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# Bonn Econ Discussion Papers

Discussion Paper 07/2011

Randomization in contracts with endogenous  
information

by

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June 2011



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# Randomization in Contracts with Endogenous Information

Stefan Terstiege\*

June 1, 2011

## Abstract

I consider a situation, where the agent can acquire payoff-relevant information either before or after the contract is signed. To raise efficiency, the principal might solicit information; to retain all surplus, however, she must prevent precontractual information gathering. The following class of stochastic contracts may solve this trade-off optimally: before signing, information acquisition is not solicited, and afterwards randomly. The key insight is that randomization makes precontractual information costlier for the agent.

*Keywords:* Information acquisition, Principal-agent, Mechanism design, Randomization

*JEL Codes:* D82, D83

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# 1 Introduction

One purpose of contracts is to resolve informational asymmetry. However, when information is not verifiable by courts, this may prove difficult since an agent possibly gains by misrepresenting what he privately knows. Incentive theory shows that contracts can induce truth-telling, but the agent must get a rent in certain states of the world. As some papers point out, this incentive-compatibility requirement does not impair the principal's expected profit, when the parties conclude the contract *ex-ante* (before the agent learns the state of the world). In this case, the principal may retain the entire surplus of the interaction, since she can appropriate in advance the rent which the agent might enjoy later (e.g. by charging a signing-fee).<sup>1</sup> Of course, such a scheme entails a loss for the agent if a state obtains where he cannot secure a rent. Thus, when the parties conclude the contract *ex-post* (when the agent is already privately informed), the principal must concede a share of the surplus to guarantee that the agent participates and reveals his private information.

This paper studies the optimal contract for a situation where the agent *himself* chooses when (if at all) to get informed. I consider a procurement relation, where a principal demands parts from an agent. While the agent's production costs are initially unknown to both parties, he himself can find out the true realization at some expense. The principal, on the other hand, can neither observe the state, nor control whether the agent observes it or whether he transmits his findings truthfully. Crucially, it is possible for the agent to acquire the information not only after the contract is signed, but also already between contract offer and signing.<sup>2</sup> He may thus check his earnings from the contract in advance, and reject it when it yields a loss.

The situation involves the following trade-off for the principal. On the one hand, she can possi-

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<sup>1</sup>For a recent contribution, see Esö and Szentes (2007) and the references cited therein. Limited liability of the agent prevents the appropriation of rents; see e.g. Sappington (1983), who considers a situation involving moral hazard.

<sup>2</sup>This assumption amounts to a lag between contract offer and signing. Situations where such lags are inevitable abound. E.g., before signing a procurement contract, a seller might first need to figure out whether the specified quantity can be produced until the specified date. Or concerning the selling of a car, the potential customer often first has to make sure that she obtains a credit at a bank to be able to pay. Similarly, a successful applicant for a job may need to discuss with his family before he is able to confirm.

bly increase the total surplus of the interaction if she tailors her demand according to true production costs. To this end, she may instruct the agent to acquire and transmit information—which requires a rent. On the other hand, the principal would like to deter precontractual information gathering (i.e., information gathering between contract offer and signing), in order to retain the entire surplus.

This paper shows that the following kind of stochastic contracts may solve the principal’s trade-off between efficiency and surplus extraction optimally: before signing, she does not solicit information from the agent, and afterwards randomly with a contractually specified probability. The key insight is that contracts for this situation implicitly fix the *price* of precontractual information for the agent. Precontractual information would effectively be for free if the principal requests information after signing with certainty, since the agent anyway bears the costs for information acquisition in that case. On the other hand, precontractual information has a positive effective price when information about the state of the world is possibly not solicited after signing and so would be useless then. In fact, a stochastic contract may elicit information from the agent with some probability, and yet leave the entire expected surplus to the principal.<sup>3</sup>

The main point of this paper is that full surplus extraction may be reconciled with incentives for information acquisition, even though the agent has the costly option to learn his type already before signing. Beyond this, the paper makes three central contributions. First, it elaborates on an insight of the seminal paper by Crémer and Khalil (1992), who also study a situation with a lag between contract offer and signing. The agent can acquire costly information in between, but obtains it at no cost after the the contract is signed. Like in my model, precontractual information gathering is just a rent-seeking activity, and is deterred by the principal.<sup>4</sup> Comparative statics shows that she would find a higher price of precontractual information more desirable. Applied to reality, where the information e.g. concerns the agent’s skill at performing a certain task, this insight implies that the principal may have an interest to conceal some details about the agent’s actual task before signing to make precontractual information gathering costlier. However, in a framework with complete contracts only a stochastic contract can actually impose uncertainty.

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<sup>3</sup>However, her contract may not maximize the surplus, since incentive-compatibility constraints must be met.

<sup>4</sup>In contrast, if several agents compete for the contract, Compte and Jehiel (2008) find that the principal possibly induces precontractual information gathering to find an agent with appropriate skills.

Second, the model closes a gap in the recent literature on mechanism design with endogenous information.<sup>5</sup> That literature examines whether and how to induce the acquisition of costly information, when information is either available only before or only after signing of the contract.<sup>6</sup> For example, in Lewis and Sappington (1997), Crémer, Khalil and Rochet (1998a) or Szalay (2009), the agent can only acquire information between contract offer and signing.<sup>7</sup> As a consequence, the principal possibly induces precontractual information gathering to increase efficiency, although her contract must then be ex-post acceptable for the agent. In other papers (Crémer, Spiegel and Zheng 2009, Krämer and Strausz 2010), information can only be collected after the contract is signed, so that just an ex-ante form of the participation constraint needs to be met. The respective optimal contracts that have been proposed in these two strands of literature are in general infeasible or suboptimal in the present situation, where the agent can gather costly information at either date. This setting seems to be more natural when information acquisition is an unobservable act.

Third, my analysis provides a new explanation for stochastic contracts. The literature already provides several theoretical justifications for stochastic contracts (see Strausz (2006), Kováč and Mylovanov (2009), Bester and Krämer (2010) and Rasul and Sonderegger (2010) for recent contributions).<sup>8</sup> It is typically pointed out that randomness in the allocation may serve as a screening device that relaxes incentive-compatibility. In the present paper, on the contrary, the optimal allocation is deterministic. Randomization only concerns the incentive to acquire information, and it is used to relax the participation constraint.

This paper is organized as follows. The next section presents the model. Section 3 considers the first-best. In section 4, I identify the set of contracts which might be offered. Section 5 reviews the cases without information before or after signing. Section 6 returns to the original situation, shows that the principal does not solicit information before signing, and states the principal's

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<sup>5</sup>See Bergemann and Välimäki (2002) for a general setting.

