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Conditioning under Cumulative Prospect Theory

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Conditioning under Cumulative Prospect Theory

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Abstract

This paper derives conditions under which the standard decomposition of unconditional expected utility into marginal probabilities and conditional expected utilities generalizes to Cumulative Prospect Theory. The results are relevant for empirical analyses in which marginal probabilities are used as explanatory variables.

Keywords: Cumulative Prospect Theory, probability weighting functions, conditioning, updating
JEL classification: D80, D84

1 Introduction

This paper is motivated by the need for modeling risky choices under alternatives to expected utility theory. So far, most tests have been based on experiments in which test subjects had to express preferences with respect to simple lotteries. Such lotteries are, however, extremely rare outside the laboratory. In real life, decision makers typically choose between complex lotteries, i.e. payoffs tied to events that are non-degenerate sets of states and are defined in terms of random variables that correlate with other state variables.[1]

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[1]In analyses of laboratory experiments, it is commonly assumed that lotteries are based on random variables that are independent of anything of "real life" relevance to the decision makers. It is in this sense that the experiments are based on simple lotteries.

In expected utility theory, complex lotteries can be modeled as compound lotteries based on a decomposition of unconditional expected utility into marginal probabilities and conditional expected utilities. Our paper specifies conditions for a similar decomposition in the framework of prospect theory. These conditions matter for empirical analyses of decision makers' behavior outside the laboratory, where *weighted* marginal probabilities are used as explanatory variables.

To fix ideas, we consider a specific example. Suppose a decision maker (DM) wishes to insure himself against an event A and faces a choice between two different insurance policies Q and R. For insurance $h \in \{Q, R\}$, the DM pays a premium c_h and receives a gross payment of t_h from the insurance company if event A occurs. In the absence of insurance, the DM's income is given by the random variable z.

In real world situations, the insured event A as well as its complement will consist of a number of states of the world. For example, the DM could be a farmer who buys insurance against the event that it rains more than a specific amount. More of his crops will be ruined if the actual rainfall is well above this amount than if it is only just above it; likewise his crops will be affected by different amounts of rain below the amount that triggers the insurance payment. In such contexts, the probability of the insured event A must be interpreted as a marginal probability since the event is a set of states. We denote this probability as p_A.

Under expected utility, we can analyze the DM's behavior based on the standard decomposition of unconditional expected utility into marginal probabilities and conditional expected utility. That is, insurance Q is weakly preferred to insurance R if

$$\begin{aligned} p_A E[u(z - c_Q + t_Q)|A] + (1 - p_A)E[u(z - c_Q)|A^{\complement}] \geq \\ p_A E[u(z - c_R + t_R)|A] + (1 - p_A)E[u(z - c_R)|A^{\complement}]. \end{aligned} \tag{1}$$

This decomposition enables us to describe the DM's choice with an econometric model in which the probability p_A is used as an explanatory variable. The decomposition is valid since, under the implicit assumption of dynamic consistency, Bayesian updating is a corollary of the subjective expected utility theorem. That is, with a state space $S = \{1, \ldots, n\}$ and prior probabilities $\pi(s)$, the posterior probabilities after arrival of information that the true state is in $A \neq \emptyset$ are given by

$$\pi_A(s) = \pi(s|A) = \begin{cases} \frac{\pi(s)}{\pi(A)} & \text{if } s \in A \\ 0 & \text{if } s \notin A. \end{cases}$$

The present paper is concerned with conditions under which a decomposition similar to (1) holds under Tversky and Kahneman's (1992) Cumulative Prospect Theory (CPT).[2] These conditions are important for empirical analyses based on models in which marginal probabilities like p_A are used as explanatory variables that determine DMs' choices via "decision weights" $w(p_A)$. The use of such models is a practical necessity since we will rarely be able to specify the joint distribution of all state variables that matter to DMs and correlate with the random variable that defines whether event A occurs.[3]

We model a DM whose updating of probability weights is with respect to a benchmark prospect, which can be interpreted as "what does the DM think would have happened if event A had not occurred." For expected utility, this benchmark can be any possible act, i.e. the updating rule does not depend on the benchmark. For probability weights, however, the prospect used as a benchmark does matter for the updating rule, just like the benchmark act matters for the updating of capacities, see Gilboa and Schmeidler (1993). In order to obtain a decomposition of unconditional CPT utility into marginal decision weights and conditional CPT utility, we need to place restrictions on the benchmark prospect.

Gilboa and Schmeidler's (1993) pessimistic updating rule for ambiguous beliefs has the benchmark act being the best possible outcome in all states. This can be interpreted as "the DM thinks that if A had not occurred, he would have gotten the best possible outcome." On the other hand, their optimistic updating rule has the benchmark act being the worst possible outcome in all states, which can be interpreted as "the DM thinks that if A had not occurred, he would have gotten the worst possible outcome." The present paper derives conditional probability weighting functions, rather than updating rules for ambiguous beliefs. We will show that certain updating rules for CPT based on the best and worst possible outcomes yield a decomposition of unconditional CPT utility into weighted marginal probabilities and conditional CPT utility, which thus parallels the standard decomposition of expected utility.

