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Does a Renewable Fuel Standard for Biofuels
Reduce Climate Costs?

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Abstract

Recent contributions have questioned whether biofuels policies actually lead to emissions reductions, and thus lower climate costs. In this paper we make two contributions to the literature. First, we study the market effects of a renewable fuel standard. Opposed to most previous studies we model the supply of fossil fuels taking into account that fossil fuels is a non-renewable resource. Second, we model emissions from land use change explicitly when we evaluate the climate effects of the renewable fuel standard. We find that extraction of fossil fuels most likely will decline initially as a consequence of the standard. Thus, if emissions from biofuels are sufficiently low, the standard will have beneficial climate effects. Furthermore, we find that the standard tends to reduce total fuel (i.e., oil plus biofuels) consumption initially. Hence, even if emissions from biofuels are substantial, climate costs may be reduced. Finally, if only a subset of countries introduce a renewable fuel standard, there will be carbon leakage to the rest of the world. However, climate costs may decline as global extraction of fossil fuels is postponed.

JEL-Code: Q200, Q320, Q400, Q420.

Keywords: biofuel, renewable fuel standard, fossil fuel.

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1 Introduction

More than 20% of energy-related CO₂ emissions come from the transport sector, and governments are therefore looking for alternatives to fossil fuels in this sector. Biofuels are currently the most employed alternative, accounting for 2-3 percent of global transport-related energy use (IEA, 2011a).

The advantage of biofuels is that they are relatively easy to introduce into the transport sector. While hydrogen and battery driven cars at the moment imply both more expensive and somewhat inferior technologies, cars run on biofuels have approximately the same performance as cars run on fossil fuels and can use the same infrastructure. The US and a number of European countries have introduced various support schemes for deployment of biofuels leading to strong growth in global biofuels production and use. Clearly, the support has not only been driven by a concern for greenhouse gas (GHG) emissions, as both the EU and the US have invoked arguments about “energy security” and the need for regional development.

Current support schemes involve the use of a myriad of policies. For instance, the EU has imposed a biofuels target of 10% in 2020, and a mixture of blending mandates, excise tax rebates to biofuels, subsidies to growing of biofuels crops, and tariffs on imported biofuels have been introduced in the EU countries (Eggert, Greaker and Potter, 2011). The US has a renewable fuel standard (RFS), which is similar to a blending mandate, and a tariff on imported biofuels in addition to various tax reliefs (Eggert, Greaker and Potter, 2011). The complex support schemes have spurred an emerging literature analyzing the combined effect of these schemes, see, e.g., DeGorter and Just (2010), Lapan and Moschini (2010) and Eggert and Greaker (2012).

Recent contributions have also questioned whether first generation biofuels actually

lead to any short-run GHG reductions. Obvious sources of emissions include the use of fertilizer when growing biofuels crops (Crutzen et al, 2008), and the use of fossil energy in the harvesting and processing of biofuels (Macedo et al, 2008). Land use change can lead to additional GHG emissions if the area of arable land is increased to accommodate increasing use of biofuels. Fargione et al. (2008) introduced the concept of carbon debt, and hold that in the worst case scenario it may take up to several hundred years to reach climate neutrality after such conversion.

In this paper we make two contributions to the literature. With very few exceptions most of the literature studying the market effects of biofuels policies have treated fossil fuel as a traditional commodity, or simply modeled fossil fuel supply as infinitely elastic. In this paper we model fossil fuel supply taking explicitly into account that fossil fuels are non-renewable natural resources. Thus, we model forward looking, competitive suppliers.¹ Furthermore, to our knowledge no previous studies have combined new knowledge on emissions from land use change with market effects of biofuels policies, assessing the effects on climate costs, that is, damage costs from climate change.

There are two strands of literature that study the effects of biofuels policies. One strand studies GHG emissions from increased use of biofuels without taking into account the interaction with the fossil fuel market. Examples of this literature are Searchinger et al (2008) and Lapola et al. (2010). They both find that increased use of biofuels may lead to increased GHG emissions due to land use change. In these analyses it is implicitly assumed that biofuels will replace fossil fuels on a one-to-one basis (based on energy content).

The second strand of literature emphasizes that one should also analyze the market

¹We do not consider market power in the oil market. This has been studied by Hochman et al. (2011) and Kverndokk and Rosendahl (2012), who consider the effects of biofuels policies taking into account OPEC behaviour. However, both these studies use static analysis.

effects of these policies. In DeGorter and Just (2009) a renewable fuel standard may lead to a decrease in total fuel sales. This happens if the elasticity of biofuels supply is lower than the elasticity of fossil fuel supply. The effect of the policy is then not only to replace fossil fuels with biofuels, but also to reduce total consumption of transport fuels, which by itself will reduce climate costs. Note that this result is reversed if the elasticity of biofuels supply is higher than the elasticity of fossil fuel supply, or if a biofuels subsidy is imposed rather than a renewable fuel standard (DeGorter and Just, 2009).

Introducing several instruments complicates the picture even further. If a renewable fuel standard is in place, adding a tax rebate for biofuels or a subsidy to biofuels can only make things worse with respect to climate costs. The subsidy then works as an implicit support to fossil fuels and, hence, GHG emissions increase (see DeGorter and Just, 2010). Lapan and Mochini (2010) compare a renewable fuel standard to a price based consumption subsidy, and find that the former welfare dominates the latter. A renewable fuel standard is identical to a tax on fossil fuels and a subsidy to biofuels for which the tax and subsidy rates are set such that the tax income is equal to the subsidy outlay (Eggert and Greaker, 2012). It then follows that a blending mandate outperforms a pure subsidy (i.e., without a tax) in cases in which there is an emission externality.

All the market effects mentioned above are derived in static models. The robustness of these results should therefore be analyzed in a model with dynamic fossil fuel supply. One such example is Chakravorty et al. (2008), who study the supply of biofuels when the stock of pollution from fossil fuels is regulated.² The results in Chakravorty et al. (2008) depend on the availability of land and the demand for land for alternative uses (e.g., food). One of the findings is that if land is abundant, biofuels behave as a textbook backstop

²Note that there are no emissions from biofuels in Chakravorty et al. (2008).

resource.

The future of biofuels seems to lie within so-called second generation biofuels, which to a large extent can be grown on pastures and currently unused land (IEA, 2011b). Thus, we follow Chakravorty et al. (2008) by treating biofuels as a textbook backstop resource. On the other hand, whereas Chakravorty et al. consider a cap on the stock of emissions, we focus on a renewable fuel standard which is the policy used both in the EU and the US.³ First, we find that the extraction period of the fossil fuel resource is extended by the introduction of a renewable fuel standard. This happens even if only a subset of countries introduces the standard, while the rest of the world continues without. Second, if fuel demand is linear, total fuel consumption will unambiguously decline initially. Finally, we show that a biofuels subsidy speeds up fossil fuel extraction and, hence, GHG emissions increase, confirming the findings from static models.

Given that we know the market effects of a renewable fuel standard, we can also study the effects upon climate costs, taking into account the effects of biofuels usage on land use changes. A biofuels standard has two opposing effects: It reduces climate costs due to the postponement of fossil fuel extraction, but increases climate costs due to higher accumulated emissions (because of more biofuels production that also involves emissions). In order to evaluate the relative strengths of these effects, we calibrate a numerical model of fossil fuel extraction and demand. We find that even for biofuels that are almost as emissions-intensive as oil, a blending mandate may have a beneficial climate effect. The reason is that the blending mandate reduces total fuel demand over the first few decades. Thus, even though cumulative fuel demand and emissions are increased, emissions are postponed, giving a beneficial climate effect.

³In our paper a renewable fuel standard is identical to a blending mandate as both policies require that biofuels constitute a given share of total fuel use in the transport sector.

Despite the beneficial climate effect, a renewable fuel standard always reduces total welfare in our numerical model runs. A renewable fuel standard implies a subsidy to biofuels alongside a tax on fossil fuels. A subsidy to biofuels hampers welfare in our model since there are no other externalities than the climate externality.

The paper is laid out as follows. In Section 2 we study the effects of a blending mandate in a one region model. In Section 3 we extend the model to the two region case. In Section 4 we treat the climate costs from fossil fuels and biofuels, respectively. In Section 5 we provide a numerical illustration of the model. Finally, in Section 6 we conclude.

2 Market effects of mandatory biofuels use

We consider a market with fossil fuels (x) and biofuels (y), which are assumed to be perfect substitutes.⁴ The stock of fossil fuels (S) is fixed, and there are no costs of extracting this stock. Unit costs of producing biofuels (b) are fixed. Hence, we treat biofuels as a textbook backstop technology.⁵ Without any climate policy, the fossil fuel price is $p(t) = p_0 e^{rt}$ until $p(T) = b$ is reached at T , when a complete switch to biofuels occurs.

The assumption that fossil fuels are given in a fixed supply with constant unit costs of extraction might seem very restrictive. As shown by e.g. Gerlagh (2011) and Hoel (2011) the effects of a carbon tax are quite different for this case and the more realistic case of extraction costs increasing in accumulated extraction. In the latter case total extraction (and hence total emissions) are determined endogenously, and are lower the higher is the level of a carbon tax. However, our focus is not on a carbon tax, but on a blending mandate. Total extraction (but not the timing of extraction) is independent of a blending mandate.

⁴We may think of fossil fuels as oil here, as we are interested in the transport sector and the competition from biofuels. Thus, we implicitly abstract from oil use in other sectors, as well as other substitutes to oil in the transport sector such as electric cars, which may become important in the future.

⁵The realism of this assumption is discussed in the concluding section.

