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**Working Paper**

## Strategic outsourcing, profit sharing and equilibrium unemployment

IZA Discussion Papers, No. 3413

**Provided in Cooperation with:**

IZA – Institute of Labor Economics

*Suggested Citation:* Koskela, Erkki; König, Jan (2008) : Strategic outsourcing, profit sharing and equilibrium unemployment, IZA Discussion Papers, No. 3413, Institute for the Study of Labor (IZA), Bonn,  
<https://nbn-resolving.de/urn:nbn:de:101:1-2008032625>

This Version is available at:

<https://hdl.handle.net/10419/35030>

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IZA DP No. 3413

## Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment

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March 2008

# Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment

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Discussion Paper No. 3413  
March 2008

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## ABSTRACT

### Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment<sup>\*</sup>

We analyze the following questions associated with outsourcing and profit sharing under imperfect labour markets. How does strategic outsourcing influence wage formation, profit sharing and employee effort when firms commit to optimal profit sharing before wage formation or decide for profit sharing after wage formation? What is the relationship between outsourcing, profit sharing, and equilibrium unemployment when profit sharing is also a part of a compensation scheme in all industries? We find that if firms will decide on profit sharing before the wage formation, higher outsourcing decreases wage whereas profit sharing has an ambiguous effect. Under flexible profit sharing wage is smaller than in the case of committed profit sharing. For equilibrium unemployment, we find that if there is also profit sharing in other industries, the effects of outsourcing and profit sharing on the unemployment rate is ambiguous both in the committed and flexible case.

JEL Classification: E23, E24, J23, J33, J82

Keywords: outsourcing, profit sharing, labour market imperfection, employee effort, equilibrium unemployment

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<sup>\*</sup> We thank seminar participants at University of Oulu, the Bank of Finland, Labour Institute for Economic Research in Helsinki, the Research Institute of the Finnish Economy in Helsinki and Ronnie Schöb for comments on earlier drafts. Koskela thanks *the Research Unit of Economic Structures and Growth* (RUESG) at the University of Helsinki, *the Yrjö Jahnesson Foundation* and *the Academy of Finland* (grant No. 1117698) for financial support and *Freie Universität Berlin* for great hospitality and König thanks *the Research Unit of Economic Structures and Growth* (RUESG) at the University of Helsinki for great hospitality.

## 1. Introduction

High wage differences across countries constitute an important explanation for the currently significant business practice of international outsourcing. For example, 1.10 € per hour in China is very low in comparison with 27 € per hour e.g. in Denmark, Germany or Norway. These wage differentials could lead to outsourcing (see e.g. Sinn (2007) for details, and also Stefanova (2006) concerning the East-West dichotomy of outsourcing). Glass and Saggi (2001) have studied the causes of outsourcing and its effects and they found that higher international outsourcing lowers the relative wage of domestic workers, while it increases the profits and thereby creates greater incentives for innovation.

It is known that higher wages affect workers' productivity which is influenced by their effort. Of course, according empirical evidence another way to stimulate the effort is profit sharing. Profit sharing is an empirically important phenomenon in many OECD countries. Pendleton et al. (2001) have presented detailed data on profit sharing schemes in 14 EU-countries. For example, among all western EU-countries in 1999/2000 a double-digit percentage of the workplaces use profit sharing in Austria, Finland, France, Germany, Ireland, Netherlands, Portugal, Spain, Sweden and United Kingdom. The lowest incidences are found in Denmark, Italy, and Greece. For further evidence regarding the incidence of profit sharing, we refer to the DICE data base, collected by CESifo, <http://www.CESifo.de>, as well as to Conyon and Freeman (2001).

In terms of profit sharing Koskela and Stenbacka (2004a) have offered a framework to analyze employment, effort, wages and profit sharing when firms face stochastic revenue shocks. Moreover, they have investigated the interaction between labour and credit market imperfections in the presence of profit sharing (see Koskela and Stenbacka (2004b). In these papers they have analyzed committed profit sharing which is decided before wage negotiation. Koskela and Stenbacka (2006) have also studied the differences between committed profit sharing and flexible profit sharing, which is decided after wage formation. They have shown that the optimal profit share under commitment is higher than under flexibility because through a profit share commitment the firms can induce wage moderation. In these papers they have also studied the relationship between profit sharing and equilibrium unemployment.

As profit sharing is now commonly incorporated in the compensation schemes, and international outsourcing has recently increased among western EU-countries and in the United States, then it is important to study their relationship and implications for workers' effort, wage formation and unemployment when profit sharing is also a part of a compensation scheme in all other industries. This is the topic, which is our focus in this paper. We assume that firms commit to outsourcing before profit sharing, wage negotiation, labour demand and effort determination. Moreover, and importantly, we also analyze the implications of two alternative time sequences in terms of profit sharing decision: (i) firms might commit to profit sharing before base wage negotiation or (ii) firms deciding about profit sharing only after knowing the result of base wage negotiation.

In our framework we analyze the following questions associated with outsourcing and profit sharing under imperfect labour markets: How does strategic outsourcing influence wage formation, profit sharing and employee effort, when firms commit to optimal profit sharing before wage formation or deciding about profit sharing after wage formation. We also analyze the relationship between outsourcing, profit sharing and equilibrium unemployment under the assumption that profit share is part of the compensation scheme in all industries. Finally, we briefly look at the long-run perspective for the optimal production mode in terms of strategic outsourcing.

First, we show that in the presence of outsourcing the wage elasticity of labour demand depends positively both on the amount of outsourcing and on the base wage, but negatively on the size of profit sharing. As a result we also show that in the case of committed profit sharing strategic outsourcing has a negative effect on wage formation. This lies in conformity with empirics and results from our assumption of perfect substitutability between outsourcing and effective domestic labour. Under flexible profit sharing the wage is smaller than in the case of committed profit sharing. We also find that the profit share under commitment in the presence of outsourcing is larger (smaller) than that associated with flexibility if the base wage depends negatively (positively) on profit sharing.

If profit sharing is a part of outside option in other industries outsourcing, profit sharing will have ambiguous effects on equilibrium unemployment. Also in the absence of outsourcing, profit sharing will have an ambiguous effect on equilibrium

unemployment. In terms of optimal long-run strategic outsourcing wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.

We proceed as follows: Section 2 presents the basic structure of theoretical framework and two different time sequences in terms of profit sharing decision in the presence of outsourcing activity. The determination of labour demand by firms and effort by workers are presented in section 3. Section 4 investigates the wage formation by monopoly labour union in the presence of strategic outsourcing and committed profit sharing, and section 5 studies the wage formation by monopoly labour union with strategic outsourcing and flexible profit sharing. Section 6 explores the implications of strategic outsourcing and different time decisions of profit sharing on equilibrium unemployment. Section 7 studies briefly the optimal long-run outsourcing given the wage formation, the profit sharing, the labour demand, and the employee effort. Finally, we present conclusions in section 8.

## **2. Basic Framework**

We consider a representative firm and assume that output depends not only on the units of domestic labour but also on the effort supplied by workers, i.e. the workers' productivity. This lies in conformity with the efficiency wage hypothesis<sup>1</sup>. Also production combines international outsourcing which is substitute for domestic effective labour. We analyze two alternative timing decisions made by the firm, the labour union and the worker.

The timing structure (I) captures the idea that the representative firm commits both to outsourcing and profit sharing in anticipation of wage determination. After wage formation, the representative firm determines employment and the representative worker decides on effort provision. The partly alternative timing structure (II) will change the timing of determination of profit sharing and wage

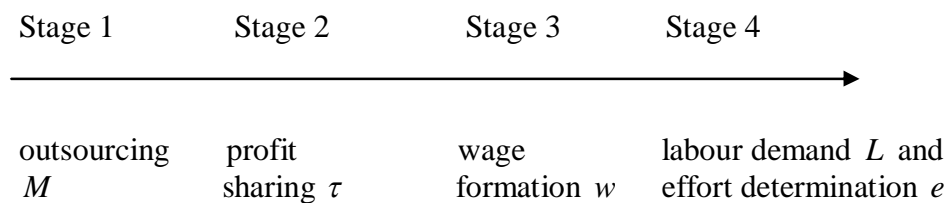
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<sup>1</sup> For a survey and several important seminal articles, see e.g. the book edited by Akerlof and Yellen (1986).