The paper is structured as follows: section 2 contains our results and section 3 gives conclusions and further discussion, including how our updating of probability weights relates

[2]Of course, since our conditions are sufficient for a decomposition like (1) to hold in the sign and rank dependent CPT model, they also suffice for the decomposition to hold in the rank-dependent utilty models of Quiggin (1982) and Yaari (1984).

[3]If we were able to specify the joint distribution of all relevant state variables, we could model DMs' choices in terms of preferences with respect to simple lotteries.

to that in Sarin and Wakker (1998).

2 Conditioning under Cumulative Prospect Theory

Let $S = \{1, \ldots, n\}$ be a finite state space and let X be a set of outcomes that describe changes with respect to the status quo or an appropriate reference level. The states in S occur with probabilities $(p_1, \ldots, p_n)$. Let $\mathscr{F}$ denote the set of prospects, i.e. functions from S to X. For an outcome $x \in X$, let $\bar{x} \in \mathscr{F}$ denote the constant prospect $(x, \ldots, x)$. The status quo is an element of the set of outcomes X, denoted by 0, and is assumed fixed.

Let $\succsim$ denote a preference relation on $\mathscr{F}$, with $\succ$ and $\sim$ denoting the asymmetric and symmetric parts respectively. We use the same notation for preference relations on the set of outcomes X, i.e. $x \succsim y$ if and only if $\bar{x} \succsim \bar{y}$. An outcome $x \succ 0$ is positive and an outcome $x \prec 0$ is negative.

We are concerned with how this preference relation is updated upon arrival of the information that the true state is in some event $A \subset S$. Our updating rules apply to prospects whose outcomes are comonotonic. A set of prospects F are comonotonic if for no $f, g \in F$ and no $s, s' \in S$, it holds that $f(s) \succ f(s')$ and $g(s') \succ g(s)$. We also restrict attention to conditioning on events A or $A^{\complement}$, for which $A^{\complement}$ is a dominating event for A: for all $s \in A$ and for all $s' \in A^{\complement}$, $f(s') \succsim f(s)$. That is, the states in the event A are (weakly) worse than the states in the complement of the event, $A^{\complement}$.

Let o be a permutation on $\{1, \ldots, n\}$ such that for the set of comonotonic prospects F under consideration, $f(o(n)) \succsim \ldots \succsim f(o(1)) \ \forall f \in F$. Hence, for non-positive prospects $0 \succsim f(o(n))$, while for the non-negative prospects $f(o(1)) \succsim 0$.

We assume throughout that preferences are represented by a Cumulative Prospect Theory (CPT) representation.[4] With the notation from Prelec (1998) this means that in general, for $x_1 \precsim \cdots \precsim x_k \precsim 0 \precsim x_{k+1} \precsim \cdots \precsim x_n$, preferences are represented by

$$V(f) = \sum_{i=1}^{k} \left[w^-\big(\sum_{j=1}^{i} p_j\big) - w^-\big(\sum_{i=1}^{i-1} p_j\big) \right] v(x_i) + \sum_{i=k+1}^{n} \left[w^+\big(\sum_{j=i}^{n} p_j\big) - w^+\big(\sum_{j=i+1}^{n} p_j\big) \right] v(x_i), \tag{2}$$

where w^- and w^+ are unique nondecreasing weighting functions satisfying $w^-(0) = w^+(0) = 0$ and $w^-(1) = w^+(1) = 1$ and $v(x)$ is a continuous and increasing ratio scale.

[4] Wakker and Tversky (1993) provide an axiomatization.

In the next two subsections, we start our analysis by considering prospects whose outcomes are either exclusively in the loss domain, i.e. non-positive, or exclusively in the gains domain, i.e. non-negative. Preferences will thus be represented by

$$V(f) = \sum_{i=1}^{n} \left[w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right] v(x_{o(i)}) \tag{3}$$

when f lies in the loss domain, and

$$V(f) = \sum_{i=1}^{n} \left[w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right] v(x_{o(i)}) \tag{4}$$

when f lies in the gains domain. For each i, the expression in brackets is the decision weight associated with outcome x_i.

