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By
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Green national accounting
for welfare and sustainability:
A taxonomy of assumptions and results

Geir B. Asheim*

October 23, 2002

Abstract

This paper summarizes assumptions made and results obtained in parts of the literature on welfare and sustainability accounting. I consider five different assumptions that can be imposed independently of each other, producing 32 different combinations. This taxonomy is used to organize results in welfare and sustainability accounting. The analysis illustrates how stronger results require stronger assumptions and thereby impose harder informational requirements.

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1 Introduction

During the more than 25 years since Martin Weitzman published his seminal paper (Weitzman, 1976) on the significance for dynamic welfare of comprehensive national accounting aggregates, there have been many important theoretical contributions on welfare and sustainability accounting. This literature shows how national accounting aggregates can be used to measure differences in welfare, both over time and across different economies, and to indicate whether development is sustainable. It is, however, often not very transparent under what assumptions different results on welfare and sustainability accounting will hold. On this background I treat the topic systematically in this paper and summarize assumptions made and results obtained in major parts of this literature.

I consider five different assumptions that can be imposed independently of each other, producing altogether 32 different combinations of assumptions. This taxonomy will be used to organize the different results. The most general analysis answering the “simplest” problems and imposing the weakest assumptions will be addressed in Sect. 3. Analysis requiring stronger assumptions, but answering more “complicated” questions will be addressed in Sects. 4 and 5. The presentation emphasizes the assumptions needed in order for results to be of interest for practical estimation, thereby organizing the discussion of informational problems that must be faced when doing empirical analysis. A discussion of methods to overcome informational constraints is contained in Sect. 6. Two tables yield an overview of assumptions and results.¹

It is a prerequisite for most of the results that the list of goods and services included in national accounting aggregates is comprehensive. The national accounts are ‘comprehensive’ if all variable determinants of current well-being are included in the vector of consumption flows, and if all variable determinants of current productive capacity are included in the vector of capital stocks. E.g., compared to NNP as normally measured, one must “green” the national accounts by introducing natural resource depletion and environmental degradation into the national accounts by (i) including such depletion and degradation of natural capital as negative components to the vector of investment goods, and (ii) adding flows of environmental amenities to the vector of consumption goods.

¹The present paper extends my earlier overview, Asheim (2000), by considering multiple consumption goods and by presenting additional results. However, it leaves out concepts that do not easily generalize to a multiple-consumption-good setting, and does not discuss the relationship between green national accounting and social cost-benefit analysis.

2 Model, assumptions, and notation

This section presents the general model that will be used throughout the paper, lists the five assumptions that will be considered, and introduces notation.

2.1 Model

Consider a setting where population is constant² and where the current instantaneous well-being at time t depends on the vector of commodities $\mathbf{C}(t) = (C_1(t), \dots, C_m(t))$ consumed at time t . To concentrate on the issue of intertemporal distribution, we abstract from how the goods and services consumed at time t are distributed among the population. Thereby we may associate the instantaneous well-being at time t with the *utility* $U(\mathbf{C}(t))$ that is derived from the vector of consumption flows, $\mathbf{C}(t)$, at time t , where U is a time-invariant, increasing, and sufficiently differentiable function. Current consumption is presumed to be observable, along with its associated vector of accounting prices. For some of the results one must have that U is concave.

That U is time-invariant means that all *variable* determinants of current well-being are included in the vector of consumption flows. In particular, non-constant flows of environmental amenities flows derived from non-constant stocks of natural capital are represented by components of the extended consumption vector \mathbf{C} . If labor supply is not fixed, then supplied labor corresponds to negative components of the vector \mathbf{C} . Thus, changes in instantaneous well-being will be measured net of the cost of turning leisure into labor effort.

The vector of *capital* goods $\mathbf{K}(t) = (K_1(t), \dots, K_n(t))$ available at time t includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other durable productive assets. Corresponding to the stock of capital of type j at time t , $K_j(t)$, there is a net *investment* flow: $I_j(t) := \dot{K}_j(t)$. Hence, $\mathbf{I}(t) = (I_1(t), \dots, I_n(t)) = \dot{\mathbf{K}}(t)$ denotes the vector of net investments. A consumption-investment pair $(\mathbf{C}(t), \mathbf{I}(t))$ at time t is attainable if and only if $(\mathbf{C}(t), \mathbf{I}(t)) \in S(\mathbf{K}(t), t)$, where S is a sufficiently smooth set that describes

²It is worthwhile also to analyze the case with a changing population. There are results available (cf., e.g., Hamilton, 2002) under exponential population growth when only per capita consumption matters, provided that one is willing to assume CRS, introduced below. Contributions where population growth need not be exponential and where instantaneous well-being also depends on population size are emerging (cf., e.g., Arrow et al., 2002b; Asheim, 2002). These result cannot as easily be integrated into the present taxonomy and are not treated here.

society's productive capacity. Current net investments are presumed to be observable, along with the associated vector of accounting prices. Some of the results require that $S(\mathbf{K}(t), t)$ is a convex set.

