

# Department of Economics

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# The Impact of Large Structural Shocks on Economic Relationships: Evidence from Oil Price Shocks

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## Abstract

This paper introduces a new model of structural breaks which assumes that structural breaks are driven by large economic shocks. The model specifies that both the timing and size of breaks are stochastic and it can be used to investigate the impact of large economic shocks on the stability of economic relationships. An application of the model to the oil-macroeconomy relationship has shown that the apparent instability of this relationship since the oil crisis in year 1973 can be attributed to large oil price shocks.

*Keywords: Structural Breaks, State Space Model, Oil Shocks*  
*JEL Codes: C13, C22, E32*

## 1 Introduction

The study of the impact of large shocks (referred to as outliers) on macroeconomic relationships has attracted considerable interest in the economic literature over the past three decades. Relatively infrequent events such as oil shocks, changes in policy regimes, turning points of business cycles and natural disasters, have been found to be associated with pronounced shifts in the structural parameters of macroeconomic relationships, known as structural breaks (see , Balke and Fomby (1993) for a survey). These shocks are assumed to be orthogonal to the explanatory variables of structural models and need not have the same dynamic effect as regular shocks. Their occurrence is considered as one of the (if not the) most significant cause for the forecasting failure of macroeconomic models, as documented by the work of Hendry among others (see, e.g., Hendry (1997)).

Motivated by the intervention analysis of Box and Tiao (1975), most of the modelling approaches of structural breaks are concentrated on detecting the presence and location of

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potential breaks in the data, as well as on estimating the structural parameters changes using intervention dummies. However, this approach assumes that the breaks are deterministic in nature and exogenous which may lack economic intuition, as breaks may reflect stochastic changes in agents' economic decisions or beliefs triggered by extraordinary events or policy changes (see Sargent (1999)). In intervention analysis the occurrence of breaks is taken as given and not viewed as part of model specification. To overcome this limitation, recent studies have suggested stochastic models of structural breaks which allow the timing of the breaks to be stochastic (see Hamilton (1989), and Lin and Terasvirta (1994), *inter alia*). As in intervention analysis, these models however assume that the magnitude of the structural parameters changes is fixed. Due to their different orientation, they do not enable us to study the effect of large structural shocks on the stability of economic relationships, which this paper is focused on.

In this paper, we suggest a new parametric model of structural breaks which allows them to be endogenously determined by large structural economic shocks. The model considers infrequent changes (shifts) in both the timing and magnitude of structural parameters which are stochastic in nature and orthogonal to the explanatory variables of the model. These changes are originated by large economic shocks which are identified by larger in size than a threshold parameter structural errors, endogenously determined by the data. When occurring, they can cause abrupt shifts in structural parameters which can resemble those picked up by the intervention analysis or any other model of structural breaks mentioned above. By allowing structural breaks to be endogenously determined by economic shocks, our model consider the breaks as part of model specification, and thus may be given an economic interpretation.

The paper applies our model to examine if the apparent instability of the oil-macroeconomy relationship documented in many studies (see Mork (1989), Hooker (1996), Hooker (1999), Hamilton (1996), and Davis and Haltiwanger (2001), *inter alia*) can be attributed to large oil-prices shocks. An answer to this question has important economic implications. If this hypothesis is true, then oil price-shocks can be thought of as endogenously determining the economy. Thus, they can not be considered as exogenously causing economic recessions. Our results support this view. They show that oil price-shocks can endogenously explain the structural instability of the oil-macroeconomy relationship which apparently becomes less important over time.

The paper is organised as follows. Section 2 presents our model and discusses its prop-

erties in comparison with other models of structural breaks. In this section, we also present an estimation procedure of our model based on the Kalman filter and conduct a small Monte Carlo to study its performance to adequately trace structural breaks in the data. Section 3 conducts the empirical application of the paper. Section 4 concludes the paper.

## 2 Modelling the impact of large structural shocks on economic relationships

### 2.1 Model set up

Consider the simple, one explanatory variable single regression model

$$y_t = \beta_t x_t + \epsilon_{1,t}, \quad \text{with} \quad (1)$$

$$\beta_t = \beta_{t-1} + I(|\epsilon_{1,t-k}| > r)\epsilon_{2,t}, \quad (2)$$

where  $\epsilon_{1,t}$  and  $\epsilon_{2,t}$  are two independent zero mean error terms, and  $I(\mathcal{A}_t)$  is an indicator function taking the value 1 if the event  $\mathcal{A}_t = \{|\epsilon_{1,t-k}| > r\}$  occurs, at time  $t - k$  where  $k > 0$ , and zero otherwise. In model (1)-(2), the structural parameter (slope coefficient)  $\beta_t$  constitutes a state variable which is governed by large in size structural errors (shocks)  $\epsilon_{1,t}$  which are larger in absolute magnitude than the value of threshold parameter  $r$ , i.e.  $|\epsilon_{1,t-k}| > r$ . These errors may reflect outliers in the levels of series which can be attributed to extraordinary unexpected events due, for instance, to monetary regime changes, oil shocks and the turning points of business cycles. Model (1)-(2) enables us to investigate whether these type of events can have a structural impact on economic relationships.

Under standard regression model assumptions, i.e. correct specification of (1), exogeneity (or predeterminedness) of  $x_t$  and orthogonality between  $x_t$  and  $\epsilon_{1,t}$ , the changes in  $\beta_t$  modelled through the transition equation (2) can be thought of as endogenous, driven by the large structural errors  $|\epsilon_{1,t-k}| > r$ . In this equation, the indicator function  $I(\mathcal{A}_t)$  plays an important role. It allows for abrupt shifts in  $\beta_t$ , in line with the common perception of structural breaks observed in reality. Both the timing (or frequency) and magnitude of these breaks are entirely stochastic and can be endogenously identified by our data. The timing of shifts depends on the magnitude of the error  $\epsilon_{1,t-k}$  relative to the threshold parameter, and thus their frequency will depend on the number of times that the event  $\mathcal{A}_t$  occurs. For instance, if, within the sample, a large shock occurs once, then model (1) implies only one shift in  $\beta_t$ , with a permanent effect. The magnitude of the structural breaks will depend on the variance of the error of the state variable  $\epsilon_{2,t}$  and the magnitude of the structural error

$\epsilon_{1,t}$ , itself. In the case that  $\epsilon_{2,t}$  has a dynamic structure, e.g., it follows an autoregressive process of lag order one

$$\epsilon_{2,t} = \rho_2 \epsilon_{2,t-1} + \eta_{2,t}, \quad (3)$$

where  $\eta_{2,t}$  is white noise, then model (1)-(2) enables us to forecast the actual size of a structural break in  $\beta_t$ , given the current information set at time  $t - 1$ , denoted as  $\mathcal{I}_{t-1}$ .<sup>1</sup>

Since model (1)-(2) shares some similarities with other existing models of structural breaks in the literature, below we clarify its main differences from the other models of structural breaks, mentioned in the introduction. Model (1)-(2) differs from standard time-varying coefficients models (see Harvey (1989)), as it allows for discontinuous, over time, changes in  $\beta_t$ . The time-varying coefficient models consider successive changes in  $\beta_t$ , every period, which cannot be recognized in the literature as breaks. Clearly our model reduces to such a model if  $r = 0$ . It also differs from the Markov regime switching model (MRS) of Hamilton (1989) and standard threshold models of structural breaks (see Lin and Terasvirta (1994), *inter alia*), as it allows for an unpredetermined number of shifts in  $\beta_t$  which are not fixed in magnitude and are driven by structural shocks. Both the MRS and standard threshold models assume that only changes which are fixed in magnitude occur in  $\beta_t$ , over the sample. According to the MRS model, these are governed by a state variable following a Markov chain process. This variable is orthogonal to the structural errors  $\epsilon_{1,t}$ , and thus it does not enable us to study the effect of structural shocks on  $\beta_t$ . The same is true for the standard threshold model of structural breaks.

