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Abstract

In this paper, we show that in a model where investors have heterogeneous preferences, the expected return of risky assets depends on the idiosyncratic coskewness beta, which measures the co-movement of the individual stock variance and the market return. We find that there is a negative (positive) relation between idiosyncratic coskewness and equity returns when idiosyncratic coskewness betas are positive (negative). Standard risk factors, such as the market, size, book-to-market, and momentum cannot explain the findings. We construct two idiosyncratic coskewness factors to capture the market-wide effect of idiosyncratic coskewness. The two idiosyncratic coskewness factors can also explain the negative and significant relation between the maximum daily return over the past one month (MAX) and expected stock returns documented in Bali, Cakici, and Whitelaw (2009). In addition, when we control for these two idiosyncratic coskewness factors, the return difference for distress-sorted portfolios found in Campbell, Hilscher, and Szilagyi (2008) becomes insignificant. Furthermore, the two idiosyncratic coskewness factors help us understand the idiosyncratic volatility puzzle found in Ang, Hodrick, Xing, and Zhang (2006). They reduce the return difference between portfolios with the smallest and largest idiosyncratic volatility by more than 60%, although the difference is still statistically significant.

JEL classification: G11, G12, G14, G33
Bank classification: Economic models; Financial markets

Résumé

Les auteurs montrent, en modélisant des investisseurs aux préférences hétérogènes, que le rendement espéré d'actifs risqués dépend du coefficient bêta de coasymétrie idiosyncrasique, qui mesure l'évolution conjointe du rendement boursier et de la variance de chaque action. Ils observent une relation négative (positive) entre la coasymétrie idiosyncrasique et les rendements des actions lorsque le coefficient bêta de coasymétrie est positif (négatif). Les facteurs de risque usuels, comme le marché, le volume, le ratio valeur comptable-valeur de marché ou le *momentum*, ne permettent pas d'expliquer ce résultat. Les auteurs élaborent deux facteurs pour représenter l'incidence de la coasymétrie idiosyncrasique sur l'ensemble du marché. Ces facteurs permettent aussi d'expliquer la relation négative significative qui lie le rendement quotidien maximal enregistré pendant le mois écoulé et les rendements boursiers espérés et dont font état Bali, Cakici et Whitelaw (2009). Une fois ces facteurs idiosyncrasiques pris en compte, l'écart de rendement entre les portefeuilles de Campbell, Hilscher et Szilagyi (2008), constitués après un tri des sociétés émettrices en fonction de leur probabilité de défaut, cesse d'être significatif. Qui plus est, ces deux facteurs aident à percer l'énigme posée par la volatilité idiosyncrasique chez Ang, Hodrick, Xing et Zhang (2006). Leur inclusion réduit en effet de plus de 60 % l'écart de rendement entre les portefeuilles présentant les niveaux de volatilité idiosyncrasique minimal et maximal; ce dernier demeure toutefois statistiquement significatif.

Classification JEL : G11, G12, G14, G33
Classification de la Banque : Modèles économiques; Marchés financiers

1 Introduction

The single factor capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has been empirically tested and rejected by numerous studies, which show that the cross-sectional variation in expected equity returns cannot be explained by the market beta alone. One possible extension is to assume that investors care about not only the mean and variance of their portfolios, but the skewness of their portfolio as well. Harvey and Siddique (2000) propose an asset pricing model where skewness is priced. In their model, the expected equity return depends on the market beta and the coskewness beta, which measures the covariance between an individual equity return and the square of the market return. Mitton and Vorkink (2008) introduce a model where investors' preference for the mean and variance is the same but the preference for skewness is heterogeneous. In their model, the idiosyncratic skewness is priced. They also show that their model can explain why many investors do not hold well-diversified portfolios.

We relax certain restrictions in the Mitton and Vorkink (2008) model in this paper. We show that in a model with heterogeneous preference for skewness, the expected return on risky assets depends on the market beta, the coskewness beta (as in Harvey and Siddique (2000)), the idiosyncratic skewness (as in Mitton and Vorkink (2008)), and the idiosyncratic coskewness beta, which measures the covariance between idiosyncratic variance and the market return.

We show empirically that when estimated idiosyncratic coskewness betas are positive, there is a negative relationship between excess returns and idiosyncratic coskewness betas. When estimated idiosyncratic coskewness betas are negative, the relationship becomes positive. In addition, when we control for risk using the market factor, the Fama-French three factors, and the Carhart four factors, the relationship between excess returns and idiosyncratic coskewness betas becomes stronger. In other words, the standard risk factors cannot explain why portfolios with low idiosyncratic coskewness betas earn high excess returns when idiosyncratic coskewness betas are positive, and why portfolios with high idiosyncratic coskewness betas earn high excess returns when idiosyncratic coskewness betas are negative. We form two long-short portfolios, which are long the portfolio with the lowest idiosyncratic coskewness beta and short the portfolio with the highest idiosyncratic coskewness beta for both groups with positive and negative idiosyncratic coskewness betas, to capture the systematic variation in excess portfolio returns sorted by idiosyncratic coskewness betas. We call them idiosyncratic coskewness factors, $ICSK_1$ for the groups with positive idiosyncratic coskewness betas, and $ICSK_2$ for the groups with negative idiosyncratic coskewness betas. The average monthly excess returns for $ICSK_1$ and $ICSK_2$ over the sample period January 1971 to December 2006 are 0.81% ($t = 1.87$) and -0.63% ($t = 2.00$) respectively.

In addition, we find that the idiosyncratic coskewness factors can help explain three anomalous findings in equity market. First, we show that the two idiosyncratic coskewness factors explain the anomalous finding that stocks with the maximum daily return over the past month (MAX) earn low expected returns. Bali, Cakici, and Whitelaw (2009) document a negative and significant relation between the maximum daily return over the past month and expected stock returns. We show that the average raw and risk-adjusted return differences between stocks in the lowest and highest MAX deciles is about 0.93% ($t = 2.51$) per month. When we regress value-weighted (MAX) portfolios returns on the two idiosyncratic coskewness factors $ICSK_1$ and $ICSK_2$, the two idiosyncratic coskewness factors reduce the monthly excess return of a long-short portfolio holding the portfolio with the lowest MAX measure and shorting the portfolio with the highest MAX measure from 0.93% to 0.26% ($t = 1.34$). The results are robust to controls for size, book-to-market and momentum.

Second, there is an anonymous negative relation between equity returns and default risk. Recent empirical studies (Dichev (1998), Griffin and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008)) document a negative relationship between default risk and realized stock returns. In addition, Campbell, Hilscher, and Szilagyi (2008) find that correcting for risk using the standard risk factors worsens the anomaly. We show that the two idiosyncratic coskewness factors can explain the anomalous finding that high stressed firms earn low equity returns. We use the Merton (1974) model to measure default risk for individual firms, and find the anomalous negative relation between default risk and equity returns. When we regress distress-sorted portfolio returns on the two idiosyncratic coskewness factors $ICSK_1$ and $ICSK_2$, we find that factor loadings on $ICSK_1$ are generally declining with distress measures, and factor loadings on $ICSK_2$ are generally increasing with distress measures. The two idiosyncratic coskewness factors reduce the monthly excess return of a long-short portfolio holding the portfolio with the lowest distress measure and shorting the portfolio with the highest distress measure from 1.42% ($t = 2.19$) to 0.64% ($t = 1.01$). Including other standard risk factors, such as the market, size, value, and momentum factors, will not significantly alter the factors loadings on the two idiosyncratic factors and the alpha of the long-short portfolio.

Third, we show that the two idiosyncratic coskewness factors can help us understand the negative relation between idiosyncratic volatility and equity returns found in Ang, Hodrick, Xing, and Zhang (2006). They find that stocks with high idiosyncratic volatility earn abysmally low average returns. This puzzling finding cannot be explained by the standard risk factors, such as the market, size, book-to-market, momentum, and liquidity. We show that the two idiosyncratic coskewness factors can explain the monthly return difference between portfolios with the lowest and second highest idiosyncratic volatility. In addition, the two idiosyncratic coskewness factors reduce the monthly return difference between portfolios with the lowest and highest idiosyncratic volatility by more than 60%. However, the return difference is still statistically significant.

This paper is organized as follows. Section 2 presents both theoretical and empirical relations between idiosyncratic coskewness betas and equity returns. Section 3 explains the findings in Bali, Cakici, and Whitelaw (2009) using idiosyncratic coskewness factors. Section 4 explains the anomalous negative relation between default risk and equity returns (Campbell, Hilscher, and Szilagyi (2008))using the idiosyncratic coskewness factors. Section 5 addresses the negative relation between equity returns and idiosyncratic volatility found in Ang, Hodrick, Xing, and Zhang (2006). Section 6 concludes.

2 Idiosyncratic Coskewness and Equity Returns

2.1 Theory

Empirical papers have documented that investors usually hold under-diversified portfolios with a small number of securities. One possible explanation is that investors care about idiosyncratic skewness in their portfolios. Barberis and Huang (2008) show that idiosyncratic skewness is priced in equilibrium under the assumption that investors have preferences based on the cumulative prospect theory[1]. Mitton and Vorkink (2008) demonstrate the same result under the assumption of heterogeneous preference for skewness. However, they allow only idiosyncratic skewness in their model. We extend their model and allow covariance between the idiosyncratic variance of individual asset returns and the market return, which is named as idiosyncratic coskewness. We then derive and test the relationship between equity returns and idiosyncratic coskewness.

We assume that the universe of stocks consists of n risky assets and a risk-free asset. The return vector of the n securities is denoted as $R = [R_1, ..., R_n]$. The covariance of asset returns is denoted Σ.

In our economy, we assume that there are two investors, a "traditional" investor and a "Lotto investor". Traditional investor utility can be approximated as a standard quadratic utility function over wealth

$$U(\mathcal{W}) = E(\mathcal{W}) - \frac{1}{2\tau} Var(\mathcal{W}), \tag{1}$$

[1]Cumulative prospect theory is a modified version of "prospect theory" developed by Kahneman and Tversky (1979). Under cumulative prospect theory, investors, departing from the predications of expected utility, evaluate risk using a value function that is defined over gains and losses, that is concave over gains and convex over losses, and that is kinked at the origin. In addition, investors use transformed rather than objective probabilities, where the transformed probabilities are obtained from objective probabilities by applying a weighting function, which overweighs the tails of the distribution it is applied to.

where $\mathcal{W}$ is the investor terminal wealth, $\tau > 0$ is the coefficient of risk aversion. Levy and Markowitz (1979) and Hlawitschka (1994) show that the quadratic utility is a reasonable approximation of standard expected utility functions. And it seems reasonable to assume that, in the population, the traditional investor behaves as a mean-variance investor. The "Lotto investor" has the same preferences as the traditional investor over mean and variance, but also has preference for skewness

$$U(\mathcal{W}) = E(\mathcal{W}) - \frac{1}{2\tau} Var(\mathcal{W}) + \frac{1}{3\phi} Skew(\mathcal{W}), \tag{2}$$

where ϕ is the investor skewness preference. As shown in Cass and Stiglitz (1970), utilities (1) and (2) can lead, under certain restrictions, to equilibrium portfolio separation. As $\phi \longrightarrow \infty$, the Lotto investor utility approaches the traditional investor utility as in Markowitz (1959). It is insightful to notice that if all investors are lotto investors, then the model would be reduced to the Kraus and Litzenberger (1976) coskewness model. Each investor maximizes his expected utility subject to his budget constraint of the form

$$\mathcal{W}_k = \mathcal{W}_{0,k} R_f + \omega_k^\intercal (R - R_f \mathbf{1}), \; k = \mathcal{T}, \mathcal{L}$$

where R_f is the return on the risk-free asset, $R - R_f\mathbf{1}$ is the vector of excess returns, $\omega_{\mathcal{T}}$ is the asset demand for the traditional investor, and $\omega_{\mathcal{L}}$ is the asset demand for the Lotto investor. The aggregate demand is $\omega_M = \omega_{\mathcal{T}} + \omega_{\mathcal{L}}$. For the traditional (hereafter $\mathcal{T}$) investor, the first-order condition of (1) is

$$E(R - R_f\mathbf{1}) - \frac{1}{\tau}\Sigma\omega_{\mathcal{T}} = 0. \tag{3}$$

For Lotto investors (hereafter $\mathcal{L}$), the first-order condition is

$$E(R - R_f\mathbf{1}) - \frac{1}{\tau}\Sigma\omega_{\mathcal{L}} + \frac{1}{\phi} E(\omega_{\mathcal{L}}^\intercal (R - ER)(R - ER)^\intercal \omega_{\mathcal{L}})(R - ER) = 0. \tag{4}$$

To isolate the effect of idiosyncratic coskewness on returns, we assume that:

$$Cov\left(\varepsilon^2, (R_i - ER_i)\right) = 0. \tag{5}$$

$$Cov\left(\varepsilon, (R_i - ER_i)(R_j - ER_j)\right) = 0 \text{ for } i, j. \tag{6}$$

where ε is defined by the return decomposition $(R_i - ER_i) = a_i(\mathcal{W}_{\mathcal{T}} - E\mathcal{W}_{\mathcal{T}}) + \varepsilon$. Under assumptions (5) and (6), we use equations (3) and (4) and decompose the expected excess return as[2]:

$$ER_i - R_f = \lambda_M \beta_{iM} + \lambda_{CSK}\beta_{iCSK} + \lambda_{ISK} Skew_i + \lambda_{ICSK}\beta_{iICSK}$$

where β_{iM}, β_{iCSK}, $Skew_i$ represent the asset's beta, the asset's coskewness, and the asset's idiosyncratic skewness respectively. λ_M, λ_{CSK}, and λ_{ICSK} represent the price of risk of the market, coskewness and idiosyncratic skewness factor. The quantity of risk β_{iICSK} which measures the co-movement between the asset's "volatility" and the market return.

$$\beta_{iICSK} = \frac{Cov(R_M, (R_i - ER_i)^2)}{Var(R_M)},$$

is referred to as the idiosyncratic coskewness beta.[3] Assumptions (5) and (6) are necessary to isolate the effect of the idiosyncratic coskewness beta on asset returns. To investigate the relation between idiosyncratic coskewness betas and expected returns we consider two assets and form a portfolio of these two assets by changing the weight on these assets from -1 to 1. We then study the return difference between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta. To perform our analysis, we fix the returns of the two assets and their idiosyncratic coskewness betas. The top left graph in Figure 1 shows the relationship between the portfolio idiosyncratic coskewness beta and the expected return when the idiosyncratic coskewness betas for

[2]See the proof in the Appendix.

