

Hlouskova, Jaroslava; Wagner, Martin

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## Finite sample correction factors for panel cointegration tests

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Jaroslava Hlouskova, Martin Wagner



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September 2009

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

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## **Abstract**

In this paper we present finite  $T$  mean and variance correction factors and corresponding response surface regressions for the panel cointegration tests presented in Pedroni (1999, 2004), Westerlund (2005), Larsson et al. (2001), and Breitung (2005). For the single equation tests we consider up to 12 regressors and for the system tests vector autoregression dimensions up to 12 variables. All commonly used specifications for the deterministic components are considered. The sample sizes considered are  $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$ .

## **Keywords**

Panel cointegration test, correction factor, response surface, simulation

## **JEL Classification**

C12, C15, C23

**Comments**

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# 1 Introduction

In this paper we present finite  $T$  mean and variance correction factors and corresponding response surface regressions for the panel cointegration tests presented in Pedroni (1999, 2004), Westerlund (2005), Larsson *et al.* (2001) and Breitung (2005). For the single equation tests we consider up to 12 regressors and for the system tests vector autoregression (VAR) dimensions up to 12 variables. The considered sample sizes are  $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$ . The results are based on 100,000 replications of approximating the moments of the functionals of Brownian motions required in the different tests by the corresponding functionals of random walk series of length  $T$ . A detailed description of the simulation of the correction factors as well as the computer code in **GAUSS** are available upon request from the authors.

It has to be noted that the use of finite sample critical values or finite sample correction factors comes at a certain price for any unit root or cointegration test, both in a time series and panel setting. For finite time series dimension the exact finite sample test statistics typically depend on all characteristics of the data generating processes. The dependence upon these nuisance parameters vanishes only as  $T \rightarrow \infty$ . Thus, the advantages of using small sample correction factors based on small sample approximations of the limiting test statistics is to a certain extent offset by the finite sample nuisance parameter dependency. With respect to this tradeoff no general results are available in either the literature or in this paper. The general tendency in the literature is to value the gains due to the small sample approximations higher than the detrimental effects incurred due to nuisance parameter dependencies. Nevertheless, the potential drawbacks have to be kept in mind when using the correction factors and response surface regressions provided in this paper.

In the following two sections we briefly describe the implemented tests with a focus on the actual computation of the test statistics and the usage of the mean and variance correction factors. The single equation tests are discussed in Section 2 and the system tests in Section 3. A detailed description of the tests including a thorough discussion of the assumptions as well as simulation performance is contained in Wagner and Hlouskova (2010). Section 4 contains the results from response surface regressions to efficiently, albeit only approximately, summarize the information from the large set of correction factors. The full set of tables with the correction factors is given in the appendix.

## 2 The single equation tests

The single equation methods are panel extensions of the Engle and Granger (1987) approach to cointegration analysis with the DGP given by

$$y_{it} = \alpha_i + \delta_i t + X'_{it}\beta_i + u_{it} \quad (1)$$

$$X_{it} = X_{it-1} + \varepsilon_{it} \quad (2)$$

observed for  $i = 1, \dots, N$  and  $t = 1, \dots, T$ . Here  $y_{it}, u_{it} \in \mathbb{R}$ ,  $X_{it}$  and  $\varepsilon_{it} \in \mathbb{R}^l$ ,  $\alpha_i, \delta_i \in \mathbb{R}$  and  $\beta_i \in \mathbb{R}^l$ . Cointegration prevails in unit  $i$ , if  $u_{it}$  is stationary in which case the single cointegrating vector is given by  $[1, -\beta'_i]'$ . To be able to write the DGP in a unified fashion for both cointegration and no cointegration we set  $\beta_i = 0$  in (1) if there is no cointegration in unit  $i$  with the corresponding process  $u_{it}$  being integrated of order 1, see below.

The null hypothesis of all considered single equation tests is that of no cointegration in all units, i.e. the  $u_{it}$  are I(1) processes for  $i = 1, \dots, N$ . In this case, when viewed as regressions to be estimated, equations (1) are spurious regressions under the null hypothesis.<sup>1</sup>

We consider the usual three cases for the deterministic variables: Case 1 without deterministic components, case 2 with only fixed effects  $\alpha_i$  and case 3 with both fixed effects  $\alpha_i$  and individual specific linear time trends  $\delta_i t$ .

### Pedroni tests

The tests developed by Pedroni (1999, 2004) are based on testing for unit roots in the residuals of equation (1),  $\hat{u}_{it}$  say, where in particular the OLS residuals of (1) can be chosen, see Phillips and Ouliaris (1990) for a discussion in the time series setting. Similarly to the time series case both regressor endogeneity and serial correlation have to be taken into account. Denote with  $v_{it} = [\Delta u_{it}, \varepsilon'_{it}]' \in \mathbb{R}^{l+1}$  the stacked stationary error processes in (1) and (2), with  $\Omega_i = \begin{bmatrix} \omega_{ui}^2 & \Omega_{u\varepsilon i} \\ \Omega'_{u\varepsilon i} & \Omega_{\varepsilon i} \end{bmatrix}$  the (full rank) long-run covariance matrices of  $v_{it}$ , with  $\omega_{u\varepsilon i}^2 = \omega_{ui}^2 - \Omega_{u\varepsilon i} \Omega_{\varepsilon i}^{-1} \Omega'_{u\varepsilon i}$  the conditional long-run variances and with  $\Lambda_i = \sum_{j=0}^{\infty} \mathbb{E} v_{it} v'_{it-j}$  the one-sided long-run variances, which are partitioned according to the partitioning of  $\Omega_i$ .

The estimates  $\hat{\omega}_{u,\varepsilon i}^2$  are given by the estimates of the long-run variances of the residuals,  $\hat{\eta}_{it}$  say, of OLS regressions of  $\Delta y_{it}$  on the differenced deterministic components and  $\Delta X_{it}$ . The

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<sup>1</sup>The fact that equations (1) are spurious regressions in case of no cointegration implies, as is well known, that e.g. OLS estimation of  $\beta$  in (1) will generally lead to a non-zero limit even if the true  $\beta = 0$  (in our notational convention).

estimates  $\hat{\omega}_{u,\varepsilon i}^2$  can be obtained by using a kernel estimator, see Andrews (1991) or Newey and West (1987) or alternatively by fitting autoregressive moving average or autoregressive models to  $\hat{\eta}_{it}$  (and computing the long-run variances model based).

The correction for serial correlation can be handled either non-parametrically (following Phillips and Perron, 1988) or by using ADF type regressions. Let us start with the non-parametric tests. Denote the residuals of the OLS regressions  $\hat{u}_{it} = \rho_i \hat{u}_{it-1} + \mu_{it}$  by  $\hat{\mu}_{it}$ . Further, denote their estimated variances by  $\hat{\sigma}_{\mu i}^2$  and their estimated long-run variances by  $\hat{\omega}_{\mu i}^2$ . Then, the serial correlation correction factors are given by  $\hat{\lambda}_i = \frac{1}{2}(\hat{\omega}_{\mu i}^2 - \hat{\sigma}_{\mu i}^2)$ . For later use we also define  $\hat{\omega}_{NT}^2 = \frac{1}{N} \sum_{i=1}^N \hat{\omega}_{\mu i}^2 / \hat{\omega}_{u,\varepsilon i}^2$ .

With the defined quantities the following pooled test statistics can be computed: the variance ratio statistic  $PP_\sigma$ , the test based on the autoregressive coefficient  $PP_\rho$ , and the test based on the  $t$ -value of the autoregressive coefficient  $PP_t$ .<sup>2</sup> The *essential* parts (i.e. without centering and scaling factors, see below) of the pooled test statistics are given by

$$PP_\sigma^o = \left( N^{-3/2} \sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-2} \sum_{t=2}^T \hat{u}_{it-1}^2 \right) \right)^{-1} \quad (3)$$

$$PP_\rho^o = N^{-1/2} \frac{\sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-1} \sum_{t=2}^T \hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i \right)}{N^{-1} \sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-2} \sum_{t=2}^T \hat{u}_{it-1}^2 \right)} \quad (4)$$

$$PP_t^o = N^{-1/2} \frac{\sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-1} \sum_{t=2}^T \hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i \right)}{\hat{\omega}_{NT} \left( N^{-1} \sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-2} \sum_{t=2}^T \hat{u}_{it-1}^2 \right) \right)^{1/2}} \quad (5)$$

The ADF-type test  $PP_{df}$  is based on autoregressions to correct for serial correlation, where two auxiliary regressions are performed

$$\Delta \hat{u}_{it} = \sum_{k=1}^{K_i} \gamma_{1ik} \Delta \hat{u}_{it-k} + \zeta_{1it} \quad (6)$$

$$\hat{u}_{it-1} = \sum_{k=1}^{K_i} \gamma_{2ik} \Delta \hat{u}_{it-k} + \zeta_{2it} \quad (7)$$

with the lag lengths  $K_i$  chosen in practice by e.g. minimizing the Akaike (1969) information criterion (AIC) in  $\Delta \hat{u}_{it} = \rho_i \hat{u}_{it-1} + \sum_{k=1}^{K_i} \tilde{\gamma}_{ik} \Delta \hat{u}_{it-k} + \tilde{\zeta}_{it}$ . Denote the OLS residuals of the above regressions (6) and (7) by  $\hat{\zeta}_{1it}$  and  $\hat{\zeta}_{2it}$ , the residuals from the regressions  $\hat{\zeta}_{1it} = \rho_i \hat{\zeta}_{2it} + \theta_{it}$  by  $\hat{\theta}_{it}$  and their estimated variances (needed later) by  $\hat{\sigma}_{\theta i}^2$ . Furthermore define  $\hat{\sigma}_{NT}^2 =$

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<sup>2</sup> $PP$  is used as acronym for Pedroni pooled test. Below we use  $PG$  as acronym for Pedroni group-mean test.

$\frac{1}{NT} \sum_{i=1}^N \sum_{t=K_i+2}^T \hat{\theta}_{it}^2$ . The essential part of the pooled ADF-type statistic is then given by

$$PP_{df}^o = N^{-1/2} \frac{\sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-1} \sum_{t=K_i+2}^T \hat{\zeta}_{1it} \hat{\zeta}_{2it} \right)}{\hat{\sigma}_{NT} \left( N^{-1} \sum_{i=1}^N \hat{\omega}_{u,\varepsilon i}^{-2} \left( T^{-2} \sum_{t=K_i+2}^T \hat{\zeta}_{2it}^2 \right) \right)^{1/2}} \quad (8)$$

Asymptotic normality using sequential limit theory, with  $T \rightarrow \infty$  followed by  $N \rightarrow \infty$  can be established for the above test statistics by applying the so called Delta method (this requires knowledge of the asymptotic means and variances of the building blocks, which are obtained for practical purposes by simulation). The mean and variance correction factors,  $\mu_{PP}(r, s, l)$  and  $\sigma_{PP}^2(r, s, l)$  depend upon the test considered ( $r \in \{\sigma, \rho, t, df\}$ ),<sup>3</sup> the deterministic variables ( $s \in \{1, 2, 3\}$ ) and upon the number of regressors  $l$ , i.e.:

$$PP_r = \frac{PP_r^o - N^{1/2} \mu_{PP}(r, s, l)}{(\sigma_{PP}^2(r, s, l))^{1/2}} \Rightarrow N(0, 1)$$

Pedroni develops three group-mean tests against the heterogeneous alternative. These are: a test based on the first order autoregressive coefficient  $PG_\rho$ , a test based on its  $t$ -value  $PG_t$  and again an ADF-type test  $PG_{df}$ . The essential parts of the test statistics are given by

$$\begin{aligned} PG_\rho^o &= N^{-1/2} \sum_{i=1}^N PG_{\rho,i}^o = N^{-1/2} \sum_{i=1}^N \frac{T^{-1} \sum_{t=2}^T (\hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i)}{T^{-2} \sum_{t=2}^T \hat{u}_{it-1}^2} \\ PG_t^o &= N^{-1/2} \sum_{i=1}^N PG_{t,i}^o = N^{-1/2} \sum_{i=1}^N \frac{T^{-1} \sum_{t=2}^T (\hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i)}{\hat{\omega}_{\mu_i} (T^{-2} \sum_{t=2}^T \hat{u}_{it-1}^2)^{1/2}} \\ PG_{df}^o &= N^{-1/2} \sum_{i=1}^N PG_{df,i}^o = N^{-1/2} \sum_{i=1}^N \frac{T^{-1} \sum_{t=K_i+2}^T \hat{\zeta}_{1it} \hat{\zeta}_{2it}}{\hat{\sigma}_{\theta_i} (T^{-2} \sum_{t=K_i+2}^T \hat{\zeta}_{2it}^2)^{1/2}} \end{aligned} \quad (9)$$

Appropriately centered and scaled group-mean test statistics converge to the standard normal distribution in the sequential limit by applying a central limit theorem to the i.i.d. (across  $N$ ) sequences, i.e.:

$$PG_r = \frac{PG_r^o - N^{1/2} \mu_{PG}(r, s, l)}{(\sigma_{PG}^2(r, s, l))^{1/2}} = N^{-1/2} \sum_{i=1}^N \frac{PG_{r,i}^o - \mu_{PG}(r, s, l)}{(\sigma_{PG}^2(r, s, l))^{1/2}} \Rightarrow N(0, 1) \quad (10)$$

with  $r \in \{\rho, t, df\}$ ,<sup>4</sup>  $s \in \{1, 2, 3\}$  and  $l$  the number of regressors.