As it stands, model (1)-(2) can generate a non-stationary pattern for  $y_t$ , as the variance of  $\beta_t$  grows with the time-interval of the data. If stationarity of  $y_t$  is a desirable property of the data, then stationarity of  $\beta_t$  would be sufficient for stationarity of  $y_t$ . There are a number of possible modifications that can be imposed on  $\beta_t$  in estimation to make it stationary (see Cogley and Sargent (2002)). A straightforward condition is that

$$\beta_t = \beta_{t-1} + I(|\beta_{t-1}^*| < \beta)I(|\epsilon_{1,t-k}| > r)\epsilon_{2,t-1}, \text{ with} \quad (4)$$

$$\beta_{t-1}^* = \beta_{t-1} + I(|\epsilon_{1,t-k}| > r)\epsilon_{2,t-1}. \quad (5)$$

where  $\hat{\beta}_{t-1|t-1}$  and  $\hat{\epsilon}_{1,t-1|t-1}$  denote the expectation of the relevant variables conditional on data available up to  $t - 1$  and  $\beta$  is some constant. This condition implies that  $\beta_t$  is bounded and stationary and, hence, for a strictly exogenous  $x_t$ , it makes the process  $y_t$  stationary.

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<sup>1</sup>Note that we can allow for serial correlation in  $\epsilon_{1,t}$ , as well, enabling, for example, forecasting of the timing of the break. However, this comes at the possible cost of losing the structural interpretation of the shock.

Further restrictions could be placed on the process so that, if the bound  $\beta$  is exceeded, the process returns to some prespecified level. We do not advocate a particular mechanism for making the process  $\beta_t$  stationary. We simply wish to indicate that there exist specifications which give both a stationary  $\beta_t$  process and a conditionally Gaussian state space model amenable to analysis via the Kalman filter, discussed in the next subsection. The exact specification of the process may be left to the empirical researcher depending on his priors on the particular issue at hand.

## 2.2 Estimation of the model

Estimation of model (1)-(2) requires an algorithm of sequentially updating estimates of the state variables  $\beta_t$ ,  $\epsilon_{1,t}$  and  $\epsilon_{2,t}$ . This can be done by using the Kalman filter. To this end, we assume Gaussianity for  $\eta_{1,t}$  and  $\eta_{2,t}$ , and that the structural error  $\epsilon_{1,t-k}$ , entering the transition equation (2) is replaced by  $\hat{\epsilon}_{1,t-k|t-1}$ , which represents the Kalman filter estimate of  $\epsilon_{1,t-k}$  given the information set  $\mathcal{I}_{t-1}$  (see Harvey (1989), *inter alia*). This set of assumptions enables valid use of the Kalman filter for estimation of the states. Note that if one specifies a condition such as (4)-(5) for  $\beta_t$  then, it should take the form

$$\beta_t = \beta_{t-1} + I(|\hat{\beta}_{t-1|t-1}^*| < \beta)I(|\hat{\epsilon}_{1,t-k|t-1}| > r)\epsilon_{2,t-1}, \text{ with} \quad (6)$$

$$\hat{\beta}_{t-1|t-1}^* = \hat{\beta}_{t-1|t-1} + I(|\hat{\epsilon}_{1,t-k|t-1}| > r)\hat{\epsilon}_{2,t-1|t-1}. \quad (7)$$

to comply with the above assumption. To carry out the estimation, first, we assume that the threshold parameter  $r$  is known. Under the above assumptions, we can write model (1)-(2) in a general state space model as

$$y_t = X_t' b_t, \quad t = 1, \dots, T \quad (8)$$

$$b_t = A_t b_{t-1} + \eta_t \quad \eta_t \sim i.i.d.N(0, \Sigma_{\eta,t}) \quad (9)$$

where  $X_t = (x_t, 1, 0)'$ ,  $b_t = (\beta_t, \epsilon_{1,t}, \epsilon_{2,t})'$ ,  $\eta_t = (0, \epsilon_{1,t}, \eta_{2,t})'$  and

$$A_t = \begin{pmatrix} 1 & I(|\hat{\epsilon}_{1,t-k|t-1}| > r) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_2 \end{pmatrix}.$$

This representation of the model corresponds to the case that the state variable  $\epsilon_{2,t}$  is given by the autoregressive process (3). Higher order autoregressive, or moving average, processes can be also considered. Note that if  $\epsilon_{2,t}$  is white noise, then  $\rho_2 = 0$  in  $A_t$ .

Below, we abstract from issues arising from the estimation of the parameters of the model and concentrate on the estimation of the state vector  $b_t$  conditional on the parameters being

known. Let us denote the estimator of  $b_t$  conditional on the information set  $\mathcal{I}_{t-1}$  as  $\hat{b}_{t|t-1}$  and that conditional on the information set up to and including time  $t$  as by  $\hat{b}_t$ . Denote the covariance matrices of the estimators  $\hat{b}_{t|t-1}$  and  $\hat{b}_t$  as  $P_{t|t-1}$  and  $P_t$ , respectively. Then, estimation of  $\hat{b}_t$  by the Kalman filter comprises sequential application of the following two sets of equations:

$$\begin{aligned}\hat{b}_{t|t-1} &= A_t \hat{b}_{t-1} \\ \hat{P}_{t|t-1} &= A_t \hat{P}_{t-1} A_t' + \Sigma_{\eta,t},\end{aligned}\tag{10}$$

known as the prediction equations, and

$$\begin{aligned}\hat{b}_t &= \hat{b}_{t|t-1} + \hat{P}_{t|t-1} x_t \left( \frac{y_t - X_t' \hat{b}_{t|t-1}}{f_t} \right) \\ P_t &= P_{t|t-1} - P_{t|t-1} X_t \left( \frac{1}{f_t} \right) X_t' P_{t|t-1},\end{aligned}\tag{11}$$

known as the updating equations, where  $f_t$  is given by  $X_t' P_{t|t-1} X_t + \sigma_t^2$  (see, Hamilton (1994), *inter alia*). For a given value of  $r$ , the log-likelihood function for the observation equation (8), denoted as  $\mathcal{L}(r)$ , can be written in terms of the prediction errors  $z_t = y_t - X_t' \hat{b}_{t|t-1}$  as

$$\mathcal{L}(r) = -\frac{T}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log f_t - \frac{1}{2} \hat{\epsilon}_{1,t}^2.\tag{12}$$

This loglikelihood function can be used to estimate recursively the unknown parameters of the model and to obtain the following sets of estimates of the state variables including  $b_t$ : (i) the conditional on the information set  $\mathcal{I}_{t-1}$ , referred to as forecasts, (ii) the conditional on  $\mathcal{I}_{t-1}$  plus the current observations of  $y_t$  and  $X_t$ , known as filter estimates, and (iii) the conditional on the information of the whole sample, denoted as  $\mathcal{I}_T$ , known as smoothed estimates. The first set of estimates can be used to predict future structural breaks in  $\beta_t$  one period ahead using information up to the period  $t-1$ . This can be done based on the prediction and updating sets of equations (10) and (11).<sup>2</sup> The filter estimates of  $b_t$  can reveal agents' perceptions about the current state of  $\beta_t$  in the economy, at time  $t$ . Finally, the set of

