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# Non-comparative versus Comparative Advertising as a Quality Signal\*

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## Abstract

Two firms produce a product with a horizontal and a vertical characteristic that we call quality. The difference in the quality levels determines how the firms share the market. Consumers do not observe quality before purchase. Under non-comparative advertising a firm signals its own quality, under comparative advertising a firm signals the quality differential. In both scenarios firms may boast at a cost. In equilibrium firms actually do so, but consumers rationally infer the true quality if firms advertise. Under comparative advertising the firms never advertise together which they may do under non-comparative advertising.

Keywords: advertising, costly state falsification, signalling.

JEL: D82, K41, K42

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# 1 Introduction

Comparative advertising was illegal in many European countries until the late 1990s. By contrast, in the US comparative advertising has been encouraged by the Federal Trade Commission since the 1970's. A 1997 EU directive (Directive 97/55/EC) changed the situation in Europe by legalizing comparative advertising subject to the restriction that it should not be misleading. European Competition Authorities now tend to agree with their American counterparts in that comparative advertising is an important tool in promoting competition. Comparative advertising increases consumers' information about alternative products. It allows consumers to evaluate the performance of particular products against other products, thus enabling more informed purchasing decisions.<sup>1</sup>

Despite its importance there has been little economic analysis on comparative advertising. We will review this literature at the end of the introduction. In this paper we address the following questions. Is the content of comparative advertisement more truthful than the content of non-comparative advertisements? Are comparative ads more informative for consumers than non-comparative ads? Is the amount of advertising higher or lower under comparative or non-comparative advertising? Can the two advertising regimes be compared using welfare criteria?

To answer these questions we consider a product with a horizontal characteristic called design and a vertical characteristic which concerns the quality of a particular feature. There are three groups of consumers: established customers who are loyal to firm 1, established customers who are loyal to firm 2, and new consumers. Only new consumers care about the product's feature; they have higher willingness-to-pay than established consumers. Each firm faces the following trade-off: either it serves both, loyal and new consumers at a low price, or it skims off the surplus on new customers by charging a

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<sup>1</sup>See Barigozzi and Peitz (2006) for more details.

high price. By assuming that there are more loyal than new customers, we ensure that firms prefer the first alternative.

As an example think of cell phones. Customers tend to be loyal because they incur switching costs when they use a different operating system. As the feature take the cell phone's camera. It is virtually impossible these days to get a cell phone without a camera even though a lot of customers never use it. Yet some consumers, say, youngsters use the camera and care about its quality. The cost of adding the camera is small and it allows firms to gain market share in this segment of the market. Similarly, a vacation resort may build additional sports facilities to attract new customers; it refrains from increasing prices in order not to lose loyal customers.

The two firms compete for new customers by advertising their quality which consumers do not observe before purchase. We compare two scenarios. In the first, firms can only engage in non-comparative advertising, i.e., a firm may send a message about its own quality but not about the competitor's one. In the second scenario, the firms can also engage in comparative advertising, aimed at influencing the perceived quality differential because this is what ultimately matters to consumers. In both scenarios the firms may attempt to mislead at a cost. The cost of misleading increases the further a firm moves away from the truth, i.e., the more a firm makes exaggerated claims. Moreover, in both scenarios firms may choose not to advertise.

In the non-comparative framework a firm advertises if its quality level is above a threshold. In the message it boasts about its quality. Consumers rationally anticipate this boasting and infer the true quality level. Thus, if firms advertise, the equilibrium is revealing, yet firms attempt to mislead. Stated differently, because the marginal cost of slightly distorting the truth is negligible but the marginal return is not, consumers expect some boasting, leading firms to do so systematically. If the quality level is below the threshold, a firm does not advertise; the cost of sending the message is higher than the gain thereof. If both firms have low quality levels, neither advertises and

they share the market equally. If one firm has a high quality level whereas the other one has a low one, the high quality firm advertises while the low quality one remains silent. The high quality firm has more customers than the low quality one. If both firms have high quality levels, both advertise. The last case may be highly inefficient: if both firms have the same high quality level, both advertise at a cost yet still share the market equally.

In the second scenario firms may also engage in comparative advertising meaning that firms send signals about the quality differential. An advertising firm wants to persuade consumers that the quality differential is in its favor. When both advertising formats are possible, consumers interpret non-comparative advertisements as implying that the quality differential is actually small; had it been high, the firm would have sent a comparative message. Therefore, firms do not use non-comparative advertising; they either send comparative or no messages at all. If the quality differential is sufficiently small, both firms do not advertise. If the quality differential is, however, large, the high quality firm advertises while the low quality one is silent. If a firm advertises, it attempts to boost the quality differential in its favor. Consumers account for this and infer the true value. In equilibrium the firms never advertise together.

Comparative advertising tends to perform better than non-comparative advertising in our set-up. Firms do not advertise if the quality differential is small and the information is of little value to consumers. If, however, the quality differential is large, the high quality firm advertises while the low quality one remains silent. There is no duplication of advertisement expenditures. By contrast, under non-comparative advertising firms advertise their high quality independently of their rival's quality level. Both firms may advertise even when the information is of little or no value to consumers.

Non-comparative advertising results in a two-sender simultaneous signalling game with a continuum of types where a type is given by the true quality. Comparative advertising yields a two-sender game with perfectly

correlated types, i.e., the actions of both firms provide information about the same quantity. Since signals are non-stochastic, the true state is inferred in both contexts if firms advertise.

Let us now review the literature. The marketing literature has discussed comparative advertising quite extensively; see Grewal et al. (1997) for a survey. There is, however, little economics literature on comparative advertising. Aluf and Shy (2001) use a Hotelling-type model. Comparative advertising shifts transportation cost to the rival's product. The model does not deal with the informative role of advertising.

Anderson and Renault (2009) consider comparative advertising with respect to horizontal characteristics. If qualities are sufficiently different, the low quality firm will disclose horizontal attributes of both products. The main difference to our approach is that advertising is costless and firms can only communicate verifiable evidence but may attempt to mislead by disclosing only what they see fit.

Barigozzi, Garella, and Peitz (2009) consider an incumbent with known quality facing an entrant with unknown quality. The entrant can choose generic advertising which is standard money burning to signal quality. Moreover, the entrant can choose comparative advertising which involves a comparison of the two firms' qualities. If the entrant uses comparative advertising, the incumbent may sue hoping to obtain damages. If the entrant uses comparative rather than generic advertising, he signals to consumers that he has a strong case. It turns out that comparative advertising can signal quality in those cases where generic advertising cannot. An important difference to our model is that only the entrant can choose to advertise.

Anderson et al. (2010 a,b) empirically study advertising in the US over-the-counter analgesics industry. Almost half the ad spending in their sample was on comparative advertisements; all firms used comparative ads. Brands with better characteristics transmit more information. Comparative ads contain significantly more information than non-comparative self-promoting ad-

vertisements. The evidence that all firms use comparative advertising is at odds with our finding that only one firm does so. One possible explanation is that in the considered markets quality is multi-dimensional and firms claim superiority in dimensions where they perform better.

More generally, our analysis is related to the industrial organization literature on advertising as quality disclosure or quality signalling. Levin, Peck, and Ye (2009) analyze a duopoly where firms can disclose their true quality by presenting costly verifiable evidence in the form of, e.g., certification from a third party. In Daughety and Reinganum (2008), a monopolist may choose between costly disclosure or signalling quality through prices.

There is an important literature, going back to Milgrom and Roberts (1986), on quality signalling via prices or advertising as money burning. By and large, however, this literature has dealt with the case of a monopolist, i.e., it has considered one-sender games. A recent exception is Daughety and Reinganum (2007) who consider signalling through prices in a duopoly. Two other exceptions, more closely related to the present analysis, are Hertzendorf and Overgaard (2001) and Fluet and Garella (2002). In these papers the duopolists know each other's quality. In the resulting equilibria, signalling is either through prices alone or through the price-advertising mix. In the present paper, signalling through prices is not feasible. Moreover, we focus on the case where both firms may jointly signal about the same quantity, namely the quality differential.

The remainder of this paper is organized as follows. In the next section we describe the model and derive the equilibrium prices. Section 3 analyzes non-comparative and section 4 comparative advertising. Section 5 compares welfare. Section 6 concludes.

## 2 The Model

Consider two firms 1 and 2. They produce products having two characteristics. The first characteristic is horizontal; we will call it design. The two firms produce different designs: firm 1 design 1 and firm 2 design 2. A computer may use the Mac OS X or the Microsoft Windows operating system; a smartphone may run the Symbian or the Android platform; a vacation resort may be located in the mountains or at the seaside; a cigar may be produced in Cuba or the Dominican Republic. The second characteristic is vertical and concerns the quality of a particular feature; for short, we will refer to it as firm  $i$ 's quality  $q_i \in [0, 1]$ ,  $i = 1, 2$ . Production costs are normalized to zero. Firm  $i$  charges the price  $p_i$ ,  $i = 1, 2$ .