The reason for this is that both with and without a blending mandate all resources with extraction costs below the backstop price b will eventually be extracted, and no resources with higher extraction costs will be extracted. Hence, assuming an exogenous total amount of fossil fuel resources is far less restrictive in the present analysis than the same assumption is when the focus is on the effects of a carbon tax.

Assume that fuel consumers are required to use at least a share α of biofuels in the total fuel use. We coin α a renewable fuel standard (RFS). Let the consumer price of fuels be given by $p^C(t)$. We then have $p^C(t) = \alpha b + (1 - \alpha)p_0 e^{rt}$. Further, let the demand for fuel be given by $D(p^C)$, with $D' < 0$. The demand for fossil fuels and biofuels is then:

$$x(t) = (1 - \alpha)D(\alpha b + (1 - \alpha)p_0 e^{rt}) \quad \text{for } t < T \quad (1)$$

$$y(t) = \alpha D(\alpha b + (1 - \alpha)p_0 e^{rt}) \quad \text{for } t < T \quad (2)$$

$$x(t) = 0 \quad \text{and} \quad y(t) = D(b) \quad \text{for } t \geq T \quad (3)$$

where T is determined by:

$$p_0 e^{rT} = b \quad (4)$$

Finally, we have the equilibrium condition:

$$\int_0^T x(t) dt = S \quad (5)$$

The endogenous variables in equations 1 - 5 are $x(t)$, $y(t)$, T and p_0 . If p_0 is known, the whole price path is known from $p(t) = p_0 e^{rt}$.

We are now ready to investigate how an increase in α affects the market equilibrium.

2.1 Effects on resource extraction

First, we examine how the extraction time T and the initial resource price p_0 are affected by a change in α . We can show the following proposition:

Proposition 1 *If the share of biofuels in an RFS system is increased, the fossil fuel resource will last longer. Moreover, the initial price of the resource falls.*

Proof: See the Appendix.

Obviously, the proposition also holds if we introduce an RFS, i.e., increase α from zero.

The intuition of this proposition is quite clear: If the resource price didn't fall, demand for fossil fuels would have to decrease in every period, which subsequently implies that there are resources left in the ground at time T when the fossil fuel price reaches the backstop price b .

Next, we examine the effects on the extraction path. We can then show:

Proposition 2 *Assume that fuel demand is concave or linear. If the share of biofuels in an RFS system is increased, extraction of fossil fuels will decline for all $t < \hat{t}$, and increase for all $t > \hat{t}$.*

Proof: See the Appendix.

This proposition is also quite intuitive. As the resource extraction is extended, average extraction per period until the "old" T must come down. The proposition states that extraction declines at every point of time until some \hat{t} . Moreover, between the "old" and the "new" T , extraction obviously increases.

If fuel demand is convex, we cannot rule out the possibility that initial extraction increases. The explanation is that with convex demand, demand may be more price elastic at low prices. Thus, if the initial consumer price (p^C) decreases in line with lower fossil fuel price (p_0), fuel demand may be stimulated substantially so that even demand for fossil fuels increases.

2.2 Effects on fuel consumption and biofuels production

In order to investigate the effects on total fuel demand, we assume that demand is a linear function of the consumer price. We can show the following:

Proposition 3 *Assume that fuel demand is linear. If the share of biofuels in an RFS system is increased, the consumer price will increase (decrease) and fuel consumption will decrease (increase) for all $t < (>)\hat{t}$, where $0 < \hat{t} < T$.*

Proof: See the Appendix.⁶

The consumer price is pulled in both directions. On the one hand, the fossil fuel price p decreases. On the other hand, a higher α increases the weighted price $p^C = \alpha b + (1 - \alpha)p$. According to the proposition, the latter effect is dominating initially, at least if the demand function is linear. Note that this holds whether the demand function is steep or not, as long as the choke price $p_{\max}^C \geq b$.

When t approaches T , both the fossil fuel price and the consumer price approaches T . Thus, for t sufficiently close to the "old" T , the consumer price must decrease when α is increased (since p drops). Hence, for linear demand, total fuel consumption declines at early dates, and increases at later dates.

⁶Note that the value of \hat{t} is generally not the same in Propositions 2, 3 and subsequent propositions where this symbol is used.

The RFS is introduced to stimulate the use of biofuels. The following proposition states how biofuels production is affected when α is increased:

Proposition 4 *If the share of biofuels in an RFS system is increased, production of biofuels will increase if either i) α is sufficiently small initially, ii) demand is sufficiently inelastic, or iii) t is sufficiently close to T . On the other hand, biofuels production will decrease initially if α is already sufficiently close to 1 and demand is linear and sufficiently elastic.*

Proof: See the Appendix.

The first result, i.e., that biofuels production expands if at least one of three conditions is fulfilled, is as expected. The last result, i.e., that biofuels production could in fact decrease, may seem counter-intuitive at first. We know from above that the initial consumer price increases when α is increased. If demand is very elastic, fuel consumption may then drop quite substantially. Furthermore, if there is already a significant biofuels consumption due to a high α , it is possible that the effect of demand reduction dominates the effect of a higher share of biofuels. Similar results have been found in static analysis of Renewable Portfolio Standards (RPS) (or tradable green certificate markets), see e.g. Amundsen and Mortensen [1].

To summarize, we have shown that introducing or strengthening an RFS system will lead to lower fossil fuel prices, and prolonged extraction period. If demand is concave or linear, fossil fuel production will decrease initially, and increase in later periods so that total extraction is unchanged. Finally, if demand is linear, the consumer price will increase initially, implying lower initial fuel consumption, but in later periods the price will drop and fuel consumption increase. Biofuels production will most likely increase, but could decrease initially if demand is sufficiently elastic and α is already high.

2.3 Effects of biofuels subsidy in addition to RFS

A number of countries, including the U.S. and the EU, have or have had subsidies to biofuels production in addition to a RFS. Such subsidies will stimulate biofuels production and consumption, but due to the binding relationship between fossil fuel and biofuels consumption given by the RFS, fossil consumption will also be stimulated. We can show this formally, and have the following proposition:

Proposition 5 *If a binding RFS is in place, a subsidy to biofuels production will reduce the extraction time for the fossil fuel resource, and increase (decrease) the use of fossil fuels for all $t < (>)\hat{t}$, where $0 < \hat{t} < T$.*

Proof: See the Appendix.

Thus, introducing subsidies to biofuels production may have quite the contrary effect of what is the purpose, at least if the subsidy is introduced for environmental reasons. In reality, such subsidies may be temporary. Nevertheless, given a binding RFS, any policy that stimulates biofuels use will also stimulate the use of fossil fuels.

3 Two region model

So far we have considered a closed one region model, which could be thought of as the whole world. However, although many countries apply a RFS for biofuels, policies are not synchronized. For instance, both the EU and the U.S. have a RFS, but the mandates differ both with respect to the fraction of biofuels in the transport market and to how long the blending mandates are executed. Further, several large countries such as China do not at present have any mandate at all. The question then arises whether the results obtained so far also hold for a single region trading with other regions with no or weaker mandates.

Hoel (2011) shows that the effects of introducing carbon prices can have quite different effects in a two country model compared to a one country model.

With more regions one would expect carbon leakage to occur. That is, regions not tightening their RFS increase their use of fossil fuels as a result of a stronger RFS in a single region. In order to analyze the strength of this effect, we extend the model to two regions, and consider the case where only one region strengthens its mandate. The initial mandate may differ between the regions. We still assume world market prices on fossil fuels and biofuels, and a given total stock of fossil fuels.

The RFS requires consumers of fossil fuels in Region i to use at least a share α_i of biofuels in the total fuel use. The consumer price in region i is therefore $p_i^C(t) = \alpha_i b + (1 - \alpha_i)p(t)$ where as above $p(t) = p_0 e^{rt}$. Biofuels is still a backstop technology, and when $p(T) = b$ is reached at T , a complete switch to biofuels occurs in both regions. Thus, the RFS is only binding as long as $b > p(t)$.

The demand for fossil fuels and biofuels is

$$x_i(t) = (1 - \alpha_i)D_i(\alpha_i b + (1 - \alpha_i)p_0 e^{rt}) \text{ for } t < T \quad (6)$$

$$y_i(t) = \alpha_i D_i(\alpha_i b + (1 - \alpha_i)p_0 e^{rt}) \text{ for } t < T \quad (7)$$

where $x_i(t) = 0$ for $t \geq T$, $y_i(t) = D_i(b)$ for $t \geq T$ and where T is determined by $p_0 e^{rT} = b$. Finally, for a given initial resource stock S we must have the equilibrium condition

$$\int_0^T x_1(t)dt + \int_0^T x_2(t)dt = S \quad (8)$$

The model is solved in the Appendix. Interestingly, we find that the results are very similar to the results with only one region: The extraction time will be extended independent of which region that tightens its RFS, and the initial price of fossil fuels declines.

Moreover, if the demand functions in the two regions are linear, fossil fuel extraction will decline for all $t < \hat{t}$, and increase for all $t > \hat{t}$ (for some $0 < \hat{t} < T$). If one or both demand functions have second order derivatives unequal to zero, the effects on the extraction path are ambiguous. Thus, Propositions 1 carry over to the case with two regions, while Proposition 2 only partly carries over.

The consumer price on transportation fuels in the region not changing its RFS must fall at all dates due to the lower fossil fuel price. Hence, we will have carbon leakage as this region will use more fossil fuels at each date due to the increased RFS rate in the other region. If the former region also has an RFS in place, it follows that it will also increase its use of biofuels. If fossil extraction declines in the initial periods, fossil fuel consumption in the region with increased RFS must fall in these periods. It is more ambiguous what happens to biofuels consumption in this region, and to fossil fuel consumption after time \hat{t} .