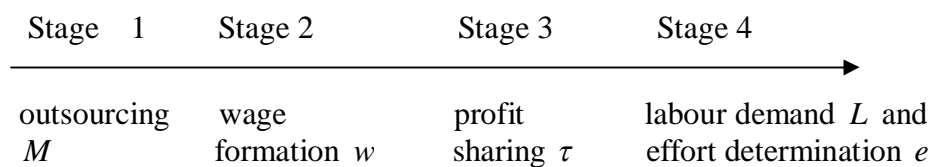
determination by keeping other timing aspects similar as in (I). In this case the representative firm is flexible in the decision of profit sharing by deciding it after wage formation. We summarize these alternative timing decisions in Figure 1.<sup>2</sup>

**Figure 1:** *Alternative time sequences of decisions in terms of employment, effort, wage formation, profit sharing and outsourcing*

(I) *Strategic outsourcing and committed profit sharing:*



(II) *Strategic outsourcing and flexible profit sharing:*



This timing structure seems plausible when the implementation of a production mode with outsourcing requires irreversible investments concerning the establishment of a network of foreign suppliers. Of course, the relative timing of wage formation in the presence of outsourcing might be different in certain circumstances. Such a reversed timing structure would be relevant if the firms flexibly adjusted their production mode, and decided whether to initiate foreign outsourcing once the domestic wage is determined.<sup>3</sup>

In the following sections we turn to an analysis of these two alternative decisions taking place at the different stages of the interaction between the

<sup>2</sup> Whether profit sharing is committed or flexible in terms of base wage formation is an important new topic for empirical research.

<sup>3</sup> Skaksen (2004) has analyzed this case using a Cobb-Douglas production function also in the case of homogenous domestic labour, but both in the absence of effort determination of workers and profit sharing of firms. Also Braun and Scheffel (2007) have developed a simple two-stage game between a monopoly union and a firm by assuming that the union sets wages before the firm decides on the degree of outsourcing and the level of production. They also abstract from effort determination of workers and profit sharing of firms. They argue that under flexible outsourcing the costs of outsourcing has an ambiguous effect on the wage set by the union.



representative firm, the monopoly labour union and the representative worker by using the backward induction and solving the game in reverse order.

### 3. Labour Demand and Employee Effort

Here we characterize the optimal labour demand by the representative firm and the effort by the representative worker in stage 4 by taking profit sharing  $\tau$ , wage formation  $w$ , and outsourcing  $M$  as given. The technology is assumed to satisfy the following revenue function<sup>4</sup>

$$R(e, L, M) = \frac{\delta}{\delta - 1} (eL + M)^{\frac{\delta - 1}{\delta}} \quad (1)$$

where  $L$  is unit of labour,  $e$  describes the effort determination,  $M$  indicates the amount of outsourcing, and  $\delta > 1$ .<sup>5</sup> We assume that outsourcing and effective labour are perfect substitutes. The representative firm makes irreversible investment  $c(M)$  with the properties that  $c'(M), c''(M) > 0$ . This captures the idea according to which the marginal cost advantages also requires irreversible investment into the establishment of networks of suppliers in low-wage countries.

The disutility of effort is assumed to satisfy the following convex function  $g(e) = \gamma e^{1/\gamma}$  with  $0 < \gamma < 1$ , i.e.  $g'(e), g''(e) > 0$ . The individual utility function for the employed worker is (2a) and for the unemployed worker (2b)

$$u = w + \frac{\tau}{L} \pi - g(e) \quad (2a)$$

---

<sup>4</sup> Specifying the inverse product demand function according to a monopolistic product market competition (for details, see the seminal paper by Dixit and Stiglitz (1977)) in a simple way as

$$p = \frac{\delta}{\delta - 1} D^{-\frac{1}{\delta}}, \quad \delta > 1, \text{ gives the following inverse elasticity of demand } -p_p D / p = \frac{1}{\delta} \text{ so}$$

that  $-D_p p / D = \delta > 1$ . By assuming  $F(e, L, M) = eL + M$  and  $F = D$  gives another suggestion for (1). In what follows we do not elaborate the potential role of product market competition for our issues.

<sup>5</sup> We assume by following Koskela and Stenbacka (2006) that effort is observable and verifiable. If effort is partly unobservable, this implies that firms are interesting to monitor workers. This is of course an important new research topic.

$$\bar{u} = b \quad (2b)$$

where  $\pi$  captures the firm's profit and  $b$  stands for the unemployed worker's exogenous outside option.

The profit function can now be expressed as

$$\pi = \frac{\delta}{\delta - 1} (eL + M)^{\frac{\delta - 1}{\delta}} - wL - c(M). \quad (3)$$

Given  $M$ ,  $w$ , and  $e$  the first-order condition for the firm's optimal labour demand can be expressed as

$$\pi_L = (eL + M)^{\frac{1}{\delta}} e - w = 0, \quad (4)$$

and the second-order condition  $\pi_{LL} = -\frac{1}{\delta} (eL + M)^{\frac{1 + \delta}{\delta}} e^2 < 0$ . The first-order condition can be re-expressed as

$$L = w^{-\delta} e^{\delta - 1} - \frac{M}{e} \quad (5)$$

where the *direct* wage elasticity of labour demand is  $\delta > 1$ . According to (5)  $L = L(\underbrace{w}_{-}, \underbrace{M}_{-}, \underbrace{e}_{+})$ , so that higher wage rate and higher outsourcing, which is substitute for domestic labour, will decrease labour demand and higher employee's effort will increase labour demand. But labour demand (5) does not directly depend on profit sharing, which lies in conformity with empirical evidence (see e.g. Wadwani and Wall (1990), Cahuc and Dormont (1997)).

The first-order condition in terms of effort determination for equation (2a) is

$$u_e = \frac{\tau}{L} \pi_e - g'(e) = 0. \quad (6)$$

Using  $g'(e) = e^{(1/\gamma)-1}$  and  $\pi_e = (eL + M)^{-\frac{1}{\delta}} L$  equation (6) implies

$$e = (\tau w)^\gamma \quad (7)$$

where  $\gamma = \frac{e_\tau \tau}{e}$  is the elasticity of effort with respect to profit sharing (see about this, Koskela and Stenbacka (2006)). Therefore the optimal effort by worker is a positive function of both base wage,  $w$ , and profit sharing,  $\tau$ , i.e.  $e_w = \frac{\gamma e}{w}$  and  $e_\tau = \frac{\gamma e}{\tau}$ , so that profit sharing and base wage enhance productivity by increasing effort provision and thereby affect labour demand indirectly.<sup>6</sup> But outsourcing will have no effect in the case of perfect substitutability between outsourcing  $M$  and employee effort  $e$ .

#### **4. Wage Formation by Monopoly Labour Union with Strategic Outsourcing and Committed Profit Sharing**

Now we continue to analyze the timing structure (I), when the firm commits to the profit share prior to the base wage formation and by taking outsourcing as given and allowing for their effects on labour demand and effort determination.

##### **4.1. Wage Formation**

By analyzing the base wage formation by monopoly labour union under strategic outsourcing and committed profit sharing in stage 3, the objective function of monopoly labour union is assumed to be

$$V = (w - b)L + \tau \pi - g(e)L + bN \quad (8)$$

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<sup>6</sup> This finding lies in conformity with empirics (see e.g. Booth and Frank (1999), Cable and Wilson (1990), Cahuc and Dormont (1997), Kruse (1992), and Wadhvani and Wall (1990)). Of course, we have to mention that these issues have not been studied to our knowledge empirically in the presence of outsourcing.

where  $b$  captures the exogenous minimum income for all labour union members  $N$ . Maximizing (8) in terms of base wage subject to labour demand (5), effort determination (7) and given outsourcing and profit sharing gives the following first-order condition

$$V_w = L + L_w(w-b) + \tau \pi_w - g(e)L_w - Lg'(e)e_w = 0, \quad (9)$$

which can be expressed as follows <sup>7</sup>

$$w = \left[ \frac{\eta}{\eta(1-\gamma\tau) - (1-\tau)} \right] b \quad (10)$$

where

$$\eta \equiv -\frac{L_w w}{L} = \delta(1-\gamma) \left( 1 + \frac{M}{eL} \right) + \gamma \quad (11)$$

is the total wage elasticity of labour demand (see Appendix 1 for details). Therefore, the total wage elasticity  $\eta$  depends on the base wage rate, the amount of outsourcing and the effort determination, but it is constant in the absence of outsourcing. It should be emphasized that the wage determination (10) is expressed in the implicit (not explicit) form, because the wage elasticity of labour demand associated with the mark-up  $A^c = \left[ \frac{\eta}{\eta(1-\gamma\tau) - (1-\tau)} \right]$  depends also on the base wage via labour demand and effort determination according to (11).