<sup>6</sup>A different strand of the literature considers the situation where information is available only prior to the contract *offer* (Crémer and Khalil 1994, Crémer, Khalil and Rochet 1998b, Kessler 1998). In that setting, the principal's contract and the agent's strategy are mutually best responses. In particular, the principal cannot induce or deter information acquisition.

<sup>7</sup>The setup of Crémer, Khalil and Rochet is most closely related to this paper's model. Their optimal contract is equivalent to the best deterministic contract for the present situation.

<sup>8</sup>Rasul and Sonderberger review the situations where stochastic contracts may be optimal.

contracting problem. In section 7, I show that stochastic contracts can be optimal. Section 8 concludes. All proofs are presented in the appendix.

## 2 The model

I use a version of the model by Baron and Myerson (1982), but allow for information to be endogenous. In contrast to Crémer, Khalil and Rochet (1998a), the agent can acquire information not only before signing, but also afterwards.

A principal (she) seeks to consume some quantity of a good which is produced by an agent (he). More specifically, the agent can produce output  $q \geq 0$  at marginal cost  $\beta \in \{\underline{\beta}, \bar{\beta}\}$ , where  $\underline{\beta} < \bar{\beta}$ . If he delivers the quantity  $q$  and receives a monetary transfer  $t$ , his payoff is  $t - \beta q$ . The principal's payoff is  $V(q) - t$ , where  $V$  is increasing, strictly concave, continuously differentiable and satisfies  $V'(0) = \infty$  as well as  $V'(\infty) = 0$ .<sup>9</sup>

Both parties do not know  $\beta$ , the agent's *cost type*. The common belief is that the low cost type  $\underline{\beta}$  obtains with probability  $p \in (0, 1)$ , and the high cost type  $\bar{\beta}$  with  $1 - p$ . However, while information is symmetric at the outset, the agent can perfectly observe his type at cost  $\gamma > 0$ .<sup>10</sup> This observation is possible both between contract offer and signing, and afterwards. I assume that information acquisition cannot be monitored by the principal or a third party, and that true production costs are not verifiable (i.e., the situation involves both hidden action and hidden information).

The principal can offer a contract to govern the exchange. If the agent does not accept the proposal, his payoff is zero. If he accepts, no party can withdraw from it any more.

The timing of the situation is as follows:

1. Nature selects production costs  $\beta$ , and the principal offers a contract.
2. The agent can privately observe  $\beta$  at cost  $\gamma$ .
3. The agent accepts or rejects the contract.

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<sup>9</sup>The latter two assumption are to guarantee some (finite) quantity of production in both states of the world.

<sup>10</sup>All results would also hold if information was cheaper after the contract is signed.



4. If he accepted and did not acquire information before signing, he may do it now at same cost; afterwards, the contract is executed. If the contract was rejected, the game ends.

### 3 First-best

As a benchmark, consider the situation where information acquisition and all available information are verifiable, and where the agent cannot gather precontractual information. A contract for this situation specifies a probability  $\alpha$  with which the agent acquires information; moreover, it specifies the quantity  $q$  to be delivered when the state of the world remains unknown, and quantities  $\underline{q}$  and  $\bar{q}$  when the agent collects information and production costs turn out to be  $\underline{\beta}$  and  $\bar{\beta}$ , respectively. Transfers are chosen such that the agent is reimbursed for information acquisition and production.

The optimal contract solves<sup>11</sup>

$$\begin{aligned} \max_{q, \bar{q}, \underline{q}, \alpha} & (1 - \alpha)[V(q) - E[\beta]q] \\ & + \alpha\{-\gamma + p[V(\underline{q}) - \underline{\beta}\underline{q}] + (1 - p)[V(\bar{q}) - \bar{\beta}\bar{q}]\}. \end{aligned}$$

At the optimum, the principal demands the quantities

$$\begin{aligned} q^{FB} &= V'^{-1}(E[\beta]), \\ \underline{q}^{FB} &= V'^{-1}(\underline{\beta}), \\ \bar{q}^{FB} &= V'^{-1}(\bar{\beta}). \end{aligned}$$

The decision whether to collect information entails a trade-off between the gain by tailoring output to production costs, and the expenses due for information acquisition,  $\gamma$ . Specifically, denote by

$$W^{FB}(0) = V(q^{FB}) - E[\beta]q^{FB}$$

the surplus from a contract that does not respond to the realized state, and let

$$W^{FB}(1) = -\gamma + p[V(\underline{q}^{FB}) - \underline{\beta}\underline{q}^{FB}] + (1 - p)[V(\bar{q}^{FB}) - \bar{\beta}\bar{q}^{FB}]$$

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<sup>11</sup>In what follows, I do not explicitly state the feasibility constraints that require  $(q, \bar{q}, \underline{q}) \geq 0$  and  $\alpha \in [0, 1]$ .

denote the surplus of a state-responsive contract. Then the choice of  $\alpha$  is

$$\alpha^{FB} = \begin{cases} 0 & \text{if } W^{FB}(0) > W^{FB}(1) \\ 1 & \text{else.} \end{cases}$$

In the first-best setting, courts can enforce information acquisition and its truthful revelation if specified in the contract. Thus, the agent gets no rent, and the principal retains the entire surplus. The optimal contract is therefore efficient.

## 4 Contracts

In the original situation, contracts specify the quantity to be delivered and a transfer, both possibly contingent on some form of communication. However, contracts cannot force the agent to acquire information and to reveal his findings truthfully to the principal.

To find out what contracts can achieve, I apply the revelation principle for multistage games (Myerson 1986), which states that any equilibrium outcome of this situation can be implemented with a direct, incentive-compatible and individually rational contract of the form

$$\{\alpha_B, \alpha_A, \{(t, q)\}, \{(\underline{t}_B, \underline{q}_B), (\bar{t}_B, \bar{q}_B)\}, \{(\underline{t}_A, \underline{q}_A), (\bar{t}_A, \bar{q}_A)\}\}.$$

According to such a contract, the principal recommends the agent before signing with probability  $\alpha_B$  to collect information, and to stay uninformed otherwise. If information acquisition is solicited, the agent is asked to reveal his type after signing. The principal implements  $(\underline{t}_B, \underline{q}_B)$  if the agent reports low production costs, and  $(\bar{t}_B, \bar{q}_B)$  otherwise.<sup>12</sup> If the principal recommends before signing to abstain from information acquisition, she submits a further recommendation after signing. With probability  $\alpha_A$ , the agent is instructed to collect information now. Again, if information acquisition is solicited, the agent is asked to reveal his type subsequently. The principal implements  $(\underline{t}_A, \underline{q}_A)$  if the agent reports low production costs, and  $(\bar{t}_A, \bar{q}_A)$  otherwise. In case the agent receives again the recommendation to abstain from information acquisition, the principal implements  $(t, q)$ . A

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<sup>12</sup>The restriction to deterministic quantity allocations is without loss: the principal's payoff is concave in the allocation. The agent is risk-neutral. Thus, the principal benefits from replacing a stochastic allocation by its expectation, and the agent does not care.

direct contract is incentive-compatible if the agent finds it best to be obedient and truthful, and it is individually rational if he accepts it.

