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Measures of Aggregate Credit Conditions and Their Potential Use by Central Banks

by Alejandro García and Andrei Prokopiw

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Abstract

Understanding the nature of credit risk has important implications for financial stability. Since authorities—notably, central banks—focus on risks that have systemic implications, it is crucial to develop ways to measure these risks. The difficulty lies in finding reliable measures of aggregate credit risk in the economy, as opposed to firm-level credit risk. In this paper, the authors examine two models recently developed for this purpose: a reduced-form model applied to credit default swap index tranches, and a structural model applied to the spread on U.S. corporate bond indexes. The authors find that these models provide information on the nature of credit events—that is, whether the event is systemic or not—and on the type of risk priced in corporate bonds (i.e., credit or liquidity risk). However, although the two models provide potentially useful information for policy-makers, at this stage it is difficult to corroborate the accuracy of the information obtained from them. Further work is needed before authorities can include conclusions drawn from the two models into their policy decisions.

JEL classification: G10, G12, G13
Bank classification: Credit and credit aggregates; Financial markets; Financial stability

Résumé

Comprendre la nature du risque de crédit a d'importantes implications pour la stabilité financière. Étant donné que les autorités – notamment les banques centrales – s'intéressent aux risques de portée systémique, il est crucial de savoir mesurer ceux-ci. La difficulté réside dans l'obtention de mesures fiables du risque de crédit dans l'ensemble de l'économie, par opposition à celui propre à une entreprise. Dans leur étude, les auteurs examinent deux modèles élaborés récemment à cette fin : un modèle de forme réduite et un modèle structurel, qu'ils appliquent, respectivement, aux tranches d'un indice de swaps sur défaillance et à l'écart sur les obligations de sociétés américaines comprises dans un indice. Ils constatent que ces modèles renseignent sur la nature – systémique ou non – des incidents de crédit et sur le type de risque – de crédit ou de liquidité – intégré au prix des obligations de sociétés. Bien que l'information fournie par les deux modèles puisse s'avérer utile aux décideurs publics, il est difficile pour le moment d'en corroborer l'exactitude. Des travaux plus approfondis seront nécessaires avant que les autorités puissent tirer des conclusions des modèles et s'en servir dans leur prise de décisions.

Classification JEL : G10, G12, G13
Classification de la Banque : Crédit et agrégats du crédit; Marchés financiers; Stabilité financière

1 Introduction

Given their focus on systemic events, central banks need to be able to assess whether developments in credit markets mainly reflect idiosyncratic events or economy-wide credit events. Therefore, central banks need tools that allow them to interpret the nature of credit events. In this paper, we examine the potential usage of two models—a reduced-form model based on Bhansali, Gingrich, and Longstaff (2008), and a structural model based on Leland and Toft (1996)—to provide information on the possibility of a major economy-wide credit event. We apply these two models to the recent period using U.S. data.

The first model extracts information about the nature of credit risk from the prices of credit default swap (CDS) indexes and CDS tranches. The model assumes that three types of event can happen: an economy-wide credit event, a medium-sized credit event, and an idiosyncratic credit event. Each event is modelled as an independent Poisson process. The parameters are estimated for each process by minimizing pricing errors. The model allows us to determine what part of the total CDS index spread is due to each type of credit event.

The second model decomposes the corporate bond spread for the U.S. broad index constructed by Merrill Lynch. We assume, as in Churm and Panigirtzoglou (2005), that the spread on the index represents the behaviour of a representative firm in the economy in order to apply the structural credit-risk model. The model uses the historical volatility of the S&P 500 to determine the asset volatility, which allows us to decompose the spread into a credit and a liquidity component.[1]

Using these models and available price data, we analyze the events over the period 16 April 2007 to 1 October 2009. Using CDS index and tranche data, we estimate the percentage of the CDS index that is pricing a large economy-wide credit event. Similar to Bhansali, Gingrich, and Longstaff (2008), we find that the percentage of the spread that accounts for an economy-wide credit event has increased dramatically during the recent period. It starts at levels close to zero basis points, and reaches 100 basis points (bps) prior to the Federal Reserve Bank of New York announcement that it would provide term financing to facilitate JPMorgan Chase's acquisition of Bear Stearns on 24 March 2008. Later declines and increases close to 100 bps occur on 16 September 2008, the day after Lehman Brothers filed for Chapter 11 bankruptcy.

Using U.S. corporate bond spread data and applying the structural model, we observe that, over the same period, liquidity risk increased substantially at the time of the Bear Stearns and Lehman events. Taken together, these models suggest that the likelihood of a major credit event increased substantially and that a large part

[1] In addition to the equity volatility, other parameters such as leverage and coupons are required for the structural model; we take them from Churm and Panigirtzoglou (2005).

of the crisis was due to the liquidity component (Figure 1).

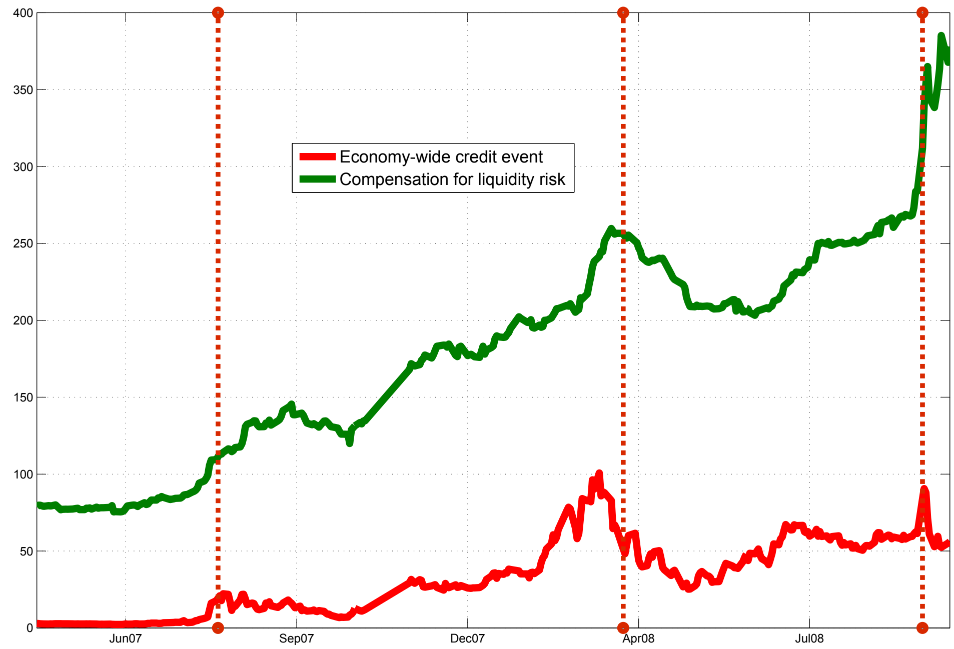

Figure 1: Economy-wide credit risk and Evolution of Liquidity Risk
The green line represents the compensation for liquidity risk in basis points for a representative investment-grade firm in the United States extracted from the spread of the North American Investment Grade Index provided by Merrill Lynch. The red line represents the compensation for an economy-wide credit event in basis points priced in the North American CDX index. The red dotted lines represent, respectively, the dates on which Bear Stearns liquidated two hedge funds that had invested in mortgage-backed securities (31 July 2007); the Federal Reserve Bank of New York announced that it would provide term financing to facilitate JPMorgan Chase's acquisition of Bear Stearns (24 March 2008); and Lehman Brothers filed for Chapter 11 bankruptcy (15 September 2008).

