

Department of Economics

A Note on Joint Estimation of Common Cycles and Common Trends in Nonstationary Multivariate Systems

George Kapetanios

Working Paper No. 483

January 2003

ISSN 1473-0278



A Note on Joint Estimation of Common Cycles and Common Trends in Nonstationary Multivariate Systems

George Kapetanios*
Queen Mary, University of London

January 7, 2003

Abstract

We provide a new method for jointly consistently estimating common trends and cycles in unit root nonstationary multivariate systems. We concentrate on the MA representation of the differenced data and we jointly impose the reduced rank restriction for the common cycles and the common trends on the MA representation coefficients.

Keywords: Common cycles and trends, Tests of rank, Cointegration JEL Codes: C32, C14

*Department of Economics, Queen Mary, University of London, Mile End Rd., London E1 4NS. Email: G.Kapetanios@qmul.ac.uk

1 Introduction

In the past two decades attention in econometrics and statistics has focused on the long run properties of nonstationary time series as represented by the concept of cointegration. A major implication of cointegration is that the number of underlying random walk components of a multivariate nonstationary system is smaller than the dimension of the system. Economic theory has been routinely used to motivate the choice of cointegrating vectors and therefore implicitly the specification of the trend components.

At the same time the specification of the short run dynamics of the multivariate system has attracted less attention because economic theory can provide fewer restrictions for the short run than for the long run. However, these dynamics underlie the cyclical behaviour of the system and are therefore of particular relevance for business cycle analysis. Following papers such as Vahid and Engle (1993) and Engle and Issler (1995) we consider the cyclical components of a multivariate series to be serially correlated stationary processes. It is then highly likely that the number of such components is smaller than the dimension of the system in analogy to the trend components.

Vahid and Engle (1993) and Engle and Issler (1995) have provided a method for estimating common cycles conditional on the long run parameter estimates of the model. No method for the joint consistent estimation problem of common trends and cycles is currently available. We provide such a method by concentrating on the moving average (MA) representation of the differenced data and jointly imposing the reduced rank restriction for the common cycles and common trends on the MA representation coefficients.

2 Theory

Let the multivariate I(1) series \mathbf{y}_t , of dimension m , accept the VAR representation given by

$$\mathbf{y}_t = \mathbf{a}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{\epsilon}_t, \quad t = 1, \dots, T \quad (1)$$

If linear combinations of the I(1) variables are I(0), i.e. if there is cointegration in the system, then the system has a vector error correction representation of the form¹.

$$\Delta \mathbf{y}_t = \mathbf{a}_0 + \mathbf{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{B}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\epsilon}_t \quad (2)$$

where $\mathbf{\Pi} = \mathbf{A}_1 + \dots + \mathbf{A}_p - \mathbf{I}$ and $\mathbf{B}_i = -\mathbf{A}_{i+1} - \dots - \mathbf{A}_p$, $i = 1, \dots, p-1$. $\mathbf{\Pi}$ will be of reduced rank, r . This implies that it can be written as $\boldsymbol{\alpha} \boldsymbol{\beta}'$ where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $m \times r$ full rank matrices. The columns of $\boldsymbol{\beta}$ will contain the linear combinations that render the I(1) variables stationary. The above system accepts an MA representation in differences. This is given by

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}(L) \boldsymbol{\epsilon}_t \quad (3)$$

where $\mathbf{C}(L) = \mathbf{I} + \mathbf{C}_1 L + \mathbf{C}_2 L^2 + \dots$. By the multivariate Beveridge Nelson decomposition, this can be written as

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + [\mathbf{C}(1) + (1-L)\mathbf{C}^*(L)] \boldsymbol{\epsilon}_t \quad (4)$$

where $\mathbf{C}^*(L) = \mathbf{I} + \mathbf{C}_1^* L + \mathbf{C}_2^* L^2 + \dots$ and $\mathbf{C}_i^* = \sum_{j>i} -\mathbf{C}_j$. In levels the MA representation becomes

$$\mathbf{y}_t = \boldsymbol{\mu} t + \mathbf{C}(1) \sum_{i=0}^{\infty} \boldsymbol{\epsilon}_{t-i} + \mathbf{C}^*(L) \boldsymbol{\epsilon}_t + \mathbf{y}_0 \quad (5)$$

