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Economides, George; Philippopoulos, Apostolis; Vassilatos, Vanghelis

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Public providers, versus private providers, of public goods: A general equilibrium study of the role of the state

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Public Providers, versus Private Providers, of Public Goods: A General Equilibrium Study of the Role of the State

George Economides
Apostolis Philippopoulos
Vangelis Vassilatos

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Abstract

This paper studies the difference between public production and public finance of public goods in a dynamic general equilibrium setup. By public finance, we mean that the public good is produced by private providers with the government financing their costs. When the model is calibrated to match fiscal data from the UK economy, the main result is that, ceteris paribus, a switch from public production to public finance can have substantial aggregate and distributional implications. Public providers cannot beat private providers in terms of aggregate efficiency. We finally design a transfer scheme that can make a switch to private provision welfare improving for all agents including public employees.

JEL-Code: H400, D900, D600.

Keywords: public goods, growth, welfare.

George Economides
Athens University of Economics and
Business / Department of International &
European Economic Studies
76 Patission street
Greece – Athens 10434
gecon@aueb.gr

Apostolis Philippopoulos
Department of Economics
Athens University of Economics and
Business
76 Patission street
Greece – Athens 10434
aphil@aueb.gr

Vangelis Vassilatos Athens University of Economics and Business 76 Patission street Greece – Athens 10434 vvassila@aueb.gr

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1. Introduction

Concerning the provision of public goods and services, an important distinction is between public production and public finance. In the former case, the goods are produced by the government itself. In particular, the government hires public employees and purchases final goods from the private sector to produce public goods and services. In the latter case, public goods and services are produced by private firms, the so-called private providers, with the government financing the cost of production of an agreed-upon quantity. Examples of goods and services that can belong to either category include public hospitals, public television and radio, public schools, environmental protection, services provided by local authorities, etc.

The issue of public goods provision has attracted increasing interest in both academic and policy circles. In academia, production and finance are two distinct ways of public goods provision. For instance, Atkinson and Stiglitz (1980, p. 482) emphasize that "the two are often confused, though both logically and in practice they are distinct". In policy, there is a big debate nowadays on the role of the state and, in particular, the idea of opening up public services to new providers. In the UK, for instance, reforms are designed to encourage "any qualified provider of public goods" (*The Observer*, 22.05.2011, p. 7) and "across much of the public sector, from health and education to local authorities and prisoner rehabilitation, the provision of public services is increasingly being contracted out to private suppliers" (*The Economist*, January 22nd, 2011, p. 41). But, at the same time, the British Deputy PM, Nick Clegg, questions private sector involvement saying that the real issue is about "diversifying providers" and that this does not extend to a belief "that private providers are inherently better than public-sector providers" (*The Guardian*, 10 February 2011, p. 15).

What are the implications of switching from public production to public finance? Can public providers beat private providers? Is this switch good for the general interest and bad for public employees? If yes, is there a mix of reforms that can be good for both private and public employees?

The present paper tries to answer these questions. To the best of our knowledge, so far there has not been an attempt to study, and quantify, the differences between public production and public finance in a dynamic general equilibrium setup. We fill this gap by studying issues of both efficiency and redistribution, where efficiency has to do with per

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¹ There is a rich taxonomy of public goods and services depending on the way of provision, financing and distribution. See e.g. Cullis and Jones (1998, chapter 5).

capita output and welfare, while redistribution refers to differences in income and welfare between private and public employees.

We build upon the neoclassical growth model. We first model the case of public production. Following e.g. Finn (1988), Ardagna (2007) and Pappa (2009), there are two distinct groups of households: those that work in the private sector and those that are employed in the public sector. The latter (called public employees), together with goods purchased from the private sector, are used as inputs in the government production function. Calibrating the model to match tax-spending data in the UK economy over 1990-2010, we solve it and specify, among other variables, the time-path of public goods as induced by the existing tax-spending policy mix. Then, using this status quo solution as a point of reference, we study what would change if, other things equal, the same time-path of public goods is produced by private firms, the so-called private providers. These firms produce the amount of public goods ordered by the government by solving a cost-minimization problem with the government financing their total cost.² We also study the case in which the same amount of public goods continues to be produced by the public sector but now, again other things equal, public firms minimize their costs like their private counterparts do in the case of public finance. These three model economies (namely, the status quo one, the one with costminimizing private providers and the one with cost-minimizing public providers) are directly comparable. In all experiments, the amount of public goods produced, and the number of households employed in the production of public goods, remain the same across regimes.

There are four main results. First, the switch from the status quo economy to an economy with cost-minimizing private providers increases per capita welfare at the cost of making public employees worse off. The latter happens because the wages of those involved in the production of public goods falls when it is private providers that supply these goods. Second, the effect of this switch on per capita output depends on the method of public financing. When the efficiency savings enjoyed by the government, coming from a more efficient way of delivering the public good, are translated into reductions in distorting taxes, then per capita output also rises. Actually, in our experiments, when the government uses the efficiency savings to cut labour taxes, output rises substantially. Third, when we assume that public providers choose inputs in a cost-minimizing way, the solution is very similar to the status quo case where the associated variables are exogenously set at their data averages.

² As the British Deputy PM, Nick Clegg, has said "there will be no for profit-providers in our publicly funded ... system" (*The Guardian*, 10 February, 2011, p. 5).

This could mean that in the UK, over 1990-2010, the public sector has exhausted its role in terms of aggregate efficiency as a provider of public goods and services. Fourth, since there are aggregate efficiency gains from switching to private providers, we show that the society can design a simple redistributive scheme that makes everybody better off, including those previously employed in the public sector.

We wish to clarify two things at the outset. First, we focus on polar cases. In the status quo economy, we assume that there is public production only. But we are aware that actually some public services have been contracted out to private suppliers already. By contrast, in the reformed economy, we assume that there are private providers only with the government just financing their costs. But we are aware that some public production is always desirable (e.g. police and courts). In any case, our main results are not expected to be affected by the presence, or not, of such public goods; one could take them as given, and then compare public production versus public finance of the remaining public goods. Second, here we do not take a stance on the optimal amount of public goods. We just take the size/mix of public spending, the share of public employment and tax rates as in the data and compute the induced amount of public goods by using a relatively standard dynamic stochastic general equilibrium model. In turn, we ask what would have happened in the case in which the same amount of public goods, socially optimal or not, was supplied by cost-minimizing firms.³

The rest of the paper is organized as follows. Section 2 models the case of public production. Section 3 models the case of public finance. Their comparison is in Section 4. Section 5 asks whether public providers can beat private providers. Section 6 looks for Pareto improving policies. Section 7 closes the paper.

2. An economy with public production of public goods

We build on the neoclassical growth model. Consider a two-sector general equilibrium model in which private firms choose capital and labor supplied by private employees to produce a private good, while the government purchases part of the private good and hires public employees to produce a public good. The latter provides utility-enhancing services to all households. The private good is converted into the public good by a production function

³ This is consistent with the Mirrlees Review in the UK (Mirrlees *et al* 2010, 2011) that also takes public spending as given and looks at the efficiency of the tax system. Here, we look at the efficiency of the system of public goods provision.

so that each can be expressed in the same units. Throughout the paper, we assume away user charges for the public good. To finance total public spending, including the cost of the public good, the government levies distorting taxes and issues bonds.

There are four agents in the economy: households that work in the private sector (called private employees), households that work in the public sector (called public employees), private firms that produce a private good and the government. All households can participate in capital and bond markets. Our model in this section is similar to that used by most of the related literature (see e.g. Finn, 1998, Cavallo, 2005, Ardagna, 2007, Pappa, 2009, Linnemann, 2009, Forni *et al*, 2009, and Fernández-de-Córdoba, 2010), in the sense that the roles of private and public employees are distinct, there is no labor mobility between the private and public sector and economic policy is exogenous.⁴

Concerning population sizes, there are $p=1,2,...,N_t^p$ identical households that work in the private sector and $b=1,2,...,N_t^b$ identical households that work in the public sector, where $N_t^b+N_t^p=N_t$ at each t. There are also $f=1,2,...,N_t^f$ identical private firms. The number of private firms equals the number of households that work in the private sector, $N_t^f=N_t^p$, or equivalently each household employed in the private sector owns one private firm. We assume that population sizes N_t , N_t^b or, equivalently, the population share $v_t^b\equiv \frac{N_t^b}{N_t}$ is exogenous (this is defined below).

2.1 Households that work in the private sector

The expected lifetime utility of each household $p = 1, 2, ..., N_t^p$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^p, e_t^p; \overline{Y}_t^g) \tag{1}$$

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⁴ Fernández-de-Córdoba *et al* (2010) provide a survey of this literature. On the other hand, see Quadrini and Trigari (2008) for job search and matching mechanisms.

where C_t^p and e_t^p are respectively p's private consumption and hours worked, \overline{Y}_t^g is per capita public goods and services, E_0 is an expectations operator and $0 < \beta < 1$ is a time preference parameter.

The period utility function is (see also e.g. Fernández-Villaverde, 2010):

$$u(C_{t}^{p}, u_{t}^{p}; \overline{Y}_{t}^{g}) = \log(C_{t}^{p} + \psi \overline{Y}_{t}^{g}) - \mu \frac{(e_{t}^{p})^{1+\xi}}{1+\xi}$$
(2)

where ψ , μ and $\xi > 0$ are standard preference parameters. Thus, $C_t^p + \psi \overline{Y}_t^g$ is composite consumption and public services influence private utility through the parameter ψ (see e.g. Christiano and Eichenbaum, 1992).