Consumers have unit demand. There are two groups of consumers: established customers who do not care about the feature and new customers who do value the feature's quality.

Let us start with the established customers. For each firm there is a mass  $M$  of loyal established consumers. They do not care about the feature. The first (second) group consists of consumers with  $U = 1$  if they purchase design 1 (resp. 2) and  $U = 0$  if they purchase design 2 (resp. 1). When he purchases the product, a consumer's net utility is  $U$  minus the price; otherwise, it is zero. If one firm sets its price equal to 1, its rival can attract all the quality indifferent consumers only by setting a price at most equal to 0. The poacher's profit with respect to quality indifferent consumers amounts to 0, which is less than the profit of  $M$  earned by setting a price of 1 and selling only to its own loyal clientele. In the absence of quality conscious consumers, the Bertrand equilibrium prices are therefore equal 1.

Let us now turn to new customers who care about design and quality and thus have higher willingness-to-pay. New customers have mass 1. A consumer's utility is given as



$$U = \begin{cases} -\theta + 2q_1 + 1 - p_1, & \text{if he buys from firm 1;} \\ \theta + 2q_2 + 1 - p_2, & \text{if he buys from firm 2;} \\ 0, & \text{if he does not buy.} \end{cases}$$

where  $\theta$  is uniform on  $[-\hat{\theta}, \hat{\theta}]$ ,  $\hat{\theta} > 1$ . The parameter  $\theta$  measures the intensity with which a consumer cares about design. If  $\theta$  is close to zero, design is not of great importance for the consumer and he cares more about quality. By contrast, if  $\theta$  is close to  $\hat{\theta}$  ( $-\hat{\theta}$ ), the consumer is a design 2 (1) aficionado for whom quality is of minor importance. The larger  $\hat{\theta}$ , the more the average new consumer cares about design.  $\theta$  is private information.<sup>2</sup>

To derive demand consider first the case where both prices are low enough so that the market is covered. Suppose for the time being that quality is observable before purchase. Which design a consumer chooses depends on prices, on his  $\theta$ , and on the difference in quality levels  $x = q_2 - q_1$ : the consumer  $\theta$  buys from firm 2 rather than from firm 1 if  $\theta + 2q_2 - p_2 \geq -\theta + 2q_1 - p_1$  or  $\theta \geq -x + .5(p_2 - p_1)$ .

Firm 1's demand (market share) from quality conscious consumers is

$$\int_{-\hat{\theta}}^{-x+.5(p_2-p_1)} \frac{1}{2\hat{\theta}} d\theta = \frac{1}{2} - \frac{x}{2\hat{\theta}} - \frac{p_2 - p_1}{4\hat{\theta}};$$

firm 2's demand is

$$\int_{-x+.5(p_2-p_1)}^{\hat{\theta}} \frac{1}{2\hat{\theta}} d\theta = \frac{1}{2} + \frac{x}{2\hat{\theta}} - \frac{p_1 - p_2}{4\hat{\theta}}.$$

To explain the effects of the quality differential, suppose both firms charge the same price. If  $q_1 = q_2$ , equivalently  $x = 0$ , both firms share the market of new consumers; if  $q_1 < q_2$  or  $x > 0$ , firm 2 has more than half of the market; if  $q_1 > q_2$  or  $x < 0$ , firm 1 has more than half of the market. The marginal impact of  $x$  on profits is  $+(-)1/2\hat{\theta}$ : the less consumers care about design (the smaller  $\hat{\theta}$ ), the higher the impact of the quality differential.

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<sup>2</sup>The average of  $|\theta|$ , i.e., the average preference intensity for one design or the other, is  $\hat{\theta}/2$ .

Next let us look at the case where firm 2 has a monopoly. Moreover, suppose  $q_2 = 1$ . We thus consider the best of all worlds for firm 2: it has a monopoly with the highest possible quality level and consumers are aware of this quality so that the firm incurs no advertising expenditures. Consumer  $\theta$  buys 2's product if  $\theta + 3 - p_2 \geq 0$ . On the market segment of quality conscious consumers firm 2 thus faces demand  $.5 - (1/2\hat{\theta})(p_2 - 3)$ . Maximizing profits with respect to this group of consumers yields  $p_2^* = \hat{\theta}/2 + 1.5 > 1$  and profits  $\pi_2^* = \hat{\theta}/8 + .75 + 9/8\hat{\theta}$ .  $\pi_2^*$  is the upper bound on profits that can be made with new customers; firm 2's profit can only be lower if firm 1 competes and/or consumers do not observe the feature's quality so that the firm has to advertise.

Finally, take the two segments together. We may now state a preliminary result.

**Lemma 1:** *If  $M > \pi_2^*$ , firms charge  $p_i = 1$ ,  $i = 1, 2$  in equilibrium.*

This result follows immediately. Each product is sold at a uniform price, i.e., firms cannot discriminate between quality conscious and quality indifferent consumers. If firm 2 charges  $p_2 = 1$ , it serves at least its established customers who generate profit  $M$ . If it charges  $p_2 > 1$ , it loses its established customers and serves, if at all, only new customers. The maximum profit it can make on this market segment is  $\pi_2^*$ . If  $M > \pi_2^*$ , firm 2 prefers to serve both market segments, which it optimally does by charging  $p_2 = 1$ ; by symmetry, this condition also ensures that firm 1 charges  $p_1 = 1$  in equilibrium. We assume  $M > \pi_2^*$  so that Lemma 1 holds.

It remains to be explained why a firm doesn't produce two versions of the product, one with the feature and one without. Here we assume that there are economies of scale in production, making one large production run cheaper than two small ones. Finally, firms add the feature to the product not to get higher prices as we just explained, but to gain market share. If, say, firm 2 adds the feature while 1 doesn't, 2 gains the market segment of

quality conscious customers; if 2 drops the feature while 1 hangs on to it, 2 loses its share of the quality conscious consumers. Therefore, if the cost of the feature is low, it is indeed optimal for both firms to add the feature to the product.

Let us now turn to the information structure. Quality conscious consumers know the designs but do not observe the quality levels: the products are experience goods and consumers find out the actual quality only after they have purchased.<sup>3</sup> We assume that  $\tilde{q}_1$  and  $\tilde{q}_2$  are independent and uniform on  $[0, 1]$ . Without any additional information consumers expect  $E(\tilde{q}_1) = E(\tilde{q}_2) = .5$  and the firms share the market. Unless  $q_1 = q_2$ , this allocation is inefficient. If consumers learn, say,  $x > 0$ , then consumers with  $\theta \in [-x, 0]$  buy from firm 2 rather than firm 1. When they buy from 1, their surplus is  $\int_{-x}^0 (-\theta + 2q_1)/2\hat{\theta} d\theta$ ; buying from 2 generates the surplus  $\int_{-x}^0 (\theta + 2(q_1 + x))/2\hat{\theta} d\theta$ . Becoming informed about  $x$  thus increases surplus by  $x^2/2\hat{\theta}$ . This expression also applies when  $x < 0$ .

Informing consumers about quality therefore not only redistributes profits, but typically also enhances efficiency. To put it differently, advertising quality in our set-up is on the one hand combative, acting to redistribute consumers among firms; on the other hand it is informative, increasing consumer surplus.<sup>4</sup>

In the sequel, we analyze how firms compete through advertising in order to increase their market share of quality conscious consumers. The timing is as follows. In stage 0, the firms learn their qualities and consumers learn their type. In stage 1, the firms simultaneously send messages about the qualities; this includes the possibility of saying nothing. In stage 2, consumers observe the messages, draw inferences, and make their purchasing decisions.

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<sup>3</sup>As in Milgrom and Roberts (1986), quality may be interpreted as the probability that a consumer is fully satisfied with the product, which is non verifiable. Therefore, a warranty is not feasible.

<sup>4</sup>For a survey of the different views on advertising see, e.g., Bagwell (2007).

### 3 Non-comparative advertising

In this section each firm may inform consumers about its own quality but not about the quality of its competitor. A firm advertises its quality by sending a message of the form  $y_i \geq 0$ ,  $i \in \{1, 2\}$ . Such a message should be thought of as an argument or evidence rendering  $q_i$  plausible: the larger  $y_i$ , the more the firm claims about its quality. Think of  $y_i$ , for example, as the results of product reviews by consumer magazines and the like, or customer satisfaction indices.