In the next section, I review the cases where information is either available only before or only after signing, and in section 6 I argue that an optimal contract never needs to induce information acquisition before signing. To provide more succinct notation for the analysis, I now introduce also contracts that can only recommend information acquisition at a particular date, i.e., either before or after signing. (It will follow from the context which date is meant.) Such contracts are of the form

$$C = \{\alpha, \{(t, q)\}, \{(\underline{t}, \underline{q}), (\bar{t}, \bar{q})\}\}.$$

Here,  $\alpha$  denotes the probability with which the principal recommends information acquisition at the particular date. Transfer and quantity are implemented analogously to the general contracts above. An incentive-compatible contract of the form  $C$  yields the principal a profit of

$$\Pi = (1 - \alpha)[V(q) - t] + \alpha\{p[V(\underline{q}) - \underline{t}] + (1 - p)[V(\bar{q}) - \bar{t}]\}.$$

If the principal recommends to stay uninformed,  $\bar{U}_0 = t - \bar{\beta}q$  denotes the agent's payoff when production costs are high, and  $\underline{U}_0 = t - \underline{\beta}q$  the payoff when they are low. Analogously, if the principal instructs the agent to collect information and if he is obedient and truthful, let  $\bar{U}_1 = \bar{t} - \bar{\beta}\bar{q}$  and  $\underline{U}_1 = \underline{t} - \underline{\beta}\underline{q}$  be the respective *gross* payoffs (i.e., excluding expenditure  $\gamma$ ).

## 5 The cases without information before/after signing

I consider next as a further benchmark the well-studied cases, where information is either available only before or only after signing of the contract. In the first case, the agent can appropriate any rent the agent might obtain ex-post, so that she retains the entire surplus. In the second case, the principal must inevitably concede a share of the surplus if she wants to elicit information, and even sometimes when she recommends to stay uninformed.

## 5.1 No information before signing

In this setting, only the recommendation after signing must be regarded. The following lemma formalizes the incentive-compatibility condition which must hold if the agent is to acquire information and reveal it truthfully. Note that a recommendation to abstain from information gathering is always incentive-compatible in this setting, since the contract is already signed when information becomes available.

**Lemma 1.** *Define  $\phi = (1 - p)p(\bar{\beta} - \underline{\beta})$  as a measure for the size of uncertainty concerning production costs, and denote by  $J(\bar{\beta}) = \bar{\beta} + \frac{p}{1-p}(\bar{\beta} - \underline{\beta})$  the high cost type's "virtual costs". To ensure incentive-compatibility after a recommendation to acquire information, the principal can without loss of generality offer a contract which satisfies*

$$\phi(\underline{q} - \bar{q}) \geq \gamma \quad (\text{IA})$$

and  $\underline{U}_1 - \bar{U}_1 = (\bar{\beta} - \underline{\beta})\bar{q} + \frac{\gamma}{p}$ .

Lemma 1 states that the low cost type obtains more payoff than the high cost type, i.e. a rent, to ensure that the acquisition and transmission of information is incentive-compatible. When the principal recommends to abstain from information acquisition, an agent with low costs will also get an extra payoff, namely

$$\underline{U}_0 - \bar{U}_0 = (\bar{\beta} - \underline{\beta})q.$$

The individual rationality condition to ensure that the agent expects a non-negative payoff at signing requires:

$$(1 - \alpha)[\bar{U}_0 + p(\bar{\beta} - \underline{\beta})q] + \alpha[\bar{U}_1 + p(\bar{\beta} - \underline{\beta})\bar{q}] \geq 0. \quad (\text{IR})$$

(IR) is an *ex-ante* participation constraint, which merely requires that the *uninformed* agent is willing to accept the contract. This constraint does not preclude a loss for the agent if his production costs are high, as long as that loss is outweighed by the payoff for an agent with low costs.

I proceed as usual and replace the transfers by the agent's payoffs. The principal's profit then reads:

$$\Pi = (1 - \alpha)[V(q) - \bar{\beta}q - \bar{U}_1] + \alpha\{-\bar{U}_1 + p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - J(\bar{\beta})\bar{q}]\}.$$

Her optimal contract solves

$$\max_{\bar{U}_0, \bar{U}_1, q, \bar{q}, \alpha} \Pi \quad s.t. \quad (IR) \text{ and } (IA).$$

At the optimum, (IR) must bind, so that the agent does not obtain a share of the surplus. The contracting problem can now be reformulated as

$$\max_{\alpha} (1 - \alpha)W^*(0) + \alpha W^*(1)$$

where

$$W^*(0) = \max_q V(q) - E[\beta]q$$

and

$$W^*(1) = \max_{q, \bar{q}} -\gamma - p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - \beta\bar{q}] \quad s.t. \quad (IA).$$

The optimal contract for this situation is efficient, although the incentive-compatibility constraint (IA) must be met if the agent is instructed to acquire information. To see this, note that instead of a direct contract the principal may offer the two-part tariff

$$T(q) = -\max\{W^{FB}(0), W^{FB}(1)\} + V(q),$$

and let the agent select any quantity  $q$ . This tariff implements the first-best information acquisition decision and the first best quantity allocation; moreover, the principal obtains the first-best surplus. Hence, (IA) does not bind. Intuitively, this scheme sells the principal's bargaining power to the agent and so eliminates the incentive-compatibility condition. Since the agent is uninformed at signing, the principal can charge as a price for her bargaining power the expected surplus of the interaction. Note that the agent will incur a loss if his production costs turn out to be high.

## 5.2 No information after signing

In this case, analyzed in detail by Crémer, Khalil and Rochet (1998a), the principal cannot instruct the agent to acquire information after signing. Thus, only recommendations before signing are relevant.

It turns out that this situation is less favorable for the principal than the case where the agent can only collect information after signing. First, even the recommendation to abstain from

information acquisition must be incentive-compatible, since precontractual information is generally valuable for the agent. Second, if the principal chooses to induce information acquisition, her contract must be individually rational given the actual state of the world.

