogenous growth models collapse the *process of innovation*: The subsequent generation, exploration and exploitation of the knowledge that constitutes a commercial opportunity into one rational decision that is entirely motivated by downstream rents.

³ In addition to producing static inefficiency due to monopoly price setting.

⁴ Jaffe and Lerner (2004) mention the case of Texas Instruments, where the patent enforcement department has grown to become the second largest profit center in the corporation.

Instead, we follow the model of the entrepreneurial economy presented by Acs et al. (2004, 2006). Our model closely resembles the basic Romer (1990) variety expansion model but in our specification, it is the entrepreneur that holds the residual claim to any monopoly rents that a new intermediate variety may generate once commercially introduced.⁵ In placing the entrepreneur centre stage, we bring back Schumpeter's (1934) original assumption that knowledge creation and commercialization are two separate activities. Furthermore, entrepreneurs and not inventors are driven by the prospect of capturing commercial rents from an innovation. These rents are the entrepreneurs' reward for seeing the commercial potential, taking the risks, investing the resources and organizing the production necessary for a new (intermediate) product or service.

To prevent our model from reverting to the exogenous Solow-esque "manna from heaven" models, we introduce a private economic incentive to *generate* new knowledge. This incentive in our model comes primarily from cost competition among final goods producers. They will invest resources in R&D to improve upon existing product lines. And we assume that in the course of that activity, they generate knowledge that is of no direct commercial value to them. That knowledge, however, presents an opportunity for entrepreneurs, who are willing and able to take the risks to develop and commercialize it. An entrepreneur will do so when the expected (risk adjusted) returns justify that investment. Without IPR protection, the knowledge spillover is costless and the investment is set equal to the wages foregone in engaging in the venture.

When intellectual property rights are easily established and enforced, however, incumbent firms will reduce the incentives for outsiders to commercialize knowledge. IPR protection then creates a knowledge filter (Acs et al. (2004)) and reduces the diffusion of knowledge in the economy. Similar arguments have been put forward in the context of North-

⁵ The knowledge commercializer, as opposed to the knowledge generator; the R&D sector in the Romer (1990) model.

South product life cycle models. Lai (1998), Sener (2004), Branstetter et al. (2007) and Glass and Wu (2007) have suggested that IPR protection may inhibit knowledge diffusion across countries and consequently, the global growth rate. To our knowledge, we present the first model that explicitly separates *invention* from *innovation* in a general equilibrium endogenous growth model.

As patent protection shifts rents from the entrepreneur to the inventor, more patent protection reduces the incentives to commercialize new knowledge, as well as creates incentives to generate more. The latter mechanism is well understood, and our model introduces an offsetting effect that explains why the relationship between innovation and IPR protection is not purely positive and there is an optimum level of protection that can, in fact, be exceeded..

This question is highly relevant for modern knowledge-based economies. In a system without protection of intellectual property, invention may well be the bottleneck in the innovative cycle. Initially, patents were awarded to benefit the Royal's favorites. The connection between invention and exclusive property rights that was then introduced, was an institutional revolution that helped spur invention and arguably paved the way for the Industrial Revolution.⁶ So it is now the inventor, not the entrepreneur, who is allowed to establish legal ownership over an invention in current patent systems. We argue that a delicate balancing act is required once such a system is in place. In most OECD countries today, entrepreneurship and not invention seems to have become the bottleneck in the innovative process (Audretsch, 2007) and the balance may well be beyond the tipping point.

By enforcing patents more strictly and allowing inventors to patent much easily, Jaffe and Lerner (2004) argue that the US patent system has now exceeded the optimum and that rents should be redistributed to the entrepreneurs. However, in their analysis, it is not the

⁶ This has been argued by Fox (1947). Greasley and Oxley (2007) actually present a compelling case that the Industrial Revolution made patenting more valuable and thus *caused* the surge in patenting, rather than the other way around.

static efficiency losses from monopoly that offset the dynamic gains. They argue that strengthening IPR-protection where it was already strong has actually hurt the innovation process by killing incentives to commercialize. Our paper embeds their narrative in a well-established general equilibrium framework with endogenous innovation-driven growth and firm decision theoretical micro-foundations. Following Schumpeter (1934), our model also places the entrepreneur at the heart of growth theory.

The structure of this paper follows: We present our model in section two and derive the equilibrium properties and implications of intellectual property rights protection in section three. In section four we examine comparative statics and the impact of stronger patent protection. We conclude in section five.

II. The Model

In our model, consumers consume a homogenous final good and producers produce this good using labor and intermediates, where production of intermediates takes place under monopolistic competition among imperfect substitutes. A key assumption in our model is the separation of knowledge *creation* from *commercialization*. The producers of final goods do R&D to generate process innovations and increase their productivity. In that process they come across opportunities for new and improved intermediate products. But in our model these more radical (intermediate) product innovations are commercially introduced by entrepreneurs who take advantage of the knowledge spillovers the R&D generates. The model is in equilibrium when all agents solve their respective choice problems rationally and the market prices adjust to equate supply and demand for final and intermediate goods, labor and capital. In the next sections, we first consider consumers, then producers, intermediate producers and entrants. The decentralized equilibrium is analyzed in section 3.

II.1 Consumers

The consumer problem below is standard in the literature (see for example Barro and Sala-I-Martin (2004)). Consumers maximize their value function:

$$V_C = \int_0^{\infty} e^{-\rho t} U(C(t)) dt \quad (1)$$

Where ρ is the subjective discount rate and $U(C(t))$ is given by $\log C(t)$, the natural log of consumption, $C(t)$. This value function is maximized subject to the intertemporal budget constraint:

$$\dot{B}(t) = r(t)B(t) + w(t) - C(t) \quad (2)$$

Where $r(t)$ is the interest rate on the stock of bonds, $B(t)$, held at time t and $w(t)$ is labor income as we normalize total labor supply to 1. Appendix A shows that the standard Ramsey-rule applies:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho \quad (3)$$

It is also shown in Appendix A that for any constant interest rate, consumers will choose consumption level:

$$C(t) = \rho \left(B(0) + \int_0^{\infty} e^{-rt} w(t) dt \right) e^{(r-\rho)t} \quad (4)$$

where $B(0)$ is the level of initial wealth and the integral represents the discounted present value of lifetime labor income. Equation (4) implies there is a positive demand for final goods at all times. To endogenize the equilibrium interest rate and wage levels, we need to specify the production side.

II.2 Producers

Producers produce the homogenous final good and maximize their profits by choosing the levels of production labor, intermediate goods and R&D labor to employ, taking as given the price level that we normalize to 1. A mass 1 number of identical firms is assumed to have the same constant returns to scale production function:

$$X_j(t) = A_j(t)^\alpha L_{Pj}(t)^\beta \sum_{i=0}^{n(t)} x_j(i,t)^{1-\alpha-\beta} \quad \text{with } 0 \leq \alpha + \beta \leq 1 \text{ and } 0 \leq \alpha, \beta \leq 1 \quad (5)$$

where $X_j(t)$ is the output of final goods producer j at time t , $L_{Pj}(t)$ is production labor that earns wage $w_P(t)$ and $x_j(i,t)$ is the quantity of intermediate i bought at price $\chi(i,t)$. All these quantities are flows. $A_j(t)$ represents the level of accumulated knowledge in the firm and $n(t)$ is the number of available varieties of intermediate goods at time t . These are stock variables. The knowledge base can be expanded by employing specialized R&D labor, L_{Rj} , that earns a wage $w_R(t)$. In the absence of intellectual property rights protection, all firms would maximize the value function:

$$V_j = \int_0^{\infty} e^{-rt} \left(X_j(t) - w_P(t)L_{Pj}(t) + w_R(t)L_{Rj}(t) - \sum_{i=0}^{n(t)} \chi(i,t)x_j(i,t) \right) \quad (6)$$

This function is maximized subject to the production function (5) and the R&D innovation function:

$$\dot{A}_j(t) = \psi A_j(t)^{1-\gamma} n(t)^\gamma L_{Rj}(t) \quad \text{with } 0 \leq \gamma \leq 1 \quad (7)$$

The presence of $A_j(t)$ in the latter reflects the intertemporal knowledge spillover. R&D is more productive when a large knowledge base has been developed in the past but at a decreasing rate. The presence of $n(t)$ represents the positive spillover effect of more variety in intermediates on process R&D. With more variety in intermediates, the final goods producing sector has more degrees of flexibility to organize the production process more efficiently and thereby generate more total factor-augmenting technical change for a given level of R&D effort. Alternatively, one can say that the relevant knowledge base for firm j 's R&D is assumed to be a Cobb-Douglas aggregate of public and private knowledge, proxied by n and A_j respectively. ψ is a scaling productivity parameter. We have chosen a linear specification in R&D labor, following Romer (1990), and thereby introduced the scale effect. Eliminating it would complicate the math and will not affect our key results.⁷