The cost of advertising consists of two components. First, advertising involves a fixed cost  $\gamma \in (0, 1/4\hat{\theta})$  which captures the cost of actually placing the ad in media.<sup>5</sup> The second component is the cost of producing the contents of the message. This cost is related to the actual quality  $q_i$ . We assume it to be of the form  $.5(y_i - q_i)^2$  if  $y_i \geq q_i$  and 0 otherwise. This variable cost captures the idea that an attempt to overstate quality is more costly than simply reporting the naked truth as it involves more fabrication; understating quality entails no cost. With the quadratic function the cost of misrepresenting the evidence increases at an increasing rate the more disconnected claims are from the truth: it becomes more difficult to produce the corresponding evidence, or advertising agencies charge more the more they embellish.<sup>6</sup> Note that the marginal cost of sending the signal  $y_i$  is decreasing in the true quality so that the single-crossing property is satisfied. If firm  $i$  chooses to advertise, its costs are therefore  $C(y_i, q_i) = \gamma + .5(y_i - q_i)^2$  if  $y_i \geq q_i$  and  $C(y_i, q_i) = \gamma$  otherwise. If firm  $i$  does not advertise, we will write  $\emptyset_i$ . In what follows, and abusing notation somewhat,  $y_i$  is either a true

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<sup>5</sup>The upper bound on  $\gamma$  ensures that firms advertise at all; see below.

<sup>6</sup>In the case of product reviews the firm has to bribe the reviewer. For example, Microsoft has given away thousands of their new Xbox 360 Slim units to the press, apparently to get good reviews; see [www.product-reviews.net/2010/06/17/microsoft-xbox-360-slim-giveaway-gift-or-bribe](http://www.product-reviews.net/2010/06/17/microsoft-xbox-360-slim-giveaway-gift-or-bribe). With customer popularity indices the firm has to produce fake customer reviews. For example, TripAdvisor is now warning you that numerous hotels are submitting phony recommendations for themselves; see [www.travelweekly.com/Travel-News/Online-Travel/TripAdvisor-faces-renewed-heat-about-integrity-of-hotel-reviews](http://www.travelweekly.com/Travel-News/Online-Travel/TripAdvisor-faces-renewed-heat-about-integrity-of-hotel-reviews).

advertising claim  $y_i \in \mathbb{R}_+$  or the no-advertising “message”  $\emptyset_i$ .

In the first stage of the game firms choose simultaneously whether or not to advertise. In the second stage consumers observe the firms’ actions and form beliefs  $E(x|y_1, y_2) = E(q_2|y_2) - E(q_1|y_1)$ . The random variables  $\tilde{q}_1$  and  $\tilde{q}_2$  are independent, meaning that the observation of the realization  $q_1$  provides no information about the realization of  $q_2$  and vice versa. We assume that this independence property carries over to the signals send by the firms. Thus, consumers’ expectation about the quality of firm 1 depends only on firm 1’s actions and is independent of what firm 2 does; similarly, consumers’ expectation about the quality of firm 2 depends only on firm 2’s actions. Note that such an equilibrium rules out even “implicit” comparative advertising.

Consumers buy from the firm maximizing expected utility, i.e., consumers with  $\theta < E(x|y_1, y_2)$  buy design 1 and the rest design 2. If firm 1 doesn’t advertise, its profit is  $.5 - E(x|\emptyset_1, y_2)/2\hat{\theta}$ ; if it sends the message  $y_1$  profit is  $.5 - E(x|y_1, y_2)/2\hat{\theta} - \gamma - .5(y_1 - q_1)^2$ . Firm 2’s profits are correspondingly  $.5 + E(x|y_1, \emptyset_2)/2\hat{\theta}$  and  $.5 + E(x|y_1, y_2)/2\hat{\theta} - \gamma - .5(y_2 - q_2)^2$ . Firms choose their advertising strategy so as to maximize expected profits. We focus on perfect Bayesian equilibria with minimum signalling costs; minimal signalling costs implies that the lowest type who signals advertises the true quality.

Let us now derive the least cost signalling strategies  $y(q_i)$ ,  $i \in \{1, 2\}$ . Suppose firms play a revealing strategy  $\hat{y}(q_i)$  for some  $q_i \geq q^0$  where  $q^0$  is a threshold yet to be determined. Revealing means  $E(q_i|\hat{y}(q_i)) \equiv q_i$ . Consider, say, firm 2 and denote by  $\pi_2(y_2, q_2; y_1)$  its profit when it sends the message  $y_2$  and its actual quality is  $q_2$ , given that its rival sends the message  $y_1$ . If firm 2 wants to mimic as  $q'_2$ , its profit is

$$\begin{aligned} \pi_2(\hat{y}(q'_2), q_2; y_1) &= \frac{1}{2} + \frac{1}{2\hat{\theta}}[q'_2 - E(q_1|y_1)] - \gamma - \frac{1}{2}(\hat{y}(q'_2) - q_2)^2 \\ &= \frac{q'_2}{2\hat{\theta}} - \frac{1}{2}(\hat{y}(q'_2) - q_2)^2 + \text{constant} \end{aligned}$$

where the constant denotes the terms that do not depend on 2’s actions or

type. Since the strategy is revealing, firm 2's profit must be maximized at  $q'_2 = q_2$ . This implies that for all  $q_2$  in the separating range

$$\left. \frac{\partial \pi_2}{\partial q'_2} \right|_{q'_2=q_2} = \frac{1}{2\hat{\theta}} - (\hat{y}(q_2) - q_2)\hat{y}'(q_2) = 0 \text{ and}$$

$$\left. \frac{\partial^2 \pi_2}{\partial q_2^2} \right|_{q'_2=q_2} = -(\hat{y}(q_2) - q_2)\hat{y}''(q_2) - (\hat{y}'(q_2))^2 \leq 0.$$

If  $\hat{y}(q_2)$  is revealing, it must solve the above first-order condition for all  $q_2 \geq q^0$ . It is easier to work with the inverse function  $q_2 = \varphi(y_2)$ , noting that the inverse exists since  $\hat{y}(q_2)$  must be strictly increasing. In terms of the inverse, the conditions are rewritten as

$$\varphi'(y_2) - 2\hat{\theta}(y_2 - \varphi(y_2)) = 0 \text{ and} \tag{1}$$

$$\varphi''(y_2) \leq 2\hat{\theta}. \tag{2}$$

The general solution to the differential equation (1) is

$$\varphi(y_2) = Ke^{-2\hat{\theta}y_2} + y_2 - \frac{1}{2\hat{\theta}} \tag{3}$$

where  $K$  is a constant. Condition (2) requires

$$Ke^{-2\hat{\theta}y_2} \leq \frac{1}{2\hat{\theta}}. \tag{4}$$

Signalling costs (excluding the fixed cost) are

$$\frac{1}{2}(y_2 - \varphi(y_2))^2 = \frac{1}{2} \left( \frac{1}{2\hat{\theta}} - Ke^{-2\hat{\theta}y_2} \right)^2.$$

Next we derive the threshold  $q^0$  at which the firm starts to advertise. Let  $y_0$  be the smallest message sent, corresponding to  $q^0$ . Signalling costs are minimized if  $y_0 = q^0$ . This implies

$$K = \frac{e^{2\hat{\theta}y_0}}{2\hat{\theta}} = \frac{e^{2\hat{\theta}q^0}}{2\hat{\theta}}.$$

Observe that (4) is then always satisfied given that  $y_2 \geq q^0$ .<sup>7</sup> We can now rewrite (3) as

$$q_2 = y_2 - \frac{1 - e^{-2\hat{\theta}(y_2 - q^0)}}{2\hat{\theta}}.$$

It is easily verified that  $y_2$  is strictly increasing in  $q_2$ , with  $y_0 = q^0$ ; see the discussion below.

When the firm does not advertise, consumers' expectation about its quality is  $E(q_2|\emptyset_2) = q^0/2$ . The firm's profit is then

$$\pi_2(\emptyset_2, q_2; y_1) = \frac{(q^0/2)}{2\hat{\theta}} + \text{constant}.$$

The threshold is such that a firm with actual quality  $q^0$  is just indifferent between advertising and not. If it does and sends the message  $y_0 = q^0$ , its profit is

$$\pi_2(y_0, q^0; y_1) = \frac{q^0}{2\hat{\theta}} - \gamma + \text{constant}.$$

Indifference between advertising and not yields  $q^0 = 4\hat{\theta}\gamma$ . A firm with quality above this threshold prefers to advertise, the converse holds if quality is below the threshold. The analysis is the same for firm 1.

