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Abstract

The lengthy half-lives of real exchange rates in the presence of high degree of exchange rate volatility has been considered as one of the most puzzling empirical regularities in international macroeconomics. This paper suggests that the measure of half-life used in the literature might be problematic and suggests an alternative measure. Empirical analysis suggests that use of the new measure may shed light on the PPP puzzle.

Keywords: PPP, Half Life, Real Exchange Rates

JEL Codes: C12, C15, C23

1 Introduction

The lengthy half-lives of real exchange rates in the presence of high degree of (nominal and real) exchange rate volatility has been considered as one of the most puzzling empirical regularities in international macroeconomics (see, Rogoff (1998), Taylor (2001), Taylor and Taylor (2004)). This conundrum has intrigued international economists working on real exchange rates since it seems to be at odds with the implications of sticky-price versions of most dynamic stochastic general equilibrium models of open economies, which typically imply that the half-life of a shock to the real exchange rate should be between one and two years. The concept of half-life is not the only possible measure for assessing the speed of mean reversion or persistence in real exchange rates[1] but has emerged as the dominant measure in the literature on real exchange rates and Purchasing Power Parity (PPP).

Nevertheless, more recently research emerged that questions various aspects of the half-life measure including uncertainty about point estimates (Rossi (2003)), the presence of bias associated with inappropriate aggregation across heterogeneous coefficients (Taylor (2001)),

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[1]See, e.g., Andrews and Chen (1994) for a discussion of measures such as the spectrum at zero frequency, the sum of the autoregressive coefficients, and the largest autoregressive root.

time aggregation of commodity prices, and downward bias in estimation of dynamic lag coefficients (Choi, Mark, and Sul (2004)) and so on. In this paper we explore the possibility that the reason for the long half-lives that give rise to the PPP puzzle may be that the measure that is used in the literature is responsible for a bias towards long half-lives. In particular, the half life measures considered in the literature invariably focus on the instantaneous effects of the shock. This measure, however, has a number of weaknesses such as, for example, non-uniqueness. We propose an alternative measure of half-lives which seems to have superior properties to that used in the international finance literature. This measure focuses on the cumulative effect of the shocks instead of the instantaneous effect.

When we employ this measure to the real exchange rates of a set of industrialized countries the emerging half-lives are between one and two years. This is consistent with the predictions of sticky price models. Thus the so-called PPP puzzle is less pronounced that initially thought, or even non-existent. The next section reviews briefly the literature on the PPP puzzle. Section 3 discusses the measurement of half-lives and their weaknesses and motivates the introduction of an alternative measure. We introduce the alternative definition of half-live and discuss its properties in section 4. In section 5 we apply this measure to US bilateral exchange rates. Section 6 considers the implication of non-linearities in the impulse responses and finally, section 7 concludes.

2 Motivation and review of the literature

The PPP puzzle consists in observing very high short run volatility of the real exchange rates on one hand and the very low speed of adjustment to PPP on the other. The high volatility in real exchange rates is usually expected to be explained in terms of monetary and financial shocks. The empirical measurements of the speed of adjustment to PPP, however, show that it is too slow to be compatible with such explanations. To examine the properties of real exchange rates and the persistence of their deviations from PPP researchers employ impulse response analysis and the concept of half-life is used to consider how long it takes for the impulse response to a unit shock to dissipate by half.[2]

Most of the recent accounts of the half-lives in real exchange rates are associated with the empirical literature on PPP. Studies focusing on groups of industrial countries include Abuaf and Jorion (1991) who find that the annual half-life in ten industrial countries is 3.3 years. Manzur (1990) and Manzur (1993) consider seven industrial countries and find that the half-lives or their real exchange rates are 5 years while Fung and Lo (1992) put

[2]This definition although apparently informative is not very clear. It is usually taken to mean that the half life of the impulse response, ϕ_i is i where $\phi_i = \phi_0/2$.

the half-lives to 6.5 years for the six industrial countries they consider. Cheung and Lai (2000) put the half-lives to a range between 2 and 5 years for industrial countries but under 3 years for developing countries.[3] Higgins and Zakrajšek (1999) focus on OECD countries and WPI-based real exchange rates and on a set of open economies, CPI-based rates finding half-lives of 2.5 and 11.5 respectively. The influential study of Frankel and Rose (1996) who focus on very broad panels finds that the half-life is 4 years for 150 countries.