In this set-up, the final goods producer has an incentive to employ R&D workers to increase productivity by accumulating firm specific knowledge A_j . In addition, we introduce the option to patent the knowledge that R&D may generate, but that is not directly relevant to the firm itself. If such patents are licensed out, the total license income, Y_j of the firm depends on the profits, $\Pi(t)$, that the licensees can generate, on the strength of the relative bargaining

⁷ Jones (2006) offers several alternatives to this specification that would not suffer from this problem, but as the issue has no bearing on our purpose we chose to stick to the Romer-specification.

position of licensor and licensee, ξ , and on the growth rate of the firm-specific knowledge base.⁸ We assume:

$$Y_j(t) = \frac{\dot{A}_j(t)}{A_j(t)} \xi \Pi(t) \quad (8)$$

The firm's problem is now a dynamic optimization problem due to the R&D investment decision - and, dropping time arguments to save on notation, it is characterized by the Hamiltonian:

$$H_j = e^{-rt} \left(A_j^\alpha L_{Pj}^\beta \sum_{i=0}^n x_j(i)^{1-\alpha-\beta} + \psi A_j^{-\gamma} n^\gamma L_{Rj} \xi \Pi - w_P L_{Pj} + w_R L_{Rj} - \sum_{i=0}^n \chi(i) x_j(i) \right) + \mu_j \left(\psi A_j^{1-\gamma} n^\gamma L_{Rj} \right)$$

Where the levels of employment and intermediate use are control variables and the stock of firm-specific knowledge is the state variable. Standard dynamic optimization yields $n+5$ first order conditions. For production labor we have:

$$\frac{\partial H_j}{\partial L_{Pj}} = 0 = e^{-rt} \left(\beta A_j^\alpha L_{Pj}^{\beta-1} \sum_{i=0}^n x_j(i)^{1-\alpha-\beta} - w_P \right) \quad (9)$$

This can be easily rewritten into a labor demand function:

⁸ We assume that the firm must generate a steady flow of new knowledge to sustain a constant level of license income to reflect the fact that patents expire and license fees fall with the age of the patent as more substitutes are available.

$$L_{Pj}^D = \left(\frac{\beta A_j^\alpha \sum_{i=0}^n x_j(i)^{1-\alpha-\beta}}{w_P} \right)^{\frac{1}{1-\beta}} = \frac{\beta X_j}{w_P} \quad (10)$$

This shows that all firms will spend exactly the same share, β , of their sales, X on production wages.⁹ Summing over all final goods producers, we obtain for total production labor demand:

$$L_P^D = \frac{\beta X}{w_P} \quad (11)$$

The total wage sum for production workers is then βX . Confronted with a fixed labor supply, this demand function generates wages that in equilibrium rise with total output.

For intermediates, the firm will choose the levels of each variety to satisfy n first order conditions:

$$\begin{aligned} \frac{\partial H_j}{\partial x_j(0)} &= 0 = e^{-rt} \left((1-\alpha-\beta) A_j^\alpha L_{Pj}^\beta x_j(0)^{-\alpha-\beta} - \chi(0) \right) \\ \frac{\partial H_j}{\partial x_j(1)} &= 0 = e^{-rt} \left((1-\alpha-\beta) A_j^\alpha L_{Pj}^\beta x_j(1)^{-\alpha-\beta} - \chi(1) \right) \\ &\dots \\ \frac{\partial H_j}{\partial x_j(n)} &= 0 = e^{-rt} \left((1-\alpha-\beta) A_j^\alpha L_{Pj}^\beta x_j(n)^{-\alpha-\beta} - \chi(n) \right) \end{aligned} \quad (12)$$

Appendix B shows that these n conditions can be used to derive the demand for variety i by final goods producer j :

⁹ Final output is homogenous and we normalize its price to 1.

$$x_j(i)^D = \frac{\chi(i)^{\frac{-1}{\alpha+\beta}}}{\sum_{i=0}^n \chi(i)^{\frac{-1}{\alpha+\beta}}} (1-\alpha-\beta)X_j \quad (13)$$

Multiplying (13) by $\chi(i)$ and summing over all varieties i shows that total expenditure on intermediates is $(1-\alpha-\beta)X_j$.¹⁰

Together with the result on the wage costs, this implies that all final goods producers j make an operating profit of αX_j . Free entry would normally eliminate such positive profits but we assume the knowledge stock is firm-specific and serves as a barrier to entry. We assume that final goods producers are perfectly symmetric and are price takers in input and output markets. Therefore, only increases in the firm's level of accumulated knowledge $A_j(t)$ and consequently $X_j(t)$ may cause increases in a firm's operating profit. Firms will then invest resources in R&D to increase their $A_j(t)$. Moreover, the R&D generates license income if the knowledge generated is commercially valuable ($\Pi(t) > 0$) and the patent system allows patent owners to capture some of the rents from commercialization ($\xi > 0$).

Formally, we model the stock of knowledge as a firm-specific state variable and its optimal path is determined by choosing the optimal level of R&D resources, $L_{Rj}(t)$. The final goods producer will increase R&D activity as long as the discounted future benefits of doing so exceed the current labor costs at the margin. As R&D is a deterministic process in our model, the firms can decide to spend on R&D exactly up to that point. The solution is formally characterized by two first order conditions, one transversality condition and the law of motion for A_j ¹¹:

¹⁰ Summing over all final goods producers j then yields the result that total expenditure on intermediates in the economy is $(1-\alpha-\beta)X$.

¹¹ Time arguments have been included in the transversality condition as the limit is taken for time to infinity.

$$\begin{aligned} \frac{\partial H_j}{\partial L_{Rj}} = 0 &= e^{-rt} (\psi A_j^{-\gamma} n^\gamma \xi \Pi - w_R) + \mu_j \psi A_j^{1-\gamma} n^\gamma \\ \frac{\partial H_j}{\partial A_j} = -\dot{\mu}_j &= e^{-rt} \left(\alpha A_j^{\alpha-1} L_{Pj}^\beta \sum_{i=0}^n x_j(i)^{1-\alpha-\beta} - \gamma \psi A_j^{-\gamma-1} n^\gamma \xi \Pi L_{Rj} \right) + (1-\gamma) \mu_j \psi A_j^{-\gamma} n^\gamma L_{Rj} \\ \lim_{t \rightarrow \infty} \mu_j(t) A_j(t) &= 0 \\ \frac{\partial H_j}{\partial \mu_j} = \dot{A}_j &= \psi A_j^{1-\gamma} n^\gamma L_{Rj} \end{aligned} \quad (14)$$

Where the first condition implies that the marginal R&D labor cost, w_R , is set equal to the marginal product of R&D labor in innovation times the shadow price of a marginal increase in A_j , μ_j , plus the marginal license income. In economic terms, firms will hire R&D labor until the marginal cost equals the sum of discounted present value of marginal benefits. Solving for that shadow price yields:

$$\mu_j = e^{-rt} \left(\frac{w_R}{\psi A_j^{1-\gamma} n^\gamma} - \frac{\xi \Pi}{A_j} \right) \quad (15)$$

Then we take the time derivative and set this expression equal to minus the right hand side in the second condition to equate the marginal return on A_j to the shadow price. Substituting the inverse law of motion (7) and the inverse production function (5) for L_{Rj} and L_{Pj} , respectively, we obtain, after rearranging, an expression that we can solve for w_R :

$$\left(r - \frac{\dot{w}_R}{w_R} + \gamma \frac{\dot{n}}{n} \right) \frac{w_R}{\psi A_j^{1-\gamma} n^\gamma} = \frac{\alpha X_j}{A_j} + \left(r - \frac{\dot{\Pi}}{\Pi} \right) \frac{\xi \Pi}{A_j} \quad (16)$$

This yields the wage level at which a positive finite amount of R&D workers will be employed by firm j . This wage level represents a horizontal demand function or arbitrage

condition. If R&D wages exceed this threshold, no R&D workers will be employed by firm j . As long as R&D wages fall short, firm j will hire additional R&D workers. This so-called bang-bang equilibrium is a result of the constant returns to R&D labor assumption that we have made. It implies that in any stable equilibrium the R&D wage must equal:

$$\bar{w}_{Rj} = \frac{(\alpha X_j + (r - \dot{\Pi} / \Pi) \zeta \Pi) \psi A_j^{-\gamma} n^\gamma}{(r - \dot{w}_R / w_R + \gamma \dot{n} / n)} \quad (17)$$

But (17) holds for all firms j and as all firms are price takers in input markets an equilibrium in the R&D labor market requires that all firms that hire R&D pay the same wage. We also know by the production function in (5) and equations (10) and (13) that X_j only varies over j due to differences in A_j and is continuous and strictly proportional to it.¹² Thus, we obtain the result that at any point in time, there is a unique level of A_j that all firms that hire R&D labor must attain. The mechanism is that the firms with $A_j = A^{max}$ also have the highest threshold wage for R&D. They will thus bid up R&D wages to this threshold level and employ a positive amount of R&D. Their level of A will then rise according to (7) and those with $A_j < A^{max}$ will not hire any R&D and their A_j remains stable. The rise in A^{max} pushes up the threshold but also increases the average A causing production wages and intermediate prices to rise. In any equilibrium with R&D only those firms that have $A_j = A^{max}$ can stay in the race, whereas others are forced to bring down their production employment and intermediate use

¹² It can be shown that the right side of (17) is actually positive in A_j when the optimal amounts of labor and intermediates have been employed. In that case, output in (5) substituting for labor and intermediates by (10) and

(13) equals: $X_j = A_j \left(\frac{\beta}{w_p} \right)^{\frac{\beta}{\alpha}} \left(\frac{1 - \alpha - \beta}{\bar{\chi}} \right)^{\frac{1 - \alpha - \beta}{\alpha}} n^{\frac{\alpha + \beta}{\alpha}}$ where $\bar{\chi}$ represents the average price for intermediates.