Each household p enters period t with predetermined holdings of physical capital and government bonds, K_t^p and B_t^p , whose returns are r_t and ρ_t respectively. The within-period budget constraint of each p is:

$$(1+\tau_t^c)C_t^p + I_t^p + D_t^p = (1-\tau_t^k)(r_t K_t^p + \pi_t^p) + (1-\tau_t^l)w_t^p e_t^p + \rho_t B_t^p + \overline{G}_t^{tr,p}$$
(3a)

where

$$I_{t}^{p} = K_{t+1}^{p} - (1 - \delta^{k}) K_{t}^{p}$$
(3b)

$$D_t^p = B_{t+1}^p - B_t^p \tag{3c}$$

and I_t^p is savings in the form of capital, D_t^p is savings in the form of government bonds, π_t^p is dividends received from private firms, w_t^p is the wage rate in the private sector, $\overline{G}_t^{tr,p}$ is government transfers to each p, $0 < \tau_t^k$, τ_t^l , $\tau_t^c < 1$ are tax rates on capital income, labor income and consumption respectively, and $0 < \delta^k < 1$ is the capital depreciation rate.

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⁵ Thus, $\overline{Y}_t^g \equiv \frac{Y_t^g}{N_{\star}}$.

⁶ We assume that only private employees receive dividends from private firms (see (3a) and (6a) below). This is unimportant because, for simplicity, there are no profits in equilibrium. This is throughout the paper.

Each p chooses $\{C_t^p, K_{t+1}^p, B_{t+1}^p, e_t^p\}_{t=0}^{\infty}$ taking prices and policy as given. The standard first-order conditions are written in Appendix A.

2.2 Households that work in the public sector (public employees)

Public employees are modeled similarly to private employees. Thus, the expected lifetime utility of each $b = 1, 2, ..., N_t^b$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^b, e_t^b; \overline{Y}_t^g) \tag{4}$$

where

$$u(C_t^b, u_t^b; \overline{Y}_t^g) = \log(C_t^b + \psi \overline{Y}_t^g) - \mu \frac{(e_t^b)^{1+\xi}}{1+\xi}$$
(5)

The within-period budget constraint of each b is:

$$(1+\tau_t^c)C_t^b + I_t^b + D_t^b = (1-\tau_t^k)r_tK_t^b + (1-\tau_t^l)w_t^g e_t^b + \rho_t B_t^b + \overline{G}_t^{tr,b}$$
(6a)

where

$$I_{t}^{b} = K_{t+1}^{b} - (1 - \delta^{k}) K_{t}^{b}$$
(6b)

$$D_{t}^{b} = B_{t+1}^{b} - B_{t}^{b} \tag{6c}$$

and w_t^g is the wage rate in the public sector and $\overline{G}_t^{tr,b}$ is government transfers to each b.

Each b chooses $\{C_t^b, K_{t+1}^b, B_{t+1}^b, e_t^b\}_{t=0}^{\infty}$ taking prices and policy as given. The first-order conditions are as in Appendix A if we replace superscript p with superscript b.

2.3 Firms in the private sector

In each period, each private firm $f = 1, 2, ..., N_t^f$ chooses capital and labor inputs, K_t^f and e_t^f , to maximize profits given by:

$$\pi_t^f = Y_t^f - r_t K_t^f - w_t^p e_t^f \tag{7}$$

where output, Y_t^f , is produced by a CRS Cobb-Douglas function:

$$Y_t^f = A_t (K_t^f)^\alpha (e_t^f)^{1-\alpha}$$
(8)

where $0 < \alpha < 1$ is a parameter and A_t is total factor productivity (this is defined below).

Each f chooses K_t^f and e_t^f taking prices as given. The standard first-order conditions of this static problem are written in Appendix B.

2.4 Government budget constraint

The within-period budget constraint of the government is (in aggregate terms):

$$G_t^g + G_t^w + G_t^{tr,p} + G_t^{tr,b} + (1 + \rho_t)B_t = B_{t+1} + T_t$$
(9a)

where G_t^g is total public spending on goods and services purchased from the private sector, G_t^w is total public wage payments, $G_t^{tr,p}$ is transfers to all households working in the private sector, $G_t^{tr,b}$ is transfers to all public employees, B_t^{r} is the beginning-of-period total stock of government bonds, and D_t^{r} denotes total tax revenues, where:

$$T_{t} \equiv \tau_{t}^{c} (N_{t}^{p} C_{t}^{p} + N_{t}^{b} C_{t}^{b}) + \tau_{t}^{k} r_{t} (N_{t}^{p} K_{t}^{p} + N_{t}^{b} K_{t}^{b}) + \tau_{t}^{k} N_{t}^{p} \pi_{t}^{p} + \tau_{t}^{l} (w_{t}^{p} N_{t}^{p} e_{t}^{p} + w_{t}^{g} N_{t}^{b} e_{t}^{b})$$

$$(9b)$$

Thus, in each period, there are nine policy instruments $(G_t^g, G_t^w, G_t^{tr,p}, G_t^{tr,b}, \tau_t^c, \tau_t^k, \tau_t^l, N_t^b, B_{t+1})$ out of which only eight can be set independently, with the ninth following residually to satisfy the government budget constraint. Following most of the related literature, we will start by assuming that the adjusting policy instrument is the end-of-period public debt, B_{t+1} , so that the other eight policy instruments can follow exogenous

processes (defined below). For convenience, concerning spending policy instruments, we will work in terms of their GDP shares, $s_t^g \equiv \frac{G_t^g}{Y_t}$, $s_t^w \equiv \frac{G_t^w}{Y_t}$, $s_t^{tr,p} \equiv \frac{G_t^{tr,p}}{Y_t}$, $s_t^{tr,b} \equiv \frac{G_t^{tr,b}}{Y_t}$, where Y_t denotes total output $(Y_t \equiv N_t^f Y_t^f)$. Similarly, concerning the number of public employees, we will work in terms of their population share, $V_t^b \equiv \frac{N_t^b}{N_t}$.

2.5 Public sector production function

Following the related literature mentioned above, we assume that public goods and services, Y_t^g , are produced by using goods purchased from the private sector, G_t^g , and public employment, L_t^g , where $L_t^g = N_t^b e_t^b$. In particular, following e.g. Linnemann (2009), we use a CRS Cobb-Douglas production function of the form:

$$Y_{t}^{g} = A_{t} (G_{t}^{g})^{\theta} (L_{t}^{g})^{1-\theta}$$
(10)

where $0 \le \theta \le 1$ is a parameter. Note that the TFP is the same as in the private sector (see (8) above). This is because we do not want our results to be driven by exogenous factors.

2.6 Decentralized competitive equilibrium (DCE) with public production

We can now solve for a DCE in which (i) all households maximize utility, (ii) all firms in the private sector maximize profits, (iii) all markets clear (see Appendix C for market-clearing conditions) and all constraints are satisfied. The DCE consists of the following eleven equations:⁸

$$\mu(e_t^p)^{\xi}(1+\tau_t^c)(C_t^p+\psi\overline{Y}_t^g) = (1-\tau_t^l)w_t^p \tag{11a}$$

$$\frac{1}{(1+\tau_t^c)(C_t^p + \psi \overline{Y}_t^g)} = \beta E_t \left[\frac{[1-\delta^k + (1-\tau_{t+1}^k)r_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^p + \psi \overline{Y}_{t+1}^g)} \right]$$
(11b)

$$\frac{1}{(1+\tau_t^c)(C_t^p + \psi \overline{Y}_t^g)} = \beta E_t \left[\frac{[1+\rho_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^p + \psi \overline{Y}_{t+1}^g)} \right]$$
(11c)

⁷ Thus,
$$\overline{G}_{t}^{tr,p} \equiv \frac{G_{t}^{tr,p}}{N_{t}^{p}}$$
 and $\overline{G}_{t}^{tr,b} \equiv \frac{G_{t}^{tr,b}}{N_{t}^{b}}$.