Another number of studies focus on European real exchange rates. Parsley and Wei (1995) find that the half life for the EMS (European Monetary System) countries is 4.25 years. The findings of Papell (1997) suggest an annual half-life of 1.9 for the European Community and of 2.8 for the EMS. Higgins and Zakrajšek (1999) suggest that the same number is 5 for Europe, when CPIs are used and 3 when WPIs are used. Finally, a number of studies focuses on single real exchange rates. For example, Frankel (1990) finds that the half-life of the Dollar-pound real exchange rate is 4.6 years. Lothian and Taylor (1996) find that the corresponding numbers are 2.8 for the Franc-pound and 5.9 for the Dollar-pound real exchange rate.

The literature has tried to improve upon those results by employing a number of methodological advances. A number of authors have pointed out the bias emerging from inappropriate pooling of cross sectional units, that typically biases the half-life upwards; Choi, Mark, and Sul (2004). This type of bias has not been received unanimously in the literature and while Imbs, Mumtaz, Ravn, and Rey (2004) attempt to correct it, Chen and Engel (2004) find that it is not important. Taylor (2001) points out to the temporal aggregation bias and finds that it leads to higher half-lives. A number of other studies focus on the uncertainty surrounding the half-life estimates. For example, Rossi (2003) constructs confidence intervals that are robust to high persistence in small sample sizes and finds that their lower bound is as low as four quarters. This finding, however, has little to offer to the discussion of the PPP puzzle given that the upper bounds are infinity. Kleijn and Dijk (2001) also find low half-lives from a Bayesian unobserved components model for the real interest rate. The most promising line of research, however, seems to be that considering the possibility of non-linearities in the real exchange rate process. Taylor (2001) finds that when non-linearities are taken into account the half-lives are significantly shorter.

While all those improvements are very useful they leave intact a major methodological

[3]This may be consistent with research trying to explain slow convergence in terms of bandwagon effects. Bandwagon effects can send a variable away from its equilibrium thereby prolonging the convergence. The result that speed of convergence in developing countries is faster may be supportive of this view as exchange rates of developing countries are less subject to speculative currency movenents.

aspect of half-life measurement, namely the concept of half-life itself. The method used for measuring the half-lives in the literature is not the only possible that one can use. Moreover, it may not be optimal since it suffers from a number of drawbacks. For example, in the case where the impulse response follows an oscillating pattern instead of a monotonically decaying one, then the current measure cannot adequately capture the persistence of deviations from PPP. But even with monotonically decaying impulse response functions, meaningful comparisons are frequently difficult when the series display varying rates of decay and the impulse responses cross each other.

In this paper we discuss the weaknesses that emerge from the standard definition of half-life and propose another definition which solves some of the problems of the standard definition such as non-uniqueness. The above problems become critical when the specific measure of half-lives is employed to assess mean reversion in real exchange rates. This implies that the presence of the PPP puzzle may be sensitive to the choice of the half-life measure used. The weaknesses of the standard measure emerge because of the focus on the instantaneous concept of half-life. We propose instead a measure that is based on the cumulative effects of the impulse responses.

When the PPP real exchanged rate is used as a benchmark for setting exchange rate parities, evaluating the degree of misalignments of actual from benchmark exchange rates,[4] using the currently popular concept of half life may not be a problematic. When the focus is on the implications of the degree of persistence in the real exchange rates, this concept may not be the most accurate. The real exchange rate puzzle that Roggof points to is related to this aspect of real exchange rates and half-life measurement. In particular, financial and monetary shocks should imply a lower degree of persistence while real shocks (say productivity, technology and tastes) should imply a high degree of persistence.

Actually, a number of theoretical explanations of real exchange rate persistence (e.g., bandwagon effects, non-linearities) seems to be consistent with a view of the half life based on the cumulative effects of the shocks. For example, non-linearities in the real exchange behavior may exist, emerging from transaction costs. One approach in reconciling theory with empirical facts (or explaining the PPP puzzle) is to stress the possibility of nonlinear real exchange rate behavior due to transaction costs. The presence of transaction costs makes adjustment costly and arbitrage takes place more difficultly.

[4]Other applications include the meauremnt of output for international comparisons.

3 Weaknesses of half-life measures

Half life measures have been discussed in the literature for the best part of the last 20 years.[5] In a majority of papers dealing with half lives, we see that the measure is inextricably linked to the $AR(1)$ model of the form

$$y_t = \rho y_{t-1} + \epsilon_t \tag{1}$$

where y_t, $t = 1, \dots, T$ is the process under investigation. Then, the half life is defined as

$$h = \frac{ln(1/2)}{ln(\hat{\rho})} \tag{2}$$

where $\hat{\rho}$ denotes the estimate of ρ. We will refer to this as Definition 1. In fact, this coincides with the more formal definition of the half life which is

$$h = i, \text{for which } \phi_i = \phi_0/2 \tag{3}$$

where

$$\phi_i = E(y_{t+i}|\epsilon_t = 1) - E(y_{t+i}|\epsilon_t = 0) \tag{4}$$

which we refer to as Definition 2 (see Mark (2001) for more details). Since for the $AR(1)$ model, $\phi_i = \rho^i$, Definition 1 follows. In what follows we will allow for non-integer i in ϕ_i.