Plugging this expression in the threshold wage in (17) and solving for the wage yields an expression that is positive and concave in A_j

levels to 0.¹³ If we assume therefore that all firms start from the same initial level of $A_j(0)=A_0$, the above implies that $A_j(t)=A^{max}(t)=A(t)$ for all j and we obtain for (17):

$$\bar{w}_R = \frac{(\alpha X + (r - \dot{\Pi} / \Pi) \zeta \Pi) \psi A^{-\gamma} n^\gamma}{(r - \dot{w}_R / w_R + \gamma \dot{n} / n)} \quad (18)$$

We have shown above that a stable labor demand in production requires an equilibrium in which wages grow at the same rate as output. Equation (18) shows that the threshold will also satisfy that constraint as long as A and n grow at the same rate and total profits grow in proportion with final output. We show below that these conditions are satisfied in the steady state so there is no long run relative wage divergence between R&D and production labor that would have an impact on relative supplies.

It would be trivial to substitute a given total supply of R&D workers into (7) to derive the optimal growth rate of $A_j(t)=A^{max}(t)=A(t)$. The starting condition $A_j(0)=A_0$ and the law of motion in (7) then determine the optimal path for $A(t)$ and the transversality condition helps to solve for $\mu_i(t)$. The model is more interesting when R&D competes for resources with other activities in the model.

Equation (18) shows the traditional innovation enhancing effect of patent protection, if we assume that R&D competes with production for labor resources. A higher level of protection, higher ζ , will cause the threshold wage level to go up for a given level of

¹³ Taken literally, this result may appear unrealistic and it yields the undesirable result that initial levels of production knowledge have to be exactly equal. However, it is worth noting that, for example, uncertainty in the R&D process and fixed costs have been assumed away. In real life, the uncertainty in R&D outcomes would create a range, rather than a precise level for the threshold wage, and fixed costs would cause firms to actually exit when employment levels fall below a critical level. Then the prediction is that a group of technology leaders will be able to survive in the market, where they must “run to stand still” and a shake-out will cause firms with less than efficient production processes to exit in the transition to the steady state. Such processes are well-known in the empirical literature on industrial dynamics (Gort and Klepper (1982), Klepper (1996)). They are present in a very stylized form in our model.

intermediate sector profits. This implies that for given levels of total employment in the final goods sector, R&D will increase and consequently, productivity improvement accelerates.¹⁴

We feel, however, that it is more interesting and realistic to have the R&D in final goods producing firms compete over skilled labor with innovative activity (entry) in the intermediate sector. With all the differences one can think of, R&D and entrepreneurship are both largely non-routine and highly skilled activities that distinguish themselves more from routine unskilled production labor than from each other. Moreover, as profits in the intermediate sector are endogenous and license fees can only be collected when knowledge spillovers are actually commercialized in the form of new varieties of intermediate goods, we now turn to the intermediate producers.

II.3 Intermediate Producers

The intermediate sector produces capital goods according to some specific process available to one intermediate firm *only*. We assume, however, that there are n varieties available to compete as imperfect substitutes and new ones are allowed to enter.

One can think of the intermediate designs as being codified in a blueprint and protected by a patent, as in Romer (1990). Entrepreneurs, however, often bring a unique combination of tacit knowledge, training, talent, access to finance and support networks etc., to their ventures, and by definition come up with a commercial opportunity that no-one recognized before.¹⁵ Therefore, we can justify the assumption that even in the absence of patent protection, every intermediate will be produced exclusively by one firm and subsequent

¹⁴ Our model can thus generate a trade-off between dynamic benefits of more innovation against the static loss of current output by assuming innovation and production compete over the same limited resources. This is a different channel trade-off than in Nordhaus (1969).

¹⁵ There are quite a few definitions of entrepreneurship in the literature. We follow the Schumpeterian tradition that defines the entrepreneur as the agent that implements an invention commercially. See e.g. Braunerhjelm (2008) for a discussion and further references.

entry with a perfect substitute is not possible. The big difference is now that knowledge is excludable but not tradable. As in Romer (1990), the producers in this sector are monopolists that set their own price and compete only with imperfect substitutes.

By the assumed symmetry in the final goods production function, all varieties face the same, iso-elastic demand curve for their variety. Also we assume that the monopolists are price takers in the market for raw capital. The problem is then identical for every intermediate producer i . They solve a static and standard profit maximization problem given by:

$$\max_{\chi(i)} : \pi(i) = \chi(i)x(i) - rK(i) \quad (20)$$

Subject to a simple production technology that implies that one unit of raw capital is required to produce one unit of intermediate good i and the total demand for intermediate i that was derived above:

$$x(i) = K(i)$$

$$x(i)^D = \frac{\chi(i)^{-\frac{1}{\alpha+\beta}}}{\sum_{i=0}^n \chi(i)^{\frac{\alpha+\beta-1}{\alpha+\beta}}} (1 - \alpha - \beta)X \quad (21)$$

Substitution into the profit function and setting the first derivative with respect to $\chi(i)$ to 0 yields:

$$\chi(i) = \frac{r}{1 - \alpha - \beta} \quad (22)$$

This does not vary over varieties i anymore. So every intermediate producer sets his price equal to this value and by the demand function all intermediates are demanded in the same quantity. This implies that in equilibrium the stock of raw capital is divided equally among all n varieties of intermediate goods:

$$x(i) = K / n \quad \forall i \quad (23)$$

Consequently the capital share in income is given by $rK = (1 - \alpha - \beta)^2 X$, whereas the monopoly rents in the intermediate sector are given by:

$$\pi(i) = \frac{(\alpha + \beta)(1 - \alpha - \beta)X}{n} \quad (24)$$
$$\sum_{i=0}^n \pi(i) = (\alpha + \beta)(1 - \alpha - \beta)X$$

These profits accrue to the entrepreneur who organized the intermediate production unit, as no other inputs or fixed (entry) costs have been assumed. But not all rents can remain with the entrepreneur. If the knowledge on which intermediate i was based spilled over from a final goods producers' R&D lab and was protected by a patent, then the patent holder can charge a license fee that reduces the flow of rents to the entrepreneur. As assumed above, the share retained by the entrepreneurs is $(1 - \zeta)$. Anticipating this, the entrepreneur will be less inclined to commercialize such knowledge if it is costly to do so. Let us therefore consider the decision to start a new intermediate goods-producing venture.

II.4 *Entry and Entrepreneurs*

The positive (expected) flow of retained rents attracts new entrants. These entrants cannot enter the existing intermediate variety markets. This can not happen even when all the relevant patents are licensed by the entrant, as we assume that these are protected by trade secrets, unique essential entrepreneurial traits or otherwise. However, the existence of these rents and the knowledge that there is a latent demand for new varieties makes it attractive to start ones own venture and enter with a new intermediate variety. As in Romer (1990), the value of a new intermediate firm that enters at time T is equal to the discounted current value of an incumbent intermediate firm's remaining flow of rents from T to infinity (assuming the impact of one additional intermediate on incumbent intermediate firms' profits is infinitely small):

$$V(T) = \int_T^{\infty} e^{-rt} \pi(i, t) dt = (\alpha + \beta)(1 - \alpha - \beta) \int_T^{\infty} e^{-rt} \frac{X(t)}{n(t)} dt \quad (25)$$

where r is the discount rate that the entrepreneur applies.¹⁶ In equation (8) we assumed that the final goods sector appropriates a share ζ of total intermediate profits for every 100% expansion of its knowledge base. This total license income has to be collected from the existing n intermediate firms. As all intermediate firms are fully symmetric, we assume all

¹⁶ We do not consider the possibility that this rate deviates from the risk-free interest rate due to the risky nature of entrepreneurial ventures and/or capital market imperfections they may face. This does not affect our results qualitatively and there are extensions to be addressed by further research. If the profit flow is at risk, for example, the discount rate includes a risk premium that captures the flow probability of losing the entire profit flow. As was argued by Jaffe and Lerner (2004), with excessive patent protection this parameter turns positive in the strength of patent protection. Of course, a patent infringement suit is usually settled out of court and does not result in the entire profit flow disappearing. However, by assuming that a high probability of losing some profits reduces the value of the firm to the entrepreneur in the same way as a low probability to lose the entire profit flow, we can still interpret the risk premium as reflecting the ease of obtaining and upholding patents in court. See for example Walsh (2003) and Aghion and Howitt (1998), who show that a positive flow probability of losing a profit flow can be incorporated by including that probability in the discount rate.