Suppose first the principal recommends to abstain from information acquisition. In this case, the individual rationality condition imposes an ex-ante participation constraint:

$$\bar{U}_0 + p(\bar{\beta} - \underline{\beta})q \geq 0. \quad (1)$$

The agent might ignore the principal's recommendation and collect precontractual information for a strategic purpose: to check the actual payoff in advance in order to accept the contract only if a state obtains where it yields a positive payoff. Incentive-compatibility requires that the *price* of precontractual information,  $\gamma$ , exceeds the value of precontractual information—the option to reject the contract if a state obtains where it yields a loss.<sup>13</sup>

$$(1 - p)\bar{U}_0 + \gamma \geq 0. \quad (2)$$

If instead the principal recommends to find out the realized state of the world, lemma 1 specifies the incentive-compatibility condition. Since information acquisition must happen before signing, individual rationality now imposes an *ex-post* participation constraint, which requires that the *informed* agent accepts the contract. This constraint precludes losses in both states of the world, and is thus more restrictive than its ex-ante counterpart. By lemma 1, it is enough only to rule out a loss for the high cost type explicitly:

$$\bar{U}_1 \geq 0. \quad (3)$$

The principal's contracting problem now takes the form

$$\max_{\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}, \alpha} \Pi \quad s.t. \quad (1), (2), (3), \text{ and } (IA).$$

In this situation, stochastic contracts are not useful; only  $\alpha \in \{0, 1\}$  can be optimal. To see this, note that the agent may wait until any uncertainty concerning the principal's recommendation

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<sup>13</sup>The formulation of constraint (2) requires that only the high cost type can incur a loss from the contract. This is without loss of generality since  $\underline{U}_0 = \bar{U}_0 + (\bar{\beta} - \underline{\beta})q \geq \bar{U}_0$ , so that the contract is only acceptable for the agent if the low cost type obtains a positive payoff.

resolves, before he takes an action. Randomization over the recommendation therefore merely amounts to randomization over the two contracts which, respectively, surely deter or surely induce information acquisition. Clearly, the principal can do better by offering the optimal deterministic contract, which I derive next.

Detering information gathering yields at most profit

$$W(0) = \max_{q, \bar{U}_0} V(q) - \bar{\beta}q - \bar{U}_0 \quad s.t. \quad (1) \text{ and } (2).$$

As Crémer, Khalil and Rochet (1998a) show, it is the price of precontractual information which determines the binding constraint(s) in this problem.

**Lemma 2.** (Crémer, Khalil and Rochet 1998a) Define  $\gamma^d = \phi \bar{q}^{FB}$  and  $\gamma^c = \phi q^{FB}$ . Denote by  $q'$  the quantity allocation of an optimal contract which deters information gathering. Then

- for  $\gamma > \gamma^c$ , just (1) binds, and production is ex-ante efficient,  $q' = q^{FB}$ .
- for  $\gamma^c \geq \gamma > \gamma^d$ , both constraints bind, and there is under-production,  $q' = \frac{\gamma}{\phi}$ .
- for  $\gamma \leq \gamma^d$ , just (2) binds, and there is under-production,  $q' = \bar{q}^{FB}$ .

Since the agent can compute his payoff from the contract before signing, he may be able to secure a share of the surplus. That is, it may be impossible for the principal to appropriate the extra payoff which an agent with low production costs obtains. Such a scheme would inevitably inflict a loss on an agent with high production costs, so that precontractual information might be worth its price  $\gamma$ .

If instead the principal induces information gathering, her maximum profit is

$$W(1) = \max_{q, \bar{q}} -\gamma - \bar{U}_1 + p[V(\underline{q}) - \underline{\beta}q] + (1-p)[V(\bar{q}) - J(\bar{\beta})\bar{q}] \quad s.t. \quad (3) \text{ and } (IA).$$

The ex-post participation constraint (3) must bind at the optimum, so that the low cost type's rent cannot be appropriated. The principal maximizes only that fraction of the surplus she retains, but not the entire surplus itself. Specifically, if information is cheap, so that (IA) does not bind, she allocates efficiently to the low cost type ( $\underline{q}' = \underline{q}^{FB}$ ), but demands an inefficiently low quantity from the high cost type ( $\bar{q}' = V^{-1}(J(\bar{\beta})) < \bar{q}^{FB}$ ). When (IA) binds, on the other hand, the quantity allocations are additionally distorted relative to the first-best to provide an incentive for information acquisition and its truthful transmission to the principal.

Finally, the principal compares the maximum profits of the two contracts to find out whether to induce or deter information acquisition, i.e., to determine the optimal  $\alpha$ . Since the respective contracts yield less profit than what the principal can obtain in the first-best ( $W(0) \leq W^{FB}(0)$  as well as  $W(1) \leq W^{FB}(1)$ ), there is no general order between  $\alpha$  and  $\alpha^{FB}$ .

I conclude this section with figure 1, which depicts the considered benchmarks. Note that the location of the various intersections depends on the parameters of the model and may well emerge differently than displayed.

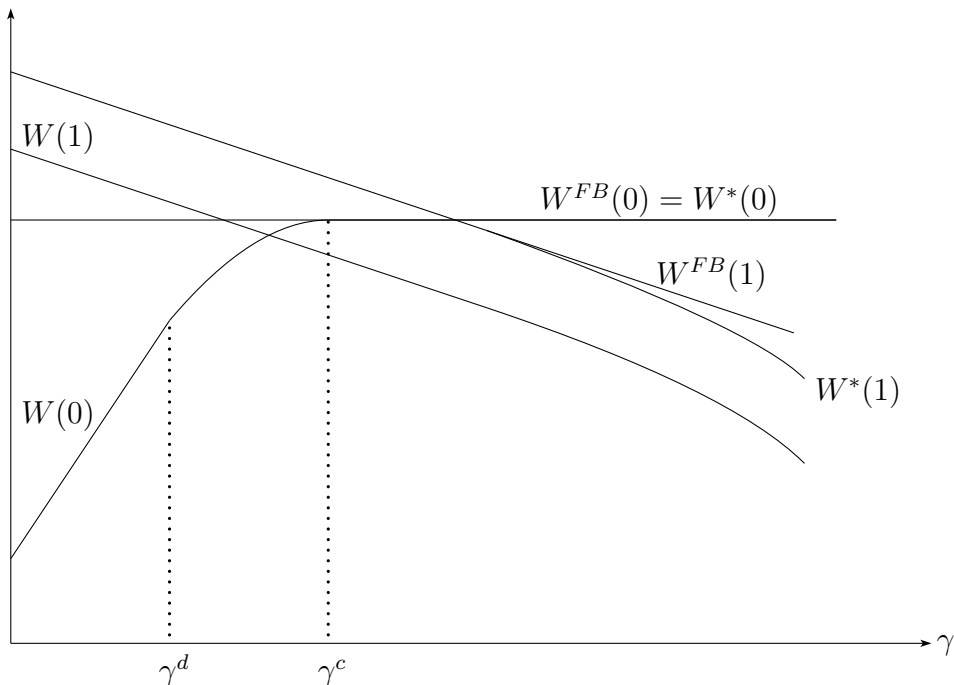


Figure 1: Maximum profits when information acquisition is deterred or induced

## 6 Principal's problem

This section develops the principal's problem of finding an optimal contract in the original setting, where information is available before as well as after signing. The key step is to realize that, in equilibrium, no party gains if information is acquired before rather than after the contract is signed, since it is equally costly at either date.