This paper is organized into four sections. Section 2 provides an introduction to CDS indexes and the reduced-form model. Section 3 provides a brief introduction to structural credit-risk models. Section 4 concludes.

2 CDS Index Tranches and Default Correlation

2.1 Basic concepts

To understand a CDS index tranche, it is necessary to first understand the characteristics of a CDS.

A single-name CDS is a contract that provides insurance against the default of a particular company.[2] The company is known as the *reference entity*, and a specific bond of the company is known as the *reference obligation*. The total value of the bond is known as the *notional principal*. There are two parties in the CDS contract: the buyer of credit protection and the seller. The former makes periodic payments to the seller of protection until either the contract matures or a *credit event* occurs. In exchange for the periodic payments, the seller of the protection agrees to pay the buyer the difference between the par value and the recovery value for the reference obligation upon a credit event.

A CDS index comprises many (for example, 100 or 125) single-name CDS contracts. Similar to a single-name CDS, a CDS index involves protection buyers and protection sellers. An important difference, however, is that the index is *tranched*, so that parties can decide to enter into a contract for credit protection that is capped by a lower and upper number of defaults. For example, the North American CDS index for investment-grade companies allows the parties to enter into contracts to trade credit protection for levels of 0–3, 3–7, 7–10, 10–15, and 15–30 per cent of losses of the portfolio on reference obligations. These categories are known as tranches. More precisely, they are known as the equity tranche (0–3 per cent loss), the junior mezzanine tranche (3–7 per cent loss), the mezzanine tranche (7–10 per cent loss), the senior tranche (10–15 per cent loss), and the super senior tranche (15–30 per cent loss). Figure 2 shows the structure of the North American CDS index tranches.

An important aspect of the tranche structure shown in Figure 2 is *subordination*. This means that the sellers of protection on the higher tranches have to pay on default only when the lower tranches have already lost all of their notional value. For example, a seller of protection of the super senior tranche has to pay only after the portfolio has lost more than 15 per cent of its value. In this sense, selling protection on the senior tranche is akin to purchasing a very high grade AAA bond: the protection seller receives quarterly payments with very low risk of incurring any losses. Another important aspect is the difference in the premiums for the tranches shown on the right side of Figure 2. The difference is that the equity (and, sometimes, the junior tranches) requires a large upfront payment (a percentage of the

[2]Here, "default" can mean a number of different credit events, such as failure to pay an obligation, bankruptcy, or financial restructuring. What qualifies as a credit event is determined by the International Swaps and Derivatives Association (ISDA).

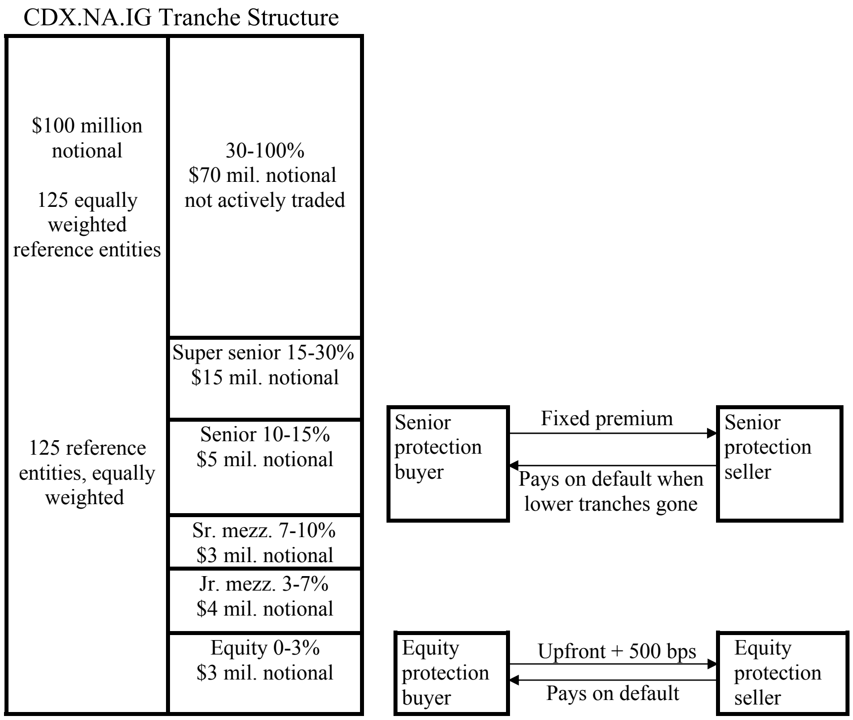

Figure 2: Structure of a CDS index tranche

The left box shows the structure of the CDS North American Investment Grade Index. The right boxes show the payment flows associated with two tranches.

tranche notional) plus a fixed spread (normally 500 bps) paid quarterly until there is a credit event or the tranche matures. This reflects the fact that the likelihood of the equity tranche to default—and thus for the protection seller to have to pay—is higher. This premium structure contrasts with the senior tranches, which require only a quarterly premium and no upfront payment, reflecting the lower likelihood of default.

Another helpful way to think of tranches is that buying protection on a tranche is the same as a spread option on the portfolio loss. That is, an a to b per cent tranche protection buyer receives payments on defaults starting at total losses of a per cent, but capped at total losses of b per cent. Figure 3 illustrates this relationship.

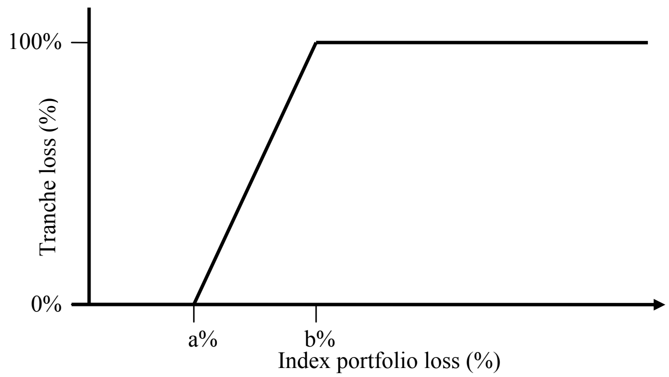

Figure 3: TRANCHE PAYOFF

Buying protection on an a to b per cent tranche is similar to a bull spread on index losses with strikes of a per cent and b per cent.