contribute an equal amount to total license fees.¹⁷ As total profits are given in equation (24) as n times π_i , we obtain as the value of a new firm to the owner at time T :

$$V_E(T) = \int_T^{\infty} e^{-rt} (1 - \zeta \dot{A} / A) \pi(i, t) dt = (\alpha + \beta)(1 - \alpha - \beta) \int_T^{\infty} e^{-rt} (1 - \zeta \dot{A}(t) / A(t)) \frac{X(t)}{n(t)} dt \quad (26)$$

We assume that the entrepreneur, as owner of the firm, can appropriate this value. We propose, as opposed to Romer (1990), that this requires the allocation of time and is therefore costly in terms of (skilled R&D) wages foregone. Moreover, we assume that entrants receive their idea as a knowledge spillover from downstream final goods producers' process R&D. One can think of this process as the spin-out of an employee from the final goods producers' R&D labs, but it is also possible that (able) others pick up on idle ideas. The entry function is given by:

$$\dot{n} = \varphi A^\delta n^{1-\delta} L_E \quad (27)$$

We have assumed constant returns to skilled entrepreneurial labor, L_E . Moreover, we assume that entry is positive in the accumulated knowledge in final goods producers process R&D, $A(t)$. As the final production process is better understood, more ideas for new and further specialized intermediates are likely to emerge. The presence of n reflects the fact that accumulated entrepreneurial experience increases the entry rate for given levels of activity and knowledge availability. Alternatively, one can interpret this specification as stating that entry is proportional to a Cobb-Douglas aggregate of accumulated public knowledge in entrepreneurship and R&D. φ is a scaling productivity parameter. δ may be interpreted as a

¹⁷ Or, equivalently run an identical ex ante risk of being charged and forced to pay license fees, such that the expected value is equal to the average license fees per intermediate firm.

parameter that reflects the knowledge filter. This concept was first coined by Acs et al. (2004) to describe the institutional, informational and otherwise existing barriers to knowledge spillover between knowledge creators and commercializers. In the context of our model, one could think of non-disclosure agreements, labor contract limitations on moving to competing firms and the defensive patenting strategies in final goods producing firms. Anything the final goods producing firms does to limit the spillover of knowledge, including legal and other action, will reduce δ . This reduces the entry of new intermediates for given increases in knowledge and levels of entrepreneurial activity.¹⁸ Equating discounted future marginal rent income to marginal (opportunity) costs at the time of entry at time T we can derive the entry arbitrage equation:

$$\frac{\partial \dot{n}(T)}{\partial L_E(T)} V_E(T) = \varphi A(T)^\delta n(T)^{1-\delta} (\alpha + \beta)(1 - \alpha - \beta) \int_T^\infty e^{-rt} (1 - \zeta \dot{A}/A) \frac{X(t)}{n(t)} dt = w_E(T) \quad (28)$$

As this trade-off is identical for entrants over time, we can replace T by t and equation (28) can be rewritten as an arbitrage condition for entrepreneurial labor.¹⁹ Dropping time arguments to save on notation, we obtain:

$$\tilde{w}_E = \frac{(\alpha + \beta)(1 - \alpha - \beta)(1 - \zeta \dot{A}/A)}{r + \dot{n}/n - \dot{X}/X} \varphi A^\delta n^{-\delta} X \quad (29)$$

As we assume that entrepreneurship competes with R&D for skilled labor, no entry will take place if the skilled wage exceeds this level. The opportunity costs are too high and all skilled

¹⁸ Note that we now have two ways in which IPR-protection can inhibit innovation: (1) through reducing the incentives to commercialize idle ideas, captured by ζ , and (2) by blocking the diffusion of such idle ideas, captured by δ . We will focus on the former as more relevant in this paper.

¹⁹ Where we assume that, at the time of entry, entrepreneurs expect output and variety expand at a constant rate (as they will in steady state), such that $X(t) = X(T)e^{\dot{X}/X * t}$ and $n(t) = n(T)e^{\dot{n}/n * t}$.

labor is employed in R&D. If it falls below this level, however, all skilled labor will switch to entrepreneurial activity. We thus have a bang-bang equilibrium due to the constant returns to L_E and L_R . Note that this implies that in such a bang-bang equilibrium, either variety n or knowledge A increases, while the other is stable. This implies that A/n changes until the threshold wages in (29) and (18) equalize. We use this property to first derive the skilled labor market and then the steady state equilibrium in section 3. We analyze the relevant comparative statics in section 4.

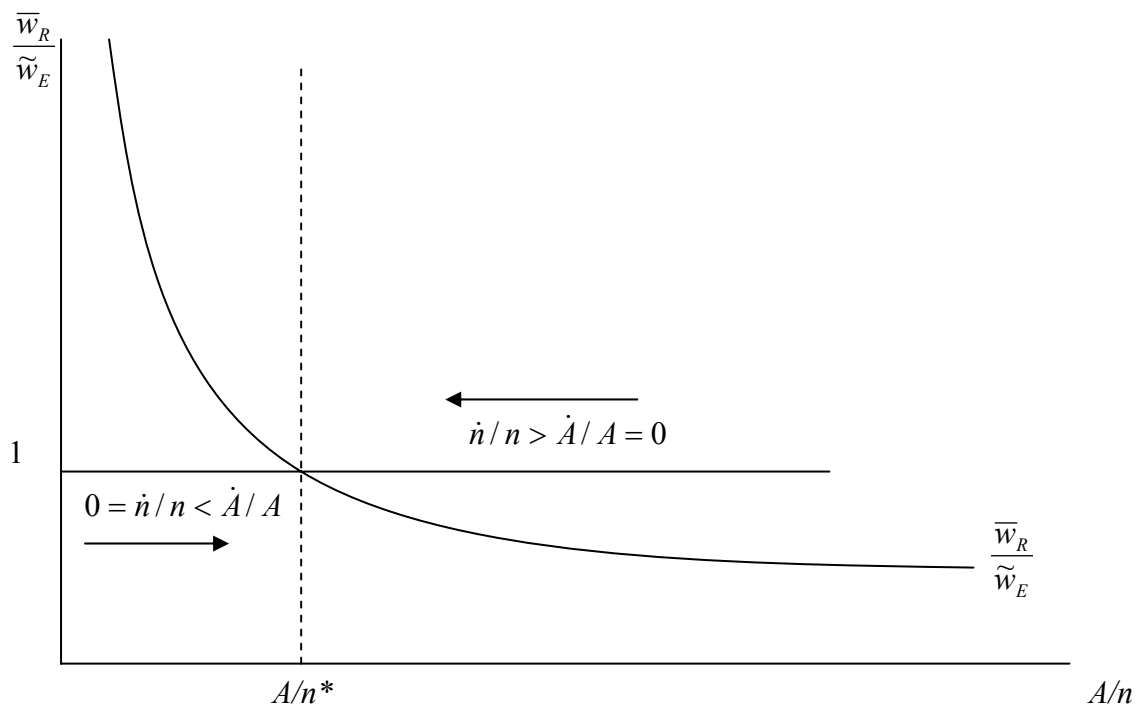
III Equilibrium

III.1 The Skilled Labor Market

The skilled labor market is in equilibrium when wages equate total exogenous supply to then-demand in R&D and entrepreneurship.²⁰ Both activities earn the same wage in equilibrium. We have $\bar{w}_R = \tilde{w}_E$ and $1 = L_R + L_E$ to determine the equilibrium, but let us first consider what happens *out of* equilibrium. If $\bar{w}_R > \tilde{w}_E$ all skilled labor is allocated to R&D and none to entrepreneurship. This implies A/n will rise. If $\tilde{w}_E > \bar{w}_R$ instead, all skilled labor is allocated to entrepreneurship and A/n will fall. Such changes in A/n will push the threshold wages towards each other. Only when $\bar{w}_R = \tilde{w}_E$ is the labor market allocation stable at positive levels of both activities. Figure 1 plots the ratio \tilde{w}_E / \bar{w}_R against A/n . The above implies that the labor market may clear at any ratio in the short run, but the corresponding allocation of labor over R&D or entrepreneurship implies that we will move towards the point where this ratio equals 1. Even then, the model is not in steady state.

²⁰ We can normalize total skilled labor supply to 1 by an appropriate choice of the scaling parameters φ and ψ .

