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STRUCTURAL UNCERTAINTY AND MONETARY STABILIZATION POLICY

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ABSTRACT

The article examines what the presence of structural uncertainty implies for the formulation of monetary stabilization policy in rational-expectations models of the Sargent-Wallace type. The policy ineffectiveness proposition of Sargent and Wallace is interpretted as a special case of Theil's certainty equivalence theorem. This theorem breaks down when there is structural uncertainty of the multiplicative-disturbance variety. Thereby a rationale for monetary stabilization policy under rational expectations is provided.

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1. Introduction

This article examines the role of monetary stabilization policy under rational expectations and structural uncertainty. The policy-ineffectiveness proposition of Sargent and Wallace (1975) is viewed as a special case of Theil's (1958) "certainty equivalence theorem". Brainard (1967) has demonstrated that this theorem breaks down in the presence of multiplicative disturbances. The "structural uncertainty" considered here is of the multiplicative-disturbance type and Brainard's analysis is shown to provide a rationale for monetary stabilization policy.

The policy-ineffectiveness proposition holds for macroeconomic models with the following properties: (i) the natural rate hypothesis[1] holds (e.g. the level of national product depends on errors in price expectations), (ii) economic agents have rational expectations (e.g., the price level anticipated by the public is equal to the mathematical expectation of the price level), (iii) all equations are linear, and (iv) all random disturbances are additive. In other words, systematic monetary policy leaves the distribution of real economic variables unaffected in models of this sort (which may be called models of the "Sargent-Wallace type"). In the non-stochative counterparts of these models, money is neutral and thus monetary policy has no effect on real variables either.

The literature on stabilization policies covers both "descriptive" and "optimizing" analyses. The former are con-

cerned with the implications of exogenously given and ad hoc policy rules (relating control variables variables to state variables) for the achievement of policy-makers' objectives (specified in terms of desired levels of state, and possibly also control, variables). In their nondeterministic guise, these analyses assume that the parameters of the model are nonstochastic but unknown to the policy maker (e.g. Cooper (1969), Mundell (1969) and Phillips (1954)). The optimizing analyses are commonly concerned with the optimization of a quadratic objective function (specified in terms of deviations of state and control variables from their respective desired levels) subject to constraints relating the state variables to the control variables. It is the optimizing analyses which are under consideration here.

Theil's certainty equivalence theorem falls within the purview of the optimizing analyses. According to this theorem, the optimal values of the control variables (and for that matter, also the state variables) are the same for a stochative linear system with only additive disturbances as for its determistic counterpart. This theorem may be applied to macroeconomic models of the Sargent-Wallace type. Suppose that the aim of systematic monetary policy is to minimize the expected aquared deviations of national product from "natural" level (at which the actual and expected price levels are equal) subject to the behavioral relations of a Sargent-Wallace-type model. Since this policy is neutral in the deterministic formulation of this model, (in which rational expectations reduce to perfect foresight) it remains neutral

once additive disturbances are included. In this sense, the policy-ineffectiveness proposition may be understood or an implication of the certainty-equivalence theorem.

In his classic article, Brainard (1967) has shown that the certainty equivalence theorem does not hold when the model contains multiplicative distrubances, i.e. when there is "structural uncertainty". In other words, the optimal policy under structural uncertainty (viz, stochastic coefficients) is not necessarily identical to that under additive risk. This article extends Brainard's result to models of the Sargent-Wallace type and thereby provides an argument for the non-neutrality of systematic monetary policy. With reference to several illustrative models, it examines what the presence of structural uncertainty implies for the formulation of optimal monetary stabilization policy.

This is the task of Section 2. Section 3 summarizes the message of this article.

2. Models of Structural Uncertainty

As noted, the monetary authority is assumed to attempt minimization of the expected aquared deviations of national product from its natural level. To set the stage, let us first show that systematic monetary policy is irrelevant to these objectives whenever all the random disturbance terms enter the model additively. Then we examine how multiplicative disturbance terms provide scope for stabilization policy through systematic monetary rules. Lastly we show that the existence of multiplicative disturbances does not guarantee the effectiveness of systematic monetary policy.

2 a. Model I.

Models of the Sargent-Wallace type are generally composed of four building blocks:

(1) an aggregate product supply function, which embodies the natural rate hypothesis,

(2) an aggregate product demand function, which may be reduced to a relation among the actual price level, the expected price level, and the money supply,

(3) a money supply function, which portrays the systematic and unsystematic components of the money supply, and

(4) a definition of rational price expectations.