The equilibrium strategies for  $i \in \{1, 2\}$  are, therefore,  $\emptyset_i$  if  $q_i < 4\hat{\theta}\gamma$  and  $y_i \geq q_i$  satisfying

$$q_i = y_i - \frac{1 - e^{-2\hat{\theta}(y_i - 4\hat{\theta}\gamma)}}{2\hat{\theta}} \quad (5)$$

when  $q_i \geq 4\hat{\theta}\gamma$ .

Equilibrium beliefs are  $E(q_i | \emptyset_i) = 2\hat{\theta}\gamma$  and

$$E(q_i | y_i) = y_i - \frac{1 - e^{-2\hat{\theta}(y_i - 4\hat{\theta}\gamma)}}{2\hat{\theta}}, \text{ for } y_i \geq 4\hat{\theta}\gamma.$$

Quality is perfectly revealed in this case.

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<sup>7</sup>Using a similar argument as in Mailath (1987), the signalling strategy derived here can be shown to be a global maximum of the firm's problem.

Out-of-equilibrium beliefs need to be defined only for a message  $y_i < 4\hat{\theta}\gamma$ . One possibility is

$$E(q_i | y_i) \leq y_i, \text{ for } y_i < 4\hat{\theta}\gamma.$$

To sum up:

**Proposition 1:** *In the least-cost equilibrium if  $q_i < 4\hat{\theta}\gamma$ , firm  $i$  chooses  $\emptyset_i$ ; consumers correctly expect  $E(q_i | \emptyset_i) = 2\hat{\theta}\gamma$ . If  $q_i \geq 4\hat{\theta}\gamma$ , firm  $i$  sends the message  $y_i \geq q_i$  solving (5); consumers infer the true quality level  $q_i$ ,  $i \in \{1, 2\}$ .*

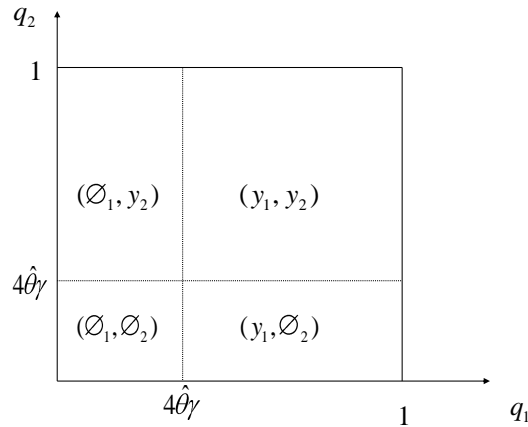


Figure 1: Non-comparative advertising

The outcome is depicted in Figure 1. If both firms have quality levels below  $4\hat{\theta}\gamma$ , neither advertises. Consumers rationally expect quality  $2\hat{\theta}\gamma$  of each firm. The more dispersed the consumers (the higher  $\hat{\theta}$ ) or the higher the fixed cost  $\gamma$ , the larger the non-advertising range. If one firm's quality level is below while the other firm's is above this threshold, the high quality one advertises while the low quality one doesn't. Consumers infer the quality of the advertising firm and expect  $2\hat{\theta}\gamma$  of the non-advertising one. When both quality levels are above the threshold, both firms advertise. Consumers infer both quality levels and thus the true quality differential. The game has prisoners' dilemma features. If, say,  $q_1 = q_2 > q^0$ , both firms advertise yet



share the market. They spend resources on boasted messages without raising consumer surplus.

Let us now look at the message  $y_i$  in more detail. Solving (5) yields

$$y_i = q_i + \frac{1}{2\hat{\theta}}(1 + \text{ProductLog}(-e^{2\hat{\theta}(4\hat{\theta}\gamma - q_i) - 1}))$$

where the ProductLog is the inverse function of  $f(w) = we^w$ .<sup>8</sup> We have  $y_i(q^0) = q^0$ , i.e., at the threshold  $q^0 = 4\hat{\theta}\gamma$  the firm states the true quality at zero variable cost. For  $q_i \in (q^0, 1]$ ,  $y_i(q_i) \in (q_i, q_i + 1/2\hat{\theta})$ . Firms boast quality; boasting increases with quality, yet at a decreasing rate. See Figure 2. Except for the threshold, if a firm advertises, it attempts to overstate its quality. For  $y_i > y_i(q^0)$ ,  $E(q_i|y_i)$  is strictly monotone in  $y_i$ : different  $q_i$ 's give rise to different  $y_i$ 's to which consumers react by computing the correct expectation. Because  $E(q_i|y_i)$  is increasing in  $y_i$  and the marginal cost of overstating is zero around the true value, it pays for firm  $i$  to exaggerate if it advertises.

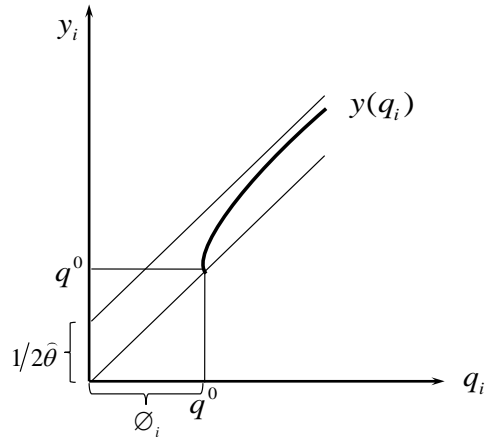


Figure 2: The least cost signalling strategy  $y(q_i)$

Two remarks are in order. First, under a properness restriction on consumers' out-of-equilibrium beliefs an equilibrium cannot be totally uninformative (Myerson (1978)). To see this, suppose on the contrary that consumers

<sup>8</sup>See, e.g., [http://en.wikipedia.org/wiki/Lambert's\\_W\\_function](http://en.wikipedia.org/wiki/Lambert's_W_function) for more on the ProductLog.

believe  $E(q_i|y_i) = 1/2$  for  $\emptyset_i$  and for all  $y_i$ . If firms actually choose not to advertise, consumers' beliefs are borne out in equilibrium. Now suppose for simplicity that, say, firm 1 has either quality  $1/4$  or  $3/4$  and can send only the two signals  $1/4$  and  $3/4$ . Sending signal  $y_1 = 3/4$  is a “big” mistake for the type  $1/4$ -firm and a “small” mistake for the type  $3/4$ -firm at the proposed equilibrium. If consumers believe that firm 1 makes the small mistake with probability  $\epsilon$  and the big mistake with probability  $\epsilon^2$ , they should expect upon observing  $y_1 = 3/4$  the quality  $(3/4 + \epsilon/4)/(1 + \epsilon)$  which converges to  $3/4$  for  $\epsilon$  going to zero.

Second, in our least cost signalling equilibrium the extent of boasting ( $y_i - q_i$ ),  $i \in \{1, 2\}$  is minimal. In particular, the amount of boasting is zero at the threshold which defines  $q^0 = 4\hat{\theta}\gamma$ . If we restrict, e.g., firms to play linear strategies, the equilibrium is given by  $y_i = q_i + 1/2\hat{\theta}$ ,  $i = 1, 2$ . For these linear strategies  $q^0 = 4\hat{\theta}\gamma + 1/4\hat{\theta}\gamma$ , i.e., the range where firms do not advertise is larger than with our least cost strategies. Since the extent of boasting is minimal given firms advertise, the range where firms do not advertise  $[0, 4\hat{\theta}\gamma]$  is also minimal. Put differently, in our equilibrium the signaling cost is minimal and, therefore, the range where firms advertise is maximal.

## 4 Comparative Advertising

Under comparative advertising, firms send messages  $z_i$ ,  $i \in \{1, 2\}$  about the difference in quality levels  $x = q_2 - q_1$ , the quantity consumers are ultimately interested in. Now firm 1 has an incentive to distort  $x$  downwards while firm 2 wishes to boost upwards. Given that  $\tilde{q}_1$  and  $\tilde{q}_2$  are independent and uniform on  $[0, 1]$ ,  $\tilde{x}$  has density

$$f(\tilde{x}) = \begin{cases} 1 + x, & \text{if } x \in [-1, 0); \\ 1 - x, & \text{if } x \in [0, 1]. \end{cases}$$

Suppose each firm may fabricate false evidence about its own product but not about the product of its competitor.<sup>9</sup> Define  $z_1 = -(y_1 - q_2)$  and  $z_2 = (y_2 - q_1)$ . Then our falsification cost function  $c(y_i, q_i)$  yields for firm 1  $c(z_1, x) = \gamma + .5(z_1 - x)^2$  for  $z_1 \leq 0$  and  $\gamma$  otherwise, and for firm 2  $c(z_2, x) = \gamma + .5(z_2 - x)^2$  for  $z_2 \geq 0$  and  $\gamma$  otherwise. Since firm 1 wants to distort downwards and firm 2 upwards, we work in the following as a shortcut with the cost function  $c(z_i, x) = \gamma + .5(z_i - x)^2$ ,  $i = 1, 2$ .

