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# Merger in Contests<sup>α</sup>

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## Abstract

Competition in some markets is a contest. This paper studies the merger incentives in such markets. Merger can be profitable. The profitability depends on the post-merger contest structure, the discriminatory power of the contest and on the number of contestants.

Keywords: contests, merger

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## 1 Introduction

Competition is sometimes well described by a contest, particularly if competition via prices is not feasible. A typical example is the market for pharma-

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ceutical products. Price competition and most other means of competition are not feasible in this market in many European countries, due to the specific payment structure of their health insurance system. However, producers make promotional effort sending sales representatives and gifts to physicians, trying to persuade them to prescribe their products instead of competing substitutes.<sup>1</sup> The effort cost cannot be recovered, even if the promotion is not successful. Similarly, firms are forced into contests in markets for products with price maintenance or price regulation by some government agency. This is well documented, for instance in the insurance industry in several European countries prior to deregulation by the European Commission in 1994.<sup>2</sup> In markets with price maintenance retailers may contest with each other and spend resources in order to attract customers to buy from them, and not from another retailer. The type of effort differs from one market to another: visiting and persuasive talking to customers in the insurance retail business, glamorous shop outlets and huge selections of goods in others.

Other important contest examples are firms competing for a monopoly as in R&D contests (see, e.g., Bagwell and Staiger 1997), contests for quasi-

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<sup>1</sup>Breyer and Zweifel (1999, p. 366) report that marketing and product information were about 20 percent of revenue through sales in pharmacies in Switzerland in the mid-eighties, almost half of these being marketing expenditure and argue that this percentage is much higher than that of other industries.

<sup>2</sup>For instance, Rees and Kessner (1999) survey regulation in the German insurance market prior to 1994. They report some evidence for price regulation that led to prices that considerably exceeded cost, leading to a contest in sales effort that was sufficiently strong to make the regulator feel a need for regulating the maximum sales expenditure. The regulator required that agents' commissions were not to exceed 11 percent of premiums, and total marketing expenditure was restricted to no more than 30 percent of premiums.

monopoly due to network externalities (Besen and Farrell 1994), litigation contests for brand names, internet addresses or other exclusive assets that yield quasi-monopoly rents, exporting firms competing for large scale projects as in Konrad (1999), or firms seeking special political favors in rent-seeking contests. In all these cases firms spend effort to attain some payoff or "prize." Firms win this prize with some probability that is a function of the various efforts of the different firms, and the effort cannot be recovered, whether a firm wins the prize or not.

One may ask whether firms gain in these contests if some of them "join forces," whereas others stay independent. For instance, two pharmaceutical firms in a market with  $n$  firms may merge, or firms in an R&D contest may merge. They may decide to fuse their research labs, or may coordinate their research expenditure in separate research labs. This type of merger question has received considerable attention for two types of market structures, Bertrand and Cournot competition. In these competition games, merger is always beneficial for the firms not involved in the merger. Merging firms may gain or lose. With Bertrand competition, firms' price choices are strategic complements and, accordingly, merger of a subgroup of firms is beneficial for them (Deneckere and Davidson 1985). The results are ambiguous for Cournot competition, merger being more likely to benefit the merging firms if the number of merging firms is large relative to the number of firms not involved in the merger (Salant, Switzer and Reynolds 1983, and Gaudet and Salant 1991) and if costs are convex (Perry and Porter 1985).

A major difference between Bertrand and Cournot games on one side and contest games on the other is the change in the competition game that is

induced by the merger. With a merger in Bertrand and Cournot competition (or perfect collusion with side payments between a subgroup of competitors), the merged firms can be considered as one single strategic player deciding about a single strategic variable. Merger therefore reduces the number of competitors in the game. The reduction in competitors typically increases overall industry profits, but reduces the share in these profits which is earned by the merged firms.<sup>3</sup>

With contests, it is less clear whether merger or cooperation between two contestants will reduce the overall number of contestants. Suppose  $n$  firms are in a contest. Each firm spends contest effort and the prize of the contest is awarded to the different firms with probabilities that depend on these efforts. If two firms merge, in some cases they may become one single contestant. This may be true, for instance, in rent-seeking or lobbying contests, and in situations in which firms' products become indistinguishable after the merger. Alternatively, two firms may continue to act as two contest participants in the game, even after the merger. For instance, in an R&D contest for a drastic innovation, the merging firms may still run two research labs, or firms may still run two separate product lines and separate organizations of sales representatives promoting the two products. The merger will make them choose the optimal contest efforts differently in these cases, since they know that they partially compete against themselves.