The five sets of different correction factors needed for the seven tests and three deterministic specifications are displayed in the appendix in Tables 5 to 19. The results from the response surface regressions are displayed in Table 1 for the pooled tests and in Table 2 for the group-mean tests in Section 4.

<sup>3</sup>The correction factors coincide for the  $PP_t$  test and the  $PP_{df}$  test.

<sup>4</sup>The correction factors coincide for the  $PG_t$  test and the  $PG_{df}$  test.

## Westerlund tests

Westerlund (2005) develops two simple non-parametric tests that extend the Breitung (2002) approach from the time series to the panel case. He proposes both a pooled test, referred to as  $WP$ , and a group-mean test, referred to as  $WG$ . As for Pedroni's tests, the residuals  $\hat{u}_{it}$  from estimating (1) by OLS are the starting point. Define  $\hat{r}_i = \sum_{t=1}^T \hat{u}_{it}^2$ ,  $\bar{r} = N^{-1} \sum_{i=1}^N \hat{r}_i$  and  $\hat{e}_{it} = \sum_{j=1}^t \hat{u}_{ij}$ . Using these quantities the essential parts of the test statistics are given by

$$WP^o = N^{-1/2} \left( \sum_{i=1}^N \frac{T^2}{\bar{r}} \left( T^{-4} \sum_{t=1}^T \hat{e}_{it}^2 \right) \right) \quad (11)$$

$$WG^o = N^{-1/2} \sum_{i=1}^N \frac{T^2}{\hat{r}_i} \left( T^{-4} \sum_{t=1}^T \hat{e}_{it}^2 \right) \quad (12)$$

Applying the Delta method to  $WP^o$  and a central limit theorem to  $WG^o$  (seen by writing  $WG^o = N^{-1/2} \sum_{i=1}^N WG_i^o$ ) leads to asymptotic standard normality under the null hypothesis in the sequential limit when using appropriate mean and variance correction factors, i.e.:

$$WP = \frac{WP^o - N^{1/2} \mu_{WP}(s, l)}{(\sigma_{WP}^2(s, l))^{1/2}} \Rightarrow N(0, 1) \quad (13)$$

$$WG = \frac{WG^o - N^{1/2} \mu_{WG}(s, l)}{(\sigma_{WG}^2(s, l))^{1/2}} \Rightarrow N(0, 1) \quad (14)$$

with  $s \in \{1, 2, 3\}$  denoting the specification of deterministic variables and  $l$  the number of regressors. The correction factors are displayed in the appendix in Tables 20 to 25 and the results from the response surface regressions are displayed in Table 3 in Section 4.

## 3 The system tests

The second strand of the panel cointegration literature is based on panel extensions of VAR cointegration analysis (see Johansen, 1995). Without imposing any homogeneity assumption the panel VAR DGP is given in *error correction form* by

$$\Delta Y_{it} = C_{1i} + C_{2i}t + \alpha_i \beta_i' Y_{it-1} + \sum_{j=1}^{p_i} \Gamma_{ij} \Delta Y_{it-j} + w_{it} \quad (15)$$

with  $Y_{it} \in \mathbb{R}^m$ ,  $C_{1i}, C_{2i} \in \mathbb{R}^m$ ,  $\alpha_i, \beta_i \in \mathbb{R}^{m \times k_i}$  with full rank,  $\Gamma_{ij} \in \mathbb{R}^{m \times m}$  and  $w_{it}$  cross-sectionally independent  $m$ -dimensional white noise processes with covariance matrices  $\Sigma_i > 0$ . To ensure that the processes described by (15) are (up to the deterministic components)

I(1) processes, the matrices  $\alpha'_{i\perp}\Gamma_i\beta_{i\perp}$  have to be invertible, where  $\alpha_{i\perp} \in \mathbb{R}^{m \times (m-k_i)}$ ,  $\beta_{i\perp} \in \mathbb{R}^{m \times (m-k_i)}$  are full rank matrices such that  $\alpha'_{i\perp}\alpha_i = 0$  and  $\beta'_{i\perp}\beta_i = 0$  and  $\Gamma_i = I_m - \sum_{j=1}^{p_i} \Gamma_{ij}$ . In this case the space spanned by the columns of the matrix  $\beta_i$  is the  $k_i$ -dimensional cointegrating space of unit  $i$ .<sup>5</sup>

In the VAR cointegration literature the following five specifications concerning the deterministic components are usually discussed. Case 1 is without any deterministic components. In case 2 restricted intercepts of the form  $C_{1i} = \alpha_i\tau_i$  are contained (in the cointegrating space) and case 3 includes unrestricted intercepts  $C_{1i}$  that induce linear time trends in  $Y_{it}$ . In case 4 unrestricted intercepts and restricted trend coefficients  $C_{2i} = \alpha_i\kappa_i$  are included. This allows for linear trends in both the data and the cointegrating relationships. Finally in case 5 unrestricted intercepts and trend coefficients are included.

The statistical analysis, i.e. parameter estimation (via reduced rank regression) as well as testing for the cointegrating rank (using the so called trace or max tests), is well-developed and known for all five listed cases (see Johansen, 1995). Therefore we do not repeat a discussion of the procedure here.

### Larsson, Lyhagen, and Löthgren test

Larsson *et al.* (2001) consider testing for cointegration in the above framework under the assumption that  $\Pi_i = \alpha_i\beta'_i = \alpha\beta' = \Pi$  for  $i = 1, \dots, N$ . The null hypothesis of their test is  $H_0: rk(\Pi_i) = k$  for  $i = 1, \dots, N$ . The test is consistent against the alternative hypothesis  $H_1: rk(\Pi_i) > k$  for a non-vanishing (as  $N \rightarrow \infty$ ) fraction of cross-section members. The construction of their test statistic is similar to Im *et al.* (2003) and hence the test statistic is given by a suitably centered and scaled version of the cross-sectional average of the individual trace statistics. Denote with  $LR_i^s(k, m)$  the trace statistic for the null hypothesis of a  $k$ -dimensional cointegrating space for unit  $i$ , where the superscript  $s$  indicates the specification of the deterministic components. Using a central limit theorem in the cross-sectional dimension and the appropriate mean and variance correction factors implies that under the null hypothesis

$$LLL^s(k, m) = N^{-1/2} \sum_{i=1}^N \frac{LR_i^s(k, m) - \mu_{LR}^s(k, m)}{(\sigma_{LR}^{2,s}(k, m))^{1/2}} \Rightarrow N(0, 1) \quad (16)$$

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<sup>5</sup>The integer  $k_i$  is often referred to as cointegrating rank. Please note that  $\beta_i$  as used in this sub-section does not coincide with  $\beta_i$  used in the description of the single equation methods, where – in case of cointegration – the single cointegrating vector of unit  $i$  is given by  $[1, -\beta'_i]'$ .

in the sequential limit with  $T \rightarrow \infty$  followed by  $N \rightarrow \infty$ . The (asymptotic) mean and variance correction factors are given by the mean and variance of the limiting distribution of the *trace statistic* of Johansen (1995). As is well known the trace statistic depends upon  $m - k$ , and the correction factors are displayed in this way in the appendix in Tables 26 to 30 for the five different specifications of the deterministic variables. The results from the corresponding response surface regressions are displayed in Table 4 in Section 4.

### Breitung test

Breitung (2005) proposes a 2-step estimation (and related test) procedure in a panel VAR where only the cointegrating spaces are assumed to be identical for all cross-section members.<sup>6</sup> In the first step of his procedure the parameters are estimated individual specifically and in the second step the common cointegrating space  $\beta$  is estimated in a pooled fashion.<sup>7</sup>

For simplicity we describe the method here for the VAR(1) model without deterministic components. In the general case lagged differences as well as (restricted) deterministic components are treated in the usual way and are concentrated out in the first step, as described in Johansen (1995). Thus, consider

$$\Delta Y_{it} = \alpha_i \beta' Y_{it-1} + w_{it} \quad (17)$$

Pre-multiplying equations (17) by  $(\alpha_i' \Sigma_i^{-1} \alpha_i)^{-1} \alpha_i' \Sigma_i^{-1}$  leads to

$$(\alpha_i' \Sigma_i^{-1} \alpha_i)^{-1} \alpha_i' \Sigma_i^{-1} \Delta Y_{it} = \beta' Y_{it-1} + (\alpha_i' \Sigma_i^{-1} \alpha_i)^{-1} \alpha_i' \Sigma_i^{-1} w_{it} \quad (18)$$

$$\Delta Y_{it}^+ = \beta' Y_{it-1} + w_{it}^+ \quad (19)$$

where the last equation defines the variables with superscript  $+$ . Note also that  $\mathbb{E} w_{it}^+ (w_{it}^+)' = (\alpha_i' \Sigma_i^{-1} \alpha_i)^{-1}$ . Now, under the assumption of its feasibility, use the normalization  $\beta = [I_k, \beta_2]'$  and partition  $Y_{it} = [(Y_{it}^1)', (Y_{it}^2)']'$  with  $Y_{it}^1 \in \mathbb{R}^k$  and  $Y_{it}^2 \in \mathbb{R}^{m-k}$ . Using this notation we can rewrite the above equation (19) as

$$\Delta Y_{it}^+ - Y_{it-1}^1 = \beta_2' Y_{it-1}^2 + w_{it}^+ \quad (20)$$

Breitung suggests to obtain the estimate of  $\beta_2$  by estimating (20) with pooled OLS using the estimate  $(\hat{\alpha}_i' \hat{\Sigma}_i^{-1} \hat{\alpha}_i)^{-1} \hat{\alpha}_i' \hat{\Sigma}_i^{-1}$  based on the Johansen estimates. Note that, given the known

<sup>6</sup>This procedure extends the Ahn and Reinsel (1990) and Engle and Yoo (1991) approach from the time series to the panel case. Since this procedure is less well-known we include a short description here.

<sup>7</sup>Note that in the first step individual specific estimates of all parameters are obtained and used, including first step estimates of  $\beta_i$ .



covariance structure of the errors in (20) (and an estimate being available), pooled feasible generalized least squares (GLS) estimation of (20) is also an option.

The test Breitung proposes for the null hypothesis of  $rk(\beta) = k$  is based on Saikkonen (1999). The difference from the Larsson *et al.* (2001) test is that Breitung's test incorporates the homogeneity restriction  $\beta_i = \beta$  for  $i = 1, \dots, N$  in the construction of the test statistic. The discussion is again for the VAR(1) case without deterministic components. Denote with  $\gamma_i \in \mathbb{R}^{m \times (m-k)}$  matrices with full column rank and consider

$$\Delta Y_{it} = \alpha_i \beta' Y_{it-1} + \gamma_i \beta'_{\perp} Y_{it-1} + w_{it} \quad (21)$$

Under the null hypothesis of a  $k$ -dimensional cointegrating space,  $\gamma_i = 0$  for  $i = 1, \dots, N$  and under the alternative (of an  $m$ -dimensional cointegrating space)  $\gamma_i$  is unrestricted (in a non-vanishing fraction of panel members to imply consistency of the test against this alternative) to allow for  $\Pi_i = \alpha_i \beta' + \gamma_i \beta'_{\perp}$  of full rank. Pre-multiply (21) with  $\alpha'_{i\perp}$  to obtain

$$\alpha'_{i\perp} \Delta Y_{it} = \alpha'_{i\perp} \gamma_i \beta'_{\perp} Y_{it-1} + \alpha'_{i\perp} w_{it} \quad (22)$$

$$\alpha'_{i\perp} \Delta Y_{it} = \phi_i (\beta'_{\perp} Y_{it-1}) + \tilde{w}_{it} \quad (23)$$

where the last equation defines the coefficients and variables. Replacing  $\alpha_{i\perp}$  and  $\beta_{\perp}$  by the estimates discussed above allows the estimation of equations (23) separately by OLS and the construction of test statistics for the hypotheses  $H_0: \phi_i = 0$  for  $i = 1, \dots, N$ . Any of the Lagrange multiplier, likelihood ratio or Wald test statistics can be used. We display here the Lagrange multiplier test statistic, which has the advantage that it only requires estimation under the null hypothesis. Denote with  $\hat{f}'_{it} = \hat{\alpha}'_{i\perp} \Delta Y_{it}$  and with  $\hat{g}'_{it} = \hat{\beta}'_{\perp} Y_{it}$ , then the Lagrange multiplier test statistic for unit  $i$  is given by

$$LM_i(k, m) = T \text{tr} \left[ \sum_{t=2}^T \hat{f}'_{it} \hat{g}'_{it-1} \left( \sum_{t=2}^T \hat{g}'_{it-1} \hat{g}'_{it-1} \right)^{-1} \sum_{t=2}^T \hat{g}'_{it-1} \hat{f}'_{it} \left( \sum_{t=2}^T \hat{f}'_{it} \hat{f}'_{it} \right)^{-1} \right] \quad (24)$$

which is sequentially computed for the different values of  $k = 0, \dots, m$ . The panel test statistic is then, as usual, given by the corresponding centered and scaled cross-sectional average (putting again the superscript  $s$  to indicate the dependence upon the deterministic components). Thus, under the null hypothesis

$$B^s(k, m) = N^{-1/2} \sum_{i=1}^N \frac{LM_i^s(k, m) - \mu_{LM}^s(k, m)}{(\sigma_{LM}^{2,s}(k, m))^{1/2}} \Rightarrow N(0, 1) \quad (25)$$

The correction factors  $\mu_{LM}^s$  and  $\sigma_{LM}^{2,s}$  coincide with the Larsson *et al.* (2001) correction factors, i.e. they are also given by the first two moments of the trace statistic and hence are as for the Larsson *et al.* (2001) tests given in Tables 26 to 30 in the appendix. Consequently, the results from the response surface regressions are those displayed in Table 4 in Section 4.

