



BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2007-8

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Bank of Canada Working Paper 2007-8

February 2007

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Acknowledgements

Thanks to Paul Gilbert, Greg Tkacz, and to Gerald Stuber for their useful comments and suggestions, and to the seminar participants at the Bank of Canada, and at the XL meeting of the Canadian Economic Association (Montreal, May 2006). Thanks also to Raphael Solomon for his useful editing remarks and suggestions.

Abstract

This paper evaluates the performance of static and dynamic factor models for forecasting Canadian real output growth and core inflation on a quarterly basis. We extract the common component from a large number of macroeconomic indicators, and use the estimates to compute out-of-sample forecasts under a recursive and a rolling scheme with different window sizes. Forecasts from factor models are compared with those from AR(p) models as well as IS- and Phillips-curve models. We find that factor models can improve the forecast accuracy relative to standard benchmark models, for horizons of up to 8 quarters. Forecasts from our proposed factor models are also less prone to committing large errors, in particular when the horizon increases. We further show that the choice of the sampling-scheme has a large influence on the overall forecast accuracy, with smallest rolling-window samples generating superior results to larger samples, implying that using “limited-memory” estimators contribute to improve the quality of the forecasts.

JEL classification: C32, E37

Bank classification: Econometric and statistical methods

Résumé

Les auteurs évaluent la capacité de modèles factoriels statiques et dynamiques à prévoir, sur une base trimestrielle, la croissance du PIB réel du Canada et l'inflation mesurée par l'indice de référence. Une fois le facteur commun extrait d'un vaste ensemble d'indicateurs macro-économiques, ils utilisent les modèles estimés pour produire des prévisions hors échantillon au moyen de deux méthodes; l'une est de type récursif, et l'autre fait appel à une fenêtre glissante de longueur variable. Les prévisions issues des modèles factoriels sont comparées à celles tirées d'un modèle autorégressif d'ordre p , d'un modèle fondé sur une courbe IS et d'un modèle formalisant une courbe de Phillips. Les auteurs constatent qu'aux horizons inférieurs à neuf trimestres, les modèles factoriels donnent de meilleures prévisions que les modèles de prévision habituels. Les modèles factoriels proposés sont également moins susceptibles de générer de larges erreurs, en particulier quand l'horizon s'allonge. Les auteurs montrent en outre que le choix du schéma d'échantillonnage influe grandement sur la qualité générale des prévisions, les fenêtres glissantes de petite taille donnant de meilleurs résultats que les gros échantillons. L'emploi d'estimateurs à « mémoire limitée » contribuerait par conséquent à améliorer la qualité des prévisions.

Classification JEL : C32, E37

Classification de la Banque : Méthodes économétriques et statistiques

1 Introduction

Rational agents and policy makers typically consider “[...] Large amounts of data about the state of the economy and the rest of the world, including private-sector expectations and plans [...]” (Svensson 2005, p. 2). While modern large-scale macroeconomic models, such as the Bank of Canada’s new projection model (Murchison and Rennison 2006), reflect various characteristics of the economy, even such large models are inherently limited in the quantity of information they can process to predict macroeconomic time series. In the case of commonly used reduced-form forecasting models, degrees-of-freedom considerations worsen matters and econometricians must rely on very parsimonious specifications to predict key economic variables such as output and inflation (on output, see Duguay 1994; and on inflation, see Demers 2003). Consequently, an enormous amount of information is left unused.

One alternative that allows the econometrician to exploit the availability of hundreds of macroeconomic time series is factor models. Following the work of Stock and Watson (1999, 2002a,b), the macroeconomic forecasting literature has witnessed a large body of empirical analyses which evaluate factor models for forecasting macroeconomic variables. Most studies conclude that factor models are indeed useful for predicting GDP or inflation, and that they generally outperform usual benchmark forecasting models.¹ The basic view underlying factor models is that macroeconomic time series tend to covary through the business cycle because they share a common component. In other words, a plethora of indicators can convey similar information about the state of the business cycle or inflationary pressures in the economy, such that the data admit a factor structure. The use of dynamic factor models, based on principal component methods, has become popular in econometrics through the work of Stock and Watson (1991, 1999), although principal components analysis has a long tradition in statistics (see, e.g., Geweke 1977).²

Factor models have been established as a distinct and simple, yet rigorous, method to predict output or inflation. Unlike structural models such as the Phillips or IS curve, which are based on economic theory, factor models are inherently data-driven. As a result, they often receive criticism for their inability to interpret or identify the forces driving the dynamics of the economy. Similar to many central banks, the Bank of Canada uses various models to predict output or inflation. Since most of these models are based on economic relationships, the benefits from further developing these types of models may be marginal.

¹For an excellent reviews and recent successful empirical work on factor models, see, e.g.: Stock and Watson (1999, 2002a, 2006); Camba-Mendez et al. (2001); and Artis et al. (2005).

²In this paper, all references to factor analysis correspond to the principal components methodology. This is to be distinguished from Time Series Factor Analysis, developed by Gilbert and Meijer (2005).

In contrast, using factor models allows central banks to diversify their forecasting strategy (Pagan 2003, p. 20). Furthermore, in a context where agents and policy makers are uncertain about the underlying structure generating the stochastic process of the economy, the improved forecast accuracy obtained from using factor models suggests that they are useful devices for improving policy making.

In this paper, we revisit the forecasting analyses of Gosselin and Tkacz (2001) and Brisson, Campbell, and Galbraith (2003), who evaluate factor-based forecasts for Canadian output and the consumer price index.³ Interestingly, both studies conclude that factor models improve the accuracy of GDP and inflation forecasts relative to various benchmarks. This paper thus reexamines and expands on the previous studies by investigating the usefulness of different factor models to predict Canadian GDP growth and core inflation. The forecasting performance of static and generalized dynamic factor models, designed in different manners, is compared with that of common benchmarks. Different forecasting schemes are used to compare the forecast accuracy based on i) various information sets, ii) various decision rules about the factor structure, and iii) various sampling schemes.

According to our empirical results, factor models can improve accuracy in forecasting Canadian GDP growth and core inflation relative to standard benchmark models, for horizons up to 8 quarters ahead. This is somewhat different from the results of Brisson, Campbell, and Galbraith (2003), who find little forecast content in factor models beyond two or three quarters. Furthermore, our factor-model forecasts appear less prone to large errors, in particular as the horizon increases. This result is similar to that found by Gosselin and Tkacz (2001) for inflation forecasts. We also find that static-factor forecasts perform quite well: incorporating information on dynamic relationships among variables in the factor estimations does not provide further accuracy gains. Over longer samples, we obtain results similar to previous studies: the bulk of the co-movement among macroeconomic variables is optimally summarized in one common factor; however, the number of factors does increase with smaller sample sizes. We show that the choice of the sampling-scheme greatly influences overall forecast accuracy, with smallest rolling-window samples generating superior results to larger samples, and expanding-window forecasts generally performing the worst. In other words, using “limited-memory” (Giacomini and White 2006) estimators contribute to improving the quality of the forecasts. This suggests that extending the sample back to the 1970s and 1980s leads to a reduction in forecast accuracy relative to including only more

³Gosselin and Tkacz (2001) analyze factor models to predict quarterly consumer price index, excluding food, energy, and the effect of changes in indirect tax. In the case of Brisson et al., they investigate various monthly and quarterly macroeconomic time series, including GDP and the consumer price index.

recent data. Lastly, to assess the degree of uncertainty around point forecasts, growth and level forecasts are compared.

The remainder of this paper is divided as follows. Section 2 presents the factor models we propose in this study. In Section 3, we present the panel of data used to estimate the factor models. In Section 4, we introduce the benchmark forecasting models. In Section 5, we report the results from an out-of-sample forecasting exercise, as well as some tests to evaluate the relative forecast accuracy of the various models examined in this paper. Finally, Section 6 concludes with brief remarks and suggestions.

2 Forecasting with Factor Models

2.1 Static factor model

Following Stock and Watson (2002a) and Gosselin and Tkacz (2001), consider first the static factor model.

Let y_t be a mean-zero, scalar time-process of interest and X_t be an N -dimensional vector of potential predictors of y_t for a horizon of up to h -steps ahead, with $t = 1, \dots, T$ and $h = 1, \dots, H$. In general, the N elements contained in X_t are mean zero with unit variance after being standardized and suitably transformed—i.e., differenced—such that they are rendered $I(0)$. Hence, we assume that the joint process (X_t, y_{t+h}) has the following factor representation:

$$X_t = \Lambda F_t + \xi_t \tag{1}$$

$$y_{t+h} = \beta_0 F_t + \beta_1 W_t + v_{t+h}, \quad \text{for } t = 1, \dots, T. \tag{2}$$

Under (1), X_t is decomposed into two unobservable orthogonal components, namely: i) the r common factors, F_t , and ii) an N -dimensional idiosyncratic component, denoted here as ξ_t , where $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_N)'$ is the factor loading matrix. Using the method of principal components, this matrix consists of the eigenvectors (scaled appropriately) corresponding to the r largest eigenvalues of the sample variance-covariance of the X matrix. Denoting estimated values with ‘ $\hat{}$ ’, the factors are then obtained by setting $\hat{F} = X' \hat{\Lambda} / N$.

Turning to (2), β_0 is simply an r -dimensional vector that maps F_t onto y_{t+h} ; W_t is a vector of other explanatory variables, possibly lagged values of y_t , with associated vector of coefficients, β_1 ; finally, v_{t+h} is a prediction error. Equation (2) can be estimated by ordinary least squares (OLS). Of course, (2) can be generalized such that F_t in (1) is replaced by $\tilde{F}_t = (F_t', \dots, F_{t-j}')'$, as proposed by Stock and Watson (2002b; hereafter denoted as SW); β_0 then represents a matrix polynomial in the lag operator. Conditional expectations at time

$t + h$ are obtained from $\hat{y}_{t+h} = \hat{\beta}_0 \hat{F}_t + \hat{\beta}_1 W_t$, which is calculated from an h -step ahead linear projection of y_{t+h} onto the t -dated predictors described above.⁴

2.2 Generalized dynamic factor model

Although the SW factor representation can be generalized to include lags of the factors, we nevertheless characterize the approach as “static”, for the fact that it relies uniquely on the contemporaneous covariances of the data contained in the time series $\{X_t\}_{t=1}^T$. Alternatively, Forni, Hallin, Lippi, and Reichlin (2000, 2005; hereafter denoted as FHLR) suggest a generalization that instead exploits the correlation structure that exists between X_t and X_{t-k} , for some $k = 0, \dots, M$. In this case, the “dynamic” estimates of the factors are obtained via an eigenvalue decomposition of the spectrum, smoothed over M different frequencies, requiring the estimation of $2M + 1$ variance-covariance matrices, instead of a single one as is done in the static case.⁵

As Boivin and Ng (2006) note, it is not clear whether the generalization proposed by FHLR is in fact superior to the SW factor model, since the former could suffer from an efficiency loss. For instance, if the data are truly generated by a generalized static factor model such as $X_t = \Lambda_0 F_t + \Lambda_1 F_{t-1} + \xi_t$, the FHLR method would use more covariance estimates than necessary. On the other hand, if $E(X_t' X_{t-k}) \neq 0$ (for $k \neq 0$), the static factor model would be misspecified. The gains from using the FHLR method therefore rely on the existence of dynamic correlation between X_t and X_{t-k} .

2.3 Selecting the number of factors and the lag structure

Estimation of factor models requires selecting the number of factors to use, values for M , as well as the autoregressive and factor lag lengths in the forecasting equation. For the static factor model, the number of factors (r) to use is determined by the information criterion developed by Bai and Ng (2002). This dictates that the optimal number of factors (r^*) is determined by the r which minimizes

$$IC_{p2}(r) = \ln(V(r, \hat{F}^r)) + r \left(\frac{N + T}{NT} \right) \ln(\min\{\sqrt{N}, \sqrt{T}\})^2, \quad (3)$$

where \hat{F}^r is the matrix of r estimated factors, and $V(r, \hat{F}^r)$ is the sum of squares of the idiosyncratic components that depends on the factor estimates and the number of factors

⁴For further technical details on this type of factor models, interested readers should consult Stock and Watson (2002b) and Schumacher (2005).