[3]Our model set up is similar to Mitton and Vorkink (2008), but the result is different because we relax their assumption that the idiosyncratic coskewness beta is zero.

both assets are positive. SET1=(0.06,0.01,0.005,0.009) contains the expected returns, and idiosyncratic coskewness betas of the two assets respectively. As shown in this graph, the difference in expected returns between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta is negative. For a different set of values, SET1=(0.06,0.01,0.009,0.006), we reach the same conclusion in the top left graph in Figure 2.

The bottom right graph in Figure 1 shows the relationship between idiosyncratic coskewness betas and expected returns when the idiosyncratic coskewness betas for both assets are negative. SET4=(0.06,0.01,-0.005,-0.009) contains the expected returns, and idiosyncratic coskewness betas of the two assets respectively. As shown in this graph, the difference in expected returns between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta is positive. For a different set of values, SET4=(0.06,0.01,-0.009,-0.005), we reach the same conclusion in the bottom right graph in Figure 2.

The top right graph in Figure 1 shows the relationship between idiosyncratic coskewness betas and expected returns when asset one has negative idiosyncratic coskewness beta and asset two has positive idiosyncratic coskewness beta, SET3=(0.06,0.01,-0.005,0.009). The bottom left graph in Figure 1 shows the relationship between idiosyncratic coskewness betas and expected returns when asset one has positive idiosyncratic coskewness beta and asset two has negative idiosyncratic coskewness beta, SET3=(0.06,0.01,0.005,-0.009). As shown in these graphs, there is no a clear relationship between the portfolio idiosyncratic coskewness beta and its expected return. We reach the same conclusion in the top right and bottom left graphs in Figure 2. This suggests that, when all assets are used regardless of the sign of their idiosyncratic coskewness betas, the relationship between excess returns and idiosyncratic coskewness betas is “hump-shaped”.

2.2 Equity Returns and Measures of Higher Moments Risk

In this section, we use the entire CRSP equity data set to investigate the relationship between equity returns and coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively. At the beginning of each month, we use the past 12-month daily data on individual stock returns to compute coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively as defined in the previous section, and form portfolios sorted by coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively. To reduce the liquidity effect on equity returns, we eliminate firms with no transaction days larger than 120. We also eliminate stocks with prices less than $1 at the end of a month. Following the same method used to compute returns for distress-sorted portfolios, we compute value-weighted returns for portfolios sorted by coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively.

Table 1 reports the results for the decile portfolios sorted by coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively. For the ten portfolios sorted by coskewness betas, there is a slight negative relation between excess equity returns and coskewness betas, which is consistent with Harvey and Siddique (2000). However, the relationship almost disappears when we control for the Fama-French factors. In addition, the relationship becomes positive when we control the Carhart four factors. For the ten portfolios sorted by idiosyncratic coskewness betas, the relationship between excess equity returns and idiosyncratic coskewness betas is hump-shaped, i.e. portfolios with both lowest and highest idiosyncratic coskewness betas have lower excess returns than the others. This hump-shaped relationship does not disappear even when we control the market factor, the Fama-French factors, or the Carhart factors. This result is consistent with our theoretical prediction. For the ten portfolios sorted by idiosyncratic skewness, there is a slight positive relation between excess equity returns and idiosyncratic skewness. However, this relationship basically disappears when we control the standard risk factors, such as the market factor, the Fama-French factors, or the Carhart factors. Table 1 confirms our theoretical finding that the idiosyncratic coskewness measure is different from the standard coskewness and idiosyncratic skewness measure. It is important to point out that empirically testing the relation between idiosyncratic skewness and returns is not a straightforward exercise. The primary obstacle is that ex ante skewness is difficult to measure. As opposed to variances and covariances, idiosyncratic skewness is not stable over time. This explains the marginal effect of idiosyncratic skewness on expected returns[4].

To further investigate the cross-sectional relation between idiosyncratic coskewness betas and idiosyncratic skewness, we run a simple OLS regression of idiosyncratic coskewness betas on idiosyncratic skewness each month using the estimated idiosyncratic coskewness betas and idiosyncratic skewness for all available firms. The time series of estimated slope coefficients and R^2s are plotted in Figure 3. It shows that there is a positive relation between cross-sectional idiosyncratic coskewness and idiosyncratic skewness during the sample period. However, the positive relation is very weak given that the average R^2s from the regressions is 1.8%. The results demonstrate that idiosyncratic coskewness betas and idiosyncratic skewness measure different aspects of equity returns.

2.3 Equity Returns Sorted by Positive and Negative Idiosyncratic Coskewness

We showed in the last section that there is a hump-shaped relation between equity returns and idiosyncratic coskewness betas. To further investigate that relationship, we divide firms into two groups according to the sign of their idiosyncratic coskewness betas. For each group, we then rank the stocks based on their past idiosyncratic coskewness betas and form ten value-weighted decile portfolios. Following the same method used to compute returns for distress-sorted portfolios, we compute value-weighted returns for idiosyncratic coskewness

[4] To avoid this obstacle, Boyer, Mitton, and Vorkink (2009) regress idiosyncratic skeweness on a set of predictor variables and use the expected component of their linear regression as a measure of expected idiosyncratic skewness. Because the goal of this paper is to investigate whether idiosyncratic coskewness betas explain the default risk puzzle, we do not investigate the empirical relationship between expected idiosyncratic skewness and expected idiosyncratic coskewness. We leave this issue for future research.

beta-sorted portfolios in each group.

Tables 2 and 3 report the results for the ten portfolios with positive and negative idiosyncratic coskewness betas respectively. Panel A reports average excess returns, in monthly percentage points, of idiosyncratic coskewness beta-sorted portfolios and the average return of a long-short-portfolio holding the portfolio with the lowest idiosyncratic coskewness beta and shorting the portfolio with the highest idiosyncratic coskewness beta. Panel A also reports alphas with respective to the CAPM, the Fama-French three-factor model, and the four-factor model proposed by Carhart (1997) that includes a momentum factor. Panel B reports estimated factor loadings in the four-factor model with adjusted R^2s. Figures 4 and 5 plot the alphas from regressions for the ten positive portfolios with positive and negatively idiosyncratic coskewness betas respectively.

The average excess returns for the first nine portfolios with positive idiosyncratic coskewness betas are almost flat. The average excess return for the tenth portfolio, which has the highest idiosyncratic coskewness beta, is much lower than those for the other nine portfolios. The average return for the long-short-portfolio which goes long the portfolio with the lowest idiosyncratic coskewness beta and short the portfolio with the highest idiosyncratic coskewness beta is 0.81% with a t-statistic of 1.87. The results weakly support the prediction that excess returns decline with idiosyncratic coskewness betas rising when idiosyncratic coskewness betas are positive.

There is also an interesting pattern in estimated factor loadings reported in Table 2. Portfolios with low idiosyncratic coskewness betas have low loadings on the market factor, negative loadings on the size factor *SMB*, and positive loadings on the value factor *HML*. Portfolios with high idiosyncratic coskewness betas have high loadings on the market factor, positive and high loadings on the size factor *SMB*, and negative loadings on the value factor *HML*. There is no clear pattern in the estimated factor loadings for the momentum factor *UMD*.

These factor loadings imply that when we correct risk using the market factor or the

Fama-French three factors, we will not be able to explain why the portfolio with the highest idiosyncratic coskewness beta has such low excess returns compared to the other nine portfolios. On the contrary, it will worsen the anomaly. In fact, alphas in the regressions with respect to the CAPM, the Fama-French three-factor model, and Carhart four-factor model are almost monotonically declining with idiosyncratic coskewness betas increasing. A long-short portfolio that holds the portfolio with the lowest idiosyncratic coskewness beta and shorts the portfolio with the highest idiosyncratic coskewness beta has a CAPM alpha of 1.21% with a t-statistic of 3.14; it has a Fama-French three-factor alpha of 1.12% with a t-statistic of 4.29; and it has a Carhart four-factor alpha of 0.98% with a t-statistic of 4.02. When we correct risk using the standard factors, we find stronger evidence to support the prediction that there is a negative relationship between excess returns and idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative.

For the ten portfolios with negative idiosyncratic coskewness betas, the average excess returns reported in Table 3 are almost monotonically increasing with idiosyncratic coskewness betas. It is consistent with the prediction that there is a positive relationship between excess returns and idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative. A long-short portfolio that holds the portfolio with the lowest idiosyncratic coskewness beta and shorts the portfolio with the highest coskewness beta has an excess return of -0.63% with a t-statistic of 2.00.

There is a clear pattern in estimated factor loadings for the market factor and the size factor *SMB* in the four-factor regression. Portfolios with low idiosyncratic coskewness betas have high loadings on the market factor and the size factor *SMB*. Portfolios with high idiosyncratic coskewness betas have low loadings on the market factor and the size factor *SMB*. There is no clear pattern in estimated factor loadings for the value factor *HML* and the momentum factor *UMD*. These loading implies that when we cannot explain the return difference using the standard risky factors. In fact, controlling those factors increases return difference for portfolios sorted by idiosyncratic coskewness betas. The same long-short portfolio has a CAPM alpha of -0.88% with a t-statistic of 2.74; it has a Fama-French three-factor alpha of -0.85% with a t-statistic of 3.76; and it has a Carhart four-factor alpha of -0.61% with a t-statistic of 2.48.

In summary, the empirical results support that the relationship between equity returns and idiosyncratic coskewness betas is positive when idiosyncratic coskewness betas are negative, and negative when idiosyncratic coskewness betas are positive. In addition, we find that the return difference between portfolios sorted by idiosyncratic coskewness betas, with either positive or negative values, cannot be explained by the standard risk factors, such as the market factor, the size factor, the value factor, and the momentum factor. In the next section, we will examine the relationship between default risk and idiosyncratic coskewness.

2.4 Idiosyncratic Coskewness Factors

We investigate two value-weighted hedge portfolios that capture the effect of idiosyncratic coskewness. As discussed in the previous section, at the beginning of each month, we use past 12 month daily equity returns to estimate idiosyncratic coskewness beta for each individual firm. We first divide firms into two groups according to the sign of the estimated idiosyncratic coskewness betas, then we form value-weighted decile portfolios based on the estimated idiosyncratic coskewness betas. We compute the excess portfolio returns in the following month (i.e. post-ranking). We construct the long-short portfolio holding the portfolio with the lowest idiosyncratic coskewness beta and shorting the portfolio with the highest idiosyncratic coskewness beta. The long-short portfolio in the group with negative idiosyncratic coskewness beta is called $ICSK_1$, and the long-short portfolio in the group with positive idiosyncratic coskewness beta is called $ICSK_2$. We use $ICSK_1$ and $ICSK_2$ to proxy for idiosyncratic coskewness factors.

The average monthly excess returns for $ICSK_1$ and $ICSK_2$ are 0.81% and -0.63% respectively over the period January 1971 to December 2006. We reject the hypothesis that the mean excess return for factor $ICSK_2$ is zero at the 5 percent level of significance. But we cannot reject the same hypothesis for factor $ICSK_1$. A high factor loading on $ICSK_1$ should be associated with high expected excess returns. In contrast, for factor $ICSK_2$, a high factor loading should be associated with low expected excess returns.

2.5 Can Idiosyncratic Coskewness Factors Explain the Fama and French Portfolios?

The failures of the CAPM model often appear in specific groups of securities that are formed on size, book-to-market ratio and momentum. To understand how idiosyncratic coskewness factors enter asset pricing, we analyze the pricing errors from other asset pricing models such as the Fama-French three-factor model, and the four-factor model proposed by Carhart (1997).

We carry out time-series regressions of excess returns,

$$r_{i,t} = \alpha_i + \sum_{j=1}^{K} \widehat{\beta}_j f_{j,t} + e_{i,t}, \text{ for } i = 1, ..., N, t = 1, ..., T, \tag{7}$$

and jointly test whether the intercepts, α_i, are different from zero using the F-test of Gibbons, Ross, and Shanken (1989) where $F \sim (N, T-N-K)$. We test the Fama-French three factor model and Carhart four-factor model for industrial portfolios, decile portfolios sorted by size, book-to-market ratio, and momentum, and decile portfolios sorted by idiosyncratic coskewness beta. The results are presented in Table 4. When we test 10 portfolios sorted by the book-to-market ratio, the inclusion of the two idiosyncratic coskewness factors reduces the F-statistics from 4.96 to 1.95 in the Fama-French model and from 3.39 to 1.31 in the Carhart model. Similar results are obtained for momentum-sorted portfolios and portfolios sorted by idiosyncratic coskewness beta. In all cases, the inclusion of the two idiosyncratic coskewness factors in either the Fama-French model or the Carhart model dramatically reduces the F-statistics. The results suggest that the two idiosyncratic coskewness factors can explain a significant part of the variation in returns even when factors based on size, book-to-market ratio, and momentum are added to the asset pricing model.