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<sup>2</sup>Note that if the structural error term follows a stochastic process, eg an AR(1) model

$$\epsilon_{1,t} = \rho_1 \epsilon_{1,t-1} + \eta_{1,t},$$

then the conditional on  $\mathcal{I}_{t-1}$  Kalman filter estimates estimates of  $b_t$  can provide forecasts of future changes in the structural coefficient  $\beta_t$  many periods ahead. This can be done by writing matrix  $A_t$  as  $A_t = \begin{pmatrix} 1 & I(|\hat{\epsilon}_{1,t-k|t-1}| > r) & 0 \\ 0 & \rho_1 & 0 \\ 0 & 0 & \rho_2 \end{pmatrix}$ . However, in this case it must be pointed out that the structural shifts in  $\beta_t$  will be no longer generated by structural errors, but from the omission of dynamic terms in model (1)-(2). To avoid such a case (and thus to assume  $\rho_1 = 0$ ), we can add dynamic terms of the regressor and regresand in the structural equation (1), so that to be correctly specified and  $\epsilon_{1,t}$  to capture the effect of large structural shocks on  $\beta_t$ .

smoothed estimates of  $b_t$  can be used to statistically appraise the impact of large structural shocks on  $\beta_t$  using information over the whole sample.

The above estimation procedure assumes that  $r$  is known. However, this may not be true in practice. In addition, from an economic analysis point of view it will be useful to estimate the threshold parameter  $r$  endogenously from the data based on our model. This allow us to evaluate the magnitude of a structural shock which can cause shifts in the slope coefficients of economic relationships. As in other threshold models, to estimate  $r$  we will adopt a grid search procedure over a range of possible values of  $r$ . According to this, the loglikelihood function  $\mathcal{L}(r)$  will be maximized for every point of the grid and the point which gives the maximum likelihood, over the grid, will be considered as an optimum estimate  $r$ . The estimates of the unknown parameters of the model and the state vector  $b_t$  corresponding to this estimate of  $r$  will constitute the maximum likelihood estimates of the state space model (8)-(9). These estimates will be consistent provided that the threshold parameter will be consistently estimated. The last result is shown in the Appendix.

### 2.3 Monte Carlo evaluation

In this subsection, we carry out a small scale Monte Carlo study to investigate the performance of our model to track structural breaks adequately, especially in samples where either the number of observations or the number of breaks is relatively small. The presence of the threshold and the fact that breaks occur infrequently raises the question of how well these breaks can be picked up by the Kalman filter. It is reasonable to expect that the state variable driving the breaks is hard to carry inference on given that there are only a few observations which will contain information about the breaks.

As the main aim of our Monte Carlo exercise is to investigate the performance of the Kalman filter, we abstract from parameter estimation and concentrate on the estimation of the state variables assuming that the parameters of the model are known. We do this for three reasons. Firstly, we know that threshold estimation is difficult even for simple threshold models (see e.g. Kapetanios (2000)). In particular, threshold estimation is slow to improve when extra observations are added to the sample, despite the superconsistency result of Chan (1993). Secondly, if the Kalman filter is shown to perform well we can reasonably expect that the performance of the threshold estimator will be similar to the case of standard threshold models. Finally, by the nature of the model, the choice of the threshold has to be restricted to extreme values of the threshold variable, as our model practically dictates the choice of the threshold value.



In our experiments, we generate data according to model (1)-(2) where  $x_t \sim IIDN(0, 1)$ ,  $\rho_2 = 0.5$ ,  $\sigma_{\epsilon_1}^2 = 4/3$  and  $\sigma_{\eta_2}^2 = 0.01$ , and  $k = 1$ . For the threshold parameter, we consider two cases:  $r = 2.5$  and  $r = 2.93$ . In the first case, our model implies that the event  $\{|\epsilon_{1,t-1}| > r\}$  occurs quite frequently, approximately every 33 periods, while in the second occurs more rarely, approximately every 92 periods. For the first case, we consider large and small samples of  $T = 1000$  and  $T = 200$  observations, respectively, while for the second we consider only large samples. The reason that we do not consider small samples for the second case is that the number of breaks is just too small to be picked up in small samples, as  $T = 200$ . This implies that we conduct in total three set of experiments. These can indicate the size of sample and the number of breaks per sample for which the performance can be considered as satisfactory. In each experiment, we run 500 replications and we report the average correlation coefficient between the true  $\beta_t$  and the smoothed and filtered estimates of  $\beta_t$ , (see Table 1). To better see how closely our model can track the changes in  $\beta_t$  over the sample, we also report pictorial results for particular replications with the confidence intervals of the smoothed estimates of  $\beta_t$ , at 95%. These replications correspond to the 25%, 50% and 75% quantiles of the empirical distribution of the correlations between the estimated and generated values of  $\beta_t$ . These results are presented in Figures 1 to 3.

<b>Table 1:</b> Monte Carlo results			
$(r, T)$	(2.5, 200)	(2.5, 1000)	(2.93, 1000)
$nb$	6	30	11
Corr. Coeff. (Filter)	0.18	0.64	0.41
Corr. Coeff. (Smoothed)	0.39	0.80	0.67
Notes: $nb$ denotes the number of breaks per $T$			

As expected, the results of the table and the figures clearly show that our model can satisfactorily capture both the timing and the magnitude of the true breaks, when there is both an adequate number of breaks per sample and the size of sample is large enough. In this case, the correlation coefficients between the smoothed estimates of the changes in  $\beta_t$  and their true values is the highest one. Note that, even if the occurrence of breaks is more rare, eg.  $nb = 11$  per  $T$ , for a reasonably large sample, eg.  $T = 1000$  our model can adequately capture the true breaks. In contrast to the smoothed, the results of the Table show that the filter estimates of  $\beta_t$  do not seem to track the true breaks very satisfactorily. For all sets of experiments, the correlation coefficient between the filtered estimates and the true values of  $\beta_t$  is smaller than that for the smoothed estimates. This should be expected because, in contrast to the filter, the smoothed estimates of  $\beta_t$  rely on the whole sample information to retrieve the breaks.

Moving on to the pictorial output we see that the method works very well for the case  $T = 1000, r = 2.5$ . The Kalman filter estimates track quite well the true break process, even for the replication corresponding to the lower 25% performance quantile. The estimated confidence interval usually contains the true process. Result are less good for the case  $T = 1000, r = 2.93$  as expected. Still, even for this experiment the estimated confidence interval usually contains the the true process. Moving on to the case where  $T = 200$  we see that the sample contains few breaks. Even in this case, the Kalman filter estimate can pick up the movement of the break process reasonably well.