Accordingly, we will pursue the problem in two separate sections, referring

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<sup>3</sup>As is typically assumed in the merger literature, we disregard any direct cost advantages or disadvantages in the merger, in order to concentrate on the strategic market structure aspects.

to fusion if two merged firms become a single contestant, and to collusion if the two merged firms continue to represent two contestants after the merger. We show that merger between a given number of contestants is, in general, more profitable for the involved firms, if the contest is “discriminatory” in the sense that, at the equilibrium contest effort levels, a contestant’s increase in contest effort has a large impact on the contestant’s win probability.

## 2 Contests

Consider a market with  $n$  symmetric firms. Suppose that these firms make efforts in a contest for some prize of size  $B$ . A few examples for this type of competition have been discussed in the introduction. Each firm  $i$  chooses contest effort  $x_i \in [0; 1)$  which is sunk and cannot be recovered, whether the firm wins the contest or not. Contest efforts determine firms’ probabilities  $q_i$  of winning the prize, according to a contest success function

$$q_i(x_1; \dots; x_n) = \frac{(x_i)^a}{\sum_{j=1}^n (x_j)^a} \text{ for } a < n = (n-1). \quad (1)$$

This function has been suggested by Tullock (1980) and is a special case of more general contest success functions  $q_i(x_1; \dots; x_n)$  but has gained support by an axiomatization in Skaperdas (1996). We call the coefficient  $a$  in (1) the discriminatory power of the contest success function. It is a measure of how much the contest outcome can be influenced by contest effort, and how much is left to chance. For instance, if  $a = 0$ , each contestant has the same chance of winning, irrespective of contest efforts. If, instead,  $a = 1$ , (1) approaches a contest success function in which the contestant who makes the highest effort wins for sure. We limit the discriminatory power to  $a \in [0; \frac{n}{n-1})$  in

order to have well-behaved optimization problems with equilibria in pure strategies and first-order conditions characterizing these equilibria.<sup>4</sup>

Firms are risk neutral. Their (expected) payoffs are

$$\pi_i = q_i B - x_i. \quad (2)$$

Firm  $i$  wins the prize  $B$  with probability  $q_i$  and spends contest effort equal to  $x_i$ . The first-order condition for firms maximizing their payoffs and symmetry can be used to calculate the contest equilibrium efforts

$$x^*(n) = \frac{aB(n-1)}{n^2}. \quad (3)$$

The equilibrium win probability is  $1/n$  for each contestant, yielding the equilibrium payoffs as

$$\pi^*(n) = \frac{B}{n} - \frac{aB(n-1)}{n^2}. \quad (4)$$

### 3 Fusion

We first consider the case in which the merger reduces the number of contestants in the contest. This case corresponds to the assumption in the literature on merger in Bertrand or Cournot markets that two firms that merge behave as one strategic player deciding about one strategic variable in the post-merger market. The alternative case in which merged firms will enter the contest success function as separate contestants will be considered later.

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<sup>4</sup>For the equilibrium (in mixed strategies) for the case of  $a > n(n-1)$  see Baye, Kovenock and deVries (1994).

Suppose  $m < n$  firms merge, implying that the number of contestants reduces from  $n$  to  $n - m + 1$ . Let  $N$  be the set of all firms and  $M$  be the set of firms that merge. Prior to the merger, the merging firms received a payoff equal to  $m\frac{B}{n}(n - m + 1)$ . After the merger their payoff equals

$$\frac{B}{n - m + 1} + \frac{aB(n - m)}{(n - m + 1)^2}.$$

Now let  $g(n; m; a)$  be the function that measures the gain (or loss) of  $m$  firms that merge in an industry composed of  $n$  firms. Accordingly,  $g(n; m; a)$  is given by

$$\begin{aligned} g(n; m; a) &= \frac{B}{n - m + 1} + \frac{aB(n - m)}{(n - m + 1)^2} - m\frac{B}{n} - \frac{aB(n - m)}{n^2} \\ &= \frac{B}{n - m + 1} + \frac{aB(n - m)}{(n - m + 1)^2} - m\frac{B}{n} - \frac{aB(n - m)}{n^2}, \end{aligned}$$

and has the following properties:

(i) For all  $n \geq 2$  it holds that  $g(n; 1; a) = 0$ : (If one firm is joined by no other in a merger, the profit doesn't change.)