**Lemma 3.** *If an optimal contract exists, there is also an optimal contract where the principal*



recommends to abstain from precontractual information gathering (i.e., with  $\alpha_B = 0$ ).<sup>14</sup>

I will thus confine attention to the class of contracts  $C$  that may only recommend to acquire information after signing.

The principal's problem combines constraints of the cases where information can be acquired either only before or only after the contract is signed. Similarly to the latter case, the agent may possibly get a recommendation to observe his type after signing, but there will be no such recommendation before. Hence, the contract must first satisfy the incentive-compatibility constraints imposed by lemma 1, to ensure that the agent is obedient and truthful when the principal solicits information. Second, the contract must be acceptable for the *uninformed* agent:

$$(1 - \alpha)[\bar{U}_0 + p(\bar{\beta} - \underline{\beta})q] + \alpha[\bar{U}_1 + p(\bar{\beta} - \underline{\beta})\bar{q}] \geq 0. \quad (\text{IR})$$

But lemma 3 adds a further incentive-compatibility constraint, which had to be considered in the case information without information after signing. Namely, the agent must not gain by precontractual information gathering; also in this setting, this would only serve the strategic purpose to reject the contract when it yields a loss given the true state of the world:<sup>15</sup>

$$(1 - p)[(1 - \alpha)\bar{U}_0 + \alpha\bar{U}_1] + (1 - \alpha)\gamma \geq 0. \quad (\text{NIA})$$

As its analog (2), this constraint requires that the price of precontractual information is too high relative to its value (i.e., the option to reject the contract if a state obtains where it yields a loss). However, that price is now  $(1 - \alpha)\gamma$ —and therefore fixed by the principal's recommendation. This is because with probability  $\alpha$  the agent will acquire information after signing, in which case the expense  $\gamma$  is due anyway. (Note that the *value* of precontractual information, on the other hand, is not fixed by the recommendation, since  $\bar{U}_0$  and  $\bar{U}_1$  are chosen by the principal.)

(IR) is an ex-ante participation constraint. However, (NIA) may in fact impose an ex-post participation constraint, which requires that the *informed* agent accepts the contract, and thus would be more restrictive. Specifically, this is the case if the principal will surely recommend to find out production costs after signing ( $\alpha = 1$ ). Precontractual information is then effectively for

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<sup>14</sup>Without the restriction to deterministic allocations, the outcome (i.e., quantity allocation and transfer) of any contract could be implemented with a contract where  $\alpha_B = 0$ .

<sup>15</sup>Due to lemma 1 and an argument as in footnote 13, the low cost type cannot incur a loss from the contract.

free, and by (NIA) even the high cost type must not incur a loss from the contract. On the other hand, when the principal will not solicit information ( $\alpha = 0$ ), precontractual information costs  $\gamma$ . In that case, it may well be sufficient to ensure just ex-ante participation (namely if  $\gamma > \gamma^d$ , cf. lemma 2). The specification of  $\alpha$  hence involves a trade-off between the efficiency gain through information acquisition on the one hand, and, via the price of precontractual information, the share of the surplus which can be appropriated, on the other hand. This paper's insight is that, to solicit information at least with some probability and yet retain the entire surplus, the principal might submit her recommendation stochastically.

Now that all constraints are identified, the principal's problem can be stated formally as

$$\max_{\bar{U}_0, \bar{U}_1, q, \bar{q}, \alpha} \Pi \quad s.t. \quad (NIA), (IR) \text{ and } (IA).$$

It turns out that if the principal had to speak out a deterministic recommendation, contracts are as in the case where information can only be gathered before signing (cf. section 5.2). Intuitively, this holds because the agent can already anticipate before signing any deterministic future recommendation. The relevant incentive-compatibility and individual rationality constraints are therefore the same in the two settings.

**Lemma 4.** *The best deterministic contract is identical to the optimal contract for the situation where information is only available before signing. In particular, with  $\alpha = 0$  (resp.  $\alpha = 1$ ), the principal's maximum profit is  $W(0)$  (resp.  $W(1)$ ).*

The next section argues that deterministic contracts may be suboptimal.

## 7 Randomization

To understand the intuition that drives the analysis, consider the following example. Let

$$V(q) = \sqrt{q}, \quad \underline{\beta} = 1, \quad \bar{\beta} = 5, \quad p = \frac{1}{2}, \quad \gamma = \frac{1}{18}.$$

This yields

$$E[\beta] = 3, \quad J(\bar{\beta}) = 9, \quad \phi = 1.$$

Suppose for the moment, the agent could only acquire information after signing (cf. section 5.1), so that the principal retains the entire surplus and offers a contract to maximize it. The optimal contract would elicit information to implement  $(\underline{q}^{FB}, \bar{q}^{FB}) = (\frac{1}{4}, 0.01)$ ; the principal earns  $W^{FB}(1) = 0.0944$  and an agent with high production costs  $\bar{U}_1 = -0.02$ . Without information (and a quantity allocation of  $q^{FB} = \frac{1}{36}$ ), the principal gets  $W^{FB}(0) = \frac{1}{12}$  and the high cost type  $\bar{U}_0 = -\frac{1}{18}$ . Now return to the original setting, where the additional constraint (NIA) must be satisfied to prevent precontractual information gathering. The first contract is no longer incentive-compatible: the value of precontractual information, to avoid an expected loss of 0.01, exceeds its (effective) price of zero. Under the second contract, precontractual information is worth  $\frac{1}{36}$  but costs  $\frac{1}{18}$ . So that contract remains feasible. As can be shown, it yields as much payoff as the best contract that solicits information with certainty. However, the leeway of (NIA) suggests that the principal might solicit information with some probability less than one to implement  $(\underline{q}^{FB}, \bar{q}^{FB})$ , but yet retain the entire surplus. And indeed, in the present example, this is feasible for any  $\alpha \leq 0.735$ . Thus, she may obtain a profit of  $0.265 \cdot \frac{1}{12} + 0.735 \cdot 0.0944 = 0.0914$  with a stochastic contract, roughly ten percent more than with any deterministic contract.

The example demonstrates the principal's trade-off between the efficiency gain through information acquisition and the price of precontractual information. Efficiency requires to elicit information and demand a cost-contingent quantity. However, if the principal recommends information acquisition with certainty, the price of precontractual information is zero (while its value to the agent is positive). In this case, the contract must satisfy the ex-post participation constraint, and consequently concedes surplus. A stochastic contract, on the other hand, generates less surplus but fixes a positive price of precontractual information—in the example, it is so high that the contract just needs to be ex-ante acceptable and consequently appropriates all surplus. Although this scheme inflicts a loss on an agent with high production costs, there is no incentive to gather precontractual information: the stochastic contract entails uncertainty, whether precontractual information will be of any use *after* signing, and that risk outweighs the risk of incurring a loss from the contract.

