An Artificial Stock Market:
Asset Pricing and Endogenous Expectations using Neural Nets

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Abstract:

In this paper the relatively new technique of neural nets is integrated in a traditional model of portfolio choice. On the basis of Arrow's State Preference Model the investment decision depends on the expectation building process which consists of two components. The individual information processing and the mutual influence upon one another. Therefore, each agent is represented by a single net but all individuals are connected with each other. On both levels the magnitude of impact for the final portfolio choice is reflected by the connection weights of the net. The aim of the heterogeneous agents is to learn the market structure in order to make forecasts of probable yield. By comparing the expected and the actual price the individuals adjust the weights according to the backpropagation algorithm. The simulation studies show, that the agents adapt to each other generating a decline in the total market error. Market entries can disturb this structure and induce erroneous forecasts of the remaining market participants. On the microeconomic level it can be seen that similar characters can profit from each other if some of them get a dominant market position.

JEL Classification: D84 Expectations; Speculations / G11 Portfolio Choice /
C45 Neural Networks

1 Introduction

Recent capital market theory more or less explicitly models the microscopic diversity. The single agent is no longer regarded as an representative market participant, acting in a full rational manner. Instead, the decision makers in such models can be described by two main characteristics: On the one hand the amount of information they are able to take into account, as well as their processing capacity is limited. On the other hand they have the ability to adapt to different situations and to learn from them. Therefore, the mechanisms that describe how agents make decisions, how prices are determined, and how changes take place out of equilibrium are of special interest (Day/Chen (1993)). On the aggregate level this leads to a system which is characterized by great interdependence and a high degree of complexity.

Neural nets are considered to be suitable in describing complex social systems. Due to the natural model, the human brain, the main point of this technique is the ability to learn (Schnabl (1995)). For this reason the increasing relevance in economics becomes obvious, whereby two main applications can be distinguished: A statistical/econometric one and a theoretic/experimental one (Blien/Lindner (1993), (Kugler/Hanusch (1995)). The former are mainly concerned with time series forecasting speculative markets. However, in comparison with conventional methods the superiority of this approach is controversially discussed.(Bol et al. (1994), Anders (1997)). Therefore, it is recommended to proof the quality of such applications in the setting of theoretical models (Riess (1994)). In this context, the basic elements are seen in the description of the decision process and the overlapping of individual decisions. The focus is set on the circumstances and the information responsible for the supply and the demand decisions. Exogenous macro- information is dissembled in individual decisions or, in other words, in micro- information. The other hand, the decisions on the microeconomic level determine the macroeconomic development and the market price respectively. By this perspective the connection between the micro- and macroeconomic level becomes a realistic one (Zimmermann (1994)).

This paper tries to integrate this methodological procedure into a traditional model of the capital market, namely Arrow's State Preference Model (SPM) (Arrow (1964)). For this purpose we first enlarge the SPM by subjective expectations of the market actors concerning the security price in the next period. The investment decision itself depends on an expectation building process which consists of two components. The first one contains of a sophisticated process analyzing the macroeconomic information. Because of the variety and quantity of such information, the investors consider alternative scenarios leading to anticipated bottom and top prices. The difference between the current price and this decision frame then leads to a more optimistic or pessimistic attitude of the traders. The second component reflects the more speculative element in the decision process. The heterogeneous agents try to find out to what extend their anticipated limits will actually prevail on the market by interacting with each other. The individual information processing and the mutual influence upon each other determine both the final price expectation and the investment decision of the agents. The aggregation of the individual decisions then leads to the security price of the next period. Therefore, the expectation building process of each agent is represented by a single neural net and according to the direct interaction all individual nets are connected with each other representing the macroeconomic level.

The aim of the agents is to learn the market structure in order to make forecasts of probable yield. The learning process is modeled by adjusting the connection weights within the individual nets reflecting the magnitude of impact of the macro information as well as of the decision frames of the other market participants. The simulation studies show, that the investors adapt to each other generating a decline in the total market error. This result holds for constant as well as for random information. Market entries of extreme optimistic or pessimistic agents can disturb this structure and induce erroneous forecasts of the remaining investors. On the microeconomic level it can be seen that similar characters can profit from each other. Precondition therefore is an income related dominant market position of some of them.

The paper is organized as follows:
Chapter 2 describes the basic model and the comparative static characteristics of the individual demand functions. The integration of this model into a neural net follows in Chapter 3. After the description of the net architecture the price adjustment function and the market exit and entry conditions are determined. The simulation runs in Chapter 4 are pointing out the agents' adaptation process and the effects on the market structure and on the market result. Chapter 5 summarizes and gives an outlook for further research.

2 The Basic Model[1]

As in the SPM we also assume that the economy consists of $k=1,...,K$ agents, with $\overline{w}_k(t-1)$ denoting the endowed wealth in period $t=1,...T$. In every period the agents make a decision about their portfolio of security holdings. For simplicity, there are only two securities to select: the risky security M and the risk-free outside security O. In contrast to the formulation above, we assume that the current consumption has just happened and is therefore independent of the investment in securities. Short sales, however, are allowed.[2] In addition to this, we assume that the prices of both securities are well-known and equal one at the beginning. Without any loss of generality, the price of the risk-free security does not change

[1] For technical details see Kugler/Sommer/Hanusch (1996) , Sommer/Kugler (1997)
[2] See Sargent (1979).

while the price of the risky security depends on the market activities. Thus, agent k's budget constraint in period t is given by:

$$p(t-1)\, m_k(t) + o_k(t) = \overline{w}_k(t-1)\,,$$

with $m_k(t)$ ($o_k(t)$) denoting the demand of security M (O) and $p(t-1)$ denoting the market price of the risky security M.