⁵For further technical details, see FHLR and Schumacher (2005).

included, namely:

$$V(r, \hat{F}^r) = \min_{\Lambda} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (X_{it} - \lambda_i^{r'} \hat{F}_t^r)^2. \quad (4)$$

The second term in (3) is a penalty for overfitting, which can lead to a loss of efficiency, and is an increasing function of both cross-sectional and time dimensions, as well as the number of factors. The criterion is evaluated for all values of $r = 1, \dots, r_{\max}$, where we set $r_{\max} = 5$. Previous empirical studies conclude that most of the predictable variation in major macroeconomic variables can be explained by very few factors (e.g., Stock and Watson 1999, 2002a, 2002b).

Having selected the optimal value r^* , the estimated static factors can then be used to determine the number of dynamic factors (q) to include in the dynamic factor model. Using the information criterion proposed by Bai and Ng (2006), a p -order VAR is estimated on the r^* static factors, where p is determined by BIC.⁶ Letting $\hat{\Sigma}$ be the estimated variance-covariance matrix from the residuals of the VAR(p), a spectral decomposition is computed to form $\hat{\Sigma}(k) = \sum_{j=1}^k \hat{c}_j \hat{\beta}_j \hat{\beta}_j'$, where \hat{c}_j is the j -th largest eigenvalue of $\hat{\Sigma}$, and $\hat{\beta}_j$ its corresponding eigenvector, with $k \leq r^*$. Defining \hat{d}_k as the vector formed from stacking the lower triangular elements of the matrix $\hat{\Sigma}(k)$, and denoting $\| \cdot \|$ as a vector norm, the k -th factor's marginal contribution to the covariance is:

$$\hat{D}_k = \| \hat{d}_{k+1} - \hat{d}_k \| / \| \hat{d}_0 \| . \quad (5)$$

We can then identify all possible values for the number of dynamic factors as those being in the set $\kappa = \{k : \hat{D}_k < \theta / \min[T, N]^{1/2-\delta}\}$, where $\theta > 0$ and $0 < \delta < 1/2$.⁷ The optimal number of dynamic factors is finally that which satisfies $q = \min\{k \in \kappa\}$.

Lastly, we need to determine the value of M . According to the Monte Carlo simulation results reported in FHLR (2000), selecting M using $\text{round}[(2/3)T^{1/3}]$ performs best. This value for M determines the maximum number of lags to include in the estimation of the autocovariance matrices for deriving the generalized dynamic factor estimates. The number of autoregressive and factor lags used in the forecasting equation (2) for both static and dynamic models is determined using AIC.

⁶Bai and Ng (2006) exploit the relation between the dynamic and static factors in order to determine the optimal value of q without having to estimate the dynamic factors. Given r^* static factors explain θ percent of the variation in the data, q^* is the optimal number of dynamic factors that would explain the same fraction of variation, up to some error (that vanishes asymptotically).

⁷Bai and Ng (2006) identify $\theta / \min[T, N]^{1/2-\delta}$ to be the maximum tolerated error arising from sampling variability in the estimation of Σ . They find that setting $\delta = 0.1$ and $\theta = 0.5$ generates robust results, and we therefore use these values in this paper.

3 Data

Our constructed panel of data, used to estimate the factors and predict GDP growth and core inflation, consists of 324 Canadian series and of 112 U.S. series, spanning the period 1973Q1 to 2005Q1. We include all data series related to the economy for the purpose of capturing as much information as possible in the factor estimates.⁸ The panel is described in the Appendix. The variables we wish to predict are real Gross Domestic Product at Market Prices and core Consumer Price Index, as used by the Bank of Canada and explained in Macklem (2001).⁹ Figure 1 plots quarterly GDP growth and core inflation for the sample period.¹⁰

4 Forecasting Methodology

4.1 Benchmark models

Because multi-step forecast comparisons are not invariant to isomorphic representations or transformations (e.g., differencing or filtering), the ranking of models and the resulting uncertainty assessments can be altered (cf., Granger and Newbold 1986; Clements and Hendry 1998). Evaluating h -step-ahead forecasts using either *change* or *level* arguments can reveal varying performance rankings, and we hence analyze forecast accuracy from both approaches.

Let Y_t denote the log level of a series of interest to be predicted. For the first approach, we consider $y_{t+h} = Y_{t+h} - Y_{t+h-1}$, so that the forecasts are evaluated on the basis of the term structure of growth rates, or the marginal growth rates. In the second approach, we look at the (cumulative) level differences over h period: $y_{t+h}^h = Y_{t+h} - Y_t$.¹¹

To perform the out-of-sample forecast evaluation, we use the *direct* forecasting method, as opposed to an *iterated* method.¹² For the marginal-growth approach, we use the following *direct* prediction equation for an autoregression of order p :¹³

$$y_{t+h} = \sum_{j=1}^p \phi_j y_{t+1-j} + \varepsilon_{t+h}. \quad (6)$$

⁸Since the dataset includes major national accounts components in current dollars as well as their implicit price indexes, we did not include the real volumes.

⁹This measure of core inflation excludes the eight most volatile items and the effects from variations of indirect taxes.

¹⁰These data were collected after the publication of the national accounts for 2005Q1.

¹¹Note that although in Section 2 equation (2) is written only in terms of y_{t+h} , we effectively performed the forecasting exercise using y_{t+h}^h as well.

¹²For more details on direct versus iterated forecasts, see Marcellino, Stock, and Watson (2006).

¹³Note that since all data have been demeaned, no intercept is included in the regressions.

For the cumulative change approach, the autoregression is written as follows:

$$y_{t+h}^h = \sum_{j=1}^p \varphi_j y_{t+1-j} + \varepsilon_{t+h}, \quad (7)$$

where the parameters ϕ_j and φ_j are autoregressive coefficients estimated by OLS, and ε_{t+h} is the resulting h -step-ahead forecast error. Predictions at time $t+h$ can thus be calculated as $\hat{y}_{t+h}^h = \sum_{j=1}^p \hat{\varphi}_j y_{t+1-j}$ and $\hat{y}_{t+h} = \sum_{j=1}^p \hat{\phi}_j y_{t+1-j}$, respectively. The important thing to note about (6) and (7) is that right-hand side information is identical in both prediction equations, but when $h > 1$, the regressands differ.

To forecast GDP growth and core inflation, the Bank of Canada staff also uses a number of variants of the IS- and Phillips-curve models. To evaluate the usefulness of the IS-curve approach, a modified version of Duguay's (1994) specification is used. The proposed model depends upon the following variables: the change in the Canada-U.S. real exchange rate, the change in the real commodity prices,¹⁴ U.S. GDP growth, and the slope of the yield curve.¹⁵ By collecting all the aforementioned exogenous variables together with the lagged values of y_t into an m -dimensional vector, say Z_t , we have a model for each approach:

$$y_{t+h} = \sum_{i=1}^m \sum_{j=1}^{p_i} \beta_{ij} Z_{i,t+1-j} + \varepsilon_{t+h}, \quad (8)$$

for the term-structure approach; and

$$y_{t+h}^h = \sum_{i=1}^m \sum_{j=1}^{p_i} \beta_{ij}^h Z_{i,t+1-j} + \varepsilon_{t+h}, \quad (9)$$

for the cumulative-change approach. the lag length for each element of Z_t is selected based on AIC and is denoted p_i ; and the β_{ij} 's and β_{ij}^h 's are the associated parameters for the i -th variable and the j -th lag of a given forecasting objective (i.e., term-structure or cumulative change).

For core inflation, a version of the Phillips-curve which relates inflation to the output gap and change in the Canada-U.S. real exchange rate is used.¹⁶ Again, after collecting all the explanatory variables into the vector Z_t , models of the same form as (8) and (9) can be used to predict core inflation.

These two econometric benchmark models are used to evaluate the marginal usefulness, or information content, of factor models.¹⁷

¹⁴Deflated using the GDP chained implicit price index.

¹⁵Defined as the spread between the 10 year yield on government bonds and the 90 day rate on commercial papers.

¹⁶Commodity prices, or oil prices, are excluded from our Phillips-curve specification.

¹⁷Because these benchmark models are estimated using different sample sizes, and because we are concerned

4.2 Design of the forecasting strategies

To construct and evaluate a particular forecasting model, the econometrician designs the forecasting experiment based on a number of critical decisions. For instance, while data can often date back to the 1960s or earlier, it is not clear that using all the available information actually improves predictive accuracy. For a complete sample of size T , the accuracy is improved only if the data generating process is homogeneous across time. In contrast, if the data generating process is heterogeneous, using too much “time information” would likely deteriorate the forecast accuracy. Efficiency gains could therefore be achieved by using only a fraction of the available information, say T^* , with $T^* < T$. The first thing one needs to determine is how much time information to use.

In this paper, we evaluate the out-of-sample forecast performance of each model/method in two distinct manners, namely the *recursive* and the *rolling* approach. The *recursive* approach consists of recursively estimating the relevant equations using $S_t (= T - P + h)$ observations to calculate the first h -step-ahead forecasts, for $h = 1, \dots, H$, after which a data point is added for each subsequent forecast until P h -step-ahead predictions are collected, where P denotes the number of one-step-ahead forecasts. This method is also referred to as the *expanding* window approach. If the data are homogeneous over time, the *recursive* method should work well, all else equal, since the econometrician is using all the available information.

Otherwise, if we suspect that the data are rather characterized as heterogeneous processes over time, we may instead prefer to base our forecasts on a “limited-memory” estimator. Under this approach, data that are no longer seen as informative are excluded (cf., Giacomini and White 2006). Hence, we propose to experiment with different values for $S_t (< T - P)$ to examine the sensitivity of the results given a chosen S_t . Selection of the effective sample size is particularly important when predictions are made using a *rolling* window approach since the conclusion may depend on the selection of a particular S_t . The effective sample size, S_t , is fixed at some value; after each iteration, the sample window rolls forward by one period. To investigate the sensitivity of our results when S_t varies, experiments with $S_t = (30, 35, \dots, 75)$ are performed. One of the main advantages of the rolling-window approach, or the use of a “limited-memory” estimator, is that it is more robust to heterogeneity in the data or to omitted structural change that causes the parameter estimates to vary over time (see, e.g., Clark and McCracken 2004; Giacomini and White 2006). The presence of heterogeneity or structural change(s) can bias parameter estimates, leading the rolling-window approach to

only with their out-of-sample predictive ability, the estimation results for these various specifications are not presented, but they are available from the authors.

provide some potential improvement. The main drawback of this method, however, is that it uses less information to infer on the parameter estimates, thereby increasing sampling variability, and consequently, the variance of the out-of-sample forecast error. These conflicting outcomes are referred to as the “bias-variance trade-off”. These sampling considerations may be more acute for larger structural models, for which a relatively large number of parameters often needs to be inferred from the data.

Throughout this exercise, the standardization of the dependent variable is done using information until time t , not T , since we are computing expectation using t -dated information only. Furthermore, the lag selection and the number of factors in factor models are re-evaluated at each period in order to allow for some flexibility in the specification and more closely reflect a real-time environment.¹⁸ Finally, for the purpose of this study, P is set to 50 for selected S_l 's.

Given the range of forecast models, methods, and strategies employed in this paper, 66 vectors of forecast errors are computed for each series of interest, and for $h = 1, 2, 4$, and 8. The Root Mean Squared Forecast Error (RMSFE) is computed using

$$RMSFE_i = \sqrt{\frac{e_i' e_i}{P - h}}, \quad (10)$$

where $e_i = \{e_{it}\}_{t=1}^P$ is the i -th vector of prediction errors associated with a particular forecast model.

Tables 1 and 2 report the RMSFEs for each h and for the various methods implemented to predict the GDP and core inflation marginal growth rates, respectively. RMSFEs are reported relative to an unconditional mean forecast, derived from an expanding window given information available at time t , not T . The purpose of using this forecast as a benchmark is to illustrate the potential improvement over a more traditional approach to forecasting, which essentially consists of using the longest sample possible.¹⁹ Tables 3 and 4 report the relative RMSFEs for the same model but for the cumulative-growth approach. As in Gosselin and Tkacz (2001), we test one version of the factor models using only the domestic variables (denoted as “Cdn”), and a second version using both the Canadian and U.S. variables (denoted as “U.S.”).

¹⁸Fully replicating the real-time environment for factor models where N is large (often greater than 400) would be, if possible at all, extremely tedious and time consuming as one would need to construct the entire real-time database.

¹⁹Here, the sample size is restricted by data availability of the variables used to construct the factors, as opposed to the variables we wish to predict (which span longer).