A first objection with Definition 1 is that it does not coincide with Definition 2 for other dynamic models such as $AR(p)$, $p > 1$ or $ARMA(p,q)$ models. Of course, it is conceptually easy to obtain the half life according to Definition 2. However, mechanically obtaining this may be difficult. As a result a number of alternative definitions based on simplifications of Definition 2 have appeared in the literature. Perhaps the most interesting one is that by Rossi (2003) which is given by

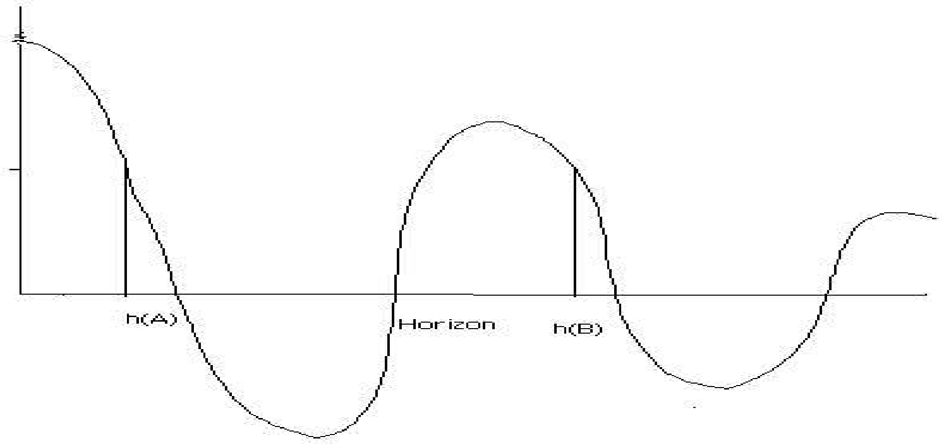
$$h = \frac{ln(1/2)b(1)}{ln(\hat{\rho})} \tag{5}$$

where $b(1)$ is the sum of the estimated AR coefficients of an $AR(p)$ model fitted onto the residuals of (1). This definition, referred to as Definition 1A arises out of assuming that the process generating the data is near unit root, i.e. that $\rho = 1 - c/T$ for some constant c.

Moving on to Definition 2 we have a common complaint in the literature. This complaint is that if the impulse response of a stationary series (or indeed a non-stationary series for which shocks are temporary such as, e.g. $ARFIMA(p,d,q)$ processes for $1/2 < d < 1$) is not monotonically declining then this definition does not necessarily give a unique half life as there may be be multiple i for which $\phi_i = 1/2\phi_0$. In this case researchers usually resort to defining half life as either the smallest i for which $\phi_i = 1/2\phi_0$ (see, e.g. Rossi (2003)) or

[5] For a recent summary see also Choi, Mark, and Sul (2004)

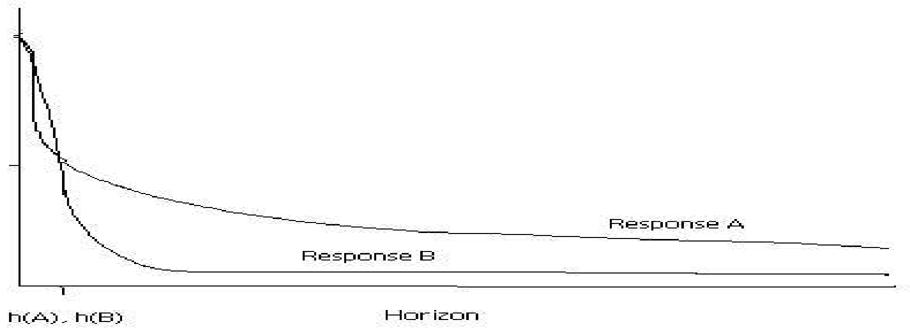
Figure 1:



alternatively the largest such i (see, e.g. Ng (2003)). This is clearly problematic. In Figure 1 we illustrate the problem pictorially using a non monotonically declining impulse response. With reference to that Figure, why would $h(A)$ be preferable to $h(B)$ as a half life measure or vice versa?