The remainder of this section shows that a stochastic contract may indeed be optimal, as indicated by the example. However, it is not insightful to solve the principal's problem explicitly.<sup>16</sup>

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<sup>16</sup>As shown below, both (NIA) and (IR) must bind at an optimum with randomization. The resulting maximiza-

Instead, I next prove existence of a solution. This will furnish a convenient criterion to examine whether the principal should randomize over her recommendations.

**Lemma 5.** *The principal's problem has a solution.*

In the following, I split the principal's problem into two steps. First, I consider  $\alpha$  as a parameter and optimize over the remaining choice variables. Let  $W(\alpha)$  denote the resulting maximum value. The second step is to find an  $\alpha$  which maximizes  $W(\alpha)$ .<sup>17</sup> However, the second step is not needed to check whether the optimal contract is stochastic. By lemma 5, it actually suffices to find an  $\alpha \in (0, 1)$  such that

$$W(\alpha) > \max\{W(0), W(1)\}.$$

This inequality necessarily requires that randomization over recommendations must yield the principal more profit than randomization over the two deterministic contracts. In the two cases without information before/after signing, that was impossible. However, the trade-off between the efficiency gain through information acquisition and the price of precontractual information only arises in the original situation with permanently available information.

**Lemma 6.** *Let  $\alpha \in (0, 1)$ . Then  $W(\alpha) > (1 - \alpha)W(0) + \alpha W(1)$  if and only if  $\gamma > \gamma^d$ .*

The proof shows that the principal's problem can be reformulated, so that  $\alpha$  only appears in the objective function and the individual rationality condition. In particular, when the principal's recommendation to acquire information gets more likely, this constraint is relaxed. But the principal only benefits from the additional leeway if the constraint is actually binding at  $\alpha = 0$  (cf. lemma 2); otherwise, randomization just yields a convex combination of the profits from the two deterministic contracts. (On the other hand, there must be a cutoff less than one, such that the individual rationality constraint gets slack if  $\alpha$  exceeds it and (NIA) becomes binding. Technically, it is this change in the binding constraints which can establish the optimality of a stochastic contract.)

It is now possible to state the main result, which generalizes the insight provided by the example. In situations where the two deterministic contracts perform roughly equally well, the  


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 tion problem is neither concave nor convex.

<sup>17</sup>This procedure is feasible: due to the assumptions on the principal's payoff function  $V$ , the first step yields a (unique) solution. A solution to the second step exists by lemma 5.

principal potentially offers a stochastic contract, designed such that she retains the entire surplus. As a final piece of notation, let  $\tilde{\gamma}$  be the intersection of  $W(0)$  and  $W(1)$ .<sup>18</sup>

**Proposition 1.** *If  $\tilde{\gamma} > \gamma^d$ , randomization is optimal for an interval containing  $\tilde{\gamma}$ .<sup>19</sup> Moreover, if randomization is optimal the ex-ante participation constraint (IR) binds, so that the entire surplus accrues to the principal.*

While the principal retains the entire surplus with the stochastic contract, this contract does not maximize the surplus—first of all just because it is stochastic. This is because the incentive-compatibility constraint (NIA) necessarily binds at the optimum, too.<sup>20</sup>

## 8 Conclusion

I studied the optimal contract offer to an uninformed agent who can acquire costly information about his type either before or after the contract is signed. In such a situation, the principal tries to deter precontractual information gathering in order to retain surplus. When she solicits information after signing with certainty, the (effective) price of precontractual information is zero since the costs of information acquisition accrue anyway. In this case, the contract must satisfy an ex-post participation constraint and consequently cedes surplus to the agent. On the other hand, when the principal with certainty does not solicit information after signing, precontractual information is costly. In that case, the contract possibly just needs to pass an ex-ante participation constraint so that the principal may retain the entire surplus. Hence, the principal trades off the efficiency gain through information acquisition against a high price of precontractual information. This trade-off is absent in the well-studied cases, where information is either not available or not costly at each date. To solve it optimally, the principal might offer a stochastic contract.

There are interesting ways to develop this paper’s ideas further. First, how does the optimal

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<sup>18</sup>This intersection must be unique since  $W(0)$  is non-decreasing while  $W(1)$  strictly decreases. Moreover, as the principal’s utility function  $V$  is concave and the feasible set of contracts convex, the respective maximum profits are concave as well and thus continuous on the interior of their domain,  $(0, \infty)$  (cf. de la Fuente 2000, theorem 2.12, p. 313 and theorem 2.14, p. 252). Since, at  $\gamma = 0$ ,  $W(0) < W(1)$ , this establishes existence of  $\tilde{\gamma}$ .

<sup>19</sup>It can be shown that the condition requires a sufficiently high value for  $p$ .

<sup>20</sup>This follows by a similar argument as in the proof of proposition 1.

auction look like when bidders can acquire information concerning their valuation both before and after they decide on participation? Crémer, Spiegel and Zheng (2009) analyze auction design in the case without information at the first date.<sup>21</sup> They find that the auctioneer should solicit information from the bidders sequentially, until she meets somebody whose valuation is sufficiently high. When information is also available before signing, the designer could raise the price of precontractual information for a particular bidder not just with a stochastic mechanism, but also by changing the order in which bidders are approached. Second, when the agent can select among several effort levels to acquire information of different precision at different costs, which level should the principal implement? This generalization adds a further dimension to the principal's trade-off. In particular, she has greater leeway to fix the price of precontractual information.

## Appendix

*Proof of lemma 1.* The obedience constraints to ensure that the agent indeed acquires information are:

$$\begin{aligned}
 p\underline{U}_1 + (1-p)\overline{U}_1 - \gamma &\geq \overline{U}_1 + p(\overline{\beta} - \underline{\beta})\overline{q} \\
 p\underline{U}_1 + (1-p)\overline{U}_1 - \gamma &\geq \underline{U}_1 - (1-p)(\overline{\beta} - \underline{\beta})\underline{q}.
 \end{aligned} \tag{O}$$

The agent is honest, i.e. he truthfully transmits his findings to the principal, if:

$$\begin{aligned}
 \overline{U}_1 &\geq \underline{U}_1 - (\overline{\beta} - \underline{\beta})\underline{q} \\
 \underline{U}_1 &\geq \overline{U}_1 + (\overline{\beta} - \underline{\beta})\overline{q}.
 \end{aligned} \tag{T}$$

(O) is equivalent to

$$\begin{aligned}
 \overline{U}_1 &\leq \underline{U}_1 - (\overline{\beta} - \underline{\beta})\underline{q} - \frac{\gamma}{p} \\
 \underline{U}_1 &\geq \overline{U}_1 + (\overline{\beta} - \underline{\beta})\overline{q} - \frac{\gamma}{1-p}.
 \end{aligned}$$

This implies (T) and is equivalent to (IA) together with

$$\underline{U}_1 - \overline{U}_1 \in \left[ (\overline{\beta} - \underline{\beta})\overline{q} + \frac{\gamma}{p}, (\overline{\beta} - \underline{\beta})\underline{q} - \frac{\gamma}{1-p} \right].$$

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<sup>21</sup>See Shi (2009) for the case without information after signing.