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<sup>7</sup> By calculating  $g'(e)e_w = e^{(1/\gamma)-1}e_w = \gamma\tau^{1-\gamma+\gamma}w^{1-\gamma+\gamma-1} = \gamma\tau > 0$ , and  $\pi_w = -L + (eL+M)^{\frac{1}{\delta}}Le_w = -L \left[ 1 - (eL+M)^{\frac{1}{\delta}}e_w \right] = -(1-\gamma)L < 0$ , we can rewrite the first-order condition as follows  $V_w = 0 \Leftrightarrow \frac{L}{w} \left[ w + \frac{L_w w}{L}(w-b) - \tau(1-\gamma)w - \frac{L_w w}{L} \tau \gamma w - \tau \gamma w \right] = 0$  which gives (10).

The base wage elasticity of labour demand depends positively on the amount of outsourcing, i.e.

$$\eta_M = \delta(1-\gamma) \frac{(eL+M)}{(eL)^2} = \frac{\delta(1-\gamma)}{eL} \left(1 + \frac{M}{eL}\right) = \frac{\eta-\gamma}{eL} > 0. \quad (12a)$$

This positive relationship results from the fact that higher outsourcing will increase the ratio between outsourcing and effective labour, i.e.  $M/eL$ . Our finding lies in conformity with empirics (see e.g. Hasan et al (2007), Slaughter (2001) and Senses (2006)).

Next we characterize the relationship between the wage elasticity of labour demand and the base wage rate, i.e. we have

$$\eta_w = \left[ \frac{\delta(1-\gamma)}{eL} \right]^2 \frac{M}{w} (eL+M) = \frac{\delta(1-\gamma)}{eL} \frac{M}{w} \left(1 + \frac{M}{eL}\right) = \delta(1-\gamma) \frac{M}{w} \frac{\eta-\gamma}{eL} > 0. \quad (12b)$$

Therefore, the wage elasticity depends positively on the base wage rate in the presence of outsourcing. In the absence of outsourcing, this effect is, however, zero, i.e.  $\eta_w|_{M=0} = 0$ .

The relationship between the base wage elasticity and profit sharing can be written as

$$\eta_\tau = \delta(1-\gamma) \left[ \frac{-M(e_\tau L + L_\tau e)}{(eL)^2} \right] = \frac{\delta^2 \gamma M}{\tau} \frac{\eta-\gamma}{eL} < 0 \quad (12c)$$

where  $e_\tau = \frac{\gamma e}{\tau} > 0$ ,  $L_\tau = w^{-\eta}(\eta-1)e^{\eta-2}e_\tau + \frac{Me_\tau}{e^2} > 0$ , so that  $e_\tau L + L_\tau e = \frac{\gamma}{\tau}\eta(eL+M)$

and  $\eta_\tau = -\frac{\eta^2(1-\gamma)\gamma}{\tau} \frac{M}{eL} \left(1 + \frac{M}{eL}\right) < 0$ . The base wage elasticity depends negatively on profit sharing, because higher profit sharing will decrease the ratio between

outsourcing and effective labour, i.e.  $M / eL$ . It is important to emphasize that there will be no effect in the absence of outsourcing, i.e.  $\eta_\tau|_{M=0} = 0$ .

We can now summarize our findings as follows.

**Proposition 1:** *In the presence of outsourcing the wage elasticity of labour demand depends positively on the amount of outsourcing and on the base wage and negatively on the size of profit sharing.*

In the absence of outsourcing the total wage elasticity is slightly different. In this case the total wage elasticity is smaller, i.e.  $\eta|_{M=0} = \delta - (\delta - 1)\gamma$  (see about this, Koskela and Stenbacka (2006)). This implies the following explicit monopoly labour union's base wage formation

$$w|_{M=0} = \frac{\delta - (\delta - 1)\gamma}{[\delta - (\delta - 1)\gamma](1 - \gamma\tau) - (1 - \tau)} b. \quad (13)$$

Next we characterize the comparative statics in a different way than in the explicit formulations (see Appendix 2 for details). After characterizing the base wage elasticity of labour demand in terms of various parameters, we now analyze the effects of these parameters on the wage formation by the monopoly labour union.

Differentiating equation (10) with respect to outsourcing gives

$$\frac{dw}{dM} = - \frac{\frac{\eta_M w(1-\tau)}{\eta}}{\left[ \eta(1-\gamma\tau) - (1-\tau) + \frac{\eta_w w(1-\tau)}{\eta} \right]} = - \frac{\frac{\eta_M w}{\eta}}{\left[ \eta \frac{(1-\gamma\tau)}{(1-\tau)} - 1 + \frac{\eta_w w}{\eta} \right]} < 0. \quad (14)$$

Therefore, higher outsourcing will decrease the base wage formation, because higher outsourcing will increase wage elasticity of labour demand. This lies in conformity with empirics under our assumption according to which there is substitutability between outsourcing and domestic labour (see e.g. Munch and Skaksen (2005)).

The derivative of equation (10) with respect to profit sharing can express as

$$\frac{dw}{d\tau} = - \frac{\frac{[\eta_\tau(1-\tau) + \eta(1-\eta\gamma)]w}{\eta}}{\left[ \eta(1-\gamma\tau) - (1-\tau) + \frac{\eta_w w(1-\tau)}{\eta} \right]} = - \frac{\left[ \frac{\eta_\tau}{\eta} + \frac{(1-\eta\gamma)}{(1-\tau)} \right] w}{\left[ \eta \frac{(1-\gamma\tau)}{(1-\tau)} - 1 + \frac{\eta_w w}{\eta} \right]} = ? \quad (15)$$

Therefore, under this framework with outsourcing the effect of committed profit sharing on the base wage formation by monopoly labour union is a priori ambiguous, because under outsourcing the profit sharing will have a negative effect to wage formation via the mark-up, but also a positive effect due to a negative effect on wage elasticity.

In the absence of outsourcing equation (15) can be re-expressed as follows

$$\left. \frac{dw}{d\tau} \right|_{M=0} = - \frac{(\delta - (\delta - 1)\gamma)[1 - \gamma(\delta - (\delta - 1)\gamma)]}{[(\delta - (\delta - 1)\gamma)(1 - \gamma\tau) - (1 - \tau)]^2} = ?, \quad (16)$$

so that profit sharing has an ambiguous effect on the mark-up of wage formation.<sup>8</sup>

We can now summarize our findings as follows.

**Proposition 2:** *In the presence of outsourcing and committed profit sharing a higher outsourcing will decrease the base wage, whereas profit sharing has an ambiguous effect on the base wage. Also in the absence of outsourcing, higher profit sharing will have a ambiguous effect on the base wage, but a negative effect if  $1 > \gamma(\delta - (\delta - 1)\gamma)$ .*

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<sup>8</sup> We have  $\delta - (\delta - 1)\gamma > 1$ , but the sign of  $1 - \gamma(\delta - (\delta - 1)\gamma)$  is a priori ambiguous, so that  $\left. \frac{dw}{d\tau} \right|_{M=0}$  is

also ambiguous. Of course, profit sharing will have a wage moderation effect in the absence of outsourcing under the assumption  $1 > \gamma(\delta - (\delta - 1)\gamma)$ . Using a U.S. data Black and Lynch (2000) have shown that the usage of profit sharing results in lower regular pay for workers, but in Wadhvani and Wall (1990) and in Kraft and Ugarkovic (2005) it has been shown in empirics by using a UK data and a German panel data that firms introducing profit sharing do not reduce base wages thus implying a supplementary (not substitutable) character of profit sharing. This issue should be studied empirically much more in the case of various countries.

