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Growth, Equilibrium, and the
Money Rate of Interest: Comment

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Diskussionsbeiträge
des Fachbereichs Wirtschaftswissenschaften
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Nr. 12

I. Introduction (the problem)

In his attempt to demonstrate the central role of the rate of monetary expansion (rate of growth of money supply), operating through its influence on anticipations and asset prices, in determining the equilibrium level of money income, MUNDELL¹⁾ primarily treats the case of a constant money supply in an economy with growth in real output. He distinguishes two extreme ways of adjusting the growing money demand to a constant money supply. He first considers a continuous fall of the level of prices of commodities and then a continuous rise in the rate of interest. Both kinds of adjustments affect the equilibrium position of the economy. This dependence of the equilibrium position on the rate of expansion of the money supply is usually neglected in macroeconomic equilibrium analysis. Thus, a frequent fallacy is pointed out. Since MUNDELL's short treatment of the second case overstrained my intuition and perhaps that of other readers, too, the aim of this note is to derive the equilibrium conditions more rigorously, with less appeal to intuition. This clarification will also show that the two cases are not as symmetrical as MUNDELL wants us to believe (p. 28), and that the second case may not be stated in the manner of MUNDELL without additional qualifications.

¹⁾ See Robert A. MUNDELL, *Monetary Theory*, Pacific Palisades, Calif., 1970; Chapt. III, p. 23-31; adapted from: *A Fallacy in the Interpretation of Macroeconomic Equilibrium*, *Journal of Political Economy* 73 (1965), pp. 61-66. Pages in my text refer to the book.

II. MUNDELL's method and results

MUNDELL considers a closed economy with three types of assets - goods, securities (bonds and equities), and money - held by a consolidated private sector (firms and households) and a consolidated public sector (banks and government). After stating the conditions of stock and flow equilibrium he investigates the nature of the equilibrium that is implied by these conditions. To simplify the exposition he assumes a money demand function with unitary interest and income elasticities. He then goes on to state: "Monetary equilibrium, together with growth and a constant stock of money..... implies that the interest rates are rising or prices are falling at a rate equal to the rate of growth of output" (p. 25/26). These immediate consequences of the constant money supply "have a first-order impact on the general equilibrium of the system. If monetary equilibrium is established by falling commodity prices, the money interest rate (the nominal yield on bonds and bills) will differ from the real interest rate (the return on equities and the marginal efficiency of capital); whereas, if it is established by falling bond prices, the cost of holding money will differ from the nominal return on bonds by the rate at which bond prices are falling. In both cases the discrepancy between the cost of holding money and the real interest rate causes a shift in the demand for money" (p. 26).

Being usually neglected, this shift allows to identify a fallacy in the conventional interpretation of macro-economic equilibrium. MUNDELL then gives a geometric solution to the problem of determining equilibrium income by the use of the familiar HICKSian LM- and

IS-schedules, representing the loci of interest rates and money incomes at which, respectively, money demand equals money supply, and real investment equals real saving. MUNDELL's method is "to regard the ordinate [of the familiar IS-LM diagram] for purposes of deriving the LM schedule as the cost of holding money, so that LM remains fixed; and to regard the ordinate, for purposes of deriving the IS-schedule, as the marginal efficiency of capital, so that IS also remains fixed. The equilibrium is then discovered by finding the level of money income at which the vertical distance between the IS and LM schedules equals the difference between the cost of holding money and the marginal efficiency of capital [= real rate of interest]" (p. 28). MUNDELL first analyses the extreme case of falling commodity prices. Here the discrepancy between the real and the money rates of interest equals the rate of deflation, the money rate of interest representing the cost of holding money. Provided the interest and income elasticities are both equal to one, equilibrium requires this gap to equal the rate of growth of output. In the other extreme case of falling bond prices, the difference between the real rate of interest, which here is equal to the money rate of interest, and the cost of holding money equals the rate at which interest rates are rising (bond prices fall) and MUNDELL - still assuming unitary elasticities of money demand - determines true equilibrium to exist at that level of income where the gap between the money (= real) rate of interest and the cost of holding money is equal to the rate of growth of real output. The cost of holding money - being here the money (= real) rate of interest less the rate at which the interest rate is rising - may be interpreted as the short-term rate of

interest, or as "the yield on bills of virtually zero duration"(p. 29). Instead of a gap between money and real rates of interest as in the case of falling commodity prices we have here, in the case of falling bond prices, a gap between short-term and long-term rates of interest.

III. Analytical treatment of the problem

Without complicating the analysis, we try to be a bit more general than MUNDELL by allowing both the interest and the income elasticity of money demand to differ from unity. Yet our point to follow is not based on this deviation. For reasons of simplicity we use a model in continuous time formulation.