2.2 Why CDS indexes are useful to measure economy-wide credit risk

While the price of a single-name CDS contains information on the probability of default of the reference entity,[3] the price of the CDS index contains information about the correlation of default between the reference entities. Economy-wide credit events are those where multiple failures occur simultaneously or very close to each other, and therefore when default correlation is high. Since correlation is priced in the tranche premiums, and correlations have a direct relationship with the type of credit event, by analyzing the tranche prices we can assess what type of credit event is priced by the market. We elaborate further on the relation between correlation and tranche pricing in the next section.

2.3 Correlation between default and tranche pricing

Two main factors drive CDS tranche premiums: *the expected number of defaults* and *the correlation between defaults*. If the expected number of defaults increases, ceteris paribus, then the premiums for **all** tranches increase, as the expected losses for each portfolio increase. That is, if the correlation remains constant, then the expected number of defaults will affect the overall tranche levels, **but will not have a significant impact on the relative price levels between tranches**.

[3]We do not elaborate on the methodology to extract probabilities of default from single-name CDS contracts, since that is not the focus of our paper. For details on this, see Hull and White (2000), and Duffie (1999).

Suppose that the expected tranche losses remain constant, but that their correlation changes. To illustrate the effect of changes in correlation, consider a hypothetical portfolio of two reference entities (A and B in Figure 4) used for a CDS index.

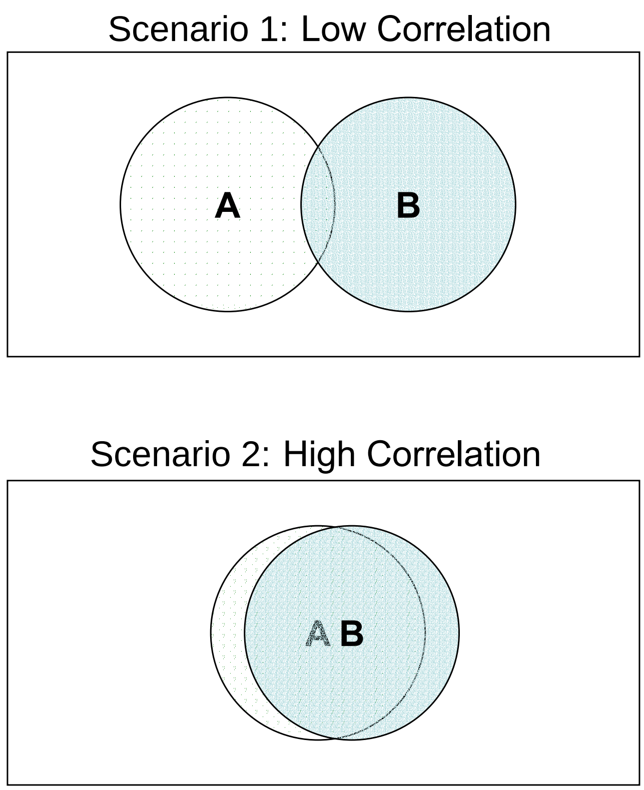

Figure 4: Correlation and Probability of Default

In Figure 4, we observe the two reference entities A and B, where the area in each circle denotes the probability of default for this reference entity, the intersection of the areas of the circles denotes the probability of default for A *and* B, and the area outside the circles but inside the rectangle denotes the probability of no default. For this portfolio, there could be two scenarios: low correlation and high correlation of default. With a *high correlation* of default (scenario 2), portfolio losses will occur less frequently (note that the area of the probability of no default is larger in scenario 2), but that the magnitude of the loss will be large, since several entities may default at once. In other words, times of high correlation are times when the risk of *economy-wide* credit events is high, but the risk of *idiosyncratic* credit events is low. In a *low-correlation* situation (scenario 1), portfolio losses occur more frequently—as illustrated by a smaller area in the rectangle—but the magnitude of the losses is small, since only one entity is likely to default at a time. In other words, when there is a low correlation, the risk of an economy-wide credit event is low, but idiosyncratic risk is higher. We can also consider times of medium correlation, where the main source of risk is small groups of defaults occurring at once, with relatively low frequency.[4]

2.4 Who benefits from high and low correlation

To better understand the role of default correlation, let us consider who benefits in a CDS contract from different degrees of correlation.

For a protection buyer in an equity tranche, a low correlation of default is "preferred," because the probability of observing losses is greater when correlation is low, and, given that it is the first tranche to pay protection, the protection buyer is almost sure to receive a protection payment. Conversely, high correlation hurts a protection buyer in an equity tranche, because defaults occur infrequently, and the protection buyer is protected only up to a small proportion of the total portfolio, but must pay premiums possibly for a longer period of time prior to getting a payment. On the other hand, for a protection buyer in a super senior tranche, a high correlation is "preferred," because, when losses occur, they are likely to be very large, eliminating the lower tranches, and thus resulting in a payment from the protection seller.

Given this relationship between correlation and tranche premiums, we can use the premiums to infer changes in default correlation for the reference entities. In other words, we determine the proportion of the premiums that compensate for idiosyncratic risk, medium-sized risk,[5] or economy-wide risk.[6]

[4] "Base correlation" is frequently reported for CDS index tranches. This is a measure of the correlation between each reference entity and an unobserved economy-wide factor. This measure, however, does not provide us with enough information to assess the changes in the correlation between the reference entities. For more information on base correlations, see O'Kane and Livesey (2004).

[5] It is tempting to refer to this medium-sized risk as sector-wide or industry-wide risk, but, here, medium refers only to the risk of small groups of companies defaulting, which may or may not be from the same sector.

[6] In our analysis, based on a portfolio of 125 investment-grade companies, an idiosyncratic event is defined as an event that involves the default of one entity, a medium-sized event as one that involves the default of 10 per cent of the reference entities, and an economy-wide event as one that involves the default of 65 per cent of the reference entities.

2.5 How CDS indexes are created

We focus on investment-grade companies in North America, and therefore we use the CDX.NA.IG indexes provided by Markit. The composition of Markit's CDX.NA.IG indexes is determined by members of the CDS IndexCo LLC through a voting process. The contributing banks propose 125 entities that satisfy Markit's requirements on ratings and liquidity, among other factors. A new series of the index is published twice a year, on 20 March and 20 September (or the next business day). In some instances, contributing banks can vote to postpone the index roll date. For example, due to turbulence in the market in September 2008, the roll date for series 11 was postponed until 2 October. Between roll dates, credit events may also occur that render an entity unsuitable for the index. In such events, new versions of the index are created after each credit event, wherein the afflicted entity is assigned an index weight of 0 per cent and is replaced by a new entity. The weights of the other original index members are not affected (they stay at 0.8 per cent in the case of CDX.NA.IG; note that all entities have the same weight).