Assume that society's actual decisions are taken according to a *resource allocation mechanism* that assigns an attainable consumption-investment pair to any vector of capital stocks \mathbf{K} and time t . Hence, for any vector of capital stocks \mathbf{K} and time t , the resource allocation mechanism determines the consumption and investment flows. The investment flows in turn maps out the development of the capital stocks. The resource allocation mechanism thereby implements a feasible path of consumption flows, investment flows, and capital stocks, for any initial vector of capital stocks and any initial time.

2.2 Assumptions

Consider a society with social preferences over infinite horizon utility paths. Let *dynamic welfare* be an index that represents these social preferences, meaning that if one path yields higher dynamic welfare than another, then it is (strictly) preferred in social evaluation. In this context results of welfare and sustainability accounting can be classified according to which of the following five assumptions are being adopted. Hence, these specific assumptions are considered because they enable us to construct the taxonomy of results presented in Sects. 3-5.

Assumption OPT (*Optimality*). Society has an optimal resource allocation mechanism implementing a price-supported³ – and thus efficient – path that maximizes dynamic welfare.

Assumption DU (*Discounted utilitarianism*). Dynamic welfare at time t is given by

$$\int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)}dt,$$

where ρ is a positive utility discount rate.

Notice that OPT can be satisfied without DU and vice versa. E.g., consider throughout this paragraph a situation where society in fact implements an efficient path with

³Formally, 'price-supported' means that there is an infinite-dimensional hyperplane, containing the implemented path, that separates all feasible paths from those that are socially preferable. Malinvaud (1953) introduced this mathematical tool to the study of dynamic infinite-horizon economies.

constant utility, but where the implementation of an optimal path according to DU would have led to non-constant utility. If, on the one hand, society's dynamic welfare is given by $\inf_{s \geq t} U(\mathbf{C}(s))$, then OPT is satisfied (since the implemented path is efficient and maximizes dynamic welfare), while DU is not satisfied (since $\int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)} dt$ does not represent the social preferences). If, on the other hand, society's dynamic welfare is given by $\int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)} dt$, then DU is satisfied, while OPT is not satisfied (since the implemented constant utility path does not maximize dynamic welfare). Weitzman (1976) assumes both OPT and DU. Dasgupta and Mäler (2000), Dasgupta (2001), and Arrow et al. (2002a) assume DU without assuming OPT, while Solow (1974) assumes OPT without assuming DU.

Assumption ST (*Stationary technology*). The set S does not depend directly on t .

This assumption is usually identified with *comprehensive accounting* and means that any *variable* determinant of current productive capacity is included in the vector of capital stocks. Weitzman (1976) along with most of the subsequent literature makes this assumption. To make accounting comprehensive in this way is a major challenge for empirical estimation. There are several reasons why ST may not be satisfied: technological progress not captured by augmented stocks, unaccounted-for stocks of natural capital, and open economies with changing terms-of-trade.

Assumption CRS (*Constant returns to scale*). S as a set valued function of \mathbf{K} is homogeneous of degree 1.

This is seldom made as an explicit assumption; neither Weitzman (1976) nor most of the subsequent literature makes it (although I have used it in some of my own papers, e.g., in Asheim, 1996). The assumption is often invoked in illustrating examples. Combined with ST it means that also *fixed* determinants of current productive capacity must be included; the fixed amount of land is a prime example of this. In a world where natural and environmental resources are important, trying to satisfy this assumption turns empirical estimation into a very demanding task. In particular, the assumption of CRS necessitates that consumption flows and capital stocks are measured along scales where 0 is defined. For flows like environmental amenities and stocks like knowledge it is unclear what this entails.

Notice that CRS can be satisfied without ST and vice versa. If, on the one hand, the technology is given by $C + I \leq A(t)K$ with $\dot{A}(t) > 0$, then CRS is satisfied, while

ST is not satisfied. If, on the other hand, the technology is given by $C + I \leq f(K)$ with f exhibiting decreasing returns to scale, then ST is satisfied, while CRS is not satisfied.

Assumption LH (*Linear homogeneity*). U as a function of \mathbf{C} is homogeneous of degree 1.

This is a generalization of an assumption made by Weitzman (1976) to multiple consumption goods. Throughout the years, Dasgupta and Mäler have argued that national accounting should not be based on this assumption.

2.3 Notation

The following notation will be used:

- \mathbf{p} : nominal consumption prices
- \mathbf{P} : real consumption prices
- \mathbf{q} : nominal investment prices
- \mathbf{Q} : real investment prices
- $\mathbf{Y} = \mathbf{P}\mathbf{C} + \mathbf{Q}\mathbf{I}$: real NNP.

It will be explained along the way what exactly is meant by “real” prices.