3 MAX Returns and Idiosyncratic Coskewness

A recent empirical paper by Bali, Cakici, and Whitelaw (2009) investigates the significance of extreme positive returns in the cross-sectional pricing of stocks. Their portfolio-level analysis and firm-level cross-sectional regressions indicate a negative and significant relation between the maximum daily return over the past month (MAX) and expected stock returns. Average raw and risk-adjusted return differences between stocks in the lowest and highest MAX deciles exceed 1% per month. Their results are robust to controls for size, book-to-market, momentum, short-term reversals, liquidity, and skewness. The idiosyncratic coskewness beta proposed in this paper directly measures the relationship between the expected return of a stock and its contribution to the skewness of the portfolio. We investigate whether our idiosyncratic coskewness factors can explain the puzzling finding in Bali, Cakici, and Whitelaw (2009). We first replicate their findings using the CRSP data set, then we examine the linkage between idiosyncratic coskewness and their anomalous findings by regressing portfolio sorted by the maximum daily return over the past month on the standard and the two idiosyncratic coskewness factors. The results are reported in table 5.

Following the same method discussed in Bali, Cakici, and Whitelaw (2009), we sort all stocks on the maximum daily return over the past month and divide them into 10 decile portfolios. The average excess returns of deciles 1 (low MAX)to 7 are approximately the same, in the range of 0.51% to 0.68% per month, but, going from decile 7 to decile 10 (high MAX), average excess returns drop significantly, from 0.51% to 0.35%, 0.15% and then to 0.42% per month. The average excess return of the portfolio with the lowest maximum daily return over the past month is 0.93% per month higher than that of the portfolio with the highest maximum daily return over the past month. In addition, the monthly return difference is 1.36%, 1.15%, and 0.97% when we control for the market factor, the

Fama-French factors, and the Carhart factors, respectively. The return differences are all statistically significant. The results are consistent with the findings in Bali, Cakici, and Whitelaw (2009). They interpret the results as "Given a preference for upside potential, investors may be willing to pay more for, and accept lower expected returns on, assets with these extremely high positive returns."

When we add the two idiosyncratic coskewness factors into the regression of returns of the 10 decile portfolios on the market factor, the monthly return difference between the portfolio with the lowest and the highest maximum daily return over the past month is reduced from 1.36% to 0.43% with a t-statistics 1.92. When we add the two idiosyncratic factors into the regressions using Fama-French 3 factors and Carhart 4 factors, the monthly return difference between the portfolio with the lowest and the highest maximum daily return over the past month is reduced from 1.15% (t = 4.66) to 0.40% (t = 1.78), and from 0.97% (t = 3.61) to 0.34% (t = 1.51), respectively. In addition, we have sorted all stocks on the average of the maximum two and three daily returns over the past month respectively. We obtain very similar results when we add the two idiosyncratic coskewness factors to the regressions.

Bali, Cakici, and Whitelaw (2009) rely on the cumulative prospect theory as modeled in Barberis and Huang (2008) to explain their findings. We provide an alternative rational explanation based on the assumption of heterogeneous preference. The "Lotto investor" who cares not only about the mean and variance of his portfolio but also about the skewness of his portfolio would bid up those lottery-like stocks to improve his portfolio allocation.

4 Default Risk and Idiosyncratic Coskewness

4.1 Equity Returns on Distressed Stocks

Recent empirical studies by Dichev (1998), Griffin and Lemmon (2002), and Campbell, Hilscher, and Szilagyi (2008) find a surprising negative relation between expected equity returns and default risk. If the stocks of financially distressed firms tend to move together, and their risk cannot be diversified away, finance theory dictates a positive relation between expected equity returns and default risk. If default risk is idiosyncratic, there is no significant relation between expected returns and default risk. These empirical findings seem to suggest that the equity market has not properly priced default risk. We examine if the two idiosyncratic coskewness factors can explain the negative relation between equity returns and default risk.

We use the Merton (1974) model to estimate the default probability for each firm (see appendix B), and examine the relationship between the likelihood of default and equity returns. We use the same method to estimate default likelihood as Vassalou and Xing (2004). Unlike Vassalou and Xing (2004), we use only industrial firms, which are more suitable for Merton's model. We also minimize liquidity effects on equity returns by eliminating illiquid stocks. At the end of each month, we sort firms according to their default measures and construct 10 portfolios as discussed in the previous section. Because highly distressed firms are more likely to be delisted and disappear from the CRSP database, it is important to carefully compute equity returns for delisted firms. CRSP reports a delisting return for the final month of a firm's life when it is available. In this case, we use delisting returns to compute portfolio returns. When delisting returns are not available, we exclude those firms from portfolios. This assumes that those stocks are sold at the end of the month before delisting, which implies an upward bias to the returns for distressed-stock portfolios (Shumway (1997)).

Table 7 reports the summary statistics of equity returns on the ten distress-sorted portfolios. The average returns are declining in general with default measures increasing. The average return is 0.92% for the portfolio with the lowest default risk, and it is -0.51% for the portfolio with the highest default risk. The volatilities of returns are increasing with default measures. The standard deviation of returns is 4.48% for the portfolio with the lowest default risk, and it is 14.95% for the portfolio with the highest default risk. In addition, returns on portfolios with low default measures exhibit negative skewness, and returns on portfolios with high default measures exhibit positive skewness. There is no clear pattern in the kurtosis of returns. Table 7 also reports the unconditional coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness of the ten distress-sorted portfolios. There is no clear pattern in the coskewness betas. However, both idiosyncratic coskewness betas and idiosyncratic skewness are in general increasing with default measures. The average size of firms in the ten portfolios is monotonically declining with default measures increasing. It suggests that controlling for the size risk factor will not explain the puzzling negative relation between equity returns and default risk.

A possible explanation for the negative relation between equity returns and default measures is that the default measure is just a proxy for other systematic risk factors. We test this hypothesis with regression results in Table 8. Panel A reports the excess returns of ten distress-sorted portfolios and a long-short-portfolio that goes long the portfolio with the lowest default risk, and short the portfolio with the highest default risk. Panel A also reports the alphas in regressions of the portfolio excess returns on the CAPM factor, Fama-French three factors, and four factors proposed by Carhart (1997) that includes a momentum factor in addition to Fama-French three factors. The returns are reported in monthly percentage points, with Robust Newey-West t-statistics below in the parentheses. Panel B, C, and D report estimated factor loadings for excess returns on the CAPM factor, Fama-French three factors, and four factors in the Carhart (1997) model. Figure 7 plots the alphas from these regressions.

The average excess returns of the 10 stress-sorted portfolios reported in Table 8 are in general declining in the default risk measure. The average excess return for the lowest-risk 5% of stocks is positive at 0.43% per month, and the average excess return for the highest-risk 1% of stocks is negative at -0.99% per month. A long-short portfolio that goes long the safest 5% of stocks, and short the most distressed 1% of stocks has an average return of 1.42% per month with a standard deviation of 14%. It implies a Sharp ratio of 0.10.

There is also a significant pattern on the factor loadings reported in Table 8. The low risk portfolios in general have smaller market betas, negative loadings on the size factor *SMB*, and negative loadings on the value factor *HML*. On the contrary, the high risk portfolios in general have bigger market betas, positive loadings on the size factor *SMB*, and positive loadings on the value factor *HML*. The results reflect the fact that most distressed stocks are small stocks with high book-to-market ratios. It implies that correcting risk using the market factor or Fama-French factors will not solve the anomaly but worsen it. In fact, the long-short portfolio that is long the safest 5% of stocks, and short the most distressed 1% of stocks has a CAPM alpha of 1.94% per month with a t-statistic of 3.16. It has a Fama-French three-factor alpha of 2.76% per month with a t-statistic of 4.90. In addition, the Fama-French three-factor alphas for all portfolios beyond 40th percentile of the default risk distribution are negative and statistically significant.

Avramov, Chordia, Jostova, and Philipov (2007) find a robust link between credit rating and momentum. They find that momentum profit exists only in low-grade firms. Distressed firms have negative momentum, which may explain their low average returns. When we correct for risk by using the Carhart (1997) four-factor model including a momentum factor, the low risk portfolios in general have low and positive loadings on the momentum factor. The high risk portfolios have high and negative loadings on the momentum factor. After controlling for the momentum factor, we find that the alpha for the long-short portfolio is cut almost in half, from 2.76% per month to 1.38% per month, which is still statistically significant.

4.2 Explaining Equity Return for Distressed Firms

We have demonstrated that in a model with heterogeneous investors who care about the skewness of their portfolios, the expected return of risky assets depends on their market betas, coskewness betas, idiosyncratic coskewness betas and idiosyncratic skewness. To capture the effect of coskewness on cross-sectional equity returns, we construct a value-weighted hedge portfolio, i.e. the coskewness factor, holding the portfolio with the lowest coskewness beta and shorting the portfolio with the highest coskewness beta. In a similar fashion, we also construct a hedge portfolio, i.e. the idiosyncratic skewness factor, to capture the effect of idiosyncratic skewness.

We have shown that the standard risk factors, such as the market factor, the Fama-French factors, and the Carhart four risk factors, cannot explain why high distressed firms earn low equity returns. In our model, expected equity returns depend on not only their CAPM betas, but also their coskewness betas, idiosyncratic coskewness betas and idiosyncratic skewness. We investigate if coskewness betas, idiosyncratic coskewness betas or idiosyncratic skewness can help explain the anomaly. We first run simple regressions of returns of distress-sorted portfolios on the market factor, the coskewness factor, the two idiosyncratic coskewness factors, and the idiosyncratic skewness factor. The results presented in Table 9 show that the equity return anomaly for distressed firms still exists when we control for any of the market factor, the coskewness factor, and the idiosyncratic skewness factor. The monthly return difference between portfolios with the lowest and highest default probabilities is 1.94% ($t = 3.16$), 1.41% ($t = 2.18$), and 1.66% ($t = 2.55$), respectively, when we control for the market factor, the coskewness factor, and the idiosyncratic skewness factor. The return differences are statistically significant at the 5% level. However, the monthly return difference between portfolios with the lowest and highest default probabilities is 0.73% ($t = 1.12$) and 0.89% ($t = 1.45$), respectively, when we control for the positive and negative idiosyncratic coskewness factors. The return differences are not statistically significant at the 5% level.

The simple regression results show that either positive or negative idiosyncratic coskewness factors can at least partially explain why equity returns are low for high distressed firms. High distressed firms will earn low equity returns if they have negative loadings on the positive idiosyncratic coskewness factor and positive loadings on the negative idiosyncratic coskewness factor. We will further test this hypothesis by regressing distress-sorted portfolio returns on two idiosyncratic coskewness factors, $ICSK_1$ and $ICSK_2$. We will also test the robustness of our results by including other risk factors, such as the Fama-French factors and the momentum factor, in the regressions. The regression results are reported in Table 10.

When we regress excess returns for distress-sorted portfolios on the two idiosyncratic coskewness factors, we find striking variations in factor loadings across portfolios. The factor loadings for factor $ICSK_1$ are almost monotonically declining with default risk increasing. In contrast, the factor loadings for factor $ICSK_2$ are almost monotonically increasing with default risk. The portfolio with the highest default risk has negative loadings on factor $ICSK_1$ and positive loadings on $ICSK_2$. They are both statistically significant at 1% level. Since a positive loading on factor $ICSK_1$ and a negative loading on $ICSK_2$ will reduce expected excess returns, controlling for the two idiosyncratic coskewness factors helps explain the equity return anomaly for distressed firms. The same result can be found in the regression of excess returns for a long-short portfolio holding the safest portfolio and shorting the most risky portfolio on the two idiosyncratic coskewness factors. The factor loading is positive for factor $ICSK_1$ and negative for factor $ICSK_2$. Both loadings are statistically significant. Controlling for the two idiosyncratic coskewness factors cuts alphas for the long-short portfolio roughly in half, from 1.42% to 0.64%, and it is not statistically significant.

To examine the robustness of our findings, we include four standard risk factors (*MKT*, *SMB*, *HML*, *UMD*)in the regression. For the ten distress-sorted portfolios and the long-short portfolio, the factor loadings on the two idiosyncratic coskewness remain similar. Alpha for the long-short portfolio is 0.73% with a t-statistic of 1.11.

The results show that the explanatory power of the two idiosyncratic coskewness factors is large for firms on both tails of the distribution of distress measures. The adjusted R^2 in the regression of returns of the long-short portfolio based on default measures on the two idiosyncratic coskewness factors is 28%. The negative loading on $ICSK_1$ and positive loading on $ICSK_2$ help reduce the alpha for the long-short portfolio based on distress measures.

5 Idiosyncratic Volatility Puzzle and Idiosyncratic Coskewness

An empirical study by Ang, Hodrick, Xing, and Zhang (2006) finds a negative relation between the expected return and a stocks's idiosyncratic volatility (IVOL) relative to the Fama and French (1993) three-factor model. This negative relationship cannot be explained by a number of standard risk factors, such as the aggregate volatility, size, book-to-market, momentum, and liquidity. Next we investigate if the two idiosyncratic coskewness factors can explain this phenomenon.

We compute idiosyncratic volatility using residuals from a regression of past 1-year daily equity returns on the Fama-French three factors. We then sort stocks into 10 decile portfolios according to the computed idiosyncratic volatility. We computed the equity returns for each portfolio for the following month. The results are presented in Table 11.

The average excess returns of deciles 1 (low IVOL) to 6 are approximately the same, in the range of 0.53% to 0.68% per month, but, going from decile 6 to decile 10 (high IVOL), average excess returns drop significantly, from 0.55% to 0.34%, 0.07%, -0.12% and then to -0.72% per month. Ang, Hodrick, Xing, and Zhang (2006) sort stocks into 5 quintile portfolios, and the average returns of the first three portfolios are approximately the same, and the average return of the last drops significantly. The average returns of the 10 decile portfolios in our study exhibit the same pattern.