To demonstrate the policy-ineffectiviness proposition, let us represent these four elements by four equations

containing only additive disturbances. All variables are expressed in logarithms. Let Q be national product (viz, the logarithm of national product), P the actual product price level at time t, P_e the price level at time t anticipated by the public at time t, and u^s a product-supply disturbance term. Then the aggregate product supply function may be written as

$$(1) \quad Q = a\,[P - P_e] + u^s$$

u^s is a random variable; its mean, QN (the natural rate of production), and its variance are both assumed constant:

$$E(u^s) = QN$$
$$\text{var}(u^s) = \sigma_s^2$$

The coefficient "a" is a positive constant. The model is static, thus, the time subscripts are suppressed.

Let M be the money supply and u^D a product-demand disturbance term. Then a simple reduced form of the aggregate product demand function is

$$(2) \quad P = M + u^D$$

u^D is a random variable with constant mean and variance:

$$E(u^D) = \bar{u}^D$$
$$\text{var}(u^D) = \sigma_D^2$$

This function differs from the one commonly found in the

literature on policy ineffectiveness. The usual function is actually a product market clearing condition, whose demand side is obtained from an IS and LM curve and whose supply side is given by the natural rate hypothesis. Equation (2) is chosen for algebraic simplicity. The central conclusion of this note (viz. that systematic monetary policy may be effective in the presence of stochastic coefficients) is not affected by our choice.

For simplicity, but without loss of generality, let the systematic money supply be a constant, $\bar{M}$ (rather than a predictable function of the endogenous variables of the model). Let u^M be a random money-supply disturbance term. Then the money supply function may be expressed as

$$(3) \quad M = \bar{M} + u^M$$

The mean of u^m is zero; its variance is constant:

$$E(u^M) = 0,$$

$$\text{var } (u^M) = \sigma_M^2$$

One version of the rational expectations hypothesis is

$$(4) \quad P_e = E\ (P \mid I)$$

Where I is the public's information set at time t. In other words, the price level (at time t) anticipated by the public (at time t) is equal to the mathematical expec-

tation (at time t) of the actual price level (at time t) conditional on I (at time t).

The policy-ineffectiveness argument emerges straightforwardly from Equations (1) - (4). By Equation (2) and (3),

$$(5) \quad P = \bar{M} + u^m + u^D$$

and thus, by Equation (4),

$$(6) \quad P_e = \bar{M}$$

Substituting Equations (5) and (6) into (1), we find

$$(7) \quad Q = a\,[u^m + u^D] + u^s$$

From this equation it is evident that the systematic money supply has no effect on the distribution of Q. Hence, systematic monetary policy cannot be used as a stabilization policy device.

2 b. Models II

To show that systematic monetary policy may require a stabilization role when there is structural uncertainty, let us reformulate the aggregate product demand function, Equation (2), in multiplicative form:[2]

$$(8) \quad P = M \cdot u^D$$

Substituting Equation (3) into (8) and taking the mathematical expectation, we obtain the anticipated price level:

$$(9) \quad P_e = \bar{M} \cdot \bar{u}^D + \bar{u}^M \cdot \bar{u}^D + \sigma_{MD}$$

where δ_{MD} is the covariance of u^M and u^D, which we assume to be constant. Substituting Equations (8) and (9) into (1), we obtain

$$(10) \quad Q = a \cdot (u^D - \bar{u}^D) \cdot \bar{M} + a \cdot [u^M \cdot u^D - \bar{u}^M \cdot \bar{u}^D - \sigma_{MD}] + u^S$$

Here the systematic money supply has a role to play in stabilizing national product. To illustrate this role, let the monetary authority have the following stabilization policy objetives:

$$(11) \quad \text{Minimize } E\,[(Q - QN)^2]$$

subject to $Q = a \cdot (u^D - \bar{u}^D) \cdot \bar{M} + a \cdot [u^M \cdot u^D - \bar{u}^M \cdot \bar{u}^D - \sigma_{MD}] + u^S$

Substituting the constraint into the objective function, taking the expectation, and minimizing with respect to $\bar{M}$, we obtain the optimal level of the money supply[3,4]:

$$(12) \quad \bar{M}^* = - \frac{\sigma_{DS} + a \cdot \sigma_{D\eta}}{a \cdot \sigma_D^2}$$

where σ_{DS} is the covariance of u^D an u^S (assumed constant), $\eta = u^M \cdot u^D$, and $\sigma_{D\eta}$ is the covariance of u^D and η (assumed constant). In other words, the optimal money supply is inversely related to the covariance of u^D and u^S and the covariance of u^D and η; it is positively related to the variance of u^D.