4 Climate costs of an RFS

We will now consider the effect of the RFS on the costs of climate change. The use of fossil fuels leads to flow emissions, while emissions from use of biofuels potentially include both flow emissions and emissions from stock changes (related to land use changes). We first discuss the climate costs of fossil fuels and biofuels separately, before we combine them and look at the effects of an RFS on climate costs.

4.1 Climate costs from using fossil fuels

Burning fossil fuels releases carbon to the atmosphere, which will gradually decline over time, as it is transferred to other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). We model this similarly to Farzin and Tahvonen (1996). The carbon in the atmosphere at any time t is artificially split into two components $A_1(t) + A_2(t)$: component 1 that remains in the atmosphere for ever and component 2 that gradually depreciates at a rate δ . For each unit emitted the share that remains in the atmosphere for ever is denoted θ . The amount of 1 unit of carbon emissions at time t remaining in the atmosphere z years after the emission date is thus $\theta + (1 - \theta)e^{-\delta z}$. If e.g. $\delta = 0.013$ and $\theta = 0.25$, 45 % of the original emissions will remain in the atmosphere after 100 years, while 27 % still remains after 300 years. These numbers are roughly in line with what is suggested by Archer (2005) and others.

At any time t , climate costs are $C(A_1(t) + A_2(t))$, where $A_1(t) + A_2(t)$ is carbon from fossil fuels; carbon from land use changes is ignored for now. Consider the climate damage caused by 1 unit of emissions from burning fossil fuels at time t . The total additional damage caused by 1 unit of carbon emissions at time t is the sum of additional damages at all dates from t to infinity caused by the additional stocks from t to infinity. To get from additional stocks at $t + z$ to additional damages at $t + z$ we must multiply the additional stocks at $t + z$ by the marginal damage at $t + z$, which is $C'(A_1(t + z) + A_2(t + z))$. The marginal damage of 1 additional unit of emissions at t , often denoted the social cost of carbon, is thus given by

$$q_x(t) = \int_0^{\infty} e^{-rz} [\theta + (1 - \theta)e^{-\delta z}] C'(A_1(t + z) + A_2(t + z)) dz \quad (9)$$

For $C'' > 0$ the social cost of carbon will vary over time. While $A_1(t)$ is increasing as long as emissions are positive, $A_2(t)$ may be declining for sufficiently low emissions. In any case, C' and hence $q_x(t)$ will change over time.⁷ To simplify the formal analysis we assume that $C'' = 0$, i.e. that damages are linear in the atmospheric stock. When C' is constant (9) may be rewritten as

$$q_x = \left[\frac{\theta}{r} + \frac{1 - \theta}{r + \delta} \right] C' \quad (10)$$

which is constant over time.

4.2 Climate costs from using biofuels

Growing and processing biofuels entails greenhouse gas (GHG) emissions, but to what extent is a controversial topic. According to life-cycle analyses the emission sources can be grouped into the following categories: (I) fossil fuel usage for harvesting, transporting and production, (II) N₂O emissions from fertilizer usage and the crop itself, (III) direct land-use change, and (IV) indirect land-use change (see e.g. Macedo et al. 2008, Khanna and Crago, 2011).

It seems reasonable that the renewable fuel standard encompasses fossil fuels used for harvesting and transporting in the agricultural sector. Thus, source (I) is to some degree included in our model already.⁸ With respect to energy for production of biofuels, this is in many cases provided from the biofuel crop itself. For instance, sugarcane ethanol production often also supplies electricity. To our knowledge such possibilities also exist

⁷Farzin and Tahvonen (1996) give a detailed analysis of how $v(t)$ might develop over time when $C'' > 0$.

⁸Ideally, source (I) could be incorporated into our model by taking into account that total fuel demand is an increasing function of $y(t)$. This could alter the extraction path of $x(t)$, but obviously not S . As the RFS policy implies that this extra fuel demand would be proportional to $x(t)$, it seems unlikely that the extraction path would be much altered either. Moreover, harvesting and transporting biofuels crops probably make up a rather small share of total transportation fuel demand.

for second generation biofuels (Schmer et al., 2008).

Furthermore, according to Crutzen et al. (2008) source (II) may be significant for some kinds of first generation biofuels, but not so much for second generation biofuels based on energy crops. Hence, in the following we will assume that the climate costs of biofuels only follow from land use changes, i.e., (III) and (IV). This is of course a simplification, and it is straightforward to add flow emissions to the expressions below.

Emissions from direct land-use change is treated by Fargione et al. (2008). Their study shows that converting different types of virgin land to crop land may give high initial emissions, coined by Fargione et al. as *carbon debt*. Biofuels crops are however often grown on existing agricultural land. Indirect land use change then refers to changes in land use that occurs through changes in the market prices for food and land. Both Searchinger et al. (2008) and Lapola et al. (2010) analyze indirect land-use change, and show that the effect upon emissions may be of great significance.

In our model we treat direct land-use change and indirect land-use change together. In particular, we assume that the global area for food crop is kept constant, and thus that a certain production level of biofuels have led to a one time boost in emissions as the growing of every new non-food crop must increase the total area of cultivated land globally. We will now turn to how this can be modeled.

4.3 Climate costs from land use changes

It is useful to first consider the case of $\theta = 1$, so that all emissions from fossil fuel use remain in the atmosphere for ever. In this case the climate cost of each unit of x is

$$q_x = \frac{C'}{r} \tag{11}$$

What is the corresponding climate cost of each unit of y ? Assume that each unit of y requires ℓ units of land, and that each unit of land converted to biofuels production will reduce the carbon sequestered on this land by an amount β . Note that β will depend on the type of initial land and on the type of biofuels production. For some types (barren land transformed to an energy forest) β may be negative. In the subsequent discussion we assume $\beta > 0$. The parameter ℓ will also vary across types of biofuels.⁹

As before, A stands for cumulative emissions from fossil fuels. The total amount of carbon in the atmosphere at date t is $A(t) + \ell\beta y(t)$. Notice that this formulation implies that any increase in y will immediately release carbon to the atmosphere. This is not an unreasonable assumption. However, the formulation also implies that any reduction in y immediately sequesters carbon from the atmosphere. This is obviously unrealistic; in reality, the sequestration takes time. Hence, our formulation makes biofuels production more climate friendly than it is in reality if we have a declining $y(t)$.

Climate damages at date t are $C(A(t) + \ell\beta y(t))$. The negative climate impact of the production of biofuels is thus modeled just as any flow pollutant, and the climate cost per unit of y is

$$q_y = \ell\beta C'$$

With $\theta = 1$ it follows that

$$\frac{q_y}{q_x} = r\ell\beta \tag{12}$$

For the more realistic case of $\theta < 1$, some of the carbon released to the atmosphere when burning fossil fuels is gradually transferred to other carbon sinks – mainly the ocean. The

⁹Alternatively, the biofuel crop is grown on already developed agricultural land. We then assume that virgin land must be converted some other place to upkeep the production of the replaced agricultural products.

atmosphere doesn't recognize any difference between carbon from fossil fuel use and carbon from land use changes. Therefore, we must make a similar assumption about a gradual transfer from the atmosphere to other sinks also for such carbon emissions. We give a formal analysis of this in the Appendix, where it is shown that (12) is valid also for this case. In this Appendix we also consider the slightly more general case of the equilibrium value of C' rising at a constant rate $m \in [0, r)$. One interpretation of $m > 0$ is that the climate damage function has the form

$$C(A, t) = C'_0 e^{mt} A$$

Climate damage increasing with income and production is in fact often assumed, see e.g. Nordhaus (2008) or EEAG (2012). For the case of $m > 0$ we show that (10) and (12) generalize to

$$q_x(t) = \left[\frac{\theta}{r - m} + \frac{1 - \theta}{r + \delta - m} \right] C'_0 e^{mt} \quad (13)$$

and

$$\frac{q_y(t)}{q_x(t)} = (r - m)\ell\beta \equiv \gamma \quad (14)$$

4.4 The effects of an RFS on climate costs

In section 2 we showed how the introduction of an RFS for biofuels affects the time paths for the use of fossil fuel and biofuels. The effect on climate costs of the introduction of an RFS will depend on the size of γ defined by (14). For the limiting case of $\gamma = 0$ the only effect on climate costs will be through the change in the time profile of fossil fuel extraction. If extraction is delayed climate costs will go down, since $e^{rt}q_x(t)$ is declining over time. By continuity, the following proposition hence follows from Proposition 2:

Proposition 6 *If γ defined by (14) is sufficiently small and demand for fuel is linear or concave, climate costs will decline as a consequence of the introduction of an RFS for biofuels.*

For larger values of γ it is not obvious how climate costs are affected by an RFS, even if fossil fuel use is postponed. The reason for this is that biofuels use increases for low values of t , but declines for higher values of t (until the date at which fossil fuel use is zero both with and without the RFS), see Proposition 1 and 4. The isolated effect of this is to increase climate costs. There are thus two opposing effects: Reduced climate costs due to the postponement of fossil fuel extraction, and increased climate costs due to advancement in time of biofuels production. Which effects is strongest will depend on γ , and the latter effect will dominate if γ is sufficiently large. We therefore have the following proposition:

Proposition 7 *If γ defined by (14) is sufficiently large, climate costs will increase as a consequence of the introduction of an RFS for biofuels.*

The effects of an RFS on climate costs is hence an empirical issue. In the next section we give some numerical estimates of the parameters determining γ defined by (14), and compare the effects of an RFS policy with the effects of an optimal tax policy.