Next, we need some structure on out-of-equilibrium beliefs. We assume that at an out-of-equilibrium information set consumers believe that it was reached with the minimum number of deviations from the equilibrium strategies. A similar restriction on beliefs, which we call the *minimality condition* has been used by Bagwell and Ramey (1991), Schultz (1999), or Emons and Fluet (2009).

Firm  $i, i = 1, 2$  may remain silent  $\emptyset_i$ , it may send a non-comparative message  $y_i$ , or a comparative advertisement  $z_i$ . The choice of the advertising format obviously has informational content. To see this, suppose firm 1 is silent while firm 2 sends  $y_2(q_2)$ . In a revealing equilibrium consumers infer  $q_2$ . Suppose, for the sake of the argument,  $q_2$  is high. Consumers have to form beliefs about  $q_1$ . They use the information that firm 2 could have used comparative advertising but preferred not to do so. If  $q_1$  is small and thus  $x$  large, 2 will most likely use comparative advertising. Accordingly, the non-comparative advertisement suggests that  $q_1$  is also large and thus  $x$  small.

In the Appendix we make these arguments more precise. To avoid the intricacies of multi-dimensional signalling, we consider the pure disclosure set-up where firms can disclose the truth at a cost  $\gamma$ , but cannot falsify as such. For this disclosure game we show that  $(\emptyset_1, q_2)$  implies that  $q_1 = 1$ . This result rests on the following reasoning. For whatever expectation consumers have about  $q_1$ , firm 2 prefers to reveal  $q_2$  if the actual  $q_1$  is above the expected

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<sup>9</sup>Suppose, for example, that if firm 1 makes false claims about 2's quality, firm 2 can sue and prove the wrong claim because it has sufficient documentation about its own product; firm 2 cannot sue wrong claims about 1's quality because it lacks the documentation.

value and  $x$  if  $q_1$  is below this expectation. Put differently, if the true  $x$  is higher than what consumers expect, firm 2 reveals  $x$ ; if it is lower, 2 tries to leave consumers in the dark by revealing only  $q_2$ . Thus, consumers expect the highest  $q_1$  for which 1 is silent which, in turn, implies that 1 is silent for all possible quality levels.

In the signalling game the reasoning is more complicated because signalling costs are not constant as are disclosure cost in the disclosure set-up. Nevertheless, to capture the idea that 2 wants to hide a low  $x$  by non-comparative advertising, we assume that consumers expect  $q_1 \geq q_2$  when they observe  $(\emptyset_1, y_2(q_2))$  and vice versa for firm 1. It turns out that with these beliefs consumers sufficiently punish non-comparative advertisers so that firms do not play  $y_i$ ,  $i = 1, 2$  in a revealing equilibrium. The formal derivation of the following results is relegated to the Appendix.

**Lemma 2:** *Suppose  $E(q_i|\emptyset_i, y_j(q_j)) \geq q_j, i = 1, 2, i \neq j$ . Then firms will not engage in non-comparative advertising in a revealing equilibrium*

Firms thus are silent or engage in comparative advertising. Suppose firm 2 follows the strategy  $\emptyset_2$  for  $x < x_2^0$  and  $z_2(x)$  otherwise; firm 1's strategy is  $\emptyset_1$  for  $x > x_1^0$  and  $z_1(x)$  otherwise. Let  $z_i(x)$  be increasing, whether  $i$  advertises alone or both firms advertise together,  $i \in \{1, 2\}$ . Again we rule out totally unrevealing equilibria. This implies  $x_2^0 < 1$  or  $x_1^0 > -1$  or both. There is thus some range where at least one firm sends a signal and  $x$  is revealed.

First we show that under comparative advertising the firms never advertise together.

**Lemma 3:** *In equilibrium the firms never advertise together, i.e.,  $x_1^0 < x_2^0$ .*

To show this we assume that firms advertise together. Yet it always pays for a firm to deviate because this either changes the expected quality differential in its favor or signalling costs fall by more than revenues.

We can now state the least-cost signalling equilibrium. The equilibrium has the following structure:

- (i) If  $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$ , neither firm advertises and beliefs are  $E(x | \emptyset_1, \emptyset_2) = 0$ .
- (ii) If  $x \geq 2\hat{\theta}\gamma$ , firm 1 plays  $\emptyset_1$  and firm 2 sends the signal  $z_2 \geq x$  solving

$$x = z_2 - \frac{1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)}}{2\hat{\theta}}. \quad (6)$$

Beliefs are

$$E(x | \emptyset_1, z_2) = z_2 - \frac{1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)}}{2\hat{\theta}}.$$

- (iii) If  $x \leq -2\hat{\theta}\gamma$ , firm 2 plays  $\emptyset_2$  and firm 1 sends the signal  $z_1 \leq x$  solving

$$x = z_1 + \frac{1 - e^{2\hat{\theta}(z_1 + 2\hat{\theta}\gamma)}}{2\hat{\theta}}. \quad (7)$$

Beliefs are

$$E(x | z_1, \emptyset_2) = z_1 + \frac{1 - e^{2\hat{\theta}(z_1 + 2\hat{\theta}\gamma)}}{2\hat{\theta}}.$$

To sum up:

**Proposition 2:** *In the least-cost equilibrium if  $x \in [-1, -2\hat{\theta}\gamma]$ , firm 1 sends the message  $z_1$  solving (7) while firm 2 doesn't advertise. If  $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$ , neither firm advertises. If  $x \in [2\hat{\theta}\gamma, 1]$ , firm 2 sends the message  $z_2$  solving (6) and firm 1 doesn't advertise. If one firm advertises, consumers infer the true quality differential; if both firms do not advertise, consumers rationally expect a quality differential of zero.*

The outcome is depicted in Figure 3. First note that unlike in the case of non-comparative advertising, the firms never advertise together. When  $|x|$  and thus the informational value to consumers is small, the firms do not advertise. Only when  $|x|$  is sufficiently large, the firm with the better quality advertises while the other firm remains silent.

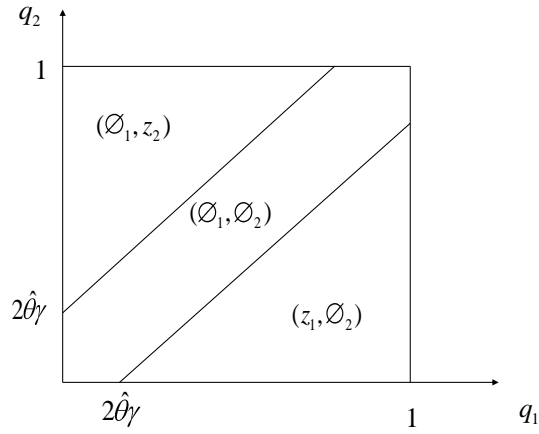


Figure 3: Comparative advertising

Here we have again that if a firm advertises, except at the threshold, it falsifies and the outcome if one firm advertises is the one with minimal falsification. The logic is the same as described in the previous section. See Figure 4.

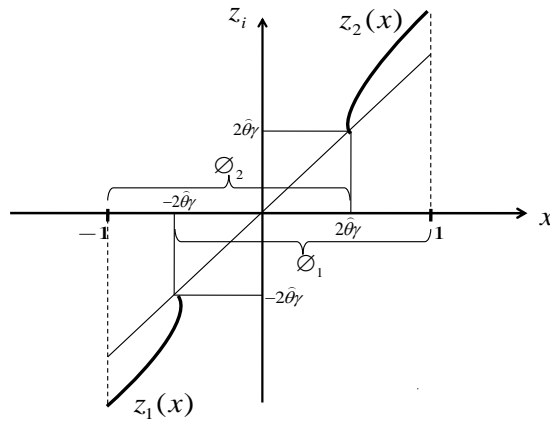


Figure 4: The least cost signalling strategies  $z_1(x)$  and  $z_2(x)$

## 5 Welfare

To compare the welfare properties of our least-cost equilibria under non-comparative and comparative advertising consider Figure 5.

Under non-comparative advertising no firm advertises in the areas  $a$  and  $b$ ; one firm advertises in the areas  $c_1$ ,  $c_2$ , and  $d$ , providing imperfect information about the quality differential  $x$ ; in the areas  $e$  and  $f$  both firms advertise and provide perfect information about  $x$ . Under comparative advertising no firm advertises in  $a$ ,  $d$ , and  $f$ ; one firm advertises in the area  $b$ ,  $c_1$ ,  $c_2$ , and  $e$ , providing perfect information about the quality differential.