## 4.2. Committed Profit Sharing

In the timing structure of decisions (I) in stage 2 the representative firm commits to profit sharing so that profit is maximized subject to labour demand (5), effort determination (7) and wage formation by the monopoly labour union (10) and by taking outsourcing as given, i.e.

$$\text{Max}_\tau \bar{\pi} = (1-\tau) \left[ \frac{\delta}{\delta-1} (eL+M)^{\frac{\delta-1}{\delta}} - wL - c(M) \right] \quad (17a)$$

s.t.

$$L = w^{-\delta} e^{\delta-1} - \frac{M}{e} \quad (17b)$$

$$e = (\tau w)^\gamma \quad (17c)$$

$$w = \left[ \frac{\eta}{\eta(1-\tau\gamma) - (1-\tau)} \right] b \quad (17d)$$

The first-order condition is  $-\pi + (1-\tau)\pi_\tau = 0$ , where the indirect profit can be expressed as  $\pi = \frac{1}{\delta-1} [w^{1-\delta} e^{\delta-1}] + \frac{wM}{e} - c(M)$ . The derivative of profit with respect to profit sharing by allowing for the wage and effort effects of profit sharing is

$$\pi_\tau = \frac{wL}{\tau} \left( \gamma - \frac{w_\tau \tau}{w} \right), \quad (18)$$

because  $L_\tau = 0$  due to the envelope theorem ( $\pi_L = 0$ ).<sup>9</sup> Next, we have to solve the optimal committed profit sharing by using equations (18) and the indirect profit in  $-\pi + (1-\tau)\pi_\tau = 0$ , so that for given outsourcing  $M$ , the optimal committed profit sharing can be presented as (see Appendix 3)

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<sup>9</sup> The derivative  $\pi_\tau = \frac{1}{\delta-1} [(1-\delta)w^{-\delta} e^{\delta-1} w_\tau + (\delta-1)w^{1-\delta} e^{\delta-2} e_\tau] + M \frac{w_\tau e - e_\tau w}{e^2}$  gives

$\pi_\tau = \frac{w}{e} \frac{1}{\tau} \left( \gamma - \frac{w_\tau \tau}{w} \right) (w^{-\delta} e^\delta - M)$ , so that we have (18).



$$\tau^c = \frac{(\delta - 1) \left( \gamma - \frac{w_\tau \tau}{w} \right)}{\left[ 1 + (\delta - 1) \left( \gamma - \frac{w_\tau \tau}{w} \right) + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} \right]}. \quad (19a)$$

This is an implicit form because concerning the RHS of (19a) employee effort, labour demand and base wage formation depend on profit sharing.

In the absence of outsourcing the optimal committed profit sharing can be re-expressed from (19a) as follows

$$\tau^c \Big|_{M=0} = \frac{(\delta - 1) \left( \gamma - \frac{w_\tau \tau}{w} \right)}{\left[ 1 + (\delta - 1) \left( \gamma - \frac{w_\tau \tau}{w} \right) \right]}. \quad (19b)$$

(see about this, Koskela and Stenbacka (2006)). We assume that in the case without outsourcing  $\frac{w_\tau \tau}{w} < 0$  holds. With that assumption a comparison between (19a) and (19b) shows that in the presence of outsourcing the optimal committed profit share is smaller than in the absence of outsourcing, i.e.  $\tau^c < \tau^c \Big|_{M=0}$  if  $\frac{w_\tau \tau}{w} > 0$  holds in the presence of outsourcing. If  $\frac{w_\tau \tau}{w} < 0$  under outsourcing, then the relation between  $\tau^c$  and  $\tau^c \Big|_{M=0}$  is a priori ambiguous. Moreover, in the denominator of (19a)  $\delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} > 0$  and it is zero in (19b). In both cases (19a) and (19b), higher wage elasticity with respect to profit sharing,  $w_\tau \tau / w$ , will have a negative effect on the optimal committed profit sharing.<sup>10</sup>

We can now summarize our findings as follows.

**Proposition 3:** *In the presence of outsourcing the optimal committed profit share is smaller than in the absence of outsourcing if in the presence of outsourcing profit share elasticity of wage formation is*

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<sup>10</sup> As we mentioned in footnote 8, the effect of profit sharing on base wage is an important new empirical research topic.

positive, but if the elasticity is negative in the presence of outsourcing then the relation is a priori ambiguous.

## 5. Wage Formation by Monopoly Labour Union with Strategic Outsourcing and Flexible Profit Sharing

We now use the timing structure (II) to analyze the wage formation before the flexible profit sharing by the representative firm. After that and by taking outsourcing as given and committed before wage and profit sharing determinations we allow for their effects on labour demand and employee effort.

### 5.1. Flexible Profit Sharing

First, we study the optimal profit sharing in stage 3 decided after outsourcing and wage formation subject to labour demand and employee effort determinations. Now the profit sharing is decided to maximize profit by taking both the base wage and the outsourcing as given, i.e.

$$\text{Max}_{\tau} \bar{\pi} = (1-\tau) \left[ \frac{\delta}{\delta-1} (eL+M)^{\frac{\delta-1}{\delta}} - wL - c(M) \right] \quad (20a)$$

s.t.

$$L = w^{-\delta} e^{\delta-1} - \frac{M}{e} \quad (20b)$$

$$e = (\tau w)^{\gamma} \quad (20c)$$

The first-order condition is similar as in the case of committed profit sharing in terms of the first-order condition, i.e.  $-\pi + (1-\tau)\pi_{\tau} = 0$ , where the indirect profit is  $\pi = \frac{1}{\delta-1} [w^{1-\delta} e^{\delta-1}] + \frac{wM}{e} - c(M)$ . But as we will show, the optimal profit sharing is slightly different in the case of flexible profit sharing decision.

To allow for the envelope theorem due to  $\pi_L = 0$ , so that  $L_{\tau}$  is not taken into account, the partial derivative of the profit in terms of profit sharing is

$\pi_\tau = w^{1-\delta} e^{\delta-2} e_\tau - \frac{wM}{e^2} e_\tau = \frac{\gamma}{\tau} \left[ w^{1-\delta} e^{\delta-1} - \frac{wM}{e} \right] = \frac{\gamma}{\tau} wL$ , so that the first-order condition

can be written as  $\frac{1}{\delta-1} wL + \frac{\delta}{\delta-1} \frac{wM}{e} - c(M) = \frac{\gamma}{\tau} (1-\tau)wL$ . This can be solved for

the optimal flexible profit sharing in the presence of outsourcing as

$$\tau^f = \frac{(\delta-1)\gamma}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]}. \quad (21a)$$

As in the case of committed optimal profit sharing (see equation (19a)), this is also an implicit form, because both employee effort and labour demand also depend on profit sharing (see equations (5) and (7)) concerning the RHS of (21a).

In the absence of outsourcing, the flexible profit share can be expressed as

$$\tau^f \Big|_{M=0} = \frac{(\delta-1)\gamma}{1 + (\delta-1)\gamma}. \quad (21b)$$

Comparison between (21a) and (21b) shows that in the presence of outsourcing the optimal flexible profit share is smaller than in the absence of outsourcing, i.e.

$$\tau^f < \tau^f \Big|_{M=0} \text{ because } \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} > 0.$$

Comparing the optimal profit share under commitment (equation (19a)) and under flexibility (equation (21a)), it is easy to show that it depends on what is the relationship between the wage rate and profit sharing. If the base wage depends negatively (positively) on profit sharing, then optimal profit share under commitment is larger (smaller) than that associated with flexibility,  $\tau^c > \tau^f$  ( $\tau^c < \tau^f$ ). Of course, in the absence of outsourcing we have higher optimal committed profit share than optimal flexible profit share, i.e.  $\tau^c \Big|_{M=0} > \tau^f \Big|_{M=0}$  by comparing equations (19b) and

(21b), when  $\frac{w_\tau \tau}{w} < 0$  holds. But if not, then the opposite occurs.

Now we analyze the effects of the parameters outsourcing and base wage on flexible profit sharing under strategic outsourcing (see Appendix 4 for details). The effect of  $M$  can be obtained by differentiating (21a) to get

$$\frac{d\tau^f}{dM} = \frac{-\frac{1}{eL} \frac{Z\tau^2}{(\delta-1)\gamma}}{\left[1 + \frac{\frac{1}{eL} X\tau^2}{[(\delta-1)\gamma]}\right]} = -\frac{Z\tau^2}{(\delta-1)\gamma(eL) + X\tau^2} < 0 \quad (22)$$

Our assumptions are  $Z = \delta\left(1 + \frac{M}{eL}\right) - (\delta-1)\frac{e}{w}\left(c'(M) + \frac{c(M)}{eL}\right) > 0$  and

$X = -\frac{\gamma}{\tau}\left[\delta\left(\delta\frac{M}{eL} - (\delta-1)\frac{c(M)}{wL}\right)(eL+M) - (\delta-1)\frac{c(M)}{wL}(eL)\right] < 0$ , and we also assume

that  $(\delta-1)\gamma(eL) + X\tau^2 > 0$ , sound to be reasonable if optimal flexible profit sharing is small enough, so that in this case optimal flexible profit sharing depends negatively on outsourcing.