⁸ $\overline{Y}_{t}^{g} \equiv \frac{Y_{t}^{g}}{N_{t}}$, $\overline{G}_{t}^{g} \equiv \frac{G_{t}^{g}}{N_{t}}$, $\overline{G}_{t}^{tr,p} \equiv \frac{G_{t}^{tr,p}}{N_{t}^{p}}$, $\overline{G}_{t}^{tr,b} \equiv \frac{G_{t}^{tr,b}}{N_{t}^{b}}$, $s_{t}^{w} \equiv \frac{G_{t}^{w}}{Y_{t}} = \frac{w_{t}^{g} L_{t}^{g}}{Y_{t}}$, $Y_{t} \equiv N_{t}^{f} Y_{t}^{f}$.

$$(1+\tau_t^c)C_t^p + K_{t+1}^p - (1-\delta^k)K_t^p + B_{t+1}^p - B_t^p = (1-\tau_t^k)r_tK_t^p + (1-\tau_t^l)w_t^p e_t^p + \rho_t B_t^p + s_t^{tr,p}Y_t^f$$
 (11d)

$$\mu(e_t^b)^{\xi} (1 + \tau_t^c) (C_t^b + \psi \overline{Y}_t^g) = (1 - \tau_t^l) w_t^g$$
(11e)

$$\frac{1}{(1+\tau_t^c)(C_t^b+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1-\delta^k + (1-\tau_{t+1}^k)r_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^b + \psi \bar{Y}_{t+1}^g)} \right]$$
(11f)

$$\frac{1}{(1+\tau_t^c)(C_t^b+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1+\rho_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^b+\psi \bar{Y}_{t+1}^g)} \right]$$
(11g)

$$Y_{t}^{f} = A_{t} (K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} K_{t}^{b})^{\alpha} (e_{t}^{p})^{1-\alpha}$$
(11h)

$$\overline{Y}_{t}^{g} = A_{t} (\overline{G}_{t}^{g})^{\theta} (v_{t}^{b} e_{t}^{b})^{1-\theta}$$

$$\tag{11i}$$

$$(s_t^w + s_t^g + s_t^{tr,p} + s_t^{tr,b})v_t^p Y_t^f + (1 + \rho_t)(v_t^p B_t^p + v_t^b B_t^b) = v_{t+1}^p B_{t+1}^p + v_{t+1}^b B_{t+1}^b + v_{t+1}^b B_{t$$

$$+\tau_{t}^{c}(v_{t}^{p}C_{t}^{p}+v_{t}^{b}C_{t}^{b})+\tau_{t}^{k}r_{t}(v_{t}^{p}K_{t}^{p}+v_{t}^{b}K_{t}^{b})+\tau_{t}^{l}(w_{t}^{p}v_{t}^{p}e_{t}^{p}+w_{t}^{g}v_{t}^{b}e_{t}^{b})$$

$$(11j)$$

$$v_t^p C_t^p + v_t^b C_t^b + v_t^p [K_{t+1}^p - (1 - \delta^k) K_t^p] + v_t^b [K_{t+1}^b - (1 - \delta^k) K_t^b] + \overline{G}_t^g = v_t^p Y_t^f$$
(11k)

where, in the above equations, we use for factor returns:9

$$r_t = \frac{\alpha Y_t^f V_t^p}{V_t^p K_t^p + V_t^b K_t^b} \tag{12a}$$

$$w_{t}^{p} = \frac{(1-\alpha)Y_{t}^{f}}{e_{t}^{p}}$$
 (12b)

$$w_t^g = \frac{S_t^w V_t^p Y_t^f}{V_t^b e_t^b} \tag{12c}$$

We therefore have eleven eleven equations in variables, $\{C_{t}^{p}, C_{t}^{b}, K_{t+1}^{p}, K_{t+1}^{b}, B_{t+1}^{p}, B_{t+1}^{b}, e_{t}^{p}, e_{t}^{b}, \rho_{t}, Y_{t}^{f}, \overline{Y}_{t}^{g}\}_{t=0}^{\infty}$ This for feasible policy, $\{s_t^g, s_t^w, s_t^{tr,p}, s_t^{tr,b}, \tau_t^c, \tau_t^k, \tau_t^l, v_t^b\}_{t=0}^{\infty}$, and total factor productivity, $\{A_t\}_{t=0}^{\infty}$. We assume that the eight exogenous policy instruments are constant and set at their data average values (see below), while $\{A_t\}_{t=0}^{\infty}$ follows a standard stochastic AR(1) process with an autoregressive parameter $0 < \rho^a < 1$ and standard deviation σ^a (see below). These equilibrium equations are log-linearized around the long run. This model serves as a benchmark and is solved numerically in section 4.¹⁰

3. The same economy with private providers of public goods

We now study what changes when, other things equal, the same amount of public goods, as found by the above solution, is produced by private firms, the so-called private providers of public services, in each time period. These private providers choose capital and labor inputs to produce the amount of public goods ordered by the government by solving a cost minimization problem with the government financing their total cost. Thus, now the government is not involved in any production itself.

The optimization problem of households $p=1,2,...,N_t^p$, which work at the private firms producing the private good, as well as the optimization problem of households $b=1,2,...,N_t^b$, which work at the private firms producing the public good ordered by the government, remain as before (see equations (1)-(3) and (4)-(6) respectively). The same applies to the problem of private firms $f=1,2,...,N^p$ producing the private good (see equations (7)-(8)). What changes is the introduction of private firms producing the public good, indexed by $g=1,2,...,N_t^b$, 11 and the new role of the government. In particular, each private provider g produces a given amount of the public good ordered by the government, \tilde{Y}^g_t/N_t^b , by choosing capital and labor in a cost-minimizing way, where $\{\tilde{Y}^g_t\}_{t=0}^{\infty}$ denotes the sequence of the total amount of public goods as found by the solution of (11a-k) in the previous regime. Notice that the values of $\{\tilde{Y}^g_t\}_{t=0}^{\infty}$ are taken as given in this new regime. Also notice that now both groups of firms, f and g, participate in factor markets (see the market-clearing conditions below). The government makes lump-sum transfers as before and

⁹ Equations (12a-b) follow from the optimality conditions of the private firm and the related market-clearing conditions, while equation (12c) follows from the policy rule $s_t^w = \frac{G_t^w}{Y_t} = \frac{w_t^g L_t^g}{Y_t} = \frac{w_t^g N^b e_t^b}{N^p Y_t^f}$.

¹⁰ Notice that the equilibrium equations are in terms of individual variables directly (i.e. private and public employees) without using any aggregation results (see also Garcia-Milà *et al*, 2010). Also recall that, with perfect capital markets and common discount factors, the allocation of the aggregate stock of capital and bonds to the two types of individuals cannot be pinned down by the equilibrium conditions (see below for details and related papers).

¹¹ We assume that the number of private firms producing the public good equals the number of households working in these firms, and we continue to assume that the number of private firms producing the private good equals the numbers of households working in these firms. All this is analogous to section 2.

finances the total cost of private providers (the latter replaces spending on public wages and goods purchased from the private sector).

In what follows, we model what changes relative to section 2.

3.1 Private firms that produce a given amount of the public good (private providers) In each period, each private provider $g = 1, 2, ..., N_t^b$ chooses capital and labor inputs, K_t^g and e_t^g , to minimize costs. The cost-minimization problem is:

$$r_t K_t^g + w_t^g e_t^g + \xi_t \left[\frac{\tilde{Y}_t^g}{N_t^b} - Y_t^g \right]$$

$$\tag{13}$$

where r_t is the rental cost of capital as defined above, w_t^g is the new wage rate received by households employed by private providers, 12 ξ_{t} is a multiplier measuring the marginal cost of production, $ilde{Y}_{t}^{g}$ is the total amount of public goods which is exogenously given by the previous problem, and Y_t^g is the private provider's output which is produced by using the same production function as in (8) above:

$$Y_t^g = A_t \left(K_t^g \right)^{\alpha} \left(e_t^g \right)^{1-\alpha} \tag{14}$$

Each firm g chooses K_t^g and e_t^g taking prices and \tilde{Y}_t^g as given. The first-order conditions are:

$$r_{t} = \xi_{t} \frac{\alpha Y_{t}^{g}}{K_{t}^{g}}$$
 (15a)

$$w_t^g = \xi_t \frac{(1-\alpha)Y_t^g}{e_t^g} \tag{15b}$$

$$\tilde{Y}_{t}^{g} / N_{t}^{b} - A_{t} (K_{t}^{g})^{\alpha} (e_{t}^{g})^{1-\alpha} = 0$$
(15c)

¹² The determination of w_t^g is different from section 2. In section 2, w_t^g was basically determined by policy (see (12c) above), while now it is determined by standard labor market forces. See also below.

where Appendix D provides details based on Mas-Colell *et al* (1995, pp. 139-143). As said, the determination of w_t^g is different from section 2. In particular, while it was determined by the policy rule for the share of public wages in section 2, now it is market determined.

3.2 Government budget constraint

The within-period budget constraint of the government changes from (9a) to:

$$N_{t}^{b}[r, K_{t}^{g} + w_{t}^{g}e_{t}^{g}] + G_{t}^{tr,p} + G_{t}^{tr,b} + (1+\rho_{t})B_{t} = B_{t+1} + T_{t}$$

$$(16)$$

where the first term on the left-hand side is the total cost of public goods produced by private firms and the other variables are as defined above.

Thus, in each period, there are eight policy instruments $(\tilde{Y}_t^g, G_t^{tr,p}, G_t^{tr,p}, T_t^c, \tau_t^k, \tau_t^l, N_t^b, B_{t+1})$ or equivalently $(\tilde{Y}_t^g, s_t^{tr,p}, s_t^{tr,b}, \tau_t^c, \tau_t^k, \tau_t^l, V_t^b, B_{t+1})$. As before, we will start by assuming that the residually determined policy instrument is the end-of-period public debt, B_{t+1} . As said, the level of public goods, \tilde{Y}_t^g , is now treated as a policy instrument and, at each t, is set at the value found in section 2 above. By contrast, in section 2, s_t^g and s_t^w were policy instruments, while \tilde{Y}_t^g was an endogenous variable.