Recall that welfare increases by  $x^2/2\hat{\theta}$  when consumers learn  $x$ . Advertising is socially desirable if the gain from informing consumers exceeds the cost. Advertising costs at least  $\gamma$  when only one firm advertises. A necessary condition for advertising to be desirable is therefore  $|x| \geq \sqrt{2\hat{\theta}\gamma}$ . Note that  $2\hat{\theta}\gamma < \sqrt{2\hat{\theta}\gamma} < 4\hat{\theta}\gamma$ .

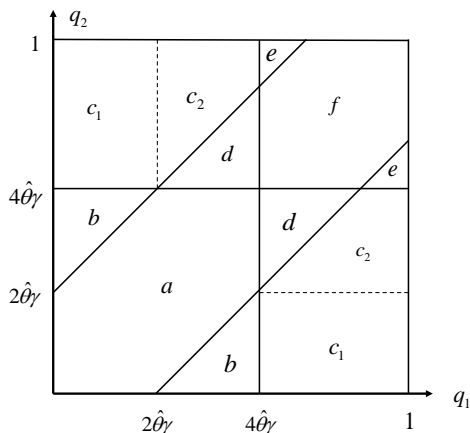


Figure 5: Welfare comparison

In areas  $a$ ,  $d$  and  $f$  advertising is inefficient: providing perfect information about  $x$  is not worth the expenditure  $\gamma$ . In area  $a$  firms don't advertise in the non-comparative and the comparative scenario, thus welfare is the same. In areas  $d$  and  $f$  firms don't advertise under comparative advertising. Under

non-comparative advertising one firm advertises in area  $d$  and both firms advertise in area  $f$ . Therefore, comparative advertising performs better than non-comparative advertising in areas  $d$  and  $f$ .

To compare welfare in the remaining areas we need the following

**Lemma 4:** *Variable signalling costs are higher for the non-comparative than for the comparative advertiser in  $c_2$  and  $e$  and lower in  $c_1$ .*

In  $c_2$  the non-comparative advertiser has the same fixed cost but a larger variable signalling cost than the comparative advertiser. Moreover, non-comparative advertising provides less information. Thus, information is better and signalling less costly under comparative advertising.

In  $e$  the information is the same under both regimes. However, each non-comparative advertiser has a higher signalling cost than the single comparative advertiser: the fixed cost is the same, and variable cost is higher. Furthermore, costs are duplicated under non-comparative advertising.

The welfare comparison is ambiguous in  $c_1$ . The non-comparative advertiser has a lower signalling cost than the comparative advertiser, but at the same time provides less information.

Finally, consider region  $b$  which can be partitioned in two subareas: one where  $x \leq \sqrt{2\hat{\theta}\gamma}$  and one where  $x > \sqrt{2\hat{\theta}\gamma}$ . In the first subarea, signalling by the comparative advertiser is not worth its cost. Hence, non-comparative advertising does better because it does not signal at all. In the second subarea, the comparison is ambiguous as is in  $c_1$ .

To sum up: Under comparative advertising for  $x \in (2\hat{\theta}\gamma, \sqrt{2\hat{\theta}\gamma})$  and  $x \in (-\sqrt{2\hat{\theta}\gamma}, -2\hat{\theta}\gamma)$  firms advertise although it is inefficient to do so. Comparative advertising has the following virtues. Whenever firms do not advertise, their decision is efficient. Furthermore, firms never advertise together, i.e., there is no duplication of advertising expenditures. For non-comparative advertising the picture is less clear cut. When no or one firm advertises, the outcome may be efficient or not. When both firms advertise, the outcome is



inefficient because signalling costs are duplicated.

For most values of the quality differential, comparative advertising does a better job than non-comparative advertising. Since under comparative advertising firms inform consumers directly about the quantity they are interested in and not just about one component thereof as under non-comparative advertising, this is after all not that surprising.

## 6 Concluding Remarks

The purpose of this paper is to analyze non-comparative and comparative advertising in a framework where firms may signal their quality. We consider the equilibria entailing minimal signalling costs, which in turn implies that the range over which firms advertise is maximal. Comparative advertising tends to perform better in our set-up than non-comparative advertising: firms do not advertise at all if the informational content is of little value to consumers; moreover, they never advertise together. By contrast, under non-comparative advertising a firm advertises if its quality level is above a threshold. When both firms have high quality, both advertise leading to a duplication of signalling costs.

We have considered a model where the market is covered so that only the quality differential matters, which obviously makes a strong case for comparative advertising. If, for example, prices are so high that neither established nor new customers with  $\theta$  close to zero buy, marginal consumers do not care about the quality differential; they care only about the quality of their favorite designs. In this case firms will only use non-comparative advertising and allowing for comparative advertising will have no effect.

Nevertheless, in markets where consumers directly compare products there is scope for comparative advertising to improve the allocation through better information and lower advertising expenditures. Our result that only one firm uses comparative advertising is obviously driven by our one-dimen-

sional quality assumption. It is at odds with the empirical findings of Anderson et al. (2010 a,b) where all firms used comparative advertising. Most likely, in the analgesics industry under consideration quality is multi-dimensional and firms claim superiority in the dimensions where they perform better. Analyzing comparative advertising with multi-dimensional quality remains an interesting topic for future research.

## Appendix

*Proof that  $E(q_1|\emptyset_1, q_2) = 1$  in the pure disclosure game.* Consider the pure disclosure set-up where firms can disclose  $q_i$  or  $x$  at a cost  $\gamma$ , or remain silent. They cannot misrepresent the evidence.

Suppose first that 1 plays  $\emptyset_1$  if and only if  $q_1 = 0$  so that  $E(q_1|\emptyset_1, q_2) = 0$ . Yet, 1 plays  $\emptyset_1$  also for  $q_1 > 0$  and small: he'd rather be seen as having zero quality than pay the fixed cost to disclose  $q_1 > 0$ , violating the only if assumption. Thus, 1 plays  $\emptyset_1$  for  $q_1 \in [0, q_1^0]$ ,  $q_1^0 \in (0, 1]$  and  $E(q_1|\emptyset_1, q_2) = 0$  is not correct.

Now consider  $E(q_1|\emptyset_1, q_2) \in (0, q_1^0]$ . Firm 2 will disclose  $q_2$  for  $q_1 \geq E(q_1|\emptyset_1, q_2)$  and  $x$  for  $q_1 < E(q_1|\emptyset_1, q_2)$ . Hence, consumers know that  $q_2 \in [E(q_1|\emptyset_1, q_2), q_1^0]$  and the expectation is correct if and only if  $E(q_1|\emptyset_1, q_2) = q_1^0$ .

For  $q_1^0 < 1$  firm 1 has to be indifferent at  $q_1^0$  between not advertising thus generating  $E(q_1|\emptyset_1, q_2)$  and disclosing  $q_1^0$  at the cost  $\gamma$ . But if  $E(q_1|\emptyset_1, q_2) = q_1^0$ , this is impossible. Therefore,  $q_1^0 = 1$ . ■

*Proof of Lemma 2.* Suppose on the contrary they do. Given  $(\emptyset_1, y_2(q_2))$ , consumers believe  $q_1 \geq q_2$  and  $x \leq 0$ . Firm 2 does better by playing  $\emptyset_2$ .

Given  $(y_1(q_1), y_2(q_2))$  consumers infer  $q_1$  and  $q_2$ . In this case the lower quality firm does better by remaining silent. If  $q_1 = q_2$ , both firms prefer not to advertise.

Now consider  $(z_1(x), y_2(q_2))$ . If  $x$  and  $q_2$  are compatible, consumers expect  $x$  and  $q_2$ . Firm 2 does better not to advertise to save the advertising costs. If  $x$  and  $q_2$  are not compatible (e.g.,  $x < q_2 - 1$ ), somebody must have made a mistake. Suppose firm 1 got it wrong. If 1 wanted to send  $\emptyset_1$  or  $y_1(q_1)$ , 2 would have played  $\emptyset_2$ . If 1 wanted to send a comparative message but picked the wrong one, 2 would also answer with  $\emptyset_2$ . Hence, by minimality consumers believe that 2 made the mistake and ignore  $y_2(q_2)$ . But this implies that 2 does better to be silent. ■

*Proof of Lemma 3.* Suppose on the contrary that there is some range  $[x_2^0, x_1^0]$  where both firms advertise and the equilibrium is revealing. At, say,  $x_1^0$  firm 1 signals  $z_1(x_1^0)$ , firm 2 sends  $z_2(x_1^0)$ , and  $E(x|z_1(x_1^0), z_2(x_1^0)) = x_1^0$ . See Figure 6.