where for simplicity we set $\mathbf{y}_0 = 0$. This is a decomposition of the series in trends and cycles as discussed by a number of authors (see e.g. Engle and Issler (1995)). Under the assumption of cointegration we have that $\mathbf{C}(1)$ has reduced rank, equal to $m - r$, giving the common trends representation by Stock and Watson (1988). Engle and Issler (1995) extend this model by suggesting that an equivalent restriction in terms of the number of cycles be imposed. This is expressed in terms of restrictions on the rank of the $\mathbf{C}^*(L)$ matrix polynomial. They suggest imposing the restriction $\mathbf{C}^*(L) = \boldsymbol{\psi} \boldsymbol{\Psi}(L)'$ where $\boldsymbol{\psi}$ and $\boldsymbol{\Psi}(L)$ are an $m \times s$ matrix and polynomial matrix of full rank respectively where $s < m$.

¹We assume that no series are integrated of order 2 or higher.

3 Joint common cycle - common trend estimation

Engle and Issler (1995) do not suggest straightforward ways of estimating either the reduced rank polynomial matrix, $\mathbf{C}^*(L)$, or the common cycles it implies apart from the special case where the number of trends and the number of cycles add up to the dimension of the system. The problem is clear and can be appreciated by examining the VAR in levels given in (1) under a plausible identification structure for the common trends common cycle model. If we restrict all coefficient matrices in the VAR representation to be of the form $\mathbf{A}_i = \mathbf{A}_1 \mathbf{A}'_{2i}$, $i = 1, \dots, p$, where \mathbf{A}_1 and \mathbf{A}_{2i} are $m \times s$ full rank matrices, as in Velu, Reinsel, and Wichern (1986) we obtain a short-run reduced rank representation for the series.² However, we also need to impose r^2 unit root restrictions on the matrix polynomial $\mathbf{A}(L)$ needed for the common trends representation. Estimation of the model under those joint restrictions clearly requires iterative numerical techniques. Additionally, the most appropriate way to impose such restrictions is not clear.

3.1 Reduced rank regression

Before presenting our method we briefly outline some results on multivariate reduced rank regressions, (see e.g. Brillinger (1981), Velu, Reinsel, and Wichern (1986), Reinsel and Velu (1998) or Camba-Mendez, Kapetanios, Smith, and Weale (2000)). The case of a single set of reduced rank regressors is well known and widely discussed. We therefore discuss the extension that is useful for our purposes and involves two sets of reduced rank regressors. Consider a multivariate regression model of the form

$$\mathbf{Y} = \mathbf{X}\mathbf{A}_x + \mathbf{Z}\mathbf{A}_z + \boldsymbol{\epsilon} \quad (6)$$

²Note that the assumption of a uniform rank for all \mathbf{A}_{2i} is not necessary but is assumed for simplicity. In the case where the ranks of \mathbf{A}_{2i} are not equal for all i , s is equal to the maximum rank over i .

where \mathbf{Y} , \mathbf{X} and \mathbf{Z} are $T \times k_1$, $T \times k_2$ and $T \times k_3$ matrices respectively. The matrices \mathbf{A}_x and \mathbf{A}_z are of reduced ranks r_x and r_z respectively. As a result they can be written as $\mathbf{A}_x = \boldsymbol{\beta}_x \boldsymbol{\alpha}'_x$ and $\mathbf{A}_z = \boldsymbol{\beta}_z \boldsymbol{\alpha}'_z$ where $\boldsymbol{\alpha}_x$ $\boldsymbol{\beta}_x$ are full rank $k_2 \times r_x$ and $k_1 \times r_x$ matrices and $\boldsymbol{\alpha}_z$ $\boldsymbol{\beta}_z$ are full rank $k_3 \times r_z$ and $k_1 \times r_z$ matrices. We want to estimate $\boldsymbol{\alpha}_x$, $\boldsymbol{\beta}_x$, $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ by minimising $\text{tr}[(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}_x\boldsymbol{\alpha}'_x + \mathbf{Z}\boldsymbol{\beta}_z\boldsymbol{\alpha}'_z)(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}_x\boldsymbol{\alpha}'_x + \mathbf{Z}\boldsymbol{\beta}_z\boldsymbol{\alpha}'_z)']$. A number of methods exists for this estimation. One method that is both non-iterative and simultaneous is given by Reinsel and Velu (1998). The method assumes initial estimates of $\boldsymbol{\beta}_x$ and $\boldsymbol{\beta}_z$ which can be obtained as follows: Estimate (6) by least squares. Conditional on the estimate of \mathbf{A}_x run a single regressor reduced rank regression to estimate $\boldsymbol{\beta}_z$ and vice versa to estimate $\boldsymbol{\beta}_x$. These estimates are denoted by $\tilde{\boldsymbol{\beta}}_x$ and $\tilde{\boldsymbol{\beta}}_z$. Conditional on these estimates, least squares estimation gives estimates of $\boldsymbol{\alpha}_x$ and $\boldsymbol{\alpha}_z$ denoted, by $\hat{\boldsymbol{\alpha}}_x$ and $\hat{\boldsymbol{\alpha}}_z$. Then, updated estimates of $\boldsymbol{\beta}_x$ and $\boldsymbol{\beta}_z$ are obtained via standard minimisation of the objective function and given by