Differentiating (21a) with respect to base wage gives

$$\frac{d\tau^f}{dw} = \frac{-\frac{1}{eL} \frac{Y\tau^2}{(\delta-1)\gamma}}{\left[1 + \frac{\frac{1}{eL} X\tau^2}{[(\delta-1)\gamma]}\right]} = -\frac{Y\tau^2}{(\delta-1)\gamma(eL) + X\tau^2} < 0 \quad (23)$$

where  $Y = (1-\gamma)\delta^2\frac{M}{eL}\frac{\gamma}{\tau w}(eL+M) + (\delta-1)\frac{c(M)}{w}\frac{e}{w}(1-\eta)$ . If optimal flexible profit

sharing is small enough, then under the assumption  $(\delta-1)\gamma(eL) + X\tau^2 > 0$ , the base wage rate will have the negative effect on flexible profit sharing, while it will have no

effect in the absence of outsourcing, i.e.  $\left.\frac{\partial\tau^f}{\partial w}\right|_{M=0} = 0$ .

We can now summarize our findings as follows.

**Proposition 4:** *In the presence of outsourcing and flexible profit sharing under reasonable assumptions higher base wage and higher outsourcing will decrease profit sharing, but in the absence of outsourcing the base wage will have no effect on flexible profit sharing.*

## 5.2. Wage Formation under Flexible Profit Sharing

We now analyze the base wage formation in stage 2 by monopoly labour union under committed outsourcing and flexible profit sharing. The objective function can be written as

$$V = (w - b)L + \tau \pi - g(e)L + bN, \quad (24)$$

where  $b$  captures the exogenous minimum income for all labour union members. Maximizing (24) in terms of the base wage subject to labour demand (5), effort determination (7), and profit sharing determination (21a), gives

$$V_w = L + L_w(w - b) + \tau \pi_w + \pi \tau_w - g(e)L_w - Lg'(e)e_w = 0, \quad (25)$$

where there is the new term  $\pi \tau_w$  compared with the case of committed profit sharing formulation (9). Using equation (23) we can follow that  $\pi \tau_w < 0$ . Therefore higher wage rate will have negative effect on flexible profit sharing, so that the base wage by monopoly labour union under committed outsourcing and flexible profit sharing is smaller than in the case of committed outsourcing and committed profit sharing. By using the earlier calculations according to which  $\pi_w = -(1 - \gamma)L$  and  $g'(e)e_w = \gamma\tau$ , we can solve the first order condition (25) as follows

$$w \left( \eta(1 - \gamma\tau) - (1 - \tau) - \tau_w \frac{\pi}{L} \right) = \eta b, \quad (26)$$

where there is the new term  $-\tau_w \frac{\pi}{L} > 0$  compared with the case of committed profit sharing. Rewriting of (26) gives the following implicit wage formation equation (see Appendix 5)

$$w = \left[ \frac{\eta}{\eta(1-\gamma\tau) - (1-\tau) - \tau_w \frac{\pi}{L}} \right] b = \left[ \frac{\eta}{\eta(1-\gamma\tau) - (1-\tau) + (1-\tau\gamma)\Gamma} \right] b \quad (27)$$

with  $\Gamma = \frac{wY}{(\delta-1)\gamma(eL) + X\tau^2}$ . If optimal flexible profit sharing is small enough, then  $\Gamma > 0$ , so that in this case in the presence of outsourcing the denominator in (27) is bigger than the one in (10), so that under  $\Gamma > 0$  the mark-up in terms of wage formation is smaller under flexible profit share.

We can summarize this as follows.

**Proposition 5:** *In the presence of outsourcing and flexible profit sharing the base wage formation is smaller than in the case of committed profit sharing if optimal flexible profit sharing is small enough.*

## 6. Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment

We now move on to explore the implications of profit sharing and outsourcing on equilibrium unemployment. Our goal is to characterize the equilibrium unemployment as a function of institutional features of labor market, defined by the benefit replacement ratio, the structure of the compensation system, and the given outsourcing.

The base wage formation by the monopoly labour union has the form

$$w_i = A_i b \quad (28)$$

in industry  $i$ , where the wage mark-up is defined by  $A_i^c = \frac{\eta}{\eta(1-\gamma\tau)-(1-\tau)} > 1$  in the case of committed profit sharing or  $A_i^f = \frac{\eta}{\eta(1-\gamma\tau)-(1-\tau)-(1-\tau)\gamma\Gamma} > 1$  in the flexible case. For simplicity, we focus on the situation with identical industries in terms of the wage mark-up, so that  $A_i = A$ . In a general equilibrium the outside option  $b$  will be re-interpreted to be the relevant outside option. Since we assume identical industries with similar compensation scheme, the outside option can be specified as

$$b = (1-u) \left( w + \tau \frac{\pi}{L} - g(e) \right) + uB, \quad (29)$$

where  $u$  denotes the unemployment rate in the case of committed profit sharing,  $B$  the unemployment benefit, and  $w$  is the base wage formation and an unemployed worker faces the probability  $(1-u)$  of being employed in another industry (for a standard justification we refer e.g. to Nickell and Layard (1999), pp. 3048-3050 and Layard et al. (2005), pp. 100-101).

Equation (29) captures the idea that all identical industries adopt profit sharing, so that an unemployed worker faces the probability  $(1-u)$  of being employed in another industry, which makes use of a similar compensation scheme. We further restrict in these outside options to the case of a constant benefit-replacement ratio  $q = B/w$  in the presence of unemployment, so that  $0 < q < 1$ .

Combining (28) and (29) and the assumption of a constant benefit-replacement ratio,  $q$ , we can rewrite the wage equation (28) as follows

$w = A(1-u)w + A(1-u) \left( \tau \frac{\pi}{L} - g(e) \right) + Auqw$ . The aggregate unemployment rate in the

committed case can now be expressed according to  $u^c = \frac{1 - \frac{1}{A^c} - g(e) + \tau \frac{\pi}{wL}}{1 - q - g(e) + \tau \frac{\pi}{wL}}$ . By

using equation (7) this can be presented in the presence of outsourcing as follows

$$u^c = \frac{1 - \frac{1}{A^c} - \gamma\tau + \frac{\tau}{\delta-1} K}{1 - q - \gamma\tau + \frac{\tau}{\delta-1} K}, \quad (30a)$$

where  $K = 1 + \delta \frac{M}{eL} + (\delta-1) \frac{c(M)}{wL}$  and the assumption is  $q < \frac{1}{A^c}$ . In the absence of outsourcing under the monopoly labour union's wage formation but committed profit sharing in all industries, we have the following equilibrium unemployment

$$u^c|_{M=0} = \frac{1 - \frac{1}{A^c|_{M=0}} - \gamma\tau + \frac{\tau}{\delta-1}}{1 - q - \gamma\tau + \frac{\tau}{\delta-1}}. \quad (30b)$$

Next, we look at the implications on equilibrium unemployment. In the presence of outsourcing differentiating (30a) with respect to outsourcing gives (see Appendix 6)<sup>11</sup>

$$\frac{du^c}{dM} = \left[ 1 - q - \gamma\tau + \frac{\tau}{\delta-1} K \right]^{-2} \left( \underbrace{\frac{dA^c}{dM}}_{?} \left( 1 - q - \gamma\tau + \frac{\tau}{\delta-1} K \right) + \underbrace{\left( \frac{\tau_M K + K_M \tau}{\delta-1} - \gamma\tau_M \right)}_{?} \underbrace{\left( \frac{1}{A^c} - q \right)}_{+} \right) = ? \quad (31)$$

The impact of outsourcing on equilibrium unemployment in this case is a priori ambiguous for the following reasons. Higher outsourcing will lower wage elasticity of labour demand and therefore will decrease the mark-up which will have a negative effect on equilibrium unemployment, but higher outsourcing will also increase profit

<sup>11</sup> Because our former assumption we can show that

$$K_M = \frac{\delta}{(eL)^2} \left[ \underbrace{eL + M}_{+} - \underbrace{(eL_w + Le_w)M}_{+} \frac{dw}{dM} - \underbrace{(eL_\tau + Le_\tau)\tau_M}_{?} \right] +$$

$$\frac{(\delta-1)}{(wL)^2} \left[ \underbrace{c'(M)wL + \frac{c(M)w}{e}}_{+} - \underbrace{c(M)(L(1-\eta)) \frac{dw}{dM}}_{?} + (wL_\tau + Lw_\tau)\tau_M \right] = ?$$



relative to wage costs so that outside option will increase and therefore will have a positive effect on equilibrium unemployment.