## 4 Response surface regressions

In this section we consider response surface regressions, as popularized in the unit root and cointegration context primarily by the work of James MacKinnon (see e.g. MacKinnon, 1996). In the time series unit root and cointegration literature response surfaces are performed for the critical values of the non-standard limiting distributions. In the panel unit root and cointegration literature often the appropriately centered and scaled limiting distributions for both  $N, T \rightarrow \infty$  are standard normal. Thus, in the nonstationary panel context it is the centering and scaling factors as functions of the time series dimension and the number of regressors (for the single equation tests) or the number of common trends (for the system tests) for which response surface regressions are performed.

The functional form of the response surface regressions we use is inspired by the response surface regressions of Banerjee and Carrion-i-Silvestre (2006). Their suggested specification also turns out to perform well for the tests considered in this paper, based on a variety of experiments. Consider any of the tests and cases for the deterministic components discussed. Then denote with  $\kappa \in \{\mu, \sigma^2\}$  either the mean or variance correction factor, which is modelled as a function of the time series length  $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$  and of  $1 \leq d \leq 12$ . As indicated above, for the single equation tests  $d = l$ , due to their dependence upon the number of regressors. For the system tests  $d = m - k$ , because for the trace test the limiting distribution depends upon the number of common trends, which is given by the number of variables ( $m$ ) minus the number of cointegrating relationships ( $k$ ). Note that for  $T = 10$  we include only values of  $d$  up to 4. This implies that the total number of observations for the response surface regressions is 136. Clearly, one could argue that also for sample sizes like  $T = 20$  or 30 including up to 12 variables is probably too large a number. However, as long as  $T \geq d$  all the regressions necessary to perform the tests can be performed and also the finite sample correction factors can be simulated.

The baseline specification of the response surface regressions is given by

$$\kappa(T, d) = \sum_{j=0}^2 (c_{0j} + c_{1j}T^{-1} + c_{2j}T^{-2} + c_{3j}T^{-3}) d^j + e(T, d) \quad (26)$$

where  $e(T, d)$  denotes the residuals. The results in the following tables are based on OLS estimation of the above equation with model reduction performed by applying stepwise least squares. In the stepwise search procedure the coefficients  $c_{00}$ ,  $c_{01}$  and  $c_{02}$ , corresponding to the asymptotic values with respect to the time series dimension, have been excluded from the set of variables for which stepwise least squares search has been performed. Given that the estimated residuals  $\hat{e}(T, d)$  show some evidence of serial correlation robust standard errors as proposed by Newey and West (1987) are used in the specification analysis. In Tables 1 to 4 we only report the coefficients included in the final equation obtained, i.e. empty entries correspond to excluded variables.

TABLE 1

*Results of response surface regressions for Pedroni's pooled tests*

|             | Case 1    |            | Case 2    |            | Case 3     |            |
|-------------|-----------|------------|-----------|------------|------------|------------|
|             | $\mu$     | $\sigma^2$ | $\mu$     | $\sigma^2$ | $\mu$      | $\sigma^2$ |
| $PP_\sigma$ |           |            |           |            |            |            |
| $c_{00}$    | -0.9335   | 42.9951    | 5.3771    | 76.3677    | 15.0585    | 236.79     |
| $c_{10}$    | 84.5553   | -12,877.37 | -101.27   | -13,822.22 | -136.04    | -33,204.3  |
| $c_{20}$    |           | 601,633.1  | 5,636.67  | 650,212.1  | 8,103.47   | 1,461,661  |
| $c_{30}$    | 1,076.93  | -4,034,887 | -38,117   | -4,307,032 | -53,972.12 | -9,839,589 |
| $c_{01}$    | 3.7642    | 19.7761    | 3.1358    | 10.9848    | 2.9211     | -45.8134   |
| $c_{11}$    |           | 7,467.13   | 70.8503   | 8,097.15   | 100.65     | 18,508.21  |
| $c_{21}$    | -1,076.83 | -326,056.6 | -3,285.56 | -355,675.2 | -4,590.12  | -776,026.7 |
| $c_{31}$    | 3,062.65  | 1,757,638  | 18,850.81 | 1,890,962  | 25,131.11  | 4,343,904  |
| $c_{02}$    | 0.0156    | 1.4981     | 0.0577    | 2.2478     | 0.0677     | 6.7963     |
| $c_{12}$    |           | -590.94    | -5.6561   | -642.44    | -7.7481    | -1,435.01  |
| $c_{22}$    | 134.52    | 24,735.32  | 313.87    | 26,923.16  | 397.56     | 55,050.31  |
| $c_{32}$    | 605.48    |            | -696.96   |            |            |            |
| $PP_\rho$   |           |            |           |            |            |            |
| $c_{00}$    | 1.2451    | 12.9593    | -2.0144   | 11.2377    | -6.7056    | 22.8178    |
| $c_{10}$    |           | 560.19     | -8.6362   | 411.06     | -7.1086    | 174.40     |
| $c_{20}$    | -659.26   |            | -454.21   |            | -316.31    |            |
| $c_{30}$    | 5,831.34  | -69,511.31 | 4,567.16  | -57,078.76 | 3,815.7    | -50,190.74 |
| $c_{01}$    | -3.8462   | 26.2931    | -3.7869   | 24.4422    | -3.654     | 22.2309    |
| $c_{11}$    | -7.4332   | -520.91    | 5.3581    | -436.32    | 18.1073    | -519.69    |
| $c_{21}$    | 568.4     |            | 439.74    | -2,269     | 342.74     |            |
| $c_{31}$    | -4,578.47 | 53,562.4   | -3,919.49 | 65,449.98  | -3,561.42  | 54,423.99  |
| $c_{02}$    | -0.0085   | -0.8045    | -0.0115   | -0.5659    | -0.0178    | -0.3875    |
| $c_{12}$    | 5.7543    | -33.79     | 5.4551    | -46.3982   | 5.2893     | -45.2887   |
| $c_{22}$    | -101.3    | 1,539.47   | -103.41   | 1,884.3    | -109.52    | 1,903.95   |
| $c_{32}$    | 682.52    | -16,061.26 | 708.33    | -18,102.03 | 749.89     | -18,992.54 |
| $PP_t$      |           |            |           |            |            |            |
| $c_{00}$    | -0.584    | 0.7882     | -1.3748   | 1.0342     | -1.9644    | 0.6830     |
| $c_{10}$    | -4.1598   |            | -0.633    | -4.1945    |            | -4.3101    |
| $c_{20}$    |           |            |           | -35.4021   | 12.1282    | 4.5137     |
| $c_{30}$    | 490.96    |            |           | 340.29     | 130.21     |            |
| $c_{01}$    | -0.5492   | 0.2181     | -0.4276   | -0.0516    | -0.3566    | 0.0050     |
| $c_{11}$    | 3.2827    |            | 4.0863    | -2.7262    | 5.6216     | -2.3971    |
| $c_{21}$    | 37.071    |            |           | 51.1496    |            | 37.6843    |
| $c_{31}$    | -568.12   |            |           | -226.69    | -134.83    | -55.7596   |
| $c_{02}$    | 0.0172    | -0.0202    | 0.0111    | 0.0012     | 0.0077     | -0.0012    |
| $c_{12}$    | 0.3716    |            | 0.2722    | 0.1301     | 0.2374     | 0.0951     |
| $c_{22}$    | -7.9692   |            | -2.1943   | -1.3963    | -4.4724    |            |
| $c_{32}$    | 86.5335   |            |           |            | 35.7399    | -20.0680   |

TABLE 2

*Results of response surface regressions for Pedroni's group-mean tests*

|           | Case 1    |            | Case 2    |            | Case 3    |            |
|-----------|-----------|------------|-----------|------------|-----------|------------|
|           | $\mu$     | $\sigma^2$ | $\mu$     | $\sigma^2$ | $\mu$     | $\sigma^2$ |
| $PG_\rho$ |           |            |           |            |           |            |
| $c_{00}$  | -1.857    | 10.0917    | -5.0569   | 20.7052    | -9.7333   | 34.5197    |
| $c_{10}$  |           | 273.18     |           |            | 14.8251   | -164.55    |
| $c_{20}$  | -631.9    |            | -417.09   | 2,991.73   | -369.35   |            |
| $c_{30}$  | 6,305.02  | -41,440.06 | 4,731.35  | -59,919.38 | 4,206.14  | -33,598.25 |
| $c_{01}$  | -4.0361   | 16.5257    | -3.9849   | 16.1141    | -3.8462   | 16.6875    |
| $c_{11}$  | 4.0981    | -320.57    | 16.5067   | -375.36    | 29.0607   | -560.98    |
| $c_{21}$  | 531.03    | -1,999.36  | 377.59    | -1,221.65  | 246.92    | 3,051.01   |
| $c_{31}$  | -5,011.37 | 53,749.31  | -4,211.11 | 55,487.74  | -3,608.18 | 34,978.93  |
| $c_{02}$  | 0.0023    | -0.0813    | -0.0004   | -0.062     | -0.0071   | -0.1171    |
| $c_{12}$  | 5.7298    | -48.7525   | 5.4381    | -48.9814   | 5.2212    | -40.9663   |
| $c_{22}$  | -115.05   | 1,749.99   | -116.04   | 1,867.89   | -119.18   | 1,776.41   |
| $c_{32}$  | 837.71    | -16,456.08 | 860.13    | -18,369.3  | 881.89    | -19,043.07 |
| $PG_t$    |           |            |           |            |           |            |
| $c_{00}$  | -1.0445   | 0.8162     | -1.6823   | 0.6907     | -2.2175   | 0.5782     |
| $c_{10}$  |           | -5.4311    |           | -3.828     | 2.7886    | -4.6666    |
| $c_{20}$  |           | 68.54      |           |            |           | 17.4594    |
| $c_{30}$  |           | -337       | 219.01    | 101.55     | 182.24    |            |
| $c_{01}$  | -0.4921   | -0.0698    | -0.4038   | -0.0336    | -0.3418   | -0.0076    |
| $c_{11}$  | 3.6238    | -0.5843    | 4.4786    | -1.5976    | 5.7424    | -1.6642    |
| $c_{21}$  |           |            | 12.7381   | 32.565     |           | 35.4064    |
| $c_{31}$  | -33.1355  |            | -270.68   | -150.81    | -168.44   | -143.70    |
| $c_{02}$  | 0.0148    | 0.0039     | 0.0101    | 0.0016     | 0.0073    | 0.0001     |
| $c_{12}$  | 0.2846    | -0.0261    | 0.3093    | 0.0457     | 0.2501    | 0.0522     |
| $c_{22}$  | -1.8769   | 1.7478     | -5.9623   |            | -5.1892   |            |
| $c_{32}$  |           | -11.9386   | 51.3486   | -11.9608   | 41.4558   | -17.6730   |