5 Numerical illustrations

5.1 Model calibration

We calibrate the model to real world data in the following way: We consider two linear fuel demand functions, with either elastic or inelastic demand. The initial price elasticities in the two cases are respectively -0.4 and -0.1 , but the elasticities are increasing with the price (due to linear demand). The demand functions are calibrated so that initial fuel

demand equals global oil consumption in 2011 if the initial price equals the average crude oil price in 2011 (BP, 2012). Fuel consumption growth (for given price) is set to 1.3% p.a., which is slightly more than what the IEA (2011a) assumes until 2035 in combination with higher oil prices.

The stock of fossil fuels (S) is set equal to remaining global oil reserves at the end of 2011, according to BP (2012). This may underestimate the ultimate recoverable amount of oil, but on the other hand we will assume constant unit extraction costs. Unit costs of biofuels are (preliminary) set to two times the crude oil price in 2011. Biofuels can be produced at lower costs today, but remember that we consider biofuels as a backstop technology with unlimited supply at a constant unit costs. We may think of this as e.g. cellulosic ethanol. Unit costs of fossil fuels are calibrated so that the initial oil price and consumption are consistent with the 2011 data. This leaves us with unit costs of respectively 83% and 79% of the oil price under elastic and inelastic demand, which seems fairly reasonable.

The initial shadow cost of carbon is set to \$50 per ton of CO₂, which is within the range of CO₂ prices suggested to reach ambitious climate targets (e.g., IPCC, 2007; Stern, 2007; IEA, 2011a).¹⁰ Converted to oil, the initial carbon cost (q_x) amounts to 19% of the initial oil price. Further, we assume a discount rate of $r = 4\%$, and an income growth of $m = 2\%$. In line with Section 3 we set $\delta = 0.013$ and $\theta = 0.25$.

Based on the discussion above, we only consider emissions from land use change for the climate costs of biofuels. Moreover, we have tried to find figures for second genera-

¹⁰Ideally, the shadow cost of carbon should be based on a global cost-benefit analysis. One prominent example of a CBA study is the Stern Review (2007). Their findings suggest that the present social cost of carbon is around \$85 per ton CO₂ if the world continues on the BaU path, and \$25–30 if the concentration of CO₂-equivalents stabilises between 450–550 ppm CO₂e. Most other CBA studies seem to find lower shadow costs of carbon. Both these studies and the Stern Review have been much criticized for various reasons, see, e.g., Weitzman (2007, 2011) who in particular emphasises the role of uncertainty.

tion biofuels of which cellulosic ethanol based on perennial grasses seems to be the most promising (Eggert, Greaker and Potter, 2011). For perennial grasses Sanderson (2006) reports ethanol yields from 2500 kg to 6000 kg per hectare per year (kg/ha*y). In our numerical illustration we use a yield of 4200 kg/ha*y, which is the average of the lower and upper bound of yields from Sanderson (2006).¹¹ Further, according to Plevin et al (2010) the emission factor for converted grassland is 75-200 tonne CO₂/ha. Since ethanol contains about 7.4 kWh/kg, the land use change emissions from ethanol is hence given as 3.2-8.6 kg CO₂ per kWh. Emissions from gasoline are 0.27 kilo CO₂/kWh. Thus, for the product $\ell\beta$ we get a value in the range [8.9, 23.7]. Clearly, higher yields may significantly reduce this number. For instance, if the yield is 6000 kg/ha*y, the interval of $\ell\beta$ becomes [6.3, 16.7]. In our simulations we mainly use $\ell\beta = 16, 3$, i.e., the average of the first interval. This implies that the shadow costs of biofuels amount to 33% of the shadow costs of oil (per energy unit), i.e., $\gamma = 0.33$ (cf. (14)). However, as there is large variation across different biofuels, as well as significant uncertainties, we will at the end of the next section consider which levels of γ that make the RFS policy climate neutral (compared to the BaU scenario).

5.2 Simulation results

We first compare the effects of blending mandates with the effects of an optimal climate policy scenario, which in our model can be implemented by imposing a Pigovian tax on the use of fossil fuels and biofuels. We search for the level of α that gives the same present value of reduced climate costs as the Pigovian tax. This turns out to be $\alpha = 0.48$ (0.61) in the case with inelastic (elastic) demand, given the calibrated model as described above.¹²

¹¹For comparison, Plevin et al (2010) reports the average the corn ethanol yield to be 3150 kg/ha*y, while IEA (2011b) reports the sugarcane ethanol yield to be 3900 kg/ha*y.

¹²Such large values of α will of course require an enormous growth in biofuels production, which will lead to substantial land use changes and presumably significant interactions with other parts of the economy (e.g., agriculture). Moreover, it is not realistic that such a large production growth may take place on

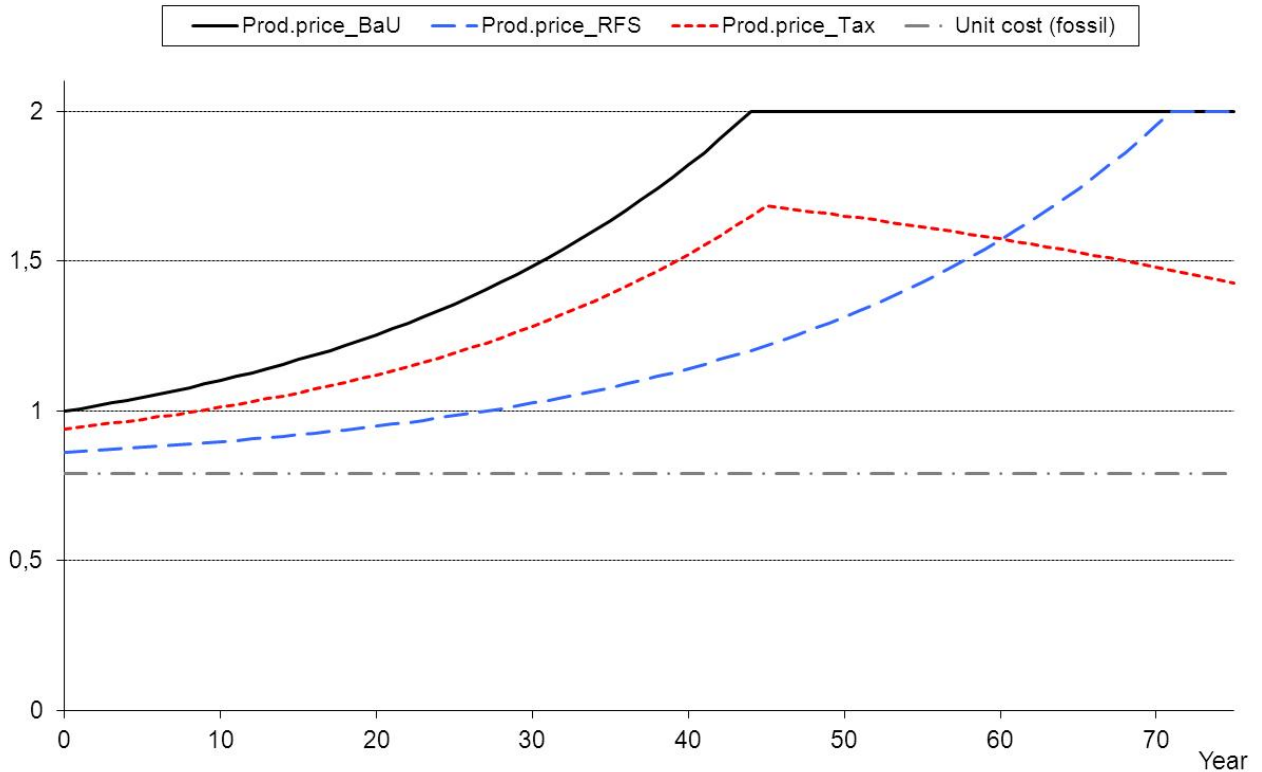


Figure 1: Producer price of oil under different scenarios

Here we discuss the case with inelastic demand only – the qualitative results are the same with elastic demand.

The two policies give very different market and welfare effects. Total welfare is reduced by 6% when choosing the RFS instead of the tax, and the RFS scenario is also reducing welfare compared to the BaU scenario. In fact, this is the case whatever the level of α is.

The RFS policy is particularly detrimental for fossil fuel producers. Whereas the Pigovian tax reduces profits of these producers by 30%, profits are reduced by two thirds under the RFS policy. The initial price of oil is reduced by almost 15% in the latter case (cf. Figure 1), which means that the initial resource rent is reduced by two thirds. In the tax scenario, the initial oil price declines by 6%.

Consumer surplus is somewhat lower in the tax scenario than under the RFS policy,

short notice. Thus, the numerical simulations should not be taken too literally.