In the presence of outsourcing differentiating (30a) with respect to profit sharing gives<sup>12</sup> (see Appendix 6)

$$\frac{du^c}{d\tau} = \left[ 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right]^{-2} \left( \underbrace{\frac{A_\tau^c}{(A^c)^2}}_? \left( 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right) + \underbrace{\left( \frac{\tau}{\delta - 1} K_\tau + \frac{K}{\delta - 1} - \gamma \right)}_? \underbrace{\left[ \frac{1}{A^c} - q \right]}_+ \right) = ? \quad (32a)$$

According to (32a), the impact of profit sharing on equilibrium unemployment in this case is a priori ambiguous for the following reasons. Higher profit sharing will have an ambiguous effect on the mark-up, but higher profit sharing will also increase profit relative to wage costs, so that outside option will increase and therefore will have a positive effect on equilibrium unemployment.

In the absence of outsourcing equation (32a) by using  $K_\tau = 0$  and  $K = 1$  can be re-expressed as

$$\left. \frac{du^c}{d\tau} \right|_{M=0} = \left[ 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} \right]^{-2} \left( \underbrace{\frac{dA^c}{d\tau}}_? \underbrace{\left( 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} \right)}_? + \underbrace{\left( \frac{1}{\delta - 1} - \gamma \right)}_? \underbrace{\left[ \frac{1}{A^c} - q \right]}_+ \right) = ? \quad (32b)$$

which is also ambiguous in terms of equilibrium unemployment.

In the case of flexible profit sharing, the base wage formation by the monopoly labour union has the form as expressed in equation (28) in industry  $i$ ,

where the mark-up is defined by  $A_i^f = \frac{\eta}{\eta(1 - \gamma\tau) - (1 - \tau) - (1 - \tau)\gamma \Gamma}$ , which is smaller

than in the case of committed profit sharing. It is important to emphasize that one

<sup>12</sup> This result holds because  $\frac{dA^c}{d\tau} = A_\tau^c + A_w^c \frac{dw}{d\tau} = ?$  (see equation (15)) and also the derivative

$K_\tau = \frac{-\delta}{(eL)^2} [e_\tau L + L_\tau e] - (\delta - 1) \frac{c(M)}{(wL)^2} [w_\tau L + L_\tau w]$  is a priori ambiguous.

cannot fix the effects of outsourcing and profit sharing on the mark-up due to the new part in the denominator of the mark-up, i.e.  $-(1-\tau)\gamma \Gamma$ . The equilibrium unemployment in the flexible case, when there is also profit sharing in the compensation scheme in other industries, is expressed in (32a)

$$u^f = \frac{1 - \frac{1}{A^f} - \gamma\tau + \frac{\tau}{\delta-1}K}{1 - q - \gamma\tau + \frac{\tau}{\delta-1}K}. \quad (33a)$$

In the absence of outsourcing under the monopoly labour union's wage formation we have the following equilibrium unemployment in the presence of flexible profit sharing in all industries

$$u^f|_{M=0} = \frac{1 - \frac{1}{A^f|_{M=0}} - \gamma\tau + \frac{\tau}{\delta-1}}{1 - q - \gamma\tau + \frac{\tau}{\delta-1}}, \quad (33b)$$

where  $A^f|_{M=0} = \frac{\delta - (\delta-1)\gamma}{[\delta - (\delta-1)\gamma](1-\tau\gamma) - (1-\tau)} > 1$  and  $\Gamma = 0$ . The implication of the

assumption of identical industries gives in the case without outsourcing  $\left. \frac{du^f}{d\tau} \right|_{M=0} = ?$

like in the case of committed profit sharing.

We can now summarize equilibrium unemployment aspects in the presence of outsourcing and profit sharing when labour markets are imperfectly competitive as follows.

**Proposition 6:** *If there is profit sharing as a part of outside option in other industries, outsourcing and profit sharing will have ambiguous effects on equilibrium unemployment under committed profit sharing. In the absence of outsourcing, profit sharing will have an ambiguous effect*

on equilibrium unemployment both under committed and flexible profit sharing.

## 7. Optimal Strategic Outsourcing

So far we have restricted to a medium or short-run perspective where the firm has committed to the magnitude of outsourcing activity prior to wage determination, profit sharing, labour demand and employee effort. Now we turn to explore the initial stage 1, where the firm commits to the outsourcing activity. It is assumed that the long-run production mode decision may internalize the effect of the share of outsourced production on wage formation depending on the time sequence decision of profit sharing.

In the long-run, the firm is assumed to have rational expectations regarding subsequent outcomes and determines the magnitude optimal committed outsourcing so as to maximize profit  $\pi = (1 - \tau) \left( \frac{\delta}{\delta - 1} [eL + M]^{\frac{\delta-1}{\delta}} - wL - c(M) \right)$  subject to labour demand (5) (allowing for the envelope theorem according to  $\pi_L = 0$ ) and effort determination (7). Moreover, in the presence of committed profit sharing profit maximization is also subject to wage formation (10) and profit sharing (19a) (allowing for the envelope theorem according to  $\pi_\tau = 0$ ), while in the presence of flexible profit sharing is also subject to profit sharing (21a) (allowing for the envelope theorem according to  $\pi_\tau = 0$ ) and wage formation (27).

Allowing the envelope theorem both in terms of the optimal profit sharing ( $\pi_\tau = 0$ ) and the optimal labour demand ( $\pi_L = 0$ ), we differentiate  $\pi = \frac{\delta}{\delta - 1} [eL + M]^{\frac{\delta-1}{\delta}} - wL - c(M)$  with respect to  $M$ . Using  $eL = w^{-\delta} e^\delta - M$  and  $wL = w^{1-\delta} e^{\delta-1} - w \frac{M}{e}$ , we can express the profit function as

$$\pi = \frac{1}{\delta - 1} w^{1-\delta} e^{\delta-1} + w \frac{M}{e} - c(M). \quad (34)$$

Differentiating (34) with respect to  $M$  and allowing both its direct effects and the indirect effects via the base wage and the effort determination, gives

$\pi_M = -w^{-\delta} e^{\delta-1} w_M + w^{1-\delta} e^{\delta-2} e_w w_M + \frac{w}{e} + \frac{M}{e} w_M - w \frac{M}{e^2} e_w w_M - c'(M) = 0$ . This can be written as follows  $\pi_M = -w_M \left[ w^{-\delta} e^{\delta-1} - \frac{M}{e} - w^{-\delta} e^{\delta-1} \gamma + \frac{M}{e} \gamma \right] + \frac{w}{e} - c'(M) = 0$ , so that we have the first-order condition

$$\pi_M = \underbrace{\left( \frac{w}{e} - w_M L(1-\gamma) \right)}_+ - c'(M) = 0. \quad (35)$$

The second-order condition is  $\pi_{MM} = \frac{w_M}{e} (2-\gamma) - w_{MM} L(1-\gamma) - c''(M) < 0$ . In addition to the direct marginal cost  $c'(M)$  there is the direct marginal profit  $\frac{w}{e}$  via outsourcing (see equation (35)) and the indirect marginal effects via the effect of outsourcing on wage, i.e.  $-w_M L(1-\gamma)$ . In the presence of committed profit sharing, outsourcing moderates base wage so that the marginal profit will increase via  $-w_M L(1-\gamma) > 0$ . But in the presence of flexible profit sharing the indirect marginal profit  $-w_M L(1-\gamma)$  in terms of outsourcing is a priori ambiguous.

We can summarize this as follows.

**Proposition 7:** *In terms of optimal long-run strategic outsourcing wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.*

## 8. Conclusions

We have analyze the following questions associated with outsourcing and profit sharing under imperfect labour markets by using the scenario without outsourcing:

How does strategic outsourcing, which we assume to be substitute for effective labour, influence wage formation, profit sharing, and employee effort when firms commit to optimal profit sharing before wage formation or decide profit sharing after wage formation. We also have studied the relationship between outsourcing, profit sharing, and equilibrium unemployment. Finally, we have characterized the long-run perspective for the optimal production mode in terms of strategic outsourcing.