Let $h, g \in \mathscr{F}$ be comonotonic prospects given by $h(1, 2, \ldots, n) = (y_1, y_2, \ldots, y_n)$ and $g(1, 2, \ldots, n) = (z_1, z_2, \ldots, z_n)$. Let $f^0 = \overline{0}$, i.e. the prospect that gives the status quo (or reference point) in all states. Let $x_* \in X$ denote the worst possible outcome in X and assume that $0 \succ x_*$. Define f_* by

$$f_*(s) = \begin{cases} x_* & \text{if } s \in A \\ 0 & \text{if } s \notin A, \end{cases}$$

i.e. the prospect that gives the worst possible outcome if A occurs and the status quo otherwise. Next, let $x^* \in X$ denote the best possible outcome in X and assume that $x^* \succ 0$. Define f^* by

$$f^*(s) = \begin{cases} 0 & \text{if } s \in A \\ x^* & \text{if } s \notin A, \end{cases}$$

i.e. the prospect that gives the status quo if A occurs and the best possible outcome otherwise.

Define conditional acts by

$$h_A f = \begin{cases} h(s) & \text{if } s \in A \\ f(s) & \text{if } s \notin A \end{cases}$$

and define conditional preference $h \succsim_A g$ by $h_A f \succsim g_A f$. I.e. $\succsim_A$ is the DM's preferences conditional on knowing that s is in A. The prospect f is the benchmark prospect.

Consider an event A such that for all $s \in A$, and for all $s' \in A^{\complement}$, $h(s') \succsim h(s)$, i.e. the event A is dominated by $A^{\complement}$. Hence, there exists $k \in \{1, \ldots, n\}$ such that $A = \{o(1), \ldots, o(k)\}$ and $A^{\complement} = \{o(k+1), \ldots, o(n)\}$. We assume that both A and $A^{\complement}$ are non-null given the DM's preferences, and thus these events are both assigned non-zero weights in the CPT-representation. We consider separately the cases in which A and $A^{\complement}$ are both in the loss domain, both in the gains domain, or constitute a loss/gains partition.

2.1 Loss domain

Assume first that both of the comonotonic prospects h and g are in the loss domain for all states. The events A and $A^{\complement}$ thus both contain states associated with non-positive outcomes, but the event A is still dominated by $A^{\complement}$.

We will show that for the loss domain we get the desired decomposition of the unconditional CPT utility when f_* is used as a benchmark prospect for updating. Notice how, with this benchmark, the resolution of uncertainty affects the DM's attitude. If the DM receives information that the true state is in the bad event A, he becomes a pessimist and acts as if he would have gotten the status quo 0 had A not happened. If instead he receives information that the true state is in the good event $A^{\complement}$, he becomes an optimist and acts as if he would have gotten the worst conceivable outcome x_* had $A^{\complement}$ not happened.

We begin with two definitions.

Definition 1 *The* **loss domain pessimistic conditional CPT utility** *of prospect h is given by*

$$E_L^P[h|s \in A] \equiv \sum_{i=1}^{k} \frac{w^-\left(\sum_{j=1}^{i} p_{o(j)}\right) - w^-\left(\sum_{j=1}^{i-1} p_{o(j)}\right)}{w^-\left(\sum_{j=1}^{k} p_{o(j)}\right)} v(y_{o(i)}).$$

Notice that the denominator is the sum of the decision weights in the numerator and equals the probability weight of the event A that consists of the states $o(1), \ldots, o(k)$, which in the loss domain give the most extreme outcomes relative to the status quo. The information that has arrived reveals that one of the worse states will occur.

Definition 2 *The* **loss domain optimistic conditional CPT utility** *of prospect h is given by*

$$E_L^O[h|s \in A^{\complement}] \equiv \sum_{i=k+1}^{n} \frac{w^-\left(\sum_{j=1}^{i} p_{o(j)}\right) - w^-\left(\sum_{j=1}^{i-1} p_{o(j)}\right)}{1 - w^-\left(\sum_{j=1}^{k} p_{o(j)}\right)} v(y_{o(i)}).$$

Again the denominator is the sum of the decision weights in the numerator, which now equals one minus the probability weight of the event $(A^{\complement})^{\complement} = A$. The information that has arrived reveals that one of the better states will occur.

Proposition 1 shows how unconditional CPT utility can be decomposed into decision weights and conditional CPT utilities in the loss domain, similar to how unconditional expected utility can be decomposed into marginal probabilities and conditional expected utility.

Proposition 1 *Suppose f_* is used as a benchmark for updating in the loss domain. Then the following holds: preferences conditional on the arrival of information that $s \in A$ are represented by $E_L^P[h|s \in A]$, preferences conditional on the arrival of information that $s \in A^{\complement}$ are represented by $E_L^O[h|s \in A^{\complement}]$, and unconditional CPT utility is given by*

$$V(h) = w^-(p_A)E_L^P[h|s \in A] + (1 - w^-(p_A))E_L^O[h|s \in A^{\complement}].$$