3 Results under ST and DU or OPT

Assume ST and DU. By ST one can assume that the possibly inefficient resource allocation mechanism in the economy is Markovian and stationary, in the sense that the implemented consumption-investment pair is a time-invariant function of the vector of capital stocks (cf. Arrow et al., 2002a). This means that the consumption-investment pair $(\mathbf{C}(t), \mathbf{I}(t))$ at any time t is determined by the vector of capital stocks at time t , and does not depend directly on t . Hence, if $\{\mathbf{C}(s)\}_{s=t}^{\infty}$ is the implemented path given the initial stock $\mathbf{K}(t) = \mathbf{K}$, then the dynamic welfare of this path,

$$V(\mathbf{K}) = \int_t^{\infty} U(\mathbf{C}(s))e^{-\rho(s-t)} ds, \quad (1)$$

is a function solely of \mathbf{K} . In particular, $dV(\mathbf{K}(t))/dt = \nabla V(\mathbf{K}(t))\mathbf{I}(t) > 0$ means that dynamic welfare is increasing at time t , where ∇ denotes a vector of partial derivatives, and where we follow Arrow et al. (2002a) by assuming that V is differentiable. Assume

furthermore that this vector of partial derivatives can be calculated up to the choice of numeraire; hence, a vector of accounting prices

$$\mathbf{q}(t) = \frac{\nabla V(\mathbf{K}(t))}{\lambda(t)}$$

is observable, where $\lambda(t) > 0$ is the price of the numeraire in terms of utils.⁴ Then the sign of the observable entity $\mathbf{q}(t)\mathbf{I}(t)$ indicates whether dynamic welfare is increasing. Notice that $\mathbf{q}(t)\mathbf{I}(t)$ represents the value of net investments, and it is often referred to as the “genuine savings indicator” (cf. Hamilton, 1994, p. 166). Its sign is of course independent of the numeraire in which $\mathbf{q}(t)$ is measured.

By differentiating (1) w.r.t. time and using the property that the resource allocation mechanism is Markovian, we obtain

$$\nabla V(\mathbf{K}(t))\dot{\mathbf{I}}(t) = \frac{dV(\mathbf{K}(t))}{dt} = \rho \int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)} ds - U(\mathbf{C}(t))$$

or

$$U(\mathbf{C}(t)) + \nabla V(\mathbf{K}(t))\dot{\mathbf{I}}(t) = \rho V(\mathbf{K}(t)). \quad (2)$$

Differentiating once more w.r.t. time yields:

$$\nabla U(\mathbf{C}(t))\dot{\mathbf{C}}(t) + \frac{d\nabla V(\mathbf{K}(t))\dot{\mathbf{I}}(t)}{dt} = \rho \nabla V(\mathbf{K}(t))\dot{\mathbf{I}}(t).$$

If we follow Asheim and Weitzman (2001) and let

$$\mathbf{p}(t) = \frac{\nabla U(\mathbf{C}(t))}{\lambda(t)}$$

denote calculated consumption prices in terms of the numeraire, we obtain

$$\mathbf{p}(t)\dot{\mathbf{C}}(t) + \frac{d\mathbf{q}(t)\mathbf{I}(t)}{dt} = r(t)\mathbf{q}(t)\mathbf{I}(t), \quad (3)$$

where $r(t) = \rho - \dot{\lambda}/\lambda$ is the interest rate associated with the numeraire. By letting real prices $\{\mathbf{P}(t), \mathbf{Q}(t)\}$ be determined locally-in-time using a Divisia consumption price index (cf. Asheim and Weitzman, 2001; Sefton and Weale, 2000), so that $\dot{\mathbf{P}}(t)\mathbf{C}(t) = 0$, it follows that

$$\dot{Y}(t) = \frac{d(\mathbf{P}(t)\mathbf{C}(t) + \mathbf{Q}(t)\mathbf{I}(t))}{dt} = R(t)\mathbf{Q}(t)\mathbf{I}(t), \quad (4)$$

where $Y(t)$ is real NNP and $R(t)$ is the real interest rate. Since $\mathbf{Q}(t)$ is proportional to $\nabla V(\mathbf{K}(t))$, (4) implies that also $\dot{Y}(t) > 0$ indicates welfare improvement.

⁴With an optimal resource allocation mechanism, $\lambda(t)$ is the marginal utility of current expenditures.

Proposition 1 *Under ST and DU, welfare improvement can be indicated by a positive value of net investments ($\mathbf{qI} > 0$), or by growth in real NNP ($\dot{Y} > 0$).*

Assume now ST and OPT. As before ST means that the resource allocation mechanism in the economy is Markovian and stationary. By OPT the implemented path is price-supported, and it follows from optimal control theory that there are investment prices $\Psi(t)$ in terms of utility, such that

$$\nabla U(\mathbf{C}(t))\dot{\mathbf{C}}(t) + \frac{d\Psi(t)\mathbf{I}(t)}{dt} = \rho(t)\Psi(t)\mathbf{I}(t),$$

where $\rho(t)$ is the supporting utility discount rate at time t , and where, as shown by Asheim and Buchholz (2002), dynamic welfare is improving if and only if $\Psi(t)\mathbf{I}(t) > 0$. By assuming that the efficiency prices $\Psi(t)$ are observable up to the choice of numeraire, and by repeating Asheim and Weitzman’s (2001) argument above—so again (4) follows when real prices are determined by a Divisia index—we obtain the following result.