The LM-curve is represented by the following equilibrium condition for the money market

$$\frac{M_t}{P_t} = c(Y_t)^\epsilon (\zeta_t)^\eta \quad (t \geq 0) \quad (1)$$

where

- M = money supply
- p = price level of commodities
- Y = real income
- ζ = opportunity cost of holding money
- ϵ = income elasticity of money demand (>0)
- η = "interest" elasticity of money demand (<0)
- c = a constant term.

We make use of the following definitions²⁾ of the money supply, the price level of commodities, real income, the opportunity cost of holding money, and the real rate of interest:

$$M_t = M_0 e^{\int_0^t g_M(\tau) d\tau} \quad (2)$$

$$P_t = P_0 e^{\int_0^t g_P(\tau) d\tau} \quad (3)$$

$$Y_t = Y_0 e^{\int_0^t g_Y(\tau) d\tau} \quad (t \geq 0) \quad (4)$$

$$\rho_t = \rho_0 e^{\int_0^t g_\rho(\tau) d\tau} \quad (5)$$

$$r_t = r_0 e^{\int_0^t g_r(\tau) d\tau} \quad (6)$$

The IS-curve is specified by

$$S(r_t, Y_t) = I(r_t, Y_t) \quad (t \geq 0) \quad (7)$$

where r_t = real rate of interest (marginal efficiency of capital)

S = real rate of saving

I = real rate of investment

In a continuous time model with bonds being claims to a continuous and perpetual income stream of constant nominal amount, the opportunity cost of holding money are to be defined as

$$\rho_t = i_t - g_i(t) \quad (t \geq 0) \quad (8)$$

2) Throughout this note we define growth rates according to

$$g_x(t) = \frac{dx(t)/dt}{x(t)} .$$

where g_i = rate of growth of the money rate of interest (rate of fall of the bond price)

and the money rate of interest (i_t) is given by Irving FISHER'S hypothesis

$$i_t = r_t + g_p^e(t) \quad (t \geq 0) \quad (9)$$

with g_p^e = expected rate of change of the commodity price level.

Since we shall be mainly concerned with steady states, for the sake of simplicity we already may specify at this point an equation that is usually taken as a property of steady states:

$$g_p^e(t) = g_p(t) \quad (t \geq 0) \quad (10)$$

As a link between real investment and growth of real output we make use of a production function:

$$Y_t = F(K_t, L_t) \quad (t \geq 0) \quad (11)$$

and define the capital stock and the labour stock as

$$K_t = K_0 e^{\int_0^t g_K(\tau) d\tau} \quad (12)$$

$$L_t = L_0 e^{\int_0^t g_L(\tau) d\tau} \quad (13)$$

where K = real capital
 L = labour

The rate of growth of capital is defined as

$$g_K(t) = \frac{I(r(t), Y(t))}{K(t)} \quad (t \geq 0) \quad (14)$$

and for the rate of growth of labour we assume:

$$g_L(t) = \text{exogenously given for all } t \geq 0 \quad (15)$$

(assumed to be constant for steady state purposes).

Thus we arrive at a system of 15 equations with 17 variables (unknowns):

$$M_t, P_t, Y_t, g_t, r_t, i_t, g_M(t), g_Y(t), g_p(t), g_g(t),$$

$$g_i(t), g_r(t), g_p^e(t), K_t, L_t, g_K(t), g_L(t), \text{ the initial}$$

conditions $M_0, p_0, Y_0, g_0, K_0, L_0$ as well as the rates

of growth of all variables and all points in time preceding t (i.e. $\tau < t$) being given (predetermined) at time t . Equations (7), (11)-(15) are used to derive equation (16)³⁾

$$g_Y(t) = g_Y(Y(t), L_0, K_0, g_L(\tau), g_K(\tau)) \quad (16)$$

$(t \geq 0)$

$\begin{matrix} \tau \leq t & \tau < t \\ 0 \leq \tau & 0 \leq \tau \end{matrix}$

In the sequel we shall use equations (1)-(10) and (16) containing the following 13 variables:

$$M_t, P_t, Y_t, g_t, r_t, i_t, g_M(t), g_Y(t), g_p(t), g_p^e(t),$$

$$g_g(t), g_i(t) \text{ and } g_r(t).$$

³⁾ Equation (16) is an analytical basis for the dashed line GG in MUNDELL's figure 3 - 1, l.c., p. 27

This equation system has two degrees of freedom. Equations (1)-(5) are used to derive

$$g_M(t) - g_p(t) = \epsilon g_Y(t) + \eta g(t) \quad (t \geq 0). \quad (17)$$

To eliminate one degree of freedom we follow MUNDELL and set

$$g_M(t) = 0 \quad \text{for all } t \geq 0. \quad (18)$$

Again following MUNDELL the second degree of freedom will be eliminated in two mutually exclusive manners

a) by setting $g_p(t) = 0$ for all $t \geq 0$ (19)
(falling commodity prices)

and

b) by setting $g_p(t) = 0$ for all $t \geq 0$ (20)
(falling bond prices).