The average excess return of the portfolio with the lowest idiosyncratic volatility is 1.28% per month higher than that of the portfolio with the highest idiosyncratic volatility. The monthly return difference is 1.74%, 1.72%, and 1.41% when we control for the market factor, the Fama-French factors, and the Carhart factors respectively. The return differences are all statistically significant. The results are very similar to those in Ang, Hodrick, Xing, and Zhang (2006). In addition, the average excess return of the portfolio with the lowest idiosyncratic volatility is 0.68% per month higher than that of the portfolio with the second highest idiosyncratic volatility. The monthly return difference is 1.15%, 0.95%, and 0.77% when we control for the market factor, the Fama-French factors, and the Carhart factors, respectively. The return differences are all statistically significant when we control for the standard risk factors.

When we add the two idiosyncratic coskewness factors into the regression of excess returns of the 10 decile portfolios on the market factor, the monthly return difference between the portfolio with the lowest and the highest idiosyncratic volatility is reduced from 1.74% to 0.47%. The return difference is still statistically significant. However, the monthly return difference between the portfolio with the lowest and the second highest idiosyncratic volatility is reduced from 1.15% to 0.01%, which is not statistically significant. When we add the two idiosyncratic factors into the regressions using Fama-French 3 factors and Carhart 4 factors, the monthly return difference between the portfolio with the lowest and the highest maximum daily return over the past one month is reduced from 1.2% to 0.78%, and from 1.41% to 0.64%, respectively. The reductions are large, but the return difference is still statistically significant. However, the monthly return difference between the portfolio with the lowest and the second highest idiosyncratic volatility is reduced from 0.95% to 0.12%, and from 0.77% to 0.08% respectively. The return differences are not statistically significant.

The results show that although the two idiosyncratic coskewness factors cannot solve the idiosyncratic volatility puzzle, they reduce the magnitude of the anonymous return difference between the portfolio with the lowest and highest idiosyncratic volatility by more than 60%.

6 Conclusion

We build a theoretical model of heterogeneous skewness preference that leads to asset-pricing relationships that differ from the standard CAPM model. We show that the expected excess return on a skewed security depends the standard risk premium in the CAPM model and the asset's idiosyncratic coskewness betas which measures the covariance of the squared idiosyncratic shock and the market return.

We empirically show that in addition to the well known idiosyncratic skewness, the idiosyncratic coskewness measure is also an important determinant for asset returns. Our measure of idiosyncratic coskewness cannot be explain by the idiosyncratic skewness, this suggests that both idiosyncratic skewness and idiosyncratic coskewness measure different higher moment risks. The idiosyncratic coskewness can explain the anomalous finding that stocks with the maximum daily return over the past month (MAX) earn low expected returns (see Bali, Cakici, and Whitelaw (2009)).

We also provide a rational explanation of the seemingly anomalous negative relation between default risk and equity returns. Although a number of theories point toward a lower return for stocks with default risk, empirical testing of the relation between default risk and measures of idiosyncratic skewness has been slow in coming. We attempt to fill this void by estimating a model of idiosyncratic coskewness and then using idiosyncratic coskewness to explain the negative relation between default risk and equity returns. We find that once we control for idiosyncratic coskewness, the negative relation between equity returns and default risk disappears.

Furthermore, the idiosyncratic coskewness seems to be related to the idiosyncratic volatility puzzle found in Ang, Hodrick, Xing, and Zhang (2006). Although the idiosyncratic coskewness factors cannot totally explain the negative relation between equity return and idiosyncratic volatility, they significantly reduced the magnitude of the return difference between portfolios with the smallest and largest idiosyncratic volatility.

Appendix A: Equity Returns and Idiosyncratic Coskewness

Since the economy should generate the same expected excess return regardless of investor preferences, the expected excess returns in (3) and (4) have to be identical. This allows us to write the equilibrium expected excess return as

$$E(R - R_f\mathbf{1}) = \frac{1}{2\tau}\Sigma(\omega_{\mathcal{L}} + \omega_{\mathcal{T}}) - \frac{1}{2\phi}E(\omega_{\mathcal{L}}^{\intercal}(R - ER)(R - ER)^{\intercal}\omega_{\mathcal{L}})(R - ER). \tag{A-1}$$

with $\Sigma = E(R - ER)(R - ER)^{\top}$. Notice that $\omega_M = \omega_{\mathcal{L}} + \omega_T$ represents the aggregate demand in this economy, hence $(\omega_M)^{\top} R$ can be treated as the return on the market portfolio, denoted by R_M. Notice that $\omega_{\mathcal{L}} = \omega_M - \omega_{\mathcal{T}}$. Given that (A-1) holds , the equilibrium expected excess return on the risky asset i is

$$\begin{aligned} E(R_i - R_f) \quad &= \quad \frac{1}{2\tau}Cov\left(R_M, R_i\right) - \frac{1}{2\phi}Cov\left(\left(R_M - ER_M\right)^2, R_i\right) \\ &\quad -\frac{1}{2\phi}Cov\left(\left(\omega_{\mathcal{T}}^{\top}\left(R - ER\right)\right)^2, R_i\right) \\ &\quad -\frac{1}{\phi}Cov\left(\left(R_M - ER_M\right)\left(\omega_{\mathcal{T}}^{\top}\left(R - ER\right)\right), R_i\right). \end{aligned} \tag{A-2}$$

To isolate the effect of idiosyncratic coskewness, we use the return decomposition $(R_i - ER_i) = a_i(\mathcal{W}_{\mathcal{T}} - E\mathcal{W}_{\mathcal{T}}) + \varepsilon$, and assumptions (5) and (6) to decompose the expected excess return as:

$$\begin{aligned} ER_i - R_f \quad &= \quad \lambda_M \frac{Cov\left(R_M, R_i\right)}{Var\left(R_M\right)} + \lambda_{CSK}\frac{Cov\left(\left(R_M - ER_M\right)^2, R_i\right)}{Var\left(R_M\right)} \\ &\quad +\lambda_{IS}\frac{E\left(R_i - ER_i\right)^3}{\sigma_i^3} + \lambda_{ISC}\frac{Cov\left(R_M, \left(R_i - ER_i\right)^2\right)}{Var\left(R_M\right)} \end{aligned}$$

with

$$\lambda_M = \frac{1}{2\tau}Var\left(R_M\right), \lambda_{CSK} = -\frac{1}{2\phi}Var\left(R_M\right), \lambda_{IS} = -\frac{1}{2\phi a_i^2}\sigma_i^3, \lambda_{ICSK} = -\frac{1}{\phi a_i}Var\left(R_M\right).$$

Appendix B.1: Measuring Default Probability with the Merton's Model

In the default risk literature, there are two approaches to measure default risk, the reduced-form and structural approaches. A reduced-form model provides the maximum likelihood estimates of a firm's default probability based on the empirical frequency of default and its correlation with various firm characteristics. A structural model provides an estimated default probability which is theoretically motivated by the classical option-pricing models (Merton (1974)). Traditional reduced-form models, such as Altman (1968) Z-score model and Ohlson (1980) O-score model, compute measures of bankruptcy by using accounting information in conditional logia models. Accounting models use information from firms' financial statement. The accounting information is about firms' past performance, rather than their future prospects. In contrast, structural models use the market value of equity to derive measures of default risk. Market prices reflect investors' expectations about firms' future performance. Therefore, they are better suited for measuring the probability that a firm may default in the future. In this paper, we use Merton's (1974) model to estimate the default probability of a firm. In Merton's model, the equity of a firm is viewed as a call option on the value of firm's assets. The firm will default when the value of the firm falls below a strike price, which is measured as the book value of firm's liabilities. The firm value is not observable and is assumed to follow a geometric Browning motion of the form:

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW, \tag{A-3}$$

where V_A is the value of firm's assets, with an instantaneous drift μ_A, and an instantaneous volatility σ_A. W is the standard Wiener process.

Let X_t denote the book value of firm's liabilities at time t, which has a maturity at time

T. The value of equity is given by the Black and Schools (1973) formula for call options:

$$V_E = V_A N(d_1) + Xe^{-rT} N(d_2), \tag{A-4}$$

where

$$d_1 = \frac{\ln(V_A/X) + (r + \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}, d_2 = d_1 - \sigma_A\sqrt{T}, \tag{A-5}$$

r is the risk-free interest rate, and N is the cumulative density function of the standard normal distribution.

Using daily equity data from the past 12 months, we adopt a maximum likelihood method developed by Duan (1998) to obtain an estimate of the volatility of firm value σ_A. Duan (1998) computes the likelihood function of equity returns by utilizing the conditional density of the unobservable firm value process. We repeat the estimation procedure at the end of every month, resulting in monthly estimates of the volatility σ_A. We always keep the estimation window to 12 months.

With an estimated σ_A, we can calculate daily values of V_A for the last 12 months, and then estimate the drift μ_A. At the end of every month t, the probability of default implied by Merton's model is given by:

$$P_{def,t} = N(\frac{\ln(V_{A,t}/X_t) + (\mu_A - \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}). \tag{A-6}$$

The $P_{def,t}$ calculated from equation (A-6) does not correspond to the true default probability of a firm in large samples since we do not use data on actual defaults. However, we use our measures to study the relationship between default risk and equity returns. The difference between our measure of default probability and true default probability may not be important as long as our measure correctly ranks firms according to their true default probability.

Appendix B.2: Data

One important parameter in Merton's model is the strike price, i.e. the book value of debt. Most firms have both long-term and short-term debts. Following KM, we calculate the book value of debt by using short-term debt plus half long-term debt. We use the COMPUSTAT annual files to obtain the firm's "Debt in One Year" and "Long-Term Debt" series for all firms. Since debt data was not available for many firms before 1970, the sample period in our study is January 1971 to December 2006. In addition, financial firms have very different capital structure than industrial firms. We exclude all financial firms (SIC codes: 6000–6999). We also exclude all utility firms (SIC codes: 4900–4999) because many utility firms were highly regulated during our sample period. We use only industrial firms (SIC codes: 1–3999 and 5000–5999) in this studies since they are more suitable for Merton's model. We obtain all industrial firms with data available simultaneously on both CRSP and COMPUSTAT databases.

We obtain the book value of debt from the COMPUSTAT annual files. To avoid the problem of delayed reporting, we lag the book value of debt by 3 months. This is to ensure that our default probability measure is based on all information available to investors at the time of calculation.

To compute the default likelihood measure, we obtain daily equity values for firms from CRSP daily files, and the risk-free interest rate from the Fama-Bliss discount bond file. We use monthly observations of the 1-year Treasury bill rate and equity data for the past 12 months to calculate monthly default measures for all firms.

When a firm is in sever financial distress, its equity is not liquid with low prices. To minimize liquidity effects on equity returns, we eliminate stocks with prices less than $1 at the portfolio construction date, and stocks with less than 120 transactions in the past 12 months. In the end, we have 10,078 firms with more than 3.5 million monthly observations in the sample.

Figure 6 plots the average default probability for industrial firms during the sample period. The shaded areas represent the NBER recession periods. The graph shows that the average default probability varies greatly and it usually peaks during recessions.

Appendix B.3: Performance of Merton's Model

To test the performance of Merton's model in predicting bankruptcy and other distress in our sample, we construct two measures based on exchange delisting as proxies for bankruptcy. One is a narrower measure of distress, called bankruptcy delisting (delisting codes: 400, 572, 574). The other is a broader measure of distress, called performance delisting (delisting codes: 400, 550 to 585). The second measure includes delisting due to not only bankruptcy and liquidation but also insufficient number of market makers, insufficient capital, surplus, and/or equity, price too low, delinquent in filing, etc. All the delisting data are obtained from CRSP.

To evaluate the predictive ability of our default measure to capture default risk, we sort firms according to their estimated default probability based on past 12-month equity data. At the end of each month from January 1971 through December 2006, default probability is re-estimated using only historical data to avoid look-ahead bias. To pay greater attention to the tail of the default risk distribution, we follow Campbell et. al. and construct 10 portfolios containing stocks in percentiles 0–5, 5–10, 10–20, 20–40, 40–60, 60–80, 80–90, 90–95, 95–99, and 99–100 (P1 and P10 denote the portfolios with the lowest and the highest default probability respectively). In the following month, we then collect the number of bankruptcy and performance delistings for each portfolio. The summary results are reported in Table 6. The evidence shows that the default risk measure based on Merton's model is a good ex ante measure of probability of bankruptcy and other distress. The number of bankruptcy and performance delistings generally increases with default risk measures from Merton's model. During the sample period, 46 out of 60 delistings due to bankruptcy and liquidation, and 213 out of 443 delistings due to performance come from the two portfolios with highest default measures. These two portfolios contains the highest-risk 5% of stocks.