4.2.1 Strategies to estimate the factors

A key advantage to using factor models is that they allow for a small number of variables to summarize systematically the information contained in a large observed dataset, thereby keeping the dimension of the forecasting model small. However, including irrelevant information may also impose a cost on the predictive accuracy. Boivin and Ng (2006) point out that including series in X_t that provide no additional information but have idiosyncratic errors that are strongly correlated with others can reduce the efficiency and precision of factor estimates.²⁰ The models described above are based on the analytical framework of *approximate* DFMs introduced by Chamberlain and Rothschild (1983), which differs from the original *exact* DFM version in that it relaxes the assumption of orthogonality across the idiosyncratic errors. However, while the asymptotic theory upon which approximate DFMs are based allows for “weak” cross-sectional correlation in the idiosyncratic errors, the precise practical limit that should be tolerated is difficult to know.²¹

Given our dataset of 324 Canadian series included in X_t , the fraction of variables with idiosyncratic errors correlated above 0.6 is 58 per cent using the static estimation of the factors, and 36 per cent under the dynamic estimation. Consequently, we follow Boivin and Ng (2006) and drop series from X_t with highly correlated idiosyncratic errors.²² In some experiments that we performed (unreported), we dropped the variables possessing errors with correlation coefficients above some specified threshold, which ranged from 0.3 to 0.8. However, we did not find that this had noticeable effects on the forecasting results. Given that the factor estimates from FHLR and SW methodologies are consistent as long as $N, T \rightarrow \infty$, any benefits from reducing the degree of error correlation may have been offset by the costs incurred from lowering N .

The number of static and dynamic factors is determined optimally at each t using the Bai and Ng (2002, 2006) information criteria described in section 2.3. In the recursive approach, it is found that a single common factor best models the co-movement among the

²⁰This may be related to an ill-conditioned matrix problem, which arises when columns or rows within a matrix are strongly correlated. This condition can affect the precision of computed eigenvalues and eigenvectors. In general, the likelihood of this condition occurring increases as the dimension of the matrix increases.

²¹Stock and Watson (2002a) require only that $\lim_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \sum_{j=1}^N |E(\xi_{it}\xi_{jt})| < \infty$, where ξ_{it} is the idiosyncratic error of component i .

²²Given two series i and j with highly correlated idiosyncratic errors, Boivin and Ng (2006) suggest dropping series i if $R_i < R_j$, where $R_i = (\sum_{i=1}^T \hat{x}_{it})^{-1} \sum_{i=1}^T \hat{\chi}_{it}$ is the relative importance of the common component, χ_{it} , in the series i .

X variables, under both static and dynamic methods.²³ The optimal number of factors does not change once U.S. variables are included. The results are quite different for the rolling-window samples, where it is found that the number of factors selected varies with each t . Furthermore, the optimal number of factors tends to increase with smaller rolling-window sample sizes.

5 Out-of-Sample Forecast Evaluation

5.1 Marginal growth rate comparison

Tables 1 and 2 report the relative RMSFEs for the marginal growth rate forecasts. Static-factor models generally appear to perform similarly or slightly better than dynamic-factor models for all sample sizes and forecast horizons considered. This suggests that the bulk of the cross-sectional correlation among the variables in X occurs contemporaneously, since there are no gains from incorporating information from the dynamic covariance matrices in the factor estimation. Furthermore, including U.S. data in the factor estimation seems to improve the forecast accuracy slightly in both static and dynamic models, though these gains disappear for rolling-window sizes larger than ($S =$) 45 for GDP growth, and larger than 35 for inflation. In general, it is clear from Tables 1 and 2 that the rolling window forecasts from all models outperform the expanding window forecasts for both GDP growth and core inflation, with accuracy improving as the sample sizes get smaller. These contrasting conclusions obtained from using different forecasting schemes effectively demonstrate the sensitivity of out-of-sample forecasting experiments, and the need for caution when forming conclusions based on any single choice of S . The sensitivity of these results is also illustrated by the declining RMSFEs with increasing sample size in Figures 2 and 3.

Relative to unconditional-mean GDP growth forecasts, none of the factor nor benchmark IS-curve model forecasts perform noticeably better (and often worse) when the sample size is larger than 60. This conclusion holds for all horizons.²⁴ The indication that incorporating

²³A study done by Boivin and Ng (2006) concludes that using less than the “true” number of factors can lead to large efficiency losses, whereas over-estimating the number of factors has little effect on the estimates or resulting forecasts. Based on these findings, we also estimate the models while fixing the number of factors to equal 3, 4, and 5. For GDP growth, using a fixed number of 5 factors for the most part worsened the model’s forecast accuracy, while using 3 or 4 factors is found to lower the 1 and 2 quarter- ahead RMSFE by 3 to 5 per cent under the static estimation, and by 5 to 13 per cent under the dynamic estimation. For inflation, using a fixed number of 3 factors provides little or no improvement, whereas using 4 or 5 factors reduces the 1 and 2 quarter-ahead RMSFE by 1 to 5 per cent under static estimation, and by 1 to 3 per cent under dynamic estimation.

²⁴For inflation, comparison with the unconditional mean forecast is extraneous as all models produce forecasts that are substantially superior. Because the unconditional mean forecast is derived using the full history at time t , behaviour in the inflation series prior to the inflation-targeting regime adds significant

longer history worsens forecast accuracy is interesting: it suggests that some heterogeneity, or structural change, may have occurred over the sample period. When heterogeneity exists in the data, information from earlier periods may be inappropriate for inferring on the present or future, and thus incorporating it may bias the forecasts. As Clements and Hendry (1998) point out, heterogeneity and misspecified structural change(s) are probably the most important reasons for forecast failure in economics. Behaviour observed in the GDP growth and inflation forecast errors from both factor and benchmark models also supports the notion of heterogeneity, with forecast error means hovering fairly close to zero for the smaller sample sizes 30 to 45, but then diverging from zero as sample sizes increase above 50.

5.1.1 GDP growth forecasts

Comparing the benchmark models, the IS curve predicts GDP growth reasonably well under the rolling-window approach, outperforming the AR model for all sample sizes up to $S = 50$ for forecasts ($h =$) 1 and 2 quarters ahead, and up to $S = 60$ for forecast horizons 4 and 8. Interestingly, its best performance occurs with a rolling window size of 30, for all forecast horizons. The IS curve does not, however, perform well under the recursive approach, producing larger forecast errors than both the AR model and the unconditional mean forecast.

Among all the factor models considered, the static factor models generate superior GDP growth forecasts at all horizons compared to the IS-curve benchmark, in the case of expanding and most rolling-window samples.

To determine whether factor-based forecasts are superior to benchmark IS-curve forecasts, the test proposed by Diebold and Mariano (1995) is used.²⁵ According to the Diebold-Mariano test results, the two models have equal forecast accuracy at horizon 1 and 2 (p-values: 0.485 and 0.209, for $h = 1, 2$, respectively), whereas the best factor forecasts significantly outperform the best benchmark forecast for $h = 4, 8$ (p-values: 0.100 and 0.008, respectively). Also worth noting is the stable performance of factor model forecasts even as the forecast horizon lengthens, suggesting that information available at time t helps predict $t + 8$.²⁶

distortion to the mean, leading forecasts based on this mean to be biased and skewed.

²⁵Diebold-Mariano tests are constructed using Hansen-Hodrick HAC standard errors with the bandwidth fixed at $h - 1$.

²⁶This is also confirmed by the (unreported) t -statistics associated with the factors, which remain well above one (in absolute value) over the different horizons.

5.1.2 Core inflation forecasts

Among the benchmark models, the Phillips curve produces superior inflation forecasts to the AR model at all horizons for almost all rolling-window sizes. Although this is not the case for the recursive forecasts, it is likely because data prior to the advent of inflation targeting in 1991 is virtually useless for forecasting inflation in the current regime. The best performance among the Phillips-curve forecasts is observed with $S = 30$ for all horizons except $h = 8$, in which case $S = 35$ gives the most accurate predictions.

Turning to factor models, a rolling window of size 30 produces the most accurate forecasts at each horizon, as in the case for GDP growth. However, the best model differs for varying horizons: for $h = 1$, the dynamic-factor model which includes U.S. data performs best; for $h = 2$, the best is the static-factor model with Canadian data only; for $h = 4$, it is the dynamic-factor model with Canadian data; and, for $h = 8$, the static-factor model including U.S. data is best.

Based on Diebold-Mariano tests, we conclude that factor-model forecasts are statistically more accurate than Phillips-curve benchmark forecasts. The p-values for the rejection of the null hypothesis of equal forecast accuracy are 0.001, 0.038, 0.016, and 0.068 at 1, 2, 4, and 8 steps-ahead, respectively.

5.2 Cumulative level change comparison

Tables 3 and 4 report the relative RMSFEs for cumulative level-change forecasts. Interestingly, the relative ranking is almost identical to that observed with the marginal growth forecasts, and factor models continue to rank first. From this experiment, the most important result to emphasize is the increasing uncertainty around the point forecasts. Figures 4 and 5 provide a clear illustration of the increasing uncertainty around the forecasts as the horizon lengthens. Uncertainty is therefore better assessed by examining the cumulative changes over h periods rather than the marginal growth rates. The cumulative change approach also confirms that the more time-information is included, the less accurate are the forecasts.

Diebold-Mariano test results indicate that factor-based inflation forecasts are more accurate for horizons 1, 2, and 4 (p-values: 0.005, 0.005, and 0.009, for $h = 1, 2$, and 4, respectively), whereas when $h = 8$, the p-value is only 0.133. Meanwhile, the null hypothesis of equal forecast accuracy cannot be rejected for the GDP forecasts (p-values: 0.565, 0.183, 0.435, and 0.438, for $h = 1, 2, 4$, and 8, respectively).

5.3 Forecast error densities

While RMSFEs inform us about the forecast errors' dispersion, they are silent about the general shape of the distribution, particularly about the tails. Figure 6, 7, 8, and 9 plot the forecast-error distributions for selected models at horizons 1, 2, 4, and 8. Even though the out-of-sample forecast-error distributions look normal for the marginal growth rates (Figures 6 and 7), important departures from normality occur in the errors from cumulative change forecasts. To better illustrate the effect of “long-memory” estimators on forecast accuracy, Figures 7 and 9 also incorporate the error densities from the recursive Phillips-curve forecasts.²⁷ Judging by the shape of the error distribution plotted in Figures 8 and 9, building correct confidence intervals around our cumulative-change forecasts would be more difficult for the benchmark model than for the factor models.

In general, we can see that relative to the benchmark, factor-model forecast errors are more closely centered around zero (unbiased), have smaller variance, and exhibit less skewness and kurtosis. The fatter tails in the benchmark forecast-error distributions also suggest that our proposed factor models are less subject to large forecast errors, especially at horizons 4 and 8.

6 Conclusion

In this paper, we have shown that factor models can provide sizeable accuracy improvements in forecasting Canadian GDP and core inflation, relative to standard benchmark models, for horizons up to 8 quarters ahead. Factor-based forecast errors have lower variance and are more closely centered around zero (unbiased) than those of the benchmark models. Furthermore, they are less prone to large errors, particularly as the horizon increases.

Static factor forecasts are found to perform quite well in general, with no clear gains from incorporating information on the dynamic relationships among variables in the factor estimations. Over longer samples, results similar to previous studies are obtained, indicating that the bulk of the co-movement among macroeconomic variables is optimally summarized in one common factor; the number of factors does, however, increase with smaller sample sizes.

Sample-size selection can influence overall forecast accuracy considerably, with smallest rolling-window samples generating superior results to larger samples, and expanding-window forecasts generally performing the worst. Our results suggest that including data that date

²⁷The recursive IS-curve forecast-error densities are not plotted because the resulting pictures are difficult to interpret, but similar conclusions hold for the cumulative GDP growth forecasts.

back to the 1970s and 1980s leads to a reduction in the forecast accuracy relative to using only more recent data.

Finally, to assess the degree of uncertainty around long-horizon forecasts of a variable of interest, our results illustrate the need to compare forecasts not only from the marginal growth rates, but also from the cumulative level change.