We have shown that in the presence of outsourcing the wage elasticity of labour demand depends positively on the amount of outsourcing and on the wage, but negatively on the size of profit sharing. As a result, it has been presented that in the case of committed profit sharing strategic outsourcing has a negative effect on wage formation. This lies in conformity with empirics and results from our assumption of perfect substitutability between outsourcing and effective domestic labour. Under flexible profit sharing the wage is smaller than in the case of committed profit sharing. We also find that the profit share under commitment in the presence of outsourcing is larger (smaller) than that associated with flexibility if the base wage depends negatively (positively) on profit sharing.

If there is profit sharing as a part of outside option in other industries outsourcing, profit sharing will have ambiguous effects on equilibrium unemployment. Also in the absence of outsourcing profit sharing will have an ambiguous effect on equilibrium unemployment. Finally, in terms of optimal long-run strategic outsourcing, wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.

There are several new research topics associated with these issues. We have focused on the case where the firm decides long-term contracts that fix the amount of outsourcing before the labour union sets the wage. Alternatively, the firm may be flexible enough to decide upon the amount of outsourcing activity only after the wage is set by the labour union. Other important issue is to study the implications of labour taxation and labor tax reforms on effort, labour demand, wage formation, profit sharing, and equilibrium unemployment in the presence of outsourcing. Moreover, other topics are to extend the framework to allow for heterogeneity of workers in the domestic country in the presence of outsourcing and to allow for wage negotiations

between labour unions and firms. Finally, it is also important to do numerical simulations and empirical research associated with various results we have presented.

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## APPENDIX 1:

The derivative of labour demand (5) with respect to the base wage is

$$L_w = -\delta w^{-\delta-1} e^{\delta-1} + w^{-\delta} (\delta-1) e^{\delta-2} e_w + \frac{M e_w}{e^2}, \quad \text{so that we have}$$

$$-L_w w = \delta w^{-\delta} e^{\delta-1} - w^{-\delta} e^{\delta-1} (\delta-1) \frac{e_w w}{e} - \frac{M}{e} \frac{e_w w}{e} = w^{-\delta} e^{\delta-1} (\delta(1-\gamma) + \gamma) - \frac{M}{e} \gamma. \quad (\text{A1})$$

This leads to (11). The effect of the base wage on the wage elasticity of labour demand is

$$\eta_w = -\delta(1-\gamma) \frac{M}{(eL)^2} (e_w L + L_w e). \quad (\text{A2})$$

We can write  $e_w L = \frac{\gamma e}{w} \left( w^{-\delta} e^{\delta-1} - \frac{M}{e} \right) = \frac{\gamma}{w} (w^{-\delta} e^{\delta} - M)$  and

$L_w e = -\delta w^{-\delta-1} e^{\delta} + (\delta-1) w^{-\delta-1} e^{\delta} \gamma + \frac{M \gamma}{w}$ . Using  $w^{-\delta} e^{\delta} = eL + M$ , we can show after

calculations that  $e_w L + L_w e = -\left( \frac{\delta(1-\gamma)}{w} \right) (eL + M)$ . Therefore, the total wage elasticity

of labour demand in terms of the base wage in the presence of outsourcing can be expressed in (12b). QED.

## APPENDIX 2:

Differentiating the implicit wage formation (10) with respect to the base wage and outsourcing gives

$$\left( 1 - \frac{[\eta(1-\gamma\tau) - (1-\tau)] \eta_w - \eta(1-\gamma\tau) \eta_w b}{[\eta(1-\gamma\tau) - (1-\tau)]^2} \right) dw =$$

$$\frac{[\eta(1-\gamma\tau) - (1-\tau)] \eta_M - \eta(1-\gamma\tau) \eta_M b}{[\eta(1-\gamma\tau) - (1-\tau)]^2} dM \quad (\text{A3})$$

which can be expressed as

$$\frac{dw}{dM} = - \frac{\frac{\eta_M(1-\tau)}{[\eta(1-\gamma\tau) - (1-\tau)]^2} b}{\left[ 1 + \frac{\eta_w(1-\tau) b}{[\eta(1-\gamma\tau) - (1-\tau)]^2} \right]} < 0. \quad (\text{A4})$$



Using equation (10), i.e.  $b = \frac{w[\eta(1-\gamma\tau)-(1-\tau)]}{\eta}$ , the relationship between the wage formation and outsourcing can be written as equation (14).

Differentiating the equation (10) with the base wage and profit sharing gives

$$\left[1 + \frac{\eta_w(1-\tau)b}{[\eta(1-\gamma\tau)-(1-\tau)]^2}\right] dw = \quad (A5)$$

$$\left(\frac{b}{[\eta(1-\gamma\tau)-(1-\tau)]^2} [(\eta(1-\gamma\tau)-(1-\tau))\eta_\tau - \eta(1-\gamma\tau)\eta_\tau + \eta^2\gamma - \eta]\right) d\tau,$$

which can be expressed as  $\frac{dw}{d\tau} = -\frac{\frac{[\eta_\tau(1-\tau) + \eta(1-\eta\gamma)]b}{[\eta(1-\gamma\tau)-(1-\tau)]^2}}{\left[1 + \frac{\eta_w(1-\tau)b}{[\eta(1-\gamma\tau)-(1-\tau)]^2}\right]}$ , where  $\eta_\tau < 0$  and

$\eta(1-\eta\gamma) > 0$  Using equation (10), i.e.  $b = \frac{w[\eta(1-\gamma\tau)-(1-\tau)]}{\hat{\eta}}$ , the relationship between wage formation and outsourcing can be written as equation (15). QED.

### APPENDIX 3:

Using the first-order condition for profit share commitment given outsourcing, i.e.  $-\pi + (1-\tau)\pi_\tau = 0$ , so that  $\pi_\tau > 0$  and we can rewrite it as follows

$$\frac{1}{\delta-1} [w^{1-\delta} e^{\delta-1}] + \frac{wM}{e} - c(M) = (1-\tau) \frac{wL}{\tau} \left(\gamma - \frac{w_\tau \gamma}{w}\right), \quad \text{which is equivalent to}$$

$$\frac{1}{\delta-1} \left[ wL + \frac{wM}{e} \right] - c(M) = (1-\tau) \frac{wL}{\gamma} \left(\gamma - \frac{w_\tau \gamma}{w}\right). \quad (A6)$$

This can be expressed as  $1 + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} = (1-\tau) \frac{1}{\tau} \left(\gamma - \frac{w_\tau \tau}{w}\right) (\delta-1)$ , so that given outsourcing  $M$  the optimal committed profit sharing can be presented as

$$\tau^c = \frac{\left(\gamma - \frac{w_\tau \tau}{w}\right) (\delta-1)}{\left[1 + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} + \left(\gamma - \frac{w_\tau \tau}{w}\right) (\delta-1)\right]}. \quad (A7)$$

QED.

#### APPENDIX 4:

By differentiating the implicit profit share function (21a) with respect to the profit sharing and the outsourcing gives the following total differential

$$\left( 1 + \frac{(\delta-1)\gamma \left[ -\frac{\delta M}{(eL)^2} (e_\tau L + L_\tau e) + \frac{(\delta-1)c(M)}{w} \frac{L_\tau}{L^2} \right]}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} \right) d\tau^f =$$

$$\frac{(\delta-1)\gamma \left[ \frac{M}{e} \left( \frac{L - ML_M}{L^2} \right) - \frac{(\delta-1)}{w} \left( \frac{c'(M)L - L_M c(M)}{L^2} \right) \right]}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} dM. \quad (\text{A8})$$

By using  $e_\tau = \frac{\gamma}{\tau} e$ ,  $L_\tau = (\delta-1)w^{-\delta} e^{\delta-1} \frac{\gamma}{\tau} + \frac{M}{e} \frac{\gamma}{\tau}$  and  $L_M = -1/e$  equation (A8) can

be re-expressed as

$$\left( 1 + \frac{(\delta-1)\gamma / eL \left[ -\delta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) + (\delta-1) \frac{c(M)}{wL} \frac{\gamma}{\tau} ((\gamma-1)eL + \delta M) \right]}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} \right) d\tau^f =$$

$$\frac{(\delta-1)\gamma / eL \left[ \delta \left( 1 + \frac{M}{eL} \right) - (\delta-1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right]}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} dM, \quad (\text{A9})$$

which gives

$$\frac{d\tau^f}{dM} = \frac{-\frac{1}{eL} \left[ \frac{(\delta-1)\gamma Z}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} \right]}{\left[ 1 + \frac{\frac{1}{eL} (\delta-1)\gamma X}{\left[ 1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]^2} \right]^2} \quad (\text{A10})$$

In (A10)  $X = -\delta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) + (\delta - 1) \frac{c(M)}{wL} \frac{\gamma}{\tau} ((\delta - 1)eL + \delta M)$ , which we re-write,

so that holds  $X = -\frac{\gamma}{\tau} \left[ \delta \left( \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} \right) (eL + M) - (\delta - 1) \frac{c(M)}{wL} (eL) \right] < 0$ . We

specify the parameter  $Z$  as  $Z = \delta \left( 1 + \frac{M}{eL} \right) - (\delta - 1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) > 0$ , which is

equivalent to  $\delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} + \delta - (\delta - 1) \frac{c'(M)e}{w} > 0$ . Using equation (21a), i.e.