In contrast to the SPM the agents form expectations about the next periods price of security M. Therefore, the expectation building process consists of two components, already mentioned by Keynes:„ ...the skill and energies of the professional investor and speculator are mainly occupied... not with making superior long-term forecasts of probable yield, but with their foreseeing changes in the conventional basis of valuation a short time ahead of the general public“ (Keynes (1936), p.154). The first component reflects the more rational part of the investors behaviour. The information processing of alternative scenarios of the economic development leads to anticipated bottom and top prices, reflecting the individuals' decision frame. The expected price movement is summarized in the following matrix $E(t)$:

$$E(t) = \begin{bmatrix} e_{11}(t), & e_{12}(t) \\ \vdots & \vdots \\ e_{K1}(t), & e_{K2}(t) \end{bmatrix}.$$

In this $K \times 2$ - matrix the single element $e_{ks}(t)$, $s \in \{1;2\}$, denotes agent k's anticipated price limit if he assumes that scenario s is realized in the next period. State one reflects the anticipated top price and state two the anticipated bottom price. The expected wealth of every agent in period t is therefore given by:

$$m_k(t)\, e_{ks}(t) + o_k(t) = w_{ks}(t)\,.$$

Furthermore, $\pi_{ks}(t)$ with $\sum_s \pi_{ks}(t) = 1$ gives the subjective probability that state s will occur in the next period. This second component reflects the individuals' belief that their limits will prevail on the market and is highly influenced by interaction. The agents anticipate the expectations and reactions of the other individuals in their own decision (Koslowski (1990, p.50)). This component reflects the more speculative element in the decision process. By distinguishing this, we follow to a certain extent the uncertainty concept of Knight: „The business man himself not merely forms the best estimate he can of the outcome of his action, but he is likely also to estimate the probability that his estimate is correct“ (Knight (1921), p.226). Therefore, in opposite to the SPM the alternative states are not longer determined exogenously and could not be regarded as an „either/or“ restriction. In this model the agents generate the state that appears in the next period endogenously. This includes the possibility that the price that is realized is not identical with the states the individuals have expected.

According to the risk utility function $V_k\,(w_{k1}(t), w_{k2}(t)) = \sum_s \pi_{ks}(t)\, U_k(w_{k1}(t), w_{k2}(t))$ each agent makes a decision about his optimal portfolio $(m_k(t), o_k(t))$. Therefore, we assume linear risk tolerance for the individual utility functions and take relative risk averse functions into consideration:

$$U_k(w_{ks}(t)) = \frac{n_k(t)}{1-n_k(t)}(\rho + \beta_k(t)\, w_{ks}(t))^{1-n_k(t)}, \qquad \text{with } n_k(t) := \frac{1}{\beta_k(t)},$$

and n_k^t denoting the relative risk aversion. For $\rho \geq 0,\ \beta > 0$ and $w \geq 0$ the usual neoclassical assumptions are always satisfied. Furthermore, we assume that $\rho = 0$ which satisfies the uniqueness of the relative risk aversion.[3] Finally, every investor *k* solves the maximization problem in period *t*:

$$\max_{(w_{k1}(t), w_{k2}(t))} V_k\,(w_{k1}(t), w_{k2}(t)) = \max_{(w_{k1}(t), w_{k2}(t))} \sum_s \pi_{ks}(t)\, U_k(m_k(t)\, e_{ks}(t) + o_k(t))\,.$$

His decision has to satisfy the budget constraint:

$$p(t-1)\; m_k(t) + o_k(t) = \overline{w}_k\,(t-1)\,.$$ [4]

In contrast to dynamic optimization problems, we look at so called „one shot decisions“ (Grinspun (1995)). This means that every agent maximizes his expected utility in each period. The solution of the maximization problem leads to the optimality condition:

$$-\frac{\pi_k(t) U_k{}'(w_{k1}(t))}{(1-\pi_k(t))\, U_k{}'(w_{k2}(t))} = T_k\,(t), \qquad \text{with } T_k\,(t) := \frac{e_{k2}(t) - p(t-1)}{e_{k1}(t) - p(t-1)} < 0\,.$$

It can easily be seen that the behaviour of agent *k* is determined by four decision variables: The anticipated limit prices $e_{k1}(t)$ and $e_{k2}(t)$; the agent's belief about the realization of this limits in the next period $\pi_k(t)$; the initial endowment $\overline{w}_k\,(t-1)$ and the current market price of the risky security *M* $p(t-1)$. The individual demand function of *M* can be derived from the wealth expectations $w_{ks}(t)$:

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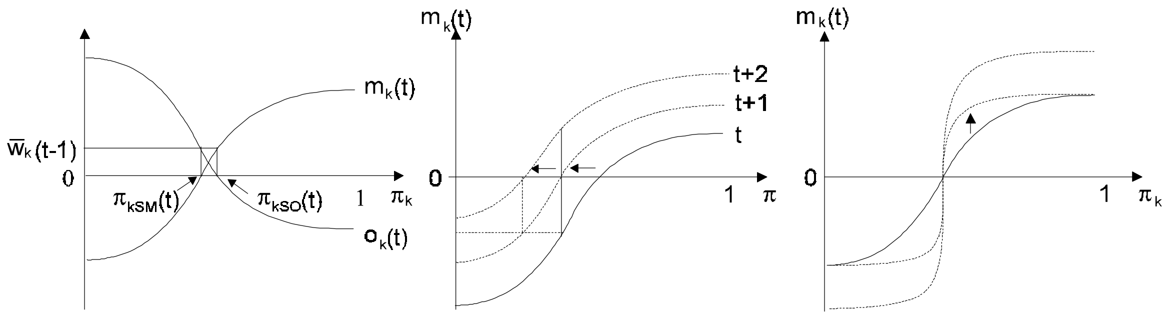
[3] By a convenient choice of the parameter β one can describe the whole class of linear risk tolerant utilities. For a detailed discussion see Ohlson (1987).