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Table 1: Relative RMSFE for GDP Growth

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 1$											
Static (Cdn)	0.971	0.649	0.875	0.753	0.939	0.868	0.837	0.934	0.964	0.926	1.020
Static (US)	0.923	0.641	0.756	0.685	0.823	0.916	0.917	0.921	0.961	0.975	0.977
Dynamic (Cdn)	0.984	0.723	0.771	0.751	0.898	0.855	0.868	0.973	0.985	0.968	1.010
Dynamic (US)	0.928	0.707	0.754	0.727	0.945	0.947	0.941	0.956	0.985	0.965	0.999
AR	0.938	0.960	0.977	0.957	0.972	0.957	0.934	0.944	0.955	0.997	0.977
IS-Curve	1.230	0.738	0.771	0.883	0.873	0.933	1.030	0.986	1.041	1.112	1.131
$h = 2$											
Static (Cdn)	0.973	0.563	0.753	0.846	0.976	0.925	0.903	1.020	1.000	1.020	1.020
Static (US)	0.952	0.719	0.763	0.726	0.918	0.994	0.985	0.962	1.000	1.050	1.010
Dynamic (Cdn)	0.999	0.726	0.628	0.786	0.978	0.956	0.912	1.050	1.040	1.040	1.020
Dynamic (US)	0.959	0.669	0.870	0.789	1.030	0.997	0.994	1.040	1.020	1.050	1.030
AR	0.972	0.981	1.020	1.010	1.000	1.010	1.000	0.993	0.995	1.040	1.030
IS-Curve	1.180	0.714	0.895	0.962	1.020	1.070	1.030	1.080	1.070	1.090	1.050

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Table 1: Relative RMSFE for GDP Growth (cont'd)

Rolling Schemes ($S =$)											
Models	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 4$											
Static (Cdn)	0.984	0.631	0.821	0.901	1.000	0.993	1.02	1.030	0.999	1.020	1.060
Static (US)	0.982	0.532	0.795	0.875	0.890	1.040	0.998	1.030	1.010	1.070	1.060
Dynamic (Cdn)	0.997	0.643	0.783	0.860	1.020	0.984	1.03	1.070	0.991	1.030	1.050
Dynamic (US)	0.985	0.701	0.884	0.919	0.911	1.040	1.04	1.030	1.030	1.060	1.050
AR	0.985	1.070	1.09	1.080	1.040	1.050	1.020	0.9979	0.993	1.020	1.030
IS-Curve	1.080	0.708	0.784	0.946	0.967	0.971	0.978	0.937	1.030	1.040	1.050
$h = 8$											
Static (Cdn)	1.040	0.536	0.756	0.911	0.973	0.991	1.04	0.943	0.985	1.080	1.090
Static (US)	1.020	0.467	0.667	0.754	0.990	1.070	1.04	0.924	0.999	1.080	1.080
Dynamic (Cdn)	1.000	0.633	0.749	0.933	0.934	0.980	1.05	0.962	0.977	1.100	1.090
Dynamic (US)	1.010	0.565	0.817	0.991	1.040	1.070	1.06	0.979	0.998	1.080	1.090
AR	0.996	1.000	1.07	1.110	1.120	1.090	1.040	1.000	0.982	1.030	1.030
IS-Curve	1.070	0.659	0.721	0.911	0.973	0.994	1.030	0.933	0.989	0.991	1.090

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Table 2: Relative RMSFE for Core CPI Inflation

Rolling Schemes ($S =$)											
Models	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 1$											
Static (Cdn)	0.309	0.172	0.183	0.216	0.229	0.239	0.259	0.282	0.283	0.289	0.285
Static (US)	0.321	0.168	0.180	0.223	0.240	0.249	0.264	0.275	0.283	0.285	0.282
Dynamic (Cdn)	0.310	0.182	0.178	0.223	0.237	0.248	0.254	0.273	0.279	0.286	0.288
Dynamic (US)	0.320	0.158	0.195	0.213	0.248	0.262	0.268	0.272	0.282	0.287	0.282
AR	0.289	0.248	0.243	0.245	0.251	0.255	0.259	0.266	0.276	0.276	0.276
Phillips-Curve	0.292	0.226	0.234	0.237	0.242	0.243	0.252	0.252	0.259	0.269	0.271
$h = 2$											
Static (Cdn)	0.332	0.173	0.180	0.219	0.237	0.224	0.256	0.278	0.285	0.297	0.291
Static (US)	0.328	0.181	0.189	0.219	0.244	0.241	0.256	0.268	0.285	0.282	0.282
Dynamic (Cdn)	0.326	0.192	0.194	0.228	0.262	0.236	0.258	0.278	0.285	0.297	0.289
Dynamic (US)	0.326	0.175	0.207	0.225	0.266	0.247	0.257	0.271	0.275	0.287	0.280
AR	0.276	0.248	0.246	0.243	0.252	0.249	0.255	0.267	0.271	0.281	0.264
Phillips-Curve	0.293	0.226	0.237	0.235	0.236	0.232	0.241	0.241	0.247	0.268	0.265

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Table 2: Relative RMSFE for Core CPI Inflation (cont'd)

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 4$											
Static (Cdn)	0.362	0.169	0.175	0.208	0.235	0.249	0.263	0.254	0.277	0.295	0.272
Static (US)	0.363	0.165	0.171	0.231	0.225	0.257	0.248	0.267	0.284	0.292	0.270
Dynamic (Cdn)	0.355	0.162	0.179	0.220	0.254	0.261	0.261	0.252	0.277	0.292	0.274
Dynamic (US)	0.362	0.183	0.203	0.237	0.246	0.257	0.258	0.263	0.274	0.288	0.284
AR	0.311	0.232	0.236	0.245	0.240	0.265	0.258	0.267	0.276	0.285	0.278
Phillips-Curve	0.332	0.210	0.214	0.238	0.234	0.259	0.252	0.265	0.281	0.286	0.264
$h = 8$											
Static (Cdn)	0.364	0.149	0.145	0.197	0.242	0.281	0.296	0.312	0.338	0.362	0.360
Static (US)	0.361	0.124	0.174	0.184	0.240	0.293	0.275	0.300	0.322	0.355	0.364
Dynamic (Cdn)	0.364	0.170	0.179	0.210	0.256	0.273	0.291	0.308	0.330	0.365	0.371
Dynamic (US)	0.352	0.163	0.180	0.202	0.276	0.304	0.276	0.322	0.316	0.351	0.360
AR	0.345	0.277	0.277	0.289	0.293	0.299	0.309	0.323	0.340	0.353	0.359
Phillips-Curve	0.337	0.230	0.228	0.243	0.260	0.265	0.290	0.296	0.307	0.371	0.376

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Table 3: Relative RMSFE for cumulative GDP Growth

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 2$											
Static (Cdn)	0.935	0.544	0.645	0.845	0.900	0.868	0.926	1.009	0.939	0.995	0.955
Static (US)	0.904	0.701	0.691	0.726	0.828	0.904	0.927	0.983	0.936	0.998	0.972
Dynamic (Cdn)	0.979	0.613	0.634	0.847	0.850	0.906	0.934	0.991	0.976	1.020	0.970
Dynamic (US)	0.918	0.698	0.789	0.777	0.913	0.905	0.936	1.014	0.965	1.033	1.004
AR	0.954	0.961	1.011	0.960	0.967	0.980	0.981	0.978	0.957	0.998	0.982
IS-Curve	1.250	0.706	0.838	0.909	0.930	0.967	1.049	1.068	1.100	1.158	1.163

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data. For $h=1$, see Table 1.

Table 3: Relative RMSFE for Cumulative GDP Growth (cont'd)

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 4$											
Static (Cdn)	0.985	0.619	0.932	0.810	0.991	0.945	0.980	1.080	1.032	1.016	1.104
Static (US)	0.976	0.581	0.729	0.742	0.925	0.966	0.955	1.055	1.054	1.084	1.058
Dynamic (Cdn)	1.014	0.707	0.870	0.839	1.017	0.967	0.968	1.120	1.032	1.018	1.104
Dynamic (US)	0.980	0.683	0.754	0.829	0.964	1.028	0.986	1.039	1.062	1.067	1.057
AR	0.993	1.062	1.076	1.054	1.021	1.055	1.045	1.025	1.010	1.041	1.044
IS-Curve	1.216	0.667	0.847	0.933	1.014	1.037	1.031	1.082	1.112	1.138	1.113
$h = 8$											
Static (Cdn)	0.994	0.495	0.621	0.942	0.905	0.986	1.050	1.062	1.042	1.063	1.125
Static (US)	0.996	0.334	0.701	0.670	0.880	1.081	1.063	1.010	1.023	1.104	1.083
Dynamic (Cdn)	1.073	0.665	0.772	1.018	0.844	1.024	1.063	1.059	1.079	1.086	1.075
Dynamic (US)	1.029	0.617	0.827	0.765	0.854	1.169	1.069	1.079	1.061	1.110	1.122
AR	0.996	1.200	1.177	1.210	1.208	1.127	1.095	1.039	0.986	1.010	1.079
IS-Curve	1.029	0.422	0.628	0.849	0.922	1.044	1.042	0.879	0.948	1.022	1.002

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Table 4: Relative RMSFE for cumulative Core CPI Inflation

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 2$											
Static (Cdn)	0.294	0.117	0.139	0.182	0.192	0.193	0.222	0.245	0.251	0.259	0.257
Static (US)	0.303	0.107	0.142	0.173	0.211	0.209	0.224	0.236	0.247	0.248	0.252
Dynamic (Cdn)	0.297	0.135	0.146	0.189	0.216	0.201	0.222	0.244	0.250	0.256	0.259
Dynamic (US)	0.305	0.130	0.150	0.179	0.228	0.221	0.222	0.234	0.248	0.250	0.248
AR	0.255	0.194	0.204	0.203	0.219	0.219	0.183	0.227	0.234	0.236	0.231
Phillips-Curve	0.268	0.173	0.192	0.202	0.202	0.201	0.184	0.212	0.221	0.232	0.229

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data. For $h=1$, see Table 3.

Table 4: Relative RMSFE for cumulative Core CPI Inflation (cont'd)

Models	Rolling Schemes ($S =$)										
	Recursive	30	35	40	45	50	55	60	65	70	75
$h = 4$											
Static (Cdn)	0.298	0.101	0.148	0.145	0.162	0.182	0.196	0.199	0.223	0.233	0.230
Static (US)	0.298	0.091	0.110	0.154	0.176	0.186	0.180	0.207	0.213	0.230	0.220
Dynamic (Cdn)	0.297	0.107	0.147	0.164	0.189	0.187	0.193	0.199	0.217	0.233	0.228
Dynamic (US)	0.298	0.116	0.119	0.163	0.188	0.195	0.189	0.203	0.209	0.226	0.220
AR	0.239	0.155	0.170	0.168	0.175	0.185	0.183	0.194	0.206	0.210	0.202
Phillips-Curve	0.250	0.134	0.149	0.161	0.168	0.170	0.184	0.187	0.195	0.209	0.199
$h = 8$											
Static (Cdn)	0.290	0.064	0.088	0.132	0.147	0.158	0.172	0.180	0.205	0.229	0.246
Static (US)	0.286	0.061	0.091	0.100	0.156	0.164	0.162	0.179	0.190	0.220	0.237
Dynamic (Cdn)	0.291	0.087	0.107	0.134	0.144	0.165	0.166	0.180	0.193	0.218	0.239
Dynamic (US)	0.280	0.062	0.107	0.121	0.160	0.168	0.175	0.187	0.185	0.214	0.236
AR	0.238	0.118	0.133	0.150	0.147	0.164	0.181	0.187	0.198	0.210	0.216
Phillips-Curve	0.244	0.084	0.105	0.138	0.162	0.152	0.180	0.176	0.185	0.211	0.231

Note: h is the forecast horizon, S is the number of observations used for the rolling exercise. Static (Cdn) and Dynamic (Cdn) are the factor models using only Canadian data, whereas Static and Dynamic (US) are the factor models using both the Canadian and the U.S. data.