TABLE 3

*Results of response surface regressions for Westerlund's pooled and group-mean tests*

|           | Case 1  |            | Case 2  |            | Case 3  |            |
|-----------|---------|------------|---------|------------|---------|------------|
|           | $\mu$   | $\sigma^2$ | $\mu$   | $\sigma^2$ | $\mu$   | $\sigma^2$ |
| <i>WP</i> |         |            |         |            |         |            |
| $c_{00}$  | 0.1444  | 0.0311     | 0.049   | 0.0021     | 0.0118  | 0.00006    |
| $c_{10}$  |         |            |         |            |         |            |
| $c_{20}$  |         |            | 0.9693  |            | 0.2957  |            |
| $c_{30}$  |         | 1.8842     |         | 0.1287     |         |            |
| $c_{01}$  | -0.0305 | -0.0069    | -0.0105 | -0.0005    | -0.002  | -0.00001   |
| $c_{11}$  |         |            |         |            |         |            |
| $c_{21}$  |         |            |         |            |         |            |
| $c_{31}$  |         |            | -3.7713 |            | -0.2667 | 0.0028     |
| $c_{02}$  | 0.0016  | 0.0004     | 0.0006  | 0.00002    | 0.00009 | 0.0000005  |
| $c_{12}$  |         |            |         |            |         |            |
| $c_{22}$  |         |            |         |            |         |            |
| $c_{32}$  |         |            | 0.2362  |            | 0.0183  | -0.0002    |
| <i>WG</i> |         |            |         |            |         |            |
| $c_{00}$  | 0.1027  | 0.0068     | 0.0333  | 0.0005     | 0.0095  | 0.00002    |
| $c_{10}$  |         |            |         |            |         |            |
| $c_{20}$  |         |            | 0.8192  |            | 0.3095  | 0.0003     |
| $c_{30}$  |         | 1.9565     |         | 0.1746     |         |            |
| $c_{01}$  | -0.0217 | -0.0016    | -0.0071 | -0.0001    | -0.0016 | -0.000005  |
| $c_{11}$  |         |            |         |            |         |            |
| $c_{21}$  |         |            |         |            |         |            |
| $c_{31}$  |         | -1.0981    | -2.9565 | -0.0956    | -0.2774 | -0.0011    |
| $c_{02}$  | 0.0012  | 0.00009    | 0.0004  | 0.000008   | 0.00007 | 0.0000003  |
| $c_{12}$  |         |            |         |            |         |            |
| $c_{22}$  |         |            |         |            |         |            |
| $c_{32}$  |         | 0.0895     | 0.1855  | 0.0078     | 0.0166  | 0.00005    |

TABLE 4

*Results of response surface regressions for the system tests*

|     | Case 1    |            | Case 2     |            | Case 3     |            | Case 4     |            | Case 5    |            |
|-----|-----------|------------|------------|------------|------------|------------|------------|------------|-----------|------------|
|     | $\mu$     | $\sigma^2$ | $\mu$      | $\sigma^2$ | $\mu$      | $\sigma^2$ | $\mu$      | $\sigma^2$ | $\mu$     | $\sigma^2$ |
| c00 | -1.0124   | -2.3879    | -0.0243    | -1.5651    | -2.184     | -4.8058    | 0.1337     | 0.4188     | -5.4796   | -5.9341    |
| c10 | -101.74   |            | -242.37    |            | -235.46    | 66.6212    | -234.05    |            | -51.3523  |            |
| c20 |           |            | 4,350.82   |            | 4,401.9    |            | 4,597.55   |            |           | 2,331.04   |
| c30 |           | -19,287.2  | -28,646.51 | -19,399.69 | -29,050.07 | -23,324.16 | -30,816.74 | -20,528.59 |           | -39,432.01 |
| c01 | -0.6331   | 0.6479     | 2.0627     | 4.5342     | 1.2779     | 3.9522     | 4.2288     | 7.2123     | 3.8053    | 6.3635     |
| c11 | 149.41    | 90.1135    | 185.51     | 70.4094    | 189.33     |            | 165.39     | -51.6168   | 121.36    |            |
| c21 | -1,655.23 | -2,463.05  | -3,121.19  | -2,142.29  | -3,199.05  | -1,139.62  | -3,321.95  |            | -1,937.43 | -2,067.07  |
| c31 | 9,940.27  | 28,674.14  | 19,432.14  | 26,410.64  | 19,887.52  | 22,045.71  | 21,530.66  | 15,799.62  | 11,998.95 | 30,393.17  |
| c02 | 1.9716    | 2.8732     | 1.9902     | 2.8791     | 1.9793     | 2.7616     | 1.979      | 2.7938     | 1.9455    | 2.7438     |
| c12 | -33.8317  | -39.4413   | -40.9204   | -42.723    | -41.1      | -35.551    | -45.8386   | -38.405    | -42.6005  | -43.0956   |
| c22 | 346.81    | 759.47     | 498.34     | 799.33     | 502.55     | 681.47     | 592.83     | 723.59     | 497.25    | 881.95     |
| c32 | -1,840.87 | -5,946.75  | -2,694.02  | -6,108.67  | -2,720.62  | -5,524.73  | -3,258.84  | -5,996.83  | -2,616.74 | -7,067.01  |

## Appendix: The correction factors

Correction factors for Pedroni tests



TABLE 5

Finite  $T$  correction factors  $\mu_{PP}(\sigma)$  and  $\sigma_{PP}^2(\sigma)$  for the pooled variance ratio test statistic of Pedroni ( $PP_0$ ) for case 1

| T/1 | 1          | 2     | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10       | 11       | 12       |
|-----|------------|-------|--------|--------|--------|--------|--------|--------|--------|----------|----------|----------|
| 10  | $\mu$      | 5.50  | 9.79   | 15.51  | 23.11  | —      | —      | —      | —      | —        | —        | —        |
|     | $\sigma^2$ | 57.50 | 173.79 | 370.51 | 671.09 | —      | —      | —      | —      | —        | —        | —        |
| 20  | $\mu$      | 4.67  | 8.18   | 12.36  | 16.98  | 22.21  | 28.00  | 34.42  | 41.43  | 49.38    | 59.51    | 70.57    |
|     | $\sigma^2$ | 38.93 | 109.40 | 208.27 | 294.40 | 379.62 | 478.52 | 596.56 | 814.25 | 1,111.49 | 1,511.97 | 2,187.00 |
| 30  | $\mu$      | 4.46  | 7.65   | 11.66  | 15.96  | 20.53  | 25.32  | 30.25  | 35.48  | 41.00    | 46.83    | 53.31    |
|     | $\sigma^2$ | 35.39 | 98.26  | 174.68 | 234.59 | 310.07 | 353.27 | 417.97 | 494.12 | 584.15   | 694.90   | 837.92   |
| 40  | $\mu$      | 4.33  | 7.49   | 11.41  | 15.57  | 19.82  | 24.29  | 28.84  | 33.59  | 38.46    | 43.36    | 48.67    |
|     | $\sigma^2$ | 33.62 | 94.48  | 169.58 | 224.45 | 264.73 | 318.12 | 370.94 | 405.96 | 473.09   | 544.80   | 622.35   |
| 50  | $\mu$      | 4.27  | 7.38   | 11.12  | 15.18  | 19.51  | 23.81  | 28.06  | 32.72  | 37.09    | 41.93    | 46.74    |
|     | $\sigma^2$ | 32.29 | 91.96  | 164.05 | 211.10 | 252.66 | 296.17 | 330.69 | 382.52 | 424.96   | 475.31   | 537.22   |
| 60  | $\mu$      | 4.21  | 7.37   | 10.99  | 15.10  | 19.17  | 23.51  | 27.86  | 32.25  | 36.63    | 41.05    | 45.58    |
|     | $\sigma^2$ | 31.61 | 90.58  | 161.11 | 201.12 | 242.89 | 283.24 | 317.34 | 372.14 | 403.43   | 444.89   | 505.81   |
| 70  | $\mu$      | 4.18  | 7.28   | 10.93  | 14.97  | 19.03  | 23.32  | 27.50  | 31.88  | 36.15    | 40.41    | 44.84    |
|     | $\sigma^2$ | 31.01 | 86.33  | 156.23 | 202.15 | 259.85 | 285.54 | 313.21 | 354.89 | 389.48   | 435.05   | 473.46   |
| 80  | $\mu$      | 4.16  | 7.21   | 10.84  | 14.87  | 18.91  | 23.14  | 27.32  | 31.48  | 35.76    | 40.16    | 44.36    |
|     | $\sigma^2$ | 31.01 | 88.55  | 150.41 | 205.53 | 232.64 | 276.59 | 317.01 | 347.14 | 370.07   | 420.47   | 458.42   |
| 90  | $\mu$      | 4.14  | 7.15   | 10.78  | 14.77  | 18.86  | 23.00  | 27.14  | 31.27  | 35.59    | 39.85    | 44.14    |
|     | $\sigma^2$ | 30.80 | 85.25  | 147.18 | 193.24 | 245.29 | 267.29 | 309.83 | 340.14 | 371.72   | 413.29   | 448.31   |
| 100 | $\mu$      | 4.13  | 7.17   | 10.82  | 14.74  | 18.75  | 22.88  | 26.89  | 31.15  | 35.57    | 39.64    | 43.76    |
|     | $\sigma^2$ | 30.07 | 85.66  | 140.02 | 205.84 | 238.03 | 280.45 | 315.90 | 337.03 | 370.26   | 414.62   | 453.37   |
| 200 | $\mu$      | 4.08  | 7.08   | 10.66  | 14.51  | 18.47  | 22.44  | 26.48  | 30.68  | 34.72    | 38.80    | 42.78    |
|     | $\sigma^2$ | 30.55 | 81.29  | 142.77 | 194.81 | 228.19 | 261.91 | 296.38 | 325.44 | 352.26   | 388.52   | 425.01   |
| 500 | $\mu$      | 4.03  | 6.95   | 10.57  | 14.43  | 18.24  | 22.29  | 26.27  | 30.40  | 34.34    | 38.38    | 42.42    |
|     | $\sigma^2$ | 29.23 | 82.00  | 141.90 | 186.05 | 237.16 | 259.98 | 287.10 | 315.50 | 345.66   | 376.86   | 417.58   |

TABLE 6

Finite  $T$  correction factors  $\mu_{PP}(\sigma)$  and  $\sigma_{PP}^2(\sigma)$  for the pooled variance ratio test statistic of Pedroni ( $PP_o$ ) for case 2

| T/\lambda | 1          | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10       | 11       | 12       |
|-----------|------------|--------|--------|--------|--------|--------|--------|--------|--------|----------|----------|----------|
| 10        | $\mu$      | 10.63  | 14.88  | 20.57  | 28.06  | —      | —      | —      | —      | —        | —        | —        |
|           | $\sigma^2$ | 100.77 | 204.13 | 358.80 | 657.46 | —      | —      | —      | —      | —        | —        | —        |
| 20        | $\mu$      | 9.52   | 13.04  | 17.11  | 21.70  | 26.57  | 32.13  | 38.49  | 45.51  | 53.72    | 63.48    | 75.66    |
|           | $\sigma^2$ | 77.03  | 133.22 | 206.09 | 280.42 | 374.59 | 491.17 | 628.79 | 839.90 | 1,134.36 | 1,584.57 | 2,281.33 |
| 30        | $\mu$      | 9.21   | 12.62  | 16.39  | 20.52  | 24.86  | 29.45  | 34.27  | 39.38  | 44.83    | 50.91    | 57.10    |
|           | $\sigma^2$ | 70.56  | 121.62 | 175.33 | 244.54 | 302.47 | 371.91 | 434.73 | 511.87 | 599.60   | 737.16   | 888.72   |
| 40        | $\mu$      | 9.04   | 12.35  | 16.14  | 20.04  | 24.23  | 28.43  | 33.00  | 37.63  | 42.37    | 47.41    | 52.61    |
|           | $\sigma^2$ | 67.62  | 116.97 | 172.92 | 228.68 | 271.34 | 331.05 | 382.52 | 434.23 | 503.15   | 572.73   | 647.89   |
| 50        | $\mu$      | 9.01   | 12.23  | 15.87  | 19.73  | 23.84  | 28.02  | 32.34  | 36.62  | 41.04    | 45.97    | 50.46    |
|           | $\sigma^2$ | 66.21  | 114.32 | 165.88 | 213.82 | 266.85 | 306.81 | 355.64 | 403.38 | 451.49   | 502.72   | 565.62   |
| 60        | $\mu$      | 8.90   | 12.10  | 15.77  | 19.55  | 23.61  | 27.65  | 31.85  | 36.12  | 40.47    | 44.87    | 49.30    |
|           | $\sigma^2$ | 65.91  | 111.03 | 161.64 | 213.57 | 251.61 | 300.67 | 335.61 | 382.41 | 429.98   | 473.55   | 521.30   |
| 70        | $\mu$      | 8.90   | 12.10  | 15.69  | 19.51  | 23.48  | 27.39  | 31.63  | 35.77  | 39.98    | 44.31    | 48.59    |
|           | $\sigma^2$ | 65.64  | 110.72 | 157.91 | 206.07 | 250.96 | 291.85 | 336.09 | 380.19 | 418.56   | 453.19   | 502.43   |
| 80        | $\mu$      | 8.85   | 12.07  | 15.60  | 19.37  | 23.41  | 27.35  | 31.39  | 35.60  | 39.80    | 43.91    | 48.22    |
|           | $\sigma^2$ | 63.80  | 110.94 | 160.31 | 201.83 | 249.74 | 295.09 | 329.99 | 365.09 | 401.07   | 447.42   | 486.48   |
| 90        | $\mu$      | 8.87   | 12.06  | 15.58  | 19.38  | 23.27  | 27.22  | 31.28  | 35.54  | 39.58    | 43.65    | 48.02    |
|           | $\sigma^2$ | 64.61  | 108.90 | 157.15 | 205.65 | 244.57 | 286.92 | 320.40 | 363.09 | 398.62   | 444.11   | 471.84   |
| 100       | $\mu$      | 8.82   | 11.98  | 15.54  | 19.31  | 23.24  | 27.20  | 31.14  | 35.31  | 39.43    | 43.60    | 47.79    |
|           | $\sigma^2$ | 63.47  | 109.70 | 153.86 | 199.64 | 239.77 | 290.35 | 322.69 | 351.60 | 385.91   | 430.43   | 470.01   |
| 200       | $\mu$      | 8.71   | 11.86  | 15.41  | 19.09  | 22.93  | 26.74  | 30.82  | 34.75  | 38.77    | 42.76    | 46.96    |
|           | $\sigma^2$ | 61.83  | 106.66 | 151.21 | 195.47 | 238.30 | 272.43 | 304.76 | 344.96 | 382.14   | 414.50   | 444.06   |
| 500       | $\mu$      | 8.66   | 11.77  | 15.30  | 19.03  | 22.81  | 26.66  | 30.60  | 34.53  | 38.54    | 42.68    | 46.64    |
|           | $\sigma^2$ | 60.90  | 104.98 | 150.42 | 191.10 | 235.82 | 275.03 | 302.66 | 342.64 | 368.90   | 400.39   | 432.66   |