Examples of recent credit events that affected the index include the placement of Fannie Mae and Freddie Mac into conservatorship, which resulted in the removal of both entities from the index and the creation of version 2 of the index for series 8, 9, and 10 on 8 September. Another event was the failure of Washington Mutual, which led to downgrades from S&P and Moody's ratings agencies. The subsequent ratings rendered the entity ineligible for the investment-grade index; therefore, version 3 was created on 29 September. Washington Mutual was replaced by ERP Operating Limited Partnership for series 11 of the index.

2.6 CDS index and price data

Figures 5 and 6 show the data for the North American Investment Grade (CDX.NA.IG) CDS index tranche series 8, 9, 10, and 11 with a 5-year maturity. We focus on the 5-year maturity because CDS index tranches are, typically, more liquid at this maturity.[7] Our sample starts on 16 April 2007 and ends on 26 November 2008. Anecdotal evidence suggests that the CDS index data for October and November 2008 are less reliable because the CDS market became less liquid at that time.

Figure 5 shows the tranche premiums for all but the equity tranche, whose evolution is reported in Figure 6. Doing so is necessary because of the different scales for the tranches: the equity tranche is in per cent and the other tranches are in basis points (bps).

[7] See Amato and Gyntelberg (2005) for more information on the liquidity of CDS index tranches.

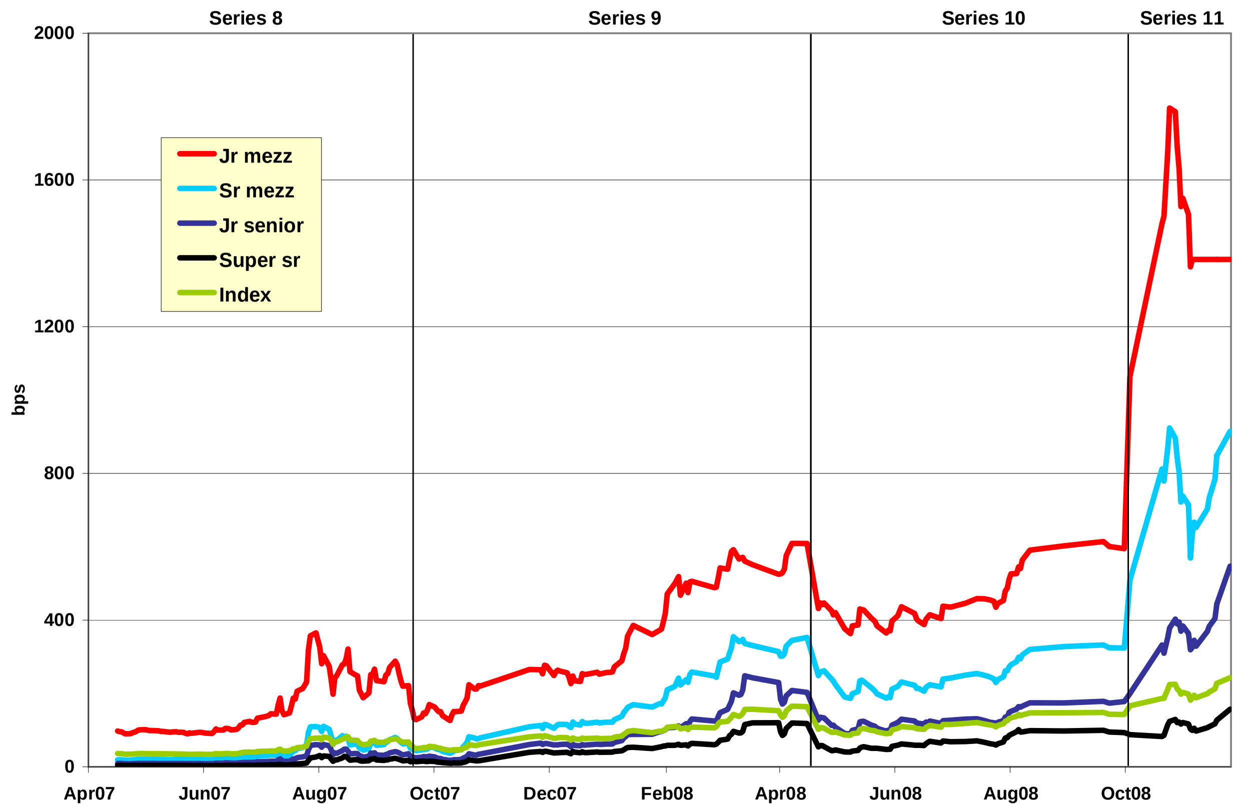

Figure 5: TRANCHE AND INDEX PREMIUMS

Daily market price data for the CDX.NA.IG series 8, 9, 10, and 11 obtained from Markit. All tranche prices are displayed, with the exception of the equity tranche.

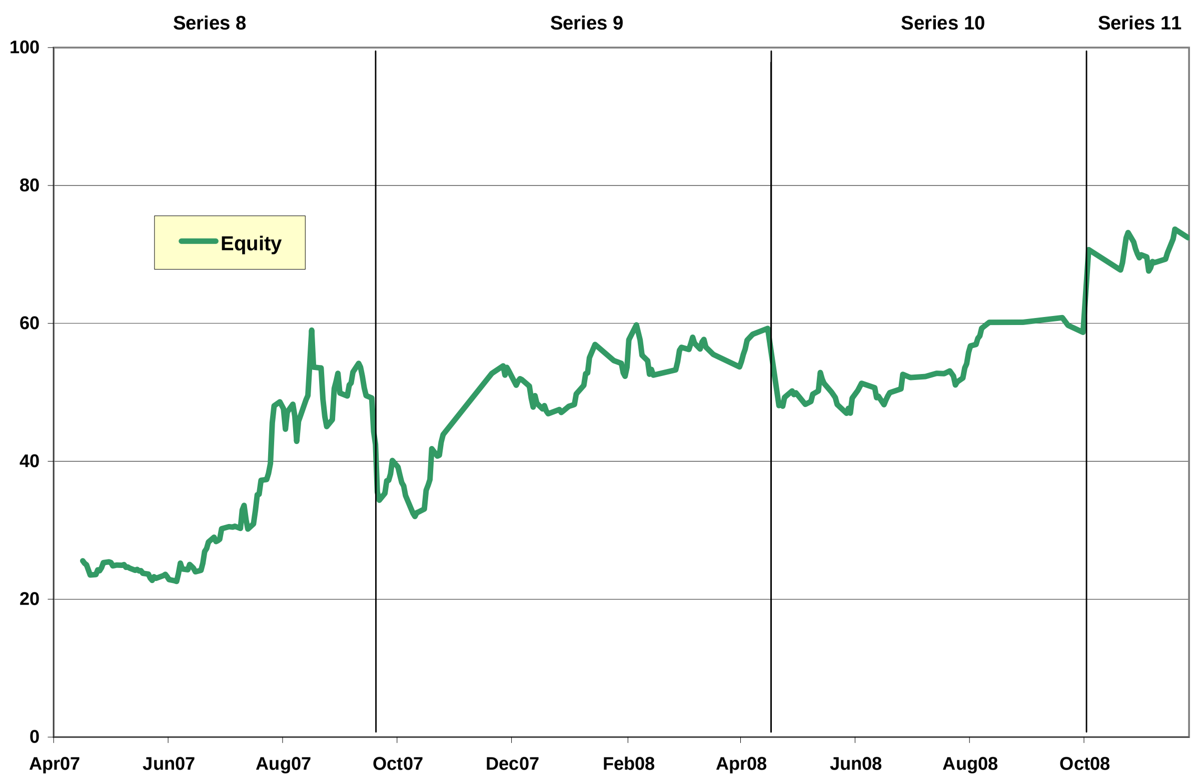

Figure 6: EQUITY TRANCHE PREMIUM

Daily market price data series 8, 9, 10, and 11 obtained from Markit.