[4] Note, that we exclude corner solutions as not economically significant, i.e. $0<\pi<1$.

$$m_k(t) = \left(\frac{1}{e_{k1}(t) - p(t-1)}\right)\left(\frac{\overline{w}_k(t-1)(1-\tau_k(t))}{\tau_k(t) - T_k(t)}\right), \qquad \text{with } \tau_k(t) := -\left[\frac{1-\pi_k(t)}{\pi_k(t)} T_k(t)\right]^{\beta_k(t)}.$$

The demand function of security *O* results from the budget constraint. The following figure 1 shows the security demand m_k and o_k as well as the effects of price movements the on individuals' attitude towards risk, and changes of the initial endowment depending on the subjective estimation π_k.

Figure 1



On the left you can see agent *k*'s general trading opportunities. He can

- hold the security *M* as well as the security *O*. This is the case for $\pi_{kSM}(t) < \pi_k(t) < \pi_{kSO}(t)$, with $\pi_{kSM}(t)\ (\pi_{kSO}(t))$ denoting the point of intersection of *M* (*O*) and the π-axis.
- invest all of his money in the security *M* (*O*) without making short sales. This means: $\pi_k(t) = \pi_{kSO}(t)\ (\pi_k(t) = \pi_{kSM}(t))$.
- make short sales of *M* (*O*) if he wants to buy more of *O* (*M*). In this case, one gets: $\pi_k(t) < \pi_{kSM}(t)\ (\pi_k(t) > \pi_{kSO}(t))$.

As already mentioned, the estimation of limit prices only reflects the investors decision frame. The market, of course, is always in motion with prices always increasing or declining. Already Keynes observed, that the decision to buy or sell stocks or bonds is not based on the absolute price but on „...the degree of divergence from what is considered a fairly safe level“ (Keynes 1936, p.201) where the latter may be taken as the spread of $p(t-1)$ within the range of $e_{k2}(t)$ and $e_{k1}(t)$. Therefore, in the middle of figure 1 one can see that price movements ceteris paribus lead to a more optimistic or pessimistic mood of the investors. The lower the price of security *M* the lower the estimation that scenario 1 $(\pi_k(t))$ will occur for the investment in this security.[5] Therefore, in the situation of a continuing price decline the investors reduce their short sales and invest in the risky security endogenously. This adaptation process of the agents' behaviour is also described by Day/Huang's „..chance of lost opportunity either to fail to buy when the market is low or fail to sell when the market is high" (Day/Huang (1990), p.302). When $p(t-1)$ is close to the anticipated top price $e_{k1}(t)$ the chance of losing a capital gain and of experiencing a capital loss is great. When $p(t-1)$ is close to the anticipated bottom price $e_{k2}(t)$, the chance of missing a capital gain by failing to buy is great. When $p(t-1)$ is more or less in the middle of this range, the perceived chance of a capital gain or loss is small or zero. But, in contrast to Day/Huang's approach we do not need an exogenously given function. These sentiments on the individual level directly result from the maximization problem and are determined by the parameter $T(t)_k$.[6] For this reason $T(t)_k$ reflects the relative mood of the investors in every period. Because of the negativeness of this value increasing (decreasing) prices diminish (raise) the value of this parameter, leading to a relatively pessimistic (optimistic) mood.

The right picture of figure 1 shows that a decrease of the relative risk aversion $n_k(t)$ changes the slope of the curve. With π_i and $\overline{w}_k(t-1)$ being constant, relatively risky agents enlarge their demand to a stronger extent than more risk averse ones. This predominantly holds for the range around the point of intersection. In contrast, an increase of the initial endowment leads to an increase of the possible demand over the whole range. For the maximal demand (short sales) of security *M* one gets:

$$m_k(t) = -\frac{\overline{w}_k(t-1)}{e_{k2}(t) - p(t-1)}\ ;\ \left(m_k(t) = -\frac{\overline{w}_k(t-1)}{e_{k1}(t) - p(t-1)} \right).$$

The implication of the income effects are pointed out in the simulation runs.

After this comparative static analysis of the basic model the following chapter gives a possible description of the individual decision process within a neural net.

[5] Because of the adequate shift of the demand function of *O* this also means that the agents need a higher estimation that state 2 will occur in the next period $(1-\pi_k(t))$ for an investment in security *O*. For simplicity we did not display the appropriate shift of *O*.