Suppose contract  $C = \{\alpha, \{(t, q)\}, \{(\underline{t}, \underline{q}), (\bar{t}, \bar{q})\}\}$  satisfies all constraints and yields  $\underline{U}_1 - \bar{U}_1 = (\bar{\beta} - \underline{\beta})\bar{q} + \frac{\gamma}{p} + r$ . Consider contract  $C' = \{\alpha, \{(t + \frac{p\alpha}{1-\alpha}r, q)\}, \{(\underline{t} - r, \underline{q}), (\bar{t}, \bar{q})\}\}$ . The two contracts implement the same quantity allocation, and the principal pays the same transfer in expectation. Hence, they are payoff-equivalent for both parties. Furthermore,  $\underline{U}'_1 - \bar{U}'_1 = (\bar{\beta} - \underline{\beta})\bar{q} + \frac{\gamma}{p}$ , so that the condition in the lemma holds. (T) and (O) are met by  $C'$ . (NIA) and (IR) only restrict  $\underline{U}' = \underline{U}$  and  $\bar{U}' = \bar{U}$  (i.e., these payoffs are identical under  $C$  and  $C'$ ), so that  $C'$  satisfies these constraints as well.

The principal's profit function,  $\Pi$ , is obtained by replacing transfers with the agent's payoffs.  $\square$

*Proof of lemma 2.* This follows from Crémer, Khalil and Rochet (1998a), proposition 2.  $\square$

*Proof of lemma 3.* Consider a feasible (i.e., incentive-compatible and individually rational) contract

$$C = \{\alpha_B, \alpha_A, \{(t, q)\}, \{(\underline{t}_B, \underline{q}_B), (\bar{t}_B, \bar{q}_B)\}, \{(\underline{t}_A, \underline{q}_A), (\bar{t}_A, \bar{q}_A)\}\}.$$

In terms of the principal's profit,  $C$  is a lottery over the contracts<sup>22</sup>

$$C' = \{\alpha_B = 0, \alpha_A, \{(t, q)\}, \{(\underline{t}_A, \underline{q}_A), (\bar{t}_A, \bar{q}_A)\}\},$$

drawn with probability  $1 - \alpha_B$ , and

$$C'' = \{\alpha_B = 1, \alpha_A, \{(\underline{t}_B, \underline{q}_B), (\bar{t}_B, \bar{q}_B)\}\},$$

drawn with probability  $\alpha_B$ . Note that  $C'$  and  $C''$  are feasible as well, since the agent may choose his optimal strategy *after* the principal submits the recommendation between contract offer and signing. The principal can thus (weakly) increase profits by offering the best contract among  $C'$  and  $C''$  with certainty. To prove the lemma, I only need to show that contract

$$\tilde{C}'' = \{\alpha_B = 0, \alpha_A = 1, \{(\underline{t}_B, \underline{q}_B), (\bar{t}_B, \bar{q}_B)\}\},$$

which implements the same pair of transfer and quantity allocation as  $C''$  but recommends to abstain from precontractual information gathering, is feasible as well.

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<sup>22</sup>To economize on notation, I omit those pairs of transfer and quantity allocation that are implemented with probability zero.

Suppose contract  $\tilde{C}''$  is offered. Since  $C''$  is individually rational, there is no (strict) incentive for the agent to acquire information already before signing, as both types would accept the contract. Hence,  $\tilde{C}''$  is incentive-compatible before signing. Since  $C''$  is incentive-compatible before signing, purchasing information must yield a non-negative payoff to the uninformed agent. Hence,  $\tilde{C}''$  is individually rational. Further, it follows that the agent prefers to collect information after signing. Since  $C''$  is incentive-compatible after signing, the agent has an incentive to report his type truthfully. Hence,  $\tilde{C}''$  is incentive-compatible after signing, and finally feasible.  $\square$

*Proof of lemma 4.* This follows by comparison of the two optimization problems.  $\square$

*Proof of lemma 5.* The proof requires additional notation. Fix  $\alpha \in [0, 1]$  in the principal's problem and maximize over the remaining choice variables. Then  $W(\alpha)$  denotes the maximum value of program

$$\mathcal{P}(\alpha) : \max_{\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}} \Pi(\alpha) \quad s.t. \quad (NIA_\alpha), (IR_\alpha) \text{ and } (IA_\alpha).$$

To prove lemma 5, it has to be shown that  $\sup_{\alpha \in [0, 1]} W(\alpha)$  is attained. By Weierstrass' theorem, this is true if  $W(\alpha)$  is continuous as a function of  $\alpha$ . This, in turn, holds by the maximum theorem (see de la Fuente 2000, theorem 2.1, p. 301) if the correspondence which assigns to each  $\alpha$  the set of feasible choice variables is compact-valued and continuous. However, this correspondence is not compact (there are no upper bounds on  $\bar{U}_0$ ,  $\bar{U}_1$ ,  $q$ , and  $\underline{q}$ ).<sup>23</sup> By adding non-binding constraints, I will replace it by another correspondence  $F^*$ , such that maximization with respect to  $F^*$  yields the same maximum value  $W(\alpha)$ , and such that  $F^*$  is compact-valued and continuous. From the maximum theorem then follows that  $W(\alpha)$  is continuous, and the proof is finished. I will use the following lemma.<sup>24</sup>

*Lemma.* Let  $g^i(\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}, \alpha) : \mathbb{R}^5 \times [0, 1] \rightarrow \mathbb{R}$  be a continuous function that is affine given  $\alpha$  for all  $i = 1, \dots, I$ , and define the correspondence  $F^* : [0, 1] \rightarrow \mathbb{R}^5$  by

$$F^*(\alpha) = \{(\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}) : g^i(\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}, \alpha) \geq 0 \text{ for all } i = 1, \dots, I\}.$$

Let  $F^*(\alpha^0)$  be compact and assume that there is some point  $(\bar{U}'_0, \bar{U}'_1, q', \bar{q}', \underline{q}') \in F^*(\alpha^0)$  such that  $g^i(\bar{U}'_0, \bar{U}'_1, q', \bar{q}', \underline{q}', \alpha^0) > 0$  for all  $i$ ; then  $F^*$  is continuous at  $\alpha^0$ .

