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## Complex dynamics in fashion life cycles

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COMPLEX DYNAMICS IN FASHION LIFE CYCLES

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### ABSTRACT

*Fashions and their revivals occur in a rather erratic manner. The paper shows that such fluctuations can be derived from the utility maximizing behaviour of rational individuals with stable preferences. It is assumed that the demand for the fashion good is determined, amongst other variables, by the social environment the individual is living in. There are positive and negative consumption externalities. If there are lags in the reaction of the individual to what other people do, then the demand for fashion goods may fluctuate over time. For certain parameter constellations, there may even be deterministic chaos.*

### INTRODUCTION

In the recent past, a lot of research effort has been devoted to the invention and analysis of economic models that exhibit strange types of dynamic behaviour. Chaos has become fashionable. This paper entertains a variation of this hypothesis: fashion can be chaotic.

Casual observation of fashion over time yields a rather erratic picture: skirt length shifting from mini to maxi to knee high; trousers with extremely wide and extremely tight legs; tissue design metamorphosis from plain to extremely wild; etc.. Moreover, some cyclical pattern can be observed in many cases, e.g. the revival of the twenties and the fifties. Fashions are not restricted to clothing but appear in areas such as language (vogue-words), religion (sects) and science (economic growth theory). To some extent, fashion and fads pose a problem for economic research, which likes to assume stable preferences and rational choice. The large and irregular movements in fashion are sometimes seen as a refutation of stable preferences, but abandoning stable preferences would imply that anything could be explained by this mechanism and that economic theory fails Popper's falsification criterion. Moreover,

one could entertain the view that fashion is dictated from Paris and Milan (or, in the case of research fashions, from MIT and Chicago) but this would imply that consumer sovereignty does not hold, which is a rather unattractive way out.

This paper is an attempt to analyse fashion in a dynamic framework as a result of consumption externalities. The paper uses the standard model of a representative agent with stable preferences who decides on how to disperse her income on the available goods. In general, fashion goods do not serve basic needs like food or sleep. By being fashionable, a person intends to satisfy wants that are higher in the hierarchy of needs like recognition and esteem. Fashion therefore is a social phenomenon and the behaviour of the consumer depends on society's feedback to his decision. It will be shown how this feedback may generate seemingly erratic patterns of consumer behaviour.

The paper is organized as follows. The next section motivates the introduction of consumption externalities into the standard model of the household. Section II contains a short exposition of a model of demand for fashion goods. Then we introduce adjustment processes and investigate their stability. In Section IV, we give a numerical example in which chaotic behaviour is possible. Some final remarks conclude the paper.

## I. CONSUMPTION EXTERNALITIES AND FASHION

The idea that patterns of consumption are influenced by the social environment the individual is living in is not new in economics. Rae's treatise on political economy, first published in 1834, contained a substantial appendix on vanity and luxury goods. See Rae (1905, 245-296). The classical reference is Veblen (1899) with his analysis of conspicuous consumption. Sombart (1913) stressed the importance of vanity and luxury goods consumption as locomotives of economic development.

Finally, Duesenberry (1949) and Leibenstein (1950) are the classical post-war references on interdependent preferences.

The assumption used in this paper is that fashion goods are characterized by interdependent preferences that cause consumption externalities. There are two general types of interdependent preferences. The bandwagon effect, on the one hand, generates a positive externality: the utility an individual derives from consuming a particular good is increased if other people consume it too. On the other hand, there is an aspiration to individuality, which causes a negative externality: the more the other people consume, the less is the satisfaction an individual person derives from her or his consumption of a particular good.<sup>1</sup> We argue that fashion goods fall in both these categories. At low levels of consumption, the bandwagon effect dominates. If I like to wear a mini skirt or a shrill tie and nobody else does, there are problems of cognitive dissonance: is my behaviour correct? In this case, the satisfaction I derive from my attire is increased when I observe that there are other people who also wear miniskirts or shrill ties. Matters are different, however, when almost everybody is dressed in the same way. There are negative externalities and my satisfaction is reduced. My desire for individuality will dominate the bandwagon effect.

Similar ideas have been expressed by Wasson (1968). There is a "drive for social approval" which may be dominated by "overadoption". Wasson uses these concepts to give an intuitive explanation of the rise and decline of demand for fashionable goods. The revival of fashions is not taken into account. Stigler and Becker (1977) use a concept similar to our individuality which they label "social distinction". Although they explain changing fashion along these lines, they rely on exogenous changes in income to induce the fashion cycle and, moreover, cannot account for re-

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<sup>1</sup> The classical negative externality is envy. The major difference between envy and the individuality effect assumed here is that the former affects the utility level in general whereas the latter affects merely the utility derived from the consumption of a particular commodity.

occurrence of fashions explicitly. These deficiencies will be overcome by the model to be analysed in the following sections.