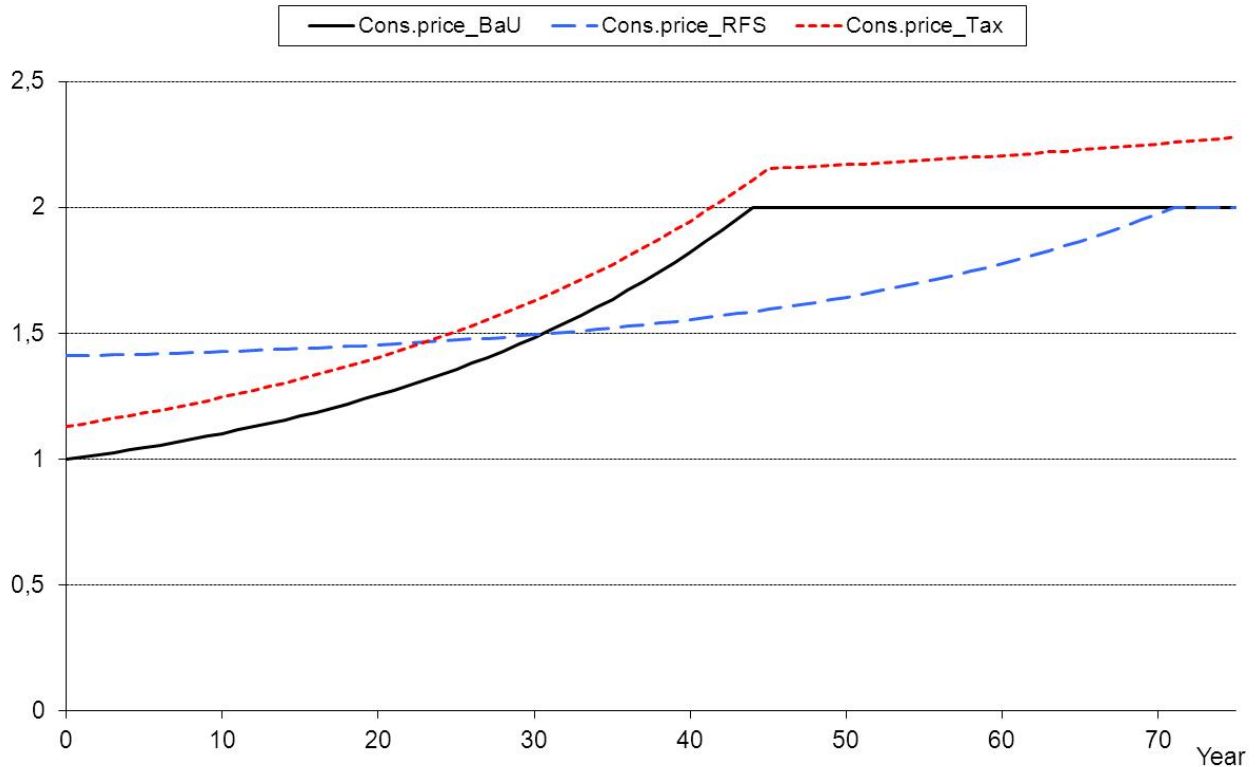


Figure 2: Consumer price of transport fuel under different scenarios

but not if the emission tax revenues are allocated back to the consumers. Thus, consumers might prefer the RFS policy if they are ignorant about public revenues, which may partly explain the popularity of blending mandates over emission taxes. On the other hand, the initial consumer price increases much more under the RFS policy than with a tax (cf. Figure 2), as the high level of α requires a large share of expensive biofuels. Thus, total fuel use is reduced much more initially under the RFS than under the tax. After about 25 years, the consumer price becomes higher under the tax policy, as the consumer price is more stable under the RFS policy due to the smaller resource rent.

Another possible explanation for the popularity of RFS might be that biofuels are thought to be environmentally friendly, or almost climate neutral, and that a blending mandate of α is assumed to reduce climate costs by close to $\alpha\%$. This is clearly not the

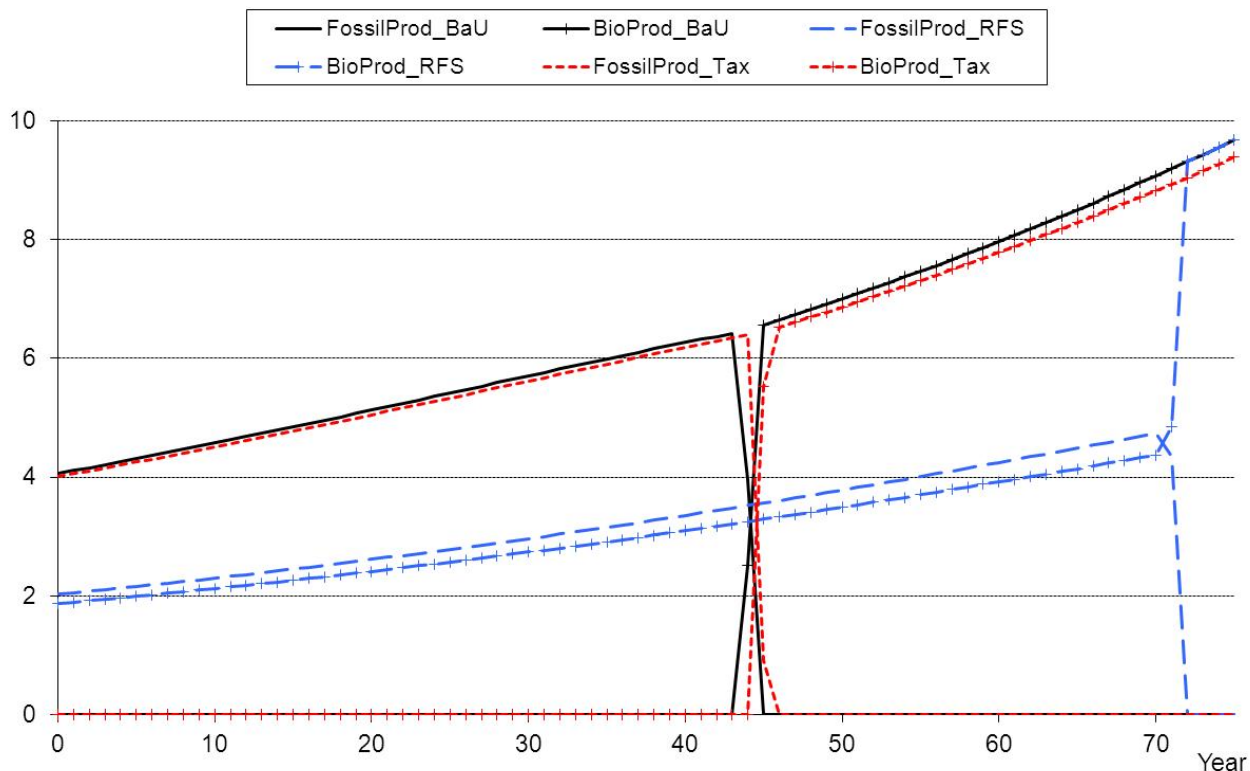


Figure 3: Fossil and biofuel production under different scenarios

case. In the optimal tax scenario, climate costs are reduced by 10%, while we have seen above that a similar reduction under RFS policy requires an α of around 50%. This is partly because emissions from biofuels are far from negligible, and partly because the RFS policy does not reduce cumulative use of fossil fuels over time. Over the first 40 years, however, fossil fuel production is almost halved, but the extraction period is extended from 44-45 years in the BaU and Tax scenarios to 71 years in the RFS scenario (cf. Figure 3).

Even if the RFS policy is welfare deteriorating, it clearly reduces climate costs given our assumed value of $\gamma = 0.33$ (i.e., $q_y/q_x = 0.33$). An interesting question, given the uncertainty of emissions from biofuels (and variation across sites), is how large must γ be before the RFS becomes counter-productive, i.e., increases climate costs? This could

clearly depend on the stringency of the RFS policy. If we consider levels of α in the range 10 – 20%, we find that the RFS policy is climate-neutral if γ is in the range 0.93 – 1. That is, even for biofuels that are almost as emissions-intensive as oil, the RFS policy may have some beneficial climate effects. The reason is that the RFS policy reduces total fuel demand over the first few decades. Thus, even though cumulative fuel demand and emissions are increased, emissions are postponed which gives a beneficial climate effect.

In Section 3 we considered a model with two regions, and discussed the effects of implementing RFS in only one of the regions. One important implication is that there will be emissions leakage to the other region. We have simulated a model version identical to the one above, except that we have split the demand region into two identical demand regions. When one region imposes an RFS with 20% biofuels, and the other region has no RFS policy, we find a leakage rate increasing from 12% initially to almost 35% just before the fossil resource is fully extracted.¹³ Thus, global emissions are postponed, and decline until the "old" depletion time period T . Climate costs are also reduced compared to BaU-levels despite leakage and an accumulated increase in emissions over time (due to more biofuels consumption). The reduction in climate costs amounts to 1.5%, versus 3.3% if both regions implement a 20% RFS share.

6 Discussion and conclusion

We have found that the extraction period of the fossil fuel resource is extended by the introduction of a renewable fuel standard. This happens even if only a subset of countries introduces a renewable fuel standard, while the rest of the world continues without. Extraction of fossil fuels will then likely decline initially. Furthermore, even for biofuels that

¹³These results refer to the inelastic demand function. With elastic demand, leakage is lower initially but higher later on.

are almost as emissions-intensive as oil, a standard may then reduce climate costs. The reason is that it tends to reduce total fuel consumption over the first decades.

We have also shown that if only a subset of countries introduce a renewable fuel standard, we will have carbon leakage to the rest of the world. However, we may still have declining climate costs as global extraction of fossil fuels is postponed, and since total fuel consumption in the country introducing the standard declines.

Note, however, that despite the beneficial climate effect, a renewable fuel standard always reduces total welfare in our numerical model runs. A renewable fuel standard implies a subsidy to biofuels alongside a tax on fossil fuels. A subsidy to biofuels hampers welfare in our model since there are no other externalities than the climate externality.

We treat biofuels as a backstop technology with constant unit costs. As shown by Chakravorty et al.(2008), this is a reasonable assumption as long as land is abundant. IEA (2011b) predicts that biofuels may provide 27% of total transport fuel in 2050. Biofuels crops must then increase from 2% of total arable land today to around 6% in 2050. Much of this increase, however, will take place on pastures and currently unused land, which is suitable for second generation biofuels. Furthermore, Schmer et al. (2008) conjectures that large improvements in both genetics and agroeconomies will increase yields dramatically. Thus, the rate of technological progress within second generation biofuels could overcome the problem with land scarcity.