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Table 1: Equity Returns on Portfolios Sorted by Coskewness Betas, Idiosyncratic Coskewness Betas, and Idiosyncratic Skewness

We sort stocks based on estimated coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness respectively, and divide them into 10 decile portfolios. 0010 denotes the portfolio with the lowest coskewness betas, idiosyncratic coskewness betas, or idiosyncratic skewness, i.e. the 0–10 percentile, and 9900 denotes the portfolio with the highest coskewness betas, idiosyncratic coskewness betas, or idiosyncratic skewness, i.e. the 90–100 percentile. The hedge portfolio that longs 0010 and shorts 9000 is denoted by $LS1090$. This table reports results from regressions of value-weighted excess returns on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B reports factor loadings from the four-factor regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	$LS1090$
Panel A. Alphas for Portfolio Sorted by Coskewness Betas											
Mean excess return	0.59	0.58	0.57	0.59	0.58	0.42	0.60	0.59	0.50	0.43	0.17
	(2.73)	(2.74)	(2.84)	(2.75)	(2.90)	(1.82)	(2.72)	(2.72)	(2.18)	(1.05)	(1.87)
CAPM alpha	0.08	0.07	0.07	0.09	0.09	−0.07	0.08	0.07	−0.02	−0.11	0.19
	(0.77)	(1.03)	(0.74)	(1.38)	(1.22)	(1.05)	(1.28)	(0.82)	(0.23)	(1.09)	(1.03)
3-factor alpha	0.03	0.02	−0.03	0.06	0.01	−0.11	0.10	0.05	0.05	−0.03	0.06
	(0.31)	(0.33)	(0.38)	(0.77)	(0.23)	(1.60)	(1.67)	(0.60)	(0.71)	(0.44)	(0.43)
4-factor alpha	−0.17	−0.07	−0.08	−0.03	−0.05	−0.16	0.11	0.11	0.16	0.19	−0.36
	(1.30)	(0.91)	(0.95)	(0.32)	(0.85)	(2.06)	(1.78)	(1.23)	(1.75)	(2.30)	(1.94)
Panel A. Alphas for Portfolio Sorted by Idiosyncratic Coskewness Betas											
Mean excess return	0.39	0.52	0.49	0.48	0.52	0.54	0.52	0.53	0.56	0.08	0.31
	(1.34)	(2.20)	(2.30)	(2.33)	(2.43)	(2.35)	(2.20)	(1.77)	(1.28)	(0.16)	(1.06)
CAPM alpha	−0.24	−0.03	0.01	0.01	0.03	0.01	−0.05	−0.12	−0.22	−0.76	0.52
	(1.65)	(0.38)	(0.10)	(0.16)	(0.44)	(0.12)	(0.61)	(0.81)	(0.82)	(2.51)	(1.83)
3-factor alpha	−0.24	−0.05	−0.07	−0.06	−0.01	−0.03	−0.07	−0.12	−0.07	−0.66	0.42
	(1.89)	(0.61)	(1.15)	(1.21)	(0.09)	(0.33)	(0.87)	(1.08)	(0.28)	(2.98)	(1.65)
4-factor alpha	−0.09	0.13	0.05	0.01	−0.03	−0.08	−0.19	−0.16	0−.08	−0.50	0.41
	(0.74)	(1.46)	(0.86)	(0.24)	(0.44)	(1.14)	(2.35)	(1.48)	(0.40)	(2.44)	(1.62)
Panel A. Alphas for Portfolio Sorted by Idiosyncratic Skewness											
Mean excess return	0.36	0.62	0.61	0.61	0.53	0.61	0.53	0.61	0.66	0.71	−0.34
	(1.79)	(2.76)	(2.78)	(2.70)	(2.25)	(2.73)	(2.38)	(2.39)	(2.33)	(2.82)	(1.98)
CAPM alpha	−0.13	0.15	0.11	0.09	0.01	0.08	−0.02	0.07	0.07	0.22	−0.34
	(1.92)	(2.07)	(2.13)	(1.39)	(0.13)	(1.37)	(0.28)	(0.66)	(0.52)	(1.50)	(1.99)
3-factor alpha	−0.11	0.14	0.12	0.09	0.00	0.07	−0.12	−0.01	−0.00	0.12	−0.23
	(1.52)	(1.95)	(2.50)	(1.40)	(0.02)	(1.12)	(1.90)	(0.07)	(0.02)	(1.01)	(1.44)
4-factor alpha	0.05	0.18	0.14	0.08	0.02	−0.06	−0.13	−0.02	−0.10	0.13	−0.08
	(0.89)	(2.19)	(2.66)	(1.25)	(0.26)	(1.05)	(1.75)	(0.24)	(0.86)	(1.03)	(0.51)

Table 2: Equity Returns on Portfolios with Positive Idiosyncratic Coskewness Betas

We sort stocks based on estimated idiosyncratic coskewness betas which have positive values, and divide them into 10 decile portfolios. 0010 denotes the portfolio with the lowest idiosyncratic coskewness beta, i.e. the 0–10 percentile, and 9900 denotes the portfolio with the highest idiosyncratic coskewness beta, i.e. the 90–100 percentile. The hedge portfolio that longs 0010 and shorts 9000 is denoted by $LS1090$. This table reports results from regressions of value-weighted excess returns on a constant, market excess return (MKT), three (MKT, SMB, HML) Fama-French factors, and four (MKT, SMB, HML, UMD) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B reports factor loadings from the four-factor regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	*LS*1090
Panel A. Portfolio Alphas											
Mean excess return	0.66	0.45	0.51	0.56	0.64	0.52	0.64	0.58	0.40	−0.15	0.81
	(3.20)	(2.12)	(2.54)	(2.32)	(2.56)	(1.79)	(2.13)	(1.54)	(0.85)	(0.29)	(1.87)
CAPM alpha	0.24	0.01	0.04	0.02	0.04	−0.13	−0.04	−0.17	−0.43	−0.98	1.21
	(2.39)	(0.12)	(0.58)	(0.31)	(0.41)	(1.00)	(0.26)	(0.81)	(1.38)	(2.95)	(3.14)
3-factor alpha	0.17	−0.03	0.02	−0.01	0.03	−0.11	−0.05	−0.05	−0.25	−0.95	1.12
	(2.28)	(0.39)	(0.31)	(0.15)	(0.28)	(0.91)	(0.39)	(0.29)	(1.00)	(3.75)	(4.29)
4-factor alpha	0.21	−0.01	0.01	0.02	0.05	−0.03	0.04	0.04	−0.10	−0.77	0.98
	(2.54)	(0.12)	(0.20)	(0.18)	(0.56)	(0.27)	(0.27)	(0.23)	(0.44)	(3.49)	(4.02)
Panel B. Four-factor loadings											
MKT	0.91	0.93	0.96	1.07	1.15	1.18	1.21	1.24	1.29	1.28	−0.37
	(58.44)	(37.32)	(37.92)	(37.44)	(37.75)	(28.07)	(31.10)	(16.74)	(18.79)	(15.42)	(3.96)
SMB	−0.27	−0.18	−0.08	−0.03	0.16	0.30	0.54	0.68	0.95	1.29	−1.57
	(9.93)	(6.08)	(3.20)	(0.48)	(4.02)	(7.07)	(8.24)	(8.34)	(11.36)	(14.46)	(15.09)
HML	0.13	0.09	0.04	0.06	−0.01	−0.10	−0.08	−0.31	−0.44	−0.26	0.40
	(2.74)	(2.39)	(1.39)	(1.07)	(0.22)	(2.06)	(1.21)	(2.79)	(2.48)	(1.29)	(1.63)
UMD	−0.04	0.02	0.01	−0.03	−0.03	−0.08	−0.09	−0.08	−0.15	−0.18	0.14
	(1.53)	(0.69)	(0.21)	(0.70)	(0.81)	(1.64)	(2.24)	(1.11)	(1.41)	(1.58)	(1.08)

Table 3: Equity Returns on Portfolios with Negative Idiosyncratic Coskewness Betas

We sort stocks based on estimated idiosyncratic coskewness betas which have negative values, and divide them into 10 decile portfolios. 0010 denotes the portfolio with the lowest idiosyncratic coskewness beta, i.e. the 0–10 percentile, and 9900 denotes the portfolio with the highest idiosyncratic coskewness beta, i.e. the 90–100 percentile. The hedge portfolio that longs 0010 and shorts 9000 is denoted by $LS1090$. This table reports results from regressions of value-weighted excess returns on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B reports factor loadings from the four-factor regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	$LS1090$
Panel A. Portfolio Alphas											
Mean excess return	−0.09	0.32	0.11	0.49	0.31	0.60	0.53	0.62	0.38	0.54	−0.63
	(0.25)	(0.92)	(0.39)	(1.70)	(1.10)	(2.61)	(2.03)	(2.80)	(1.68)	(2.42)	(2.00)
CAPM alpha	−0.76	−0.32	−0.53	−0.12	−0.30	0.06	0.02	0.14	−0.06	0.12	−0.88
	(2.72)	(1.15)	(3.25)	(0.78)	(2.19)	(0.44)	(0.21)	(1.17)	(0.59)	(1.04)	(2.74)
3-factor alpha	−0.81	−0.35	−0.52	−0.13	−0.36	−0.01	−0.02	0.08	−0.14	0.04	−0.85
	(3.92)	(1.46)	(3.66)	(0.87)	(2.58)	(0.05)	(0.13)	(0.75)	(1.48)	(0.37)	(3.76)
4-factor alpha	−0.56	−0.15	−0.41	−0.06	−0.28	−0.03	0.13	0.09	−0.12	0.05	−0.61
	(2.53)	(0.54)	(2.88)	(0.38)	(1.98)	(0.25)	(1.17)	(0.78)	(1.29)	(0.41)	(2.48)
Panel B. Four-factor loadings											
MKT	1.06	1.07	1.11	1.10	1.15	1.08	1.02	0.98	0.95	0.89	0.17
	(15.91)	(24.70)	(22.34)	(30.31)	(22.82)	(27.42)	(32.68)	(26.15)	(48.18)	(31.38)	(2.05)
SMB	0.99	0.73	0.57	0.40	0.29	0.02	−0.08	−0.09	−0.21	−0.17	1.16
	(10.54)	(9.64)	(7.46)	(8.03)	(4.45)	(0.24)	(1.30)	(1.65)	(4.59)	(5.35)	(11.57)
HML	−0.11	−0.09 −	0.12	−0.06	0.04	0.12	0.04	0.10	0.16	0.15	−0.26
	(0.99)	(1.24)	(1.75)	(1.08)	(0.54)	(1.22)	(1.00)	(1.75)	(2.80)	(2.81)	(1.84)
UMD	−0.24	−0.19	−0.10	−0.07	−0.08	0.03	−0.15	−0.01	−0.02	−0.06	−0.23
	(2.95)	(2.22)	(2.60)	(1.89)	(2.51)	(0.53)	(3.57)	(0.30)	(0.50)	(0.16)	(2.23)

Table 4: Test Intercepts from the Fama-French Model and the Carhart Model

This table reports the results from multivariate tests on intercepts from time-series regressions with the three FamaFrench factors and the Carhart four factors including the momentum factor. We also include the two idiosyncratic coskewness factors in the regressions. The test-statistic is the Gibbons–Ross–Shanken F-test statistic distributed as $F \sim (N, T-N-K)$, where T is the number of observations, N is the number of portfolios and K is the number of factors. The p-values of the test statistics are presented in parentheses. The sample period is January 1971 to December 2006.

Portfolios	Number of Portfolios	F-test for Fama-French Three Factors	F-test for Fama-French Three Factors and Two ICSK Factors	F-test for Carhart Four Factors	F-test for Carhart Four Factors and Two ICSK Factors
industrial	30	15.45	11.80	12.68	10.00
		(0.00)	(0.00)	(0.00)	(0.00)
industrial	48	20.86	13.19	14.91	10.69
		(0.00)	(0.00)	(0.00)	(0.00)
Size	10	5.80	2.64	5.19	2.61
		(0.00)	(0.00)	(0.00)	(0.00)
Book/Market	10	4.96	1.95	3.39	1.31
		(0.00)	(0.04)	(0.00)	(0.22)
Momentum	10	14.90	7.88	8.85	5.52
		(0.00)	(0.04)	(0.00)	(0.22)
Positive idiosyncratic	10	8.99	1.88	6.39	1.83
coskewness beta		(0.00)	(0.06)	(0.00)	(0.05)
Negative idiosyncratic	10	8.37	2.50	4.06	1.51
coskewness beta		(0.00)	(0.01)	(0.00)	(0.13)

Table 5: Explaining Equity Returns on Portfolios Sorted by Maximum Daily Returns

We sort all stocks on the maximum daily return over the past month and divide them into 10 decile portfolios. The hedge portfolio that longs the portfolio with smallest maximum daily return and shorts the portfolio with largest maximum daily return is denoted by $LS1090$. Panel A reports alphas from regressions of value-weighted excess returns on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors respectively. Panel B reports alphas from regressions of value-weighted excess returns on the above mentioned standard risk factors and two idiosyncratic coskewness factors. The sample period is January 1971 to December 2006. The table shows alphas (in monthly percent units) from these regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	$LS1090$
Panel A. Alphas when controlling standard risk factors											
Mean excess return	0.51	0.55	0.65	0.59	0.67	0.68	0.51	0.35	0.15	−0.42	0.93
	(2.93)	(2.79)	(3.05)	(2.56)	(2.62)	(2.42)	(1.57)	(0.92)	(0.37)	(0.96)	(2.51)
CAPM alpha	0.17	0.13	0.17	0.07	0.10	0.06	−0.15	−0.41	−0.62	−1.19	1.36
	(1.90)	(1.64)	(2.36)	(0.97)	(1.22)	(0.64)	(1.04)	(2.29)	(3.07)	(4.65)	(4.34)
3-factor alpha	0.00	0.02	0.11	0.09	0.14	0.07	−0.02	−0.29	−0.48	−1.15	1.15
	(0.05)	(0.30)	(1.93)	(1.08)	(1.69)	(0.72)	(0.16)	(2.00)	(2.98)	(5.47)	(4.66)
4-factor alpha	−0.01	−0.00	0.10	0.13	0.21	0.10	0.04	−0.22	−0.37	−0.98	0.97
	(0.13)	(0.03)	(1.45)	(1.27)	(2.50)	(1.17)	(0.36)	(1.54)	(2.35)	(4.22)	(3.61)
Panel B. Alphas when controlling standard and two idiosyncratic coskewness factors											
CAPM + 2 ICSK alpha	−0.01	−0.04	0.06	0.03	0.14	0.18	0.11	0.05	−0.07	−0.45	0.43
	(0.15)	(0.56)	(1.07)	(0.32)	(1.77)	(2.10)	(0.82)	(0.38)	(0.53)	(2.57)	(1.92)
3-factor + 2 ICSK alpha	−0.15	−0.13	0.03	0.07	0.16	0.15	0.12	0.05	−0.11	−0.55	0.40
	(1.97)	(1.51)	(0.51)	(0.83)	(1.88)	(1.60)	(0.80)	(0.39)	(0.68)	(2.77)	(1.78)
4-factor + 2 ICSK alpha	−0.14	−0.13	0.03	0.10	0.22	0.18	0.16	0.07	−0.06	−0.49	0.34
	(1.71)	(1.46)	(0.42)	(1.07)	(2.55)	(1.88)	(1.14)	(0.51)	(0.38)	(2.40)	(1.51)

Table 6: Predictive Performance of Merton's Model

This table reports the number of bankruptcy and performance delistings as a function of default risk rank based on Merton's model. At the end of each month firms are assigned into portfolios according to their probability of default measures. We construct 10 portfolios containing stocks in percentiles 0-5, 5-10, 10-20, 20-40, 40-60, 60-80, 80-90, 90-95, 95-99, and 99-100 of the distribution of the default measure. Portfolio 1 contains firms with the lowest default probability measure. BD is the number of bankruptcy and liquidation delistings, and PD is the number of performance delistings. The sample period is January 1971 to December 2006.