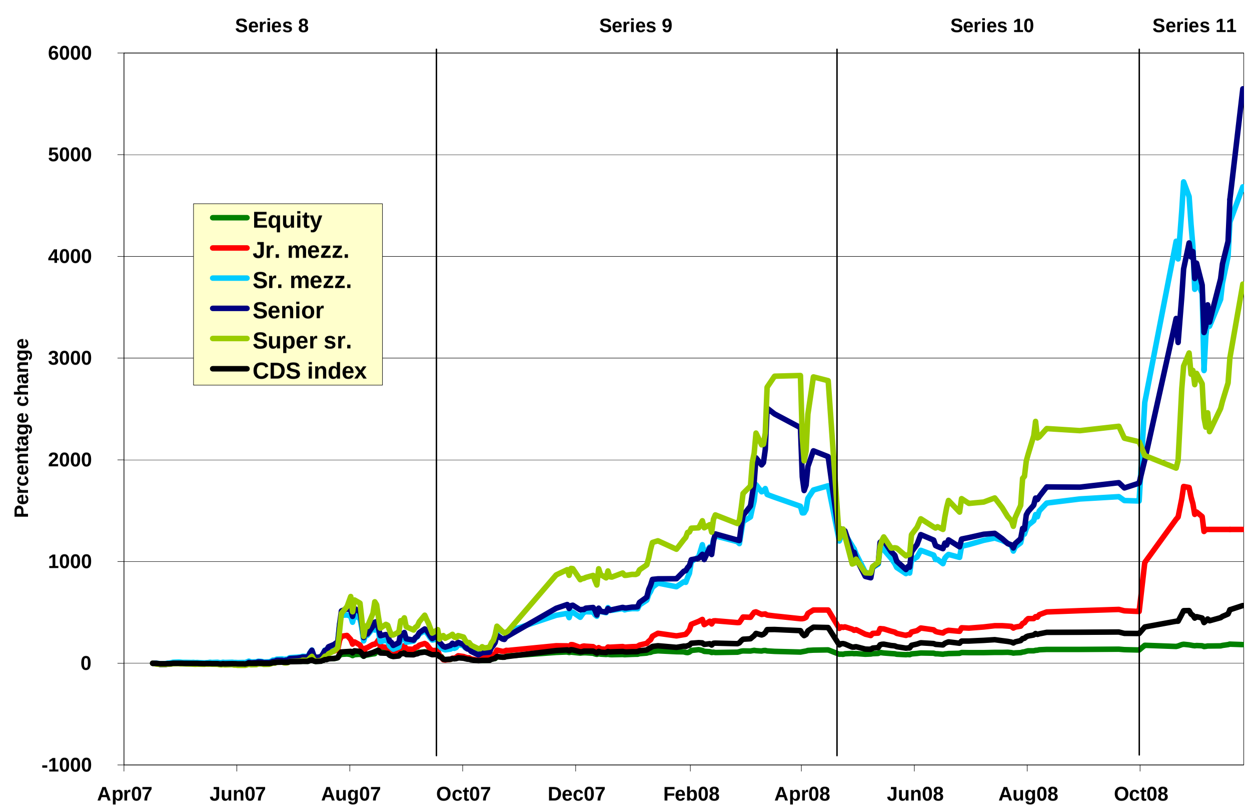

Figure 7: Percentage price changes in Tranche and Index Spreads
Daily market price data for the CDX.NA.IG series 8, 9, and 10 obtained from Markit.

From Figures 5 and 6 we observe that the tranche prices increased dramatically in the fall of 2008. However, it is difficult to determine which tranche increased more relative to the others. To identify the relative price changes, we construct Figure 7.

In Figure 7, we show the percentage price increase with respect to the first observation in the quoted spreads for all tranches. We also show the value of the quotes for each series during the time the index was on the run. This means that, during the entire sample, we have data for series 8 from 16 April 2007 to 21 September 2007; for series 9 from 21 September 2007 to 21 April 2008; for series 10 from 21 April 2008 to 3 October 2008; and for series 11 from 3 October to 3 November 2008. A simple observation of Figure 7 allows us to note that:

- the CDS spreads exhibited large increases during August 2007 and March, April, August, October, and November 2008, suggesting increased corporate default risk during those times[8]
- overall, the senior tranches show the largest percentage increase since April 2007, while the equity tranche experienced very little/almost no change

As explained earlier, higher correlation increases senior tranche premiums, but not equity tranche premiums. The relative price changes observed suggest that, as CDS spreads increased, not only did individual credit concerns increase, but correlation increased as well, thereby indicating that the likelihood of an economy-wide credit event was higher.

2.7 Decomposition of index spread

To confirm that the large increase in the price of senior tranches indeed illustrates an increased likelihood of the occurrence of an economy-wide credit event, we use the Bhansali, Gingrich, and Longstaff (2008) model, which can be summarized as follows:[9]

(i) Minimize the difference between observed tranche prices and the theoretical prices.

(ii) Theoretical prices are modelled by equating the expected payments from the premium stream to the expected payments from the protection payment side.

[8] Those periods were associated with individual events that led to the broader credit market turmoil: the bailout of Bear Sterns (March 2008), the government rescue of Fannie Mae and Freddy Mac (July and August 2008), the bankruptcy of Lehman Brothers (September 2008), and the acceleration of the credit market crisis and the meltdown in equity markets (November 2008).

[9] See Appendix A for technical details of the model.

(iii) Since payments for both streams depend on the portfolio losses (L), a model for the losses is specified.

(iv) Portfolio losses (L) are modelled by three independent default processes (N_1, N_2, and N_3). The estimation of L depends on two parameters: default intensities (λ), which characterize the default processes, and jump sizes (γ), which determine the size of the credit event. λs are estimated for each day, whereas γs are estimated using the entire sample data. Therefore, if the sample size changes, γ changes as well.

(v) Each process (N_i) represents the occurrence of a type of credit event—idiosyncratic, medium-sized, and economy-wide—and the type of event is given by the jump size of the defaults.

(vi) Each type of credit event accounts for a portion of the overall CDS index spread. The corresponding spread for each credit event is a function of γ_i and λ_i.