The reason why the optimal money supply is unrespon-

sive to the authority's choice of production target is that systematic monetary policy cannot affect the mean of the distribution of Q. To derive this result substitute Equation (12) into (10) and take the expectation of Q:

$$(13) \quad E(Q) = QN$$

A different money supply rule emerges if the multiplicative disturbance occurs in the money supply function, rather than in product demand function:[5]

$$(14) \quad M = \bar{M} \cdot u^M$$

where it is assumed that $E(u^M) = \bar{u}^M$, a constant which may differ from zero.

Thus, the macroeconomic model under consideration comprises Equations (1), (2), (14), and (4). Substituting (14) into (2), we get

$$(15) \quad P = \bar{M} \cdot u^M + u^D,$$

and taking the expectation, we find the anticipated price level:

$$(16) \quad P_e = \bar{M} \cdot \bar{u}^M + \bar{u}^D$$

Inserting (15) and (16) into (1), we obtain

$$(17) \quad Q = a \cdot (u^M - \bar{u}^M) \cdot \bar{M} + a \cdot (u^D - \bar{u}^D) + u^S$$

Maximizing the monetary authority's objective function

subject to constraint (17), we obtain the following money supply rule:

$$(18) \qquad \bar{M}^* = - \frac{\sigma_{MS} + a \cdot \sigma_{MD}}{a \cdot \sigma_M^2},$$

where σ_{MD} is the covariance of u^M and u^D and σ_{MS} is the covariance of u^M and u^S (both covariances assumed constant.

Needless to say, the optimal money supply rules (12) and (18) are closely related. The role of the random variable u^D in the former rule is adopted by u^M in the latter. Thus, whereas in rule (12) the optimal money supply depends inversely on the variance of u^D and politively on the covariance of u^D and u^S, in rule (18) it depends inversely on the variance of u^M and positively on the covariance of u^M and u^S. Furthermore, the role of $u^M \cdot u^D = \eta$ in the former rule is adopted by u^D in the latter. Thus, in the former rule the optimal money supply is inversely related to the covariance of u^D and η, while in the latter rule it is inversely related to the covariance of u^M and u^D.

2 c. Model III.

Thus far, the inclusion of a multiplicative disturbance term in our macroeconomic models turned systematic monetary policy into an effective stabilization device. However, the existence of multiplicative disturbances is <u>not</u> a

sufficient condition for the effectiveness of monetary rules. To demonstrate this negative result, suppose that the multiplicative disturbance occurs in the product supply function (rather than in the product demand of money supply functions). In particular, assume that the coefficient "a" (in Equation (1)) is a random variable with constant mean and variance:

$$E(a) = \bar{a} > 0$$

$$\text{var}(a) = \sigma_a^2.$$

With this modification, our new macroeconomic model comprises Equations (1)- (4). The actual and anticipated price levels are given by Equations (5) and (6). National product is given by Equation (7). Here it is apparent that systematic monetary policy has no influence on production.

3. Conclusions

The primary message of this paper is that, even if the rational expectations hypothesis and the natural rate hypothesis both hold, systematic monetary rules may nevertheless have a role to play in stabilization policy, provided that the linear macroeconomic model contains stochastic coefficients. Whereas the absence of stochastic coefficients guarantees the ineffectiveness of systematic monetary policy, the presence of these coefficients does not ensure that such policy is effective.

Footnotes

1. Here the natural-rate hypothesis is interpreted narrowly: production is assumed to depend solely on the difference between the actual and the expected price level. If the natural rate hypothesis is amended to include the effects of capital accumulation on production (as in Fischer (1979)) or the effects of the real interest rate on production (as in Fair (1978)) and if there is a real money balance effect on consumption, then changes in money supply rules can influence real variables. In accordance with the certainty equivalence theorem, this result holds regardless of whether the model is deterministic or contains additive disturbances.

2. This case implies ex post (but not ex ante) non-neutrality of money. Thereby the systematic monetary policy is able to influence the variance of national product.

3. Recall that $\bar{M}$ is the logarithm of the optimal systematic money supply. Thus, Equation (12) does not violate a nonnegativity condition on the absolute level of the money supply.

4. Note that the case for monetary stabilization policy does not hinge on the existence of nonzero covariances between u^D and u^S and between u^D and η.

5. This case implies ex post (but not ex ante) non-separability of systematic and unsystematic monetary policy. In other words, variations in the unsystematic component of the money supply influences the effect of the systematic component on national product.

6. The origin of these regularities may be clarified by rewriting Equation (10) as

$$Q = a\cdot(u^D - \bar{u}^D)\cdot\bar{M} + a\cdot\ [u^M\cdot u^D - E(u^M\cdot u^D)] + u^S$$

and comparing it with Equation (17).

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