Figure 1: The Skilled Labor Market



The position of the convex curve still depends on the various growth rates in the model, as can be verified when we take the ratio of (18) over (29) and substitute for the growth rate and level of total profits using (24):

$$\frac{\bar{w}_R}{\tilde{w}_E} = \frac{\psi}{\varphi} \left(\frac{A}{n} \right)^{-\gamma-\delta} \frac{(\alpha + \beta)(1 - \alpha - \beta)(1 - \xi \dot{A}/A)}{\alpha + (\alpha + \beta)(1 - \alpha - \beta)(r - \dot{X}/X)\xi} \frac{r + \dot{n}/n - \dot{X}/X}{r - \dot{w}_R/w_R + \gamma \dot{n}/n} \quad (30)$$

Out of steady state equilibrium, the labor market will thus ensure that first A/n is at A/n^* , but due to the fact that (30) depends on the growth rates of output, skilled wages, the interest rate and the growth rate of n , this A/n^* is not necessarily the steady state ratio. A steady state is reached when knowledge stocks have adjusted to such levels that A and n grow at the same positive rate and A/n is stable at A/n^{**} . We analyze the steady state below.

III.2 The Steady State

The model is in steady state equilibrium when all variables expand at a constant rate and the skilled labor market allocation is stable. From the arbitrage equations (18) and (29) and the analysis of the labor market above we can derive that the allocation of skilled labor is stable when A and n expand at the same rate.²¹ Output, by the production function (5) and the fact that all intermediates are used at level K/n , will then grow at rate:

$$\frac{\dot{X}}{X} = \alpha \frac{\dot{A}}{A} + (\alpha + \beta) \frac{\dot{n}}{n} + (1 - \alpha - \beta) \frac{\dot{K}}{K} \quad (31)$$

Using the fact that output in steady state grows at the same rate as both wages, total wage income and consumption, we then know that asset income must also grow at that rate by the dynamic budget constraint of consumers. Hence, for a constant interest rate, asset and raw capital accumulation must also take place at the growth rate of output. Using this fact and equation (31) we obtain:

$$\frac{\dot{X}}{X} = \frac{\alpha}{\alpha + \beta} \frac{\dot{A}}{A} + \frac{\dot{n}}{n} \quad (32)$$

And as a stable labor allocation requires a constant ratio A/n the steady state growth rates will be equal to:

²¹ Substituting for profits and computing the growth rates for (18) and (29) immediately shows that in any steady state equilibrium, the skilled wage will grow at rate: $\frac{\dot{w}_R}{w_R} = \frac{\dot{X}}{X} - \gamma \left(\frac{\dot{A}}{A} - \frac{\dot{n}}{n} \right) = \frac{\dot{w}_E}{w_E} = \frac{\dot{X}}{X} + \delta \left(\frac{\dot{A}}{A} - \frac{\dot{n}}{n} \right)$.

Equation (11) has also shown that a stable steady state demand for production workers implies that growth rate of unskilled wages equals the growth rate of output. Both wage levels grow at the same rate as output for a stable ratio of A/n .

$$\frac{\dot{K}}{K} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\dot{B}}{B} = \frac{\dot{w}_P}{w_P} = \frac{\dot{w}_R}{w_R} = \frac{\dot{w}_E}{w_E} = r - \rho = \frac{\dot{n}}{n} \left(\frac{2\alpha + \beta}{\alpha + \beta} \right) \quad (33)$$

This solves the model if we can obtain the steady state growth rate of n (and A). The first steady state condition follows from rewriting equation (30) for the steady state. The ratio in equation (30) is 1 in equilibrium and can be solved for A/n :

$$\frac{A}{n} = \left(\frac{\psi}{\varphi} \frac{\Omega \Xi(\dot{n}/n)}{\Phi} \frac{1}{\Gamma(\dot{n}/n)} \right)^{\frac{1}{\gamma+\delta}} \quad (33)$$

Where we define auxiliary parameters $\Omega \equiv (\alpha + \beta)(1 - \alpha - \beta)$, $\Phi \equiv \alpha + \Omega\rho\zeta$, and functions $\Xi(\dot{n}/n) = 1 - \zeta\dot{n}/n$ and $\Gamma(\dot{n}/n) = \frac{\rho + \gamma\dot{n}/n}{\rho + \dot{n}/n}$ to save on notation. Equation (33) solves in parameters only for the special case that $\zeta=0$ (no license income) and $\rho=0$ (no time preference). Using the condition that in steady state variety expansion, \dot{n}/n equals productivity growth, \dot{A}/A we can derive a second steady state relation between entrepreneurial activity and R&D labor using equations (7) and (27):

$$\frac{L_R}{L_E} = \frac{\varphi}{\psi} \left(\frac{A}{n} \right)^{\delta+\gamma} \quad (34)$$

Using the labor market clearing condition $1 = L_R + L_E$ we can compute the steady state level of entrepreneurial and R&D activity. We thus obtain in the steady state allocation of skilled labor:

$$L_E^* = \frac{1}{1 + \frac{\varphi}{\psi} \left(\frac{A}{n}\right)^{\delta+\gamma}}$$

$$L_R^* = \frac{\frac{\varphi}{\psi} \left(\frac{A}{n}\right)^{\delta+\gamma}}{1 + \frac{\varphi}{\psi} \left(\frac{A}{n}\right)^{\delta+\gamma}} \quad (35)$$

Plugging the level of entrepreneurship in (35) into the entry function in equation (27), dividing both sides by n and using (33) to solve for the rate of variety expansion yields:

$$\dot{n}/n^* = \frac{(\psi\Phi)^{\frac{\delta}{\gamma+\delta}} (\varphi\Omega)^{\frac{\gamma}{\gamma+\delta}}}{\Omega(\Gamma(\dot{n}/n)\Xi(\dot{n}/n))^{\frac{\delta}{\gamma+\delta}} + \Xi(\Gamma(\dot{n}/n)\Xi(\dot{n}/n))^{\frac{-\gamma}{\gamma+\delta}}} \quad (36)$$

This determines the growth rate in steady state, by the fact that the right side is a function of that growth rate but cannot be solved analytically.²² Equation (36) allows us to make the following proposition:

Proposition I: *There exists a positive unique and stable steady state equilibrium growth rate.*

The proof is in Appendix C.

²² Note that the analytical solution can be computed for the special case $\rho=0$, such that $\Gamma=\gamma$ and $\zeta=0$ such that $\Phi=1$. As that would imply no discounting and no license income, and we are primarily interested in the impact of stronger patent protection, that special case is less relevant for the purpose of this paper.

VI Comparative Statics and the Impact of stronger IPR-Protection

VI.1 The key result

We can now investigate the impact of stronger intellectual property rights protection on the steady state rate of innovation in our model and formulate our key proposition.

Proposition II:

Strengthening the level of patent protection as captured by an increase in ζ in our model will only generate increases in the overall rate of innovation *if* the initial level of protection is low enough. More patent protection is beneficial for economic growth as long as:

$$\zeta < \frac{1 - \alpha}{\psi^{\frac{\delta}{\gamma+\delta}} \varphi^{\frac{\gamma}{\gamma+\delta}} \frac{\delta}{\gamma} + \frac{\rho}{\delta}}.$$

Corollary I:

An increase in patent protection when initial levels of patent protection are already high will result in a reduction of overall innovation. This negative effect will certainly arise when:

$$\zeta < \frac{1 - \alpha\gamma}{\psi^{\frac{\delta}{\gamma+\delta}} \varphi^{\frac{\gamma}{\gamma+\delta}} \frac{\delta}{\gamma} + \frac{\rho\gamma}{\delta}}$$

Appendix D provides the proofs.

The threshold level for ζ in Proposition II and Corollary I are reached faster when the output elasticity of knowledge in final goods production, α , is large. Intuitively, this means patent protection is less likely to be beneficial when private incentives to R&D are already strong.

The effects of the knowledge spillover parameters in the R&D and the entry functions, γ and δ , are ambiguous but the threshold also shows that more productive high skilled labor, higher φ and ψ , and more impatient consumers, higher ρ , unambiguously reduce the optimal level of patent protection. The intuition for these results is that more productive labor in innovation increases the rate of innovation without patents. Therefore higher productivity reduces the effectiveness of patents to increase R&D activity through shifting incentives from innovation to invention. Finally, impatience reduces innovation with and without patents in two ways. The consumers' willingness to finance the investments in R&D is reduced. This reduces the benefits of strong patent protection for the incumbents. Moreover, increasing the rental cost of capital reduces the profitability of the intermediate sector. This reduces the incentives to invest in commercialization. Strong patent protection will reduce those incentives even more. Consequently the optimal level of protection is lower when consumers are less patient.

VI.2 Discussion

We have introduced parameter ζ to represent the strength, length and breadth of patent protection. This parameter determines how much of the commercial rents of innovation the original generator of knowledge can expropriate from the commercializer of that knowledge. We argue that this parameter captures the essence of the patent system and the strength of patent protection. We envision patents as an instrument of the legislature to redistribute commercial rents from innovation between the creator and commercializer of knowledge. Stronger patents imply stronger bargaining power for the knowledge creator and hence, allow him to extract a larger share of the rents. Longer patenting spells, the patentability of a broader knowledge base in earlier stages of development, the bias in patent infringement courts and the lower costs of patenting all work to increase the share of the knowledge creator

versus the potential commercializer. Recent reforms in the US patent system (see Lerner and Jaffe (2004)) are therefore largely covered by an increase in our parameter ξ . We have shown that there may be an offsetting effect of strengthening patent protection on the rate of innovation and growth, when invention and innovation draw on the same scarce resources.