We have to distinguish three cases.

i) Let  $z_2(x_1^0) \leq z_2(x_1^0 + \epsilon)$  with  $\epsilon$  small. Here there is no signal  $z_i$  which is sent by party  $i$  for different two different values of  $x$ . The argument depends on whether firm 2's signalling costs go up or down when he switches from  $z_2(x_1^0)$  to  $z_2(x_1^0 + \epsilon)$  given  $x = x_1^0$ . For  $\epsilon$  sufficiently small, the sign of the cost change is the same if  $x = x_1^0 + \epsilon$ .

Consider first the case where the 2's cost does not increase when he switches from  $z_2(x_1^0 + \epsilon)$  to  $z_2(x_1^0)$ . Let the true state be  $x_1^0 + \epsilon$ . Along the equilibrium path  $(z_2(x_1^0 + \epsilon), \emptyset_1)$  and consumers correctly infer  $x_1^0 + \epsilon$ .

If the firm 2 deviates to  $z_2(x_1^0)$ , consumers observe  $(z_2(x_1^0), \emptyset_1)$  which is off the equilibrium. By the minimality condition consumers think that either 1 deviated while 2 played his equilibrium strategy and the underlying  $x = x_1^0$ ; or they think that 1 played his equilibrium strategy  $\emptyset_1$  while 2 deviated and the underlying  $x \in (x_1^0, 1]$ . Consumers assign equal probability to both possibilities so that  $E(x|z_2(x_1^0), \emptyset_1) > x_1^0$ . Since beliefs do not depend on  $\epsilon$ , it is possible to choose an  $\epsilon$  such that  $E(x|z_2(x_1^0), \emptyset_1) > x_1^0 + \epsilon$ . Since 2's cost does not increase and his revenue increases, he will deviate.

Now consider the case where 2's costs decrease when he switches from  $z_2(x_1^0)$  to  $z_2(x_1^0 + \epsilon)$ . Let the true state be  $x_1^0$ . Along the equilibrium path  $(z_2(x_1^0), z_1(x_1^0))$  and consumers correctly infer  $x_1^0$ .

Suppose 2 switches to  $z_2(x_1^0 + \epsilon)$  so that consumers observe  $(z_2(x_1^0 + \epsilon), z_1(x_1^0))$  which is off the equilibrium. Then consumers think that 2 deviated and  $x = x_1^0$  or 1 deviated and  $x = x_1^0 + \epsilon$ . Thus,  $E(x|z_2(x_1^0 + \epsilon), z_1(x_1^0)) = x_1^0 + .5\epsilon$ . 2's costs decrease, his revenue does not decrease, hence he will deviate.

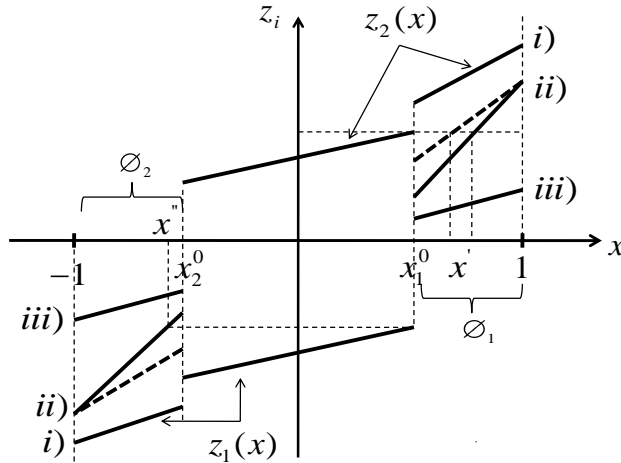


Figure 6: Signalling strategies  $z_1(x)$  and  $z_2(x)$  if  $x_2^0 < x_1^0$

ii) Let  $z_2(x_1^0) > z_2(x_1^0 + \epsilon)$  with  $\epsilon$  small and  $z_2(1) \geq z_2(x_1^0)$ . Here there are signals  $z_i$  which are sent by party  $i$  for two different values of  $x$ .

Consider first the case where  $z_2(x_1^0) < z_2(x_1^0 + \epsilon)$  or, by symmetry,  $z_1(x_1^0) > z_1(x_1^0 - \epsilon)$  which is given by the dotted line in Figure 6. Here  $z_1(x_1^0)$  is played by 1 only at  $x_1^0$  in equilibrium. Let  $x' > x_1^0$  be such that  $z_2(x') = z_2(x_1^0)$ . Now suppose 2 lowers the signal to  $z_2(x_1^0) - \epsilon$  so that consumers observe  $(z_2(x_1^0) - \epsilon, z_1(x_1^0))$  which is off the equilibrium path. Then they think with equal probability that either 2 deviated and  $x = x_1^0$  or 1 deviated and  $x = x_1^0 - \epsilon$  or  $x = x' - \delta$ ; let consumers assign probabilities  $b > 0$  and  $(1 - b)$  to the two possibilities. Hence,  $E(x|z_2(x_1^0) - \epsilon, z_1(x_1^0)) = .5x_1^0 + .5(b(x_1^0 - \epsilon) + (1 - b)(x' - \delta))$ . For  $\epsilon$  going to zero, so does  $\delta$  and  $\lim_{\epsilon \rightarrow 0} E(x|z_2(x_1^0) - \epsilon, z_1(x_1^0)) > x_1^0$ . 2's revenue increases, hence he will deviate.

Now consider the case where  $z_2(x_1^0) > z_2(x_1^0 + \epsilon)$  or, by symmetry,  $z_1(x_1^0) < z_1(x_1^0 - \epsilon)$ . Here any equilibrium signal  $z_i$  under joint advertising is also played when  $i$  advertises alone. Let  $x'' < x_1^0$  be such that  $z_1(x'') = z_1(x_1^0)$ .

Suppose the true state is  $x_1^0$ . Along the equilibrium path  $(z_2(x_1^0), z_1(x_1^0))$  and consumers infer  $x_1^0$ . If 2 lowers his signal to  $z_2(x_1^0) - \epsilon$ , consumers observe  $(z_2(x_1^0) - \epsilon, z_1(x_1^0))$  which is off the equilibrium. Then consumers think with probability .5 that 2 deviated and  $x = x_1^0$  or  $x = x''$ ; consumers assign probabilities  $b$  and  $(1 - b)$  to these possibilities. With probability .5 they think that 1 deviated and  $x = x_1^0 - \epsilon$  or  $x = x' - \delta$ . Thus,  $E(x|z_2(x_1^0) - \epsilon, z_1(x_1^0)) = .5(bx_1^0 + (1 - b)x'') + .5(b(x_1^0 - \epsilon) + (1 - b)(x' - \delta))$ . We have  $\lim_{\epsilon \rightarrow 0} E(x|z_2(x_1^0) - \epsilon, z_1(x_1^0)) = bx_1^0 + .5(1 - b)(x' + x'')$ . If this exceeds  $x_1^0$ , 2 deviates.

If 1 lowers his signal to  $z_1(x_1^0) - \epsilon$ , consumers observe  $(z_2(x_1^0), z_1(x_1^0) - \epsilon)$  which is off the equilibrium. Then they think that 1 deviated and  $x = x_1^0$  or  $x = x'$ ; or they think 2 deviated and  $x = x_1^0 - \epsilon$  or  $x = x'' - \delta$ . Hence,  $E(x|z_2(x_1^0), z_1(x_1^0) - \epsilon) = .5(bx_1^0 + (1 - b)x') + .5(b(x_1^0 - \epsilon) + (1 - b)(x'' - \delta))$ . Again we have  $\lim_{\epsilon \rightarrow 0} E(x|z_2(x_1^0), z_1(x_1^0) - \epsilon) = bx_1^0 + .5(1 - b)(x' + x'')$ . If this is less than  $x_1^0$ , 1 will deviate.

iii) Let  $z_2(x_1^0) > z_2(x_1^0 + \epsilon)$  with  $\epsilon$  small and  $z_2(1) < z_2(x_1^0)$ . Here again there is no signal  $z_i$  which is sent by party  $i$  for two different values of  $x$  and the argument is along the same lines as in i). ■

*Proof of Proposition 2.* a)  $z_2$  as defined in (6) is a best response to  $\emptyset_1$  when  $x \geq 2\hat{\theta}\gamma$ . The argument is similar the case of non-comparative advertising, except that the threshold is different. When firm 2 does not advertise, its profit is

$$\pi_2(\emptyset_1, \emptyset_2) = \frac{1}{2}.$$

It starts advertising at the threshold  $x_2^0 = 2\hat{\theta}\gamma$ , in which case it sends the message