Proof: With the benchmark prospect f_*, we have

$$\begin{aligned}
V(h_A f_*) &= V(y_{o(1)}, \ldots, y_{o(k)}, 0, \ldots, 0) \\
&= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) \\
&\quad + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(0) \\
&= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) + \left(1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) \right) v(0)
\end{aligned}$$

and

$$\begin{aligned}
V(g_A f_*) &= V(z_{o(1)}, \ldots, z_{o(k)}, 0, \ldots, 0) \\
&= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) + \left(1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) \right) v(0).
\end{aligned}$$

Thus,

$$
\begin{aligned}
h_A f_* \gtrsim g_A f_* \quad &\Leftrightarrow \quad V(h_A f_*) \geq V(g_A f_*) \\
&\Leftrightarrow \quad \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) + \left(1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) \right) v(0) \\
&\qquad \geq \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) + \left(1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) \right) v(0) \\
&\Leftrightarrow \quad \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) \\
&\qquad \geq \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) \\
&\Leftrightarrow \quad \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\
&\qquad \geq \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(z_{o(i)}),
\end{aligned}
$$

so preferences conditional on arrival of the information that $s \in A$ are represented by $E_L^P[h|s \in A]$.

Now consider conditioning on the complement $A^{\complement} = \{o(k+1), \ldots, o(n)\}$. We have

$$
\begin{aligned}
V(h_{A^{\complement}} f_*) &= V(x_*, \ldots, x_*, y_{o(k+1)}, \ldots, y_{o(n)}) \\
&= w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)})
\end{aligned}
$$

and

$$
\begin{aligned}
V(g_{A^{\complement}} f_*) &= V(x_*, \ldots, x_*, z_{o(k+1)}, \ldots, z_{o(n)}) \\
&= w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
h_{A^{\complement}} f_* \succsim g_{A^{\complement}} f_* \quad &\Leftrightarrow \quad V(h_{A^{\complement}} f_*) \geq V(g_{A^{\complement}} f_*) \\
&\Leftrightarrow \quad w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) \\
&\quad\; \geq w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) \\
&\Leftrightarrow \quad \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) \\
&\quad\; \geq \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) \\
&\Leftrightarrow \quad \sum_{i=k+1}^{n} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\
&\quad\; \geq \sum_{i=k+1}^{n} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(z_{o(i)}),
\end{aligned}
$$

so preferences conditional on arrival of the information that $s \in A^{\complement}$ are represented by $E_L^O[h|s \in A^{\complement}]$.

Since $p_A = \sum_{j=1}^{k} p_{o(j)}$, it now follows that in the loss domain

$$
\begin{aligned}
& w^-(p_A) E_L^P[h|s \in A] + (1 - w^-(p_A)) E_L^O[h|s \in A^{\complement}] \\
= \;& w^-(\sum_{j=1}^{k} p_{o(j)}) \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\
& + (1 - w^-(\sum_{j=1}^{k} p_{o(j)})) \sum_{i=k+1}^{n} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\
= \;& \sum_{i=1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}),
\end{aligned}
$$

which equals the unconditional CPT utility in (3). Hence, we have the desired decomposition. ■

An immediate consequence of Proposition 1 is the following corollary, which describes updating of the probability weighting function for the loss domain.

Corollary 1 *If the DM starts with a prior probability weighting function $w^-(\cdot)$, his conditional probability weighting function $w_A^-(\cdot)$ given the information that $s \in A$ is*

$$w_A^-(\cdot) = \begin{cases} \frac{w^-(\cdot)}{w^-\left(\sum_{j=1}^k p_{o(j)}\right)} & \text{if } s \in A \\ 0 & \text{if } s \in A^{\complement}, \end{cases}$$

while his conditional probability weighting function $w_{A^{\complement}}^-(\cdot)$ given the information that $s \in A^{\complement}$ is

$$w_{A^{\complement}}^-(\cdot) = \begin{cases} 0 & \text{if } s \in A \\ \frac{w^-(\cdot)}{1-w^-\left(\sum_{j=1}^k p_{o(j)}\right)} & \text{if } s \in A^{\complement}. \end{cases}$$

2.2 Gains domain

Assume now that both of the comonotonic prospects h and g are in the gains domain for all states, i.e. have exclusively non-negative outcomes. For the gains domain we obtain the desired decomposition of the unconditional CPT utility when f^* is used as a benchmark prospect for updating. Again the benchmark determines how the resolution of uncertainty affects the DM's attitude. If the DM receives information that the true state is in the bad event A, he becomes a pessimist and acts as if he would have gotten the best conceivable outcome x^* had A not happened. On the other hand, if he receives information that the true state is in the good event $A^{\complement}$, he becomes an optimist and acts as if he would have gotten the status quo 0 had $A^{\complement}$ not happened.