TABLE 7

Finite  $T$  correction factors  $\mu_{PP}(\sigma)$  and  $\sigma_{PP}^2(\sigma)$  for the pooled variance ratio test statistic of Pedroni ( $PP_o$ ) for case 3

| T/1 | 1          | 2      | 3      | 4      | 5        | 6      | 7      | 8      | 9        | 10       | 11       | 12       |
|-----|------------|--------|--------|--------|----------|--------|--------|--------|----------|----------|----------|----------|
| 10  | $\mu$      | 22.32  | 28.05  | 36.10  | 47.79    | —      | —      | —      | —        | —        | —        | —        |
|     | $\sigma^2$ | 251.95 | 427.16 | 791.75 | 1,600.04 | —      | —      | —      | —        | —        | —        | —        |
| 20  | $\mu$      | 19.75  | 23.63  | 27.92  | 32.96    | 38.31  | 44.55  | 51.62  | 60.51    | 70.33    | 82.87    | 98.76    |
|     | $\sigma^2$ | 160.67 | 224.18 | 300.09 | 396.77   | 522.58 | 704.03 | 929.39 | 1,267.69 | 1,773.52 | 2,542.41 | 3,887.98 |
| 30  | $\mu$      | 19.22  | 22.71  | 26.51  | 30.54    | 35.08  | 39.96  | 45.08  | 50.55    | 56.37    | 62.90    | 69.83    |
|     | $\sigma^2$ | 144.41 | 195.81 | 252.77 | 313.29   | 381.38 | 460.17 | 550.14 | 659.27   | 802.37   | 956.24   | 1,160.74 |
| 40  | $\mu$      | 18.80  | 22.21  | 26.00  | 29.95    | 33.88  | 38.28  | 42.93  | 47.52    | 52.58    | 57.70    | 63.01    |
|     | $\sigma^2$ | 137.96 | 178.94 | 233.97 | 282.55   | 339.69 | 401.52 | 462.96 | 534.01   | 607.66   | 689.08   | 803.83   |
| 50  | $\mu$      | 18.64  | 22.02  | 25.59  | 29.48    | 33.36  | 37.52  | 41.80  | 46.29    | 50.84    | 55.41    | 60.28    |
|     | $\sigma^2$ | 133.94 | 175.09 | 219.98 | 267.18   | 319.45 | 367.59 | 422.50 | 478.31   | 545.80   | 607.79   | 678.06   |
| 60  | $\mu$      | 18.53  | 21.88  | 25.46  | 29.02    | 33.03  | 37.15  | 41.22  | 45.53    | 49.81    | 54.29    | 58.80    |
|     | $\sigma^2$ | 130.39 | 174.47 | 220.18 | 263.49   | 313.30 | 355.17 | 402.17 | 453.16   | 504.30   | 559.72   | 609.49   |
| 70  | $\mu$      | 18.39  | 21.70  | 25.25  | 28.95    | 32.73  | 36.71  | 40.90  | 44.94    | 49.28    | 53.61    | 58.01    |
|     | $\sigma^2$ | 130.27 | 172.69 | 215.20 | 258.83   | 303.69 | 346.92 | 390.63 | 439.43   | 482.76   | 528.20   | 578.88   |
| 80  | $\mu$      | 18.31  | 21.58  | 25.14  | 28.83    | 32.66  | 36.57  | 40.57  | 44.75    | 48.78    | 53.16    | 57.39    |
|     | $\sigma^2$ | 127.01 | 170.69 | 214.63 | 258.06   | 297.01 | 344.40 | 380.54 | 423.14   | 469.83   | 508.55   | 558.83   |
| 90  | $\mu$      | 18.36  | 21.55  | 25.03  | 28.78    | 32.56  | 36.47  | 40.49  | 44.39    | 48.60    | 52.69    | 56.85    |
|     | $\sigma^2$ | 127.63 | 168.78 | 210.86 | 257.50   | 291.11 | 336.55 | 380.21 | 420.31   | 459.72   | 502.56   | 546.65   |
| 100 | $\mu$      | 18.28  | 21.48  | 25.01  | 28.72    | 32.44  | 36.30  | 40.32  | 44.40    | 48.40    | 52.50    | 56.66    |
|     | $\sigma^2$ | 126.71 | 165.63 | 210.66 | 253.20   | 291.40 | 332.92 | 377.97 | 416.28   | 454.09   | 491.40   | 535.19   |
| 200 | $\mu$      | 18.07  | 21.36  | 24.76  | 28.52    | 32.12  | 35.85  | 39.70  | 43.65    | 47.57    | 51.62    | 55.53    |
|     | $\sigma^2$ | 123.87 | 161.11 | 203.16 | 243.44   | 283.76 | 322.41 | 358.18 | 398.53   | 428.24   | 474.08   | 501.23   |
| 500 | $\mu$      | 17.97  | 21.16  | 24.62  | 28.21    | 31.91  | 35.72  | 39.46  | 43.29    | 47.25    | 51.27    | 55.08    |
|     | $\sigma^2$ | 121.99 | 162.68 | 201.61 | 235.50   | 276.57 | 319.81 | 355.36 | 385.14   | 422.98   | 453.64   | 480.05   |
|     |            |        |        |        |          |        |        |        |          |          |          | 526.84   |

TABLE 8

*Finite T correction factors  $\mu_{PP}(\rho)$  and  $\sigma_{PP}^2(\rho)$  for the pooled autoregressive coefficient test statistic of Pedroni ( $PP_\rho$ ) for case 1*

| T/l |            | 1     | 2     | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -2.76 | -5.54 | -7.85  | -9.66  | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 21.04 | 37.37 | 40.04  | 32.17  | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -2.76 | -5.94 | -8.90  | -11.53 | -13.87 | -15.95 | -17.80 | -19.38 | -20.79 | -22.12 | -23.23 | -24.31 |
|     | $\sigma^2$ | 22.61 | 46.56 | 60.15  | 58.85  | 52.25  | 46.84  | 40.76  | 39.70  | 36.36  | 33.74  | 32.30  | 28.73  |
| 30  | $\mu$      | -2.79 | -6.03 | -9.35  | -12.37 | -15.13 | -17.64 | -19.96 | -22.11 | -24.07 | -25.88 | -27.58 | -29.15 |
|     | $\sigma^2$ | 23.79 | 52.57 | 67.63  | 64.50  | 69.73  | 63.51  | 60.18  | 56.85  | 54.45  | 52.57  | 51.09  | 49.19  |
| 40  | $\mu$      | -2.79 | -6.12 | -9.59  | -12.77 | -15.81 | -18.63 | -21.28 | -23.76 | -26.02 | -28.22 | -30.32 | -32.19 |
|     | $\sigma^2$ | 24.49 | 55.45 | 73.68  | 77.82  | 75.09  | 78.77  | 77.42  | 72.41  | 72.92  | 72.61  | 69.45  | 68.13  |
| 50  | $\mu$      | -2.78 | -6.17 | -9.68  | -13.05 | -16.30 | -19.27 | -22.04 | -24.83 | -27.37 | -29.85 | -32.18 | -34.36 |
|     | $\sigma^2$ | 24.59 | 56.81 | 79.69  | 81.65  | 81.94  | 83.83  | 83.10  | 85.79  | 84.07  | 82.57  | 84.06  | 83.09  |
| 60  | $\mu$      | -2.77 | -6.28 | -9.78  | -13.25 | -16.54 | -19.70 | -22.74 | -25.64 | -28.39 | -31.00 | -33.54 | -35.89 |
|     | $\sigma^2$ | 24.62 | 59.07 | 82.57  | 84.70  | 85.72  | 87.33  | 90.94  | 93.23  | 94.24  | 95.29  | 98.01  | 96.24  |
| 70  | $\mu$      | -2.77 | -6.27 | -9.86  | -13.34 | -16.80 | -20.03 | -23.17 | -26.22 | -29.08 | -31.82 | -34.50 | -37.20 |
|     | $\sigma^2$ | 24.25 | 56.11 | 80.22  | 89.90  | 97.42  | 96.87  | 98.27  | 101.12 | 103.13 | 105.37 | 107.84 | 107.14 |
| 80  | $\mu$      | -2.77 | -6.26 | -9.86  | -13.48 | -16.95 | -20.28 | -23.54 | -26.59 | -29.61 | -32.57 | -35.35 | -38.11 |
|     | $\sigma^2$ | 24.64 | 60.03 | 84.30  | 92.63  | 91.82  | 98.15  | 106.88 | 109.70 | 107.85 | 113.27 | 115.36 | 117.82 |
| 90  | $\mu$      | -2.78 | -6.25 | -9.91  | -13.58 | -17.06 | -20.47 | -23.75 | -26.89 | -30.12 | -33.12 | -36.05 | -38.83 |
|     | $\sigma^2$ | 24.85 | 59.91 | 80.01  | 91.85  | 104.73 | 103.42 | 108.83 | 114.26 | 116.51 | 121.57 | 122.33 | 126.70 |
| 100 | $\mu$      | -2.79 | -6.29 | -10.01 | -13.62 | -17.17 | -20.59 | -23.90 | -27.22 | -30.51 | -33.57 | -36.51 | -39.49 |
|     | $\sigma^2$ | 24.73 | 59.72 | 79.75  | 99.46  | 102.86 | 115.42 | 115.12 | 118.50 | 120.87 | 126.87 | 133.36 | 134.48 |
| 200 | $\mu$      | -2.78 | -6.35 | -10.15 | -13.94 | -17.68 | -21.31 | -24.98 | -28.66 | -32.19 | -35.68 | -39.16 | -42.55 |
|     | $\sigma^2$ | 26.04 | 60.45 | 88.65  | 103.95 | 110.50 | 119.34 | 129.80 | 139.39 | 146.30 | 154.60 | 165.56 | 173.37 |
| 500 | $\mu$      | -2.78 | -6.35 | -10.24 | -14.12 | -17.95 | -21.85 | -25.63 | -29.54 | -33.35 | -37.08 | -40.96 | -44.63 |
|     | $\sigma^2$ | 25.56 | 63.45 | 93.60  | 107.61 | 128.75 | 131.49 | 141.45 | 152.83 | 165.74 | 177.31 | 187.24 | 200.99 |