Appendix: The Data Set

I. NATIONAL ACCOUNTS (CURRENT DOLLARS)

1. GROSS DOMESTIC PRODUCT AT MARKET PRICES - VALUE
2. GROSS NATIONAL PRODUCT AT MARKET PRICES - VALUE
3. PERSONAL EXPENDITURE ON GOODS AND SERVICES
4. DURABLE GOODS
5. SEMI-DURABLE GOODS
6. NON-DURABLE GOODS
7. SERVICES
8. GOVERNMENT EXPENDITURE
9. RESIDENTIAL STRUCTURES
10. BUSINESS FIXED INVESTMENT
11. NON-RESIDENTIAL STRUCTURES
12. MACHINERY & EQUIPMENT
13. FINAL DOMESTIC DEMAND
14. NET EXPORTS
15. FINAL SALES
16. BUSINESS INVESTMENT IN INVENTORIES: NON-FARM
17. BUSINESS INVESTMENT IN INVENTORIES: FARM
18. DOMESTIC DEMAND
19. EXPORTS OF GOODS & SERVICES
20. TOTAL DEMAND
21. DEDUCT: IMPORTS OF GOODS & SERVICES
22. LABOUR INCOME
23. CORPORATION PROFITS BEFORE TAXES
24. INVENTORY VALUATION ADJUSTMENT
25. NET INC OF NON-FARM UNINC BUSINESS, INCLUDING RENT
26. ACCRUED NET INC OF FARM OPERATORS FROM PRODUCTION
27. OTHER NET INCOME
28. CAPITA CONSUMPTION ALLOWANCES
29. NET LENDIN (GOVERNMENT BALANCE)
30. FEDERAL
31. PRO. & MUNICIPALITIES
32. C.P.P. & Q.P.P.

33. INCOM (PERSONAL)

34. CURREN TRANSFERS TO GOVERNMENT

35. DISPOSABLE INCOME

II. BUSINESS INVESTMENT

1. NEW COMMERCIAL VEHICLE SALES

III. HOUSING MARKET

1. DWELLING STARTS - ALL AREAS - TOTAL S.A.A.R. UNITS

2. DWELLING STARTS - URBAN AREAS - TOTAL S.A.A.R. UNITS

3. DWELLING STARTS - URBAN AREAS - SINGLES S.A.A.R. UNITS

4. DWELLING STARTS - URBAN AREAS - MULTIPLES S.A.A.R. UNITS

5. DWELLING STARTS - URBAN AREAS - ATLANTIC PROVINCES S.A.A.R. UNITS

6. DWELLING STARTS - URBAN AREAS - QUEBEC S.A.A.R. UNITS

7. DWELLING STARTS - URBAN AREAS - ONTARIO S.A.A.R. UNITS

8. DWELLING STARTS - URBAN AREAS - PRAIRIE PROVINCES S.A.A.R. UNITS

9. DWELLING STARTS - URBAN AREAS - BRITISH COLUMBIA S.A.A.R. UNITS

IV. BUILDING PERMITS AND NEWLY COMPLETED BUT UNOCCUPIED DWELLINGS

1. BUILDING PERMITS TOTAL UNITS ALL AREAS (SAAR)

2. BUILDING PERMITS SINGLE UNITS ALL AREAS (SAAR)

3. BUILDING PERMITS MULTIPLE UNITS ALL AREAS (SAAR)

4. HOUSES AND DUPLEXES - DWELLINGS COMP. BUT UNOC UNITS - S.A.

5. ROW AND APARTMENTS - UNOC. DWELLINGS - TOTAL METRO. AREAS UNITS - S.A.

6. ROW AND APARTMENTS - UNOC. DWELLINGS - MONTREAL DWELLING UNITS - S.A.

7. ROW AND APARTMENTS - UNOC. DWELLINGS - TORONTO DWELLING UNITS - S.A.

8. ROW AND APARTMENTS - UNOC. DWELLINGS - VANCOUVER DWELLING UNITS - S.A.

V. BALANCE OF PAYMENTS

1. TOTAL CURRENT ACCOUNT BALANCE

2. TOTAL CURRENT ACCOUNT BALANCE (AS % OF GDP)

3. GOODS AND SERVICES BALANCE

4. GOODS BALANCE

5. SERVICES BALANCE

6. INVESTMENT INCOME BALANCE

7. DIRECT INVESTMENT INCOME BALANCE

8. PORTFOLIO INVESTMENT INCOME BALANCE

9. OTHER INVESTMENT INCOME BALANCE

10. TRANSFERS BALANCE
11. TOTAL CURRENT ACCOUNT RECEIPTS
12. GOODS AND SERVICES RECEIPTS
13. GOODS RECEIPTS
14. SERVICES RECEIPTS
15. INVESTMENT INCOME RECEIPTS
16. DIRECT INVESTMENT INCOME RECEIPTS
17. PORTFOLIO INVESTMENT INCOME RECEIPTS
18. OTHER INVESTMENT INCOME RECEIPTS
19. TRANSFERS RECEIPTS
20. TOTAL CURRENT ACCOUNT PAYMENTS
21. GOODS AND SERVICES PAYMENTS
22. GOODS PAYMENTS
23. SERVICES PAYMENTS
24. INVESTMENT INCOME PAYMENTS
25. DIRECT INVESTMENT INCOME PAYMENTS
26. PORTFOLIO INVESTMENT INCOME PAYMENTS
27. OTHER INVESTMENT INCOME PAYMENTS
28. TRANSFER PAYMENTS
29. CAPITA ACCOUNT, NET FLOW
30. FINANCIA ACCOUNT, NET FLOW
31. CANADIA ASSETS, NET FLOW
32. CANADIA DIRECT INVESTMENT ABROAD
33. CANADIA PORTFOLIO INVESTMENT
34. FOREIG PORTFOLIO BONDS
35. FOREIG PORTFOLIO STOCKS
36. OTHE CANADIAN INVESTMENT
37. LOANS
38. DEPOSITS
39. OFFICIA INTERNATIONAL RESERVES
40. CANADIA LIABILITIES, NET FLOW
41. FOREIG DIRECT INVESTMENT IN CANADA
42. FOREIG PORTFOLIO INVESTMENT
43. CANADIA PORTFOLIO BONDS
44. CANADIA PORTFOLIO STOCKS

45. CANADIA MONEY MARKET
46. OTHE FOREIGN INVESTMENT
47. DEPOSITS
48. OTHE LIABILITIES
49. TOTAL CAPITAL AND FINANCIAL ACCOUNTS, NET FLOW

VI. OTHER

1. CONSUMER ATTITUDES - FOR ALL RESPONDENTS - CANADA % S.A.

VII. SURVEY OF EMP., PAYROLLS & HOURS

1. EMPLOYMENT - COMMERCIAL INDUSTRIES S.A.
2. EMPLOYMENT - SERVICE PRODUCING (INCL. UTILITIES) - COMMERCIAL S.A.
3. EMPLOYMENT - NON-COMMERCIAL INDUSTRIES S.A.

VIII. LABOUR INCOME

1. WAGES AND SALARIES
2. COMMERCIAL (EXCL. FORESTRY)
3. MANUFACTURING
4. NON-COMMERCIAL
5. AGRICULTURE, FORESTRY, FISHING AND HUNTING
6. SUPPLEMENTARY LABOUR INCOME
7. LABOUR INCOME LESS MILITARY
8. LABOUR INCOME

IX. UNIT LABOUR COSTS

1. LABOUR INCOME PER UNIT OF REAL OUTPUT - TOTAL ECONOMY
2. WAGES & SALARIES PER UNIT OF OUTPUT - NON-FARM SECTOR (EXCL. FORESTRY)
3. WAGES & SALARIES PER UNIT OF OUTPUT - NON-FARM COMMERCIAL (EXCL. FORESTRY)

X. CONSUMER PRICE INDEX

1. C.P.I. TOTAL
2. C.P.I. FOOD
3. C.P.I. BEEF
4. C.P.I. CEREAL & BAKERY PRODUCTS
5. C.P.I. DAIRY PRODUCTS INCL. BUTTER
6. C.P.I. ENERGY
7. C.P.I. FUEL OIL AND OTHER LIQUID FUEL
8. C.P.I. PIPED GAS
9. C.P.I. ELECTRICITY

10. C.P.I. GASOLINE AND OTHER FUELS
11. C.P.I. EXCL. FOOD AND ENERGY
12. C.P.I. GOODS EXCL. FOOD AND ENERGY
13. C.P.I. GOODS EXCL. MOTOR VEHICLES
14. C.P.I. NON-DURABLES EXCL. FOOD AND ENERGY
15. C.P.I. SEMI-DURABLE GOODS
16. C.P.I. DURABLE GOODS
17. C.P.I. AUTO AND TRUCK PURCHASE
18. C.P.I. DURABLES EXCL. MOTOR VEHICLES
19. C.P.I. TOTAL SERVICES
20. C.P.I. SHELTER SERVICES
21. C.P.I. RENTALS

XII. INDUSTRIAL PRODUCT PRICE INDEX

1. I.P.P.I. - ALL MANUFACTURING INDUSTRIES S.A.
2. I.P.P.I. - FOOD AND BEVERAGE INDUSTRIES S.A.
3. I.P.P.I. - BEEF (FRESH OR FROZEN) EXCL. GROUND S.A.
4. I.P.P.I. - PORK (FRESH OR FROZEN) S.A.
5. I.P.P.I. - TOTAL EXCL. FOOD & BEVERAGES S.A.

XIII. CAPACITY UTILIZATION RATE

1. TOTAL NON-FARM GOODS PRODUCING INDUSTRIES
2. LOGGING AND FORESTRY
3. MINING, QUARRYING AND OIL WELLS
4. MANUFACTURING INDUSTRIES
5. FOOD INDUSTRIES
6. BEVERAGE INDUSTRIES
7. TOBACCO PRODUCTS INDUSTRIES
8. RUBBER PRODUCTS INDUSTRIES
9. PLASTIC PRODUCTS INDUSTRIES
10. LEATHER AND ALLIED PRODUCTS INDUSTRIES
11. PRIMARY TEXTILE INDUSTRIES
12. TEXTILE PRODUCTS INDUSTRIES
13. CLOTHING INDUSTRIES
14. PAPER AND ALLIED PRODUCTS INDUSTRIES
15. PRINTING, PUBLISHING AND ALLIED INDUSTRIES

16. REFINED PETROLEUM AND COAL PRODUCTS INDUSTRIES
17. CHEMICAL AND CHEMICAL PRODUCTS
18. WOOD INDUSTRIES
19. FURNITURE AND FIXTURE INDUSTRIES
20. PRIMARY METAL INDUSTRIES
21. FABRICATED METAL PRODUCTS INDUSTRIES
22. MACHINERY INDUSTRIES
23. TRANSPORTATION EQUIPMENT INDUSTRIES
24. ELECTRICAL AND ELECTRONIC PRODUCTS INDUSTRIES
25. NON-METALLIC MINERAL PRODUCTS INDUSTRIES
26. OTHER MANUFACTURING INDUSTRIES
27. ELECTRIC POWER AND GAS DISTRIBUTION SYSTEMS
28. CONSTRUCTION INDUSTRIES

XIV. GOVERNMENT REVENUE AND EXPENDITURE

1. DIRECT TAXES - PERSONS
2. FEDERAL
3. PROVINCIAL
4. C.P.P. & Q.P.P.
5. DIRECT TAXES - FROM CORPORATIONS & GOVT BUSINESS ENTERPRISES
6. FEDERAL
7. PROVINCIAL
8. INDIRECT TAXES
9. FEDERAL
10. PROVINCES & MUNICIPALITIES
11. OTHER REVENUES
12. OTHER REVENUES - FEDERAL
13. PROVINCES & MUNICIPALITIES
14. C.P.P. & Q.P.P.
15. CURRENT TRANSFERS FROM GOVERNMENT
16. FROM PROVINCIAL LEVEL
17. FED. TO PROV. & MUNIC.
18. PROV. TO MUNIC.
19. REVENUE BY LEVEL - FEDERAL
20. REVENUE BY LEVEL - PROVINCES & MUNICIPALITIES

21. REVENUE BY LEVEL - C.P.P. & Q.P.P.
22. GROSS CURRENT EXPENDITURE ON GOODS AND SERVICES
23. FEDERAL - DEFENCE
24. PROVINCES & MUNICIPALITIES
25. C.P.P. & Q.P.P.
26. INTEREST ON THE PUBLIC DEBT
27. INTEREST ON THE PUBLIC DEBT - FEDERAL
28. PROVINCES & MUNICIPALITIES
29. OTHER EXPENDITURES
30. OTHER EXPENDITURES - FEDERAL
31. C.P.P. & Q.P.P.
32. TOTAL
33. TRANSFERS TO OTHER LEVELS
34. TO GOVERNMENT
35. TO LOCAL LEVEL
36. EXPENDITURE BY LEVEL - FEDERAL
37. C.P.P. & Q.P.P.
38. SURPLUS OR DEFICIT (NET LENDING) - WITHOUT Q.P.P.
39. FEDERAL
40. PROVINCE & MUNICIPALITIES
41. C.P.P. & Q.P.P.