Proposition 2 *Under ST and OPT, welfare improvement can be indicated by a positive value of net investments ($\mathbf{qI} > 0$), or by growth in real NNP ($\dot{Y} > 0$).*

The fundamental equation in both these results is (4), stating that

$$\text{change in real NNP} = \text{real interest rate} \cdot \text{the real value of net investments.}$$

It is this equation that allows the “futurity” in any welfare evaluation of any dynamic situation” (Samuelson, 1961, p. 53) to be captured by *current* national accounting aggregates.⁵

Say that development is *sustainable* at the current time, if the utility derived from the current vector of consumption flows can potentially be sustained forever. What does Props. 1 and 2 tell us about the following question: Is the value of net investments (or, equivalently, real NNP growth) an indicator of sustainable development? The answer depends on the circumstances.

Assume that ST is combined with OPT, and that the social preferences take sustainability into account, e.g., through the constraint that, at any time, current utility should not exceed the maximum sustainable utility level given the current capital stocks. By OPT, the agents in society expect that development will indeed be sustainable, and these expectations will be reflected by the relative investment prices. In such circumstances, non-decreasing welfare may well correspond to development being sustainable.

⁵Under his stronger set of assumptions, this equation was used by Weitzman (1976, eq. (14)).

Hence, since ST is satisfied, it follows from Prop. 2 that a non-negative value of net investments (or equivalently, non-negative rate of real NNP growth) may serve as an exact indicator of sustainability.

In Asheim and Buchholz (2002, Sect. 6.2) we provide an explicit example of this within the context of the model of capital accumulation and resource depletion introduced by Dasgupta and Heal (1974) and Solow (1974). In the setting of this model we show that the growth rate of real NNP decreasing towards zero indicates that unconstrained development is no longer sustainable. Hence, the information on welfare changes offered by the growth rate of real NNP (or equivalently, the sign of the value of net investments) can be useful for the management of society's assets, given that unsustainable paths are deemed socially unacceptable.

If instead society adheres to DU, then—even if ST and OPT hold—sustainability need not be indicated in this manner, since DU does not necessarily lead to sustainable development and the ratio of investment prices may be affected by this. In context of the Dasgupta-Heal-Solow model it was established by Asheim (1994) and Pezzey (1994) that the value of net investments can be positive at the same time as utility exceed the maximum sustainable level.

However, Pezzey (2002) has recently established a *one-sided sustainability test* under ST, OPT, and DU: It is a *necessary* condition for sustainable development that the value of net investments (or, equivalently, real NNP growth) is non-negative. The following result is a version of Pezzey (2002, Prop. 2).

Proposition 3 *Under ST, OPT and DU, the current level of utility cannot be sustained forever if the value of net investments is negative ($\mathbf{qI} < 0$), or if growth in real NNP is negative ($\dot{Y} < 0$).*

To see this, notice that if ST and DU are assumed, then it follows from (2) that $U(\mathbf{C}(t)) + \nabla V(\mathbf{K}(t))\mathbf{I}(t)$ is a Hicks (1946)–Weitzman (1976) stationary equivalent of future utility

$$\int_t^\infty (U(\mathbf{C}(s)) + \nabla V(\mathbf{K}(s))\mathbf{I}(s))e^{-\rho(s-t)}ds = V(\mathbf{K}(t)) \quad (5)$$

since $\int_t^\infty e^{-\rho(s-t)}ds = 1/\rho$. Moreover, if OPT is added, then

$$V(\mathbf{K}(t)) \geq \int_t^\infty \bar{U} \cdot e^{-\rho(s-t)}ds \quad (6)$$

where \bar{U} is the maximum level of utility that can be sustained forever from time t on, given the initial stocks at time t . It follows from (5) and (6) that $U(\mathbf{C}(t))$ exceeds the maximum sustainable level if, at t , $\mathbf{qI} = (\nabla V(\mathbf{K})\mathbf{I})/\lambda < 0$, or equivalently, $\dot{Y} < 0$.

Finally, notice that if ST and DU—but not OPT—hold, then the value of net investments and real NNP growth are quite unreliable indicators of sustainability. Consider, e.g., a society where traditional growth is promoted through high investment in reproducible capital goods, but where incorrect (or lack of) pricing of natural capital leads to depletion of natural and environmental resources that is excessive both from the perspective of short-run efficiency and long-run sustainability. Then utility growth in the short to intermediate run will, if the discount rate ρ is large enough, lead to current growth in dynamic welfare. Hence, both the value of net investments and real NNP growth will be positive. At the same time, the resource depletion may seriously undermine the long-run livelihood of future generations, so that current utility far exceeds the level that can be sustained forever.⁶

The local-in-time character of these results means as NNP as a linear index (cf. Hartwick, 1990) has significance for welfare and sustainability even though no linearity assumptions (like CRS and LH) are made. The next two sections will present stronger assumptions, on the basis of which national accounting aggregates can be used for global welfare comparisons.

The major information problem that must be faced to utilize the results on this section, is how to make accounting comprehensive when – at the outset – not all capital goods that contribute to increased productive capacity are included, i.e., the technology is not stationary. To apply Prop. 1, one must in addition be able to calculate the vector of partial derivatives of the welfare function, V , to determine accounting prices. If, instead, OPT holds, then the vector of relative investment prices, $\mathbf{q}(t)$, correspond to actual market prices or can be calculated as efficiency prices using standard techniques.