One difficulty of the model is that the γs change as the sample size increases, which implies that the historical decomposition of the spread would also change when a better fit can be found. To make the analysis of results consistent with a given value of gammas even when the sample is updated, we use the average jump sizes (γs) estimated by Bhansali, Gingrich, and Longstaff (2008) and estimate only the default intensities (λ). For further details on the implementation, see Appendix B. Following this approach, we obtain the results of the decomposition shown in Figure 8.[10] As the figure shows, there has been a significant increase in the economy-wide risk factor since April 2007. Using our sample data, in 2007, on average, the idiosyncratic component accounted for 76.16 per cent of the index, the medium-sized credit event for 6.54 per cent, and the economy-wide credit event for 17.32 per cent. In 2008, the corresponding numbers were 46.29, 14.92, and 38.83 per cent, respectively.

This decomposition supports the economy-wide factor increase during the recent crisis.

[10] Additional details are provided in Bhansali, Gingrich, and Longstaff (2008), and Longstaff and Rajan (2006).

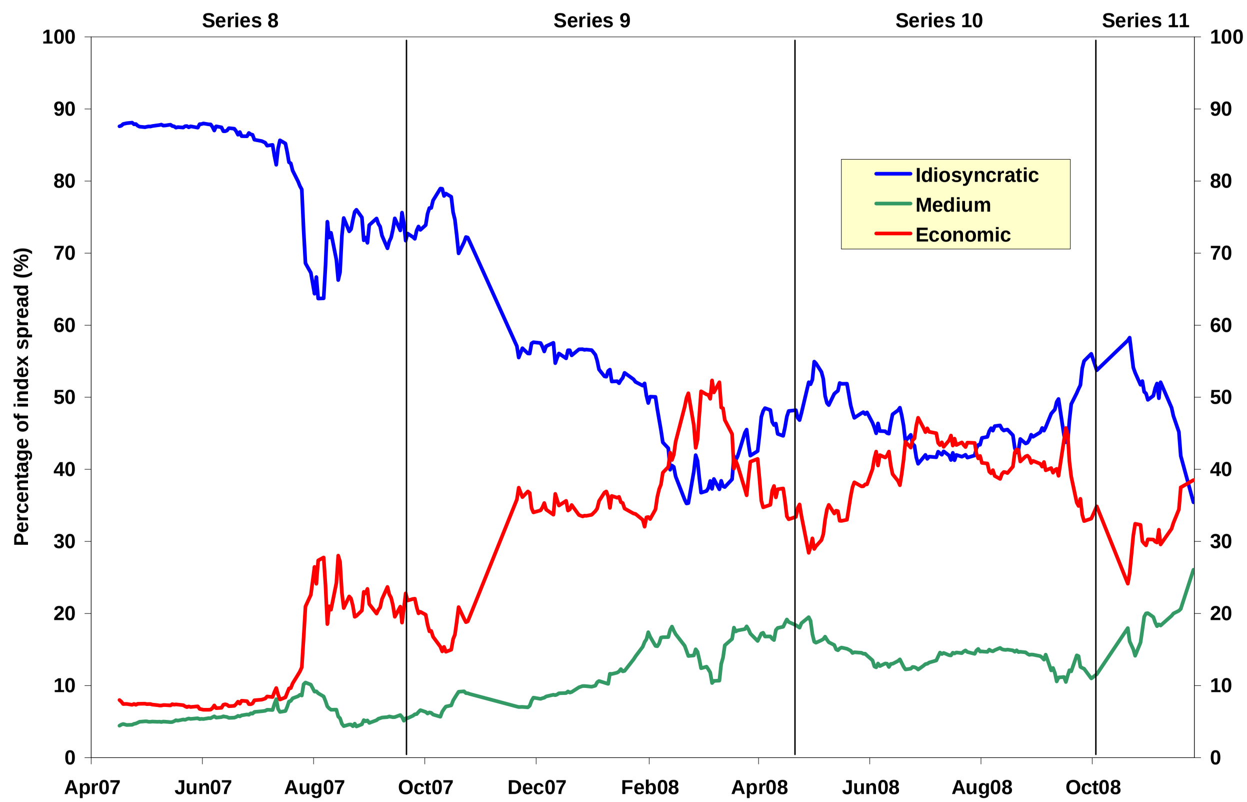

Figure 8: Decomposition of cdx.na.ig series 8, 9, 10, and 11

Decomposition is shown as a percentage of the index premium, not as absolute basis points.

3 Structural Credit-Risk Model

In this section, we employ the Leland and Toft (1996) model to decompose the corporate spread for the Merrill Lynch U.S. corporate bond index. This model belongs to the family of structural models.[11] These models assume that default on a corporate bond depends on the relationship between the assets and liabilities of the firm: when a firm's assets drop below its liabilities, the firm defaults. More formally, the model assumes that the asset value evolves according to Equation (1),

$$\frac{dV}{V} = [\mu(V,t) - \delta]dt + \sigma dz. \quad (1)$$

Equation (1) models the change in the asset value ($\frac{dV}{V}$), as a stochastic process with

(i) a *mean* that depends positively on the cost of capital (μ) and negatively on the value of payouts (δ) that the firm makes to security shareholders and,

(ii) a *volatility* parameter (σ) that is constant and adjusted with the increment (dz) of a standard Brownian motion.

As stated before, default occurs when the value of the assets reaches a threshold (V_B), where V_B is a function of the risk-free rate (r), asset volatility (σ), time to maturity of debt (T), bankruptcy costs (α), and leverage (P), among other parameters. These parameters are estimated following the assumptions of Churm and Panigirtzoglou (2005) for the United States. An important assumption is that the equity volatility used to obtain the asset volatility for the representative firm is taken from the historical volatility of the S&P 500.[12] With all this information, the model provides closed-form solutions for the price of debt and equity. We obtain a proxy for the model-implied spread that accounts for credit risk. The difference between the model-implied spread and the observed spread is attributed to liquidity.

Figure 9 shows the decomposition of the U.S. corporate bond index into the credit and liquidity components. The large increases in the corporate bond spread during the 2007–08 sample are mostly due to liquidity risk. However, as the crisis persisted, its impact on the economy became more obvious (i.e., a recession occurred), and the credit component started to increase.

[11] Structural models are inspired by Merton (1974).

[12] For more information on the implementation of this model and the choice of parameter values for the United States, see Churm and Panigirtzoglou (2005).