3.3 Decentralized competitive equilibrium (DCE) with cost-minimizing private providers

We solve for a DCE in which (i) all households maximize utility, (ii) all private firms that produce the private good maximize profits and all private firms that produce the public good minimize costs, (iii) all markets clear (see Appendix E for the new market-clearing conditions) and all constraints are satisfied. The new DCE consists of the following eleven equations:

$$\mu(e_t^p)^{\xi}(1+\tau_t^c)(C_t^p+\psi \overline{Y_t}^g) = (1-\tau_t^l)w_t^p \tag{17a}$$

$$\frac{1}{(1+\tau_{t+1}^{c})(C_{t}^{p}+\psi \bar{Y}_{t}^{g})} = \beta E_{t} \left[\frac{[1-\delta^{k}+(1-\tau_{t+1}^{k})r_{t+1}]}{(1+\tau_{t+1}^{c})(C_{t+1}^{p}+\psi \bar{Y}_{t+1}^{g})} \right]$$
(17b)

$$\frac{1}{(1+\tau_{t}^{c})(C_{t}^{p}+\psi\overline{Y}_{t}^{g})} = \beta E_{t} \left[\frac{[1+\rho_{t+1}]}{(1+\tau_{t+1}^{c})(C_{t+1}^{p}+\psi\overline{Y}_{t+1}^{g})} \right]$$
(17c)

$$(1+\tau_t^c)C_t^p + K_{t+1}^p - (1-\delta^k)K_t^p + B_{t+1}^p - B_t^p = (1-\tau_t^k)r_tK_t^p + (1-\tau_t^l)w_t^p e_t^p + \rho_t B_t^p + s_t^{tr,p}Y_t^f$$
 (17d)

$$\mu(e_t^b)^{\xi} (1 + \tau_t^c) (C_t^b + \psi \overline{Y}_t^g) = (1 - \tau_t^l) w_t^g$$
(17e)

$$\frac{1}{(1+\tau_t^c)(C_t^b+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1-\delta^k + (1-\tau_{t+1}^k)r_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^b + \psi \bar{Y}_{t+1}^g)} \right]$$
(17f)

$$\frac{1}{(1+\tau_t^c)(C_t^b+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1+\rho_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^b+\psi \bar{Y}_{t+1}^g)} \right]$$
(17g)

$$Y_{t}^{f} = A_{t} \left[K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} (K_{t}^{b} - K_{t}^{g}) \right]^{\alpha} (e_{t}^{p})^{1-\alpha}$$
(17h)

$$\bar{Y}_{t}^{g} \equiv \frac{\tilde{Y}_{t}^{g}}{N_{t}} = \frac{N_{t}^{b} A_{t} (K_{t}^{g})^{\alpha} (e_{t}^{b})^{1-\alpha}}{N_{t}}$$
(17i)

$$v_{t}^{b}(r_{t} K_{t}^{g} + w_{t}^{g} e_{t}^{b}) + (s_{t}^{tr,p} + s_{t}^{tr,b})v_{t}^{p}Y_{t}^{f} + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{b}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{b}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t+1}^{p}B_{t+1}^{p} + v_{t+1}^{b}B_{t+1}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}) = \frac{N_{t+1}}{N_{t}}[v_{t}^{p}B_{t}^{p} + v_{t}^{b}B_{t}^{p}] + (1 + \rho_{t})(v_{t}^{p}B_{t}^{p}$$

$$+\tau_{t}^{c}(v_{t}^{p}C_{t}^{p}+v_{t}^{b}C_{t}^{b})+\tau_{t}^{k}r_{t}(v_{t}^{p}K_{t}^{p}+v_{t}^{b}K_{t}^{b})+\tau_{t}^{l}(w_{t}^{p}v_{t}^{p}e_{t}^{p}+w_{t}^{g}v_{t}^{b}e_{t}^{b})$$

$$(17j)$$

$$v_t^p C_t^p + v_t^b C_t^b + v_t^p [K_{t+1}^p - (1 - \delta^k) K_t^p] + v_t^b [K_{t+1}^b - (1 - \delta^k) K_t^b] = v_t^p Y_t^f$$
(17k)

where, in the above equations, we use for factor returns (see Appendix F):

$$r_{t} = \frac{\alpha Y_{t}^{f}}{K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} (K_{t}^{b} - K_{t}^{g})}$$
(18a)

$$w_{t}^{p} = \frac{(1-\alpha)Y_{t}^{f}}{e_{t}^{p}}$$
 (18b)

$$\frac{w_t^g}{w_t^p} = \frac{e_t^p K_t^g}{e_t^b [K_t^p + \frac{V_t^b}{V_t^p} (K_t^b - K_t^g)]}$$
(18c)

Therefore, in this new system, we have eleven equations in eleven variables, $\{C_t^p, C_t^b, K_{t+1}^p, K_{t+1}^b, K_{t+1}^g, B_{t+1}^p, B_{t+1}^b, e_t^p, e_t^b, \rho_t, Y_t^f\}_{t=0}^{\infty}$. This is for any feasible policy, $\{\tilde{Y}_t^g, s_t^{tr,p}, s_t^{tr,b}, \tau_t^c \tau_t^k \tau_t^l, v_t^b\}_{t=0}^{\infty}$, and total factor productivity, $\{A_t\}_{t=0}^{\infty}$. As said above, $\{\tilde{Y}_t^g\}_{t=0}^{\infty}$ is set as found in the previous regime, while the other six exogenous policy instruments are assumed to be constant and set at their data average values (see below). Total factor productivity, $\{A_t\}_{t=0}^{\infty}$, follows the same AR(1) process as above. These new equilibrium

equations are log-linearized around the new long run. The model is solved numerically in section 4 below.

4. Numerical solutions and comparison of the two model economies

We now solve and compare the above two model economies.

4.1 How we work

We work in two steps. We first solve the model in section 2, when this model is calibrated to match some stylized facts, and in particular the tax-spending policy mix, of the UK economy over 1990-2010. This solution will give us, among other things, the time path of public goods induced by the existing UK tax-spending policy mix. This status quo economy will then be used as a point of reference for evaluating policy reforms. In particular, in the second step, using the same parameterization, we solve the model economy in section 3, where it is private providers, rather than the government itself, that produce the same time path of public goods. We will compare the status quo economy to the reformed economy both in the long run and in the transition path. The way we work follows most of the literature on policy reforms (see e.g. Lucas, 1990, who compares the macroeconomic allocation implied by the existing US tax mix to that under optimal Ramsey policy according to which the capital tax rate is set to be zero).

4.2 Parameter values and policy variables used in the solution

Table 1 reports the baseline parameter values for technology and preference, as well as the values of exogenous policy variables, used to solve the status quo model economy in section 2. The time unit is meant to be a year.

Our parameterization is standard with most parameter values for technology and preference being borrowed from Angelopoulos *et al* (2011) who have recently calibrated an aggregate DSGE model to annual data for the UK economy. When we have no a priori information about a technology or preference parameter value, or when different authors use different values, we will consider a range of values. In general, we can report that all main results are robust to parameter values.

Public spending and tax rate values are those of sample averages of the UK economy over 1990-2010. The data are obtained from OECD, Economic Outlook, no. 88. We report

that our main results do not change when we consider alternative time periods, e.g. 1970-2010 or 1996-2010.

Table 1 around here (parameter values and policy variables)

Let us discuss, briefly, the values summarized in Table 1. The labour share in the private production function, $1-\alpha$, is set at 0.601, which is the value in Angelopoulos *et al.* (2011). The long-run scale parameter in the technology function, A, is set at 1. The time preference rate is set at 0.99. The weight given to public goods and services in composite consumption, ψ , is set at 0.1, as is usually the case in similar studies. The other preference parameters related to hours worked, μ and ξ , are set at 5 and 1 respectively; these parameter values jointly imply hours worked within usual ranges. The capital depreciation rate is set at $\delta = 0.05$.

In the baseline calibration, the productivity of public employment, vis-à-vis the productivity of goods purchased from the private sector, in the public sector production function, $1-\theta$, is set at 0.493. This value is the sample average of public wage payments, as share of total public spending on inputs used in the production of public goods (see also e.g. Linnemann, 2009, for similar practice). But we will also experiment with other values of $1-\theta$.

Public employees as a share of total population, v^b , are 0.1904 in the data. Public spending on wage payments and transfers, as shares of output, are respectively $s_t^w = 0.109$ and $s_t^m = 0.2199$ in the data. We then assume that total transfers are allocated to private and public employees according to their shares in population, $v^b = 0.1904$ and $v^p = 1-0.1904$ (see below for other cases considered). The output share of public spending on goods and services purchased from the private sector, s_t^g , is then calculated residually from total public spending minus spending on public wage payments, transfers and interest payments; this is found to be 0.1119 in the data. The effective tax rates on consumption, capital and labor are respectively $\tau^c = 0.1852$, $\tau^k = 0.3875$ and $\tau^l = 0.2685$ over 1990-2008; the data are taken from Angelopoulos *et al* (2011), who have followed the methodology of Conesa *et al* (2007) in constructing effective tax rates for the UK economy.

4.3 Long-run solutions

The long-run solutions of the two model economies follow from (11a-k) and (17a-k) respectively if we assume that variables do not change and there are no shocks. Notice that the computation of the long-run solution is complicated by the fact that the system is "underidentified" in the sense that there are nine equations and eleven endogenous variables. This happens because, in the long run, the two agents' (i.e. private and public workers') Euler conditions for capital (see equations (11b) and (17b) written in the long run) are reduced to one equation only. The same applies to the two Euler conditions for bonds (see equations (11c) and (17c) written in the long run). Thus, the model can pin down the total long-run stocks of capital and bonds but not their allocations to the two types of agents. To circumvent this usual problem, we assume that the allocation of total capital and total bonds to the two types of agents is according to their shares in population. Thus, the long-run stock of capital, denoted as K, is allocated as $K = N^p K^p + N^b K^b$, and the long-run stock of bonds, denoted as K, is allocated as $K = N^p K^p + N^b B^b$.