XV. MOTOR VEHICLE SALES

1. PASSENGER CAR SALES—NORTH AMERICAN MANUFACTURED
2. PASSENGER CAR SALES—OVERSEAS MANUFACTURED
3. TOTAL PASSENGER CAR SALES
4. TOTAL - NEW COMMERCIAL VEHICLE SALES
5. PASS. CARS - NORTH AMERICAN MFRD. - UNITS, S.A.
6. PASSENGER OVERSEAS MANUFACTURED VEHICLE SALES, S.A.
7. TOTAL COMMERCIAL VEHICLE SALES, S.A.
8. TOTAL PASSENGER CAR SALES - UNITS, S.A.

XVI. INTEREST RATES

1. TREASURY BILL AUCTION - AVERAGE YIELDS: 3 MONTH
2. TREASURY BILL AUCTION - AVERAGE YIELDS: 6 MONTH
3. GOVERNMENT OF CANADA MARKETABLE BONDS, AVERAGE YIELD: 1-3 YEAR

4. GOVERNMENT OF CANADA MARKETABLE BONDS, AVERAGE YIELD: 3-5 YEAR
5. GOVERNMENT OF CANADA MARKETABLE BONDS, AVERAGE YIELD: 5-10 YEAR
6. GOVERNMENT OF CANADA MARKETABLE BONDS, AVERAGE YIELD: OVER 10 YEARS
7. PRIME CORPORATE PAPER RATE: 1 MONTH
8. PRIME CORPORATE PAPER RATE: 3 MONTH
9. BANKERS' ACCEPTANCES: 1 MONTH
10. CHART. BANK ADMIN. INTEREST RATES - NON-CHEQUABLE SAVINGS DEPOSITS
11. CHART. BANK ADMIN. INTEREST RATES: 5-YEAR PERSONAL FIXED TERM
12. CHART. BANK ADMIN. INTEREST RATES: PRIME BUSINESS
13. MORTGAGE LENDING RATES: AVERAGE RESIDENTIAL MORTGAGE - 5 YEAR
14. FEDERAL RESERVE BANK OF NEW YORK - DISCOUNT RATE
15. FEDERAL FUNDS RATE
16. GOVERNMENT 5 YEAR BOND YIELDS
17. 90 DAY CERTIFICATES OF DEPOSIT
18. PRIME RATE CHARGED BY BANKS
19. 90 DAY PREMIUM (+) OR DISCOUNT (-) U.S. DOLLAR IN CANADA

XVII. MONETARY AGGREGATES

1. CURR. OUTSIDE BKS., S.A.
2. GROSS M1 - CURRENCY & GROSS DEMAND DEPOSITS, AOW, SA
3. CDN. \$ DEPS.NON-PERS. NOTICE, UNADJ.
4. CDN. \$ DEPS.-PERS. NOTICE, UNADJ.
5. CDN. \$ DEPS.-PERS. SAV.-FIXED TERM, UNADJ.
6. M2-CURR. & ALL CHEQ. NOT. & PERS. TERM DEPS., S.A.
7. GROSS M1,ALL NOTICE DEP. & CONTINUITY ADJUSTMENTS - M1++
8. M2+,CSB'S & NON MONEY MARKET MUTUAL FUNDS - M2++ - S.A.
9. TOT. DEPS. - TRUST AND MORTGAGE LOAN COMPANIES REPORTED BY CHART. BANKS
10. TOT. DEPS. AT C.U. & C.P., S.A.

XVIII. CREDIT AGGREGATES

1. HOUSEHOLD CREDIT (S.A.)
2. RESIDENTIAL MORTGAGE CREDIT (S.A.)
3. CONSUMER CREDIT (S.A.)
4. RES. MTG. CREDIT: O/S BAL. OF MAJOR PRIV. INSTIT. LENDERS, TOTAL, S.A.
5. TOTAL SHORT-TERM BUSINESS CREDIT (S.A.)
6. TOTAL BUSINESS CREDIT (S.A.)

7. TOTAL HOUSEHOLD & BUSINESS CREDIT (S.A.)

XIX. EXCHANGE RATES

1. U. S. DOLLAR (NOON)
2. NOON 90 DAYS FORWARD CAN/US EXCHANGE RATE
6. NOON CAN/JAPAN EXCHANGE RATE
7. NOON CAN/SWITZERLAND EXCHANGE RATE
8. NOON CAN/UK EXCHANGE RATE
9. CLOSING 90 DAYS FORWARD CAN/US EXCHANGE RATE

XX. STOCK MARKET

1. TORONTO & MONTREAL STOCK EX.-VALUE OF SHARES TRADED
2. TORONTO & MONTREAL STOCK EX.-VOLUME OF SHARES TRADED
3. U.S. COMMON STOCKS - DOW JONES INDUSTRIAL (30) - HIGH
4. U.S. COMMON STOCKS - DOW JONES INDUSTRIAL (30) - LOW
5. U.S. COMMON STOCKS - DOW JONES INDUSTRIAL (30) - CLOSE
6. N.Y. STOCK EX. - VALUE OF SHARES TRADED
7. N.Y. STOCK EX. - VOLUME OF SHARES TRADED
3. TORONTO STOCK EX. - COMPOSITE (300)-CLOSE
4. TORONTO STOCK EX. - CLOSING QUOT. AT MONTH-END - STOCK DIVIDEND YIELD

XXI. NATIONAL ACCOUNTS (IMPLICIT PRICE INDEXES, 1997=100)

1. PERSONAL EXPENDITURE ON CONSUMER GOODS & SERVICES
2. DURABLE GOODS
3. SEMI-DURABLE GOODS
4. NON-DURABLE GOODS
5. SERVICES
6. GOVERNMENT EXPENDITURE
7. RESIDENTIAL STRUCTURES
8. BUSINESS FIXED INVESTMENT
9. NON-RESIDENTIAL STRUCTURES
10. MACHINERY & EQUIPMENT
11. FINAL DOMESTIC DEMAND
12. FINAL SALES
13. DOMESTIC DEMAND
14. EXPORTS OF GOODS & SERVICES
15. TOTAL DEMAND

16. IMPORTS OF GOODS & SERVICES

17. GROSS DOMESTIC PRODUCT AT MARKET PRICES

18. RATIO OF EXPORTS TO IMPORTS

XXII. NATIONAL ACCOUNTS (FIXED-WEIGHTED PRICE INDEXES, 1997=100)

1. GROSS DOMESTIC PRODUCT AT MARKET PRICES

2. RATIO OF EXPORTS TO IMPORTS

XXIII. GDP AT BASIC PRICES (CHAINED 1997 DOLLARS)

1. ALL INDUSTRIES

2. AGRICULTURE, FORESTRY, FISHING, HUNTING & TRAPPING

3. FORESTRY & LOGGING

4. MINING AND OIL & GAS EXTRACTION

5. MANUFACTURING

6. CONSTRUCTION

7. RESIDENTIAL BUILDING CONSTRUCTION

8. UTILITIES

9. TRANSPORTATION & WAREHOUSING

10. INFORMATION & CULTURAL INDUSTRIES

11. WHOLESALE TRADE

12. RETAIL TRADE

13. FIN., INSU., REAL ESTATE, RENTING, LEASING & MGT. OF Co. AND ENTREPRISES

14. SERVICES: PROF. SCI. & TECH., ACCO., FOOD, & OTHER (EXCL. PUBLIC ADMIN.)

15. EDUCATIONAL SERVICES, & HEALTH CARE & SOCIAL ASSISTANCE

16. PUBLIC ADMINISTRATION

17. BUSINESS SECTOR INDUSTRIES

18. NON-BUSINESS SECTOR INDUSTRIES

19. GOODS PRODUCING INDUSTRIES

20. SERVICE PRODUCING INDUSTRIES

21. INDUSTRIAL PRODUCTION

XXIV. MERCHANDISE TRADE (NOMINAL)

1. EXPORTS TOTAL OF ALL MERCHANDISE

2. TOTAL EXCL. MOTOR VEHICLES & MOTOR VEHICLE PARTS

3. ENERGY

4. NON-ENERGY COMMODITIES

5. MOTOR VEHICLES & MOTOR VEHICLE PARTS

6. MACHINERY & EQUIPMENT
7. IMPORTS TOTAL OF ALL MERCHANDISE
8. TOTAL EXCL. MOTOR VEHICLES & MOTOR VEHICLE PARTS
9. ENERGY
10. NON-ENERGY COMMODITIES
11. MOTOR VEHICLES & MOTOR VEHICLE PARTS
12. MACHINERY & EQUIPMENT
13. OTHER CONSUMER GOODS

XXV. U.S. DATA

1. GROSS DOMESTIC PRODUCT
2. GROSS DOMESTIC PRODUCT (CHAINED)
3. CHAIN-TYPE PRICE INDEX - GDP
4. PERSONAL CONSUMPTION EXPENDITURES - TOTAL
5. PERSONAL CONSUMPTION EXPENDITURES - DURABLE GOODS
6. PERSONAL CONSUMPTION EXPENDITURES - NONDURABLE GOODS
7. PERSONAL CONSUMPTION EXPEND - SERVICES
8. GOVERNMENT CONSUMPTION EXPENDITURES & GROSS INVESTMENT - TOTAL
9. FEDERAL GOVT CONSUMPTION EXPENDITURES & GROSS INVESTMENT
10. STATE & LOCAL CONSUMPTION EXPENDITURES & GROSS INVESTMENT
11. PRIVATE DOMESTIC INVESTMENT, FIXED NONRES. - TOTAL
12. PRIVATE DOMESTIC INVESTMENT, FIXED NONRES. - STRUCTURES
13. PRIVATE DOMESTIC INVESTMENT, FIXED NONRES. - EQUIPMENT & SOFTWARE
14. PRIVATE DOMESTIC INVESTMENT, FIXED RESIDENTIAL
15. FINAL SALES TO DOMESTIC PURCHASERS
16. NET EXPORTS OF GOODS & SERVICES
17. EXPORTS OF GOODS & SERVICES
18. IMPORTS OF GOODS & SERVICES
19. FINAL SALES OF DOMESTIC PRODUCTS
20. CHANGE IN PRIVATE INVENTORIES - TOTAL
21. CHANGE IN PRIVATE INVENTORIES - NONFARM
22. CHANGE IN PRIVATE INVENTORIES - FARM
23. PROPRIETORS' INCOME WITH IVA & CCADJ - FARM
24. COMPENSATION OF EMPLOYEES
25. CORPORATE PROFITS BEFORE TAX (EXCL. IVA & CCADJ)