$$\hat{\boldsymbol{\beta}}'_x = \hat{\boldsymbol{\alpha}}'_x \left[1/T(\mathbf{Y}' - \hat{\boldsymbol{\alpha}}'_z \tilde{\boldsymbol{\beta}}'_z \mathbf{Z}') \mathbf{X} \right] [1/T \mathbf{X}' \mathbf{X}]^{-1}$$

$$\text{vec}(\hat{\boldsymbol{\beta}}'_z) = [(1/T \mathbf{Z}' \mathbf{Z} \otimes \mathbf{I}) - (\mathbf{R} \otimes \mathbf{Q})] \text{vec}(\mathbf{P})$$

where $\mathbf{R} = 1/T \mathbf{Z}' \mathbf{X} (1/T \mathbf{X}' \mathbf{X})^{-1} 1/T \mathbf{X}' \mathbf{Z}$, $\mathbf{Q} = (\hat{\boldsymbol{\alpha}}'_z \hat{\boldsymbol{\alpha}}_x) (\hat{\boldsymbol{\alpha}}'_x \hat{\boldsymbol{\alpha}}_z)$ and $\mathbf{P} = \hat{\boldsymbol{\alpha}}'_z (1/T \mathbf{Y}' \mathbf{Z}) - (\hat{\boldsymbol{\alpha}}'_z \hat{\boldsymbol{\alpha}}_x) \hat{\boldsymbol{\alpha}}'_x (1/T \mathbf{Y}' \mathbf{X}) (1/T \mathbf{X}' \mathbf{X})^{-1} (1/T \mathbf{Y}' \mathbf{Z})$. These estimates are \sqrt{T} -consistent and asymptotically normally distributed.

3.2 Estimation of Common Trends and Cycles

In what follows we suggest a consistent method of estimating jointly a common cycle common trend model. We start by estimating the unrestricted VAR in levels in (1). This provides an estimate for the error sequence $\hat{\boldsymbol{\epsilon}}_t$. Under the presence of cointegration, $\boldsymbol{\epsilon}_t - \hat{\boldsymbol{\epsilon}}_t$ is $O_p(T^{-1/2})$ for all t . If no cointegration occurs we obtain as a corollary of superconsistency that $\boldsymbol{\epsilon}_t - \hat{\boldsymbol{\epsilon}}_t$ is $O_p(T^{-1})$. Due care should be taken to ensure that the normalisations needed

to achieve parameter identification are consistent with each other for the successive reduced rank estimators.

Given the above we demean the differenced data and run the following regression with two sets of reduced rank regressors

$$\Delta \mathbf{y}_t = \mathbf{C}(1)\hat{\boldsymbol{\epsilon}}_t + \mathbf{C}_0^* \Delta \hat{\boldsymbol{\epsilon}}_t + \mathbf{C}_1^* \Delta \hat{\boldsymbol{\epsilon}}_{t-1} + \dots + \mathbf{C}_q^* \Delta \hat{\boldsymbol{\epsilon}}_{t-q} + \hat{\boldsymbol{\eta}}_t \quad (7)$$