TABLE 9

Finite  $T$  correction factors  $\mu_{PP}(\rho)$  and  $\sigma_{PP}^2(\rho)$  for the pooled autoregressive coefficient test statistic of Pedroni ( $PP_\rho$ ) for case 2

| T/l |            | 1     | 2     | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -5.48 | -7.48 | -9.26  | -10.68 | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 16.46 | 22.68 | 22.23  | 21.75  | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -5.72 | -8.42 | -10.99 | -13.32 | -15.37 | -17.23 | -18.90 | -20.35 | -21.66 | -22.87 | -23.96 | -24.91 |
|     | $\sigma^2$ | 22.83 | 34.37 | 40.60  | 41.30  | 41.88  | 39.79  | 36.47  | 33.91  | 31.94  | 29.66  | 27.69  | 26.60  |
| 30  | $\mu$      | -5.84 | -8.78 | -11.71 | -14.43 | -17.00 | -19.30 | -21.49 | -23.46 | -25.35 | -27.07 | -28.63 | -30.13 |
|     | $\sigma^2$ | 25.37 | 40.89 | 50.86  | 57.36  | 57.11  | 58.21  | 56.37  | 53.58  | 51.56  | 49.35  | 49.00  | 45.94  |
| 40  | $\mu$      | -5.85 | -8.95 | -12.11 | -15.04 | -17.93 | -20.51 | -23.09 | -25.40 | -27.61 | -29.70 | -31.70 | -33.51 |
|     | $\sigma^2$ | 26.82 | 43.97 | 56.30  | 66.54  | 67.02  | 72.55  | 69.10  | 69.53  | 68.18  | 67.18  | 66.53  | 63.80  |
| 50  | $\mu$      | -5.92 | -9.08 | -12.30 | -15.43 | -18.49 | -21.34 | -24.07 | -26.70 | -29.14 | -31.55 | -33.73 | -35.86 |
|     | $\sigma^2$ | 28.08 | 47.13 | 61.70  | 70.62  | 77.22  | 80.68  | 80.63  | 80.47  | 82.29  | 81.40  | 81.16  | 82.05  |
| 60  | $\mu$      | -5.93 | -9.11 | -12.46 | -15.69 | -18.87 | -21.92 | -24.84 | -27.57 | -30.29 | -32.80 | -35.25 | -37.62 |
|     | $\sigma^2$ | 28.28 | 48.85 | 64.47  | 75.85  | 81.34  | 86.77  | 88.36  | 91.17  | 93.95  | 93.11  | 93.66  | 95.47  |
| 70  | $\mu$      | -5.94 | -9.24 | -12.57 | -15.95 | -19.19 | -22.30 | -25.39 | -28.24 | -31.10 | -33.80 | -36.39 | -38.92 |
|     | $\sigma^2$ | 28.95 | 49.39 | 66.00  | 77.28  | 85.18  | 93.77  | 95.81  | 101.17 | 103.79 | 103.61 | 106.31 | 107.24 |
| 80  | $\mu$      | -5.95 | -9.28 | -12.63 | -16.04 | -19.45 | -22.62 | -25.78 | -28.84 | -31.75 | -34.57 | -37.31 | -39.99 |
|     | $\sigma^2$ | 28.80 | 50.86 | 68.20  | 79.76  | 90.45  | 100.04 | 103.86 | 105.08 | 109.02 | 112.19 | 115.73 | 115.65 |
| 90  | $\mu$      | -5.97 | -9.31 | -12.74 | -16.23 | -19.62 | -22.91 | -26.12 | -29.31 | -32.28 | -35.19 | -38.09 | -40.77 |
|     | $\sigma^2$ | 29.54 | 51.33 | 68.94  | 83.35  | 93.87  | 101.73 | 106.19 | 111.71 | 114.86 | 121.24 | 120.69 | 125.18 |
| 100 | $\mu$      | -6.00 | -9.29 | -12.82 | -16.31 | -19.78 | -23.16 | -26.35 | -29.63 | -32.70 | -35.73 | -38.77 | -41.49 |
|     | $\sigma^2$ | 30.10 | 52.53 | 69.56  | 83.67  | 93.94  | 105.44 | 111.35 | 115.67 | 119.04 | 124.47 | 130.80 | 132.69 |
| 200 | $\mu$      | -5.99 | -9.43 | -13.08 | -16.78 | -20.44 | -24.04 | -27.71 | -31.24 | -34.74 | -38.23 | -41.74 | -45.06 |
|     | $\sigma^2$ | 30.81 | 54.91 | 76.24  | 93.00  | 108.65 | 118.31 | 128.08 | 141.83 | 151.01 | 159.67 | 167.31 | 174.69 |
| 500 | $\mu$      | -5.99 | -9.50 | -13.22 | -17.04 | -20.91 | -24.76 | -28.58 | -32.36 | -36.18 | -40.03 | -43.68 | -47.42 |
|     | $\sigma^2$ | 31.02 | 56.74 | 80.44  | 97.27  | 116.91 | 133.62 | 142.39 | 159.57 | 169.89 | 178.96 | 192.00 | 206.90 |

TABLE 10

Finite  $T$  correction factors  $\mu_{PP}(\rho)$  and  $\sigma_{PP}^2(\rho)$  for the pooled autoregressive coefficient test statistic of Pedroni ( $PP_\rho$ ) for case 3

| T/l |            | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -8.66  | -9.96  | -11.10 | -12.14 | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 13.70  | 15.49  | 16.13  | 16.06  | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -9.59  | -11.75 | -13.80 | -15.73 | -17.45 | -19.02 | -20.43 | -21.79 | -22.92 | -24.01 | -24.99 | -25.87 |
|     | $\sigma^2$ | 24.85  | 30.55  | 33.79  | 34.82  | 34.88  | 35.20  | 32.80  | 29.93  | 28.57  | 26.63  | 25.57  | 24.94  |
| 30  | $\mu$      | -9.96  | -12.47 | -14.93 | -17.27 | -19.57 | -21.69 | -23.61 | -25.45 | -27.10 | -28.71 | -30.18 | -31.52 |
|     | $\sigma^2$ | 30.13  | 39.52  | 46.35  | 50.13  | 51.10  | 51.61  | 50.83  | 49.96  | 48.56  | 46.64  | 44.71  | 43.53  |
| 40  | $\mu$      | -10.10 | -12.83 | -15.57 | -18.27 | -20.76 | -23.25 | -25.54 | -27.69 | -29.81 | -31.70 | -33.51 | -35.25 |
|     | $\sigma^2$ | 33.27  | 43.29  | 53.05  | 58.44  | 62.65  | 64.91  | 65.22  | 66.83  | 65.15  | 64.28  | 64.03  | 61.82  |
| 50  | $\mu$      | -10.19 | -13.08 | -15.96 | -18.84 | -21.58 | -24.28 | -26.82 | -29.25 | -31.60 | -33.75 | -35.88 | -37.89 |
|     | $\sigma^2$ | 35.33  | 47.06  | 57.30  | 64.37  | 71.62  | 75.31  | 77.55  | 78.87  | 80.47  | 79.73  | 79.82  | 77.96  |
| 60  | $\mu$      | -10.26 | -13.26 | -16.24 | -19.21 | -22.17 | -25.03 | -27.75 | -30.38 | -32.90 | -35.31 | -37.65 | -39.86 |
|     | $\sigma^2$ | 36.38  | 50.49  | 62.65  | 71.26  | 79.65  | 82.64  | 87.53  | 89.94  | 90.98  | 92.33  | 92.89  | 92.40  |
| 70  | $\mu$      | -10.30 | -13.29 | -16.41 | -19.50 | -22.50 | -25.51 | -28.38 | -31.19 | -33.87 | -36.47 | -39.05 | -41.34 |
|     | $\sigma^2$ | 38.03  | 52.47  | 65.25  | 75.64  | 84.38  | 90.35  | 94.46  | 98.34  | 100.74 | 102.65 | 104.54 | 105.49 |
| 80  | $\mu$      | -10.34 | -13.35 | -16.55 | -19.73 | -22.88 | -25.94 | -28.94 | -31.84 | -34.65 | -37.44 | -40.07 | -42.63 |
|     | $\sigma^2$ | 38.25  | 54.43  | 67.87  | 78.88  | 87.49  | 96.83  | 100.23 | 104.67 | 108.64 | 110.99 | 114.24 | 113.63 |
| 90  | $\mu$      | -10.40 | -13.46 | -16.67 | -19.94 | -23.15 | -26.32 | -29.41 | -32.32 | -35.32 | -38.14 | -40.87 | -43.60 |
|     | $\sigma^2$ | 38.99  | 54.95  | 69.03  | 82.23  | 91.06  | 99.91  | 105.26 | 112.20 | 114.96 | 119.97 | 124.00 | 124.81 |
| 100 | $\mu$      | -10.41 | -13.48 | -16.78 | -20.08 | -23.32 | -26.53 | -29.73 | -32.83 | -35.84 | -38.74 | -41.58 | -44.36 |
|     | $\sigma^2$ | 39.07  | 55.42  | 70.66  | 83.99  | 93.82  | 103.20 | 111.73 | 117.25 | 121.31 | 125.55 | 130.09 | 134.37 |
| 200 | $\mu$      | -10.51 | -13.79 | -17.21 | -20.82 | -24.33 | -27.87 | -31.42 | -34.84 | -38.39 | -41.80 | -45.08 | -48.42 |
|     | $\sigma^2$ | 41.68  | 59.75  | 77.91  | 94.44  | 109.66 | 120.36 | 133.30 | 143.45 | 152.21 | 163.89 | 171.56 | 178.23 |
| 500 | $\mu$      | -10.54 | -13.92 | -17.50 | -21.20 | -24.96 | -28.70 | -32.47 | -36.22 | -40.01 | -43.78 | -47.41 | -51.20 |
|     | $\sigma^2$ | 42.79  | 63.72  | 82.80  | 100.19 | 118.49 | 135.85 | 148.41 | 162.62 | 178.04 | 187.21 | 197.97 | 215.35 |

TABLE 11

*Finite T correction factors  $\mu_{PP}(t)$  and  $\sigma_{PP}^2(t)$  for the pooled t-value test statistic of Pedroni ( $PP_t$ ) for case 1*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -0.83 | -1.15 | -1.23 | -1.17 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 1.06  | 0.85  | 0.64  | 0.52  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -0.92 | -1.41 | -1.69 | -1.85 | -1.93 | -1.94 | -1.92 | -1.86 | -1.77 | -1.65 | -1.51 | -1.36 |
|     | $\sigma^2$ | 1.24  | 1.09  | 0.89  | 0.66  | 0.51  | 0.42  | 0.37  | 0.35  | 0.34  | 0.32  | 0.31  | 0.29  |
| 30  | $\mu$      | -0.96 | -1.49 | -1.85 | -2.09 | -2.25 | -2.36 | -2.42 | -2.46 | -2.47 | -2.45 | -2.41 | -2.36 |
|     | $\sigma^2$ | 1.33  | 1.25  | 0.97  | 0.69  | 0.61  | 0.49  | 0.43  | 0.39  | 0.36  | 0.34  | 0.33  | 0.32  |
| 40  | $\mu$      | -0.97 | -1.53 | -1.93 | -2.21 | -2.42 | -2.57 | -2.69 | -2.78 | -2.83 | -2.87 | -2.89 | -2.89 |
|     | $\sigma^2$ | 1.39  | 1.31  | 1.06  | 0.82  | 0.64  | 0.56  | 0.49  | 0.44  | 0.40  | 0.38  | 0.36  | 0.35  |
| 50  | $\mu$      | -0.98 | -1.56 | -1.97 | -2.28 | -2.52 | -2.71 | -2.85 | -2.97 | -3.07 | -3.14 | -3.19 | -3.23 |
|     | $\sigma^2$ | 1.42  | 1.34  | 1.13  | 0.85  | 0.67  | 0.59  | 0.51  | 0.47  | 0.43  | 0.40  | 0.38  | 0.37  |
| 60  | $\mu$      | -0.98 | -1.58 | -2.00 | -2.33 | -2.58 | -2.79 | -2.97 | -3.11 | -3.23 | -3.32 | -3.40 | -3.46 |
|     | $\sigma^2$ | 1.44  | 1.40  | 1.18  | 0.88  | 0.70  | 0.60  | 0.54  | 0.49  | 0.46  | 0.43  | 0.41  | 0.39  |
| 70  | $\mu$      | -0.98 | -1.59 | -2.03 | -2.36 | -2.64 | -2.86 | -3.04 | -3.21 | -3.34 | -3.45 | -3.55 | -3.64 |
|     | $\sigma^2$ | 1.42  | 1.33  | 1.14  | 0.92  | 0.77  | 0.64  | 0.58  | 0.52  | 0.48  | 0.46  | 0.44  | 0.41  |
| 80  | $\mu$      | -0.99 | -1.60 | -2.04 | -2.39 | -2.67 | -2.91 | -3.11 | -3.28 | -3.43 | -3.56 | -3.67 | -3.77 |
|     | $\sigma^2$ | 1.44  | 1.44  | 1.22  | 0.96  | 0.73  | 0.65  | 0.62  | 0.55  | 0.50  | 0.47  | 0.45  | 0.43  |
| 90  | $\mu$      | -0.99 | -1.60 | -2.05 | -2.41 | -2.70 | -2.95 | -3.15 | -3.33 | -3.50 | -3.64 | -3.76 | -3.87 |
|     | $\sigma^2$ | 1.46  | 1.44  | 1.16  | 0.95  | 0.83  | 0.68  | 0.61  | 0.57  | 0.52  | 0.49  | 0.46  | 0.44  |
| 100 | $\mu$      | -1.00 | -1.61 | -2.07 | -2.43 | -2.72 | -2.97 | -3.19 | -3.38 | -3.55 | -3.70 | -3.83 | -3.95 |
|     | $\sigma^2$ | 1.45  | 1.43  | 1.14  | 1.01  | 0.81  | 0.75  | 0.64  | 0.59  | 0.53  | 0.51  | 0.49  | 0.46  |
| 200 | $\mu$      | -1.00 | -1.64 | -2.12 | -2.50 | -2.83 | -3.11 | -3.36 | -3.60 | -3.80 | -4.00 | -4.17 | -4.34 |
|     | $\sigma^2$ | 1.55  | 1.45  | 1.26  | 1.05  | 0.86  | 0.76  | 0.70  | 0.65  | 0.60  | 0.57  | 0.56  | 0.54  |
| 500 | $\mu$      | -1.01 | -1.65 | -2.15 | -2.55 | -2.89 | -3.20 | -3.47 | -3.73 | -3.96 | -4.18 | -4.39 | -4.58 |
|     | $\sigma^2$ | 1.52  | 1.52  | 1.33  | 1.08  | 0.98  | 0.82  | 0.74  | 0.69  | 0.66  | 0.63  | 0.60  | 0.59  |