Figure 1:  $T = 1000, r = 2.5$

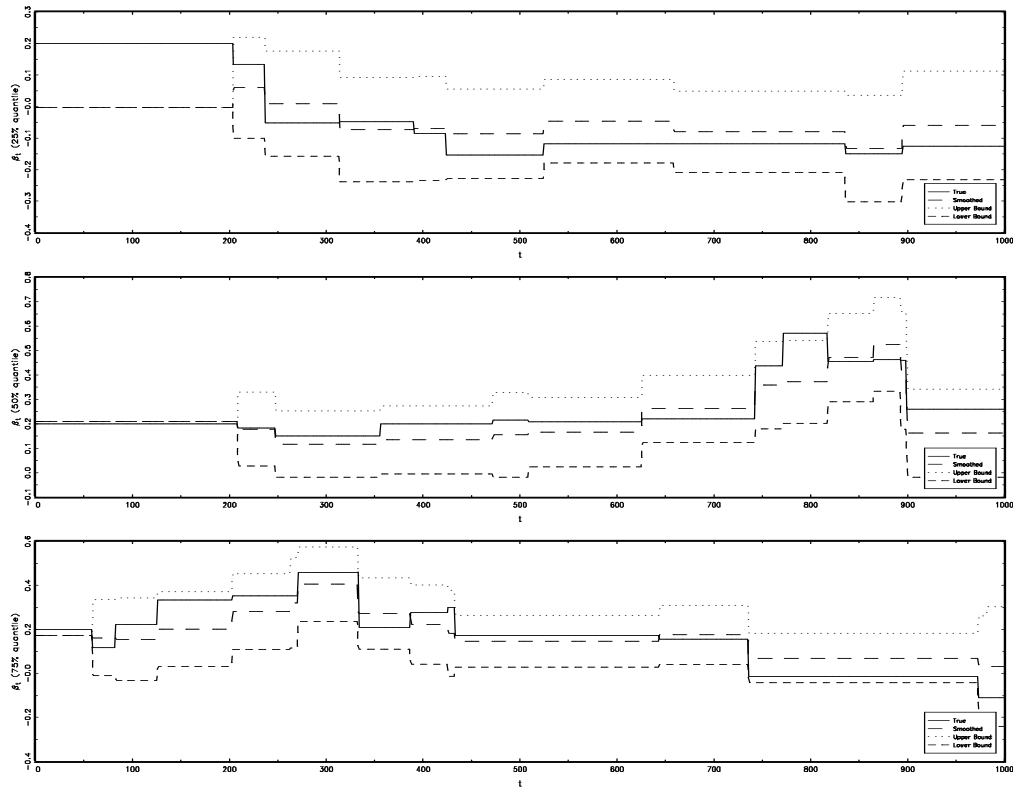
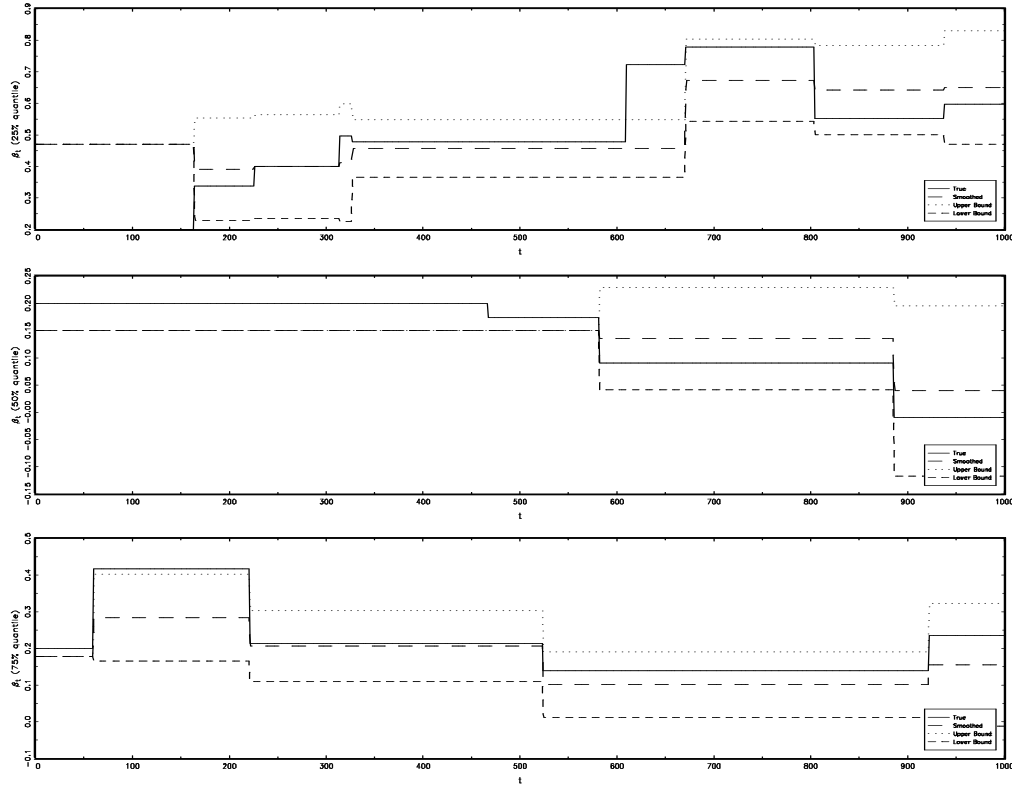


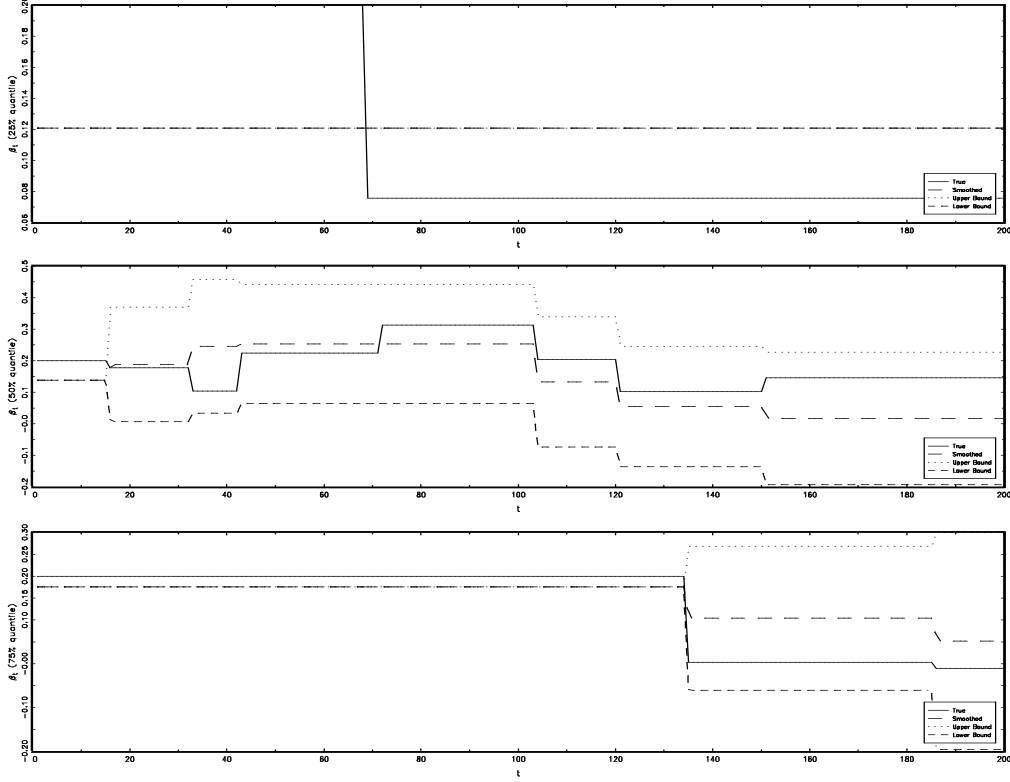
Figure 2:  $T = 1000$ ,  $r = 2.93$



### 3 The impact of oil price-shocks on the oil-macroeconomy relationship

The study of the impact of large oil price-shocks on economic activity has attracted considerable interest in the economic literature over the last two decades. The suggestion that oil price-shocks contribute directly to economic activity downturns is controversial, because of the instability or non-linearity of the oil-macroeconomic relationship (see, Hooker (1996), Hooker (1999), Lee and Ni (2002), and Hamilton (2003), *inter alia*). Oil price-shocks tend to have asymmetric effects on economic activity which seem to become less important, over recent years. Several authors have supported the view that these effects must no longer be treated as exogenous, but may reflect changes in consumers' or firms' economic beliefs and behaviour on what an oil-price shock means for the future. In particular, Lee, Ni, and Ratti (1995) argue that the effect of an oil-price shock on the economy depends on how surprising the shock is relative to the recent changes in the oil-macroeconomic relationship. Hamilton