We also assume that biofuels can replace fossil fuels fully when all fossil fuels are extracted. Whether a total replacement of fossil fuels is possible at reasonable costs seems to depend on the rate of technological development, for instance, if the experiments with algae based biofuels will be successful (IEA, 2011b).

Our paper should be seen as a first attempt to include both dynamic optimization

and emission from land use change when looking at biofuels policies. Later contributions should consider replacing the constant unit cost of biofuels assumption with more realistic biofuels supply schedules, among other taking into account that land quality may vary. One would then also likely let the carbon sequestered on the converted land vary with total production.

Our study has focused on the transport sector, and implicitly disregarded oil used in other sectors. Thus, future research should also consider incorporating fuel demand in other sectors. Our analysis of the two region model can in fact be alternatively interpreted as a simple representation of a two sector model, where Region 2 represents demand in non-transport sectors where the RFS policy does not apply. The numerical results above then indicate that the climate costs of the RFS policy will still be reduced, especially as the higher use of oil in Region 2 will lead to less use of other energy goods.

In the analysis above we have considered a time invariant blending mandate. It could be argued that a more realistic scenario would be to introduce a gradually increasing share of biofuels, i.e., that α is increasing over time. If so, fossil producers could find it profitable to enhance their initial extraction as future policies are (expected to be) even more detrimental to them than current policies. We have briefly tested this in our numerical model, considering linear increases in the blending rate.¹⁴ The simulations suggest that initial extraction and emissions (including those from biofuels) tend to increase if demand is elastic, but decrease if demand is inelastic. Accumulated climate costs decline in all our simulations, given our benchmark assumptions. Thus, using the terminology used by Gerlagh (2011), there may be a weak green paradox if a blending mandate is gradually introduced (if demand is sufficiently elastic), but probably not a strong green paradox.

¹⁴That is, $\alpha = kt$, $\alpha \leq \hat{\alpha}$ for different values of k and $\hat{\alpha}$.

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7 Appendix

Proof of Proposition 1:

We first differentiate 4 with respect to α :

$$\frac{dp_0}{d\alpha} e^{rT} + rp_0 e^{rT} \frac{dT}{d\alpha} = 0 \Leftrightarrow \frac{dp_0}{d\alpha} = -rp_0 \frac{dT}{d\alpha} \quad (15)$$

Next, we insert from 1 into 5 and differentiate:

$$-\int_0^T D(p^C(t)) dt + (1-\alpha) \int_0^T \left(b - p_0 e^{rt} + (1-\alpha) e^{rt} \frac{dp_0}{d\alpha} \right) D'(p^C(t)) dt + (1-\alpha) D(b) \frac{dT}{d\alpha} = 0$$

We notice that the first term equals $-S/(1-\alpha)$. Inserting for $\frac{dp_0}{d\alpha}$ then gives:

$$-\frac{S}{(1-\alpha)} + (1-\alpha) \int_0^T [(b - p_0 e^{rt}) D'(p^C(t))] dt - (1-\alpha)^2 rp_0 \frac{dT}{d\alpha} \int_0^T [e^{rt} D'(p^C(t))] dt + (1-\alpha) D(b) \frac{dT}{d\alpha} = 0$$

$$\frac{dT}{d\alpha} \left[(1-\alpha) D(b) - (1-\alpha)^2 rp_0 \int_0^T [e^{rt} D'(p^C(t))] dt \right] = \frac{S}{(1-\alpha)} - (1-\alpha) \int_0^T [(b - p_0 e^{rt}) D'(p^C(t))] dt$$

$$\frac{dT}{d\alpha} = \frac{1}{(1-\alpha)^2} \frac{S - (1-\alpha)^2 \Gamma}{D(b) - (1-\alpha) rp_0 \Lambda} > 0 \quad (16)$$

where $\Gamma = \int_0^T [(b - p_0 e^{rt}) D'(p^C(t))] dt < 0$ and $\Lambda = \int_0^T [e^{rt} D'(p^C(t))] dt < 0$.

This further gives:

$$\frac{dp_0}{d\alpha} = \frac{-rp_0}{(1-\alpha)^2} \frac{S - (1-\alpha)^2 \Gamma}{D(b) - (1-\alpha) rp_0 \Lambda} < 0 \quad (17)$$

Hence, we have proved the proposition. ■

Proof of Proposition 2:

We differentiate $x(t)$ with respect to α :

$$\begin{aligned} \frac{dx(t)}{d\alpha} &= -D(p^C(t)) + (1 - \alpha) (b - p_0 e^{rt}) D'(p^C(t)) - e^{rt} r p_0 \frac{S - (1 - \alpha)^2 \Gamma}{D(b) - (1 - \alpha) r p_0 \Lambda} D'(p^C(t)) \\ &= -D(p^C(t)) + \left[(1 - \alpha) (b - p_0 e^{rt}) - p_0 e^{rt} r \frac{S - (1 - \alpha)^2 \Gamma}{D(b) - (1 - \alpha) r p_0 \Lambda} \right] D'(p^C(t)) \quad (18) \end{aligned}$$

The first term is negative, whereas the last term can be either positive or negative. Notice that the bracket in front of $D'(p^C(t))$ decreases over time. If the bracket is positive at $t = 0$, we obviously have that the whole expression is negative, i.e., $x(0)$ decreases when α increases. If the bracket is negative at $t = 0$ (and thus for all t), the bracket will increase in absolute value over time. If the demand function is concave, i.e., $D''(p^C(t)) \leq 0$, then $D'(p^C(t))$ will also increase in absolute value over time (since $p^C(t)$ increases over time). Thus, the second term will increase over time, and so will the first term, too. Hence, $dx(t)/d\alpha$ will increase over time. But then we must have $dx(0)/d\alpha < 0$ - otherwise accumulated resource extraction over time will increase, which is not possible. Moreover, if the demand function is concave, it follows that fossil extraction will decline for all $t < \hat{t}$ and increase for all $t > \hat{t}$ (for some $0 < \hat{t} \leq T$). ■

Proof of Proposition 3:

We differentiate $p^C(t)$ with respect to α :

$$\frac{dp^C(t)}{d\alpha} = b - p_0 e^{rt} + (1 - \alpha) e^{rt} \frac{-r p_0}{(1 - \alpha)^2} \frac{S - (1 - \alpha)^2 \Gamma}{D(b) - (1 - \alpha) r p_0 \Lambda} \quad (19)$$

We notice that $\frac{dp^C(t)}{d\alpha}$ is decreasing over time. We also know that $\frac{dp^C(t)}{d\alpha} < 0$ for t sufficiently close to T , since T increases with α . Thus, if we can show that $\frac{dp^C(0)}{d\alpha} > 0$, we

have proved the proposition.

With linear demand we have:

$$\Gamma = \left[T - \frac{1}{r}(1 - e^{-rT}) \right] b\overline{D}' \text{ and } \Lambda = \frac{1}{r}(e^{rT} - 1)\overline{D}'.$$

Thus, we get:

$$\frac{dp^C(0)}{d\alpha} = p_0 \left[e^{rT} - 1 - r \frac{S - (1 - \alpha)^2 \Gamma}{D(b)(1 - \alpha) - (1 - \alpha)^2 r p_0 \Lambda} \right] \quad (20)$$

Next, we derive the following expression for S , where we use that $D(p^C) = \overline{D}'(-p_{\max}^C + p^C)$

(p_{\max}^C is the choke price, i.e., $D(p_{\max}^C) = 0$):

$$\begin{aligned} S &= \int_0^T x(t) dt = (1 - \alpha) \int_0^T D(\alpha b + (1 - \alpha)p_0 e^{rt}) dt \\ &= (1 - \alpha) \overline{D}' \int_0^T [-p_{\max}^C + (\alpha b + (1 - \alpha)p_0 e^{rt})] dt \\ &= (1 - \alpha) \overline{D}' \left[(-p_{\max}^C T + \alpha b T) + (1 - \alpha) \frac{b}{r} (1 - e^{-rT}) \right] \end{aligned} \quad (21)$$

We insert this and the expressions for Γ and Λ into 20 (note that $D(b) = \overline{D}'(b - p_{\max}^C)$):

$$\begin{aligned} \frac{dp^C(0)}{d\alpha} &= p_0 [e^{rT} - 1 \\ &\quad - r \frac{(1 - \alpha) \overline{D}' [(-p_{\max}^C T + \alpha b T) + (1 - \alpha) \frac{b}{r} (1 - e^{-rT})] - (1 - \alpha)^2 [T - \frac{1}{r}(1 - e^{-rT})] b \overline{D}'}{\overline{D}'(b - p_{\max}^C)(1 - \alpha) - (1 - \alpha)^2 r p_0 \frac{1}{r}(e^{rT} - 1)\overline{D}'}] \\ &= p_0 \left[e^{rT} - 1 - r \frac{(-p_{\max}^C T + \alpha b T) + (1 - \alpha) \frac{b}{r} (1 - e^{-rT}) - (1 - \alpha) [T - \frac{1}{r}(1 - e^{-rT})] b}{(b - p_{\max}^C) - (1 - \alpha)b(1 - e^{-rT})} \right] \\ &= p_0 \frac{\Phi}{\alpha b + (1 - \alpha)b e^{-rT} - p_{\max}^C} = \frac{p_0}{b} \frac{\Phi}{\alpha + (1 - \alpha)e^{-rT} - \frac{p_{\max}^C}{b}} \end{aligned} \quad (22)$$

where

$$\begin{aligned}
\Phi &= \alpha b e^{rT} + (1 - \alpha)b - p_{\max}^C e^{rT} - \alpha b - (1 - \alpha)b e^{-rT} + p_{\max}^C + p_{\max}^C rT - \alpha b rT - (1 - \alpha)b \\
&\quad + (1 - \alpha)b e^{-rT} + (1 - \alpha)b rT - (1 - \alpha)b + (1 - \alpha)b e^{-rT} \\
&= b \left[\frac{p_{\max}^C}{b} (rT + 1 - e^{rT}) + \alpha (e^{rT} - 1 - rT) + (1 - \alpha) (e^{-rT} + rT - 1) \right] \\
&\leq b [(rT + 1 - e^{rT}) + \alpha (e^{rT} - 1 - rT) + (1 - \alpha) (e^{-rT} + rT - 1)] \\
&= b(1 - \alpha) [(rT + 1 - e^{rT}) + (e^{-rT} + rT - 1)] = b(1 - \alpha) [2rT - e^{rT} + e^{-rT}] < 0
\end{aligned}$$