It is not our aim to examine all details of the solution of these two equation systems, but to limit ourselves to some aspects of the solutions which are at stake in MUNDELL's paper.

Falling commodity prices

Exploiting (17), (18), and (19) we arrive at

$$- g_p(t) = \epsilon g_Y(t) \quad (t \geq 0) \quad (21)$$

Assuming $g_Y(t) > 0$ it then follows from $\epsilon > 0$ that

$$g_p(t) < 0 \quad (t \geq 0) \quad (22)$$

By taking derivatives with respect to time, we find from (8)

$$g_p(t) = \frac{\left(\frac{di(t)}{dt}\right) - \left(\frac{dg_i(t)}{dt}\right)}{g(t)} \quad (t \geq 0) \quad (23)$$

Again using (19), it follows from (23) that

$$\left(\frac{di(t)}{dt}\right) = \left(\frac{dg_i(t)}{dt}\right) \cdot (t \geq 0) \quad (24)$$

Now, if we further assume the system to be in a steady state, then we may set

$$\left(\frac{dg_i(t)}{dt}\right) = 0 \quad (25)$$

and thus arrive at

$$g_i(t) \equiv \cancel{\left(\frac{di(t)}{dt}\right)} i(t) = 0, \quad (26)$$

which leads us to

$$g(t) = i(t) \quad (27)$$

From this result, together with (9), (10) and (21), we finally derive

$$r(t) - g(t) = -g_p(t)$$

$$\boxed{g(t) = r(t) - \epsilon g_Y(t)} \quad (28)$$

We emphasize that (28) stands for any t for which a steady state is reached, while the existence of steady states does not require t to go to infinity ($t \rightarrow \infty$).

Choosing $\epsilon = 1$, we have MUNDELL's result for the case of falling commodity prices.

Falling bond prices

Using (17), (18), and (20) we find

$$g_Y(t) = -\frac{\eta}{\epsilon} g_g(t) \quad t \geq 0. \quad (29)$$

From (20), (9) and (10) we have

$$i(t) = r(t) \quad \text{for all } t \geq 0 \quad (30)$$

which tells us that

$$g_i(t) = g_r(t) \quad t \geq 0. \quad (31)$$

Taking derivatives with respect to time, we derive from (8):

$$\frac{\left(\frac{dg(t)}{dt}\right)}{g(t)} = \frac{\left(\frac{di(t)}{dt}\right)}{i(t) - g_i(t)} - \frac{\left(\frac{dg_i(t)}{dt}\right)}{i(t) - g_i(t)} \quad (32)$$

which may be written as

$$g_g(t) \equiv \frac{\left(\frac{dg(t)}{dt}\right)}{g(t)} = \frac{g_i(t)}{1 - \frac{g_i(t)}{i(t)}} - \frac{\left(\frac{dg_i(t)}{dt}\right)}{\frac{i(t)}{g_i(t)} - 1} \quad (t \geq 0)$$

$$g_g(t) = w_1(t) g_i(t) + w_2(t) \delta(t) \quad (33)$$

where $\delta(t) \equiv (dg_i(t)/dt)/g_i(t) \quad (34)$

$$(t \geq 0)$$

Thus the rate of growth of the opportunity cost of holding money ($g_p(t)$) is a weighted average of the rate of growth of the bond rate (money rate of interest) g_i and its relative acceleration $\delta(t)$, the weights being

$$w_1(t) = \frac{i(t)}{i(t) - g_i(t)} \quad (35a)$$

$$w_2(t) = \frac{-g_i(t)}{i(t) - g_i(t)} \quad (35b)$$

$(t \geq 0)$

It is important to know, that for steady states we necessarily have $(dg_i(t)/dt) = 0$,

which for $g_i(t) > 0$ implies $\delta(t) = 0$. (36)

Combining (29) and (33), we find

$$g_Y(t) = -\frac{\eta}{\epsilon} \left[w_1(t)g_i(t) + w_2(t)\delta(t) \right] \quad (37)$$

and by rearrangement of (37) we arrive at

$$g_i(t) = -\frac{\epsilon}{w_1(t)\eta} g_Y(t) - \frac{w_2(t)}{w_1(t)} \cdot \delta(t) \quad (38)$$