Perhaps more fundamentally, this definition is suspect on more basic grounds. To appreciate the point we examine the two impulse responses in Figure 2. The two impulse responses have the same half life. However, few would support the case that the same proportion of the shock has been dissipated for the two impulse responses. The problem seems to be that Definition 2 considers only points in the impulse response in isolation and not the whole of the impulse response. A further problem arises if we consider the case of a non-stationary process. Assume that for a non-stationary process the effect of a shock (impulse response) settles for long horizons at a non zero value which is less that half the initial effect of the shock. Perversely, this means that the half life measure according to definition 2 will be finite. Clearly, a permanent shock cannot have a finite half life. Again the failure of intuition and formal definition is due to the consideration of points in the impulse response in isolation. Is it possible to come up with an alternative definition that addresses all the above issues? We provide one definition in the next section.

Figure 2:



4 An alternative definition of half life

Before suggesting a possible solution to the questions raised in the previous section we should point out that no half life measure will be able to convey the informational content of an impulse response since it is only a summary statistic. Hence, there will always be cases where any half life measure will not do justice to the underlying impulse response. Nevertheless, the half-life measure has the advantage that it is readily interpreted in terms of time units and the debate on real exchange rate convergence to PPP values has been casted in terms of this measure.

The concept of half life originates from experimental sciences where it arises in a multitude of contexts. Perhaps the most widely familiar definition to laymen is taken from nuclear physics. There, it is defined as the amount of time it takes for half of the atoms in a sample of radioactive isotope to decay. Note the discrepancy with Definition 2 which taken to a physics context would define half life as the point in time at which half the amount of atoms instantaneously decay compared to the amount of atoms that instantaneously decay at the start of the decay process.

An intuitive analogy to our context then may be the following: Define the impulse response as a function of i. We denote this as $\phi(i)$ to provide a distinction in focus from standard impulse responses. Then, the half life is the point h^* at which

$$\int_0^{h^*} |\phi(i)| di = \int_{h^*}^{\infty} |\phi(i)| di \tag{6}$$

In words, h^* is the point in time at which half the absolute cumulative effect of the shock has dissipated. We refer to this definition as Definition 3. The use of $|\phi(i)|$ rather than $\phi(i)$ solves the problem arising out of the possibility of negative as well as positive impulse responses. The use of the integral firstly guarantees uniqueness of the measure and secondly accords with the intuition behind shock dissipation. How does Definition 3 compare with, say, Definition 1? Simple algebra indicates that if the model is $AR(1)$, Definitions 1 and 3 coincide. We do not claim that the definition we suggest is novel because it follows immediately from the one in experimental sciences.

An immediate concern relates to the calculation of half life according to Definition 3. In particular, we are concerned with calculating h^* given the estimates of the coefficients of an $AR(p)$. Denote these coefficients by $\rho_1, \ldots, \rho_p$. Define the matrix coefficient of the companion form of the AR(p) model by

$$A = \begin{pmatrix} \rho_1 & \rho_2 & \ldots & \rho_p \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$

Then, denote the ordered eigenvalues of A by $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_1$. Hamilton (1994) shows that if the eigenvalues are distinct then

$$\phi(i) = \sum_{j=1}^{p} c_j \lambda_j^i \tag{7}$$

where

$$c_j = \frac{\lambda_j^{p-1}}{\prod_{k=1,k\neq i}^{p} (\lambda_j - \lambda_k)} \tag{8}$$

Then, simple algebra implies that h^* solves the equation

$$2\sum_{j=1}^{p} \frac{c_j \lambda_j^{h^*}}{ln(\lambda_j)} = \sum_{j=1}^{p} \frac{c_j}{ln(\lambda_j)} \tag{9}$$

This is not trivial to solve for h^*. Nevertheless, numerical methods can be readily used to solve for h^*.