Figure 3:  $T = 200$ ,  $r = 2.5$



(1996) claims that the key question is whether an oil-price increase is big enough compared with the decreases in the preceding periods in order to affect the economy. All these suggestions have a certain plausibility. To investigate them formally, in this section we consider the following multivariate extension of model (1)-(2)

$$\Delta y_t = a_1 + \beta_t P_{t-1} + \sum_{i=1}^p \gamma_i \Delta y_{t-i} + v_t \quad (13)$$

$$P_t = a_2 + \sum_{i=1}^q \delta_i P_{t-i} + \epsilon_{1,t} \quad (14)$$

$$\beta_t = \beta_{t-1} + I(|\hat{\epsilon}_{1,t-k|t-1}| > r) \epsilon_{2,t-1} \quad (15)$$

$$\epsilon_{2,t} = \rho_2 \epsilon_{2,t-1} + \eta_{2,t}, \quad (16)$$

where  $\Delta y_t$  denotes real industrial production growth and  $P_t$  denotes oil prices in US dollars deflated by the domestic (USA) CPI index.

We consider the industrial production as a proxy of economic activity, instead of GDP used in other studies (e.g., Hamilton (1983)), for two main reasons. The first is that industrial production is expected to be more sensitive to oil price-shocks than the GDP, as it can directly reflect production and cost optimal decisions triggered by oil price-shocks. The second reason is a statistical one. Since industrial production is measured every month, it provides a bigger number of observations than the GDP series, measured quarterly. As our Monte Carlo analysis has shown, this will enable us to more accurately identify possible breaks in the structural coefficient  $\beta_t$  originated by large oil price-shocks. Our choice to include the level of real oil price in the LHS of the structural equation (13) is primarily motivated by theoretical reasons, as it is the level rather the change in firms' input prices which is expected to affect production activity (see Carrath, Hooker, and Oswald (1995), Phelps (1994) and Hooker (1996)).

The system of equations (13)-(14) enables us to examine a number of interesting economic questions on the oil-macroeconomy relationship using the same econometric framework. First, we can formally examine if large oil price-shocks, measured by  $I(|\hat{\epsilon}_{1,t-k|t-1}| > r)$ , can cause changes in the structural coefficient  $\beta_t$  of the oil-macroeconomic relationship (13), as suggested in the literature. Note that, if large oil price-shocks do not cause any change in  $\beta_t$ , then our model predicts that  $\beta_t = \beta_0$ , for all  $t$ , and  $\sigma_{\eta_2} = 0$ . Second, we can endogenously estimate from the data the magnitude of the threshold parameter  $r$  above which large oil price-shocks can cause structural changes in  $\beta_t$ . Third, estimation of the state variable  $\beta_t$ , together with its confidence intervals, will enable us to assess how persistent the changes in  $\beta_t$ , caused by the oil price shocks, can be over the sample.

Before estimating system (13)-(14), in Table 2 we present estimates of a popular nonlinear specification of the oil-macroeconomy relationship advocated by Mork (1989), given by

$$\Delta y_t = a_1 + \beta P_{t-1} + \sum_{i=1}^k \theta_i \Delta P_{t-i} I_{\{\Delta P_{t-i} > 0\}} + \sum_{i=1}^p \gamma_i \Delta y_{t-i} + v_t, \quad (17)$$

where the variable  $\Delta P_{t-i} I_{\{\Delta P_{t-i} > 0\}}$  captures nonlinear and asymmetric effects of positive oil price-shocks, indexed by  $I_{\{\Delta P_i > 0\}}$ , on the oil-macroeconomy relationship. Our sample consists of monthly observations and covers the period 1957:02-2002:12. The lag orders of the regressors  $\Delta P_{t-i} I_{\{\Delta P_{t-i} > 0\}}$  and  $\Delta y_{t-i}$  in (17) are chosen based on the maximization of the

Bayesian information criterion. To see if the oil-macroeconomic relationship has changed after the middle of eighties, as claimed in the literature (see Hooker (1996) and Hamilton (1996)), the table presents results for the whole sample 1957:02-2002:12 and the two subsamples 1957:02-1985:12 and 1986:01-2002:12, see Panels A, B and C, respectively. For each set of estimates, the table also reports the  $p$ -values of some standard misspecification test statistics for neglected serial correlation, nonlinearity and ARCH effects.

<b>Table 2:</b> Estimates of system (13)-(14)						
$\alpha_1$	$\beta$	$\theta_1$	$\theta_2$	$\gamma_1$	$\gamma_2$	$\gamma_3$
Panel A: Whole sample 1957:02-2002:12						
-0.002 (0.0013)	-0.003 (0.0007)	-0.006 (0.0018)	-0.007 (0.002)	1.40 (0.05)	-0.32 (0.10)	-0.09 (0.006)
$p$ -values: $p_{LM(4)}=0.068$ , $p_{NL(4)}=0.00$ , $p_{ARCH(4)}=0.00$						
Panel B: Subsample 1957:01-1985:12						
0.001 (0.002)	-0.0042 (0.003)	-0.003 (0.003)	-0.005 (0.002)	1.41 (0.06)	-0.35 (0.11)	-0.07 (0.07)
$p$ -values: $p_{LM(4)}=0.051$ , $p_{NL}=0.00$ , $p_{ARCH(4)}=0.00$						
Panel C: Subsample 1986:01-2002:12						
-0.002 (0.004)	-0.0023 (0.0012)	-0.003 (0.003)	-0.007 (0.004)	1.07 (0.06)	0.16 (0.10)	-0.24 (0.065)
$p$ -values: $p_{LM(4)}=0.051$ , $p_{NL}=0.00$ , $p_{ARCH(4)}=0.00$						

Notes:

- (i) Standard errors are in parentheses.
- (ii) LM(4) is the LM test statistic for serial correlation using four lags.
- (iii) NL is the White et al (1993) neural network test of nonlinearity.
- (iv) ARCH(4) is Engle's LM test statistic for ARCH effects up to order four.