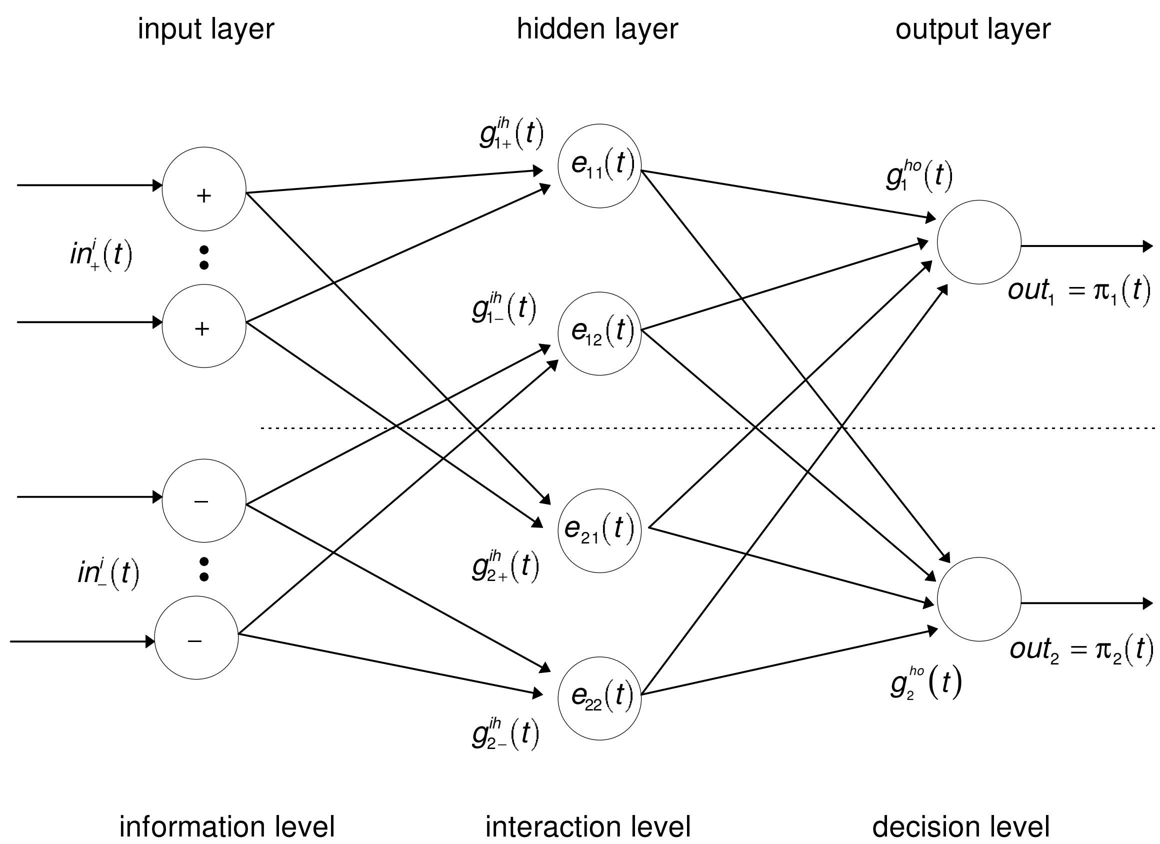
[6] Formally one gets for the point of intersection of the security demand function *M*: $\pi_{kSM}(t) = \frac{T_k(t)}{T_k(t)-1}$.

3 A Neural Net of the Capital Market

3.1 The Net Architecture

The description of the basic model emphasizes that the decision making process is determined by two components. Consequently, each agent is described by a Multilayer-Perceptron. The figure 2 shows for two investors how the information processing as well as the interaction can be modeled within this type of net.

Figure 2



The input layer is used to receive information $in^i(t)$, with $i = 1,2,\dots I$ consisting of two categories. An set indicating a positive $in_+^i(t) > 0$ and a set indicating a negative economic development $in_-^i(t) < 0$. For agent k the magnitude of the impacts each of the information has on his decision is reflected by the connection weights $g_{k+}^{ih}(t)$ for the positive ones, and $g_{k-}^{ih}(t)$ for the negative ones, respectively. At the beginning the weights are randomly drawn from the interval $[0,1]$. The sum of the weighted inputs $neth_{k(.)}(t)$ then leads to the anticipated bottom and top prices as follows:

$$e_{k1}(t) = 1 + \Delta p_{k+}(t), \quad \text{with } \Delta p_{k+}(t) = \frac{1}{1 + e^{-neth_{k+}(t)}} - 0{,}5, \quad \text{and } neth_{k+}(t) = \sum_i g_{k+}^{ih}(t)\, in_{+}^{i}(t);$$

$$e_{k2}(t) = 1 + \Delta p_{k-}(t), \quad \text{with } \Delta p_{k-}(t) = \frac{1}{1 + e^{-neth_{k-}(t)}} - 0{,}5, \quad \text{and } neth_{k-}(t) = \sum_i g_{k-}^{ih}(t)\, in_{-}^{i}(t).$$

In every period the sigmoid function of the hidden layer yields: $e_{k1}(t) \in [1\,;1{,}5[$ and $e_{k2}(t) \in [0{,}5\,;1[$. Therefore, the price limits are only determined by analyzing the exogenously given information, which is not related to the market activities. The price of the last period, however is an additional endogenous information. Both together lead to the individual mood mentioned above.

The second component of the decision process is formed by the connection weights $g_k^{ho}(t)$ from the hidden layer to the output layer. At the beginning they are also randomly drawn from the interval $[0,1]$. These weights reflect the importance of the other investors' decision frames for the final portfolio choice of agent *k* denoted by $neto_k(t)$. Based on this interaction level the agent *k* will derive his subjective estimation from his anticipation of other agents expectations. Formally one gets:

$$\pi_k(t) = \frac{1}{1 + e^{-neto_k(t)}}, \quad \text{and } neto_k(t) = \sum_h g_k^{ho}\, \Delta p_{k(.)}.$$

Therefore, in every period one gets: $0 < \pi_k(t) < 1$. The determination of $\pi_k(t)$ has two consequences for the individual decision process. On the one hand the anticipated top and bottom prices are transformed in a concrete price expectation $ep_k(t)$ according to:

$$ep_k(t) = \pi_k(t)\, e_{k1}(t) + (1 - \pi_k(t))\, e_{k2}(t).$$

On the other hand the agents decide on their final portfolio choice $m_k(t)\,; o_k(t)$. They are going to hold a positive stock of *M*, if the expected price exceeds the current price level. Otherwise they invest in security *O*.[7]

The aim of the agents is to learn the market structure in order to make forecasts of probable yield. Therefore. the learning process is influenced by the divergence of the expected and

[7] For the point of intersection $\pi_{kSM}(t)$ one gets: $ep_k(t) = p(t-1)$.