4 Results under ST, DU, and LH

Turn now to global welfare comparisons,

- either in one society over time, where $\mathbf{K}' = \mathbf{K}(t')$ is the vector of capital stocks at time t' and $\mathbf{K}'' = \mathbf{K}(t'')$ is the vector of capital stocks at time t'' ,
- or across different societies, where \mathbf{K}' is the vector of capital stocks in the one society and \mathbf{K}'' is the vector of capital stocks in the other society.

⁶This should be borne in mind when, e.g., Arrow et al. (2002a) under ST and DU identify the term “sustainable development at t ” with non-negative value of net investments at t , instead of making use of the definition considered here, namely that utility at t can potentially be sustained forever.

Which of the vectors of capital stocks, \mathbf{K}' or \mathbf{K}'' , corresponds to higher welfare?

It follows from the previous section that, under ST and DU,

$$V(\mathbf{K}'') - V(\mathbf{K}') = \int_{\mathbf{K}'}^{\mathbf{K}''} \nabla V(\mathbf{K}) d\mathbf{K}$$

is a measure of welfare differences that is independent of the path between \mathbf{K}' and \mathbf{K}'' . So if \mathbf{q} corresponding to different values of \mathbf{K} can be measured in a numeraire that is in a fixed proportion to utils, then a global measure of welfare differences would be available. However, to be useful for empirical estimation this essentially requires that utils are measurable. As the following argument suggests, LH is sufficient for utils to be measurable.

If the utility function U is homothetic, then a Divisia consumption price index is path independent, so that real prices can be determined globally.⁷ Moreover, if LH is satisfied, so that U is linearly homogeneous, then these real prices are measured in a numeraire that is in a fixed proportion to utils. W.l.o.g. we may set the factor of proportionality equal to one, so that $\mathbf{P} = \nabla U(\mathbf{C})$ and $\mathbf{Q} = \nabla V(\mathbf{K})$, and implying that

$$U(\mathbf{C}) = \nabla U(\mathbf{C})\mathbf{C} = \mathbf{P}\mathbf{C}.$$

It now follows from (2) that

$$Y = \mathbf{P}\mathbf{C} + \mathbf{Q}\mathbf{I} = \rho V(\mathbf{K}). \quad (7)$$

Furthermore,

$$V(\mathbf{K}'') - V(\mathbf{K}') = \int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q} d\mathbf{K}.$$

This yields the following result.

Proposition 4 *Under ST, DU and LH, a positive welfare difference can be indicated by a positive real value of stock differences ($\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q} d\mathbf{K} > 0$), or by a positive difference in real NNP ($Y'' - Y' > 0$).*

The fundamental equation in this result is (7), stating that

$$\text{real NNP} = \text{real interest rate} \cdot \text{the present value of future consumption.}$$

This is Weitzman's (1976) main result, which we here have established without invoking OPT, but instead assuming that the vector of partial derivatives of V can be calculated.

⁷Cf. Hulten (1987) for a discussion of the properties of a Divisia index.

Notice that the rhs. of (7) is not *wealth* in the sense of the current value of stocks, unless we make further assumptions (see below). Notice also that $\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}d\mathbf{K}$ is not a difference in wealth, but rather a “wealth-like magnitude”, to use Samuelson’s (1961) term.

A major information problem that must be faced to utilize the result on this section, is how to measure utility if the utility function is not linearly homogeneous.

The addition of LH does not help when it comes to indicating sustainability: Under ST and DU the value of net investments and real NNP growth are quite unreliable indicators of sustainability.

5 Results without ST

If ST is not satisfied, then *not all* variable determinants of current productive capacity are included in the vector of capital stocks. Still, by imposing the linearity conditions CRS and LH in addition to OPT and DU, positive results can be obtained.

Assume OPT and CRS. Then it follows that the wealth, in the sense of the current value of stocks, equals the present value of future consumption:

$$\mathbf{q}(t)\mathbf{K}(t) = \int_t^\infty \mathbf{p}(s)\mathbf{C}(s)e^{-\int_t^s r(\tau)d\tau} ds,$$

where $r(s)$ is the nominal discount rate at time s . The result is simple to establish in the framework of Dixit et al. (1980), and has a clear intuition: CRS means that all flows of future earnings can be treated as currently existing capital.

Notice that OPT is needed for this result. To see this, consider Solow’s (1974) maximin path in the Dasgupta-Heal-Solow model of capital accumulation and resource depletion when CRS is satisfied, and let society’s dynamic welfare be given by DU. Then the present value of future consumption as evaluated by DU is constant, while the value of the stocks increases since the value of net investments is zero (cf. Hartwick, 1977) and there are positive anticipated capital gains on the resource.