## II. THE MODEL

We look at a society consisting of many identical individuals. They are price takers. There are two commodities one of which is a fashion good. Let their relative prices be constant. This assumption is justified, e.g., for a small open economy. Let  $p$  be the price of the fashion good in terms of the normal good and let  $y$  and  $x$  be the quantities of the fashion good and the normal good. Moreover, let income be normalized to one. Then the budget constraint is:

$$(1) \quad x + py = 1.$$

Preferences are interdependent among individuals. This implies that the consumption of the fashion good by the average individual,  $Y$ , is an argument of the utility function in addition to the individual's consumption of the two goods. Of course,  $y = Y$ , but this is not taken into account by individuals. The utility function is, for reasons of expositional simplicity, separable in fashion and non-fashion goods. Let

$$(2) \quad u(x; y, Y)$$

be the utility function. It is assumed to have the following properties:

$$(3) \quad u_x > 0, u_y > 0, u_{xy} = 0, u_{xY} = 0$$

where subscripts denote the partial derivatives. Moreover, we assume that the utility function is strictly quasiconcave in  $(x, y)$ . The partial derivatives  $u_Y$  and  $u_{yY}$  have the same signs. They are positive if there is a bandwagon effect and negative if the desire for individuality dominates.

Individual utility maximisation yields

$$(4) \quad p u_x = u_y.$$

Due to the assumption of strict quasiconcavity, this is a sufficient condition for an optimum. A socially optimal pattern of consumption could be achieved by subsidising or taxing the purchase of the fashion good depending on whether the bandwagon or the individuality effect dominates. See Layard (1980) for instance.

### III. ADJUSTMENTS TOWARDS THE EQUILIBRIUM

If every individual knows what the other individuals consume at the same time, there are no problems of adjustment. The first-order condition (4) and the budget constraint define the demand for the two goods under consideration. Matters are different if there are adjustment lags. Each individual observes what the other persons wear, then decides what she or he wants to buy and finds out in the next period that other people have made the same decision. This may be possible if there are adjustment costs, e.g. in the case of long-lived consumption goods such as cars or ties, or if shopping costs are involved.<sup>2</sup>

The adjustment proceeds as follows. At time  $t$ , the individual observes what other people have done at time  $t-1$ . Taking into account that all individuals are identical, i.e.  $y_{t-1} = Y_{t-1}$ , the first order condition can be rewritten:

$$(5) \quad p u_x(x_t, y_t, y_{t-1}) - u_y(x_t, y_t, y_{t-1}) = 0.$$

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<sup>2</sup> It would be desirable to model the investment decision more explicitly by looking at the intertemporal maximisation problem that the individual faces. In the presence of adjustment costs, there the demand for the fashion good will depend on its lagged values.



$x_t$  can be eliminated via the budget constraint (1). Total differentiation of (5) with respect to  $y_t$  and  $y_{t-1}$  then yields:

$$(6) \quad [p^2 u_{xx} + u_{yy}] dy_t = -u_{yY} dy_{t-1}$$

where the arguments of the utility function are the same as in (5). Equation (6) is a nonlinear difference equation. Two cases can be distinguished. If there is a bandwagon effect, then  $u_{yY}$  is positive and  $dy_t/dy_{t-1}$  is positive too: the larger consumption has been yesterday the larger is it today. The opposite is true in the case of the individuality effect. If many individuals consumed a lot of the fashion good yesterday, the individual chooses a low consumption today. Thus, the adjustment process alternates between high and low levels of consumption whereas in the case of the bandwagon effect it is monotonous. The path is locally stable if the absolute value of  $dy_t/dy_{t-1}$  is less than one and unstable otherwise.

As we have argued in Section I, fashion goods are characterized by positive externalities (bandwagon effects) if aggregate consumption is low, and negative externalities (individuality) if aggregate consumption is large. This implies that the transition function from  $t$  to  $t-1$  is bell-shaped. It is known that with these sort of functions, erratic dynamic behaviour, so-called chaos, may occur. See for instance Benhabib and Day (1981) and Lorenz (1989). In order to be able to analyse the determinants of dynamic behaviour in more detail, we turn to a numerical example.

#### IV. A NUMERICAL EXAMPLE

We assume a very simple quadratic utility function

$$(7) \quad u(x; y, Y) = 2x - x^2 + 2y(Y - Y^2)$$

This function is strictly quasiconcave in  $(x, y)$ . Its partial derivatives with respect to  $x$  and  $y$  are

$$(8a) \quad u_x = 2 - 2x,$$

$$(8b) \quad u_y = 2(Y - Y^2).$$

Using the first-order condition (4) and substituting for  $x$  from the budget constraint (1), one obtains the demand function for the fashion good:

$$(9) \quad y = p^{-2}(Y - Y^2).$$

The individual's consumption of the fashion good is a decreasing function of the price. The maximum consumption of shrill ties occurs if the average consumption of this commodity activity is  $1/2$ . For values less than  $1/2$ , there is a bandwagon effect. Otherwise, the individuality effect dominates.