Using the parameter and policy values in Table 1, and the allocation rules above, the solutions of the two model economies are reported in Table 2a.

Table 2a around here (solution when public debt is the residual instrument in the long run)

Columns 1 and 2 in Table 2a report respectively the solution of the status quo economy in section 2 and the solution of the reformed economy in section 3. Notice that our long-run solution for the status quo economy can mimic well most of the averages in the actual data. For instance, our long-run solution for the public wage to private wage ratio is found to be $w^g/w^p = 0.8112$ in column 1, which is close to that in the actual data over our sample period, 0.8884. We also report that our long-run output shares of consumption, capital, etc, are close to their average values in the data.

We can now proceed to compare the status quo economy to the reformed economy. Recall that, in the latter, the same amount of public goods is supplied by cost-minimizing private providers. Recall also that the superscript b denotes those households that are

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¹³ Resorting to some extraneous assumption to get an allocation share to each individual in models with different agents and perfect capital markets (or to some portfolio share in models where assets are perfect

involved in the production of the public good, either as public employees in the status quo economy, or as workers at the cost-minimizing private providers/firms in the reformed economy, while the superscript p denotes those households that work in private firms producing the private good.

Per capita long-run utility, u, defined as the weighted average of the utility of p households and the utility of b households where the weights are their shares in population, rises as we switch from the status quo to the reformed economy. This is driven by a substantial increase in the utility of p households, u^p . By contrast, the utility of bhouseholds, u^b , falls as we switch to the reformed economy. The fall in the utility of b households is mainly due to lower wages. In particular, the ratio of public to private wages, w^g/w^p , falls from 0.8112 in column 1 to only 0.3931 in column 2 of Table 2a. Lower labor income explains in turn the fall in consumption and hours of work of b households. It also explains why the total labor cost of public good production, as share of output, falls from 0.1090 in the data (see column 1) to 0.0362 in the reformed economy (see column 2). Since this cost is always financed by the government, a more efficient way of delivering the public good allows the government to make efficiency savings. In turn, since the residual policy instrument is, by assumption, the stock of public debt, these efficiency savings allow the government to afford a higher debt burden through the long-run government budget constraint. As reported in Table 2a, the endogenously determined output share of public debt, b/y, rises from 309.57% in column 1 to a very high number in column 2.

Notice that long-run per capita output, y, falls as we switch to the reformed economy. This seemingly paradoxical result arises simply because we have assumed that it is public debt that adjusts to close the government budget. As said above, in this case, efficiency savings only allow the government to afford the financing of a much higher debt burden. Thus, although we move to a more efficient way of delivering the public good, we do not use, by assumption, the resources saved in a way that benefits the supply side of the economy. At the same time, the decrease in public wages creates an adverse demand effect on output. The combination of those two effects, namely, the absence of supply-side benefits and the adverse effect on the demand side, explains the drop in y, even if we have switched to a more efficient way of delivering the public good in column 2.

substitutes) is usual in the literature (see e.g. Judd, 1985, and Garcia-Milà et al, 2010, in models with different agents, and Mendoza and Tesar, 1998, in a two-country model with different assets).

To confirm the above, we study two alternative, more interesting, long-run ways of public financing. In Table 2b, the residual policy instrument is assumed to be the consumption tax rate, while, in Table 2c, the residual policy instrument is the labor tax rate. In both cases, the public debt-to-output ratio is now exogenously set at its average value in the data, 0.8. As shown in Table 2b, efficiency savings allow the government to afford a much lower consumption tax rate. The fall in consumption taxes allows a large increase in the consumption of p households and this offsets the adverse demand effect coming from the fall in consumption of p households. As our numerical solutions show, one effect almost offsets the other so that output remains more or less as in the status quo economy (compare columns 1 and 2 in Table 2b). In Table 2c, efficiency savings allow the government to afford a much lower labor tax rate. Since labor taxes are particularly distorting (see also e.g. Angelopoulos p0. Angelopoulos p1. For the UK), their reduction stimulates long-run output substantially (it rises from 0.7514 in column 1 to 0.7877 in column 2). Thus, we now have supply-side benefits that lead to a larger pie (as we show below, this larger pie allows the government to afford Pareto improving redistributive policies).

Table 2b around here (solution when the consumption tax is the residual instrument in the long run)

Table 2c around here (solution when the labor tax is the residual instrument in the long run)

4.4 Transition dynamics and lifetime utility

The above referred to the long run. We now compare expected discounted lifetime utility under the status quo economy to that under private providers. To do so, we work in two steps. In the first step, we check that, when log-linearized around their long-run solutions, both model economies in sections 2 and 3 are saddle-path stable. Note that, within each regime, transition dynamics arise because of shocks to total factor productivity (these shocks are the only source of uncertainty in our model). In the second step, plugging the equilibrium time-path solutions into the welfare criterion in (1)-(2) above, we compute expected discounted lifetime utility under all regimes. This is done for each type of household.

Results for expected discounted lifetime utility, under the three different ways of long-run public financing, are reported in the last three rows of Tables 2a, 2b and 2c respectively. As can be seen, the transition results are qualitatively as the long-run results. Namely, in all cases studied, a switch from the status quo economy (see column 1) to an economy with private provision/public finance (see column 2) is good for private employees and the aggregate economy but this is at the welfare cost of those employed in the public sector.

4.5 Robustness

We now check the sensitivity of our results to changes in the assumed parameter values and, especially, the value of the unknown parameter, $1-\theta$, measuring the productivity of public employment in the public sector production function, equation (10). We report that all results above are robust to parameter values used.

For instance, keeping everything else as in the baseline parameterization of Table 1, we now arbitrarily set $1-\theta=0.3$ and $1-\theta=0.7$. Results for these two new cases are reported in Tables 3a-c and 4a-c respectively.

Tables 3a-c around here (like Tables 2a-c with $1-\theta = 0.3$)

Tables 4a-c around here (like Tables 2a-c with $1-\theta = 0.7$)

4.6 Summary of this section

In all cases studied, the switch from the status quo economy to an economy with private providers increases aggregate welfare at the cost of making public employees worse off. On the other hand, the effects of this reform on aggregate output depend on the method of public financing; when the efficiency savings, coming from a more efficient way of delivering the public good, are translated into reductions in distorting taxes, aggregate output also rises.

5. Can public providers beat private providers?

¹⁴ Impulse response functions are available upon request. We also report that our main results do not change if we also allow the exogenous policy instruments to be stochastic.

So far, strictly speaking, we have been "unfair" to the public sector. In particular, we have compared the status quo economy to an economy with private providers, but the presumption has been that, in the status quo economy, public production decisions were ad hoc and as in the data, while, in the reformed economy, public production decisions were made by cost-minimizing private firms. Although comparisons of this type are common in the related literature on policy reforms (see e.g. Lucas, 1990), one is wondering what would happen when we compare the cases in which, not only private providers, but also public providers/enterprises minimize their costs, always with the general taxpayer (i.e. the government) financing these costs. We turn to this question now.

5.1 Cost-minimizing public providers

The economy with cost-minimizing public providers is as in section 2 but now, in addition, in each period, the public provider/enterprise chooses its two productive inputs, G_t^g and L_t^g , or equivalently the associated spending shares, s_t^g and s_t^w , to minimize costs. Using the same public sector production function as in (10) above, the cost-minimization problem is:

$$G_t^g + w_t^g L_t^g + \xi_t [\tilde{Y}_t^g - A_t (G_t^g)^{\theta} (L_t^g)^{1-\theta}]$$
(19)

where w_t^g is the wage rate, ξ_t is a multiplier measuring the marginal cost of producing the public good and \tilde{Y}_t^g is the total amount of public goods which is again given by the solution of the model in section 2.

It is straightforward to show that the first-order conditions imply:

$$\frac{s_t^g}{s_t^w} = \frac{\theta}{1 - \theta} \tag{20}$$

which says that the ratio of public spending on the two inputs should be equal to the ratio of their productivities.

In the new DCE, we have twelve equations, (11a-k) and (20), in twelve variables, $\{C_t^p, C_t^b, K_{t+1}^p, K_{t+1}^b, B_{t+1}^p, B_{t+1}^b, u_t^p, u_t^b, \rho_t, Y_t^f, s_t^g, s_t^w\}_{t=0}^{\infty}$. This is for any feasible policy, $\{\tilde{Y}_t^g, s_t^{tr,p}, s_t^{tr,b}, \tau_t^c \ \tau_t^k \ \tau_t^l, v_t^b\}_{t=0}^{\infty}$, and total factor productivity, $\{A_t\}_{t=0}^{\infty}$. These equilibrium

equations are log-linearized around the long run. Solutions of this model economy, under the three different ways of long-run public financing, and under the three different values of $1-\theta$, are reported in column 3 in all Tables above, namely Tables 2a-c, 3a-c and 4a-c.

The general message is that the regime in column 3 is very similar to the status quo regime in column 1. Any differences between the results in column 3 and the results in column 1 are minor and are driven by differences in $1-\theta$. More importantly, the comparison of results in columns 1 and 3, on one hand, to those in column 2, on the other hand, indicate that private provision/public finance is the best choice in all cases studied.