We have the following definitions of conditional CPT utility for the gains domain:

Definition 3 *The* **gains domain optimistic conditional CPT utility** *of prospect h is given by*

$$E_G^O[h|s \in A^{\complement}] \equiv \sum_{i=k+1}^{n} \frac{w^+\left(\sum_{j=i}^n p_{o(j)}\right) - w^+\left(\sum_{j=i+1}^n p_{o(j)}\right)}{w^+\left(\sum_{j=k+1}^n p_{o(j)}\right)} v(y_{o(i)}).$$

Again the denominator is the sum of the decision weights in the numerator, which here equals the probability weight of the event $A^{\complement}$. Note that $A^{\complement}$ consists of the states $o(k+1), \ldots, o(n)$, which for the gains domain result in the most extreme outcomes relative to the status quo. The information that has arrived reveals that one of the better states will occur.

Definition 4 *The* **gains domain pessimistic conditional CPT utility** *of prospect h is given by*

$$E_G^P[h|s \in A] \equiv \sum_{i=1}^{k} \frac{w^+\big(\sum_{j=i}^n p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^n p_{o(j)}\big)}{1 - w^+\big(\sum_{j=k+1}^n p_{o(j)}\big)} v(y_{o(i)}).$$

Here the denominator equals one minus the probability weight of the event $A^{\complement}$. The information that has arrived reveals that one of the worse states will occur.

Proposition 2 shows how unconditional CPT utility can be decomposed into marginal decision weights and conditional CPT utilities in the gains domain. The proof is similar to that of Proposition 1 and is therefore relegated to the appendix.

Proposition 2 *Suppose f^* is used as a benchmark for updating in the gains domain. Then the following holds: preferences conditional on the arrival of information that $s \in A$ are represented by $E_G^P[h|s \in A]$, preferences conditional on the arrival of information that $s \in A^{\complement}$ are represented by $E_G^O[h|s \in A^{\complement}]$, and unconditional CPT utility is given by*

$$V(h) = (1 - w^+(p(A^{\complement}))E_G^P[h|s \in A] + w^+(p(A^{\complement}))E_G^O[h|s \in A^{\complement}].$$

Proof: See the appendix.

In parallel to Proposition 1, an immediate consequence of Proposition 2 is the following corollary, which describes updating of the probability weighting function for the gains domain.

Corollary 2 *If the DM starts with a prior probability weighting function $w^+(\cdot)$, his conditional probability weighting function $w_A^+(\cdot)$ given the information that $s \in A$ is*

$$w_A^+(\cdot) = \begin{cases} \frac{w^+(\cdot)}{1 - w^+\big(\sum_{j=k+1}^n p_{o(j)}\big)} & \text{if } s \in A \\ 0 & \text{if } s \in A^{\complement}, \end{cases}$$

while his conditional probability weighting function $w_{A^{\complement}}^+(\cdot)$ given the information that $s \in A^{\complement}$ is

$$w_{A^{\complement}}^+(\cdot) = \begin{cases} 0 & \text{if } s \in A \\ \frac{w^+(\cdot)}{w^+\big(\sum_{j=k+1}^n p_{o(j)}\big)} & \text{if } s \in A^{\complement}. \end{cases}$$

2.3 Loss and Gains Partition

Now, we consider the case of comonotonic prospects h and g that have outcomes in the loss domain for all states in an event A, and have outcomes in the gains domain for all states in the event $A^{\complement}$. Then, updating under the benchmarks f_* and f^* in the loss and gains domains, respectively, still yields the desired decomposition of the unconditional CPT, even though the DM does not use the same benchmark throughout. The result is obtained because the DM always takes the status quo outcome as the counterfactual, irrespective of whether she learns that the true state is in the bad event A or that it is in the good event $A^{\complement}$. That is, since f_* and f^* are of the form

$$\hat{f}_*(s) = \begin{cases} x_i & \text{if } s \in A \\ 0 & \text{if } s \notin A, \end{cases} \quad \text{and} \quad \hat{f}^*(s) = \begin{cases} 0 & \text{if } s \in A \\ x_j & \text{if } s \notin A, \end{cases}$$

for some $x_i, x_j \in S$, we obtain that

$$h_A f_* = \begin{cases} h(s) & \text{if } s \in A \\ 0 & \text{if } s \notin A \end{cases} \quad \text{and} \quad h_{A^{\complement}} f^* = \begin{cases} 0 & \text{if } s \in A \\ h(s) & \text{if } s \notin A. \end{cases}$$

Of course, the decomposition in Proposition 3 could also have been obtained by simply using f^0 (i.e. the prospect that gives the status quo in all states) as a benchmark.