(ii) For all  $n \geq 2$  and for all  $a > 0$  it holds that  $g(n; n; a) = \frac{B}{n}a(n - 1) > 0$ : (Merger to monopoly is always profitable.)

(iii) For all  $n \geq 2$  it holds that  $\frac{\partial g(n; m; a)}{\partial m} \bigg|_{m=1} = \frac{B}{n^3}(2a + n - 2an + an^2 - n^2) \leq 0$  if  $a \leq \frac{n(n-1)}{(n-1)^2+1} < \frac{n}{n-1}$  for  $n \geq 2$ :

(iv) For all  $n \geq 4$  and for all  $a \geq 0; \frac{n}{n-1}$  it holds that

$$\frac{\partial^2 g(n; m; a)}{\partial m^2} = 2B \frac{n - m + 1 - a(n - m - 2)}{(n - m + 1)^4} > 0;$$

i.e.,  $g(n; m; a)$  is strictly convex (and also continuous) with regard to  $m$ :



With the help of properties (i) ; (iv) we can prove the following

**Proposition 1** (A) If there are 3 firms in the premerger equilibrium, then a bilateral merger is profitable if and only if  $a \geq \frac{6}{7}, \frac{3}{2}$  :

(B) For any number  $n$  of firms, there is a critical discriminatory power  $a_0(n)$  such that merger with  $m \leq n-1$  is never profitable for all contests with  $a \leq a_0(n)$ .

(C) Let  $a \geq 0, \frac{n(n_i-1)}{(n_i-1)^2+1}$  and  $n \geq 4$ : Then the following two statements hold true: If a merger by a specified number of firms is not profitable for the merging firms, a merger by a smaller number of firms is also not profitable. If a merger by a specified number of firms is profitable for them, a merger by a larger number of firms is also profitable.

(D) If  $a \geq \frac{n(n_i-1)}{(n_i-1)^2+1}, \frac{n}{n_i-1}$  then for any number  $n \geq 4$  of firms in the premerger equilibrium, a merger by any number  $m = 2; 3; \dots; n$  of firms is profitable.

**Proof:** For part (A) note that  $g(3; 2; a) = \frac{B}{36} (7a - 6)$ . For part (B) note that  $\lim_{a \rightarrow 0} g(n; m; a) = -\frac{(n_i-m)(m_i-1)}{n(n_i-m+1)}B < 0$ . The proof of part (C) follows the lines of proof of result D in Salant, Switzer, and Reynolds (1983): properties (i) and (iii) imply that  $g(n; m; a)$  becomes negative for small  $m > 1$  if  $a < \frac{n(n_i-1)}{(n_i-1)^2+1}$ : According to property (iv),  $g(n; m; a)$  is continuous and strictly convex with regard to  $m$ . Thus, because of property (ii), there is a unique  $y^n < n$  such that  $g(n; y^n; a) = 0$  and the result follows. Finally, for the proof of (D), note that in this case properties (i); (ii); (iii) and (iv) imply that  $g(n; m; a) > 0$  for all  $m = 2; 3; \dots; n$ : □

Proposition 1 shows that the profitability of the type of merger considered here depends on the discriminatory power of the contest. If the discriminatory power is small, merger of many firms can be profitable whereas merger of few firms is not. If the discriminatory power of the contest is very small, fusion is profitable only if all firms merge. If, in contrast, the discriminatory power is high, merger—of any number of firms—is always profitable.

Intuitively, fusion in contests has two effects. First, it increases total profit of the industry, because total contest effort is reduced with a reduction in the number of contestants. Second, the share of this profit that goes to the merging group of firms is reduced. If the discriminatory power is very small, e.g., close to zero, then total effort in the equilibrium becomes negligible in comparison to the contest prize, and industry profit becomes almost equal to  $B$  and increases only infinitesimally as a result of merger. In that case  $m$  merger candidates earn almost  $mB/n$  prior to merger, while, if they merge, their joint payoff decreases to roughly  $B/(n - m + 1)$ : Hence, the second effect dominates and merger is not profitable. On the other hand, with large discriminatory power, the effect of merger on total industry profit becomes more important. The first effect dominates the second and merger becomes profitable.