5.2 Summary of this section

When we assume that public providers choose inputs in a cost-minimizing way, the solution is similar to that when the associated policy variables are exogenously set at their data averages. This could mean that in the UK, over 1990-2010, the public sector has exhausted its role in terms of aggregate efficiency as a provider of public goods. By contrast, contracting out the production of public goods to private providers can lead to important efficiency gains.

6. Pareto improving reforms and transfers

As we have seen, although per capita welfare increases when we move from the status quo economy to the economy with public finance, public employees clearly become worse off since they lose their role and turn into employees in private firms providing the public good. This means that such reforms, although good for the general interest, are unlikely to be implemented, especially, when public sector employees, or their trade unions, have a strong influence in blocking reforms.

The question is whether the society can take advantage of the efficiency savings, generated by private provision/public finance, and design a transfer scheme that improves

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¹⁵ In Tables 2a-c and 3a-c, the welfare of public employees falls as we move from the status quo regime in column 1, where public purchases of goods and services, as well as the public wage bill are set as in the data, to the regime in column 3, where these choices are made in a cost-minimizing way. This is intuitive. The opposite happens in Tables 4a-c, where s_t^w and hence the welfare of public employees rise as we move from column 1 to column 3. This might look paradoxical but it happens simply because of the optimality condition (20). Since $1-\theta=0.7$ is relatively high (or $\theta=0.3$ is relatively low), it is optimal to choose a relatively high s_t^w (or a relatively low s_t^g).

the welfare of both types of agents, namely both private and public employees, relative to the status quo economy.

6.1 Endogenzing transfers

We search for a transfer scheme that, in combination with private provision/public finance of public goods, makes everybody equally well off. In particular, instead of assuming that transfers are exogenously allocated to the two groups according to their population shares, we now endogenize this scheme by solving for an allocation of transfers that makes both agents equally off in the long run of the reformed economy in section 3.¹⁶ Results are reported in column 4 in Tables 2a-c, 3a-c and 4a-c. As can be seen, when we compare this economy (in column 4) to the status quo economy (in column 1), there is room for substantial welfare gains for both types of agents. This holds in all Tables (2a-c, 3a-c and 4a-c). This provides an argument for social contracts (see below).

6.2 Summary of this section

A switch to private provision/public finance, in combination with redistributive government transfers, is Pareto improving relative to the status quo economy.

7. Conclusions

We studied a much debated reform of the state - the idea of opening up public services to new providers - in a dynamic stochastic general equilibrium setup. We showed that substantial aggregate gains are possible if the society switches to private provision/public finance of public goods. It is remarkable that this happens even when the amount of public goods produced, and the number of households employed in the production of public goods, remain the same as in the status quo economy. We also showed that one can design redistributive schemes that allow everybody, including public employees, to benefit from such a switch.

Our results are another example of the importance of social contracts (see also the discussion in Garcia-Milà *et al*, 2010). In our model, social contracts that terminate the

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¹⁶ See e.g. Park and Philippopoulos (2003) for other redistributive transfer mechanisms in a dynamic general equilibrium model.

monopoly of the public sector as a producer of public goods, in combination of transfers that compensate those previously employed by the state, can benefit everybody.

APPENDIX

Appendix A: First-order conditions of household p in section 2

The first-order conditions include the budget constraints and:

$$\mu(e_t^p)^{\xi}(1+\tau_t^c)(C_t^p+\psi \overline{Y}_t^g) = (1-\tau_t^l)w_t^p \tag{A.1}$$

$$\frac{1}{(1+\tau_t^c)(C_t^p+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1-\delta^k + (1-\tau_{t+1}^k)r_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^p + \psi \bar{Y}_{t+1}^g)} \right]$$
(A.2)

$$\frac{1}{(1+\tau_t^c)(C_t^p+\psi \bar{Y}_t^g)} = \beta E_t \left[\frac{[1+\rho_{t+1}]}{(1+\tau_{t+1}^c)(C_{t+1}^p+\psi \bar{Y}_{t+1}^g)} \right]$$
(A.3)

Appendix B: First-order conditions of private firm f in section 2

$$r_{t} = \frac{\alpha Y_{t}^{f}}{K_{t}^{f}}$$

$$w_t^p = \frac{(1-\alpha)Y_t^f}{e_t^f}$$

Appendix C: Market-clearing conditions in section 2

In the labor market:

$$N_t^f e_t^f = N_t^p e_t^p$$
 or $e_t^f = e_t^p$

or
$$e_t^J = e_t^I$$

$$L_t^g = N_t^b e_t^b$$

In the capital market:

$$N_t^f K_t^f = N_t^p K_t^p + N_t^b K_t^b$$

In the bond market:

$$\boldsymbol{B}_{t} = \boldsymbol{N}_{t}^{p} \boldsymbol{B}_{t}^{p} + \boldsymbol{N}_{t}^{b} \boldsymbol{B}_{t}^{b}$$

In the dividend market:

$$N_t^f \pi_t^f = N_t^p \pi_t^p$$
 or $\pi_t^f = \pi_t^p$

In the goods market (economy's resource constraint):

$$N_{t}^{p}C_{t}^{p} + N_{t}^{b}C_{t}^{b} + N_{t}^{p}I_{t}^{p} + N_{t}^{b}I_{t}^{b} + G_{t}^{g} = N_{t}^{f}Y_{t}^{f}$$

Appendix D: Cost minimization of private provider g in section 3

We follow Mas-Colell *et al* (1995, pp. 139-143). The first-order conditions imply:

$$r_{t} = \xi_{t} \frac{\alpha \tilde{Y}_{t}^{g}}{K_{t}^{g} N_{t}^{b}}$$
 (D.1a)

$$w_t^g = \xi_t \frac{(1-\alpha)\tilde{Y}_t^g}{e_t^g N_t^b} \tag{D.1b}$$

$$\xi_{t} = \frac{(r_{t})^{\alpha} (w_{t}^{g})^{1-\alpha}}{A_{t} \alpha^{\alpha} (1-\alpha)^{1-\alpha}} = \frac{r_{t} K_{t}^{g} + w_{t}^{g} e_{t}^{g}}{Y_{t}^{g}} = \frac{(r_{t} K_{t}^{g} + w_{t}^{g} e_{t}^{g}) N_{t}^{b}}{\tilde{Y}_{t}^{g}}$$
(D.1c)

where (D.1c) follows if we use (D.1a)-(D.1b) to get expressions for K_t^g and e_t^g respectively, and use these expressions for K_t^g and e_t^g in the production function, $Y_t^g = A_t (K_t^g)^\alpha (e_t^g)^{1-\alpha} = \frac{\tilde{Y}_t^g}{N_t^b} \,.$

In turn, we use (D.1c) to substitute out the multiplier, ξ_t , in (D.1a) and (D.1b):

$$K_t^g = \frac{Y_t^g}{A_t} \left(\frac{r_t}{\alpha}\right)^{\alpha - 1} \left(\frac{w_t^g}{1 - \alpha}\right)^{1 - \alpha} \tag{D.2a}$$

$$e_t^g = \frac{Y_t^g}{A_t} \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t^g}{1-\alpha}\right)^{-\alpha} \tag{D.2b}$$

so that the total cost of each firm can be written as:

$$r_{t} K_{t}^{g} + w_{t}^{g} e_{t}^{g} = \frac{Y_{t}^{g} (r_{t})^{\alpha} (w_{t}^{g})^{1-\alpha}}{A_{t}} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right] =$$

$$= \frac{\tilde{Y}_{t}^{g} (r_{t})^{\alpha} (w_{t}^{g})^{1-\alpha}}{N_{t}^{b} A_{t}} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right]$$
(D.3)

Note that profits are zero for this firm (thanks to CRS). To show this, consider profits:

$$Y_{t}^{g} - r_{t} K_{t}^{g} - w_{t}^{g} e_{t}^{g} = Y_{t}^{g} - \frac{Y_{t}^{g} (r_{t})^{\alpha} (w_{t}^{g})^{1-\alpha}}{A_{t}} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right]$$
(D.4)

so that (thanks to linearity) the first-order condition is:

$$1 = \frac{(r_t)^{\alpha} (w_t^g)^{1-\alpha}}{A_t} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right]$$
 (D.5)

but, if this condition holds, total profits are also zero in each period.

Appendix E: Market-clearing conditions in section 3

In the labor market:

$$N_t^p e_t^f = N_t^p e_t^p$$
 or $e_t^f = e_t^p$

$$N_t^b e_t^g = N_t^b e_t^b$$
 or $e_t^g = e_t^b$

In the capital market:

$$N_{t}^{p}K_{t}^{f}+N_{t}^{b}K_{t}^{g}=N_{t}^{p}K_{t}^{p}+N_{t}^{b}K_{t}^{b}$$

In the bond market:

$$B_t = N_t^p B_t^p + N_t^b B_t^b$$

In the goods market (economy's resource constraint):

$$N_{t}^{p}C_{t}^{p} + N_{t}^{b}C_{t}^{b} + N_{t}^{p}I_{t}^{p} + N_{t}^{b}I_{t}^{b} = N_{t}^{p}Y_{t}^{f}$$

where recall that that the privately produced public good is provided without charge as in section 2.