the realized market prices. Formally, the forecasting error $err_k(t)$ is computed according to the backpropagation algorithm (Werbos (1974), Rumelhart/McClelland (1986)):

$$err_k(t) = \frac{1}{T}\sum_t \frac{1}{2}\left(ep_k(t) - p(t)\right)^2 .$$

The formalism of this learning algorithm indicates that each investor considers the past decisions as a kind of „training examples“. At the end of each period *t* the agents look upon all well known decisions of the former periods. The sum of the remaining errors is then used for the weight adjustment. Therefore, the traders take into account that „precedents have an important influence on later action“ (Choi (1993), p.52). For the adjustment of the connection weights on the interaction level one gets:

$$\Delta g_k^{ho}(t) = \eta_k\ \delta_k^o(t)\ \Delta p_{k(.)}(t), \quad \text{with } \delta_k^o(t) = \Delta p_{k(.)}(t)\left(1 - \Delta p_{k(.)}(t)\right) err_k(t).$$

On the information level the change of the magnitude of impact is given by:

$$\Delta g_{k+}^{ih}(t) = \eta_k\ \delta_{k+}^h(t)\ in_{k+}(t), \quad \text{with } \delta_{k+}^h(t) = in_{k+}(t)\left(1 - in_{k+}(t)\right)\sum\nolimits_o g_k^{ho}\ \delta_k^o(t),$$

$$\Delta g_{k-}^{ih}(t) = \eta_k\ \delta_{k-}^h(t)\ in_{k-}(t), \quad \text{with } \delta_{k-}^h(t) = in_{k-}(t)\left(1 - in_{k-}(t)\right)\sum\nolimits_o g_k^{ho}\ \delta_k^o(t).$$

Here, η_k is reflecting the „learning rate“ of agent *k*. The higher the value of η_k, the greater the weights adjustment. Ceteris paribus relatively high values lead to strong variations with regard to the significance of the other market participants and to the input information. Therefore, the main character of the individuals is determined by the random starting weights. The learning rates then give the willingness to change the individual positions. The simulation runs will point out the importance of this rate.

3.2 The price adjustment function and the entry and exit conditions

The adjustment of the connection weights lead to a re-evaluation of the anticipated limit prices and the belief that they will prevail on the market. Having selected the desired investment decision $m_k(t)$ the comparison with the existing holdings $m_k(t-1)$ indicates whether an agent wishes to raise or to reduce his stock according to $\Delta m_k(t) = m_k(t) - m_k(t-1)$. If the aggregate bids and offers are exactly equal, i.e. $\sum_k \Delta m_k(t) = 0$, all orders will be fulfilled by trading at the current ruling price $p(t-1)$. Because of the agents' heterogeneity, usually this does not happen. One can expect a surplus either of bids or of offers. Suppose, for example, that there are more bids to buy on the market than offers to sell in period *t*. Then, all desired sales $\Delta m_{kA}(t) < 0$ are executed, but the purchases $\Delta m_{kN}(t) > 0$ can only be partially fulfilled. Thus, here we use a simple rationing scheme according to earlier versions of the SFI artificial stock market and do not consider the existence of a market maker (Palmer et al. (1994)). For an excess demand this leads to:

$$m_{kN}(t) = m_k(t-1) + \alpha_N \, \Delta m_{kN}(t) \quad \text{with } \alpha_N = \frac{\sum_k \left|\Delta m_{kA}(t)\right|}{\sum_k \Delta m_{iN}(t)},$$

$$m_{kA}(t) = m_k(t-1) + \Delta m_{kA}(t).$$

And for an excess supply, respectively, we get:

$$m_{kA}(t) = m_k(t-1) + \alpha_A \, \Delta m_{kA}(t) \quad \text{with } \alpha_A = \frac{\sum_k \Delta m_{kN}(t)}{\sum_k \left|\Delta m_{kA}(t)\right|},$$

$$m_{kN}(t) = m_k(t-1) + \Delta m_{kA}(t),$$

where α_ε is denoting the restriction variable and $m_{k\varepsilon}(t)$, $\varepsilon = [N, A]$, is reflecting the realized stock at time *t.* As short sales of *M* and *O* are explicitly allowed, we shall introduce the rule that they have to be satisfied at the end of each period, independent whether a gain or loss situation will result. Thus, at the beginning of the following trading period all agents either hold security *M* or security *O*, depending on the last periods short selling. This restriction enables higher gains because of higher trading quantities but have the „price" of higher losses if the market price goes in the opposite direction. Aggregating all bids and offers in period *t* leads to a price adjustment characterized by the following equation:

$$p(t) = p(t-1)\left[1 + r(\textstyle\sum_k \Delta m_k(t))\right]$$

with $r > 0$ being a constant. Again, this is the basis for the adjustment of the connection weights used in period (t+1).

In such a frame market exits can happen if:

a) at the end of a period a price is realized which was not anticipated according to the individual information processing i.e. $p(t) < e_{k2}(t)$ or $e_{k1}(t) < p(t)$

b) bad investment decisions lead to the loss of the whole endowment i.e. $\overline{w}_k(t-1) \leq 0$. Therefore, agents don't have the ability to go into debt.