5 An Empirical Application to US Real Exchange Rates

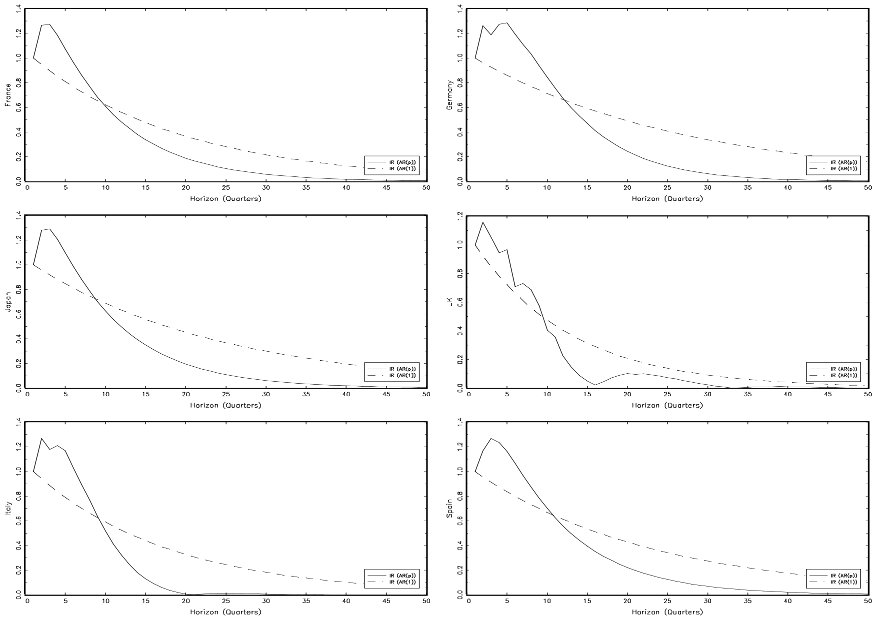
We investigate the half life of quarterly US real exchange rates using both the proposed and available half life definitions. We construct the bilateral real exchange rate q of the i-th currency against the US Dollar at time t as $q_{i,t} = s_{i,t} + p_{j,t} - p_{i,t}$, where $s_{i,t}$ is the corresponding nominal exchange rate (*i-th* currency units per one US dollar), $p_{j,t}$ the price level in the United States, and $p_{i,t}$ the price level of the *i-th* country. That is, a rise in $q_{i,t}$ implies a real appreciation of the US Dollar against the *i-th* currency. Data are quarterly, spanning from 1957Q1 to 1998Q4. We use the average quarterly nominal exchange rates and the price levels are consumer price indices (not seasonally adjusted). All variables are in logs. All data are from the International Monetary Fund's *International Financial Statistics* in CD-ROM.

We use the Chang (2002) unit root test based on nonlinear IV estimation. We consider the case of a constant and a trend and 4 lag augmentations. According to this test the US real exchange rates with respect to France, Japan, the UK, and Italy are found to be stationary at the 5% significance level. Evidence, using panel data methodologies, reported in Chortareas and Kapetanios (2004), suggests that the real exchange rates of Germany and Spain are stationary as well and, so, we include these in our analysis. We note that in the panel data analysis of Chortareas and Kapetanios (2004) it is found that all of the above series, but Germany, are stationary even if the DF test is used instead of the Chang (2002) test and, therefore, our results do not appear overly sensitive to the choice of the unit root test. In any case, in half life analysis, results from unit root tests are usually discounted as many standard unit root tests have low power. We then estimate an AR(1) model to construct a half life measure according to Definition 1. We estimate an AR(p) model for each series and use that to get half life measures according to definitions 1A and 3. Table 1 presents the chosen lags (using Akaike's information criterion) and the estimated half lives, where the measure according the Definition 3 has been obtained numerically. It is clear that Definition 3 provides plenty of evidence that the half life puzzle identified repeatedly in the literature is due to an inappropriate definition of half life. Definition 1A incorporates an assumption that the series is highly persistent (near unit root). Hence, it is not surprising that it produces the highest half life measure of the three.

Table 1

Country	Lag	1	1A	3
France	2	3.282	8.689	1.603
Germany	4	4.649	11.399	1.786
Japan	2	4.167	11.172	1.620
UK	7	2.103	5.981	1.162
Italy	4	2.951	7.208	1.146
Spain	3	3.882	10.217	1.726

Figure 3:



To further analyse these real exchange rate half lives and confirm the intuitive appeal of Definition 3 we plot the impulse responses implied by the AR(p) and AR(1) models in Figure 3.

We see that the impulse responses cross each other at around 0.55. implying that a half life measure according to Definition 2 would be close to that provided by Definition 1. In fact, for the case of the UK the two impulse responses cross at a point uncannily close to 0.5. For that case the AR(p) and AR(1) model would give equal half life measures according to Definition 2. Few people would claim though that the shock dissipates equally fast for these two impulse responses. Hence, the use of Definition 3 looks increasingly justified, both on theoretical and empirical grounds.

The evidence indicating a speedy mean-reversion is consistent with recent analyses showing that the PPP puzzle is less pronounced than initially thought. In particular, Chortareas and Kapetanios (2004) employ a methodology that allows them to identify the stationary real exchange rates within panels without trading off any of the panel advantages and this has strong implication for the analysis of the PPP puzzle. On one hand,hen univariate tests are used to consider PPP one typical obtains limited evidence of stationarity rendering discussions about convergence to PPP meaningless. On the other hand the existing panel methodologies test a joint null of non-stationarity and this does not prevent us from the possibility that the panel includes a number of non-stationary series. To the extent that we cannot identify them, the resulting half-life measures will include real exchange rates that do not converge to their PPP values and this can only biases them upwards. Chortareas and Kapetanios (2004) by being able to consider only the stationary real exchange rates within panels find that find that half-lives to PPP when the US dollar and the DM are used as numeraire currency can be shorter by up to one and two-and-a-half years respectively.