Here we have used that $p_{\max}^C \geq b$ and $2rT + e^{-rT} - e^{rT} < 0$ for any rT . We see that the denominator in 22 is negative. Hence, we have shown that the whole expression is positive for any $p_{\max}^C \geq b$, so that $\frac{dp^C(0)}{d\alpha} > 0$. ■

Proof of Proposition 4:

We differentiate $y(t)$ with respect to α :

$$\begin{aligned}
\frac{dy(t)}{d\alpha} &= D(p^C(t)) + \alpha \left(b - p_0 e^{rt} + (1 - \alpha) e^{rt} \frac{dp_0}{d\alpha} \right) D'(p^C(t)) \\
&= D(p^C(t)) + \alpha (b - p_0 e^{rt}) D'(p^C(t)) - \alpha r p_0 e^{rt} \frac{S - (1 - \alpha)^2 \Gamma}{(1 - \alpha)D(b) - (1 - \alpha)^2 r p_0 \Lambda} D'(p^C(t)) \quad (23)
\end{aligned}$$

The first term is positive. The second and third terms are zero if either $\alpha = 0$ or $D' = 0$. Thus, if α is sufficiently low initially, or if demand is sufficiently inelastic, $y(t)$ will increase. Furthermore, if t is sufficiently close to T , we know from above that the consumer price falls, implying that $y(t)$ must increase. Hence, we have shown the first part of the proposition.

Next, let us show that $y(0)$ decreases if demand is linear and sufficiently elastic, and

α is sufficiently large initially. We use the derivations in 22, and insert for Γ , Λ and $D(p^C) = \overline{D}'(-p_{\max}^C + p^C)$. Then we get:

$$\begin{aligned}
\frac{dy(t)}{d\alpha} &= \overline{D}'[-p_{\max}^C + \alpha b + (1 - \alpha)p_0 e^{rt} + \alpha b - \alpha p_0 e^{rt} \\
&\quad - \alpha r p_0 e^{rt} \frac{(-p_{\max}^C T + \alpha b T) + (1 - \alpha)\frac{b}{r}(1 - e^{-rT}) - (1 - \alpha)[T - \frac{1}{r}(1 - e^{-rT})] b}{(b - p_{\max}^C) - (1 - \alpha)b(1 - e^{-rT})}] \\
&= \overline{D}'[-p_{\max}^C + 2\alpha b + (1 - 2\alpha)be^{r(t-T)} \\
&\quad - \alpha be^{r(t-T)} \frac{(-p_{\max}^C rT + \alpha brT) - (1 - \alpha)[rT - 2(1 - e^{-rT})] b}{-p_{\max}^C + \alpha b + (1 - \alpha)be^{-rT}}] \tag{24}
\end{aligned}$$

If we let α go towards one, we get:

$$\frac{dy(t)}{d\alpha} \rightarrow \overline{D}'[-p_{\max}^C + 2b - be^{r(t-T)}(1 + rT)] \tag{25}$$

If p_{\max}^C is sufficiently close to b , we see that the bracket is positive for $t = 0$, and hence

$\frac{dy(0)}{d\alpha}$ is negative. ■

Proof of Proposition 5:

Introducing (or increasing) a unit subsidy to biofuels production has the same market effect as reducing the size of b . Thus, we examine the effects of changing b . Following the same procedure as in the proof of Proposition 1, we get:

$$\frac{dT}{db} = -\alpha \frac{\int_0^T D'(p^C(t)) dt}{D(b) - (1 - \alpha)rp_0\Lambda} > 0 \tag{26}$$

Thus, T decreases when b declines, or when a subsidy is introduced.

Next, differentiating 1 with respect to b , we get:

$$\frac{dx(t)}{db} = (1 - \alpha)D'(p^C(t)) \left[\alpha - (1 - \alpha)\alpha r p_0 \frac{dp_0}{db} e^{rt} \right] \quad (27)$$

The only variable that changes over time is e^{rt} . Thus, the paranthesis must decrease over time. We know that if $x(t)$ increases for some t , it must decrease at some other time (since S is fixed). Hence, there must be a \hat{t} where the paranthesis is equal to zero. Then we have that the whole expression must be negative for all $t < \hat{t}$ and positive for all $t > \hat{t}$. In other words, a subsidy to biofuels increases (decreases) fossil fuel consumption and extraction for all $t < (>)\hat{t}$.

The case of two regions

The RFS rate is now region-specific, α_i . Equations (1)-(3), as well as the consumer price, are also then region-specific, while (4) is unchanged. Let S_1 and S_2 denote accumulated resource use in the two regions, i.e., $S_1 = \int_0^T x_1(t)dt$ and $S_2 = \int_0^T x_2(t)dt$. We then have:

$$S_1 + S_2 = S \quad (28)$$

We are now ready to look at the effects of an increase in α_i . We insert from (1) into (5), and then into (28):

$$\sum_i (1 - \alpha_i) \int_0^T D_i(\alpha_i b + (1 - \alpha_i)p_0 e^{rt}) dt = S$$

and differentiate with respect to α_i

$$-\int_0^T D_i(p_i^C(t)) dt + (1 - \alpha_i) \int_0^T \left(b - p_0 e^{rt} + (1 - \alpha_i) e^{rt} \frac{dp_0}{d\alpha_i} \right) D_i'(p_i^C(t)) dt + (1 - \alpha_i) D_i(b) \frac{dT}{d\alpha_i} +$$

$$(1 - \alpha_j) \int_0^T (1 - \alpha_j) e^{rt} \frac{dp_0}{d\alpha_i} D'_j(p_j^C(t)) dt + (1 - \alpha_j) D_j(b) \frac{dT}{d\alpha_i} = 0$$

Inserting for $\frac{dp_0}{d\alpha}$ from (15), which still holds but is region-specific, gives:

$$0 = - \int_0^T D_i(p_i^C(t)) dt + (1 - \alpha_i) \int_0^T [(b - p_0 e^{rt}) D'_i(p_i^C(t))] dt +$$

$$\left[(1 - \alpha_i) D_i(b) + (1 - \alpha_j) D_j(b) - (1 - \alpha_i)^2 r p_0 \int_0^T e^{rt} D'_i(p_i^C(t)) dt - r p_0 (1 - \alpha_j) \int_0^T e^{rt} D'_j(p_j^C(t)) dt \right] \frac{dT}{d\alpha_i}$$

which can be rearranged:

$$\frac{dT}{d\alpha_i} = \frac{1}{1 - \alpha_i} \frac{S_i - \Gamma_i}{(1 - \alpha_i) D_i(b) + (1 - \alpha_j) D_j(b) - \Lambda_i - \Lambda_j} > 0 \quad (29)$$

where $\Gamma_i = (1 - \alpha_i)^2 \int_0^T [(b - p_0 e^{rt}) D'_i(p_i^C(t))] dt < 0$, $\Lambda_i = (1 - \alpha_i)^2 r p_0 \int_0^T e^{rt} D'_i(p_i^C(t)) dt < 0$ and $\Lambda_j = r p_0 (1 - \alpha_j) \int_0^T e^{rt} D'_j(p_j^C(t)) dt < 0$. Note that Γ_i , Λ_i and Λ_j can all be treated as constants (given the new paths for $p_i^C(t)$ and $p_j^C(t)$ and the new T). Since $\frac{dp_0}{d\alpha_i} = -r p_0 \frac{dT}{d\alpha_i}$, it follows that $\frac{dp_0}{d\alpha_i} < 0$.

For the change in total fossil fuel extraction we have:

$$\begin{aligned} \frac{dx_i(t)}{d\alpha_i} + \frac{dx_j(t)}{d\alpha_i} &= -D_i(p_i^C(t)) + (1 - \alpha_i) D'_i(p_i^C(t)) (b - p_0 e^{rt}) \\ &\quad + [(1 - \alpha_i)^2 D'_i(p_i^C(t)) + (1 - \alpha_j)^2 D'_j(p_j^C(t))] e^{rt} \frac{dp_0}{d\alpha_i} \end{aligned} \quad (30)$$

The two first terms are negative: Increasing the RFS decreases the use of fossil fuels

for a given consumer price on transportation fuel and increases the consumer price on transportation fuel for a given price on fossil fuels. On the other hand, the last term is positive as the price on fossil fuels falls, having a downward effect on the consumer price in both regions. We know that extraction must increase at some point since extraction now last longer. It must then decline at other points since the amount of resource is given.