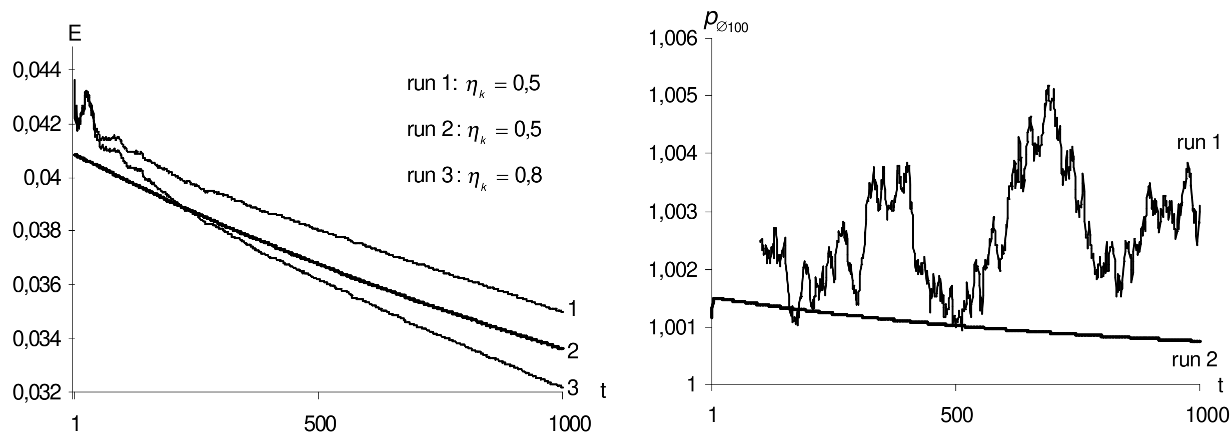
Each exit directly is followed by a market entry. The newcomer starts with an initial endowment of $\overline{w}_k(t-1) = 1$ and the connection weights are also randomly drawn from the intervals mentioned above, whereby $e_{k2}(t) < p(t) < e_{k1}(t)$ must be satisfied. This means, that the main character of the new market participant is influenced by the current price level. High (low) prices lead to high values of $e_{k1}(t)$ (low values of $e_{k2}(t)$) Therefore, in relation to the existing traders the newcomers have higher (lower) values of $T_k(t)$ and are in a more optimistic (pessimistic) mood in tendency.

4 The model simulation

The following simulations predominantly focus on the agents' adaptation processes and on the dynamics of the market behaviour reflected in the price movements of security *M*. In detail, we take a look at the influence of various „learning rates" η_k on the individual forecasting error err_k, the total market error $E = \sum_k err_k$, and the individual endowment $\overline{w}_k$. This analysis is done under constant as well as under random information. Therefore, the information sets consist of two positive and two negative ones available for all agents. The inputs are drawn from the intervals $in_+^i(t) \in [0{,}45\,;0{,}55]$ and $in_-^i(t) \in [-0{,}55\,;-0{,}45]$. The population of investors consists of *k*=50 heterogeneous agents characterized by the connection weights from the intervals mentioned above. Furthermore, at the beginning of the simulation runs we assume the following condition values : $\overline{w}_k = 1$; $o_k = 0{,}5\,; m_k = 0{,}5\,; n_k = 0{,}5\,;\ p = 1\,;\ r = 0{,}0001$. In all runs the trading time is fixed to 1000 periods.

In figure 3 we have illustrated three simulations showing the market development in dependence of the input information and the „learning rate". All of the runs are based on identical starting weights and in the case of various „learning rates" on the same information development

Figure 3

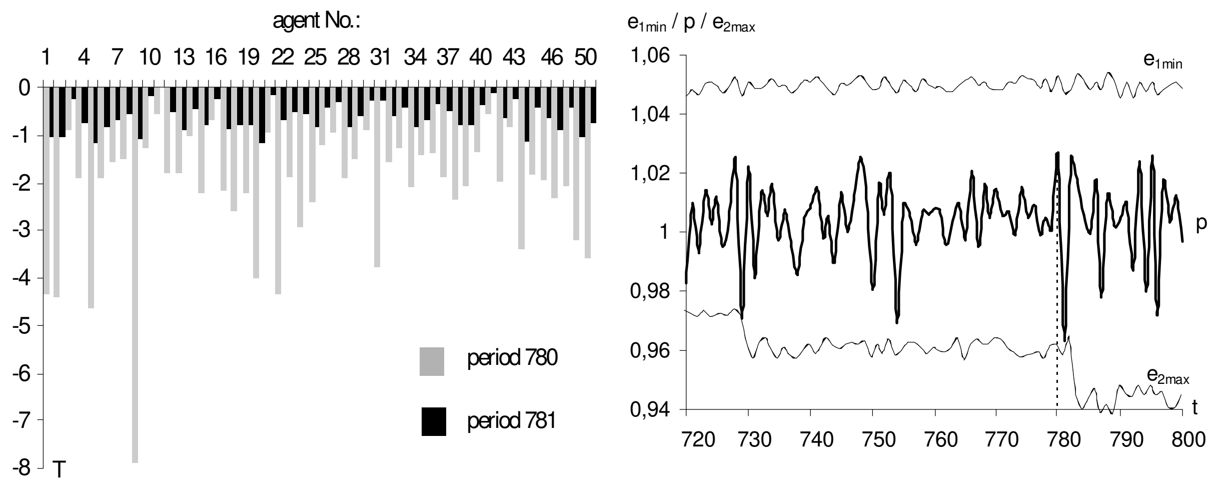


On the left one can see that the total market error is declining in the long run. The agents adapt to each other by adjusting their connection weights. Therefore, the majority of the investors get a better insight into the market features and achieve a better forecasting. In the case of the same learning rates of the agents $\eta_k = 0{,}5$ periodical random information lead to higher errors (run 1) than a constant information set (run 2). It can also be seen that higher „learning rates“ of the traders $\eta_k = 0{,}8$ foster the adaptation process. After a certain time this leads to lower forecasting errors compared to constant conditions (run 3).

On the right the price movements are shown which result from run 1 and run 2. A stable environment leads to a market equilibrium where all orders are fulfilled by trading at the current price level (run 2). In contrast, changes in information lead to fluctuating prices, as shown by the moving average of order 100 (run 1). The investors' behaviour generates alternating periods of generally rising or generally falling prices, so-called „bull“ and „bear“ markets. Higher „learning rates“, however, do not influence the price movement significantly. The stronger adaptation process become equally noticeable on the supply and on the demand side.

In the following figure 4 the price building process under random information is shown more detailed for another simulation run.