Portfolio	Number of Bankruptcy Delistings (BD)	Number of Performance Delistings (PD)
1	0	2
2	0	3
3	1	11
4	0	23
5	1	27
6	2	42
7	3	55
8	7	67
9	20	111
10	26	102

Table 7: Summary Statistics of Returns on Distress-Sorted Portfolios

This table reports the mean, standard deviation, skewness, and kurtosis of monthly returns, in percentage points, on ten distress-sorted portfolios. It also reports unconditional coskewness betas, idiosyncratic coskewness betas, and idiosyncratic skewness of the ten portfolios, as well as the average size, in million dollars, of firms in each portfolio. At the end of each month firms are assigned into portfolios according to their probability of default measures. We construct 10 portfolios containing stocks in percentiles 0-5, 5-10, 10-20, 20-40, 40-60, 60-80, 80-90, 90-95, 95-99, and 99-100 of the distribution of the default measure. Portfolio 1 contains firms with the lowest default probability measure. Value-weighted realized returns in the next month are calculated for the ten portfolios. The sample period is January 1971 to December 2006.

	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900
Mean	0.92	0.98	1.17	1.03	1.01	0.88	0.81	0.36	0.41	−0.51
Stdev	4.48	4.40	4.36	4.87	5.81	6.57	7.64	9.02	10.70	14.96
Skew	−0.29	−0.48	−0.16	−0.37	−0.41	−0.27	0.15	0.49	0.86	0.81
Kurt	5.42	4.94	4.65	5.53	5.28	5.13	7.53	9.82	13.36	7.65
Coskew(10^{-4})	0.31	−0.69	1.69	0.05	−0.95	−1.60	−1.70	−2.84	−2.89	0.76
Idio. Coskew(10^{-2})	0.01	0.00	0.00	0.00	0.01	0.04	0.09	0.22	0.30	0.65
Idio. Skew	−0.26	−0.41	0.21	0.48	0.16	0.34	1.05	1.01	1.71	0.95
Mean size	4.90	4.33	2.80	1.43	0.71	0.35	0.18	0.11	0.10	0.07

Table 8: Equity Returns on Distress-Sorted Portfolios

We sort all stocks on the predicted default probability from Merton's Model and divide them into 10 portfolios on percentile cutoffs. The ten portfolios containing stocks in percentiles 0–5, 5–10, 10–20, 20–40, 40–60, 60–80, 80–90, 90–95, 95–99, and 99-100 of the distribution of the default measure. The portfolio contains the 0 to 5th percentile is denoted by 0005, and the portfolio contains the 99th to 100th percentile is denoted by 9900. The hedge portfolio that longs the 0005 and shorts 9900 is denoted by $LS0599$. This table reports the regression results from regressions of value-weighted excess returns on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B, C, and D report factor loadings from the CAPM, three-factor and four-factor regressions respectively. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900	*LS*0599
Panel A. Portfolio Alphas											
Mean excess return	0.43	0.50	0.69	0.55	0.53	0.40	0.33	−0.12	−0.08	−0.99	1.42
	(1.79)	(2.20)	(3.48)	(2.55)	(2.13)	(1.30)	(0.94)	(0.28)	(0.14)	(1.43)	(2.19)
CAPM alpha	−0.01	0.04	0.22	0.02	−0.07	−0.25	−0.36	−0.86	−0.89	−1.95	1.94
	(0.08)	(0.40)	(3.14)	(0.25)	(0.56)	(1.50)	(1.54)	(2.95)	(2.18)	(3.41)	(3.16)
3-factor alpha	0.20	0.16	0.24	−0.08	−0.33	−0.55	−0.79	−1.30	−1.41	−2.56	2.76
	(2.18)	(1.78)	(3.98)	(0.83)	(2.95)	(3.52)	(3.80)	(4.67)	(3.93)	(4.67)	(4.90)
4-factor alpha	0.01	−0.01	0.20	−0.01	−0.08	−0.09	−0.19	−0.54	−0.44	−1.37	1.38
	(0.11)	(0.09)	(2.82)	(0.14)	(0.76)	(0.84)	(0.99)	(2.25)	(1.43)	(2.28)	(2.27)
Panel B. CAPM factor loading											
MKT	0.87	0.89	0.92	1.03	1.18	1.27	1.35	1.46	1.60	1.88	−1.01
	(21.07)	(27.72)	(36.35)	(39.63)	(25.88)	(21.09)	(18.50)	(15.32)	(12.69)	(10.34)	(4.81)
Panel C. Three-factor loadings											
MKT	0.83	0.87	0.95	1.07	1.24	1.33	1.42	1.46	1.62	1.81	−1.01
	(22.74)	(25.67)	(43.42)	(53.99)	(42.76)	(22.70)	(21.09)	(14.11)	(13.10)	(9.06)	(4.36)
SMB	−0.25	−0.15	−0.13	0.04	0.30	0.40	0.60	0.90	1.01	1.55	−1.81
	(6.81)	(3.46)	(5.11)	(0.96)	(4.14)	(2.85)	(3.21)	(3.88)	(3.91)	(6.20)	(6.57)
HML	−0.30	−0.16	−0.01	0.15	0.38	0.44	0.61	0.59	0.71	0.78	−1.08
	(5.10)	(3.72)	(0.23)	(2.73)	(4.92)	(3.37)	(3.78)	(2.74)	(2.35)	(4.01)	(4.72)
Panel D. Four-factor loadings											
MKT	0.85	0.89	0.95	1.06	1.20	1.26	1.34	1.36	1.48	1.65	−0.79
	(29.04)	(31.64)	(44.59)	(50.64)	(40.07)	(29.63)	(27.92)	(18.05)	(17.85)	(12.57)	(5.46)
SMB	−0.26	−0.16	−0.13	0.04	0.03	0.40	0.61	0.92	1.03	1.56	−1.83
	(8.49)	(5.02)	(4.92)	(1.17)	(6.91)	(5.10)	(5.17)	(6.40)	(6.43)	(8.20)	(8.91)
HML	−0.26	−0.13	0.00	0.14	0.32	0.34	0.48	0.41	0.49	0.51	−0.77
	(4.96)	(3.03)	(0.04)	(2.85)	(6.06)	(4.95)	(5.16)	(3.25)	(2.42)	(3.21)	(4.63)
UMD	0.18	0.16	0.04	−0.06	−0.25	−0.45	−0.57	−0.73	−0.94	−1.16	1.34
	(3.35)	(5.56)	(1.21)	(1.84)	(5.06)	(9.09)	(6.31)	(6.32)	(5.61)	(5.67)	(5.71)

Table 9: Explaining Equity Returns on Distress-Sorted Portfolios 1

This table reports the regression results from regressions of value-weighted excess returns of distress-sorted portfolios on the market factor, coskewness factor, two idiosyncratic coskewness factors ($ICSK_1$, $ICSK_2$), and idiosyncratic skewness factor respectively. The sample period is January 1971 to December 2006. Alphas (in monthly percent units) from these regressions are reported. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900	LS0599
	Portfolio Alphas										
CAPM alpha	−0.01	0.04	0.22	0.02	−0.07	−0.25	−0.36	−0.86	−0.89	−1.95	1.94
	(0.08)	(0.40)	(3.14)	(0.25)	(0.56)	(1.50)	(1.54)	(2.95)	(2.18)	(3.41)	(3.16)
Coskewness alpha	−0.08	−0.02	0.19	0.03	0.02	−0.09	−0.16	−0.61	−0.55	−1.50	1.41
	(0.70)	(0.15)	(2.56)	(0.39)	(0.18)	(0.51)	(0.67)	(1.98)	(1.35)	(2.46)	(2.18)
Posi. Idio. Coskew. alpha	−0.18	−0.07	0.13	0.04	0.09	0.02	0.01	−0.32	−0.21	−0.90	0.73
	(1.54)	(0.74)	(1.88)	(0.41)	(0.63)	(0.11)	(0.02)	(0.92)	(0.45)	(1.47)	(1.12)
Nega. Idio. Coskew. alpha	−0.14	−0.06	0.15	0.05	0.08	0.00	−0.01	−0.39	−0.28	−1.03	0.89
	(1.20)	(0.63)	(2.33)	(0.52)	(0.66)	(0.02)	(0.05)	(1.32)	(0.69)	(1.77)	(1.45)
Idio. Skewness alpha	−0.02	0.02	0.21	0.04	−0.00	−0.14	−0.22	−0.73	−0.74	−1.68	1.66
	(0.19)	(0.23)	(2.74)	(0.54)	(0.01)	(0.88)	(0.94)	(2.49)	(1.86)	(2.72)	(2.55)

Table 10: Explaining Equity Returns on Distress-Sorted Portfolios 2

This table reports the regression results from regressions of value-weighted excess returns of distress-sorted portfolios on two idiosyncratic coskewness factors ($ICSK_1$, $ICSK_2$), and six factors (*MKT, SMB, HML, UMD,* $ICSK_1$, $ICSK_2$). The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B and C report factor loadings and R^2-adjusted from the two-factor and six-factor regressions respectively. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900	$LS0599$
Panel A. Portfolio Alphas											
two-factor alpha	−0.18	−0.08	0.13	0.04	0.10	0.05	0.04	−0.28	−0.15	−0.82	0.64
	(1.57)	(0.86)	(1.88)	(0.47)	(0.76)	(0.25)	(0.17)	(0.85)	(0.34)	(1.38)	(1.01)
six-factor alpha	−0.12	−0.07	0.16	−0.02	−0.09	−0.01	−0.08	−0.34	−0.13	−0.85	0.73
	(1.27)	(0.88)	(2.25)	(0.28)	(0.93)	(0.11)	(0.41)	(1.29)	(0.39)	(1.30)	(1.11)
Panel B. Two-factor loadings											
$ICSK_1$	0.12	0.05	0.06	0.01	−0.06	−0.11	−0.15	−0.30	−0.35	−0.56	0.68
	(3.07)	(1.74)	(5.03)	(0.31)	(0.83)	(1.26)	(1.08)	(1.95)	(1.81)	(3.08)	(3.14)
$ICSK_2$	−0.02	−0.05	−0.02	0.02	0.07	0.11	0.17	0.18	0.25	0.36	−0.38
	(0.68)	(2.16)	(1.62)	(1.61)	(2.53)	(3.65)	(3.72)	(3.16)	(3.19)	(3.30)	(2.95)
R^2−adjusted	0.20	0.12	0.15	0.01	0.10	0.18	0.18	0.25	0.24	0.24	0.28
Panel C. Six-factor loadings											
MKT	−0.10	−0.08	−0.03	0.07	0.21	0.24	0.30	0.28	0.38	0.47	−0.56
	(3.93)	(2.94)	(1.80)	(3.00)	(6.34)	(5.36)	(5.58)	(3.74)	(4.25)	(4.57)	(5.12)
SMB	−0.04	−0.05	−0.06	0.06	0.33	0.27	0.42	0.58	0.52	0.71	−0.76
	(0.96)	(1.02)	(1.53)	(1.37)	(6.24)	(3.84)	(3.65)	(4.31)	(3.07)	(2.10)	(2.18)
HML	−0.31	−0.05	−0.02	0.13	0.31	0.37	0.52	0.50	0.61	0.72	−1.03
	(8.19)	(3.49)	(0.70)	(3.15)	(6.81)	(5.37)	(5.92)	(4.17)	(3.66)	(4.26)	(6.08)
UMD	0.16	0.15	0.03	−0.06	−0.25	−0.43	−0.55	−0.70	−0.88	−1.06	1.22
	(3.78)	(5.41)	(1.01)	(1.79)	(4.92)	(8.40)	(5.74)	(6.05)	(5.55)	(5.82)	(6.09)
$ICSK_1$	0.13	0.05	0.04	0.02	0.02	−0.05	−0.07	−0.17	−0.24	−0.41	0.54
	(8.00)	(2.15)	(2.65)	(0.91)	(0.69)	(1.91)	(1.12)	(2.83)	(2.47)	(3.02)	(3.81)
$ICSK_2$	−0.01	−0.03	−0.01	0.01	0.01	0.04	0.08	0.06	0.12	0.19	−0.20
	(0.35)	(1.53)	(0.51)	(0.80)	(0.35)	(1.71)	(2.63)	(1.23)	(1.98)	(2.04)	(1.97)
R^2−adjusted	0.45	0.30	0.17	0.11	0.51	0.60	0.55	0.53	0.51	0.40	0.48

Table 11: Explaining Equity Returns on Portfolios Sorted by Idiosyncratic Volatility