TABLE 12

*Finite T correction factors  $\mu_{PP}(t)$  and  $\sigma_{PP}^2(t)$  for the pooled t-value test statistic of Pedroni ( $PP_t$ ) for case 2*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -1.39 | -1.43 | -1.38 | -1.24 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 0.57  | 0.52  | 0.46  | 0.47  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -1.56 | -1.80 | -1.96 | -2.04 | -2.06 | -2.05 | -2.00 | -1.91 | -1.80 | -1.68 | -1.53 | -1.37 |
|     | $\sigma^2$ | 0.70  | 0.63  | 0.55  | 0.47  | 0.44  | 0.40  | 0.36  | 0.35  | 0.33  | 0.32  | 0.31  | 0.32  |
| 30  | $\mu$      | -1.62 | -1.92 | -2.16 | -2.32 | -2.44 | -2.51 | -2.55 | -2.56 | -2.55 | -2.52 | -2.47 | -2.40 |
|     | $\sigma^2$ | 0.75  | 0.71  | 0.64  | 0.57  | 0.49  | 0.44  | 0.40  | 0.38  | 0.36  | 0.34  | 0.33  | 0.32  |
| 40  | $\mu$      | -1.64 | -1.99 | -2.27 | -2.47 | -2.63 | -2.75 | -2.84 | -2.90 | -2.95 | -2.97 | -2.97 | -2.97 |
|     | $\sigma^2$ | 0.80  | 0.76  | 0.67  | 0.62  | 0.53  | 0.50  | 0.44  | 0.41  | 0.38  | 0.37  | 0.35  | 0.34  |
| 50  | $\mu$      | -1.67 | -2.03 | -2.32 | -2.56 | -2.75 | -2.90 | -3.02 | -3.12 | -3.20 | -3.25 | -3.30 | -3.32 |
|     | $\sigma^2$ | 0.85  | 0.80  | 0.73  | 0.65  | 0.58  | 0.54  | 0.47  | 0.44  | 0.42  | 0.39  | 0.38  | 0.36  |
| 60  | $\mu$      | -1.68 | -2.05 | -2.36 | -2.62 | -2.83 | -3.01 | -3.15 | -3.27 | -3.37 | -3.45 | -3.52 | -3.57 |
|     | $\sigma^2$ | 0.84  | 0.84  | 0.75  | 0.68  | 0.61  | 0.56  | 0.50  | 0.46  | 0.45  | 0.42  | 0.39  | 0.39  |
| 70  | $\mu$      | -1.69 | -2.08 | -2.39 | -2.67 | -2.89 | -3.08 | -3.24 | -3.37 | -3.49 | -3.59 | -3.68 | -3.75 |
|     | $\sigma^2$ | 0.86  | 0.83  | 0.77  | 0.69  | 0.62  | 0.58  | 0.52  | 0.50  | 0.47  | 0.44  | 0.42  | 0.40  |
| 80  | $\mu$      | -1.69 | -2.09 | -2.41 | -2.69 | -2.93 | -3.13 | -3.31 | -3.46 | -3.59 | -3.70 | -3.80 | -3.89 |
|     | $\sigma^2$ | 0.86  | 0.85  | 0.79  | 0.70  | 0.65  | 0.61  | 0.55  | 0.51  | 0.48  | 0.46  | 0.44  | 0.41  |
| 90  | $\mu$      | -1.70 | -2.10 | -2.44 | -2.73 | -2.97 | -3.18 | -3.36 | -3.53 | -3.67 | -3.79 | -3.90 | -3.99 |
|     | $\sigma^2$ | 0.88  | 0.86  | 0.80  | 0.73  | 0.67  | 0.61  | 0.56  | 0.53  | 0.49  | 0.48  | 0.44  | 0.44  |
| 100 | $\mu$      | -1.71 | -2.10 | -2.45 | -2.75 | -3.00 | -3.22 | -3.40 | -3.58 | -3.72 | -3.86 | -3.99 | -4.08 |
|     | $\sigma^2$ | 0.89  | 0.88  | 0.80  | 0.74  | 0.67  | 0.63  | 0.58  | 0.54  | 0.50  | 0.48  | 0.47  | 0.45  |
| 200 | $\mu$      | -1.72 | -2.15 | -2.52 | -2.84 | -3.12 | -3.37 | -3.61 | -3.82 | -4.01 | -4.19 | -4.36 | -4.51 |
|     | $\sigma^2$ | 0.91  | 0.92  | 0.87  | 0.80  | 0.74  | 0.68  | 0.64  | 0.61  | 0.59  | 0.56  | 0.54  | 0.52  |
| 500 | $\mu$      | -1.72 | -2.17 | -2.56 | -2.90 | -3.20 | -3.48 | -3.73 | -3.97 | -4.19 | -4.40 | -4.59 | -4.77 |
|     | $\sigma^2$ | 0.92  | 0.95  | 0.91  | 0.83  | 0.78  | 0.74  | 0.68  | 0.66  | 0.63  | 0.60  | 0.58  | 0.58  |

TABLE 13

*Finite T correction factors  $\mu_{PP}(t)$  and  $\sigma_{PP}^2(t)$  for the pooled t-value test statistic of Pedroni ( $PP_t$ ) for case 3*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -1.62 | -1.48 | -1.30 | -1.07 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 0.39  | 0.40  | 0.43  | 0.48  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -1.97 | -2.05 | -2.10 | -2.11 | -2.07 | -2.01 | -1.93 | -1.81 | -1.68 | -1.54 | -1.37 | -1.20 |
|     | $\sigma^2$ | 0.45  | 0.43  | 0.41  | 0.38  | 0.36  | 0.35  | 0.34  | 0.32  | 0.31  | 0.30  | 0.30  | 0.29  |
| 30  | $\mu$      | -2.08 | -2.25 | -2.38 | -2.47 | -2.53 | -2.57 | -2.57 | -2.56 | -2.52 | -2.47 | -2.41 | -2.33 |
|     | $\sigma^2$ | 0.50  | 0.49  | 0.47  | 0.43  | 0.40  | 0.38  | 0.36  | 0.34  | 0.33  | 0.32  | 0.31  | 0.31  |
| 40  | $\mu$      | -2.14 | -2.35 | -2.52 | -2.66 | -2.77 | -2.86 | -2.91 | -2.96 | -2.98 | -2.98 | -2.97 | -2.95 |
|     | $\sigma^2$ | 0.53  | 0.52  | 0.50  | 0.47  | 0.44  | 0.41  | 0.39  | 0.38  | 0.36  | 0.34  | 0.33  | 0.33  |
| 50  | $\mu$      | -2.17 | -2.41 | -2.61 | -2.78 | -2.92 | -3.04 | -3.13 | -3.21 | -3.26 | -3.30 | -3.33 | -3.34 |
|     | $\sigma^2$ | 0.56  | 0.54  | 0.52  | 0.49  | 0.47  | 0.45  | 0.43  | 0.40  | 0.39  | 0.37  | 0.36  | 0.34  |
| 60  | $\mu$      | -2.19 | -2.45 | -2.67 | -2.86 | -3.03 | -3.17 | -3.28 | -3.38 | -3.46 | -3.52 | -3.58 | -3.61 |
|     | $\sigma^2$ | 0.57  | 0.57  | 0.55  | 0.53  | 0.50  | 0.47  | 0.45  | 0.43  | 0.41  | 0.39  | 0.38  | 0.36  |
| 70  | $\mu$      | -2.21 | -2.47 | -2.71 | -2.92 | -3.09 | -3.25 | -3.39 | -3.50 | -3.60 | -3.68 | -3.76 | -3.81 |
|     | $\sigma^2$ | 0.59  | 0.59  | 0.57  | 0.54  | 0.52  | 0.49  | 0.46  | 0.44  | 0.42  | 0.41  | 0.39  | 0.38  |
| 80  | $\mu$      | -2.22 | -2.49 | -2.74 | -2.96 | -3.15 | -3.32 | -3.47 | -3.60 | -3.71 | -3.81 | -3.90 | -3.97 |
|     | $\sigma^2$ | 0.59  | 0.61  | 0.59  | 0.56  | 0.53  | 0.52  | 0.48  | 0.46  | 0.44  | 0.42  | 0.41  | 0.40  |
| 90  | $\mu$      | -2.23 | -2.51 | -2.77 | -3.00 | -3.20 | -3.38 | -3.54 | -3.67 | -3.80 | -3.91 | -4.00 | -4.09 |
|     | $\sigma^2$ | 0.60  | 0.61  | 0.59  | 0.58  | 0.55  | 0.53  | 0.49  | 0.48  | 0.46  | 0.44  | 0.43  | 0.41  |
| 100 | $\mu$      | -2.24 | -2.52 | -2.79 | -3.02 | -3.23 | -3.42 | -3.59 | -3.74 | -3.87 | -3.99 | -4.09 | -4.19 |
|     | $\sigma^2$ | 0.60  | 0.62  | 0.60  | 0.59  | 0.56  | 0.53  | 0.52  | 0.49  | 0.47  | 0.45  | 0.44  | 0.42  |
| 200 | $\mu$      | -2.27 | -2.59 | -2.88 | -3.16 | -3.40 | -3.62 | -3.83 | -4.01 | -4.20 | -4.36 | -4.51 | -4.65 |
|     | $\sigma^2$ | 0.63  | 0.65  | 0.65  | 0.64  | 0.62  | 0.59  | 0.58  | 0.56  | 0.53  | 0.53  | 0.51  | 0.49  |
| 500 | $\mu$      | -2.29 | -2.62 | -2.94 | -3.23 | -3.50 | -3.74 | -3.97 | -4.19 | -4.40 | -4.59 | -4.77 | -4.95 |
|     | $\sigma^2$ | 0.65  | 0.69  | 0.68  | 0.67  | 0.66  | 0.65  | 0.62  | 0.61  | 0.60  | 0.57  | 0.55  | 0.56  |

TABLE 14

*Finite T correction factors  $\mu_{PG}(\rho)$  and  $\sigma_{PG}^2(\rho)$  for the group-mean autoregressive coefficient test statistic of Pedroni ( $PG_\rho$ ) for case 1*