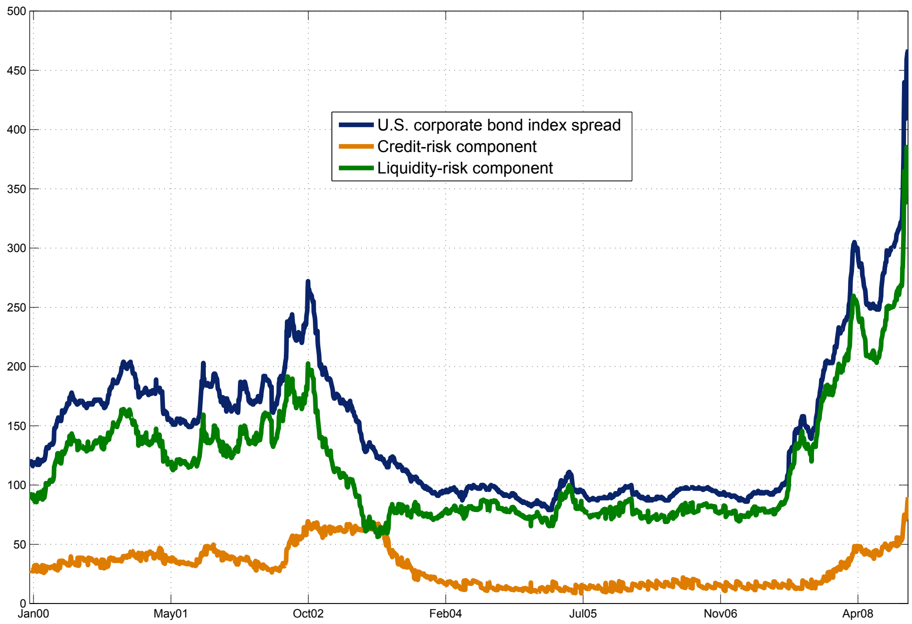

Figure 9: Decomposition of U.S. Corporate Bond Index Spread
Decomposition using a structural credit-risk model. Units in basis points.

4 Conclusion

Our analysis shows that there have been significant changes in the nature of credit risk for corporate bonds since the onset of the credit crisis in August 2007. Namely, economy-wide risk is currently contributing to a larger component of the CDS spreads than idiosyncratic risk. In other words, correlation has increased. In addition, liquidity risk was the major source of risk during the 2007–08 crisis. This suggests that these approaches can provide useful information to policy-makers to help them better understand credit conditions and the nature of credit, and possibly develop policies to address crisis periods if it is deemed desirable.

Although the information we obtain from these models permits a plausible interpretation of events, a number of issues need to be clarified before policy-makers can reliably draw solid conclusions from these models. For instance, latent factors captured from the CDS index data are not related to observable variables that affect default, and it is therefore difficult to interpret them. A way to address this issue would be to try to link default intensities to economic variables. With respect to the decomposition of the corporate spread using a structural model, one concern is to accentuate whether the parameters, including the volatility from equity prices, are reflective of the representative corporation in the economy. One way to confirm the results would be to perform a decomposition at the firm level for a large number of firms in the economy, and then compare the results with those obtained from the analysis of the index. This task is very demanding, since one would have to choose a representative sample of corporations in the economy and then estimate the model for each one. For the United States, this could entail a large number of corporations. For countries with a smaller number of corporations that have access to capital markets, a firm-level analysis could help confirm the results obtained from the application of the models discussed in this paper.

This paper has taken a first step in examining some existing models that could help characterize credit conditions at an aggregate level. Although these models provide relevant information, more work is required to enhance their reliability.

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Appendices

A Description of the Model

In this section, we present a brief summary of the methodology used to decompose the CDS index spread. The methodology follows the model proposed by Bhansali, Gingrich, and Longstaff (2008).

A.1 Index decomposition

The decomposition performed in section 2.7 is done by modelling the total portfolio loss percentage by a linear combination of three independent Poisson processes:[13]

$$L(t) = \gamma_1 N_1(t) + \gamma_2 N_2(t) + \gamma_3 N_3(t), \tag{2}$$

where γ_1, γ_2, and γ_3 give jump sizes for each of the loss components, representing small, medium, and large jumps in portfolio loss. For our analysis, we use $\gamma_1 = 1/125, \gamma_2 = 15/125$, and $\gamma_3 = 100/125$.[14] Let λ_i denote the intensity parameter for the Poisson process $N_i(t)$ for $i = 1, 2, 3$.

The probability $P_{i,j}(t)$ of having j jumps for the ith Poisson process up to time t is given by

$$P_{i,j}(t) = \frac{e^{-\lambda_i t}(\lambda_i t)^j}{j!}. \tag{3}$$

We can compute the expected tranche losses $E[L(t)]$ by

$$\begin{aligned} E[L_{a,b}(t)] &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_3=0}^{\infty} P(N_1(t) = j_1, N_2(t) = j_2, N_3(t) = j_3)(\gamma_1 j_1 + \gamma_2 j_2 + \gamma_3 j_3) \\ &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_3=0}^{\infty} \frac{e^{-(\lambda_1+\lambda_2+\lambda_3)t}(\lambda_1 t)^j(\lambda_2 t)^j(\lambda_3 t)^j}{j_1! j_2! j_3!}(\gamma_1 j_1 + \gamma_2 j_2 + \gamma_3 j_3). \end{aligned} \tag{4}$$

[13] Poisson processes are commonly used to model the occurrence of rare events. A single Poisson process models events that occur independently of each other; hence the use of three different Poisson processes, each with different coefficients.

[14] We found that small variations in the γs did not affect the root mean square error drastically. In Bhansali, Gingrich, and Longstaff (2008), the estimation involved iterating the entire procedure (i.e., run through the data set many times) to optimize over the γs. However, this has the downside that, when new data come in, the fitting from previous days also changes. We adopt the approach described because we are more interested in detecting a change in market conditions than in obtaining the best possible overall fit. Nevertheless, we compared the loss in fit from using constant γs close to, but different than, those found by Bhansali, Gingrich, and Longstaff (2008), and found very little loss in accuracy but a large gain in computational time.

To compute tranche losses, we may think of a tranche as a spread option on the total loss. In other words, if a tranche has an attachment point of a per cent and a detachment point of b per cent, then the tranche loss per cent $L_{a,b}(t)$ is given by

$$L_{a,b}(t) = \frac{\max(L(t) - b, 0) - \max(L(t) - a, 0)}{b - a}. \tag{5}$$

Let $f_{a,b}(x) = \frac{\max(x-b,0)-\max(x-a,0)}{b-a}$ be the payoff function depicted in Figure 3. Then $L_{a,b}(t) = f_{a,b}(L(t))$. In this way, we can compute the expected tranche loss $E[L_{a,b}(t)]$ by

$$\begin{aligned} E[L_{a,b}(t)] &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_3=0}^{\infty} P(N_1(t) = j_1, N_2(t) = j_2, N_3(t) = j_3) f_{a,b}(\gamma_1 j_1 + \gamma_2 j_2 + \gamma_3 j_3) \\ &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_3=0}^{\infty} \frac{e^{-(\lambda_1+\lambda_2+\lambda_3)t}(\lambda_1 t)^j (\lambda_2 t)^j (\lambda_3 t)^j}{j_1! j_2! j_3!} f_{a,b}(\gamma_1 j_1 + \gamma_2 j_2 + \gamma_3 j_3). \end{aligned} \tag{6}$$