where q is chosen to be a function of the sample size as discussed below. $\mathbf{C}(1)$ has rank $m - r$ and each \mathbf{C}_i^* can be written as $\mathbf{G}\mathbf{H}_i'$ where both constituent matrices have full rank $s < m$. In anticipation of what follows define: $\Delta \mathbf{y} = (\Delta \mathbf{y}_1, \dots, \Delta \mathbf{y}_T)'$, $\hat{\boldsymbol{\epsilon}}_{q,t} = (\hat{\boldsymbol{\epsilon}}'_t, \Delta \hat{\boldsymbol{\epsilon}}'_{t-1}, \dots, \Delta \hat{\boldsymbol{\epsilon}}'_{t-q})'$, $\hat{\boldsymbol{\epsilon}}_q = (\hat{\boldsymbol{\epsilon}}_{q,1}, \dots, \hat{\boldsymbol{\epsilon}}_{q,T})'$, $\boldsymbol{\epsilon}_{q,t} = (\boldsymbol{\epsilon}'_t, \Delta \boldsymbol{\epsilon}'_{t-1}, \dots, \Delta \boldsymbol{\epsilon}'_{t-q})'$, and $\boldsymbol{\epsilon}_q = (\boldsymbol{\epsilon}_{q,1}, \dots, \boldsymbol{\epsilon}_{q,T})'$.

We now provide an asymptotic justification of our method in terms of parameter estimate consistency for the parameter set $\hat{\boldsymbol{C}}_q = (\hat{\mathbf{C}}(1), \hat{\mathbf{C}}_1^*, \dots, \hat{\mathbf{C}}_q^*)'$. We split the argument in two parts. In the first part we assume that the error sequence $\boldsymbol{\epsilon}_t$ is known. We get consistency of $\hat{\boldsymbol{C}}$ under this assumption and then generalise to the actual setup. q is assumed to tend to infinity as T tends to infinity, but at a rate less than $T^{1/3}$ (see Berk (1974)). No more stringent conditions are needed for consistency. If one wants to investigate parameter estimate rates of convergence and asymptotic normality then more restrictive conditions are needed (it suffices for that to have q rise by at least $T^{1/r}$ for some $r > 3$). We concentrate on consistency of OLS estimates of the parameters to simplify the analysis. As both OLS estimates and reduced rank estimates are continuous functions of the moment matrices of the regressors and regressands, consistency of one estimator implies consistency of the other and vice versa³.

Assuming a known error sequence⁴, and $\boldsymbol{\mu} = 0$ for simplicity, we have that

$$\hat{\boldsymbol{C}}_q^{OLS} - \mathbf{C}_q = (\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \Delta \mathbf{y} - \mathbf{C}_q = (\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q \mathbf{C}_q - \mathbf{C}_q + (\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_{-q} \mathbf{C}_{-q} = (\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_{-q} \mathbf{C}_{-q}$$

³In the reduced rank case, consistency, of course, only holds for assumed ranks equal to or larger to true ones.

⁴This implies $\boldsymbol{\eta}_t = 0$ in (7).

where $\boldsymbol{\epsilon}_{-q,t} = (\Delta \boldsymbol{\epsilon}'_{t-q-1}, \dots)'$, $\boldsymbol{\epsilon}_{-q} = (\boldsymbol{\epsilon}_{-q,t}, \dots)'$ and $\mathbf{C}_{-q} = (\mathbf{C}_{q+1}^*, \dots)'$. By the fact that the data generation process is a VAR in levels, $\|\mathbf{C}_{q+1}^*\| \sim c^q$ where $|c| < 1$ and $\|\cdot\|$ is the supremum matrix norm. This and the assumption that q tends to infinity at rate of less than $T^{1/3}$ leads to consistency of the OLS estimator for the case where the error sequence is known. More specifically we have that $\|(\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_{-q} \mathbf{C}_{-q}\|$ converges to zero in probability. To see this note the following: By Lemma 3 of Berk (1974) we have that the norm of the difference between $(\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1}$ and its population counterpart converges to zero if q^3/T converges to zero. Further, the norm of $\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_{-q} \mathbf{C}_{-q}$ converges in probability to zero if $q(\|\mathbf{C}_{q+1}^*\| + \|\mathbf{C}_{q+2}^*\| + \dots)$ converges to zero which is the case if q tends to infinity and $\|\mathbf{C}_{q+1}^*\| \sim c^{q+1}$, $|c| < 1$, for large q , which we assume. Combining the above two results with the fact that the norm of a product is dominated by the product of the norms gives the result.