## 4 Collusion

In many contests a merger between firms does not reduce the number of contestants. For instance, consider R&D contests. Let  $N$  be the set of all firms, with  $\#N = n$ , let  $M$  be the set of firms that merge, with  $\#M = m$ ,

and let  $U = N \setminus M$  be the set of unmerged firms. The merged firms may keep separate research labs working on the same research problem. Similarly, takeovers or mergers in insurance markets often do not imply that these companies amalgamate their sales departments. Invariance of the number of contest players with respect to merger is likely to occur if the merging firms stay as separate entities with a joint headquarter, or if merger is simply a contract between the firms according to which the firms make all strategic choices cooperatively and maximize their joint profit. When asking whether firms can gain from merger in this case, we can straightforwardly draw on the literature on the profitability of merger with other types of competition. Two types of effects are at work. First, the merging firms take into account that an increase in contest effort in, say, firm  $i \in M$  reduces the win probability of all other firms, including the win probability of all other firms in  $M$ : This latter effect will be internalized, making firm  $i$  behave less aggressively in the contest, and so will all merging firms. Second, the less aggressive behavior of merged firms changes the contest behavior of all other firms.

If a merger does not change the number  $n$  of contestants, it nevertheless changes the objective functions of the merged firms. Instead of (2) they maximize their joint payoff

$$\sum_{i \in M} (q_i B_i - x_i), \quad (5)$$

whereas all firms  $k \in U$  not participating in the merger continue to maximize their profits as given in (2). The respective system of first-order conditions, which by using symmetry reduces to a system of two equations in two unknowns, is not explicitly solvable. However, a comparative static analysis is possible. If the firms in  $M$  maximize (5), they take into account that

$\frac{\partial q_i}{\partial x_j} < 0$  for  $i \neq j$  holds, and therefore, coordination makes them reduce their effort compared to  $x^a(n)$ : Let us consider the effect of a symmetric marginal reduction in effort choices by the firms in  $M$  on their equilibrium profits. Firms in  $U$  will react to this (anticipated) reduction in effort. Define  $\bar{x}$  as the symmetric equilibrium level of effort chosen by all  $k \in U$  such that

$$\bar{x} = \bar{x}(x) \leftarrow \arg \max_{x_k \geq 0} f_{q_k} B_{-i} \mid x_k \geq x_i = x \ \forall i \in M \text{ and } x_j = \bar{x} \ \forall j \in U \setminus k.$$

We call  $\bar{x}(x)$  the symmetric reaction function of the unmerged firms (firms in  $U$ ) for effort choices of the merged firms (firms in  $M$ ). This reaction function is implicitly given by the first-order condition for a firm in  $U$  and can be written, making use of symmetry, as

$$a\bar{x}^{a-1}((n-i-m-i-1)\bar{x}^a + m\bar{x}^a)B = ((n-i-m)\bar{x}^a + m\bar{x}^a):$$

At the fully non-cooperative equilibrium  $x^a(n)$ , the slope of the function  $\bar{x}(x)$  is obtained by total differentiation of this condition and equals

$$\frac{d\bar{x}(x)}{dx} \Big|_{x=x^a} = -i \frac{am(n-i-2)}{(n-i-am)(n-i-1) + am}. \quad (6)$$

**Remark 1** The slope of the reaction function  $\bar{x}(x)$  as in (6) is strictly negative for all  $n \geq 3$  and  $m \leq n-i-1$ . Furthermore,  $\lim_{a \downarrow 0} \bar{x}^{10}(x^a) = 0$ .

To see this, note that  $a \leq \frac{n}{n_i-1}$ . The intuition for the result in Remark 1 is as follows. As can be seen by inspecting  $\frac{d}{dx_i} \frac{1}{4}_i = 0$  for  $a \rightarrow 0$ , each contestant's marginal benefit from spending additional effort becomes infinitely small. Hence, a contestant would not like to spend much, even if other contestants would increase their effort considerably.