6 Relaxing the linearity assumption when constructing impulse responses

Recent work in the macroeconometric literature has been moving away from the paradigm of stationary linear processes, usually parametrized using the Box-Jenkins framework of ARMA models Our previous analysis was contingent on such a framework. Such work includes random nonstationary processes and nonlinear processes. Focusing on covariance stationary processes, the increased focus on nonlinearity has been productive in a number of ways. Firstly, nonlinear models have been shown to provide a superior fit to a number of macroeconomic series. Secondly, impulse response analysis has illuminated a number of issues such as asymmetry for economic phenomena such as the business cycle.

Work on impulse responses for nonlinear processes has been carried out by Koop, Pesaran, and Potter (1996) and Potter (2000) . That body of work is firmly set in a parametric context even though the underlying ideas can easily extend to nonparametric contexts. Therein, lies a possibly serious issue concerning the validity of impulse response analysis. Once the restrictive assumption of linearity has been relaxed, the choice of the nonlinear model becomes paramount. It is clear that misspecification of the model can lead to equally if not greater inferential problems compared to restricting the analysis to linear models.

Unfortunately, model selection in a nonlinear world is much more difficult compared to the same task in the ARMA framework. The main difficulty lies in actually defining the space of parametric models to consider. The problem appears intractable given the infinity of parametric nonlinear models that can be used to fit a time series. A possible way out is provided by nonparametric analysis. In particular, in this section we will argue that obtaining an impulse response from a nonparametric analysis may provide useful information on such issues as the persistence of series. Of course a nonparametric analysis has serious costs. Firstly, it is clearly inefficient compared to the true parametric model. This is a well known cost which we will not comment upon further. Secondly, the nonparametric analysis we suggest will be based on the Wold representation of a covariance stationary stochastic process. As Potter (2000) argued using such a representation may obscure interesting local features such as asymmetry. Nevertheless, as the Wold representation is valid even for nonlinear processes, the obtained impulse response will be informative for global features such as persistence.

Our suggestion in more detail, is as follows. Let us extend the specification of the model by assuming that

$$y_t = f(y_{t-1}, \ldots, y_{t-p}; v_t; \theta) \tag{10}$$

where v_t is an i.i.d. zero mean process with finite variance and θ is a vector of parameters. Nevertheless, the form of $f(.;.;.)$ is not known and is difficult to retrieve. Additionally, since we do not assume additivity of the error term, it is not clear how one can obtain impulse responses using nonparametric regression analysis. Nevertheless, as long as y_t is covariance stationary, the following Wold representation exists

$$y_t = \sum_{i=1}^{\infty} c_i u_{t-i} \tag{11}$$

where u_t is white noise. Note that $u_t \neq v_t$ is not i.i.d. As Potter (2000) states, impulse response analysis using this representation may obscure local features such as asymmetry to shocks. Nevertheless, global features such as the persistence of the process will still be correctly represented. The only genuine nonparametric alternative to the Wold representation is the use of a Volterra expansion of the form

$$y_t = \sum_{i=0}^{\infty} c_i v_{t-i} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij} v_{t-i} v_{t-j} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} c_{ijk} v_{t-i} v_{t-j} v_{t-k} + \ldots$$

This is clearly a hopelessly overparametrized representation of little practical use. We suggest estimation of the Wold representation and use of the estimated c_i as impulse responses. To carry out estimation we use the algorithm suggested in the proof of Theorem 2.10.1 of Fuller (1986) which proves the existence of the Wold representation. This algorithm is equivalent to estimation of the infinite AR representation of y_t and use of the residual of that as an estimate of u_t. More specifically, the infinite AR representation given by

$$y_t = \sum_{i=1}^{\infty} d_i y_{t-i} + u_t$$

is guaranteed to exist as long as $\sum_i i^s c_i < \infty$ for some $s > 1$ by, e.g., Hannan and Kavalieris (1986) . Then, we estimate

$$y_t = \sum_{i=1}^{p_T} d_i y_{t-i} + u_t$$

Then, $\hat{u}_t$ and its lags are used as a regressor in

$$y_t = \sum_{i=1}^{p_T} c_i \hat{u}_{t-i} + z_t$$

to get estimates of c_i, denoted $\hat{c}_i$. As long as $p_t \to \infty$ at rate T^θ where $1/(2(s+1)) < \theta < 1/4$ then $\hat{c}_i$ is consistent for c_i.[6] Once $\hat{c}_i$ are obtained a nonparametric estimate of the half life can be easily obtained too.