To see what happens at $t = 0$, we rearrange (30) we obtain the following expression for

$$\frac{dx_i(t)}{d\alpha_i} + \frac{dx_j(t)}{d\alpha_i}:$$

$$\begin{aligned} & -D_i(p_i^C(t)) - r(1 - \alpha_j)^2 D_j'(p_j^C(t)) p_0 e^{rt} \frac{S_i - \Gamma_i}{(1 - \alpha_i)D_i(b) + (1 - \alpha_j)D_j(b) - \Lambda_i - \Lambda_j} \\ & + (1 - \alpha_i) \left[b - p_0 e^{rt} - r p_0 e^{rt} \frac{S_i - \Gamma_i}{(1 - \alpha_i)D_i(b) + (1 - \alpha_j)D_j(b) - \Lambda_i - \Lambda_j} \right] D_i'(p_i^C(t)) \quad (31) \end{aligned}$$

The first term is negative, and will become less negative over time since the consumer-price on transportation fuel $p_i^C(t)$ must increase over time. The next term is positive, and it must increase over time as long as the demand function is concave, i.e., $D_j''(p_j^C(t)) \leq 0$. Hence, if the sum of the first and the second term is positive, the sum will stay positive and increase in value for all t until T .

The bracket in the last term decreases over time. If the bracket is negative at $t = 0$, the whole term is positive initially. Moreover, it will become more and more positive over time as long as $D_i''(p_i^C(t)) \leq 0$. If the bracket is positive at $t = 0$, the whole term is negative initially. At some time $\bar{t} < T$ the bracket will become negative, and then the second term will become more and more positive over time as long as $D_i''(p_i^C(t)) \leq 0$. In

the time interval $[0, \hat{t})$ the terms in brackets will decrease towards zero, while the derivative $D'_i(p_i^C(t))$ will stay constant or become more negative (as long as $D''_i(p^C(t)) \leq 0$).

There are only two ways in which the whole expression in (31) can be positive for $t = 0$. The sum of the first and second term can be positive and the last term can be positive. However, then the whole expression will stay positive for all $t < T$. This is inconsistent with the fact that extraction time increases. Thus, this case can be ruled out.

The last term could be negative, but still the whole expression could be positive for $t = 0$. This implies that the sum of the first and second term is positive initially, and that this sum is larger than the absolute value of the second term. However, we know that the sum of the first and second term is increasing in t . Hence, in order for the whole expression to become negative at some point, the second term must become more negative. This cannot happen if $D''_i(p^C(t)) = 0$. Hence, it follows that if $D''_i(p^C(t)) = 0$, total fossil extraction will decline for all $t < \hat{t}$ and increase for all $t > \hat{t}$ (for some $0 < \hat{t} < T$). For the other cases $\frac{dx_i(0)}{d\alpha_i} + \frac{dx_j(0)}{d\alpha_i}$ is ambiguous.

For the effects on the consumer price $p_i^C(t)$ in Region i we have:

$$\frac{\partial p_i^C(t)}{\partial \alpha_i} = b - p_0 e^{rt} + (1 - \alpha_i) e^{rt} \frac{dp_0}{d\alpha_i}$$

Note that the sum of the two first terms are positive (for $t < T$), while the last term is negative. We know that $\frac{dp^C(t)}{d\alpha} < 0$ for t sufficiently close to T , since T increases with α . What about the effect at $t = 0$? Using that $\frac{dp_0}{d\alpha_i} = -rp_0 \frac{dT}{d\alpha_i}$ and that $p_0 = be^{-rT}$ we have:

$$\begin{aligned}
\frac{\partial p_i^C(0)}{\partial \alpha_i} &= b \left[1 - e^{-rT} - rT e^{-rT} \frac{(1 - \alpha_i)}{T} \frac{dT}{d\alpha_i} \right] \\
&= b \left[1 - e^{-rT} - r e^{-rT} \frac{S_i - \Gamma_i}{(1 - \alpha_i)D_i(b) + (1 - \alpha_j)D_j(b) - \Lambda_i - \Lambda_j} \right] \\
&> b \left[1 - e^{-rT} - r e^{-rT} \frac{S_i - \Gamma_i}{(1 - \alpha_i)D_i(b) - \Lambda_i} \right] > 0,
\end{aligned}$$

where we use the findings for one region in Section 1.4 (cf. (20)) in the last inequality.

Hence, total fuel consumption in Region i declines initially. Initial consumption of fossil fuels in this region must then also decline.

For the region not changing its blending mandate, we have:

$$\frac{\partial p_j^C(t)}{\partial \alpha_i} = (1 - \alpha_j) e^{rt} \frac{dp_0}{d\alpha_i} < 0$$

It follows that Region j will use more of the resource at all times. Thus, we have:

$\frac{dS_j}{d\alpha_i} > 0$ and $\frac{dS_i}{d\alpha_i} < 0$. It also follows that Region j will use more biofuels in each period, given that $\alpha_j > 0$.

Proof of equation (14)

Consider the following simple optimization problem, where the resource constraint is ignored:

$$\max \int_0^\infty e^{-rt} [F(x(t), y(t)) - C(A_1(t) + A_2(t))] dt \quad (32)$$

where x and y stand for fossil fuel use and biofuel use, respectively. The net benefit (ignoring climate effects) function F is strictly increasing and concave, and the climate cost function C has the properties $C' > 0$ and $C'' \geq 0$. Total carbon in the atmosphere is given by $A_1 + A_2$, and is a state variable in the optimization problem.

We assume that x can be chosen freely at any time. In order to avoid jumps in the state variables, we assume y is a state variable developing according to

$$\dot{y}(t) = u(t) \text{ where } u(t) \in [-\bar{u}, \bar{u}]$$

Any change in y must thus occur via the control variable u differing from zero. We assume \bar{u} is large, so that jumps in y are "almost possible".

Previously, we assumed that any jump in y was associated with a corresponding jump on the amount of carbon in the atmosphere. Now any change in y will be captured by the change in the atmosphere being gradual and proportional to u :

$$\dot{A}_1(t) = \theta [x(t) + \ell\beta u(t)]$$

$$\dot{A}_2(t) = (1 - \theta) [x(t) + \ell\beta u(t)] - \delta A_2(t)$$

Before proceeding, it is useful to consider the development of carbon in the atmosphere when x and y are constant equal to x^* and y^* . In this case the long-run equilibrium is characterized by $u = 0$ and A_2 constant equal to

$$A_2^* = \frac{1 - \theta}{\delta} x^*$$

while $A_1(t)$ will be given by

$$A_1(t) = A_1(0) + \theta \left\{ \int_0^t x(t) dt + \ell\beta (y^* - y(0)) \right\}$$

Hence,

$$A(t) = A_1(0) + \frac{1-\theta}{\delta}x^* + \theta \left\{ \int_0^t x(t)dt + \ell\beta(y^* - y(0)) \right\}$$

which will be growing over time unless $x^* = 0$. If $x^* = 0$, A^* will be higher the higher is the level of biofuel production.

Returning to the maximization problem (32), the current value Hamiltonian is (formulated with positive costate variables and ignoring time references)

$$\begin{aligned} H = & F(x, y) - C(A_1 + A_2) \\ & + \mu u - \lambda_1 \theta [x(t) + \ell\beta u(t)] - \lambda_2 \{(1 - \theta) [x(t) + \ell\beta u(t)] - \delta A_2(t)\} \end{aligned}$$

The optimal solution must satisfy

$$F_x(x(t), y(t)) - [\theta\lambda_1(t) + (1 - \theta)\lambda_2(t)] = 0 \text{ for } x(t) > 0 \quad (33)$$

$$\mu(t) - \ell\beta [\theta\lambda_1(t) + (1 - \theta)\lambda_2(t)] = 0 \text{ for } u(t) \in (-\bar{u}, \bar{u}) \quad (34)$$

$$\dot{\mu}(t) = r\mu(t) - F_y(x(t), y(t)) \quad (35)$$

$$\dot{\lambda}_1(t) = r\lambda_1(t) + C'(A_1(t) + A_2(t)) \quad (36)$$

$$\dot{\lambda}_2(t) = (r + \delta)\lambda_2(t) + C'(A_1(t) + A_2(t)) \quad (37)$$

$$\text{Lim}_{t \rightarrow \infty} e^{-rt} \mu(t) y(t) = 0 \quad (38)$$

$$\text{Lim}_{t \rightarrow \infty} e^{-rt} \lambda_i(t) A_i(t) = 0 \quad i = 1, 2 \quad (39)$$

The equations above imply that

$$\mu(t) = \int_0^\infty e^{-rz} F_y(x(t+z), y(t+z)) dz \quad (40)$$

$$\theta \lambda_1(t) + (1 - \theta) \lambda_2(t) = q_x(t) \text{ given by (9)}$$

For an interior solution at any time when y does not "jump" we thus have

$$F_x(x(t), y(t)) = q_x(t) \quad (41)$$

$$\mu(t) = \ell \beta q_x(t) \quad (42)$$

There is not much one can say about the ratio F_y/F_x for the general case of $C'' > 0$. We therefore restrict ourselves to the special case of the equilibrium value of C' rising at a constant rate $m \in [0, r)$. One interpretation of the case $m > 0$ was given in section 3. A second interpretation could be that the functions in the optimization problem are such that the solution gives a constant growth rate of C' .

Inserting $C' = C'_0 e^{mt}$ into (9) gives

$$q_x(t) = \left[\frac{\theta}{r - m} + \frac{1 - \theta}{r + \delta - m} \right] C'_0 e^{mt} \quad (43)$$

From (42) it follows that $\mu(t)$ must grow at the rate m , and from (40) this implies that F_y grows at the rate m and that

$$\mu(t) = \frac{F_y(x(t), y(t))}{r - m}$$

Together with (42) we thus get

$$F_y(x(t), y(t)) \equiv q_y(t) = (r - m)\ell\beta q_x(t)$$

and hence

$$\frac{q_y(t)}{q_x(t)} = (r - m)\ell\beta$$

■