Using the envelope theorem and the fact that  $\frac{\partial \pi_i}{\partial x_k} = \frac{1}{n_i - 1}$  for  $i \neq k$  at the fully non-cooperative Nash equilibrium with efforts (3), the profit increase of each firm in the merging group  $M$  from a joint reduction in their contest effort  $x$  starting in  $(x^a; x^a)$  can be calculated and equals

$$i \frac{d\pi_i}{dx} = \frac{1}{n_i - 1} ((m_i - 1) + (n_i - m_i)) \pi_i^1(x^a). \quad (7)$$

Hence, a joint decrease in their effort increases their profit if the direct effect of reduced effort within the group outweighs the equilibrium reaction by the unmerged firms. This expression (7) to be positive is a necessary condition for collusion to be profitable. It resembles the condition that has been derived in Gaudet and Salant (1991) who consider Cournot competition. The profitability effect of collusion is ambiguous in general. However, by Remark 1, we have

**Proposition 2** A marginal joint reduction in effort among colluding firms increases their profit if the discriminatory power of the contest is sufficiently small.

The intuition for the result in Proposition 2 is as follows. Collusion on contest effort leads to a reduction in effort for the set of colluders. If this reduction in effort does not trigger an increase in other contestants' efforts, collusion is beneficial. As has been noted in Remark 1, other contestants' reaction to the colluding set's effort reduction is very moderate if the discriminatory power of the contest is sufficiently small.

Note that the profit change of an unmerged firm  $k \in U$  which results from a joint reduction of the contest effort  $x$  of firms in  $M$  equals

$$i \frac{d\pi_j}{dx} = \frac{1}{n_i - 1} (m + (n_i - m_i - 1)) \pi_i^1(x^a).$$

Thus, a joint reduction of the contest effort of firms in  $M$  is always beneficial for the firms not participating in the merger.

## 5 Conclusions

In this paper we consider the profitability of merger in contests. We consider situations in which the merger leads to a reduction in the number of contest participants (fusion) and situations in which the merger merely leads to coordinated effort choices within a subgroup of contestants, but does not change the number of contestants. This type of merger is called collusion. The profitability effect of merger for the merging group of firms is different for the two types of merger. Whereas high discriminatory power of the contest makes fusion profitable, a necessary condition for collusion to be profitable is more likely to be fulfilled if the discriminatory power of contests is low.

When firms merge, it can be expected that they have a choice as regards whether the merger leads to a fusion or to mere collusion between merged entities. The structure of the contest will therefore be important for this decision, fusions being more likely if the discriminatory power of the contest is sufficiently high.

## References

- [1] Bagwell, Kyle, and Robert W. Staiger, 1997, Strategic export subsidies and reciprocal trade agreements, the natural monopoly case, Japan and the World Economy, 9, 491-510.

- [2] Baye, Michael R., Dan Kovenock, and Casper G. deVries, 1994, The solution to the Tullock rent-seeking game when  $R$  is greater than 2: mixed-strategy equilibria and mean dissipation rates, *Public Choice*, 81, 363-380.
- [3] Besen, Stanley M., and Joseph Farrell, 1994, Choosing how to compete: strategies and tactics in standardization, *Journal of Economic Perspectives*, 8, 117-131.
- [4] Breyer, Friedrich, and Peter Zweifel, 1999, *Gesundheitsökonomie*, Springer, Heidelberg.
- [5] Deneckere, Raymond, and Carl Davidson, 1985, Incentives to form coalitions with Bertrand competition, *Rand Journal of Economics*, 16, 473-486.
- [6] Gaudet, Gerard, and Stephen W. Salant, 1991, Increasing the profits of a subset of firms in oligopoly models with strategic substitutes, *American Economic Review*, 81, 658-665.
- [7] Konrad, Kai A., 1999, Trade contests, *Journal of International Economics* (forthcoming).
- [8] Perry, Monty K., and R.H. Porter, 1985, Oligopoly and the incentive for horizontal merger, *American Economic Review*, 75, 219-227.
- [9] Rees, Ray, and Ekkehard Kessner, 1999, European insurance markets, regulation and efficiency, *Economic Policy*, 29, 363-397.

- [10] Salant, Stephen W., Sheldon Switzer, and Robert J. Reynolds, 1983, Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium, *Quarterly Journal of Economics*, 98, 185-199
- [11] Skaperdas, Stergios, 1996, Contest success functions, *Economic Theory*, 7, 283-290.
- [12] Tullock, Gordon, 1980, Efficient rent seeking, in: J.M. Buchanan, R.D. Tollison and G. Tullock, eds., *Toward a Theory of Rent-seeking Society* (Texas A&M University Press, College Station) 97-112.