We sort all stocks on the idiosyncratic volatility relative to the Fama and French (1993) 3-factor model and divide them into 10 decile portfolios. The hedge portfolio that longs the portfolio with smallest idiosyncratic volatility and shorts the portfolio with the second largest idiosyncratic volatility is denoted by $LS1080$. The hedge portfolio that longs the portfolio with smallest idiosyncratic volatility and shorts the portfolio with largest idiosyncratic volatility is denoted by $LS1090$. Panel A reports alphas from regressions of value-weighted excess returns on a constant, market excess return (MKT), three (MKT, SMB, HML) Fama-French factors, and four (MKT, SMB, HML, UMD) factors respectively. Panel B reports alphas from regressions of value-weighted excess returns on the above mentioned standard risk factors and two idiosyncratic coskewness factors. The sample period is January 1971 to December 2006. The table shows alphas (in monthly percent units) from these regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	$LS1080$	$LS1090$
Panel A. Alphas when controlling standard risk factors												
Mean excess return	0.56	0.53	0.68	0.65	0.56	0.55	0.34	0.07	−0.12	−0.72	0.68	1.28
	(3.02)	(2.55)	(3.09)	(2.25)	(1.85)	(1.49)	(0.86)	(0.17)	(0.25)	(1.43)	(1.56)	(2.74)
CAPM alpha	0.17	0.04	0.13	0.02	−0.12	−0.21	−0.46	−0.77	−0.98	−1.56	1.15	1.74
	(1.99)	(0.58)	(1.57)	(0.23)	(0.91)	(1.05)	(1.92)	(2.84)	(2.99)	(4.19)	(2.91)	(3.95)
3-factor alpha	0.10	−0.03	0.08	0.01	−0.09	−0.10	−0.35	−0.64	−0.85	−1.63	0.95	1.72
	(1.69)	(0.44)	(1.03)	(0.07)	(0.86)	(0.64)	(2.17)	(3.40)	(3.12)	(6.70)	(3.13)	(6.33)
4-factor alpha	0.07	−0.05	0.11	0.05	0.01	−0.01	−0.18	−0.49	−0.70	−1.34	0.77	1.41
	(1.26)	(0.94)	(1.30)	(0.55)	(0.09)	(0.04)	(1.33)	(2.71)	(2.74)	(4.71)	(2.75)	(4.55)
Panel B. Alphas when controlling standard and two idiosyncratic coskewness factors												
CAPM + 2 ICSK alpha	−0.03	−0.11	0.14	0.11	0.16	0.25	0.10	−0.05	−0.04	−0.50	0.01	0.47
	(0.51)	(1.62)	(1.71)	(1.15)	(1.98)	(2.06)	(0.76)	(0.40)	(0.25)	(2.39)	(0.08)	(2.13)
3-factor + 2 ICSK alpha	−0.02	−0.13	0.05	0.04	0.07	0.17	0.00	−0.16	−0.14	−0.80	0.12	0.78
	(0.27)	(2.21)	(0.49)	(0.40)	(0.91)	(1.50)	(0.01)	(1.25)	(0.62)	(3.46)	(0.48)	(2.98)
4-factor + 2 ICSK alpha	−0.03	−0.14	0.07	0.08	0.13	0.22	0.10	−0.10	−0.11	−0.66	0.08	0.64
	(0.43)	(2.23)	(0.78)	(0.77)	(1.60)	(1.89)	(0.83)	(0.68)	(0.50)	(2.73)	(0.35)	(2.37)

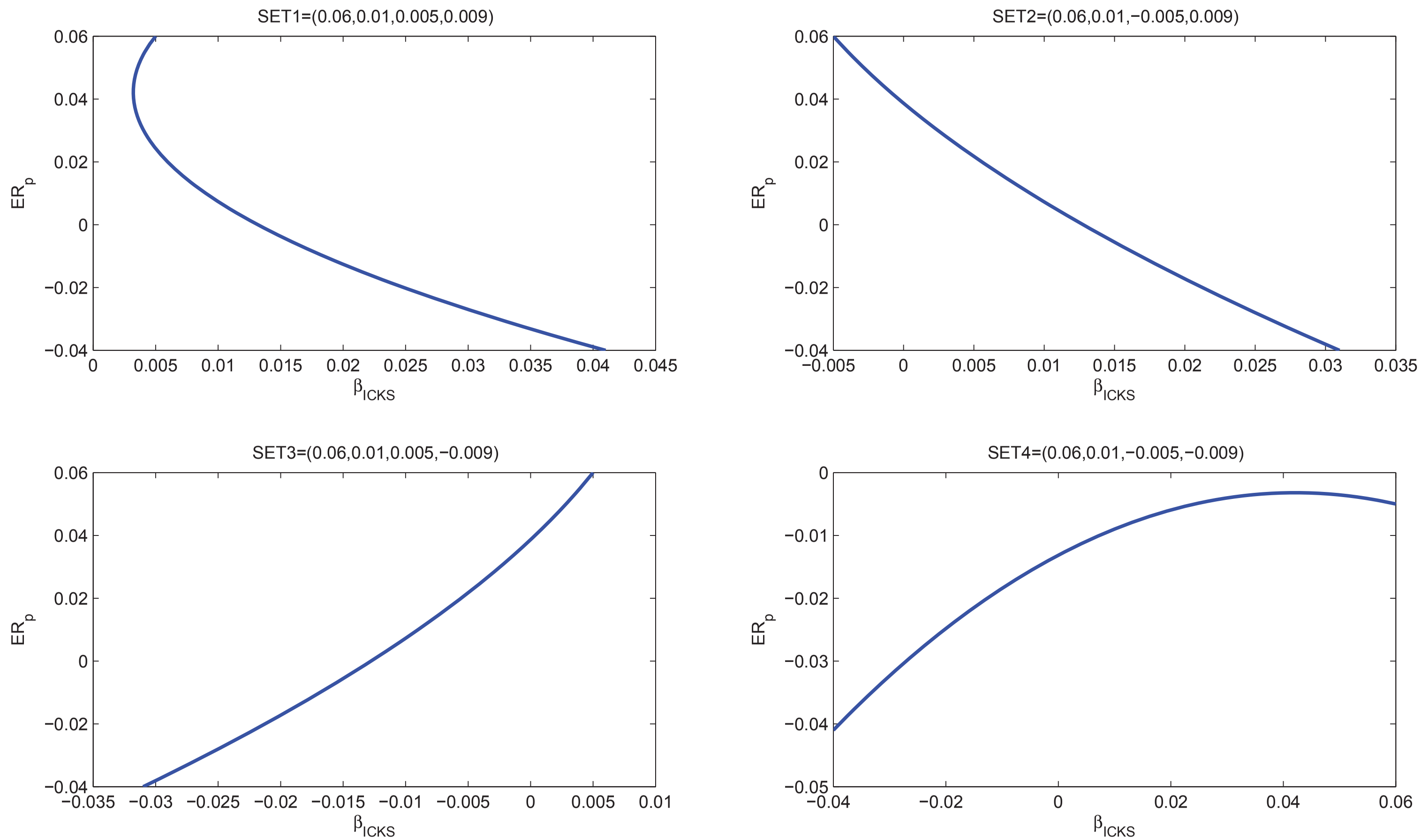

Figure 1: Relation between Expected Returns and Idiosyncratic Coskewness Betas 1

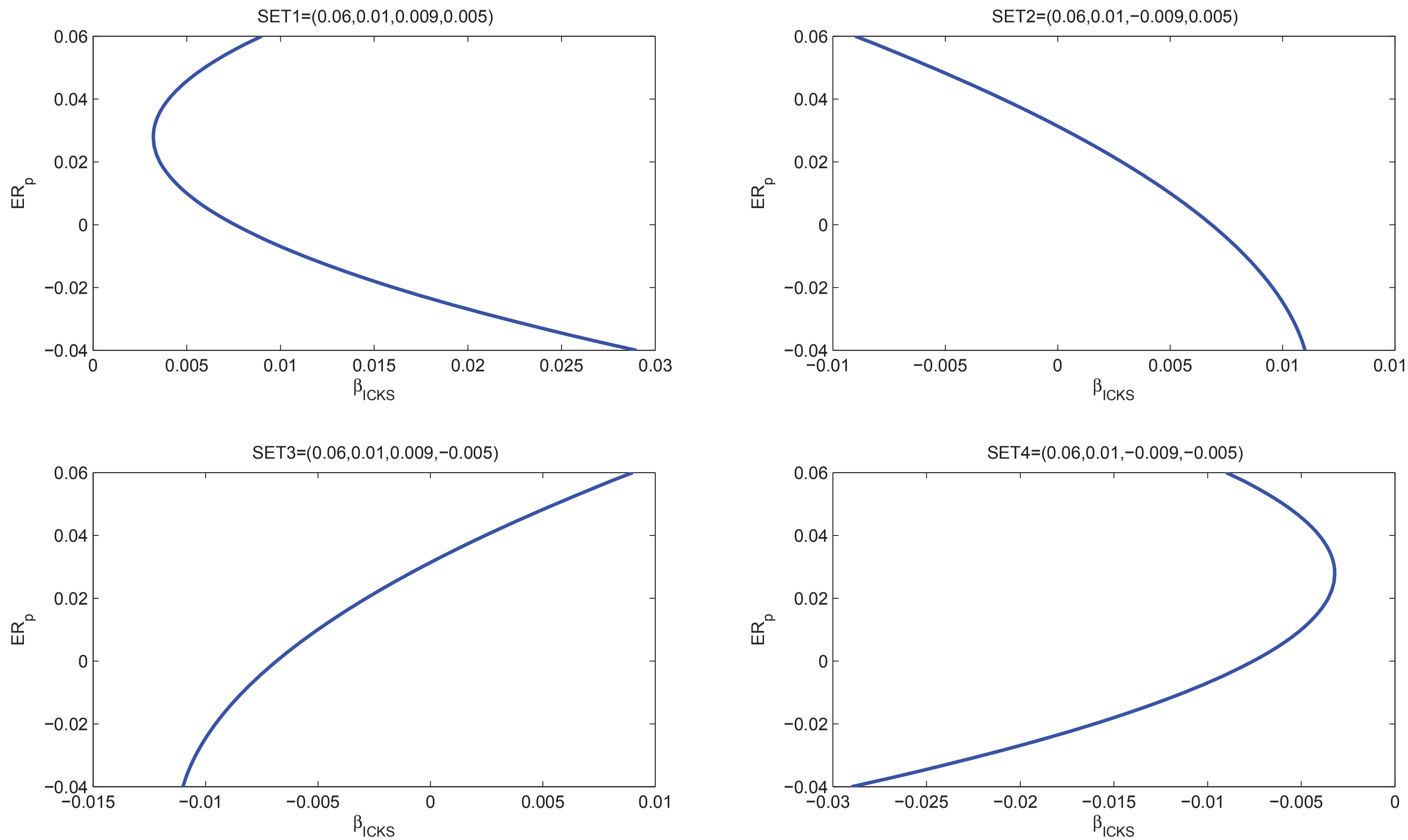

Figure 2: Relation between Expected Returns and Idiosyncratic Coskewness Betas 2

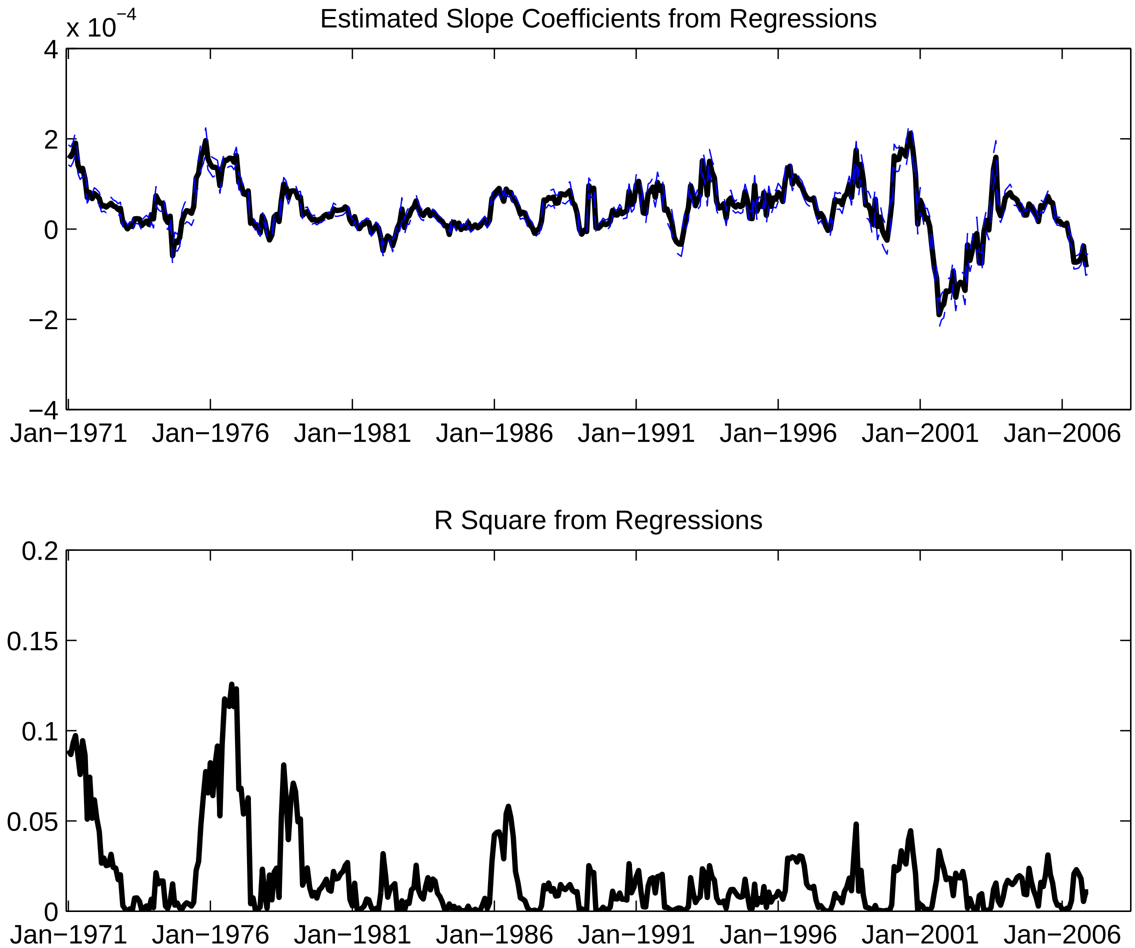

Figure 3: Cross-Sectional Relation Between Idiosyncratic Coskewness on Idiosyncratic Skewness)