These results strongly contradict the traditional idea-based growth models of Romer (1990) and others like him, who do not separate knowledge creation from commercialization. In the absence of this separation, one would conclude that internalization of spillovers through (re)enforcing intellectual property rights of R&D labs is always a good idea. Less spillover implies more appropriability and more R&D, which cause higher growth in the modern growth literature. This is not a merely of academic interest, as these models lend strong and perhaps oversimplified support to claims made by patent lawyers, firms with large R&D labs and developed countries in WTO rounds. Our model demonstrates that support for more and better patent protection needs to at least be qualified.

As we have argued and shown above, our result emerges when commercialization and invention are no longer assumed to collapse into one decision. When commercialization of new opportunities has to take place outside the existing and inventing firm, then barriers to the knowledge spillover may reduce growth. The risks of being sued for patent infringement and losing that case in court can overturn the initial benefits of being able to legally protect monopoly profits.²³ This problem is aggravated when the patent office allows inventors to patent ideas and knowledge which they never intended to commercialize themselves. The public policy implications of this model are therefore straightforward but also unconventional. To facilitate the spilling over of knowledge, governments should stop enforcing non-disclosure agreements in labor contracts, should stop enforcing defensive patenting, stop patenting knowledge unless a working prototype of a commercial product can be shown,

²³ Particularly in industries where the need for formal and legal protection is not so high.

encourage the dissemination of knowledge and labor mobility between entrepreneurship and wage-employment and try to facilitate the generation *and* diffusion of corporate R&D output.

Following the traditional endogenous growth theorists, we argue there is a case for R&D to be stimulated, for example through subsidies, but we add to that usual result the qualification that the subsidy must be used as leverage to promote commercialization of results inside *and* outside the firm. In this way, government can reduce deadweight losses (subsidizing R&D investments that incumbent firms would have undertaken anyway) and maximize resulting economic growth and innovation.

V Conclusions

We have presented an endogenous growth model in which monopoly rents provide the incentive to innovate. In our model rents motivate the commercialization of existing knowledge rather than the generation of new ideas. The model has entrepreneurs invest resources in commercialization and capture the rents from innovation. They do not, however, produce the opportunities themselves. Incumbent firms do R&D to maintain competitiveness through efficiency improvements on their final output and in our model the commercial opportunities spill over from this R&D. We then analyze the impact of stronger IPR protection and patents in the context of our model.

The implications of this amended model are more than trivial. R&D spillovers contribute to growth but as spinouts are growth enhancing, non-disclosure agreements and patenting may turn out to be growth inhibiting. Patent protection increases incentives to create and patent knowledge but reduces incentives to commercialize it. The latter effect may overtake the former and reduce the aggregate rate of growth. When IPR protection and patents shift a share of the rents from knowledge commercializers to knowledge generators, the resulting rate of innovation in the economy follows an inverted U-shape in the level of protection.

New growth theory correctly asserts that the knowledge generated by commercial R&D can be a source of steady state growth, but inaccurately considers it a sufficient precondition or even the most important one. Protecting and giving incentives for the generation of knowledge is useful and necessary, but doing so through mechanisms like patents and IPR may shift the balance of power in the ex post bargain over rents too much in favor of knowledge creators. This can reduce incentives to commercialize to the extent that

economic growth falls. As both the inventor and the innovator generate large positive spillovers to society, a more balanced approach to IPR protection is required.

Knowledge is only valuable to society when it is commercialized in new products and services. The patent system was never intended to enable large firms' legal departments to bully small competitors out of adjacent market niches. Or to enable individual inventors that lack the motivation, talent or means to commercialize their ideas, to discourage others from doing so. As Lerner and Jaffe (2004) have argued forcefully, however, that is exactly what the most recent reforms in the US patent system have accomplished.

In our model we have abstracted from uncertainty and have introduced IPR protection at a very high abstraction level as part of the bargain between knowledge creator and commercializer. That bargain and the relative bargaining power of the parties involved may contain many other possible legal, institutional and economic elements. Possible extensions at this point include the role of intermediaries such as venture capitalists and university technology transfer offices. In future work, we aim to be more explicit on the issue of risk and to derive more precisely how the ex-ante value of new ventures is shared among parties involved in the innovation process.

Appendix A: The full dynamic optimization problem of Consumers.

The Hamiltonian to this problem:

$$H_c = e^{-\rho t} \log(C(t)) + \mu(t)(r(t)B(t) + w(t) - C(t)) \quad (\text{A1})$$

Yields the first order conditions:

$$\frac{\partial H_c}{\partial C(t)} = 0 = \frac{e^{-\rho t}}{C(t)} - \mu(t)$$

$$\frac{\partial H_c}{\partial B(t)} = -\dot{\mu}(t) = r(t)\mu(t) \quad (\text{A2})$$

$$\lim_{t \rightarrow \infty} \mu(t)B(t) = 0$$

$$\frac{\partial H_c}{\partial \mu(t)} = \dot{B}(t) = r(t)B(t) + w(t) - C(t)$$

Taking the first two conditions, solving the first for $\mu(t)$, taking the time derivative and substituting into the second yields:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho \quad (\text{A3})$$

For any constant $r(t)=r$ we then obtain²⁴:

²⁴ The assumption of a stable equilibrium interest rate is consistent with a steady state equilibrium later on but convenient to also make here. The interest rate cannot have a positive or negative growth rate as it would imply bond prices going to 0 or infinity, which is not consistent with rational expectations. It is a very common assumption in the literature. See for example Barro and Sala-I-Martin (2004) for a derivation of the result that equilibrium interest rates are constant.

$$C(t) = C(0)e^{(r-\rho)t} \quad (\text{A4})$$

Now we can use the third and fourth condition to derive $C(0)$ and express final goods demand in variables that are given to the consumer. First rewrite condition four to:

$$\dot{B}(t) - rB(t) = w(t) - C(t) \quad (\text{A5})$$

Then multiply both sides with integrating factor e^{-rt} and solve for $C(0)$:

$$\begin{aligned} e^{-rt} \frac{dB(t)}{dt} - re^{-rt} B(t) &= e^{-rt} w(t) - e^{-rt} C(t) \\ \frac{d(e^{-rt} B(t))}{dt} &= e^{-rt} w(t) - e^{-rt} C(t) \\ d(e^{-rt} B(t)) &= e^{-rt} w(t) dt - e^{-rt} C(t) dt \\ \int_0^{\infty} d(e^{-rt} B(t)) &= \int_0^{\infty} e^{-rt} w(t) dt - \int_0^{\infty} e^{-rt} C(t) dt \end{aligned} \quad (\text{A6})$$

Which by using the third (transversality) condition in (A2) and the expression for consumption in (A4) yields:

$$-B(0) = \int_0^{\infty} e^{-rt} w(t) dt - C(0) \int_0^{\infty} e^{-\rho t} dt \quad (\text{A7})$$

Such that:

$$C(0) = \rho \left(B(0) + \int_0^{\infty} e^{-rt} w(t) dt \right) \quad (\text{A8})$$

To the consumers initial wealth, interest rate, discount rate and life time wage income are given, so this determines the optimal consumption path:

$$C(t) = \rho \left(B(0) + \int_0^{\infty} e^{-rt} w(t) dt \right) e^{(r-\rho)t} \quad (\text{A9})$$

Appendix B: Derivation of demand for intermediate i .

The n conditions in (12) allow one to derive the demand for intermediate good i in terms of the relative price and quantity of the n^{th} intermediate:

$$x_j(i)^D = x_j(n)\chi(n)^{1/\alpha+\beta} \chi(i)^{-1/\alpha+\beta} \quad (\text{B1})$$

Substituting this demand function into the production function and rewriting in terms of total output yields:

$$\begin{aligned} \sum_{i=0}^n x_j(i)^{1-\alpha-\beta} &= \sum_{i=0}^n x_j(n)^{1-\alpha-\beta} \chi(n)^{(1-\alpha-\beta)/(\alpha+\beta)} \chi(i)^{(1-\alpha-\beta)/(\alpha+\beta)} \\ &= x_j(n)^{1-\alpha-\beta} \chi(n)^{\frac{1-\alpha-\beta}{\alpha+\beta}} \sum_{i=0}^n \chi(i)^{\frac{\alpha+\beta-1}{\alpha+\beta}} = \frac{X_j}{A_j^\alpha L_{Pj}^\beta} \end{aligned} \quad (\text{B2})$$

From the n^{th} order condition we also know that for all i :

$$A^\alpha L_P^\beta = x_j(n)^{\alpha+\beta} \frac{\chi(n)}{1-\alpha-\beta} \quad (\text{B3})$$

So combining (B2) and (B3) and solving for $x_j(n)$ we get:

$$x_j(n)^D = \frac{\chi(n)^{\frac{-1}{\alpha+\beta}}}{\sum_{i=0}^n \chi(i)^{\frac{1-\alpha-\beta}{\alpha+\beta}}} (1-\alpha-\beta) X_j \quad (\text{B4})$$

And by the symmetry in the production function this implies that all varieties i have that demand function:

$$x_j(i)^D = \frac{\chi(i)^{\frac{-1}{\alpha+\beta}}}{\sum_{i=0}^n \chi(i)^{\frac{-1}{\alpha+\beta}}} (1-\alpha-\beta) X_j \quad (\text{B5})$$

Appendix C: Proof of Proposition I, the existence, uniqueness and stability of the steady state equilibrium.