$z_2^0 = x_2^0$ . Its profit is then

$$\pi_2(\emptyset_1, z_2^0) = \frac{1}{2} + \frac{x_2^0}{2\hat{\theta}} - \gamma.$$

Equating the two yields  $x_2^0 = 2\hat{\theta}\gamma$ . At  $x = 2\hat{\theta}\gamma$ , the firm is therefore indifferent between advertising and not. We assume that it does. At  $x > 2\hat{\theta}\gamma$ , it is easily seen that it is strictly better off by advertising.

b)  $\emptyset_2$  is a best response to  $\emptyset_1$  when  $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$ . From the argument used to derive the threshold  $z_2^0$ , it is easily seen that profit is larger with  $\emptyset_2$  than with some message  $z_2 \geq 2\hat{\theta}\gamma$ . We therefore need only consider the case where the message sent is some  $z_2 < 2\hat{\theta}\gamma$ . Applying the minimality condition, consumers infer that 1 played its equilibrium strategy while 2 deviated. They therefore infer that the true differential belongs to  $(-2\hat{\theta}\gamma, 1]$ . Any  $E(x | \emptyset_1, z_2) \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$  supports the equilibrium because firm 2's profit is then

$$\pi_2(\emptyset_1, z_2) = \frac{1}{2} + \frac{E(x | z_2, \emptyset_1)}{2\hat{\theta}} - \gamma < \frac{1}{2},$$

i.e., its profit is smaller than the profit if it does not advertise.

c)  $\emptyset_2$  is firm 2's best response to  $z_1$  as defined as (7) when  $x \leq -2\hat{\theta}\gamma$ . Consider first the play of some  $z_2 < 2\hat{\theta}\gamma$ . Applying the minimality condition, consumers infer that firm 2 deviated while 1 played its equilibrium strategy. Their beliefs are therefore determined by (7), i.e., firm 2's deviation has no influence on beliefs. Hence firm 1 is better off with  $\emptyset_2$  since it avoids the cost  $\gamma$ .

Consider now the play of some  $z_2 \geq 2\hat{\theta}\gamma$ . Consumers do not know which firm has deviated but think that at most one did. Let them assign probability 1/2 to a deviation by firm 1. Their beliefs are then

$$E(x | z_1, z_2) = \frac{1}{2} \left( z_1 + \frac{1 - e^{2\hat{\theta}(z_1 + 2\hat{\theta}\gamma)}}{2\hat{\theta}} \right) + \frac{1}{2} \left( z_2 - \frac{1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)}}{2\hat{\theta}} \right).$$

Because 1 is playing its equilibrium strategy,

$$E(x | \hat{z}_1(x), z_2) = \frac{1}{2} \left( z_2 - \frac{1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)}}{2\hat{\theta}} \right) + \frac{1}{2}x.$$

Firm 2's profit is then

$$\pi_2(\hat{z}_1(x), z_2) = \frac{1}{2} + \frac{E(x | z_2, \hat{z}_1(x))}{2\hat{\theta}} - \gamma - \frac{1}{2}(z_2 - x)^2.$$

If it plays the equilibrium strategy, its profit is

$$\pi_2(\hat{z}_1(x), \emptyset_2) = \frac{1}{2} + \frac{x}{2\hat{\theta}}.$$

We need to show that  $\pi_2(\hat{z}_1(x), \emptyset_2) \geq \pi_2(\hat{z}_1(x), z_2)$  for all  $x \leq -2\hat{\theta}\gamma$  and  $z_2 \geq 2\hat{\theta}\gamma$ . Define

$$\begin{aligned}\varphi(x, z_2) &= 2\hat{\theta}(\pi_2(\hat{z}_1(x), z_2) - \pi_2(\hat{z}_1(x), \emptyset_2)) \\ &= \frac{1}{2} \left( z_2 - \frac{1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)}}{2\hat{\theta}} - x \right) - 2\hat{\theta}\gamma - \hat{\theta}(z_2 - x)^2.\end{aligned}$$

We want to show that  $\varphi(x, z_2) < 0$  when  $x \leq -2\hat{\theta}\gamma$  and  $z_2 \geq 2\hat{\theta}\gamma$ . Differentiating with respect to  $z_2$  yields

$$\begin{aligned}\varphi_2(x, z_2) &= \frac{1}{2} \left( 1 - e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)} \right) - 2\hat{\theta}(z_2 - x) \quad \text{and} \\ \varphi_{22}(x, z_2) &= 2\hat{\theta} \left( \frac{1}{2} e^{-2\hat{\theta}(z_2 - 2\hat{\theta}\gamma)} - 1 \right) < 0.\end{aligned}$$

At  $z_2 = 2\hat{\theta}\gamma$ ,  $\varphi_2(x, z_2) < 0$ . Since  $\varphi_{22} < 0$ , this implies that 2 would never want to send a signal  $z_2 > 2\hat{\theta}\gamma$ . Thus, it remains to evaluate the sign of

$$\varphi(x, 2\hat{\theta}\gamma) = \frac{1}{2} \left( 2\hat{\theta}\gamma - x \right) - 2\hat{\theta}\gamma - \hat{\theta} \left( 2\hat{\theta}\gamma - x \right)^2.$$

Observe that

$$\begin{aligned}\varphi(-2\hat{\theta}\gamma, 2\hat{\theta}\gamma) &= -\hat{\theta} \left( 4\hat{\theta}\gamma \right)^2 < 0 \quad \text{and} \\ \varphi(-1, 2\hat{\theta}\gamma) &= \frac{1}{2} \left( 2\hat{\theta}\gamma + 1 \right) - 2\hat{\theta}\gamma - \hat{\theta} \left( 2\hat{\theta}\gamma + 1 \right)^2 < 0.\end{aligned}$$

Furthermore

$$\varphi_1(x, 2\hat{\theta}\gamma) = -\frac{1}{2} + 2\hat{\theta} \left( 2\hat{\theta}\gamma - x \right) > 0.$$

It follows that  $\varphi(x, 2\hat{\theta}\gamma) < 0$  for all  $x \leq -2\hat{\theta}\gamma$ , which in turn implies  $\varphi(x, z_2) < 0$  over the relevant domain.

Regarding the best responses of firm 1, the same arguments can be made. The only difference is that 1 wants  $x$  to be perceived as small. ■

*Proof of Lemma 4.* Let  $s_q = y_2 - q_2$ .  $s_q^2$  is thus the variable signalling cost. Substituting in (5) yields

$$s_q(q_2) := \frac{1 - e^{-2\hat{\theta}(s_q + q_2 - 4\hat{\theta}\gamma)}}{2\hat{\theta}}, \quad \text{for } q_2 \geq 4\hat{\theta}\gamma.$$

Likewise, let  $s_x = z_2 - x$ . Substituting in (6) yields

$$s_x(x) := \frac{1 - e^{-2\hat{\theta}(s_x + x - 2\hat{\theta}\gamma)}}{2\hat{\theta}}, \quad \text{for } x \geq 2\hat{\theta}\gamma.$$

Next we show that  $s_x(t) > s_q(t)$  for  $t \geq 2\hat{\theta}\gamma$ . Let  $t$  and  $t'$  be such that  $s_x(t) = s_q(t')$ . Thus

$$\frac{1 - e^{-2\hat{\theta}(\hat{s}_q(t') + t' - 4\hat{\theta}\gamma)}}{2\hat{\theta}} = \frac{1 - e^{-2\hat{\theta}(\hat{s}_x(t) + t - 2\hat{\theta}\gamma)}}{2\hat{\theta}},$$

which yields

$$t' = t + 2\hat{\theta}\gamma.$$

Since  $s_x(\cdot)$  and  $s_q(\cdot)$  are increasing functions, it follows that for  $t' > t + 2\hat{\theta}\gamma$ ,  $\hat{s}_q(t') > \hat{s}_x(t)$ . Conversely,  $t' < t + 2\hat{\theta}\gamma$  implies  $\hat{s}_q(t') < \hat{s}_x(t)$ .

Consider now what happens in the areas  $c$  and  $e$  of Figure 5. For the comparative advertiser,  $s_x = s_x(x)$ . For the non comparative advertiser,  $s_q = s_q(q_2)$ . Since  $q_2 = x + q_1$ ,  $s_q \geq s_x$  if  $q_1 \geq 2\hat{\theta}\gamma$ , in which case variable signalling costs are at least as large for the non-comparative advertiser in the areas  $c_2$  and  $e$  of Figure 5. Conversely,  $s_q < s_x$  if  $q_1 < 2\hat{\theta}\gamma$ . In  $c_1$  signalling costs are lower with non- than with comparative advertising. ■

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