$1 + (\delta - 1)\gamma + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} = \frac{(\delta - 1)\gamma}{\tau}$ , we can re-express (A10) as (22).

By differentiating the implicit profit share function (21a) with respect to the profit sharing and the base wage gives the following total differential

$$\left( \frac{(\delta - 1)\gamma \left[ -\frac{\delta M}{(eL)^2} (e_\tau L + L_\tau e) + \frac{(\delta - 1) c(M)}{w} \frac{L_\tau}{L^2} \right]}{1 + \frac{(\delta - 1)\gamma + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL}}{\tau}} \right) d\tau^f = \frac{(\delta - 1)\gamma \left[ \delta \frac{-M}{(eL)^2} (e_w L + L_w e) + (\delta - 1) \frac{c(M)}{(wL)^2} (L + L_w w) \right]}{\left[ 1 + (\delta - 1)\gamma + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} \right]^2} dw. \quad (\text{A11})$$

By using  $e_\tau = \frac{\gamma}{\tau} e$ ,  $L_\tau = (\delta - 1)w^{-\delta} e^{\delta-1} \frac{\gamma}{\tau} + \frac{M}{e} \frac{\gamma}{\tau}$ ,  $e_w = \frac{\gamma}{w} e$  and

$L_w = w^{-\delta-1} e^{\delta-1} [(\delta - 1)\gamma - \delta] + \frac{M}{e} \frac{\gamma}{w}$ , equation (A11) can be re-expressed as

$$\left( \frac{(\delta - 1)\gamma / eL \left[ -\delta^2 \frac{M}{eL} \frac{\gamma}{\tau w} (eL + M) + (\delta - 1) \frac{c(M)}{w} \frac{e}{w} (1 - \eta) \right]}{1 + \frac{(\delta - 1)\gamma + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL}}{\tau}} \right) d\tau^f = \frac{(\delta - 1)\gamma / eL \left[ (1 - \gamma) \delta^2 \left( 1 + \frac{M}{eL} \right) - (\delta - 1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right]}{\left[ 1 + (\delta - 1)\gamma + \delta \frac{M}{eL} - (\delta - 1) \frac{c(M)}{wL} \right]^2} dw, \quad (\text{A12})$$

which gives

$$\frac{d\tau^f}{dw} = \frac{-\frac{1}{eL} \left[ \frac{(\delta-1)\gamma Y}{\left[1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL}\right]^2} \right]}{\left[ 1 + \frac{\frac{1}{eL}(\delta-1)\gamma X}{\left[1 + (\delta-1)\gamma + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL}\right]^2} \right]^2} \quad (\text{A13})$$

In (A13)  $Y = (1-\gamma)\delta^2 \frac{M}{eL} \frac{\gamma}{\tau w} (eL+M) + (\delta-1) \frac{c(M)}{w} \frac{e}{w} (1-\eta)$ . For  $Y > 0$ , which means that  $(1-\gamma)\delta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL+M) > \frac{c(M)e}{w} (\delta-1)(\eta-1)$  where  $\eta-1 = (1-\gamma) \left( \delta-1 + \delta \frac{M}{eL} \right)$ , so that  $\delta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL+M) > \frac{c(M)e}{w} (\delta-1) \left( \delta-1 + \delta \frac{M}{eL} \right)$ . Using (21a) we get (23). QED.

#### APPENDIX 5:

According to equation (23),  $\frac{d\tau^f}{dw} = \tau_w^f = -\frac{Y \tau^2}{(\delta-1)\gamma(eL) + X \tau^2} < 0$ , so that  $-\tau_w \frac{\pi}{L} > 0$ ,

where  $\frac{\pi}{L} = \frac{w}{(\delta-1)} \left[ 1 + \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right]$ . Now using equation (21a) we have

$\frac{\pi}{L} = \frac{w\gamma(1-\tau)}{\tau}$ , so that we can re-express

$$-\tau_w \frac{\pi}{L} = \frac{w(1-\tau)\gamma Y}{(\delta-1)\gamma(eL) + X\tau^2}, \quad (\text{A14})$$

where  $Y = (1-\gamma)\delta^2 \frac{M}{eL} \frac{\gamma}{\tau w} (eL+M) + (\delta-1) \frac{c(M)}{w} \frac{e}{w} (1-\eta) > 0$  and

$$X = -\frac{\gamma}{\tau} \left[ \delta \left( \delta \frac{M}{eL} - (\delta-1) \frac{c(M)}{wL} \right) (eL+M) - (\delta-1) \frac{c(M)}{wL} (eL) \right] < 0. \quad \text{QED.}$$

#### APPENDIX 6:

In the presence of outsourcing differentiating (31a) with respect to  $M$  gives -

by using the notation  $K = 1 + \delta \frac{M}{eL} + (\delta - 1) \frac{c(M)}{wL}$ , so that  $u^c = \frac{1 - \frac{1}{A^c} - \gamma\tau + \frac{\tau}{\delta - 1} K}{1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K}$ .

$$\frac{du^c}{dM} = \left[ 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right]^{-2} \left( \underbrace{\frac{dA^c}{(A^c)^2}}_{?} \left( 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right) - \underbrace{\left( \frac{K_M \tau + \tau_M K}{\delta - 1} - \tau_M \gamma \right)}_{?} \underbrace{\left( \frac{1}{A^c} - q \right)}_{+} \right) = ? \quad (\text{A15})$$

where  $\frac{dA^c}{dM} = A_M^c + A_w^c \frac{dw}{dM} + A_\tau^c \tau_M = ?$ ,  $\frac{d\tau}{dM} = \tau_M = ?$  (see equation (19a) and

$$K_M = \frac{\delta}{(eL)^2} \left[ \underbrace{eL + M}_{+} - \underbrace{(eL_w + Le_w)M}_{+} \frac{dw}{dM} - \underbrace{(eL_\tau + Le_\tau)\tau_M}_{?} \right] +$$

$$\frac{(\delta - 1)}{(wL)^2} \left[ \underbrace{c'(M)wL + \frac{c(M)w}{e}}_{+} - \underbrace{c(M) \left( L(1 - \eta) \frac{dw}{dM} + (wL_\tau + Lw_\tau)\tau_M \right)}_{?} \right] = ?$$

In the presence of outsourcing differentiating (31a) with respect to profit sharing gives

$$\frac{du^c}{d\tau} = \left[ 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right]^{-2} \left( \left( 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right) \left( \frac{dA^c}{(A^c)^2} + \frac{\tau}{\delta - 1} K_\tau + \frac{K}{\delta - 1} - \gamma \right) - \left( 1 - \frac{1}{A^c} - \gamma\tau + \frac{\tau}{\delta - 1} K \right) \left( \frac{\tau}{\delta - 1} K_\tau + \frac{K}{\delta - 1} - \gamma \right) \right) =$$

$$\left[ 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right]^{-2} \left( \frac{dA^c}{(A^c)^2} \left( 1 - q - \gamma\tau + \frac{\tau}{\delta - 1} K \right) + \left( \frac{\tau}{\delta - 1} K_\tau + \frac{K}{\delta - 1} - \gamma \right) \left( \frac{1}{A^c} - q \right) \right) = ?, \quad (\text{A16})$$

where  $\frac{dA^c}{d\tau} = A_\tau^c + A_w^c \frac{dw}{d\tau} = ?$  (see equation (15)) and also the derivative

$$K_\tau = \frac{-\delta}{(eL)^2} [e_\tau L + L_\tau e] - (\delta - 1) \frac{c(M)}{(wL)^2} [w_\tau L + L_\tau w] = ? \text{ QED.}$$