Proposition 3 *Suppose f_* is used as a benchmark for updating in the loss domain and f^* is used as a benchmark for updating in the gains domain. Then the following holds for a partition $\{A, A^{\complement}\}$ of the state space S, where A and $A^{\complement}$ consist exclusively of states in the loss and gains domains, respectively:*

Preferences conditional on the arrival of information that $s \in A$ are represented by $E_L^P[h|s \in A]$, preferences conditional on the arrival of information that $s \in A^{\complement}$ are represented by $E_G^O[h|s \in A^{\complement}]$, and unconditional CPT utility is given by

$$V(h) = w^-(p(A))E_L^P[h|s \in A] + w^+(p(A^{\complement}))E_G^O[h|s \in A^{\complement}].$$

Proof: The proof is similar to that of Proposition 1 and is given in the appendix.

The following corollary describes the updating of the probability weighting functions for a partition $\{A, A^{\complement}\}$ of the state space S into the sets of states associated with the loss and gains domains when f_* and f^* are used as benchmarks in the loss- and the gains domain, respectively.

Corollary 3 *If the DM starts with prior probability weighting functions $w^-(\cdot)$ and $w^+(\cdot)$, his conditional probability weighting function $w_A^-(\cdot)$ given the information that $s \in A$ is*

$$w_A^-(\cdot) = \begin{cases} \frac{w^-(\cdot)}{w^-\left(\sum_{j=1}^{k} p_{o(j)}\right)} & \text{if } s \in A \\ 0 & \text{if } s \in A^{\complement}, \end{cases}$$

while his conditional probability weighting function $w_{A^{\complement}}^+(\cdot)$ given the information that $s \in A^{\complement}$ is

$$w_{A^{\complement}}^+(\cdot) = \begin{cases} 0 & \text{if } s \in A \\ \frac{w^+(\cdot)}{w^+\left(\sum_{j=k+1}^{n} p_{o(j)}\right)} & \text{if } s \in A^{\complement}. \end{cases}$$

2.4 Repeated Conditioning

Propositions 1 through 3 all concern updating following the arrival of a single piece of information. If information arrives in several consecutive pieces, the propositions can be applied repeatedly.

To illustrate, let D and D' be disjoint sets such that $D \cup D' = A$ and D is dominated by D', and let C and C' be disjoint sets such that $C \cup C' = A^{\complement}$ and C is dominated by C'. Suppose that information arrives in two pieces: first it is revealed whether the state is in A or $A^{\complement}$ and subsequently whether the state is in D (C) or D' (C'). If the reference point is such that A and $A^{\complement}$ are both in the loss domain and the DM is updating according to Proposition 1, he uses f_* as his benchmark act after the arrival of the first piece of information. Conditional on the first piece of information, the universe is either $D \cup D'$ or $C \cup C'$, both of which are still entirely in the loss domain. The benchmark for updating after arrival of the second piece of information is therefore f_* as well. A similar argument shows that f^* will be used as the benchmark throughout if A and $A^{\complement}$ are both in the gains domain.

Repeated conditioning also further motivates the result in Proposition 3, i.e. that the DM can use f_* as the benchmark act for updating in the loss domain and f^* for updating in the gains domain if $A/A^{\complement}$ is a loss/gain partition. Had the DM instead used f^0 for updating conditional on the first piece of information, then the results in Corollaries 1 and 2 could not be used for updating conditional on the second piece of information unless the benchmark act is changed after the first round of updating. The updating rules specified in Proposition 3 can instead be used consistently – no change of benchmark is required.

There does exist an act that can be used as a benchmark for updating throughout and still results in conditional preferences being represented as in Propositions 1 through 3. This act is defined by

$$f^{**}(s) = \begin{cases} x_* & \text{if } s \in A \\ x^* & \text{if } s \notin A, \end{cases}$$

that is, the very worst outcome in A and the very best outcome in $A^{\complement}$. Updating with f^{**} as the benchmark results in the following proposition:

Proposition 4 *Suppose f^{**} is used as a benchmark for updating. Then unconditional CPT utility and the representation of conditional preferences are as given in Proposition 1 when A and $A^{\complement}$ are both in the loss domain, as given in Proposition 2 when A and $A^{\complement}$ are both in the gains domain, and as given in Proposition 3 when A and $A^{\complement}$ constitute a loss/gain partition.*

Proof: The proof follows the lines of the proofs of Propositions 1 through 3 by noting that the terms concerning the non-occuring event still cancel out when the benchmark act is replaced by f^{**}. ■

3 Discussion and Conclusion

Sarin and Wakker (1998) introduce a notion of revealed likelihood for rank-dependent expected utility models. They define revealed likelihood for comonotonic acts through decision weights and dominating events. For updating of revealed likelihood, they suggest the following definition of revealed conditional likelihood:

$$\pi(Q|R) = \pi(Q, D|R, D') = \frac{\pi(Q \cap R, D)}{\pi(R, D')} = \frac{\nu((Q \cap R) \cup D) - \nu(D)}{\nu(R \cup D') - \nu(D')},$$

where D is a dominating event for $Q \cap R$, D' is a dominating event for R, π is the revealed likelihood, and ν is a capacity.