Next, let $D(t)$ denote the discount curve. The expected payments for the premium leg of a tranche with maturity T is given by

$$C_{a,b} \int_0^T D(t)(1 - E[L_{a,b}(t)])\, dt, \tag{7}$$

where $C_{a,b}$ is the coupon for the a to b per cent tranche. This should equal the value of the protection leg given by

$$\int_0^T D(t) E[dL_{a,b}(t)]. \tag{8}$$

In the case of the index coupon C, equating Equations (7) and (8) and expanding the integrals results in

$$C \int_0^T D(t)(1 - \gamma_1\lambda_1 t - \gamma_2\lambda_2 t - \gamma_3\lambda_3 t)dt = \int_0^T D(t)(\gamma_1\lambda_1 + \gamma_2\lambda_2 + \gamma_3\lambda_3)dt. \tag{9}$$

Solving for C gives

$$C = \frac{\gamma_1\lambda_1 + \gamma_2\lambda_2 + \gamma_3\lambda_3}{1 - (\gamma_1\lambda_1 - \gamma_2\lambda_2 - \gamma_3\lambda_3)A}, \tag{10}$$

where

$$A = \frac{\int_0^T D(t) t\, dt}{\int_0^T D(t)\, dt}. \tag{11}$$

Hence we can compute λ_1 given λ_2, λ_3, γ_i, and C by the formula

$$\lambda_1 = \frac{\frac{C}{1+AC} - \gamma_2\lambda_2 - \gamma_3\lambda_3}{\gamma_1}. \tag{12}$$

With the estimates of the λs we can decompose the spread of the index credit derivatives into the three types of credit risk. Where the economy-wide risk event corresponds to $S_{ecw} = \frac{\gamma_3\lambda_3}{1-(\gamma_1\lambda_1+\gamma_2\lambda_2+\gamma_3\lambda_3)A}$, the medium-shock event corresponds to $S_{mid} = \frac{\gamma_2\lambda_2}{1-(\gamma_1\lambda_1+\gamma_2\lambda_2+\gamma_3\lambda_3)A}$ and the idiosyncratic shock corresponds to $S_{ids} = \frac{\gamma_1\lambda_1}{1-(\gamma_1\lambda_1+\gamma_2\lambda_2+\gamma_3\lambda_3)A}$.

A.2 Default probabilities

Note that a portfolio loss occurs by time t if either $N_1(t) > 0, N_2(t) > 0$, or $N_3(t) > 0$. Hence a loss occurs with probability $1 - P(N_1(t) = 0, N_2(t) = 0, N_3(t) = 0)$. By independence,

$$\begin{aligned} P(N_1(t) = 0, N_2(t) = 0, N_3(t) = 0) &= P(N_1(t) = 0)P(N_2(t) = 0)P(N_3(t) = 0) \\ &= e^{-\lambda_1 t}e^{-\lambda_2 t}e^{-\lambda_3 t} \\ &= e^{-(\lambda_1+\lambda_2+\lambda_3)t}. \end{aligned}$$

Therefore, the probability of a loss within the next year equals $1 - e^{-(\lambda_1+\lambda_2+\lambda_3)}$. In a similar fashion, the probability of having at least one medium- or economy-sized shock in the next year is $1 - e^{-(\lambda_2+\lambda_3)}$, and the probability of having an economic shock is $1 - e^{-\lambda_3}$.

B Decomposition Implementation

To implement this model, we approximate $L_{a,b}$ using finite sums. Since $N_1(t)$ will represent losses due to a single reference, we allow this to occur up to 50 times. Further, we let $N_2(t)$ jump at most 10 times, representing sector-wide losses, and let $N_3(t)$ jump at most 3 times, representing economy-wide losses. Thus, we have

$$E[L_{a,b}(t)] \approx \sum_{j_1=0}^{50}\sum_{j_2=0}^{10}\sum_{j_3=0}^{3} \frac{e^{-(\lambda_1+\lambda_2+\lambda_3)t}(\lambda_1 t)^j(\lambda_2 t)^j(\lambda_3 t)^j}{j_1!j_2!j_3!} f_{a,b}(\gamma_1 j_1+\gamma_2 j_2+\gamma_3 j_3). \quad (13)$$

Since payments are made quarterly, on a given date, let M denote the number of quarters until maturity for the CDX series that we wish to study. Then the integrals in Equations (7) and (8) can be approximated, for instance, as Riemann sums:

$$C_{a,b}\sum_{i=0}^{M} D(\frac{i}{4})(1-E[L_{a,b}(\frac{i}{4})])\frac{1}{4}, \quad (14)$$

and

$$\sum_{i=0}^{M} D(\frac{i}{4})(E[L_{a,b}(\frac{i}{4})]-E[L_{a,b}(\frac{i-1}{4})]), \quad (15)$$

where $E[L_{a,b}(t)]$ is zero for $t \leq 0$. Alternatively, one could use a trapezoidal or other rule to approximate the integral.

To fit the model to the data, we begin with a time series of the tranche and index spreads. First, we choose parameters for gamma that represent the type of credit event: $\gamma_1 = 1/125$ (idiosyncratic risk), $\gamma_2 = 15/125$ (sector risk), and $\gamma_3 = 100/125$ (economy-wide risk). Then, for each day, for a choice of parameters λ_2, λ_3, we can compute λ_1 using Equation (12). On a given day, let M denote the remaining of the given series. From this we can then compute the tranche spreads by the formula

$$C_{a,b} = \frac{\sum_{i=0}^{M} D(\frac{i}{4})(1-E[L_{a,b}(\frac{i}{4})])\frac{1}{4}}{\sum_{i=0}^{M} D(\frac{i}{4})(E[L_{a,b}(\frac{i}{4})]-E[L_{a,b}(\frac{i-1}{4})])}. \quad (16)$$

Since an equity (e.g., 0–3 per cent) tranche is quoted in terms of $C_{0,3}$ per cent upfront payment plus a running spread of 500 basis points, we have

$$C_{0,3} + 0.05\int_0^T (D(t)(1-E[L_{0,3}(t)])\,dt = \int_0^T D(t)dE[L_{0-3}(t)]. \quad (17)$$

We can then evaluate the integral similarly to Equations (14) and (15).

For a given day, we observe tranche quotes $C^{obs}_{0,3} - C^{obs}_{15,30}$, and the entire index quote C^{obs}. For a given set of values for λ_2 and λ_3, we can use C^{obs} and Equation (12) to compute λ_1. Then, using Equation (16), and the discrete form of Equation (17), we can compute theoretical tranche prices $C^{th}_{0,3} - C^{th}_{15,30}$. We then perform the following optimization:

$$\min_{\lambda_2,\lambda_3} \left(\frac{C^{obs}_{0,3} - C^{th}_{0,3}}{C^{obs}_{0,3}} \right)^2 + \ldots + \left(\frac{C^{obs}_{15,30} - C^{th}_{15,30}}{C^{obs}_{15,30}} \right)^2 . \qquad (18)$$