The results of the table are consistent with evidence provided in the literature (e.g., Hooker (1996)). They show that positive oil-price shocks, captured by  $\Delta P_{t-i}I_{\{\Delta P_{t-i}>0\}}$ , significantly affect the oil-macroeconomy relationship until 1986:01. After this date, they become totally unimportant. In fact, the sensitivity of real industrial growth to the level of real oil price seems to have considerably reduced after that year, as the estimates of  $\beta$  significantly drop, in absolute value, after that date. Although the above results are in accordance with the literature, there is a number of issues concerning the adequacy of the non-linear regression (17) to resemble the correct specification of the oil-macroeconomy relationship. The misspecification tests for non-linearity, denoted as NL, and ARCH effects reported in the table reveal that the non-linear regression (17) may not constitute the correct specification of the data, especially in capturing non-linearities of the oil-macroeconomy relationship. In addition, the sum of estimates of the autoregressive coefficients  $\gamma_i$  is close to unity which suggests that the dynamic effects of oil prices or their associated shocks on industrial production growth may not be so important and persistent, compared with the industrial production own shocks.

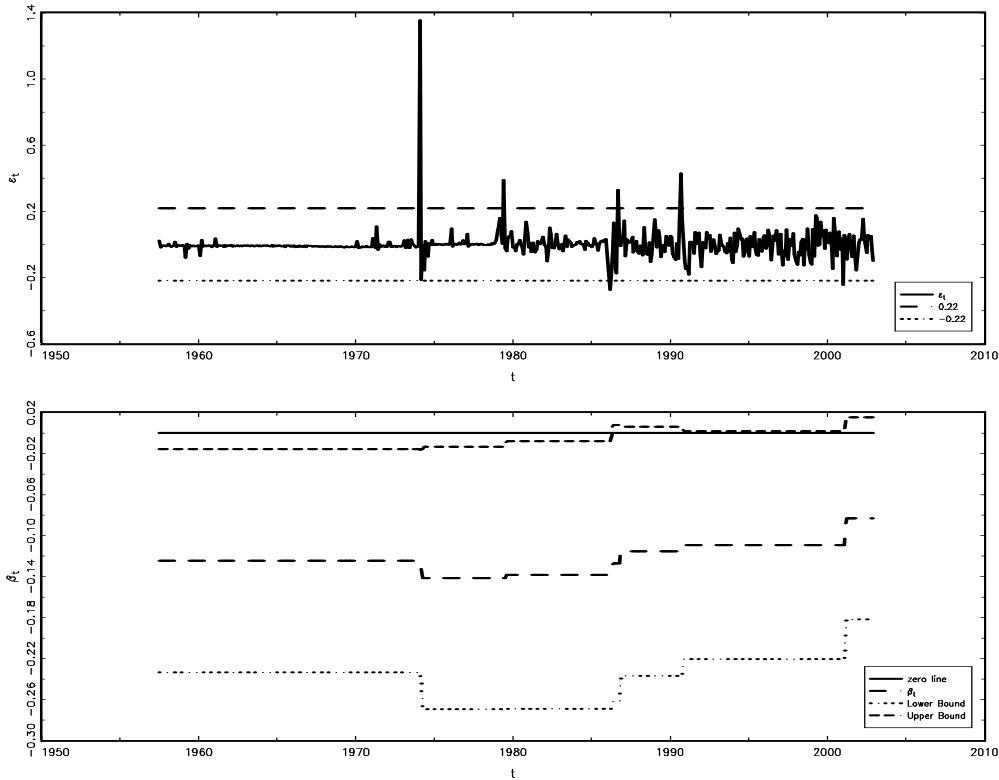
<b>Table 3:</b> Estimates of system (13)-(14)					
$\alpha_1$	$\beta_0$	$\sigma_{\eta_2}$	$\sigma_v$	$\gamma_1$	$\gamma_2$
-0.142	-0.124	0.083	0.775	0.391	0.086
(0.165)	(0.066)	(0.002)	(0.024)	(0.043)	(0.043)
$p$ -values: $p_{LM(4)}=0.99$ , $p_{NL(4)}=0.07$ , $p_{ARCH(4)}=0.03$					
$a_2$	$\delta_1$	$\delta_2$			
-0.024	1.142	-0.154			
(0.013)	(0.042)	(0.042)			
$p$ -values: $p_{LM(4)}=0.98$ , $p_{NL}=0.01$ , $p_{ARCH(4)}=0.58$					
Note: Standard errors are in parentheses					

In Table 3 we present estimates of the parameters of the structural system of equations (13)-(14). These are Kalman filter estimates which correspond to an optimal estimate of the threshold parameter  $r = 0.22$  (2.2%) found by our data, based on the grid search method described in Subsection 2.2. The table also reports the  $p$ -values of the misspecification test statistics LM(4), NL(4) and ARCH(4), reported in Table 2. In the lower panel of Figure 4, we present the smoothed estimates of the state variable  $\beta_t$ , together with their upper and lower 90% confidence interval bounds, while the upper panel of that Figure presents the estimates of the oil-shocks  $\epsilon_{1,t}$ , together with a positive and negative value of the threshold parameter  $r$ . The above results are based on a choice of the lag order of the dynamic terms and the oil-price shock in equations (13)-(14) given by  $p = q = k = 2$ . As in estimating (17), these were chosen based on a minimization of the Bayesian information criterion.

The general conclusion which can be drawn from the results of the table and the inspection of the figures is that large oil price-shocks (above 2.2% in absolute value) can explain the instability of the oil-macroeconomy relationship across the sample. This conclusion can be supported from the estimates of model (13) which constitutes a better specification of the true oil-macroeconomy relationship than the non-linear regression (17), as there is little evidence of non-linearities or any other misspecification in the residuals. Furthermore, the sum of the autoregression coefficients  $\gamma_i$  now is found to be far away from unity which implies that, in contrast to model (17), our model is able to signify dynamic effects of oil price-shocks on industrial production growth.

<b>Table 4:</b> Detection of outliers						
Date:	1974:01	1979:05	1986:07	1986:08	1990:07	2000:11
$\lambda_{\max}$	16.27	4.51	6.058	3.85	4.03	3.92
Notes: $\lambda_{\max}$ is Tsay's test statistic for detecting outliers						
A critical value of three is chosen.						

Figure 4: Empirical Results



The estimates of oil price-shocks  $\epsilon_{1,t}$  and the threshold parameter  $r$  reveal that, in total, there are six large oil price-shocks which drive the structural changes in the oil-macroeconomy relationship (13), over the sample. These correspond to the following dates: 1974:01, 1979:05, 1986:03, 1986:09, 1990:09 and 2001:01. The shocks of dates 1974:01, 1979:05 and 1990:09 are positive and correspond to the significant falls of oil production associated with the Arab-Israel war, the Iranian revolution and Persian Gulf war, respectively. The shocks of 1986:03 and 2000:01 are negative and correspond to the dramatic oil price fall in years 1986 and 2000, associated with the oil market collapses in those years. These type of shocks are considered by many authors as a correction for the substantial oil price increases occurred during the previous years (see Hooker (1996) and Hamilton (2003)). By the same token, someone can characterise the positive oil price shock of 1986:09 as a correction to the dramatic oil price fall in the early eighties. To see which ones of the above shocks can be classified as outliers from the statistical point of view, in Table 4 we present estimates of the additive



outliers found in the residuals of the oil price equation (14), together with their corresponding dates. These are identified based on the sequential algorithm suggested by Tsay (1988) and Balke (1993).<sup>3</sup> The results of this table indicate that the dates of the large oil price-shocks  $\epsilon_{1,t}$ , identified using our model, are closely related to those detected through the outliers sequential algorithm of Tsay.