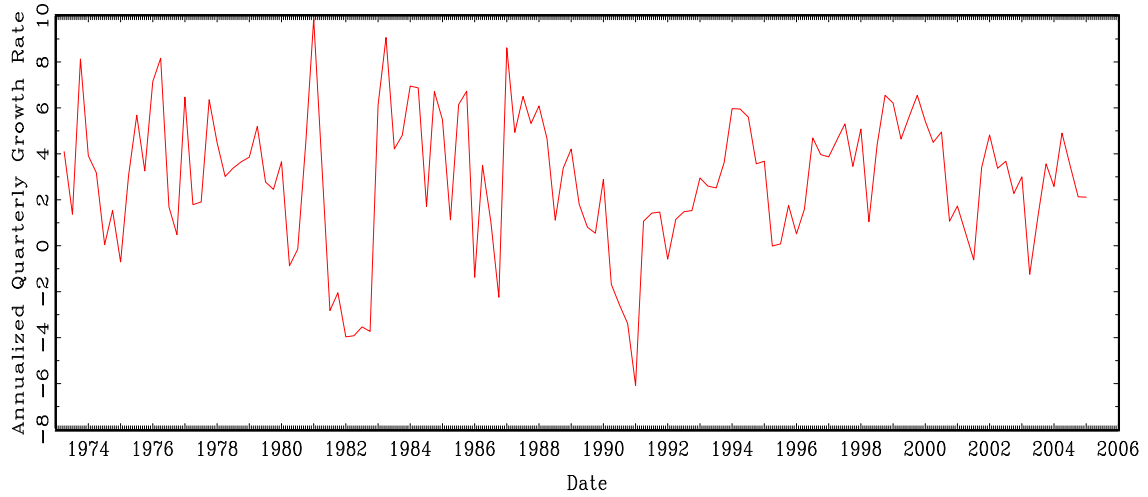
26. DIVIDENDS
27. INVENTORY VALUATION ADJUSTMENT, CORPORATE
28. NET INTEREST & MISC PAYMENTS, DOMESTIC - INDUSTRIES
29. INDIRECT BUSINESS TAX & NONTAX LIABILITY
30. GOVERNMENT CURRENT SURPLUS OR DEFICIT, NIPA BASIS
31. FEDERAL GOVERNMENT CURRENT SURPLUS OR DEFICIT, NIPA BASIS
32. STATE AND LOCAL GOVERNMENT CURRENT SURPLUS OR DEFICIT, NIPA BASIS
33. PERSONAL INCOME - TOTAL
34. PERSONAL INCOME - PERSONAL CURRENT TAXES
35. DISPOSABLE PERSONAL INCOME
36. INDUSTRIAL PROD INDEX - PRODUCTS, TOTAL (G17)
37. INDUSTRIAL PROD INDEX - FINAL PRODUCTS TOTAL (G17)
38. INDUSTRIAL PROD INDEX - MANUFACTURING (G17)
39. INDUSTRIAL PROD INDEX - DURABLE MANUFACTURING (G17)
40. INDUSTRIAL PROD INDEX - NONDURABLE MANUFACTURING (G17)
41. INDUSTRIAL PROD INDEX - EQUIPMENT (G17)
42. INDUSTRIAL PROD INDEX - MATERIALS (G17)
43. CAPACITY UTILIZATION - MANUFACTURING, MINING, & UTILITIES (G17)
44. CAPACITY UTILIZATION - MANUFACTURING (G17)
45. RETAIL UNIT SALES OF NEW PASSENGER CARS - TOTAL
46. RETAIL UNIT SALES OF NEW PASSENGER CARS - DOMESTIC
47. RETAIL UNIT SALES OF NEW PASSENGER CARS - IMPORTS
48. RETAIL UNIT SALES OF TRUCKS - 14,000 LBS. GVW & UNDER, DOMESTIC
49. MOTOR VEHICLES ASSEMBLIES - TOTAL
50. MOTOR VEHICLES ASSEMBLIES, AUTO ASSEMBLIES - TOTAL
51. MOTOR VEHICLES ASSEMBLIES, TRUCKS (G17)
52. SHIPMENTS - MANUFACTURING (M31)
53. MONEY SUPPLY - CURRENCY, DEMAND DEPOSITS, OTHER CHECKABLE DEP.
54. MONEY SUPPLY - M2
55. MONEY SUPPLY - M3
56. ASSETS - LOANS & LEASES IN BANK CREDIT AT CB'S
57. ASSETS - CONSUMER LOANS AT CB'S
58. ASSETS - COMMERCIAL & INDUSTRIAL LOANS AT CB'S
59. ASSETS - REAL ESTATE LOANS AT CB'S
60. HOUSING STARTS, PRIVATE INCLUDING FARM - TOTAL

61. HOUSING PERMITS, PRIVATE (C20)
62. NEW ONE-FAMILY HOMES SOLD
63. EXISTING SINGLE-FAMILY HOMES SOLD - TOTAL
64. VACANCY RATE, RENTAL HOUSING UNITS
65. VACANCY RATE, HOMEOWNER HOUSING UNITS
66. CPI (ALL URBAN) - ALL ITEMS
67. CPIU - FOOD
68. CPIU - ENERGY
69. CPIU - ALL ITEMS LESS FOOD
70. CPIU - ALL ITMES LESS FOOD & ENERGY
71. CPIU - SERVICES LESS ENERGY SERVICES
72. PRODUCER PRICE INDEX - FINISHED GOODS
73. PRODUCER PRICE INDEX - FINISHED CONSUMER GOODS
74. PRODUCER PRICE INDEX - CAPITAL EQUIPMENT
75. PRODUCER PRICE INDEX - INTERMEDIATE MATERIALS,SUPPLIES AND COMPONENTS
76. PRODUCER PRICE INDEX - CRUDE MATERIALS
77. CIVILIAN LABOR FORCE - TOTAL (ESIT)
78. EMPLOYED - CIVILIAN TOTAL (ESIT)
79. UNEMPLOYMENT RATE - CIVILIAN (ESIT)
80. PARTICIPATION RATE - CIVILIAN LABOR FORCE, TOTAL (ESIT)
81. HOURS, PRODUCTION WORKERS - TOTAL PRIVATE
82. OUTPUT PER WORKER
83. OUTPUT PER HOUR OF ALL PERSONS, INDEX - NONFARM BUSINESS SECTOR (PC)
84. COMPENSATION PER HOUR INDEX - NONFARM BUSINESS SECTOR (PC)
85. UNIT LABOR COSTS INDEX - NONFARM BUSINESS SECTOR (PC)
86. RECEIPTS, FEDERAL GOVT - TOTAL
87. RECEIPTS, S&L GOVT - TOTAL
88. FEDERAL GOVERNMENT CURRENT TAXES
89. STATE & LOCAL GOVERNMENT CURRENT TAXES
90. FEDERAL GOV'T TAXES ON CORPORATE INCOME
91. STATE & LOCAL GOV'T TAXES ON CORPORATE INCOME
92. FEDERAL GOV'T TAXES ON PROD'N & IMPORTS
93. STATE & LOCAL GOV'T TAXES ON PROD'N & IMPORTS
94. FEDERAL GOV'T CONTRIBUTIONS FOR GOV'T SOCIAL INS.
95. STATE & LOCAL GOV'T CONTRIBUTIONS FOR GOV'T SOCIAL INS.

96. FEDERAL GOV'T CURRENT EXPENDITURES
97. STATE & LOCAL GOV'T CURRENT EXPENDITURES
98. GOV'T CONSUMPTION EXPENDITURES & GROSS INVESTMENT - FEDERAL
99. GOV'T CONSUMPTION EXPENDITURES & GROSS INVESTMENT - STATE & LOCAL
100. FEDERAL GOV'T CURRENT TRANSFER PAYMENTS
101. U.S. INT'L TRANSACTIONS - BALANCE ON GOODS (BOP)
102. EXPORTS - GOODS, BALANCE OF PAYMENTS BASIS (BOP)
103. U.S. BALANCE ON SERVICES, INVENTORY INCOME & TRANS. (EX. MILITARY GRANTS)
104. U.S. BALANCE ON SERVICES (BOP)
105. U.S. BALANCE ON INVESTMENT INCOME
106. U.S. UNILATERAL TRANSFERS (EX MILITARY GRANTS), NET (BOP)
107. U.S. BALANCE ON CURRENT ACCOUNT (BOP)
108. U.S. BALANCE ON CURRENT ACCOUNT (BOP), % OF GDP
107. U.S. - FEDERAL FUNDS RATE
108. U.S. - GOVERNMENT 5 YEAR BOND YIELD (CONSTANT MATURITY)
109. U.S. - 90 DAY CERTIFICATES OF DEPOSIT (ADJUSTED)
110. U.S. - PRIME RATE CHARGED BY BANKS

Figure 1: **GDP growth and Core CPI Inflation: 1973Q1-2005Q1**

GDP growth



Core CPI Inflation

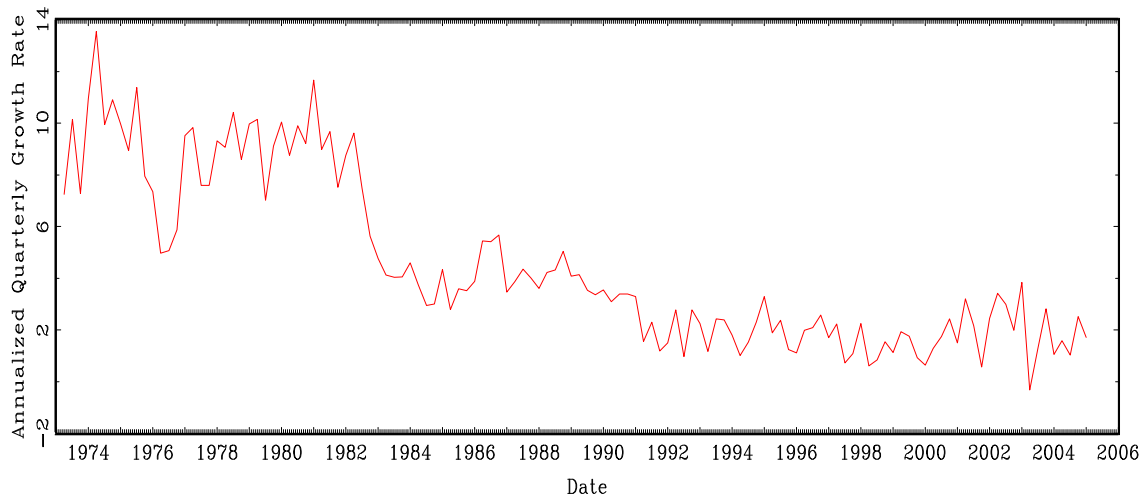


Figure 2: GDP Growth RMSFE: Factor vs. Benchmark Model

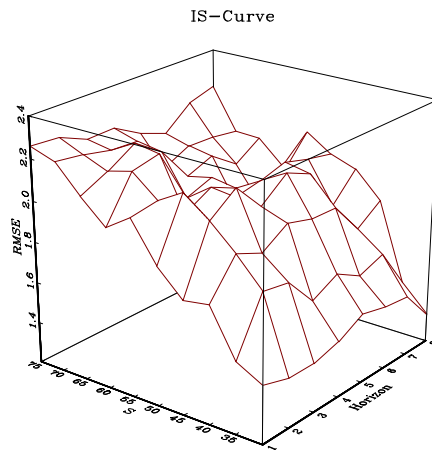
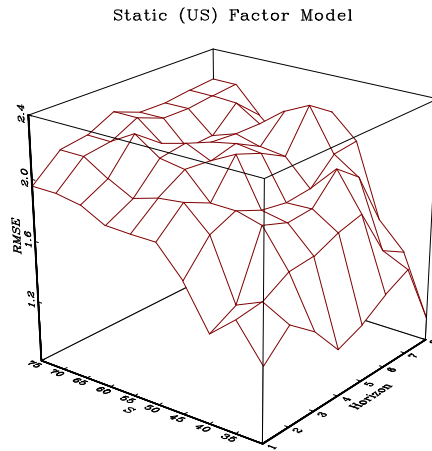


Figure 3: Core Inflation RMSFE: Factor vs. Benchmark Model

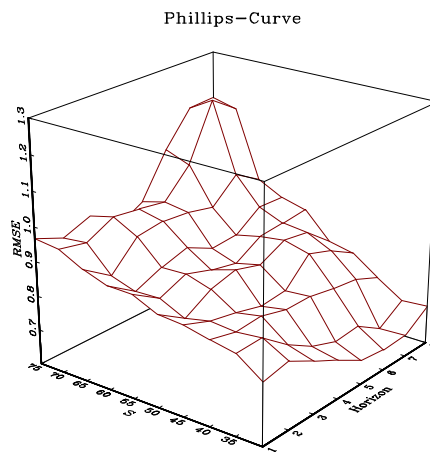
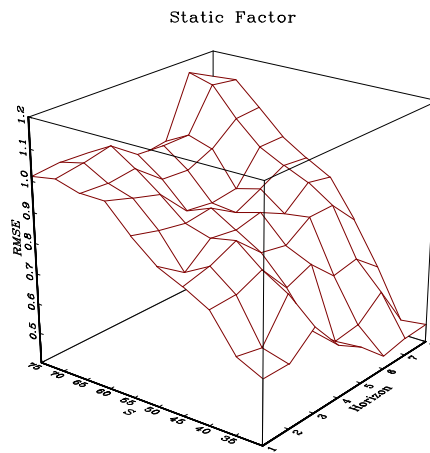


Figure 4: Cumulative GDP Growth RMSFE: Factor vs. Benchmark Model

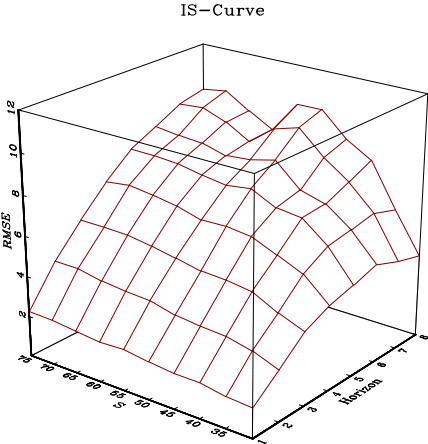
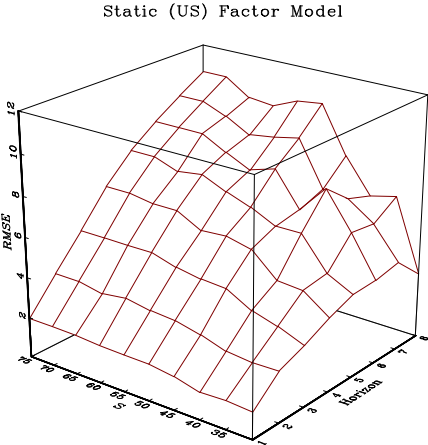