Appendix F: Factor returns in section 3

From the profit maximization and the cost minimization problems of the two firms, we have:

$$r_{t} = \frac{\alpha Y_{t}^{f}}{K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} (K_{t}^{b} - K_{t}^{g})} = \xi_{t} \frac{\alpha Y_{t}^{g}}{K_{t}^{g}} = \xi_{t} \frac{\alpha \tilde{Y}_{t}^{g}}{K_{t}^{g} N_{t}^{b}}$$

$$w_t^p = \frac{(1-\alpha)Y_t^f}{e_t^p}$$

$$w_t^g = \xi_t \frac{(1-\alpha)\tilde{Y}_t^g}{e_t^b N_t^b}$$

The conditions for the return to capital imply:

$$\xi_{t} = \frac{Y_{t}^{f} N_{t}^{b} K_{t}^{g}}{\tilde{Y}_{t}^{g} [K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} (K_{t}^{b} - K_{t}^{g})]}$$

so that:

$$r_t = \frac{\alpha Y_t^f}{K_t^p + \frac{V_t^b}{V_t^p} (K_t^b - K_t^g)}$$

$$w_t^p = \frac{(1-\alpha)Y_t^f}{e_{\cdot}^p}$$

$$\frac{w_{t}^{g}}{w_{t}^{p}} = \frac{e_{t}^{p} K_{t}^{g}}{e_{t}^{b} [K_{t}^{p} + \frac{V_{t}^{b}}{V_{t}^{p}} (K_{t}^{b} - K_{t}^{g})]}$$

Table 1
Baseline parameterization

Parameters		
and policy	Description	Value
instruments		
α	Share of capital in private production	0.399
$1-\theta$	Share of public employment in public production	0.493*
$\delta^{\scriptscriptstyle k}$	Capital depreciation rate	0.05
β	Rate of time preference	0.99
Ψ	Public consumption weight in utility	0.1
μ	Preference parameter on work hours in utility	5
ξ	Elasticity of work hours in utility	1
S ^w	Public wage payments as share of GDP	0.1090
s ^g	Public purchases as share of GDP	0.1119
s tr	Public transfers as share of GDP	0.2199
$ au^c$	Tax rate on consumption	0.1852
$ au^k$	Tax rate on capital income	0.3875
$ au^l$	Tax rate on labor income	0.2685
v^b	Public employees as share of population	0.1904
A	Long-run TFP	1
$ ho^a$	Autoregressive parameter of TFP	0.9
σ^a	Standard deviation of TFP	

Notes: * We also experiment with $1 - \theta = 0.3$ (see Tables 3a-c) and $1 - \theta = 0.7$ (see Tables 4a-c).

Table 2a Solution when public debt is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	- 0.9634	- 0.8174	- 0.9635	- 0.8546
u^b	- 1.0912	- 1.1693	- 1.0916	- 0.8546
и	- 0.9877	- 0.8844	- 0.9879	- 0.8546
c^p	0.5185	0.5692	0.5184	0.5555
c^b	0.4413	0.3405	0.4411	0.4581
e^p	0.3581	0.3266	0.3581	0.3345
e^b	0.3404	0.2128	0.3404	0.1894
w^g/w^p	0.8112	0.3931	0.8107	0.4683
у	0.7357	0.6710	0.7357	0.6873
y g	0.0732	0.0732	0.0732	0.0732
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7834	0.6847	0.7812
k / y	4.0663	4.3222	4.0663	4.3756
<i>b</i> / <i>y</i>	3.0957	19.6714	3.0889	19.2809
s ^w	0.1090 (exogenous)	0.0362	0.1089	0.0375
s g	0.1119 (exogenous)	-	0.1120	-
$s^{t,p}$	0.8096* s ^{tr}	0.8096* s ^{tr}	0.8096* s ^{tr}	0.6476* s ^{tr}
$s^{t,b}$	$0.1904* s^{tr}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.3524* s ^{tr}
U^{p}	- 96.3480	- 81.8854	- 96.8391	- 85.6252
U^b	- 109.1243	- 117.0656	- 109.5463	- 85.6126
U	- 98.7806	- 88.5837	- 99.2586	- 85.6228

Notes: (i) We use the baseline parameterization in Table 1. (ii) $U^p = \sum_{t=0}^{1100} \beta^t u_t^p$, $U^b = \sum_{t=0}^{1100} \beta^t u_t^b$ and $U = v^p U^p + v^b U^b$.

Table 2b Solution when the consumption tax is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	- 0.9577	- 0.7469	- 0.9576	-0.8021
u^b	- 1.0909	- 1.2892	- 1.0915	-0.8021
и	- 0.9830	- 0.8501	- 0.9831	-0.8021
c^p	0.5262	0.6515	0.5263	0.6286
c^b	0.4457	0.3124	0.4454	0.4902
e^p	0.3631	0.3632	0.3631	0.3739
e^b	0.3461	0.2443	0.3460	0.2043
w^g/w^p	0.8092	0.3265	0.8084	0.4275
У	0.7461	0.7462	0.7461	0.7683
y ^g	0.0743	0.0743	0.0743	0.0743
,		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7866	0.6847	0.7839
k / y	4.0663	4.2679	4.0663	4.3227
<i>b</i> / <i>y</i>	0.8	0.8	0.8	0.8
	(exogenous)	(exogenous)	(exogenous)	(exogenous)
$ au^c$	0.1513	- 0.0676	0.1513	- 0.0618
s^w	0.1090 (exogenous)	0.0310	0.1089	0.0330
s g	0.1119 (exogenous)	-	0.1119	-
$s^{t,p}$	0.8096* s ^{tr}	0.8096* s ^{tr}	0.8096* s ^{tr}	0.6362* s ^{tr}
$s^{t,b}$	0.1904* s ^{tr}	$0.1904* s^{tr}$	0.1904* s ^{tr}	0.3638* s ^{tr}
U^p	- 95.7719	- 74.8024	- 96.2548	- 80.3566
U^b	- 109.0989	- 129.0222	- 109.5306	- 80.3466
U	- 98.3094	- 85.1258	- 98.7825	- 80.3547

Notes: See the notes of Table 2a.

Table 2c Solution when the labor tax is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	- 0.9548	- 0.7136	- 0.9549	-0.7812
u^b	- 1.0910	- 1.4100	- 1.0915	-0.7812
и	- 0.9808	- 0.8462	- 0.9809	-0.7812
c^p	0.5302	0.7000	0.5302	0.6703
c^b	0.4479	0.2845	0.4477	0.5053
e^p	0.3657	0.3834	0.3657	0.3961
e^b	0.3490	0.2676	0.3489	0.2129
w^g/w^p	0.8082	0.2881	0.8076	0.4066
у	0.7514	0.7877	0.7514	0.8137
y ^g	0.0749	0.0749	0.0749	0.0749
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7882	0.6847	0.7851
k / y	4.0663	4.2362	4.0663	4.2984
<i>b</i> / <i>y</i>	0.8	0.8	0.8	0.8
	(exogenous)	(exogenous)	(exogenous)	(exogenous)
$ au^l$	0.2358	- 0.0540	0.2359	-0.00430
s ^w	0.1090 (exogenous)	0.0284	0.1089	0.0309
s g	0.1119 (exogenous)	-	0.1120	-
$s^{t,p}$	0.8096* s ^{tr}	$0.8096* s^{tr}$	$0.8096* s^{tr}$	$0.5539* s^{tr}$
$s^{t,b}$	$0.1904*s^{tr}$	0.1904* s ^{tr}	$0.1904* s^{tr}$	0.4461* s ^{tr}
U^{p}	- 95.4891	- 71.4572	- 95.9885	- 78.2488
U^b	- 109.1058	- 141.0739	- 109.5255	- 78.2413
U	- 98.0817	- 84.7122	- 98.5660	- 78.2474

Notes: See the notes of Table 2a.

Table 3a Solution when public debt is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u ^p	-0.9629	-0.8191	-0.9379	-0.8559
u^b	-1.0905	-1.1643	-1.3081	-0.8559
и	-0.9872	-0.8848	-1.0084	-0.8559
c^p	0.5183	0.5682	0.5264	0.5547
c^b	0.4412	0.3438	0.3196	0.4594
e^p	0.3580	0.3269	0.3525	0.3348
e^b	0.3403	0.2178	0.2764	0.1944
w^g/w^p	0.8113	0.4065	0.4804	0.4822
у	0.7354	0.6717	0.7242	0.6878
y ^g	0.0766	0.0766	0.0766	0.0766
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7823	0.6725	0.7800
k / y	4.0663	4.3538	4.0663	4.3993
<i>b</i> / <i>y</i>	3.0957	19.3737	5.6874	19.5990
s ^w	0.1090 (exogenous)	0.0383	0.0532	0.0396
s g	0.1119 (exogenous)	-	0.1242	-
$s^{t,p}$	0.8096* s ^{tr}	0.8096* s ^{tr}	0.8096* s ^{tr}	0.6507* s ^{tr}
$S^{t,b}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.1904* s ^{tr}	0.3493* s ^{tr}
U^{p}	-96.2942	-82.0878	-94.3049	-85.7895
U^b	-109.0579	-116.6119	-131.3597	-85.7964
U	-98.7244	-88.6612	-101.3601	-85.7908

Notes: (i) We use the baseline parameterization in Table 1 except that now $1 - \theta = 0.3$. (ii) $U^p = \sum_{t=0}^{1100} \beta^t u_t^p$,

$$U^{b} = \sum_{t=0}^{1100} \beta^{t} u_{t}^{b}$$
 and $U = v^{p} U^{p} + v^{b} U^{b}$.