This graph plots estimated slope coefficients and R2 from regressions of idiosyncratic coskewness betas on idiosyncratic skewness each month. The sample period is January 1971 to December 2006. Idiosyncratic coskewness betas on idiosyncratic skewness are calculated using past 12-month daily returns for each firm

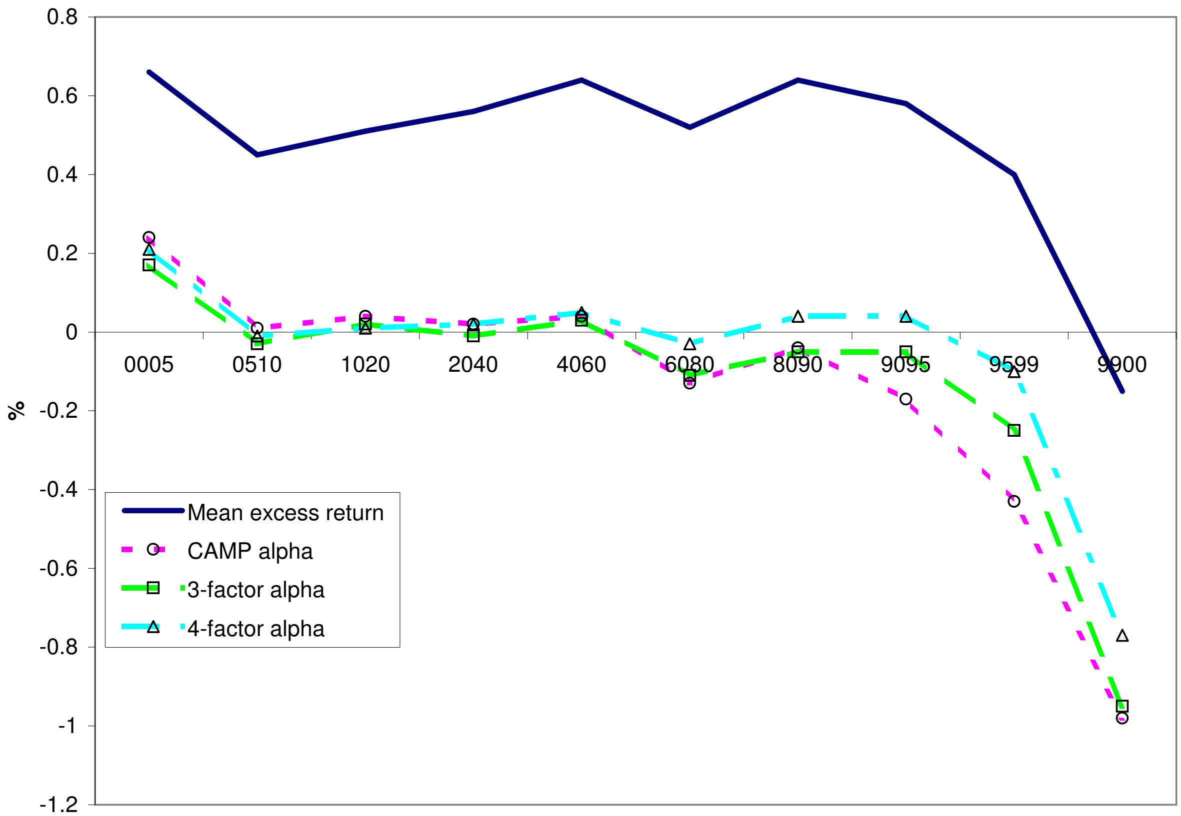

Figure 4: Alphas of Portfolios Sorted by Idiosyncratic Coskewness Betas (Positive Values)

This graph plots monthly excess returns of 10 portfolios sorted by idiosyncratic coskewness betas (positive values), and alphas with respect to the CAPM, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.

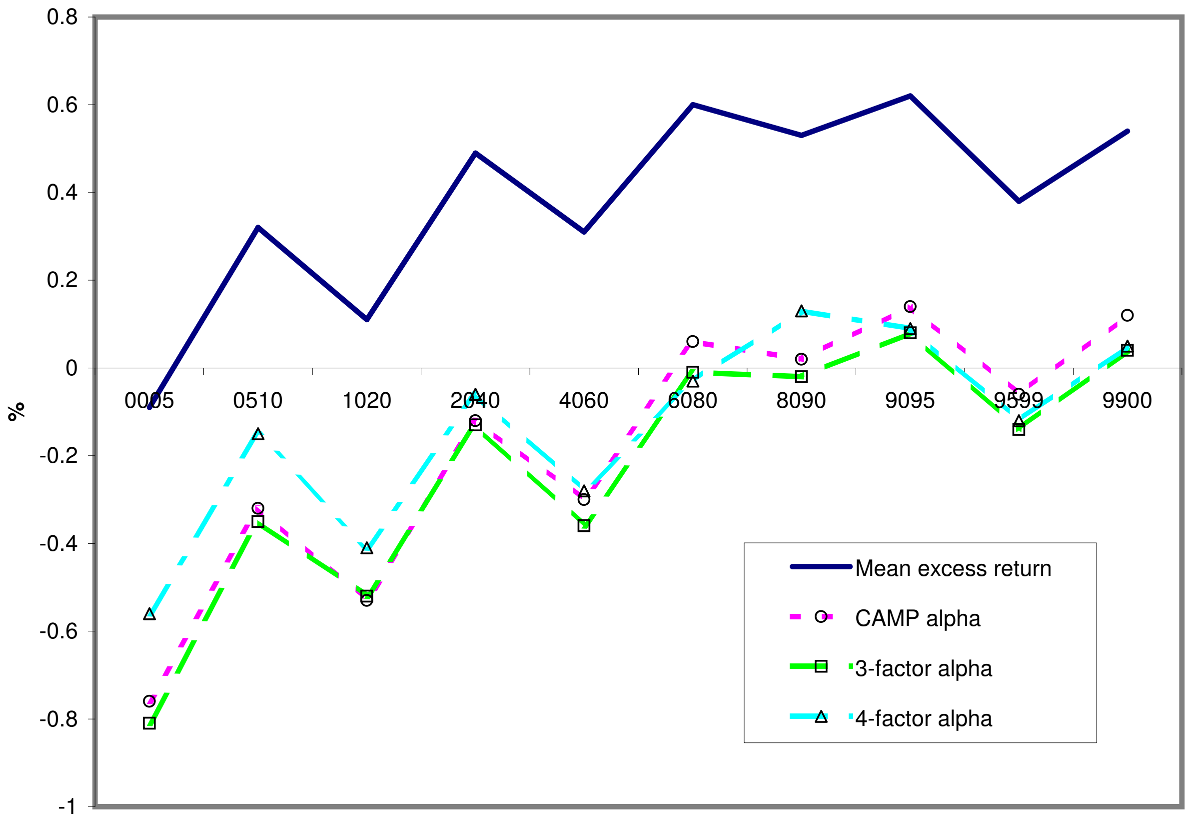

Figure 5: Alphas of Portfolios Sorted by Idiosyncratic Coskewness Betas (Negative Values)

This graph plots monthly excess returns of 10 portfolios sorted by idiosyncratic coskewness betas (negative values), and alphas with respect to the CAPM, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.

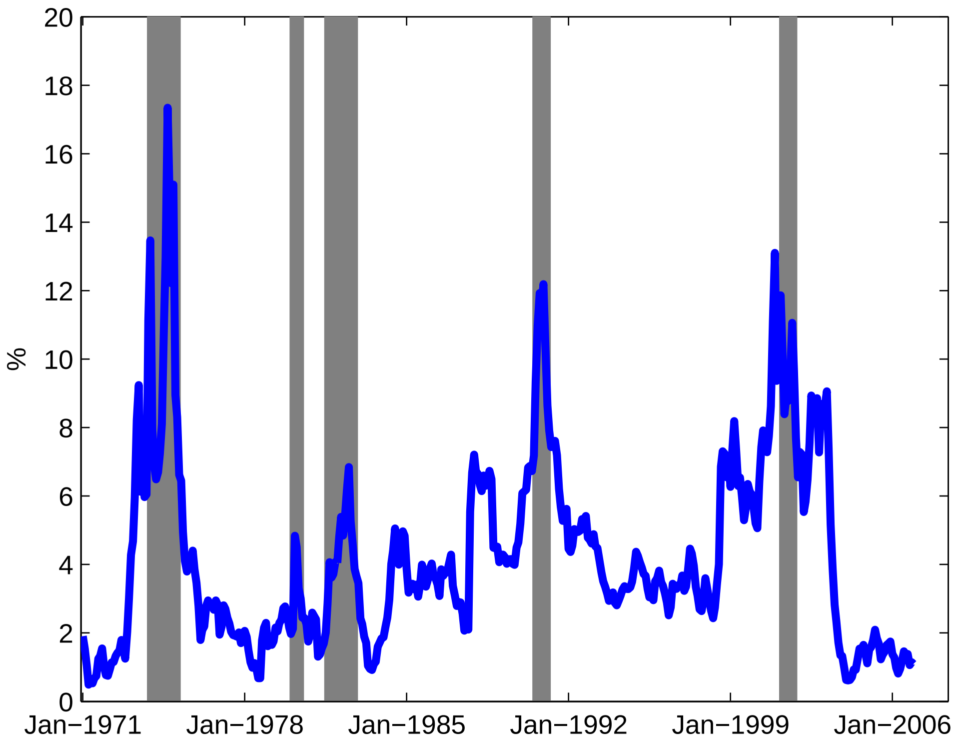

Figure 6: Average Default Probability

This graph plots monthly average default probabilities. The shaded areas denote recession periods, as defined by NBER. The sample period is January 1971 to December 2006.

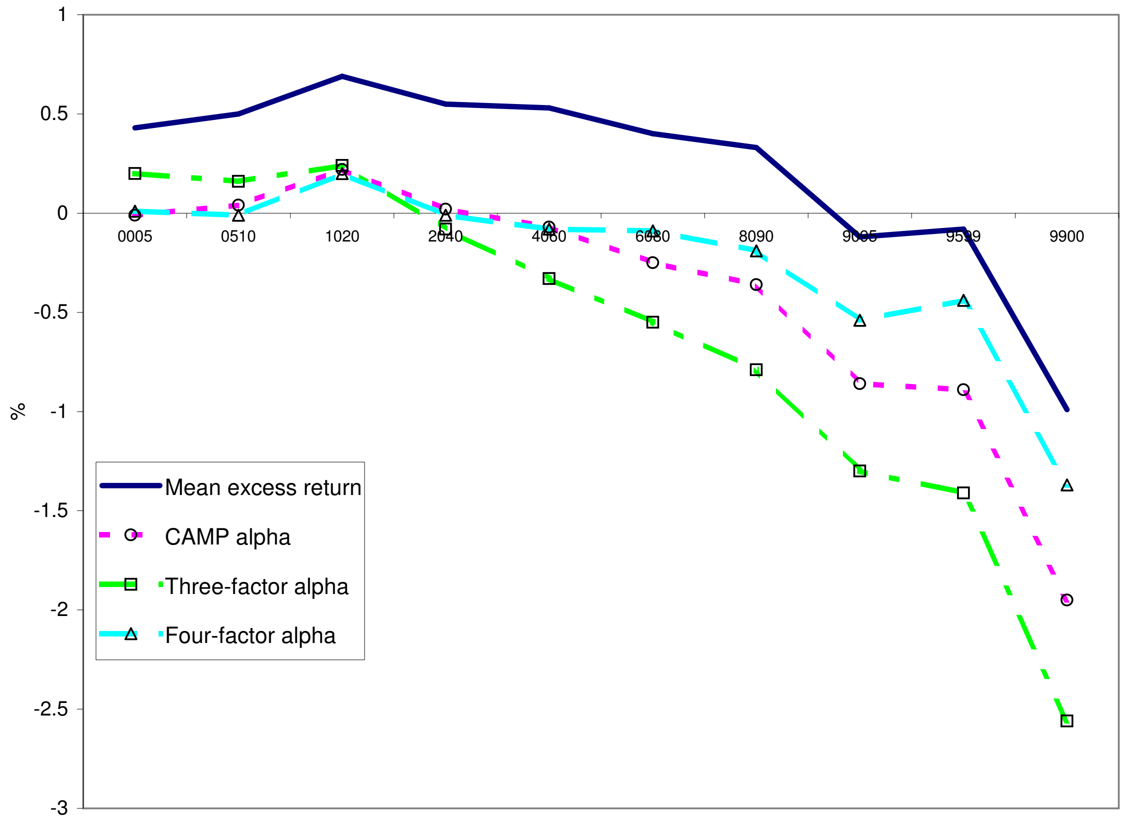

Figure 7: Alphas of Distress-Sorted Portfolios

This graph plots monthly excess returns of ten distress-sorted portfolios, and alphas with respect to the CAPM, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.