The second step is to show that $\hat{\mathbf{C}}_q^{OLS} - \hat{\mathbf{C}}_q^{OLS} = o_p(1)$ where $\hat{\mathbf{C}}_q^{OLS}$ is obtained by using $\hat{\boldsymbol{\epsilon}}_t$ rather than $\boldsymbol{\epsilon}_t$. We have that

$$\hat{\mathbf{C}}_q^{OLS} - \hat{\mathbf{C}}_q^{OLS} = (\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} \boldsymbol{\epsilon}'_q \Delta \mathbf{y} - (\hat{\boldsymbol{\epsilon}}'_q \hat{\boldsymbol{\epsilon}}_q)^{-1} \hat{\boldsymbol{\epsilon}}'_q \Delta \mathbf{y}$$

It is sufficient to show that (i) $1/T(\boldsymbol{\epsilon}'_q \Delta \mathbf{y} - \hat{\boldsymbol{\epsilon}}'_q \Delta \mathbf{y}) = o_p(1)$ and (ii) $1/T((\boldsymbol{\epsilon}'_q \boldsymbol{\epsilon}_q)^{-1} - (\hat{\boldsymbol{\epsilon}}'_q \hat{\boldsymbol{\epsilon}}_q)^{-1}) = o_p(1)$. We prove (i).

$$\|1/T(\boldsymbol{\epsilon}'_q \Delta \mathbf{y} - \hat{\boldsymbol{\epsilon}}'_q \Delta \mathbf{y})\| = \|1/T \sum_{t=1}^T (\boldsymbol{\epsilon}_{q,t} \Delta \mathbf{y}'_t - \hat{\boldsymbol{\epsilon}}_{q,t} \Delta \mathbf{y}'_t)\| \leq q \max_t \|\boldsymbol{\epsilon}_t - \hat{\boldsymbol{\epsilon}}_t\| \|1/T \sum_{t=1}^T \Delta \mathbf{y}'_t\|$$

$1/T \sum_{t=1}^T \Delta \mathbf{y}'_t$ converges to its non zero expectation, each of $\boldsymbol{\epsilon}_t - \hat{\boldsymbol{\epsilon}}_t$ is at most $O_p(T^{-1/2})$ and therefore the whole term is $o_p(1)$. (ii) follows from similar arguments as above and the facts that, firstly for positive definite matrices A and B , if $A - B = o_p(1)$ then $A^{-1} - B^{-1} = o_p(1)$, and secondly, if $A - \hat{A} = o_p(1)$ and $B - \hat{B} = o_p(1)$ then $AB - \hat{A}\hat{B} = o_p(1)$.

The above analysis has concentrated on providing consistent joint estimates of common cycles and common trends and has not considered neither

the asymptotic distribution of the estimates nor the question of their asymptotic efficiency. On the issue of the asymptotic distribution we note that if standard errors of the parameter estimates are required then they may be derived using the distribution of coefficients of reduced rank regression models with two sets of reduced rank regressors given in Theorem 3.2 of Reinsel and Velu (1998, pp. 81). Note that all regressors involved here are stationary and ergodic and therefore no asymptotic distributional issue arises out of the presence of nonstationarity in \mathbf{y}_t . Of course, in our case the number of regressors increases with the sample size but results in Berk (1974) can be used to obtain conditions on the rate of increase of the number of regressors q to give consistent estimation of the moment matrices needed to justify use of Theorem 3.2 of Reinsel and Velu (1998)⁵.