Figure 5: Cumulative Core Inflation RMSFE: Factor vs. Benchmark Model

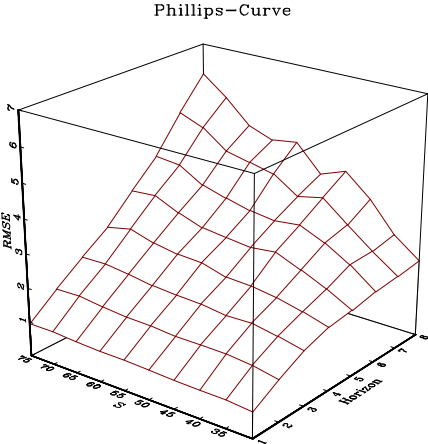
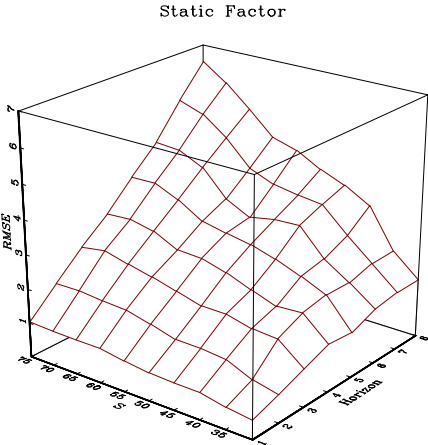


Figure 6: GDP growth Forecast Error Densities

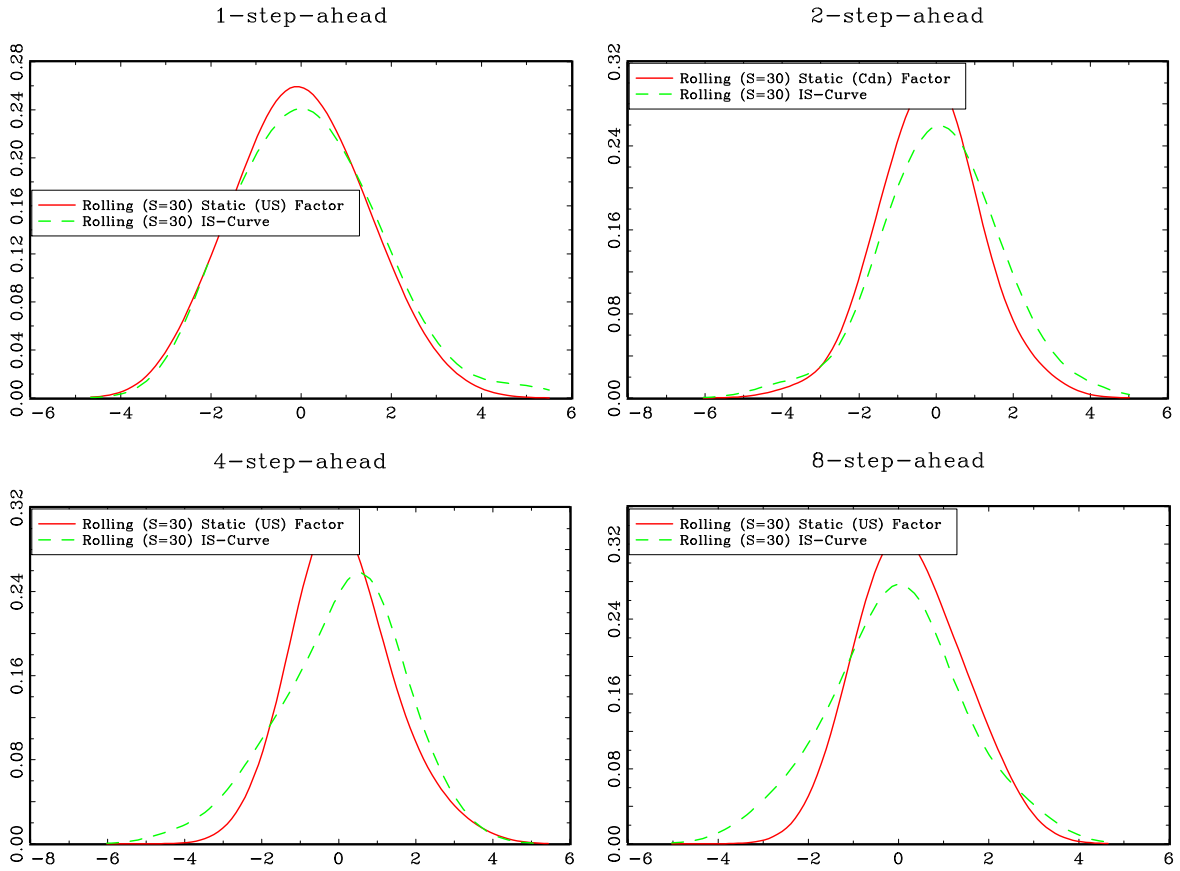


Figure 7: Core Inflation Forecast Error Densities

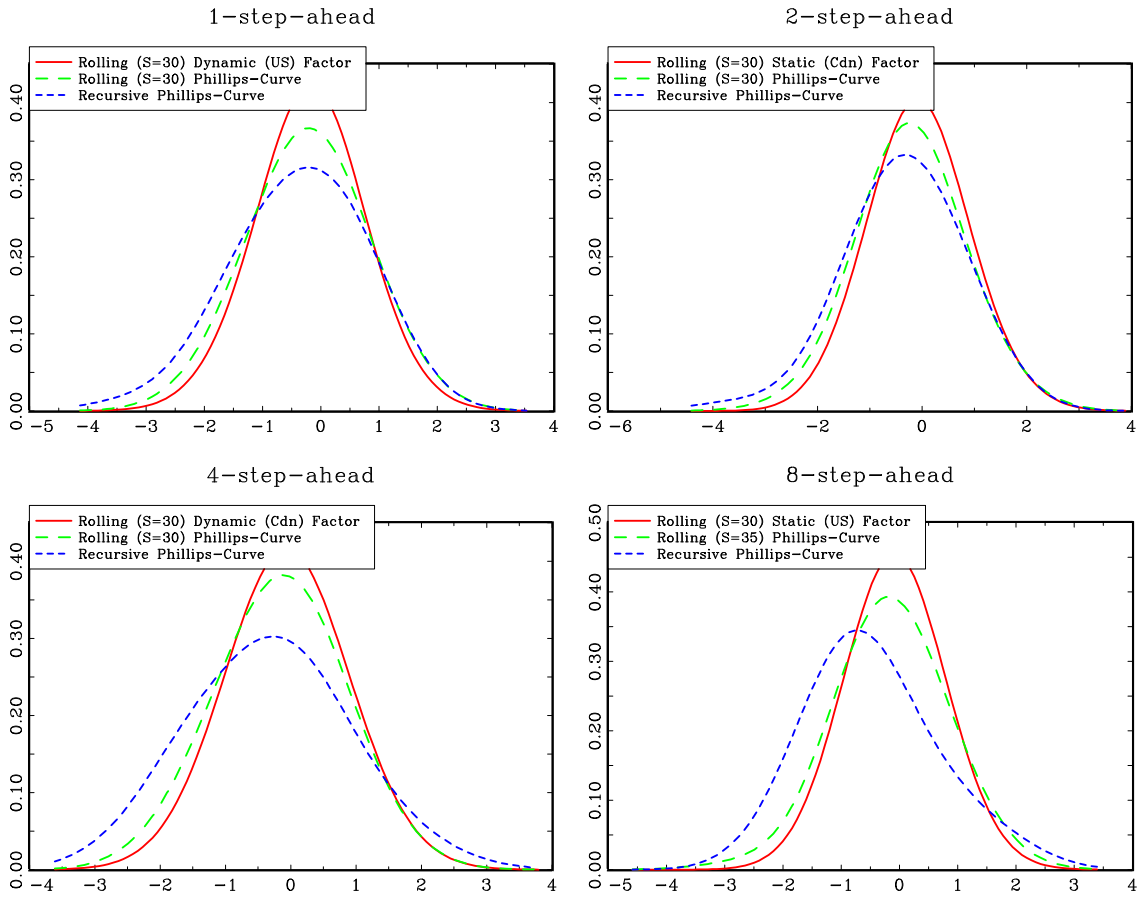


Figure 8: Cumulative GDP growth Forecast Error Densities

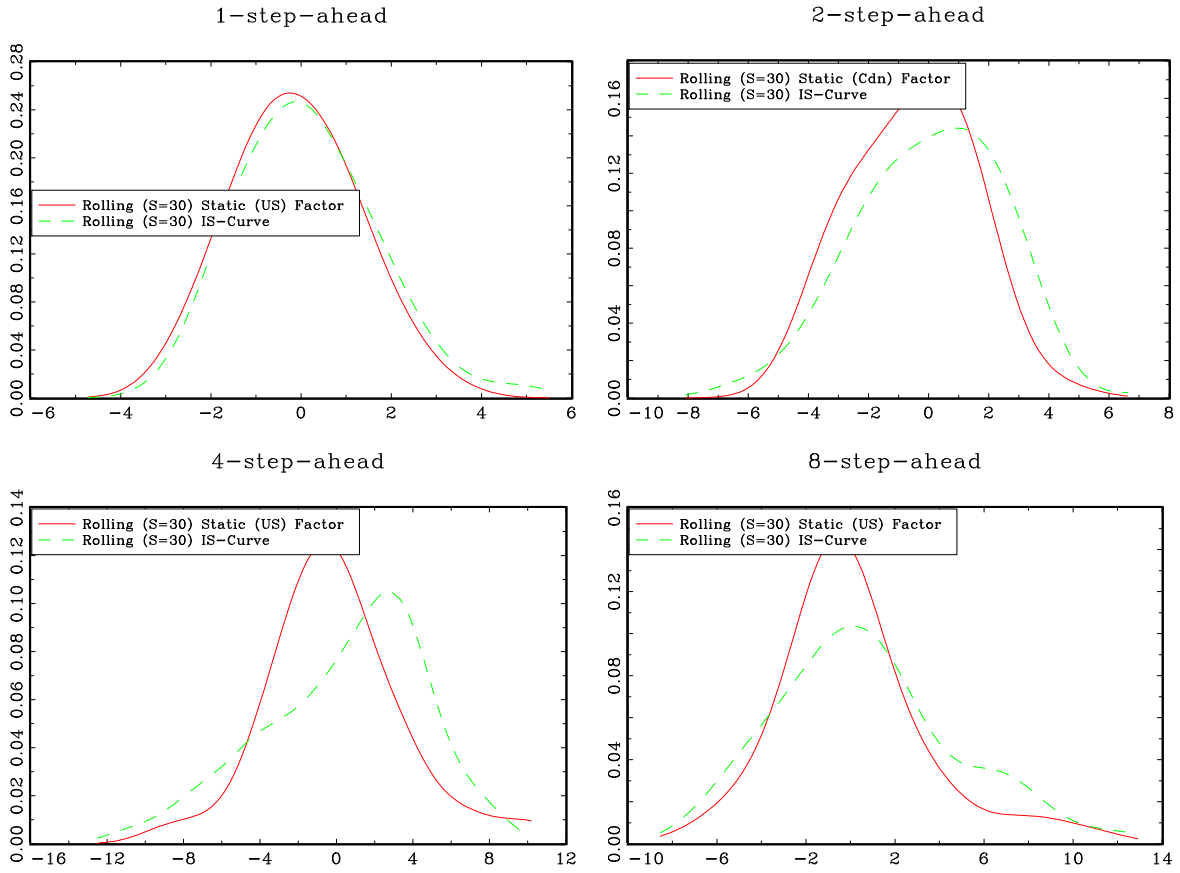


Figure 9: Cumulative Core Inflation Forecast Error Densities