Our updating rules for the gains domain are consistent with Sarin and Wakker's definition. For the loss domain, however, Sarin and Wakker's definition only coincides with our updating rule if $w^-(1-p) = 1 - w^-(p)$ for all $p \in [0,1]$. This, of course, does not hold for probability weighting functions that exhibit subcertainty, and e.g. does not hold for the parameterizations proposed in Tversky and Kahneman (1992) and Prelec (1998). If,

however, we replace the dominating events in Sarin and Wakker's definition with the more extreme events relative to the status quo, we have consistency in the loss domain as well.

To illustrate the relevance of our results for empirical work, consider again the insurance example from the introduction, i.e. a comparison of two insurance policies Q and R that represent insurance against the same event A, but to different extents and at different costs. Suppose that the two insurance options rank the states such that A is clearly dominated by $A^{\complement}$, and that the labels Q and R are assigned such that $t_Q - c_Q > t_R - c_R$. Suppose further that the DM uses bankruptcy as his reference point, and thus regards all outcomes as being in the gains domain. We can then use the result in Proposition 2 to evaluate the insurance options:

$$\begin{aligned} Q \succeq R \quad &\Leftrightarrow \quad (1 - w^+(p_{A^{\complement}}))E_G^P[Q|s \in A] + w^+(p_{A^{\complement}})E_G^O[Q|s \in A^{\complement}] \\ &\qquad \geq (1 - w^+(p_{A^{\complement}}))E_G^P[R|s \in A] + w^+(p_{A^{\complement}})E_G^O[R|s \in A^{\complement}] \\ &\Leftrightarrow \quad \frac{w^+(p_{A^{\complement}})}{1 - w^+(p_{A^{\complement}})} \left(- \frac{E_G^O[Q|s \in A^{\complement}] - E_G^O[R|s \in A^{\complement}]}{E_G^P[Q|s \in A] - E_G^P[R|s \in A]} \right) \leq 1 \end{aligned}$$

The last equation suggest an econometric specification in which the probability $p_{A^{\complement}}$ is used as an explanatory variable, based on a parametric specification for the "odds ratio" $w^+(p_{A^{\complement}})/(1 - w^+(p_{A^{\complement}}))$. Similar specifications could be obtained based on Propositions 1 and 3 for different sets of assumptions.

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4 Appendix

Proof of proposition 2: With the benchmark prospect f^*, we have

$$\begin{aligned} V(h_A f^*) &= V(y_{o(1)}, \ldots, y_{o(k)}, x^*, \ldots, x^*) \\ &= \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) \\ &\quad + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(x^*) \\ &= \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) + w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) v(x^*) \end{aligned}$$

and

$$\begin{aligned} V(g_A f^*) &= V(z_{o(1)}, \ldots, z_{o(k)}, x^*, \ldots, x^*) \\ &= \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(z_{o(i)}) + w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) v(x^*). \end{aligned}$$

Thus,

$$
\begin{aligned}
& h_A f^* \gtrsim g_A f^* \\
\Leftrightarrow \quad & V(h_A f^*) \geq V(g_A f^*) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) + w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) v(x^*) \\
& \geq \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(z_{o(i)}) + w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) v(x^*) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) \geq \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(z_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \geq \sum_{i=1}^{k} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(z_{o(i)}),
\end{aligned}
$$

so preferences conditional on arrival of the information that $s \in A$ are represented by $E_G^P[h|s \in A]$.

Now consider conditioning on the complement $A^{\complement} = \{o(k+1), \ldots, o(n)\}$. We have

$$
\begin{aligned}
V(h_{A^{\complement}} f^*) &= V(0, \ldots, 0, y_{o(k+1)}, \ldots, y_{o(n)}) \\
&= \sum_{i=1}^{k} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(0) \\
&\quad + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) \\
&= \left(1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) \right) v(0) + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)})
\end{aligned}
$$

and

$$
\begin{aligned}
V(g_{A^{\complement}} f^*) &= V(0, \ldots, 0, z_{o(k+1)}, \ldots, z_{o(n)}) \\
&= \left(1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big) \right) v(0) + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(z_{o(i)}).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& h_{A^{\complement}} f^* \succsim g_{A^{\complement}} f^* \\
\Leftrightarrow \quad & V(h_{A^{\complement}} f^*) \geq V(g_{A^{\complement}} f^*) \\
\Leftrightarrow \quad & \left(1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)\right) v(0) + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(y_{o(i)}) \\
& \geq \left(1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)\right) v(0) + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(z_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(y_{o(i)}) \\
& \geq \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(z_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \\
& \geq \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(z_{o(i)}),
\end{aligned}
$$

so preferences conditional on arrival of the information that $s \in A^{\complement}$ are represented by $E_G^O[h|s \in A^{\complement}]$.