Figure 4 indicates that the instability of the oil-macroeconomy relationship become apparent after 1974:03, a few months after the first large oil price-shock occurred in November 1973. Before this date, the estimates of  $\beta_t$  are found be stable, given by  $\beta_0 = -0.124$ , and not significantly different from zero. The estimates of  $\beta_t$  indicate that the first oil price-shock of year 1973 caused a significant drop in industrial production growth. This lasted for a substantial number of periods, until the significant drop of oil prices occurred in 1986. As expected by the economic theory, the negative oil price-shock of 1986 caused less adverse effects on  $\beta_t$  than the positive shock of 1973. However, the effects of the oil price-shock of 1986 on production activity were so strong and pervasive, so that none of the positive oil price-shocks occurred in the subsequent years 1986:11 and 1990:11 caused any significant changes in  $\beta_t$ . These results are consistent with evidence provided by Hooker (1996) that the correlation between oil prices and industrial production growth was substantially reduced in the middle of 1980's. Our estimates show that this can be attributed to the fact that, in contrast to large negative oil price shocks (see also the effect of the oil-price shock of 2000:01 on  $\beta_t$ ), the large positive oil price-shocks occurred after 1986 do not seem to cause adverse effects on industrial production. The latter may be attributed to firms' costs adjustments to reduce the negative impact of an oil-price increase on production (see Lillien (1982) and Hooker (1996)) or to firms' beliefs that the positive oil price shocks after year 1986 will not have had any permanent effect on oil price in the future (see Hamilton (2003)).

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<sup>3</sup>This procedure work as follows. First, we estimate the AR(2) model for the real oil price  $p_t$  and extract the residuals and their variance. Then, we search for an outlier in the residuals based on the statistic  $\lambda_{\max} = \max_{1 \leq t \leq T} \{|\lambda_t|\}$ , where

$$\lambda_t = \rho^2(\epsilon_{1,t} - \sum_{i=1}^{T-t} \delta_i \epsilon_{1,t+i}) / (\rho \sigma_{\epsilon_2}),$$

with  $\rho^2 = (1 + \sum_{i=1}^{T-t} \delta_i^2)^{-1}$ . If an outlier is found, we remove its effects on  $p_t$  using a dummy variable and we recalculate the residuals and their variance. We continue searching and adjusting until no more outliers are detected. As in Balke and Fomby (1993), a critical value of three is chosen to detect an outlier.

<b>Table 5:</b> Estimates of (13)-(14) including $\Delta P_t I_{\{\Delta P_t > 0\}}$							
$\alpha_1$	$\beta_0$	$\sigma_{\eta_2}$	$\sigma_v$	$\theta_1$	$\theta_2$	$\gamma_1$	$\gamma_2$
-0.105	-0.11	0.083	0.775	0.303	0.466	0.391	0.086
(0.187)	(0.073)	(0.002)	(0.024)	(0.837)	0.843	(0.043)	(0.042)
<i>p</i> -values: $p_{LM(4)}=0.99$ , $p_{NL(4)}=0.07$ , $p_{ARCH(4)}=0.03$							
$a_2$	$\delta_1$	$\delta_2$					
-0.024	1.142	-0.154					
(0.013)	(0.042)	(0.042)					
<i>p</i> -values: $p_{LM(4)}=0.98$ , $p_{NL(4)}=0.07$ , $p_{ARCH(4)}=0.58$							

Although the misspecification test statistics reported in Table 3 indicate that (13) is correctly specified, especially regarding to omitted nonlinearities, it will be interesting to investigate at this point if this parameterization of the oil-macroeconomy relationship remains robust to the inclusion of the variable  $\Delta P_t I_{\{\Delta P_t > 0\}}$ , defined in the nonlinear relationship (17). This variable can directly capture the effects of positive oil price-shocks on the industrial production growth. Therefore, in Table 5 we report estimates of the structural system (13)-(14) including the variable  $\Delta P_t I_{\{\Delta P_t > 0\}}$  with one and two periods lags as regressors, with slope coefficients denoted  $\theta_1$  and  $\theta_2$ , respectively. Testing for the significance of  $\theta$ 's can show if our model remains robust to the type of non-linearity in oil-macroeconomic relationship encountered by  $\Delta P_t I_{\{\Delta P_t > 0\}}$ . Since this type of testing considers a specific type of non-linearity under the alternative hypothesis, it is expected to have better power than the NL statistic for nonlinearity, which considers more general patterns of nonlinearity in the data. The results of the table clearly indicate that none of the  $\theta$ 's are found to be significant, while the estimates of the remaining parameters of the model hardly change from the previous ones, reported in Table 4. These results therefore suggest that the apparent non-linearities of the oil-macroeconomy relationship found in the literature to be captured by  $\Delta P_t I_{\{\Delta P_t > 0\}}$  may be attributed to structural changes in this relationship originated by large oil price shocks.

## 4 Conclusion

This paper has introduced a new approach of modelling breaks in the structural coefficients of economic relationships which are allowed to be driven by large structural economic shocks. These shocks can be originated by oil price-shocks, monetary regime changes and turning points of business cycles. The latter have been found to be associated with persistent shifts in the structure of economic relationships. Our approach enables us to empirically investigate if large economic shocks have an impact on economic relationships. This can be done endogenously by the data, as part of our model specification. The nature of the breaks that we consider are entirely stochastic in both timing and magnitude. The last property of our

model enables us to study the persistency and size effects of different in magnitude large shocks on economic relationships.

The paper has applied our modelling approach of structural breaks to investigate if the apparent instability of the oil-macroeconomy relationship found in the literature can be attributed to large oil price-shocks. Answering this question has important economic implications, as oil is considered as one of the most significant production factors which can cause economic recessions. Our results suggest that, although the large oil price increase in year 1973 caused a dramatic reduction in the industrial production activity, the oil-price increases followed after the middle of eighties did not have any impact on the production activity. These results support the view that oil prices-shocks are not necessarily responsible for economic recessions occurred in US economy since the early seventies.

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## 5 Appendix

In this appendix, we give a proof of the consistency of the threshold parameter  $r$ , which can be estimated via a grid search procedure. To simplify matters we suggest estimation of the threshold parameter via minimisation of the sum of squares function  $S(\psi) = \sum_{i=1}^T z_t^2$  where  $z_t$  are given by

$$z_t = y_t - x_t \hat{\beta}_{t|t-1} - \hat{\epsilon}_{t|t-1}$$

and are the prediction errors of the model. Harvey (1989) (pp. 129) states that for univariate models such a minimisation is equivalent to maximum likelihood estimation. For simplicity we also assume  $k = 1$  without loss of generality.