From (38) and (8) we see that

$$g(t) = i(t) + \frac{\epsilon}{\eta w_1(t)} g_Y(t) + \frac{w_2(t)}{w_1(t)} \cdot \delta(t) \quad (39)$$

an expression that even under MUNDELL's narrowing assumptions $\epsilon = 1, \eta = -1$ deviates from his result

$$g(t) = i(t) - g_Y(t) \quad (40)$$

Under the specified conditions of unitary elasticities his result is equivalent to ours if and only if

$$w_1(t) = 1 \quad \text{and} \quad w_2(t) = 0. \quad (41)$$

There are two situations in which (41) holds. One is the stationary case of $g_i(t) = 0$ for all $t \geq 0$. In the nonstationary case of falling bond prices we may assume:

$$\bar{C} \geq g_i(t) > 0 \quad \text{for all} \quad t \geq 0 \quad (42)$$

(\bar{C} = a finite upper limit).

This leads us to

$$\lim_{t \rightarrow \infty} i = \infty \quad (43)$$

which implies

$$\lim_{t \rightarrow \infty} w_1(t) = 1 \quad (44)$$

$$\lim_{t \rightarrow \infty} w_2(t) = 0. \quad (45)$$

Excluding stationarity, we are limited to the asymptotic case. Thus we have reached the point where from (38) we realize that in the present case of falling bond prices the existence of a steady state is restricted to an infinite t . For $t \rightarrow \infty$ we have from (31), (33), (36), (38), (42), (44) and (45)

$$\varepsilon_r = \varepsilon_i = \varepsilon_p = -\frac{\varepsilon}{\eta} \varepsilon_y \quad (46)$$

IV. Generalizations

Starting again from (17) and without using (18) we may easily arrive at more general expressions which allow the growth rate of the money supply to deviate from zero:

$$g_g(t) = \frac{1}{\eta} \left[g_M(t) - g_p(t) - \epsilon g_Y(t) \right] \quad (17')$$

(t ≥ 0)

a) Changing commodity prices

For steady states, using (19), (27), (17'), (9) and (10), we immediately find

$g(t) = r(t) + \left[g_M(t) - \epsilon g_Y(t) \right] \quad \text{for any } t \geq 0. \quad (47)$
--

b) Changing bond prices

Applying (8), (20), (33) and (17') we find

$$g(t) = i(t) - \frac{1}{\eta w_1(t)} \left[g_M(t) - \epsilon g_Y(t) \right] + \frac{w_2(t)}{w_1(t)} \delta(t) \quad (48)$$

for any t ≥ 0.

Again, we have a nonstationary steady state only for t → ∞ where

$$g_r = g_i = g_g = \frac{1}{\eta} \left[g_M - \epsilon g_Y \right] \quad (49)$$

and together with (30) we arrive at

$g(t) = r(t) - \frac{1}{\eta} \left[g_M(t) - \epsilon g_Y(t) \right] \quad (50)$ <p>for t → ∞</p>
--

c) Combinations

Combinations of the two cases may be described by the relation

$$g_p(t) = \alpha g_g(t) \quad \alpha \leq 0, t \geq 0. \quad (51)$$

Substituting (51) for $g_p(t)$ in (17), we may derive from (33)

$$g_i(t) = \frac{g_M(t) - \epsilon g_Y(t)}{(\eta + \alpha) w_1(t)} - \frac{w_2(t)}{w_1(t)} \delta(t) \quad (52)$$

which, via (8), (9), (10), (51) and (33), finally leads us to

$$g(t) = r(t) + \left[\alpha w_1(t) - 1 \right] \cdot \frac{g_M(t) - \epsilon g_Y(t)}{(\eta + \alpha) w_1(t)} + \frac{w_2(t)}{w_1(t)} \delta(t) \quad (53)$$

(52)-(53) holding for all $t \geq 0$.

Applying (44), (45), (42) and (36), the limit of $g(t)$ for $t \rightarrow \infty$ is an immediate result of (53) and the only nonstationary ($g_i > 0$) steady state solution.

V. Final Remark

Our results clearly point towards an asymmetry between the two cases considered. In the case of falling(changing) commodity prices a steady state may be realised at any finite t . We have no restrictions as to the time parameter t . Furthermore, as equations (28) and (47) show, the size of the interest elasticity of the demand for money in this case does not enter the steady state solution for the opportunity cost of holding money.

This is different in the case of changing bond prices. Here, a nonstationary steady state exists only asymptotically ($t \rightarrow \infty$), and the interest elasticity of the money demand plays a decisive role for the steady state value of the opportunity cost of holding money (see (39) and (50)).

The qualifications of MUNDELL's results as to the size of t ($t \rightarrow \infty$) and as to the relevance of the interest elasticity that proved necessary in the case of changing bond prices equally apply to the general case of com-
bining both changing commodity and changing bond prices (see (53)).