Since $p(A^{\complement}) = \sum_{j=k+1}^{n} p_{o(j)}$, it now follows that in the gains domain

$$
\begin{aligned}
& \big(1 - w^+(p_{A^{\complement}})\big) E_G^P[h|s \in A] + w^+(p_{A^{\complement}}) E_G^O[h|s \in A^{\complement}] \\
= \quad & \big(1 - w^+(\sum_{j=k+1}^{n} p_{o(j)})\big) \sum_{i=1}^{k} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{1 - w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \\
& + w^+(\sum_{j=k+1}^{n} p_{o(j)}) \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \\
= \quad & \sum_{i=1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(y_{o(i)}),
\end{aligned}
$$

which equals the unconditional CPT utility in (4). Hence, we have the desired decomposition. ■

Proof of proposition 3: With the benchmark prospect f_*, we have

$$\begin{aligned} V(h_A f_*) &= V(y_{o(1)}, \ldots, y_{o(k)}, 0, \ldots, 0) \\ &= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) + w^+\big(\sum_{i=k+1}^{n} p_{o(i)}\big) v(0) \end{aligned}$$

and

$$\begin{aligned} V(g_A f_*) &= V(z_{o(1)}, \ldots, z_{o(k)}, 0, \ldots, 0) \\ &= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) + w^+\big(\sum_{i=k+1}^{n} p_{o(i)}\big) v(0). \end{aligned}$$

Thus,

$$\begin{aligned} h_A f_* \gtrsim g_A f_* &\Leftrightarrow V(h_A f_*) \geq V(g_A f_*) \\ &\Leftrightarrow \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(y_{o(i)}) \\ &\quad \geq \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(z_{o(i)}) \\ &\Leftrightarrow \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\ &\quad \geq \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(z_{o(i)}), \end{aligned}$$

so preferences conditional on arrival of the information that $s \in A$ are represented by $E_L^P[h|s \in A]$.

Now consider conditioning on the complement $A^{\complement} = \{o(k+1), \ldots, o(n)\}$. We have

$$\begin{aligned} V(h_{A^{\complement}} f^*) &= V(0, \ldots, 0, y_{o(k+1)}, \ldots, y_{o(n)}) \\ &= w^-\big(\sum_{i=1}^{k} p_{o(i)}\big) v(0) + \sum_{i=k+1}^{n} \left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big) \right) v(y_{o(i)}) \end{aligned}$$

and

$$\begin{aligned} V(g_{A^{\complement}}f^*) &= V(0,\ldots,0,z_{o(k+1)},\ldots,z_{o(n)}) \\ &= w^-\big(\sum_{i=1}^{k} p_{o(i)}\big)v(0) + \sum_{i=k+1}^{n}\left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(z_{o(i)}). \end{aligned}$$

Thus,

$$\begin{aligned} h_{A^{\complement}}f^* \succsim g_{A^{\complement}}f^* &\Leftrightarrow V(h_{A^{\complement}}f^*) \geq V(g_{A^{\complement}}f^*) \\ &\Leftrightarrow \sum_{i=k+1}^{n}\left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(y_{o(i)}) \\ &\quad \geq \sum_{i=k+1}^{n}\left(w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)\right) v(z_{o(i)}) \\ &\Leftrightarrow \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \\ &\quad \geq \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(z_{o(i)}), \end{aligned}$$

so preferences conditional on arrival of the information that $s \in A^{\complement}$ are represented by $E_G^O[h|s \in A^{\complement}]$.

Since $p_A = \sum_{j=1}^{k} p_{o(j)}$ and $p(A^{\complement}) = \sum_{j=k+1}^{n} p_{o(j)}$, it now follows that

$$\begin{aligned} & w^-(p_A)E_L^P[h|s\in A] + w^+(p_{A^{\complement}})E_G^O[h|s\in A^{\complement}] \\ = \; & w^-(\sum_{j=1}^{k} p_{o(j)}) \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(y_{o(i)}) \\ & + w^+(\sum_{j=k+1}^{n} p_{o(j)}) \sum_{i=k+1}^{n} \frac{w^+\big(\sum_{j=i}^{n} p_{o(j)}\big) - w^+\big(\sum_{j=i+1}^{n} p_{o(j)}\big)}{w^+\big(\sum_{j=k+1}^{n} p_{o(j)}\big)} v(y_{o(i)}) \\ = \; & \sum_{i=1}^{k}\left[w^-\big(\sum_{j=1}^{i} p_j\big) - w^-\big(\sum_{i=1}^{i-1} p_j\big)\right] v(y_{o(i)}) + \sum_{i=k+1}^{n}\left(w^+\big[\sum_{j=i}^{n} p_j\big) - w^+\big(\sum_{j=i+1}^{n} p_j\big)\right] v(y_{o(i)}), \end{aligned}$$

which equals the unconditional CPT utility in (2). Hence, we have the desired decomposition.■