Following the proof of consistency of the threshold parameter estimates by Chan (1993) we see that three conditions need to be satisfied for consistency. Firstly, we need to show that the data  $y_t$  are geometrically ergodic and hence covariance stationary (Condition C1). Secondly we need to show that (Condition C2)

$$E_{\theta^0}(z_t|t-1)^2 < E_{\theta}(z_t|t-1)^2 \quad \forall \theta \neq \theta^0 \quad (18)$$

and thirdly we need to show that (Condition C3)

$$\lim_{\delta \rightarrow 0} E \left( \sup_{\theta \in B(\theta^0, \delta)} |z_t(\theta^0) - z_t(\theta)| \right) = 0 \quad (19)$$

where  $B(a, b)$  is an open ball of radius  $b$  centered around  $a$ . C1 is needed for obtaining a law of large numbers needed for Claim 1 of Chan (1993). C1 can be obtained in a number of ways for a strictly exogenous geometrically ergodic processes  $x_t$ . For that we simply need geometric ergodicity of  $\beta_t$ . This can be easily obtained using the drift condition of Tweedie (1975). This condition states that a process is ergodic under regularity conditions satisfied by assuming a disturbance with positive density everywhere if the process tends towards the center of its state space at each point in time. More specifically, an irreducible aperiodic Markov chain  $\beta_t$  is geometrically ergodic if there exists constants  $\delta < 1$ ,  $B, L < \infty$ , and a small set  $C$  such that

$$E [\|\beta_t\| \mid \beta_{t-1} = \beta] < \delta \|\beta\| + L, \quad \forall \beta \notin C, \quad (20)$$

$$E [\|\beta_t\| \mid \beta_{t-1} = \beta] \leq B, \quad \forall \beta \in C, \quad (21)$$

where  $\|\cdot\|$  is a norm. The concept of the small set is the equivalent of a discrete Markov chain state in a continuous context. For example, a model for  $\beta_t$  that is easily seen to satisfy the above is

$$\beta_t = I(|\beta_{t-1}^*| > \beta) \beta_1 + I(|\beta_{t-1}^*| < \beta) \beta_{t-1} + I(|\beta_{t-1}^*| < \beta) I(|\epsilon_{1,t-1}| > r) \epsilon_{2,t-1} \quad (22)$$

where  $\beta_{t-1}^* = \beta_{t-1} + I(|\epsilon_{1,t-1}| > r)\epsilon_{2,t-1}$  and  $\beta > \beta_1$  are finite constants. This model simply restricts the process  $\beta_t$  to return to a prespecified level  $\beta_1$  if its expected value at time  $t - 1$  exceeds  $\beta$ . A wide variety of other models are possible.

We need condition C2 to get a similar expression to (3.7) of Chan (1993) and condition C3 to prove Lemma 1 of Chan (1993). Condition C3 is a stochastic equicontinuity type condition and is particularly important in view of the discontinuity involved with respect to the threshold parameter.

To prove condition C2 we focus on the following model.

$$y_t = X\alpha_t \quad (23)$$

$$\alpha_t = A_t(\theta)\alpha_{t-1} + u_t \quad (24)$$

We assume that the parameters of interest appear only in the matrix  $A_t$ . We have that

$$z_t(\theta) = y_t - \hat{y}_{t|t-1}(\theta) = X\alpha_t - X\alpha_{t|t-1}(\theta) = \quad (25)$$

$$XA_t(\theta^0)\alpha_{t-1} + Xu_t - X\hat{\alpha}_{t|t-1}(\theta) = XA_t(\theta^0)\alpha_{t-1} - XA_t(\theta)\hat{\alpha}_{t-1|t-1}(\theta) + Xu_t \quad (26)$$

It is clear that the value of  $\theta$  enters recursively through  $\hat{\alpha}_{t-i|t-i}(\theta)$ . But for showing C2 it suffices to show that  $E_{\theta^0}(z_t|t-1)^2 < E_{\theta}(z_t|t-1)^2$  for the case where  $\theta^0$  enters in  $\hat{\alpha}_{t-1|t-1}(\theta)$  both for  $z_t(\theta)$  and  $z_t(\theta^0)$ .

So let us define

$$\tilde{z}_t(\theta) = XA_t(\theta^0)\alpha_{t-1} - XA_t(\theta)\hat{\alpha}_{t-1|t-1}(\theta^0) + Xu_t \quad (27)$$

and

$$\tilde{z}_t(\theta^0) = XA_t(\theta^0)\alpha_{t-1} - XA_t(\theta^0)\hat{\alpha}_{t-1|t-1}(\theta^0) + Xu_t = XA_t(\theta^0)(\alpha_{t-1} - \hat{\alpha}_{t-1|t-1}(\theta^0)) + Xu_t \quad (28)$$

If we show that  $E(\tilde{z}_t(\theta)|t-1)^2 > E(\tilde{z}_t(\theta^0)|t-1)^2$  then C2 is proven. But, noting that  $A_t$  depends only on data available up to  $t - 1$  and that

$$E((\alpha_{t-1} - \hat{\alpha}_{t-1|t-1}(\theta^0))(\alpha_{t-1} - \hat{\alpha}_{t-1|t-1}(\theta^0))') = P_{t-1|t-1}$$

we get

$$E(\tilde{z}_t(\theta^0)|t-1)^2 = A_t(\theta^0)'P_{t-1|t-1}A_t(\theta^0) + \Sigma_u$$

Also

$$\tilde{z}_t(\theta) = XA_t(\theta^0)\alpha_{t-1} - XA_t(\theta^0)\hat{\alpha}_{t-1|t-1}(\theta^0) + XA_t(\theta^0)\hat{\alpha}_{t-1|t-1}(\theta^0) - XA_t(\theta)\hat{\alpha}_{t-1|t-1}(\theta^0) + Xu_t =$$

$$XA_t(\theta^0)(\alpha_{t-1} - \hat{\alpha}_{t-1|t-1}) + \hat{\alpha}_{t-1|t-1}(XA_t(\theta^0) - XA_t(\theta)) + Xu_t$$

Noting that  $\hat{\alpha}_{t-1|t-1}$  is fixed given data at  $t - 1$  gives

$$\begin{aligned} E(\tilde{z}_t(\theta)|t-1)^2 &= A_t(\theta^0)'P_{t-1|t-1}A_t(\theta^0) + \Sigma_u + \\ (XA_t(\theta^0) - XA_t(\theta))\alpha_{t-1|t-1}\alpha'_{t-1|t-1}(XA_t(\theta^0) - XA_t(\theta))' \end{aligned}$$

Hence, C2 is proven.

We move on to condition C3. We show this result for  $z_2$  assuming without loss of generality that the initial conditions are given by  $\alpha_0 = 0$  and  $P_0 = 0$ . Then it is easy to show the same result for any  $t$  working recursively. So we have to show that

$$\lim_{\delta \rightarrow 0} E(\sup_{\theta \in B(\theta^0, \delta)} |z_2(\theta^0) - z_2(\theta)|) = 0$$

or equivalently that

$$\lim_{\delta \rightarrow 0} E(\sup_{\theta \in B(\theta^0, \delta)} |\tilde{z}_2(\theta^0) - \tilde{z}_2(\theta)|) = 0$$

Given (27) and (28) we need to show that

$$\lim_{\delta \rightarrow 0} E(\sup_{\theta \in B(\theta^0, \delta)} |\alpha_{1|1}(A_1(\theta^0) - A_1(\theta))|) = 0$$

or

$$\lim_{\delta \rightarrow 0} E(\sup_{\theta \in B(\theta^0, \delta)} |(A_1(\theta^0) - A_1(\theta))|) = 0$$

We use a simple model for  $A_t$  to illustrate the proof although more complicated models can be similarly treated. We use  $A_t(\theta) = A_t(r) = I(|\epsilon_t| > r)$ . Then we need to show that

$$\lim_{\delta \rightarrow 0} E(\sup_{r \in B(r^0, \delta)} (I(|\epsilon_1| > r) - I(|\epsilon_1| > r^0))) = 0$$

This is simply equal to  $Pr(|\epsilon_t| \in (r, r^0))$  where we have assumed with loss of generality that  $r > r^0$ . But

$$\lim_{r \rightarrow r^0} Pr(|\epsilon_t| \in (r, r^0)) = 0$$

proving C3.



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