Table 3b Solution when the consumption tax is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	-0.9571	-0.7494	-0.9163	-0.8040
u^b	-1.0903	-1.2795	-1.3348	-0.8040
и	-0.9825	-0.8504	-0.9960	-0.8040
c^p	0.5261	0.6493	0.5484	0.6266
c^b	0.4456	0.3171	0.3161	0.4915
e^p	0.3630	0.3630	0.3631	0.3736
e^b	0.3459	0.2491	0.2881	0.2093
w^g/w^p	0.8093	0.3392	0.4621	0.4409
у	0.7459	0.7459	0.7461	0.7676
y ^g	0.0777	0.0777	0.0777	0.0777
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7857	0.6758	0.7828
k / y	4.0663	4.2854	4.0663	4.3431
<i>b</i> / <i>y</i>	0.8	0.8	0.8	0.8
	(exogenous)	(exogenous)	(exogenous)	(exogenous)
$ au^c$	0.1513	-0.0646	0.1048	-0.0585
s ^w	0.1090 (exogenous)	0.0329	0.0518	0.0349
s g	0.1119 (exogenous)	-	0.1209	-
$s^{t,p}$	0.8096* s ^{tr}	$0.8096* s^{tr}$	0.8096* s ^{tr}	$0.6391*s^{tr}$
$s^{t,b}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.1904* s ^{tr}	0.3609* s ^{tr}
U^p	-95.7180	-75.0874	-92.1387	-80.5827
U^b	-109.0320	-128.0927	-134.0285	-80.5823
U	-98.2530	-85.1796	-100.1145	-80.5826

Notes: See the notes of Table 3a.

Table 3c Solution when the labor tax is the residual public finance instrument

	foliation when the labor tax is the residual public finance instrument				
Variable	Status quo economy	Cost-minimizing private providers	Cost-minimizing public providers	Cost-minimizing private providers plus redistributive	
				transfers	
u^p	-0.9543	-0.7164	-0.9030	-0.7844	
u^b	-1.0903	-1.3954	-1.3560	-0.7844	
и	-0.9802	-0.8457	-0.9892	-0.7844	
c^p	0.5301	0.6972	0.5628	0.6672	
c^b	0.4478	0.2903	0.3129	0.5109	
e^p	0.3656	0.3831	0.3699	0.3957	
e^b	0.3489	0.2721	0.2959	0.2172	
w^g / w^p	0.8082	0.3004	0.4497	0.4218	
у	0.7512	0.7870	0.7599	0.8130	
y g	0.0783	0.0783	0.0783	0.0783	
,		(exogenous)	(exogenous)	(exogenous)	
c / y	0.6848	0.7874	0.6780	0.7841	
k / y	4.0663	4.2516	4.0663	4.3182	
b/y	0.8	0.8	0.8	0.8	
	(exogenous)	(exogenous)	(exogenous)	(exogenous)	
$ au^l$	0.2358	-0.0494	0.1799	-0.0378	
s^w	0.1090	0.0302	0.0509	0.0327	
	(exogenous)				
s g	0.1119	-	0.1187	-	
	(exogenous)				
$s^{t,p}$	$0.8096* s^{tr}$	0.8096* s ^{tr}	$0.8096* s^{tr}$	0.5544* s ^{tr}	
$s^{t,b}$	$0.1904*s^{tr}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.4456* s ^{tr}	
U^p	-95.4351	-71.7637	-90.7988	-78.6098	
U^b	-109.0386	-139.6605	-136.1383	-77.6047	
U	-98.0252	-84.6913	-99.4314	-78.4184	
			1		

Notes: See the notes of Table 3a.

Table 4a Solution when public debt is the residual public finance instrument

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	-0.9640	-0.8158	-0.9939	-0.8534
u^b	-1.0919	-1.1745	-0.8011	-0.8534
и	-0.9883	-0.8841	-0.9571	-0.8534
c^p	0.5186	0.5701	0.5092	0.5563
c^b	0.4415	0.3371	0.6406	0.4567
e^p	0.3582	0.3262	0.3647	0.3343
e^b	0.3405	0.2075	0.3829	0.1842
w^g/w^p	0.8112	0.3792	1.3170	0.4537
у	0.7359	0.6702	0.7493	0.6867
y ^g	0.0697	0.0697	0.0697	0.0697
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7845	0.7129	0.7824
k / y	4.0663	4.3109	4.0663	4.3518
<i>b</i> / <i>y</i>	3.0957	19.9708	0.1392	19.5990
s ^w	0.1090 (exogenous)	0.0341	0.1954	0.0353
s g	0.1119 (exogenous)	-	0.0838	-
$s^{t,p}$	0.8096* s ^{tr}	0.8096* s ^{tr}	0.8096* s ^{tr}	$0.6445*s^{tr}$
$s^{t,b}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.1904* s ^{tr}	$0.3555*s^{tr}$
U^{p}	-96.4031	-81.6855	-99.8450	-85.4613
U^b	-109.1922	-117.5351	-80.1622	-85.4458
U	-98.8382	-88.5112	-96.0974	-85.4584

Notes: (i) We use the baseline parameterization in Table 1 except that now $1 - \theta = 0.7$. (ii) $U^p = \sum_{t=0}^{1100} \beta^t u_t^p$,

$$U^b = \sum_{t=0}^{1100} \beta^t u_t^b$$
 and $U = v^p U^p + v^b U^b$.

Table 4b Solution when the consumption tax is the residual public finance instrument

Solution when the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance instrumed in the consumption tax is the residual public finance in the consumption tax is				
Variable	Status quo economy	Cost-minimizing private providers	Cost-minimizing public providers	Cost-minimizing private providers plus redistributive
				transfers
u^p	-0.9582	-0.7443	-1.0089	-0.8002
u^b	-1.0916	-1.2993	-0.7869	-0.8002
и	-0.9836	-0.8500	-0.9666	-0.8002
c^p	0.5264	0.6537	0.4999	0.6305
c^b	0.4459	0.3076	0.6512	0.4889
e^p	0.3633	0.3633	0.3631	0.3743
e^b	0.3462	0.2392	0.3841	0.1990
w^g / w^p	0.8092	0.3135	1.3733	0.4135
У	0.7463	0.7465	0.7461	0.7690
y ^g	0.0708	0.0708	0.0708	0.0708
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7875	0.7087	0.7849
k / y	4.0663	4.2507	4.0663	4.3024
<i>b</i> / <i>y</i>	0.8	0.8	0.8	0.8
	(exogenous)	(exogenous)	(exogenous)	(exogenous)
$ au^c$	0.1513	-0.0706	0.2119	-0.0651
s^w	0.1090	0.0292	0.2053	0.0311
	(exogenous)			
s g	0.1119	-	0.0880	-
~	(exogenous)			
$s^{t,p}$	$0.8096*s^{tr}$	$0.8096* s^{tr}$	$0.8096* s^{tr}$	$0.6333*s^{tr}$
$s^{t,b}$	$0.1904*s^{tr}$	$0.1904* s^{tr}$	$0.1904* s^{tr}$	0.3667* s ^{tr}
U^p	-95.8271	-74.5196	-101.3694	-80.1331
U^b	-109.1674	-129.9837	-78.7532	-80.1098
U	-98.3671	-85.0800	-97.0633	-80.1287

Notes: See the notes of Table 4a.

 $\label{eq:Table 4c} \textbf{Solution when the labor tax is the residual public finance instrument}$

Variable	1 Status quo economy	2 Cost-minimizing private providers	3 Cost-minimizing public providers	4 Cost-minimizing private providers plus redistributive transfers
u^p	-0.9554	-0.7107	-1.0213	-0.7791
u^b	-1.0917	-1.4255	-0.7813	-0.7791
и	-0.9813	-0.8468	-0.9756	-0.7791
c^p	0.5304	0.7029	0.4909	0.6729
c^b	0.4481	0.2785	0.6541	0.5038
e^p	0.3659	0.3838	0.3601	0.3967
e^b	0.3491	0.2626	0.3835	0.2074
w^g/w^p	0.8081	0.2752	1.4136	0.3927
у	0.7517	0.7885		0.8151
<i>y</i> ^g	0.0713	0.0713	0.0713	0.0713
		(exogenous)	(exogenous)	(exogenous)
c / y	0.6848	0.7890	0.7055	0.7860
k / y	4.0663	4.2206	4.0663	4.2793
<i>b</i> / <i>y</i>	0.8	0.8	0.8	0.8
	(exogenous)	(exogenous)	(exogenous)	(exogenous)
$ au^l$	0.2358	-0.0484	0.3031	-0.0482
s^w	0.1090	0.0266	0.2128	0.0290
	(exogenous)			
s g	0.1119 (exogenous)	-	0.0912	-
$s^{t,p}$	0.8096* s ^{tr}	$0.8096* s^{tr}$	0.8096* s ^{tr}	0.5485* s ^{tr}
$s^{t,b}$	0.1904* s ^{tr}	0.1904* s ^{tr}	0.1904* s ^{tr}	0.4515* s ^{tr}
U^p	-95.5443	-71.1438	-102.6201	-78.0124
U^b	-109.1746	-142.5787	-78.1981	-77.9893
U	-98.1395	-84.7450	-97.9702	-78.0080
		1	l l	

Notes: See the notes of Table 4a.

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