When we add the assumptions of DU and LH, so that one can determine real prices in terms of utils and the value of consumption equals utility as discussed in Section 4, it follows from this result that

$$\mathbf{Q}(t)\mathbf{K}(t) = \int_t^\infty \mathbf{P}(s)\mathbf{C}(s)e^{-\rho(s-t)} ds = \int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)} ds = V(\mathbf{K}(t)). \quad (8)$$

Moreover, by time-differentiation we get that

$$\mathbf{Q}(t)\dot{\mathbf{K}}(t) + \dot{\mathbf{Q}}(t)\mathbf{K}(t) = \rho \int_t^\infty U(\mathbf{C}(s))e^{-\rho(s-t)} ds - U(\mathbf{C}(t))$$

or (applying that $U(\mathbf{C}) = \mathbf{P}\mathbf{C}$ and $\mathbf{I} = \dot{\mathbf{K}}$)

$$Y(t) + \dot{\mathbf{Q}}(t)\mathbf{K}(t) = \mathbf{P}(t)\mathbf{C}(t) + \mathbf{Q}(t)\mathbf{I}(t) + \dot{\mathbf{Q}}(t)\mathbf{K}(t) = \rho\mathbf{Q}(t)\mathbf{K}(t). \quad (9)$$

Combining (8) and (9) leads to the following result.

Proposition 5 *Under OPT, DU, CRS, and LH, a positive welfare difference can be indicated by a positive difference in wealth ($\mathbf{Q}''\mathbf{K}'' - \mathbf{Q}'\mathbf{K}' > 0$), or by a positive difference in real NNP plus anticipated capital gains ($(Y'' + \dot{\mathbf{Q}}''\mathbf{K}'') - (Y' + \dot{\mathbf{Q}}'\mathbf{K}') > 0$).*

The fundamental equation in this result is (9), stating that

$$\text{real NNP} + \text{anticipated capital gains} = \text{real interest rate} \cdot \text{real wealth}.$$

Given that DU and LH imply a constant rate of interest equal to the utility discount rate ρ , the anticipated capital gains capture the effects of a non-stationary technology.⁸ Hence, if we add ST to the list of assumption, then we arrive at

$$\text{real NNP} = \text{real interest rate} \cdot \text{real wealth}.$$

However, the needed assumptions – OPT, DU, ST, CRS, and LH – makes this not a very interesting result as a basis for empirical estimation.

Under OPT, DU, CRS, and LH, a variant of Pezzey's (2002) one-sided sustainability test (cf. Prop. 3) can be obtained.

Proposition 6 *Under OPT, DU, CRS and LH, the current level of utility cannot be sustained forever if wealth decreases ($d(\mathbf{Q}\mathbf{K})/dt < 0$), or if growth in the sum of real NNP and anticipated capital gains is negative ($d(Y + \dot{\mathbf{Q}}\mathbf{K})/dt < 0$).*

The proof is similar as the one for Prop. 3: If OPT, DU, CRS and LH are assumed, then it follows from (8) and (9) that $Y + \dot{\mathbf{Q}}\mathbf{K}$ is a Hicks-Weitzman stationary equivalent of future utility

$$\int_t^\infty (Y(s) + \dot{\mathbf{Q}}(s)\mathbf{K}(s))e^{-\rho(s-t)}ds = V(\mathbf{K}(t)) \quad (10)$$

since $\int_t^\infty e^{-\rho(s-t)}ds = 1/\rho$. Moreover, by OPT, (6) holds, where \bar{U} is the maximum level of utility that can be sustained forever from time t on, given the initial stocks at time t . Since, under LH, $U(\mathbf{C}) = \mathbf{P}\mathbf{C}$ and $Y = \mathbf{P}\mathbf{C} + \mathbf{Q}\dot{\mathbf{K}}$, it follows from (6) and (10) that $U(\mathbf{C}(t))$ exceeds the maximum sustainable level if, at t , $d(\mathbf{Q}\mathbf{K})/dt = \mathbf{Q}\dot{\mathbf{K}} + \dot{\mathbf{Q}}\mathbf{K} < 0$, which is, by (9), is equivalent to $(d(Y + \dot{\mathbf{Q}}\mathbf{K}))/dt < 0$.

Hence, under OPT, DU, CRS and LH,

⁸See, e.g., Vincent et al. (1997).

- welfare improvement can be measured by increasing wealth (or by growth in real NNP plus anticipated capital gains), and
- it is a *necessary* condition for sustainable development that wealth (or, equivalently, real NNP plus anticipated capital gains) does not decline.

However, the results depend on LH and CRS, which are strong and controversial linearity assumptions.

6 Methods for satisfying informational demands

When doing practical estimation, the assumptions above represent serious informational demands. What techniques can be used to satisfy these informational demands?

All results require that one must be able to account for changes in society’s productive capacity. Such changes may be caused by

- accumulation of ordinary reproducible capital,
- technological change or human capital accumulation,
- reduced resource availability,
- in the case of open economies, changing terms-of-trade.

The assumption of ST means that all such changes are captured by the vector of investments flow, where the size of these flows can be measured and valued at efficiency or accounting prices. What can be done if the assumption of ST is not satisfied, so that it is *not* the case that all changes in society’s productive capacity correspond to stock changes that can be measured and valued?

One—purely formal, but in principle important—method consists of letting time be an additional state variable; i.e., an additional capital component. This reformulates the problem as one of measuring the “value of passage of time”.