Figure 4



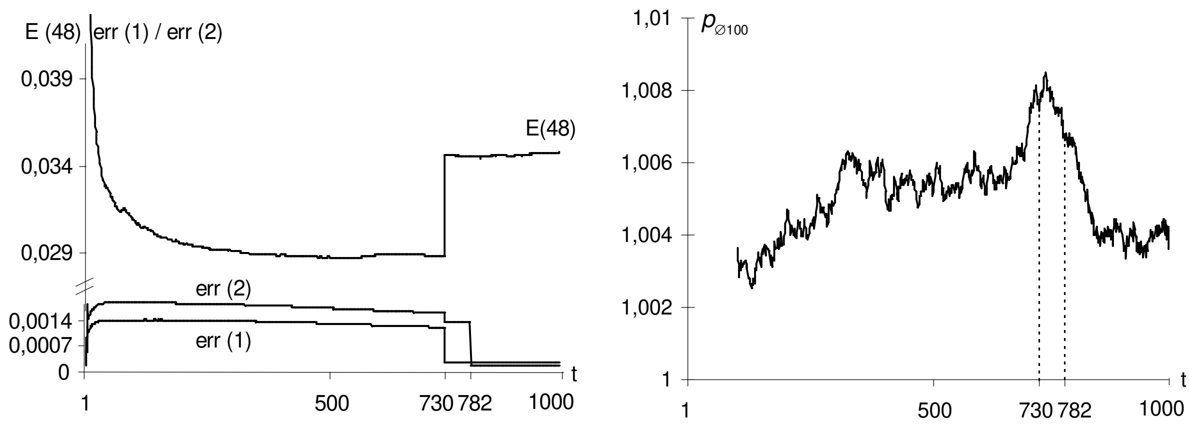
As already mentioned, the parameter T_k reflects the individuals' mood. The left side represents the market distribution of this parameter in period 780 and in period 781. The right side shows the price movement p and the expectations development of the two marginal characters $e_{1\min}$ and $e_{2\max}$ for the period 720 to 800. For period 780 one can recognize a high price level. In this case, the agents are characterized by relative low T-values and are, therefore in a more pessimistic mood. In this situation two possible developments can occur, depending on the random input information:

1) like in the present run, more or less constant information leads to a predominant investment in security *O*. Consequently, the price of security *M* is declining with an endogenous change towards a more optimistic market structure represented by relatively high values of *T* in period 781.
2) high positive or, from the amount, low negative information values would lead to relatively high anticipated limit prices $e_{k2}(t)$ and $e_{k1}(t)$ of the agents, generating a more optimistic market situation. By interacting with each other this would result in a continuing market demand for security *M* holding up the current price level in period 781.

From the illustration on the right side of figure 4 a market exit also becomes obvious. By determining the starting weights we created a marginal character which leaves the market in period 730. At this time, a price is realized which was not anticipated by this trader $\left(e_{2\max}(730) < p(730)\right)$. One cannot see from the right side of figure 4 that another agent lost his whole endowment and leaves the market in period 782. However, both exits are followed by entries according to the conditions mentioned above. This leads to the entries clearly reducing of the lowest estimated bottom price existing in the market $\left(e_{2\max}\right)$.[8] One should notice, that the newcomers are not necessarily those agents with the lowest anticipated limit price. Because of the interaction between all agents the newcomers can cause a reduction of e_{k2} anticipated by an existing trader.

Figure 5 shows the effects which the new population will generate concerning the forecasting errors and the price behaviour.

Figure 5



[8] At the end of period 730 a price p(730) =0,975 is realized; at the end of period 782 there is p(782) =0,981.

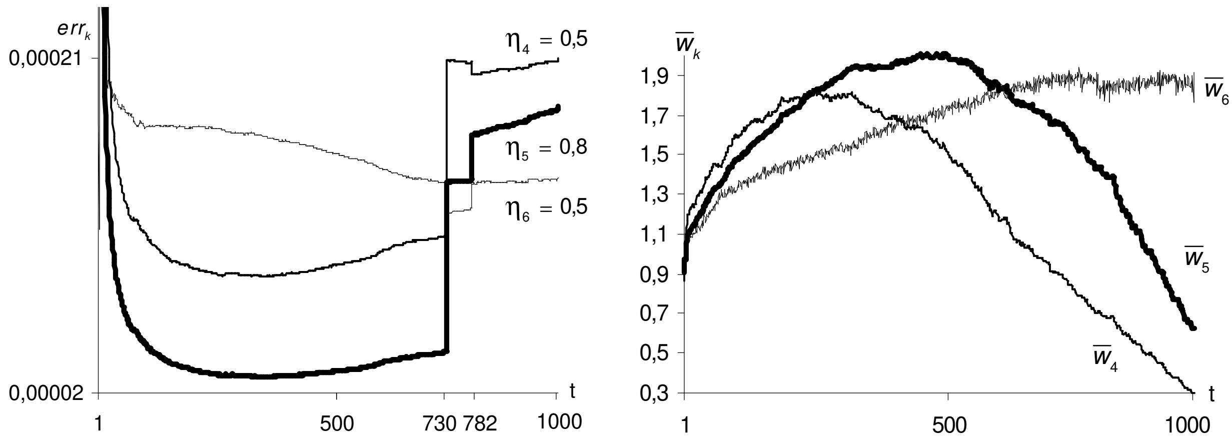
The left side indicates the development of the forecasting errors. The exiting agents which are replaced by the two newcomers are market by $err(1)$ and $err(2)$. The remaining 48 investors are distinguished by $E(48)$. It becomes obvious that already one new agent can disturb the existing market structure due to the small number of agents in the population and due to income effects. At the date of exit the leaving agent No.1 has only a very small endowment left for his disposal and agent No.2 has even lost all his money. In contrast, the newcomers start with an initial endowment of $\overline{w}_k(t-1)=1$ and thus get a relatively strong market position. How they effect the adapted investors depends on their basic attitude. For instance in period 730 a rather optimistic trader enters the market, while the second newcomer in period 782 better fits into the existing market structure.[9] Therefore, the first one is interfering with the relationship between the agents far more than the second one, leading to an increase in the forecasting errors. On the individual level it can be seen that both new agents achieve better forecasts than the exiting ones. The first newcomer is using his relatively strong market position. His optimistic mood results in a large demand for security M and leads to a price movement towards his expectation value $e_{11}(t)$. Nevertheless, looking at the development of the whole market the more homogeneous newcomer No.2 achieves comparatively better forecasts.