We can show the uniqueness of the steady state equilibrium by investigating the properties of equation (36) in the text:

$$\dot{n}/n^* = \frac{(\psi\Phi)^{\frac{\delta}{\gamma+\delta}} (\varphi\Omega)^{\frac{\gamma}{\gamma+\delta}}}{\Omega(\Gamma(\dot{n}/n)\Xi(\dot{n}/n))^{\frac{\delta}{\gamma+\delta}} + \Xi(\Gamma(\dot{n}/n)\Xi(\dot{n}/n))^{\frac{-\gamma}{\gamma+\delta}}} \quad (36)$$

The left hand side of this equation is a simple 45-degree line. A unique steady state equilibrium can be established when we show that the right hand side intersects that line once and only once in the positive quadrant. First consider the properties of functions $\Gamma(\cdot)$ and $\Xi(\cdot)$ defined in the text. $\Gamma(\cdot)$ falls monotonously from 1 to γ as the growth rate increases from 0 to infinity. As $\Xi(\cdot)$ cannot fall below 0 (as that would imply that license incomes exceed total intermediate profits) we know that $\Xi(\cdot)$ falls from 1 to 0 as the growth rate increases from 0 to $1/\xi$. This implies that $\Gamma(\cdot)\Xi(\cdot)$ falls from 1 to 0 as the growth rate increases from 0 to $1/\xi$. The

right hand side of (36) equals $\frac{(\psi\Phi)^{\frac{\delta}{\gamma+\delta}} (\varphi\Omega)^{\frac{\gamma}{\gamma+\delta}}}{\Omega + \Phi}$, a positive constant for $\dot{n}/n = 0$. It equals

$\frac{(\psi\Phi)^{\frac{\delta}{\gamma+\delta}}(\varphi\Omega)^{\frac{\gamma}{\gamma+\delta}}}{0+\infty} = 0$ for $\dot{n}/n = 1/\zeta$. As (36) is continuous in $\Gamma(.)*\Xi(.)$ we have therefore

shown that an uneven number and at least one equilibrium exists. The equilibrium, however, is not necessarily unique and stability remains to be shown.

First consider the restriction for uniqueness. For multiple steady states it is required that the slope of the right hand side of equation (36) switches sign at least twice. Once is insufficient as the right hand side starts from a positive intercept. To intersect the 45 degree line more than once the right hand side needs to fall, then rise and fall again or alternatively rise, fall, rise and fall again. As $\Gamma(.)*\Xi(.)$ falls monotonously over the entire domain, this implies that the right hand side of (36) needs to switch sign in $\Gamma(.)*\Xi(.)$. Defining $\Psi \equiv \Gamma(.)*\Xi(.)$ and taking the derivative of denominator in the right hand side of (36) with respect to Ψ yields:

$$\frac{\delta}{\gamma+\delta}\Omega\Psi^{\frac{-\gamma}{\gamma+\delta}} - \frac{\gamma}{\gamma+\delta}\Phi\Psi^{\frac{-\gamma}{\gamma+\delta}-1} \quad (C1)$$

Which can be shown to switch sign at most once over its domain $\Psi \in \{0,1\}$ at:

$$\Psi = \frac{\gamma\Phi}{\delta\Omega} \quad (C2)$$

Thereby we show that there is one unique steady state equilibrium in the model. By the fact that the right hand side of (36) intersects the 45 degree line only once in the positive quadrant, we also know that it must intersect it from above. And as the right hand side of (36) represents the implied growth rate of n when the high skilled labor market is in equilibrium, an actual out of steady state growth rate to the left of the intersection point implies a rate of

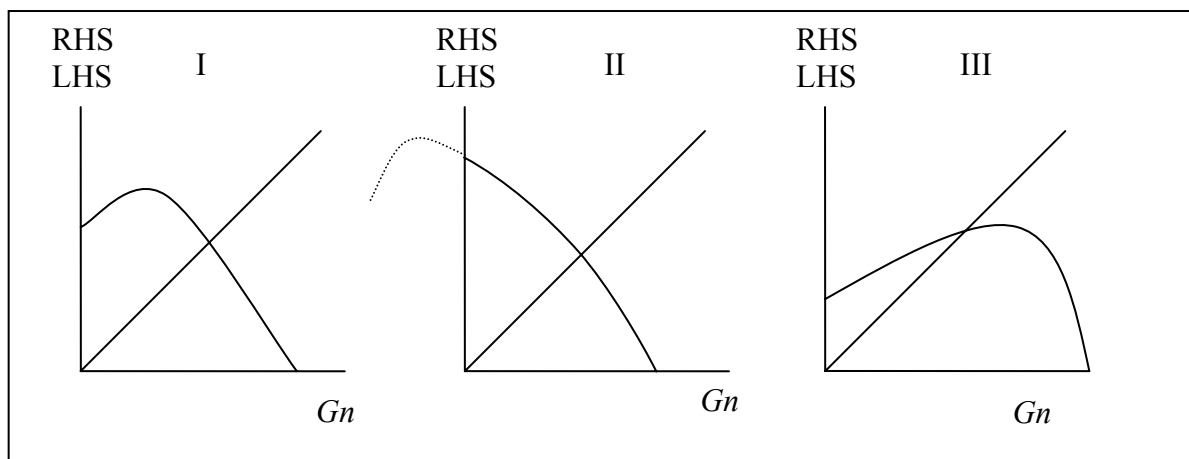
variety expansion that exceeds the steady state growth rate. This implies A/n will fall and the knowledge spillovers to entrepreneurs and R&D workers adjust to re-establish the equality of variety expansion and productivity growth rates. This mechanism implies the unique steady state growth rate is also stable.

Q.E.D.

Appendix D: Proof of Proposition II and Corollary I, the comparative statics of increasing patent protection.

To investigate the effect of an increase in ξ we need to consider its effect on the right hand side of (36). As appendix C has shown, this is a continuous curve over the domain 0 to $1/\xi$ that intersects the 45 degree line once and goes from a positive vertical intercept at $\{0, \frac{(\psi\Phi)^{\frac{\delta}{\gamma+\delta}}(\varphi\Omega)^{\frac{\gamma}{\gamma+\delta}}}{\Omega + \Phi}\}$ to a positive horizontal intercept at $\{1/\xi, 0\}$, switching slope sign at most once (from positive to negative) in the positive quadrant. There are now three general possibilities, illustrated in figure D.1.

Figure D.1



It is immediately clear that the horizontal intercept will shift inwards for higher levels of patent protection. Ceteris Paribus this causes the steady state growth rate to fall unambiguously in cases I and II and will first increase and then decrease the growth rate in case III. However, there is also an impact on the vertical intercept and the position of the curve when ζ increases.

First consider the impact on the point where *RHS* reaches a maximum. That point was defined in (C2) by:

$$\Psi = \frac{\gamma\Phi}{\delta\Omega} \tag{C2}$$

Where Ψ is negatively dependent on the growth rate of n through $\Gamma(\cdot)$ and $\Xi(\cdot)$ and negatively on ζ through $\Xi(\cdot)$. Φ is positively affected by an increase in ζ . This implies that the growth rate at which the right hand side switches sign must fall for an increase in ζ . This implies that the equilibrium growth rate can only rise for an increase in ζ when there is an increase in the maximum of *RHS*. By plugging (C2) into the right hand side of (36), however, we obtain after some rearranging:

$$RHS = \psi^{\frac{\delta}{\gamma+\delta}} \varphi^{\frac{\gamma}{\gamma+\delta}} \frac{\delta}{\gamma} \quad (D1)$$

And it is obvious that this maximum value is not dependent on ξ . From this we can also conclude that the vertical intercept increases in ξ in cases I and III and drops in case II. This concludes the graphical analysis and allows us to state proposition II. Only in case III an increase in ξ will cause an increase in the steady state growth rate. Case III is characterized by the restriction that the maximum of RHS in (D1) is less than the corresponding value of LHS , which is equal to the growth rate that satisfies the condition in (C2). Recalling the definitions of $\Gamma(\cdot)$ and $\Xi(\cdot)$ and Φ and Ω we can rewrite (C2) into:

$$\frac{\rho + \gamma \dot{n}/n}{\rho + \dot{n}/n} (1 - \xi \dot{n}/n) = \frac{\gamma(\alpha + (\alpha + \beta)(1 - \alpha - \beta)\xi\rho)}{\delta(\alpha + \beta)(1 - \alpha - \beta)} \quad (D2)$$

As the first fraction on the left hand side must take a value between 1 and γ , the growth rate that satisfies this condition, S , satisfies:

$$\frac{1}{\xi} - \left(\frac{\alpha}{\xi} + \frac{\rho}{\delta} \right) \leq S \leq \frac{1}{\xi} - \gamma \left(\frac{\alpha}{\xi} + \frac{\rho}{\delta} \right) \quad (D3)$$

By taking the minimum value that S can attain we can be sure that we are in situation III when that minimum value exceeds the maximum value of RHS . We are definitely in situation III, where more patent protection increases the steady state rate of innovation if:

$$\xi < \frac{1 - \alpha}{\psi^{\frac{\delta}{\gamma+\delta}} \varphi^{\frac{\gamma}{\gamma+\delta}} \frac{\delta}{\gamma} + \frac{\rho}{\delta}} \quad (D4)$$

Which is what we state in proposition II. The proof for Corollary I follows from reversing the argument above and deriving the condition for which we are certain that the effect of increased patent protection on the steady state rate of innovation is negative.