A first attempt at a practical solution is to assume that the value of passage of time does not change over time. Then time does not contribute to changes in the value of net investments. Hence, time need not be included when calculating growth in real NNP. On the other hand, one must calculate how the real value of consumption changes over time, where the consumption vector must include e.g. environmental amenities.

Growth in real NNP will also give a right qualitative result if the value of passage of time is in fixed proportion to total NNP. The Dasgupta-Heal-Solow model of capital accumulation and resource depletion illustrates this possibility: With fixed factor shares,

the value of resource depletion is a fixed proportion of total NNP. Therefore, real NNP growth measures welfare improvement even if resource depletion is not included, while the measurable value of net investments can be grossly inaccurate if resource depletion is left out.

A model where technological progress is endogenous—in the sense that human capital accumulation equals the fraction of net output that is used for neither consumption nor accumulation of ordinary reproducible capital—illustrates another possibility. Assume that net output (= real NNP) is observable, but that it is not possible to observe how net output not used to augment the stock of ordinary capital is split between consumption, on the one hand, and investment in human capital, on the other. Then, real NNP growth can be used for indicating welfare improvement (and sustainability), while the sign of the measurable value of net investments may not give a correct indication since human capital accumulation cannot be distinguished from consumption.

Hence, in spite of the theoretical equivalence between real NNP growth and the value of net investments expressed in Props. 1 and 2 (as well as Prop. 3), there seems to be cases where the informational requirements are smaller when using real NNP growth as an indicator for welfare improvement (and sustainability). On the other hand, the value of net investments does not need valuation of the components of the extended consumption vector \mathbf{C} capturing environmental amenities.⁹

A second attempt at a practical solution is to try directly to measure the value of passage of time, using forward-looking terms. Such method has been suggested by e.g. by Aronsson et al. (1997), Pezzey (2002), Sefton and Weale (1996), and Vellinga and Withagen (1996). In particular, Sefton and Weale (1996) show how to take into account changing terms-of-trade being faced by a resource exporter.

A third attempt at a practical solution is to assume a constant interest rate (being implied by DU and LH), and then combine this assumption with OPT and CRS. As pointed out in Sect. 5, anticipated capital gains will, under this set of assumptions, capture the effects of changes in society’s productive capacity. When using this method, one must, as mentioned in Sect. 2, face the practical problems that arise when trying to satisfy the CRS assumption, especially in a world where natural and environmental resources are important. Moreover, unanticipated (“windfall”) capital gains as well as capital gains arising from changing interest rates must be excluded. The method seems to be of practical interest when estimating the sustainable income arising from a raw

⁹There does not seem to be much empirical work that tries to crosscheck \mathbf{QI} and \dot{Y} as measures of welfare and sustainability. See, however, Hanley et al. (1999).

material exporting country’s resource endowment.

Sects. 4 and 5 list DU and LH among the assumptions that are sufficient for global comparisons; i.e., for comparisons between two points in time that are not adjacent or between two societies that are not similar. We have seen that the assumption of LH ensures that the value of consumption equals the utility derived from consumption. How can utility be measured if the utility function is not homogeneous of degree 1?

To investigate this, allow the utility function introduced in Sect. 2 to be a strictly concave function. Let us also—for the purpose of the analysis of the following paragraphs—denote this utility function by \tilde{U} , so $\tilde{U}(\mathbf{C})$ is the well-being derived from the vector of consumption flows \mathbf{C} . A technical way to transform \tilde{U} into a utility function U that is homogeneous of degree 1 is to add an additional consumption good, say good 0, where $C_0 \equiv 1$. Then we can define U by

$$U(C_0, \mathbf{C}) := C_0 \cdot \tilde{U}(\mathbf{C}/C_0),$$

implying that U is homogeneous of degree 1.

The problem of applying the global welfare comparison analysis of Sect. 3 is now “reduced” to determining the price change of the added consumption good 0, so that one can determine a Divisia consumption price index satisfying $\dot{P}_0(t) + \dot{\mathbf{P}}(t)\mathbf{C}^*(t) = 0$, or, equivalently,

$$\dot{P}_0(t) = -\dot{\mathbf{P}}(t)\mathbf{C}^*(t), \quad (11)$$

entailing that prices are in fixed proportion to utility, and it holds w.l.o.g. that

$$P_0(t) = \partial U(1, \mathbf{C}(t))/\partial C_0 \quad \text{and} \quad \mathbf{P}(t) = \nabla U(1, \mathbf{C}(t)) = \nabla \tilde{U}(\mathbf{C}(t)). \quad (12)$$

Notice that (12) implies that

$$P_0(t) = \frac{\partial U(1, \mathbf{C})}{\partial C_0} = \tilde{U}(\mathbf{C}(t)) - \nabla \tilde{U}(\mathbf{C}(t))\mathbf{C}(t) = \tilde{U}(\mathbf{C}(t)) - \mathbf{P}(t)\mathbf{C}(t).$$

Hence, by (11), $\dot{\mathbf{P}}(t)\mathbf{C}^*(t)$ corresponds to the per time unit loss of “consumers’ surplus”. By following Weitzman (2001) one can argue that this change in “consumers’ surplus” is in principle observable in a market economy (see also Li and Löfgren, 2002).