If the model is a finite order VAR model then the suggested method will not provide asymptotically efficient estimates since the true form of the model is not taken under account. However, this drawback is compensated by the fact that the method we suggest is robust to the case where the true model is a VAR model of infinite order or indeed any model where the differenced data accept an infinite MA representation⁶. To see this note that the MA representation of the differenced data still holds under an infinite order VAR model. This implies that the Beveridge Nelson decomposition, in (4), which forms the basis of the analysis, holds. If we use data dependent methods such as information criteria to fit the initial vector autoregression then, if that vector autoregression is of infinite order, the lag order chosen will be growing with the sample size at rate $\ln(T)$ (see Ng and Perron (1995) and for a complete treatment see Hannan and Deistler (1988)). Then, the estimated error sequence $\hat{\epsilon}_t$ will be consistent for the true disturbance terms. In this context cointegration is defined as the occurrence of reduced rank in the $\mathbf{C}(1)$

⁵Essentially, the conditions needed for consistency of parameter estimates need to be tightened so that $q = O(T^{1/r})$, for some $r > 3$ to get asymptotic normality. (see also Ng and Perron (1995))

⁶This of course occurs, under regularity conditions, whenever the differenced data are stationary using the Wold decomposition.

matrix following Bierens (1997). Then, the same reduced rank analysis of the MA representation, as above, applies.

3.3 Numbers of trends and cycles

The above analysis assumed that the ranks $m - r$ and s are known. Clearly, this is not the case in practical applications. We suggest two methods of determining these ranks. The first is the standard method for determining ranks in reduced rank regression models using two sets of reduced rank regressors. Partial canonical correlation analysis is used for this as suggested by Reinsel and Velu (1998). However, this method essentially determines one rank conditional on the other. Alternatively, for joint determination of the ranks, information criteria may be used. To do that we estimate the model for the grid of all possible ranks where both the rank of $\mathbf{C}(1)$ and that of $\mathbf{C}^*(L)$ are allowed to vary independently between 1 and m . A penalty term depending on the information criterion is then added to the sum of squared residuals to provide the objective function to be minimised. Any of the usual information criteria may be used (e.g. Akaike's, Bayesian information criterion, Hannan-Quinn information criterion) taking into account the fact that Akaike's criterion may choose larger ranks asymptotically since it is not a consistent criterion.

4 Conclusion

In this note we have suggested a new method for joint determination of common trends and common cycles in cointegrated multivariate systems. No other joint estimation method exists in the literature. The method uses the infinite MA representation of the differenced series and applies reduced rank regression methods to estimate the trend and cycle parameters. A method to determine jointly the number of trends and cycles has also been suggested. It is worth noting that the method, based on the infinite MA representation

is applicable in more general settings and in particular it is applicable for VAR models with an infinite number of lags.

References

- BERK, K. N. (1974): “Consistent Autoregressive Spectral Estimates,” *Annals of Statistics*, 2(3), 489–502.
- BIERENS, H. (1997): “Nonparametric Cointegration Analysis,” *Journal of Econometrics*, 77, 379–404.
- BRILLINGER, D. R. (1981): *Time Series: Data Analysis and Theory*. Holden Day.
- CAMBA-MENDEZ, G., G. KAPETANIOS, R. J. SMITH, AND M. R. WEALE (2000): “Tests of Rank in Reduced Rank Regression Models,” Forthcoming in *Journal of Business and Economic Statistics*.
- ENGLE, R. F., AND J. V. ISSLER (1995): “Estimating Common Sectoral Cycles,” *Journal of Monetary Economics*, 35, 83–113.
- HANNAN, E. J., AND M. DEISTLER (1988): *The Statistical Theory of Linear Systems*. John Wiley.
- NG, S., AND P. PERRON (1995): “Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag,” *Journal of the American Statistical Association*, 90, 268–281.
- REINSEL, G. C., AND R. P. VELU (1998): *Multivariate Reduced Rank Regression*. Springer-Verlag.
- STOCK, J. H., AND M. WATSON (1988): “Testing for Common Trends,” *Journal of the American Statistical Association*, 83, 1097–1107.
- VAHID, F., AND R. F. ENGLE (1993): “Common Trends and Common Cycles,” *Journal of Applied Econometrics*, 8, 341–360.

VELU, R. P., G. C. REINSEL, AND D. W. WICHERN (1986): "Reduced Rank Models for Multiple Time Series," *Biometrika*, 73, 105–118.

**This working paper has been produced by
the Department of Economics at
Queen Mary, University of London**

**Copyright © 2003 George Kapetanios
All rights reserved.**

**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096 or Fax: +44 (0)20 8983 3580
Email: j.conner@qmul.ac.uk
Website: www.econ.qmul.ac.uk/papers/wp.htm**