To evaluate the new method we have carried out a small Monte Carlo experiment. We consider two nonlinear models: A threshold autoregressive (TAR) model and an exponential smooth transition autoregressive (ESTAR) model. The first is given by

$$y_t = \gamma_1 I(|y_{t-1}| < r) y_{t-1} + \gamma_2 I(|y_{t-1}| \geq r) y_{t-1} + \epsilon_t$$

and the second by

$$y_t = \delta_1 y_{t-1} + \delta_2 (1 - e^{-y_{t-1}^2}) y_{t-1} + \epsilon_t$$

We also consider two specifications for each. These are $(\gamma_1, \gamma_2, r) \in \{(1, 0.6, 3), (1.2, 0.7, 4)\}$ for the TAR model and $(\delta_1, \delta_2) \in \{(0.95, -0.4), (1.4, -0.6)\}$ for the ESTAR model. All specifications are highly persistent. Figure 4 reports the true ($T = \infty$) and the average estimated impulse responses for a horizon of up to 10 periods. The true response is obtained by using a sample of 10000 observations. We see that for persistent nonlinear processes, the estimates $\hat{c}_i$ are downward biased mirroring the downward bias of AR coefficient estimates for persistent AR processes. In order to avoid this problem we introduce a bootstrap procedure to estimate the bias of the $\hat{c}_i$. More specifically, we have considered the moving block bootstrap to estimate the bias. Result on the average estimated impulse responses using the bootstrap are reported in Figure 5. We thus see that the bootstrap helps in that respect removing the bias even for samples of 50 observations.

We have carried out the above computations for the series we considered in the previous section up to horizon 25 setting $p_T = 25$. Longer lags are inadvisable given the size of the sample. In any case experimentation with longer lags did not lead to substantially different results. The block bootstrap is implemented with block size 30 and 199 bootstrap replications. Results on the nonparametric impulse responses are presented in Figure 6. Estimates of the nonparametric half lives are given in Table 2 where column NL corresponds to the nonlinear definition and NLBC to the nonlinear definition corrected for biases using the bootstrap.

[6] For a proof see Kapetanios (2003) .

Figure 4:

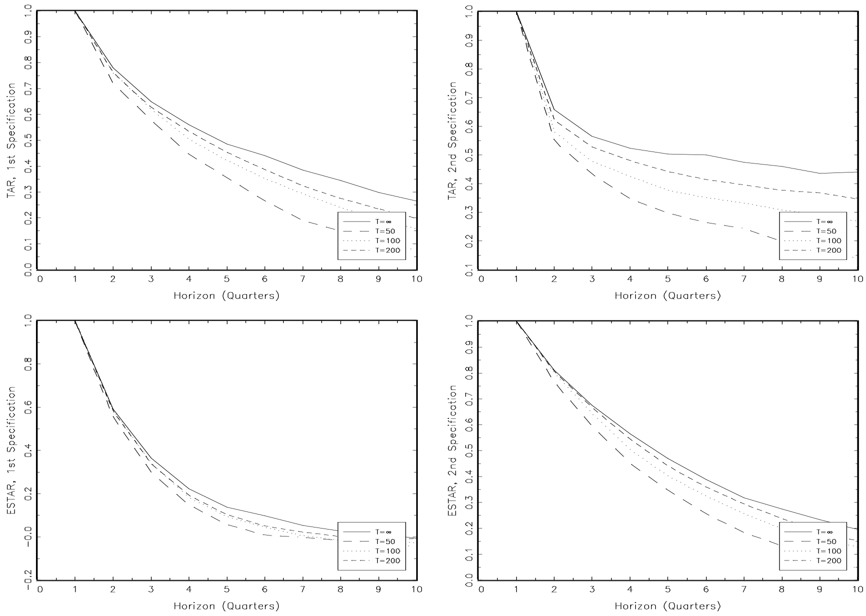


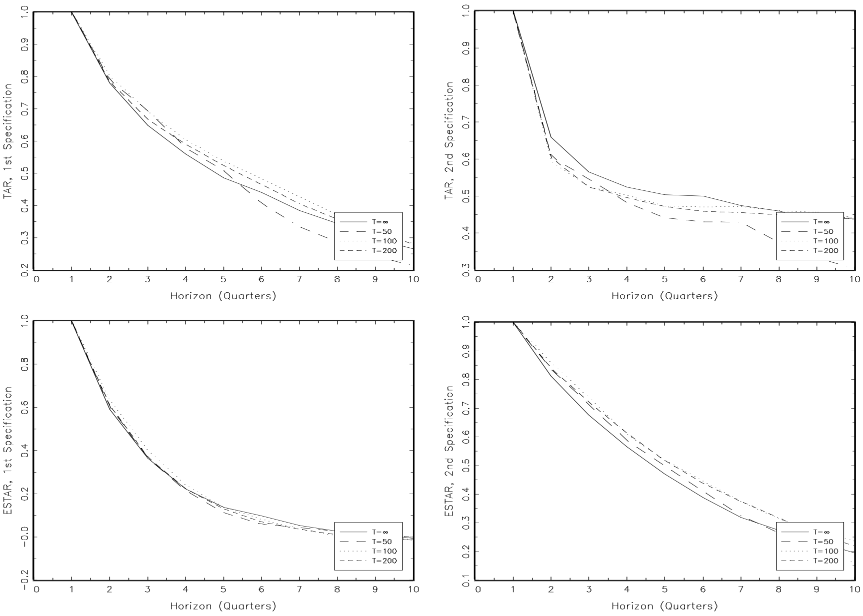
Table 2		
Country	NL	NLBC
France	1.617	1.865
Germany	2.173	2.132
Japan	2.909	2.635
UK	1.742	1.929
Italy	1.587	1.871
Spain	2.394	2.307

7 Conclusion

We address a number of questions pertaining to the measurement of the speed of real exchange rate convergence to PPP using the concept of half lives. The incompatibility of the observed lengthy half-lives with high degrees of exchange rate volatility has been considered one of the major puzzles in international macroeconomics. We find that the choice of methodology for measuring half lives is not innocuous to the results that one obtains, and this has in turn implications for the degree to which the process of real exchange rates convergence to PPP can be considered puzzling.

While the consensus in the literature has been that the half-lives are between 3 and 5 years, more recent analyses that adopt modern methodologies find evidence of considerably

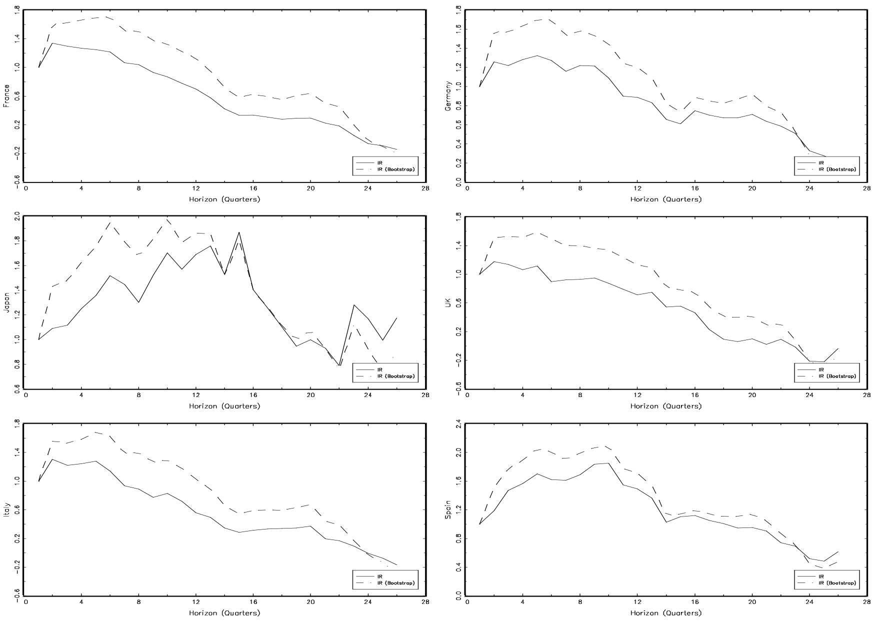
Figure 5:



shorter half-lives. Particular emphasis has been placed on the uncertainty surrounding the half-life estimates (e.g., Rossi (2003)), and on the role of non-linearities (e.g., Taylor (2001)). Being able to focus on stationary only series has lead to evidence that the PPP puzzle is milder than initially thought (Chortareas and Kapetanios (2004)). Notwithstanding those developments the literature still relies on an instantaneous concept of half-life. We suggest that this concept suffers from a number of drawbacks such as non-uniqueness and we propose the use of an alternative measure that has better properties. This measure is focusing on the cumulative effect of the impulse responses. The resulting half-lives for a number of major currencies against the US dollar appear to be below two years and therefore are consistent with the predictions of the sticky price models. We also take into account the possibility that the real exchange rate follows a non-linear process and we provide the corresponding half lives (correcting also for possible biases).

Our results indicate that the PPP puzzle may not be so puzzling if we take into account the possibility that conclusions on the length of the convergence to PPP process can be sensitive to the choice of the half-life measure.

Figure 6:



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