Global comparisons across space are not only more difficult because they require utility to be measurable. The assumption of ST also become more demanding when comparing two different societies. Recall that ST requires all *variable* determinants of productive capacity to be included in the vector of capital stocks. Since two different societies are likely to be less similar than the same society at two different times along the time axis, the vector of capital stocks must include more components when making comparisons across space.

7 Summary of results

I have considered five assumptions – OPT, DU, ST, CRS, and LH – which can be made separately of each other. This makes altogether 32 different combinations of assumptions, as illustrated in the two tables.

Table 1 summarizes results in welfare accounting. For 12 combinations, namely those that satisfy ST and either OPT or DU, it has been shown that a local-in-time welfare improvement can be indicated both by a positive value of net investments and by growth in real NNP. The fundamental equation is that change in real NNP equals the real interest on the real value of net investments. For a subset of 4 combinations, namely those that satisfy DU, ST, and LH, it has been shown that a positive global welfare difference can be indicated both by a positive real value of stock differences and by a positive difference in real NNP. The fundamental equation is that real NNP equals the real interest on the present value of future consumption. For two combinations, namely those that satisfy OPT, DU, CRS, and LH, it has been shown that a global welfare difference can be indicated both by a positive wealth difference and by a positive difference in real NNP plus anticipated capital gains. Only if all five assumptions are made – namely OPT, DU, ST, CRS, and LH – is it the case that both *(i)* real NNP is the real interest on real wealth and *(ii)* differences in wealth have welfare significance.

Table 2 summarizes results in sustainability accounting. For combinations that satisfy only DU and ST, it appears that the value of net investments (or, equivalently, real NNP growth) is a quite unreliable indicator of whether current utility can potentially be sustained forever. For 8 combinations, namely those that satisfy OPT and ST, it has been argued that a non-negative value of net investments (or equivalently, non-negative rate of real NNP growth) may serve as an exact indicator of sustainability, provided that the social preferences take sustainability into account. For 4 combinations, namely those that satisfy OPT, DU, and ST, it has been shown that a negative value of net investments (or equivalently, negative rate of real NNP growth) implies that development is unsustainable. For two combinations, namely those that satisfy OPT, DU, CRS, and LH, it has been shown that decreasing real wealth implies that development is unsustainable.

The analysis of this paper illustrates how stronger results require stronger assumptions, and thereby impose harder informational requirements.

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Table 1: Overview of assumptions and results in welfare accounting.

		Not CRS	Not ST	CRS	Not CRS	ST	CRS
OPT	Not						
	LH						
	Not						
	DU						
OPT	Not						
	LH				$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.		$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.
	Not						
	DU				$\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}d\mathbf{K} > 0,$ $Y'' > Y'$ greater welfare		$\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}d\mathbf{K} > 0,$ $Y'' > Y'$ greater welfare
OPT	Not			$\mathbf{qK} =$ $\int \mathbf{pC}e^{-\int r d\tau} dt$	$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.		$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.
	Not						
	DU			$\mathbf{QK} =$ $\int \mu U(\mathbf{C})dt$	$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.		$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.
	Not						
OPT	LH			$\mathbf{qK} =$ $\int \mathbf{pC}e^{-\int r d\tau} dt$	$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.		$\mathbf{qI} > 0, \dot{Y} > 0$ welf. improvem.
	Not						
	DU			$\mathbf{Q}''\mathbf{K}'' > \mathbf{Q}'\mathbf{K}'$ greater welfare	$\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}d\mathbf{K} > 0,$ $Y'' > Y'$ greater welfare		$\mathbf{Q}''\mathbf{K}'' > \mathbf{Q}'\mathbf{K}',$ $Y'' > Y'$ greater welfare
	LH						

Table 2: Overview of assumptions and results in sustainability accounting.

		Not CRS	Not ST	CRS	Not CRS	ST	CRS
OPT	Not						
	DU						
	—						
	LH						
DU	Not						
	LH						
	—						
	LH						
LH	Not				$\mathbf{qI} \geq 0, \dot{Y} \geq 0$ may ind. sust.	$\mathbf{qI} \geq 0, \dot{Y} \geq 0$ may ind. sust.	
	DU				$\mathbf{qI} \geq 0, \dot{Y} \geq 0$ may ind. sust.	$\mathbf{qI} \geq 0, \dot{Y} \geq 0$ may ind. sust.	
	—						
	LH				$\mathbf{qI} < 0, \dot{Y} < 0$ \Rightarrow unsustain.	$\mathbf{qI} < 0, \dot{Y} < 0$ \Rightarrow unsustain.	
LH	Not				$\mathbf{qI} < 0, \dot{Y} < 0$ \Rightarrow unsustain.	$d(\mathbf{QK})/dt < 0,$ $\dot{Y} < 0$ \Rightarrow unsustain.	
	LH			$d(\mathbf{QK})/dt < 0$ \Rightarrow unsustain.	$\mathbf{qI} < 0, \dot{Y} < 0$ \Rightarrow unsustain.	$d(\mathbf{QK})/dt < 0,$ $\dot{Y} < 0$ \Rightarrow unsustain.	