Introducing lagged adjustments as discussed in the preceding section, one obtains

$$(10) \quad y_t = p^{-2}(y_{t-1} - y_{t-1}^2).$$

This quadratic difference equation can produce several kinds of dynamic behaviour. The equation is rather well-analysed and, therefore, we can draw on results known from the literature when we look at the dynamics of the demand for fashion goods. There is a trivial equilibrium  $y^* = 0$ . The other equilibrium,

$$(11) \quad y^* = 1 - p^2,$$

is of more interest. Of course  $p < 1$  has to hold for an interior optimum, i.e.  $y^* > 0$ . Lorenz (1989, ch. 4) shows that this equilibrium is stable as long as  $p^{-2} < 3$ . For  $p^{-2} = 3$ , a bifurcation occurs and we get a cycle of period two: demand is jumping back and forth between two values. The next bifurcation occurs at  $p^{-2} = 3.449$ , and there are cycles of period 4. As  $p^{-2}$  is increased additional bifurcations occur, and at  $p^{-2} = 3.5699$ , the behaviour of demand becomes completely erratic: there is chaos in the market. Such a case is depicted in Figure 1, where  $p^{-2} = 3.6$  is assumed. If

$p^{-2} > 4$ , the adjustment process becomes completely unstable and  $y$  leaves the interval  $[0,1]$  and our model would no longer apply.

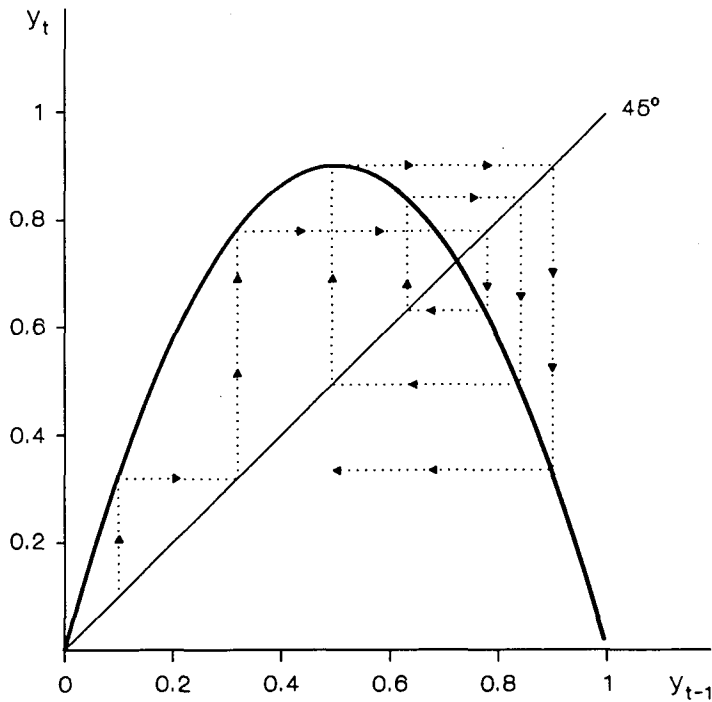


Figure 1

In the numerical example, deterministic chaos is possible if the relative price of the fashion good is rather low. In this case, the opportunity cost of the fashion good is low. The response of the individual to a deviation of aggregate consumption from its equilibrium value is relatively strong and this causes the instability. Due to the simultaneous occurrence of bandwagon and individuality effects, the adjustment trajectory is trapped in the interval  $[0,1]$ .

A similar model has been established by Benhabib and Day (1981). They consider time-interdependent preferences of an individual and end up with a difference equation very similar to (9). From an algebraic point of view, our analysis does not differ much from theirs. The interpretation, however, is rather different. They ex-

plain erratic behaviour as an individual phenomenon. Our paper, in contrast, explains this type of behaviour as a social phenomenon, which is generated by externalities in consumption.

An interpretation problem occurs if the non-fashion good is considered. Since its demand is derived as a residual in the model, it can be chaotic too. As an alternative, one could consider two fashion goods that are characterized by interdependent preferences. The expenditure constraint would then refer to the share of the budget which is spent on fashion goods. But why should this share be constant? In order to avoid such an a-priori restriction, one should consider a model with more than two goods. But such a model would yield a system of interdependent difference equations and, therefore, would not be analytically tractable anymore.

#### V. FINAL REMARKS

Fashion life cycles are often quite irregular. The paper has shown that fashions and their revivals that are seemingly random may actually be completely deterministic. The rational behaviour of utility-maximizing individuals may lead to chaotic trajectories. In this respect, our model extends earlier results obtained by Wasson (1968) and Stigler and Becker (1977). The main ingredients of the model are product-specific externalities: a bandwagon effect and a desire for individuality. Moreover, observation lags or adjustment costs are necessary to generate the dynamics of the consumption path. Of course, one should aim at dealing with adjustment costs more explicitly and future research in this area is desirable. With such an extension, our model would be much more realistic but also much more complex and difficult to interpret. Therefore, we think that this paper is only a first, albeit an important, step towards a deeper understanding of the irregularities of fashion life cycles. This does not only apply to shrill ties and miniskirts but also to fashions in academic research. The present paper itself is a contribution to one of them.

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