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<sup>23</sup> $\bar{q}$  is bounded above due to  $(IA_\alpha)$ .

<sup>24</sup>See de la Fuente (2000) theorem 2.2, p. 303.



Denote the feasible set in problem  $\mathcal{P}(\alpha)$ , as given by  $(\text{NIA}_\alpha)$ ,  $(\text{IR}_\alpha)$ ,  $(\text{IA}_\alpha)$  and the implicit non-negativity constraints on quantity allocations, by  $F(\alpha)$ . From the main text follows that, for  $k > 0$ ,

$$\bar{U}_0 \leq k, \quad \bar{U}_1 \leq k, \quad q \leq q^{FB} + k, \quad \text{and} \quad \underline{q} \leq \max\{\underline{q}^{FB}, \bar{q}^{FB} + \frac{\gamma}{\phi}\} + k$$

would be non-binding as constraints in  $P(\alpha)$ . Add them to  $P(\alpha)$ , which yields a bounded feasible set. Denote it by  $F^*(\alpha)$ . Being an intersection of closed half-spaces,  $F^*(\alpha)$  is closed, and therefore compact. It can be described by level sets of continuous functions  $g^i(\bar{U}_0, \bar{U}_1, q, \bar{q}, \underline{q}, \alpha)$ , where the functions are affine given  $\alpha$ . At  $(\bar{U}'_0, \bar{U}'_1, q', \bar{q}', \underline{q}') = (0, 0, q^{FB}, \bar{q}^{FB}, \max\{\underline{q}^{FB}, \bar{q}^{FB} + \frac{\gamma}{\phi}\})$  all constraints are satisfied with strict inequality. Hence, the correspondence  $F^*$  is continuous by the lemma. Replacing the feasible set  $F(\alpha)$  in problem  $\mathcal{P}(\alpha)$  by  $F^*(\alpha)$  yields the same maximum value  $W(\alpha)$ , because the solution with respect to  $F(\alpha)$  is contained in  $F^*(\alpha)$ , and  $F^*(\alpha)$  itself is contained in  $F(\alpha)$ .  $\square$

*Proof of lemma 6.* First, note that it is without loss of generality to set  $\bar{U}_1 = 0$  (if necessary,  $\bar{U}_0$  can be replaced by  $X = \bar{U}_0 + \frac{\alpha}{1-\alpha}\bar{U}_1$ ). The optimal contract then solves<sup>25</sup>

$$\begin{aligned} \max_{\bar{U}_0, q, \bar{q}, \underline{q}, \alpha} \quad & (1 - \alpha)[V(q) - \bar{\beta}q - \bar{U}_0] + \alpha\{-\gamma + p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - J(\bar{\beta})\bar{q}]\} \\ \text{s.t.} \quad & (1 - p)\bar{U}_0 + \gamma \geq 0 \quad (\text{NIA}_0) \\ & \bar{U}_0 + p(\bar{\beta} - \underline{\beta})(q + \frac{\alpha}{1-\alpha}\bar{q}) \geq 0 \quad (\text{IR}^+) \\ & \text{and (IA)} \end{aligned}$$

I now work with the reformulated problem to prove the lemma. By definition,

$$\begin{aligned} W(\alpha) = \max_{\bar{U}_0, q, \bar{q}, \underline{q}} \Pi = & (1 - \alpha)[V(q) - \bar{\beta}q - \bar{U}_0] \\ & + \alpha\{-\gamma + p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - J(\bar{\beta})\bar{q}]\} \\ & + \lambda_1(\alpha)[(1 - p)\bar{U}_0 + \gamma] \\ & + \lambda_2(\alpha)[\bar{U}_0 + p(\bar{\beta} - \underline{\beta})(q + \frac{\alpha}{1-\alpha}\bar{q})] \\ & + \lambda_3(\alpha)[\phi(\underline{q} - \bar{q}) - \gamma], \end{aligned}$$

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<sup>25</sup>Constraint  $(\text{IR}^+)$  is not well-defined at  $\alpha = 1$ . Let it require  $\bar{U}_0 \in \mathbb{R}$  in this case.

where the  $\lambda$ s denote non-negative Lagrange multipliers. Also by definition,

$$\begin{aligned}
(1 - \alpha)W(0) + \alpha W(1) &= (1 - \alpha) \left[ \max_{q, \bar{U}_0} V(q) - \bar{\beta}q - \bar{U}_0 \right. \\
&\quad \left. + \lambda_1(0)[(1 - p)\bar{U}_0 + \gamma] \right. \\
&\quad \left. + \lambda_2(0)[\bar{U}_0 + p(\bar{\beta} - \underline{\beta})q] \right] \\
&+ \alpha \left[ \max_{\underline{q}, \bar{q}} -\gamma + p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - J(\bar{\beta})\bar{q}] \right. \\
&\quad \left. + \lambda_3(1)[\phi(\underline{q} - \bar{q}) - \gamma] \right].
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
(1 - \alpha)W(0) + \alpha W(1) &= \max_{\bar{U}_0, \underline{q}, \bar{q}, q} (1 - \alpha)[V(q) - \bar{\beta}q - \bar{U}_0] \\
&\quad + \alpha \{-\gamma + p[V(\underline{q}) - \underline{\beta}q] + (1 - p)[V(\bar{q}) - J(\bar{\beta})\bar{q}]\} \\
&\quad + \lambda_1(0)[(1 - p)\bar{U}_0 + \gamma] \\
&\quad + \lambda_2(0)[\bar{U}_0 + p(\bar{\beta} - \underline{\beta})q] \\
&\quad + \lambda_3(1)[\phi(\underline{q} - \bar{q}) - \gamma].
\end{aligned}$$

Whenever  $\lambda_2(0) > 0$  or, equivalently,  $\gamma > \gamma^d$  (cf. lemma 2), the strict inequality holds as asserted in the lemma. Otherwise, both sides are equal.  $\square$

*Proof of proposition 1.* Suppose  $\tilde{\gamma} > \gamma^d$ . Let  $\alpha \in (0, 1)$  and  $\gamma = \tilde{\gamma}$ . Then,  $W(0) = W(1) = (1 - \alpha)W(0) + \alpha W(1) < W(\alpha)$ , where the inequality follows from lemma 6. By continuity of  $W(\alpha)$  in  $\gamma$  (which follows by the same argument as in footnote 18), this holds for a whole interval containing  $\gamma^*$ . To see the second claim, note that if the constraint is slack, the principal's maximum profit is affine in  $\alpha$ , so that (generically) the choice of  $\alpha$  is not optimal.  $\square$

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