As the right hand side of the condition is a positive constant, lower initial levels of patent protection make it more likely that the economy will benefit from increasing patent protection. It can also be verified that a lower output elasticity of knowledge in final goods production, α , increases that probability (as it reduces the private incentives to do R&D in the absence of license income). Also less productive skilled labor, φ and ψ , strengthens the case for more protection. The intuition is that this lower productivity decreases the maximum attainable growth rate and therefore. The effect of δ and γ are ambiguous. More patient consumers (lower ρ) also improves the case for patent protection.

References

- Acemoglu, D., S. Johnson and J. Robinson (2001), "The Colonial Origins of Comparative Development: An Empirical Investigation", *American Economic Review*, Vol. 91(5), pp. 1369-1401.
- Acs, Z., D. Audretsch, P. Braunerhjelm and B. Carlsson (2004), "The Missing Link: The Knowledge Filter and Entrepreneurship in Endogenous Growth", *Center for Economic Policy Research Discussion Paper*, No. 4783.
- Acs, Z., D. Audretsch, P. Braunerhjelm and B. Carlsson (2006), "The Knowledge Spillover Theory of Entrepreneurship", *Center for Economic Policy Research Discussion Paper*, No. 5326.
- Aghion, P. and P. Howitt (1992), "A Model of Growth through Creative Destruction", *Econometrica*, Vol. 60, pp. 323-51.
- Aghion, P. and P. Howitt (1998), *Endogenous Growth Theory*, The MIT-press, Cambridge, MA.
- Allred, B. and W. Park (2007), "The Influence of Patent Protection on Firm Innovation Investment in Manufacturing Industries", *Journal of International Management*, Vol. 13, pp. 91-109.
- Audretsch, D. (2007), *The Entrepreneurial Society*, Oxford University Press, Oxford.
- Barro, R. (1996), "Democracy and Growth", *Journal of Economic Growth*, Vol. 1(1), pp. 1-27.
- Barro, R. and X. Sala-I-Martin (2004), *Economic Growth*, 2nd Ed., The MIT-press, Cambridge, MA.
- Bessen, J. and E. Mashkin (2006), "Sequential Innovation, Patents and Imitation", *Economic Working Papers*, No. 0025, Institute for Advanced Study, School of Social Science, Cambridge, MA.
- Branstetter, L., R. Fishman and F. Foley (2006), "Do Stronger Intellectual Property Rights Increase International Technology Transfer? Empirical Evidence from US Firm-Level Panel Data", *Quarterly Journal of Economics*, Vol. 121(1), pp. 321-349
- Braunerhjelm, P. (2008), "Entrepreneurship, Knowledge, and Economic Growth", *Foundations and Trends in Entrepreneurship*, Vol. 4(5), pp. 451-533.
- Fox, H. (1947), *Monopolies and Patents*, University of Toronto Press, Toronto.
- Furukawa, Y. (2007), "The Protection of Intellectual Property Rights and Endogenous Growth: Is Stronger always Better?", *Journal of Economic Dynamics and Control*, Vol. 31, pp. 3644-3670.
- Gallini, N. (1992), "Patent Policy and Costly Imitation", *Rand journal of Economics*, Vol. 23(1), pp. 52-63.

- Gallini, N. (2002), "The Economics of Patents: Lessons from Recent US Patent Reform", *Journal of Economic Perspectives*, Vol. 16(2), pp. 131-154.
- Gallini, N. and S. Scotchmer (2001), "Intellectual Property: When is it the best Incentive System?", in: A. Jaffe, J. Lerner and S. Stern (Eds.), *Innovation Policy and the Economy; Volume 2*, The MIT-press, Cambridge, MA.
- Gilbert, R. and C. Shapiro (1990), "Optimal Patent Length and Breath", *Rand Journal of Economics*, Vol. 21(1), pp. 106-112.
- Glass, A. and X. Wu (2007), "Intellectual Property Rights and Quality Improvement", *Journal of Development Economics*, Vol. 82(2), pp. 393-415.
- Gort, M. and S. Klepper (1982), "Time Paths in the Diffusion of Product Innovations", *The Economic Journal*, Vol. 92(367), pp. 630-653.
- Gould, D. and W. Gruben (1996), "The Role of Intellectual Property Rights in Economic Growth", *Journal of Development Economics*, Vol. 48, pp. 323-350.
- Greasley, D. and L. Oxley (2007), "Patenting, Intellectual Property Rights and Sectoral Outputs in Industrial Revolution Britain, 1780-1851", *Journal of Econometrics*, Vol. 139, pp. 340-354.
- Green, J. and S. Scotchmer (1995), "On the Division of Profit in Sequential Innovation", *Rand Journal of Economics*, Vol. 26(1), pp. 20-33.
- Grossman, G. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, The MIT-press, Cambridge, MA.
- Horii, R. and T. Iwaisako (2007), "Economic Growth with Imperfect Protection of Intellectual Property Rights", *Journal of Economics*, Vol. 90(1), pp. 45-85.
- Iwaisako, T. and K. Futagami (2003), "Patent Policy in an Endogenous Growth Model", *Journal of Economics*, Vol. 78, pp. 239-258.
- Jaffe, A. (2000), "The US Patent System in Transition; Policy Innovation and the Innovation Process", *Research Policy*, Vol. 29(4-5), pp. 531-557.
- Jaffe, A., and J. Lerner (2004), *Innovation and its Discontents: How our Broken Patent System is Endangering Innovation and Progress, and What to Do About It*, Princeton University Press, Princeton, NJ.
- Jones, C. (2006), "Growth and Ideas", in: Aghion, P. and S. Durlauf (Eds.), *Handbook of Economic Growth*, Vol. 1, Part 2, Elsevier Publishers, New York.
- Klepper, S. (1996), "Entry, Exit, Growth and Innovation over the Product Life Cycle", *American Economic Review*, Vol. 86, No. 3, pp. 562-583.
- Kullti, K. and T. Takalo (2008), "Optimal fragmentation of Intellectual Property Rights", *International Journal of Industrial Organization*, Vol. 26 (1), pp 137-149.

Jena Economic Research Papers 2008 - 069

- Kwan, Y. and E. Lai (2003), "Intellectual Property Rights Protection and Endogenous Economic Growth", *Journal of Economic Dynamics and Control*, Vol. 27, pp. 853-873.
- Lai, E. (1998), "International Property Rights Protection and The Rate of Product Innovation", *Journal of Development Economics*, Vol. 55, pp. 133-153.
- Maurer, S. and S. Scotchmer (2002), "Independent-Invention Defence in Intellectual Property", *Economica* Vol. 69, pp. 535-547.
- Nordhaus, W. (1969) *Invention Growth and Welfare: A Theoretical Treatment of Technological Change*, The MIT-press, Cambridge MA.
- Romer, P. (1986), "Increasing Returns and Long-Run Growth", *Journal of Political Economy*, Vol. 94 (5), pp. 1002-1037.
- Romer, P. (1990), "Endogenous Technological Change", *Journal of Political Economy*, Vol. 98, pp. S71-S102.
- Sala-I-Martin, X. (1996), "I Just Ran Two Million Regressions", *American Economic Review*, Vol. 87(2), pp. 178-183.
- Sener, F. (2004), "Intellectual Property Rights and Rent Protection in a North-South Product-Cycle Model", *Mimeo*, Union College, Schenectady, NY.
- Schumpeter, J. (1934), *The Theory of Economic Development*, Harvard University Press, Cambridge, MA.
- Scotchmer, S. (1991), "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law", *Journal of Economic Perspectives*, Vol. 5(1), pp. 29-41.
- Scotchmer, S. (1996), "Protecting Early Innovators: Should Second-Generation Products be Patentable?", *Rand Journal of Economics*, Vol. 27(2), pp. 322-331.
- Walsh, C. (2003), *Monetary Theory and Policy*, 2nd Ed., The MIT-Press, Cambridge, MA.