On the right side of figure 5, the consequences of the market entries on the price movement are illustrated. In the short run, the relatively optimistic newcomer causes a further increase of the moving average price of order 100. By interacting with each other the erroneous forecasts of the remaining investors lead to relatively strong adjustments of the connection weights and to a rethinking of a „fairly safe price level “. However, this development cannot hold in the long run. The continuing price increase causes a predominant pessimistic market structure leading to a continuing price decline. One can also recognize, that the second newcomer does not cause a significant re-orientation of the existing traders due to his more homogeneous character.

The next figure 6 gives a more detailed insight into what happened on the microeconomic level during the previous simulation run. On the left the values of the forecasting errors of three agents are illustrated exemplary. The right side shows their belonging endowments.

[9] $T_{\varnothing}$ (730) = -2,159; T_1 (730) = -0,320
$T_{\varnothing}$ (782) = -1,732; T_2 (782) = -0,605

Figure 6



The agents No. 4 and No. 5 start with identical weights and, therefore, have the same distinguishing features at period one. The only difference consists in the values of their „learning rate“ η_k. The value of Agent No.4 is $\eta_4 = 0{,}5$, that of Agent No.5 is $\eta_5 = 0{,}8$. In the course of time both investors get a better insight into the market structure. Notice, that the higher „learning rate“ of agent No.5 leads to a better understanding of the market, reflected by a more precise forecasting series. However, it can also be seen that after approximately 500 periods a increase in the error values of both series takes place. Contrary, to that the errors of agent No.6 are declining continuously more or less. This result is mainly caused by the market power of so called „big players“. During this simulation run two investors, namely No. 12 and No. 27 are able to increase their endowment immensely.[10] The character of the traders is similar to that of Agent No.6. Therefore, he can profit from the income related market power whereas agent No.4 and No.5 loose influence. On the right side of the income development of the three traders No. 4, No. 5, and No. 6 is illustrated. One can see that the trading profits of the two identical characters are rising at the beginning and are followed by high losses later on. On the other hand, investor No.6 achieves a smaller but continuous increase of his endowed wealth during the whole trading period. This leads to the implication that similar individuals may profit from each other, but will not necessarily reach a dominant market position in course of time. In valuing this result, however, one should have in mind that in this model specification high values of the starting weights lead to a wide range of the estimated limit prices as well as to a strong significance of certain agents. Therefore, distinct specifications of π will bring up differences in individuals' demand and their endowment, respectively.

[10] $\overline{w}_{12}(1000) = 4{,}843$; $\overline{w}_{27}(1000) = 7{,}821$.

Looking at the development of the individual forecasting errors another result can be deduced. Higher „learning rates“ not necessarily go along with better forecasts. The various values of the characters No.4 and No.5 form a different behaviour and the change of the population effects them to a different extend. Both newcomers cause the most erroneous forecasting of agent No.5. The agents which are getting out of the market have a comparatively small significance in his decision process because of the great weight adjustments. Therefore, he neglects the relatively strong position of the newcomers leading to high forecasting errors and another reduction of his endowment. Due to this, the use of variable „learning rates“ would yield better results (Zimmermann (1994)).

5 Summary and Outlook

The neural net approach which is often used for time series forecasting still plays a minor role in theoretical economic modeling. One reason for this can be seen in the rudimentary connection between economic theories and the modeling opportunities of neural nets.

The purpose of this paper was an integration of this technique into a concrete capital market model. Therefore, we enlarged the State Preference Model of Arrow by subjective expectations. Thus, the well founded structure of this model allows a transparent description of the individual behaviour which is determined by two components: the general mood of the agents, reflecting their willingness for an investment, and the subsequent estimation to what extend this mood will prevail on the market. Therefore, each agent considers exogenous information as well as the information processing of the other traders. The magnitude of impact of both components is reflected by the connection weights within the net architecture. In addition to the decision process all agents pass a learning process which is based on their forecasting error. This error then leads to an adaptation of the connection weights and to a re-evaluation of the decision parameters, respectively. In the course of time, the agents adapt to each other resulting in a decline of the total market forecasting error. However, populations with higher „learning rates“ achieve a greater error reduction than populations with a lower adaptation ability. But, on the individual level, high „learning rates“ can lead to a disregard of less successful investors. If they are leaving the market, the newcomers can cause erroneous forecasts and high losses of the remaining ones. Especially agents, who have a relatively weak market position or do not belong to a powerful investor group are susceptible to such a development.

The consideration of heterogeneous agents and the explicit modeling of interaction allows a detailed examination of the market behaviour and a clear description of the feedback effects between the micro- and macroeconomic level. Furthermore, the structure of the model allows the integration of additional phenomenon, observed in reality i.e. endogenous contagion processes by an income oriented adaptation of the connection weights. Therefore, successful traders will be imitated and get a dominant market position in addition to their existing income related market power. Furthermore, a more complex information structure can be seen in an endogenous influence by the market activities as well as by considering a heterogeneous information processing. In going this way, each investor focus certain information supposing that they actually come to dominate the future. The integration of such ideas together with socioeconomic experiments can lead to a possible theoretical foundation of existing time series forecasting models.

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