| T/l |            | 1     | 2     | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -4.91 | -7.52 | -9.49  | -10.96 | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 13.30 | 13.21 | 12.03  | 10.44  | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -5.33 | -8.57 | -11.33 | -13.68 | -15.82 | -17.65 | -19.29 | -20.75 | -22.04 | -23.24 | -24.27 | -25.23 |
|     | $\sigma^2$ | 18.30 | 22.99 | 24.97  | 25.46  | 25.48  | 24.43  | 23.22  | 22.15  | 20.68  | 19.22  | 18.07  | 16.59  |
| 30  | $\mu$      | -5.49 | -8.94 | -12.09 | -14.92 | -17.50 | -19.82 | -21.99 | -23.97 | -25.80 | -27.49 | -29.06 | -30.56 |
|     | $\sigma^2$ | 20.57 | 27.82 | 32.70  | 35.96  | 37.15  | 37.97  | 38.51  | 37.82  | 37.05  | 36.12  | 34.77  | 33.82  |
| 40  | $\mu$      | -5.56 | -9.21 | -12.56 | -15.61 | -18.47 | -21.12 | -23.63 | -25.97 | -28.12 | -30.22 | -32.18 | -33.96 |
|     | $\sigma^2$ | 21.88 | 31.22 | 37.89  | 42.46  | 46.13  | 48.55  | 50.32  | 51.05  | 51.62  | 51.60  | 50.61  | 50.22  |
| 50  | $\mu$      | -5.64 | -9.36 | -12.76 | -16.03 | -19.12 | -21.99 | -24.64 | -27.29 | -29.70 | -32.06 | -34.29 | -36.40 |
|     | $\sigma^2$ | 22.98 | 33.17 | 40.92  | 47.44  | 52.18  | 56.81  | 59.58  | 61.52  | 63.19  | 63.59  | 63.72  | 64.34  |
| 60  | $\mu$      | -5.67 | -9.47 | -13.02 | -16.33 | -19.51 | -22.54 | -25.49 | -28.28 | -30.92 | -33.41 | -35.89 | -38.16 |
|     | $\sigma^2$ | 23.63 | 34.48 | 43.94  | 51.40  | 57.41  | 62.10  | 66.54  | 69.84  | 72.18  | 74.18  | 75.78  | 76.85  |
| 70  | $\mu$      | -5.68 | -9.52 | -13.12 | -16.54 | -19.90 | -23.02 | -26.02 | -29.00 | -31.78 | -34.41 | -37.02 | -39.62 |
|     | $\sigma^2$ | 23.85 | 35.67 | 45.69  | 53.96  | 61.41  | 67.07  | 71.97  | 76.44  | 80.35  | 82.29  | 85.84  | 87.40  |
| 80  | $\mu$      | -5.70 | -9.57 | -13.18 | -16.76 | -20.06 | -23.33 | -26.54 | -29.50 | -32.39 | -35.29 | -37.97 | -40.67 |
|     | $\sigma^2$ | 24.10 | 36.52 | 47.01  | 56.60  | 63.64  | 70.19  | 77.30  | 82.48  | 86.28  | 90.26  | 92.54  | 96.50  |
| 90  | $\mu$      | -5.72 | -9.58 | -13.25 | -16.88 | -20.31 | -23.60 | -26.84 | -29.93 | -33.00 | -35.96 | -38.79 | -41.53 |
|     | $\sigma^2$ | 24.48 | 37.15 | 47.63  | 57.83  | 66.37  | 74.57  | 80.73  | 87.99  | 92.10  | 96.92  | 100.98 | 103.53 |
| 100 | $\mu$      | -5.73 | -9.61 | -13.37 | -16.96 | -20.47 | -23.83 | -27.08 | -30.31 | -33.49 | -36.51 | -39.39 | -42.28 |
|     | $\sigma^2$ | 24.55 | 37.38 | 49.21  | 58.89  | 68.96  | 77.32  | 84.53  | 91.22  | 96.94  | 102.00 | 106.12 | 111.35 |
| 200 | $\mu$      | -5.81 | -9.80 | -13.70 | -17.47 | -21.19 | -24.86 | -28.50 | -32.11 | -35.60 | -39.03 | -42.50 | -45.83 |
|     | $\sigma^2$ | 25.97 | 40.28 | 54.19  | 66.51  | 79.02  | 90.74  | 102.48 | 111.58 | 121.99 | 130.65 | 140.24 | 148.82 |
| 500 | $\mu$      | -5.83 | -9.88 | -13.86 | -17.75 | -21.61 | -25.54 | -29.35 | -33.25 | -37.00 | -40.77 | -44.55 | -48.31 |
|     | $\sigma^2$ | 26.75 | 41.79 | 56.44  | 70.76  | 85.11  | 99.77  | 113.86 | 128.33 | 140.89 | 154.79 | 165.24 | 179.11 |

TABLE 15

Finite  $T$  correction factors  $\mu_{PG}(\rho)$  and  $\sigma_{PG}^2(\rho)$  for the group-mean autoregressive coefficient test statistic of Pedroni ( $PG_\rho$ ) for case 2

| T/l |            | 1     | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -7.08 | -9.00  | -10.54 | -11.77 | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 11.71 | 11.36  | 10.13  | 8.94   | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -7.92 | -10.66 | -13.11 | -15.23 | -17.11 | -18.79 | -20.29 | -21.60 | -22.81 | -23.90 | -24.89 | -25.80 |
|     | $\sigma^2$ | 19.76 | 22.99  | 24.21  | 24.22  | 23.85  | 23.00  | 21.63  | 20.40  | 19.31  | 17.81  | 16.63  | 15.51  |
| 30  | $\mu$      | -8.31 | -11.35 | -14.21 | -16.81 | -19.22 | -21.37 | -23.42 | -25.24 | -26.98 | -28.59 | -30.07 | -31.46 |
|     | $\sigma^2$ | 24.07 | 29.96  | 33.44  | 35.75  | 37.04  | 37.26  | 37.27  | 36.59  | 35.66  | 34.59  | 33.53  | 32.43  |
| 40  | $\mu$      | -8.46 | -11.73 | -14.82 | -17.70 | -20.41 | -22.89 | -25.32 | -27.51 | -29.61 | -31.59 | -33.48 | -35.19 |
|     | $\sigma^2$ | 26.67 | 34.26  | 39.46  | 43.73  | 46.61  | 48.40  | 49.53  | 49.94  | 50.06  | 50.15  | 49.50  | 48.44  |
| 50  | $\mu$      | -8.61 | -11.97 | -15.21 | -18.24 | -21.21 | -23.95 | -26.56 | -29.06 | -31.42 | -33.68 | -35.78 | -37.83 |
|     | $\sigma^2$ | 28.34 | 37.06  | 44.32  | 49.31  | 53.74  | 57.28  | 59.42  | 61.24  | 62.66  | 62.92  | 63.50  | 63.48  |
| 60  | $\mu$      | -8.67 | -12.13 | -15.47 | -18.67 | -21.75 | -24.70 | -27.48 | -30.14 | -32.74 | -35.17 | -37.50 | -39.80 |
|     | $\sigma^2$ | 29.31 | 39.39  | 47.09  | 53.92  | 59.75  | 64.21  | 67.41  | 70.59  | 72.32  | 74.09  | 74.79  | 76.30  |
| 70  | $\mu$      | -8.74 | -12.28 | -15.67 | -19.03 | -22.16 | -25.20 | -28.20 | -30.98 | -33.73 | -36.32 | -38.83 | -41.29 |
|     | $\sigma^2$ | 30.36 | 40.93  | 49.75  | 58.15  | 63.87  | 69.30  | 73.67  | 77.93  | 81.03  | 83.03  | 85.25  | 87.38  |
| 80  | $\mu$      | -8.77 | -12.38 | -15.82 | -19.16 | -22.52 | -25.63 | -28.70 | -31.64 | -34.49 | -37.22 | -39.91 | -42.48 |
|     | $\sigma^2$ | 31.13 | 42.27  | 51.62  | 59.89  | 67.51  | 73.32  | 79.68  | 83.68  | 87.66  | 91.30  | 94.43  | 95.60  |
| 90  | $\mu$      | -8.83 | -12.43 | -15.96 | -19.43 | -22.76 | -25.98 | -29.10 | -32.23 | -35.13 | -37.98 | -40.76 | -43.40 |
|     | $\sigma^2$ | 31.49 | 43.24  | 53.33  | 62.70  | 70.70  | 77.83  | 83.99  | 89.42  | 94.11  | 98.59  | 101.49 | 104.60 |
| 100 | $\mu$      | -8.88 | -12.46 | -16.05 | -19.55 | -22.99 | -26.30 | -29.43 | -32.62 | -35.61 | -38.59 | -41.56 | -44.22 |
|     | $\sigma^2$ | 32.28 | 44.15  | 54.66  | 64.73  | 73.14  | 81.10  | 87.31  | 93.60  | 99.19  | 103.79 | 109.05 | 111.77 |
| 200 | $\mu$      | -8.97 | -12.74 | -16.49 | -20.26 | -23.95 | -27.52 | -31.15 | -34.67 | -38.14 | -41.55 | -45.02 | -48.30 |
|     | $\sigma^2$ | 34.17 | 48.00  | 61.26  | 73.46  | 86.09  | 97.09  | 107.89 | 117.79 | 127.42 | 134.85 | 143.97 | 151.98 |
| 500 | $\mu$      | -9.04 | -12.92 | -16.77 | -20.67 | -24.56 | -28.42 | -32.23 | -36.06 | -39.84 | -43.65 | -47.32 | -51.07 |
|     | $\sigma^2$ | 35.79 | 50.70  | 65.24  | 79.92  | 93.91  | 107.60 | 120.80 | 135.53 | 148.01 | 159.64 | 173.89 | 187.52 |

TABLE 16

Finite  $T$  correction factors  $\mu_{PG}(\rho)$  and  $\sigma_{PG}^2(\rho)$  for the group-mean autoregressive coefficient test statistic of Pedroni ( $PG_\rho$ ) for case 3

| T/l |            | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-----|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10  | $\mu$      | -9.72  | -10.95 | -12.01 | -12.92 | -      | -      | -      | -      | -      | -      | -      | -      |
|     | $\sigma^2$ | 9.59   | 9.12   | 8.30   | 7.29   | -      | -      | -      | -      | -      | -      | -      | -      |
| 20  | $\mu$      | -11.43 | -13.58 | -15.53 | -17.33 | -18.93 | -20.40 | -21.68 | -22.91 | -23.95 | -24.93 | -25.84 | -26.64 |
|     | $\sigma^2$ | 21.56  | 23.33  | 23.83  | 23.39  | 22.66  | 21.64  | 20.34  | 18.99  | 17.75  | 16.55  | 15.36  | 14.31  |
| 30  | $\mu$      | -12.15 | -14.71 | -17.12 | -19.38 | -21.54 | -23.55 | -25.34 | -27.08 | -28.63 | -30.11 | -31.51 | -32.77 |
|     | $\sigma^2$ | 28.83  | 33.01  | 35.35  | 36.78  | 36.98  | 37.09  | 36.25  | 35.51  | 34.53  | 33.35  | 32.17  | 30.79  |
| 40  | $\mu$      | -12.49 | -15.27 | -18.04 | -20.65 | -23.07 | -25.45 | -27.62 | -29.67 | -31.68 | -33.47 | -35.21 | -36.83 |
|     | $\sigma^2$ | 33.14  | 38.74  | 43.52  | 46.38  | 48.38  | 49.37  | 49.98  | 50.27  | 49.88  | 49.54  | 48.74  | 47.52  |
| 50  | $\mu$      | -12.71 | -15.70 | -18.61 | -21.43 | -24.10 | -26.73 | -29.15 | -31.49 | -33.74 | -35.81 | -37.85 | -39.76 |
|     | $\sigma^2$ | 36.16  | 43.74  | 49.47  | 53.63  | 56.99  | 59.67  | 61.37  | 62.39  | 62.90  | 63.42  | 63.63  | 62.69  |
| 60  | $\mu$      | -12.88 | -15.99 | -19.02 | -21.97 | -24.88 | -27.64 | -30.27 | -32.83 | -35.25 | -37.58 | -39.81 | -41.93 |
|     | $\sigma^2$ | 38.55  | 46.60  | 53.26  | 59.15  | 63.94  | 67.13  | 70.16  | 72.37  | 74.03  | 75.68  | 75.44  | 75.57  |
| 70  | $\mu$      | -12.99 | -16.12 | -19.31 | -22.38 | -25.33 | -28.28 | -31.07 | -33.78 | -36.36 | -38.91 | -41.40 | -43.64 |
|     | $\sigma^2$ | 40.25  | 48.80  | 57.20  | 63.73  | 69.06  | 74.20  | 78.24  | 80.81  | 83.05  | 85.66  | 86.93  | 88.53  |
| 80  | $\mu$      | -13.06 | -16.26 | -19.54 | -22.72 | -25.80 | -28.85 | -31.73 | -34.56 | -37.30 | -39.98 | -42.56 | -45.02 |
|     | $\sigma^2$ | 41.18  | 51.38  | 60.10  | 67.90  | 73.83  | 80.13  | 84.01  | 88.30  | 91.53  | 93.99  | 96.20  | 97.64  |
| 90  | $\mu$      | -13.16 | -16.43 | -19.69 | -22.99 | -26.15 | -29.29 | -32.29 | -35.16 | -38.07 | -40.81 | -43.49 | -46.14 |
|     | $\sigma^2$ | 42.33  | 52.88  | 61.97  | 70.25  | 78.00  | 84.40  | 89.26  | 93.74  | 98.36  | 101.00 | 104.65 | 107.30 |
| 100 | $\mu$      | -13.20 | -16.47 | -19.85 | -23.17 | -26.40 | -29.57 | -32.72 | -35.75 | -38.69 | -41.51 | -44.31 | -47.03 |
|     | $\sigma^2$ | 43.01  | 54.09  | 64.23  | 72.92  | 80.92  | 88.01  | 93.72  | 99.31  | 103.73 | 108.48 | 111.93 | 116.28 |
| 200 | $\mu$      | -13.46 | -16.95 | -20.51 | -24.17 | -27.72 | -31.27 | -34.80 | -38.20 | -41.70 | -45.09 | -48.33 | -51.60 |
|     | $\sigma^2$ | 47.43  | 60.47  | 73.07  | 85.14  | 96.12  | 107.48 | 117.94 | 127.18 | 136.67 | 145.11 | 153.91 | 159.01 |
| 500 | $\mu$      | -13.60 | -17.24 | -20.97 | -24.75 | -28.58 | -32.33 | -36.11 | -39.84 | -43.63 | -47.39 | -50.99 | -54.80 |
|     | $\sigma^2$ | 50.00  | 64.85  | 79.48  | 94.51  | 108.19 | 121.00 | 134.85 | 147.31 | 160.24 | 173.52 | 183.94 | 197.79 |



TABLE 17

*Finite T correction factors  $\mu_{PG}(t)$  and  $\sigma_{PG}^2(t)$  for the group-mean t-value test statistic of Pedroni ( $PG_t$ ) for case 1*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -1.08 | -1.33 | -1.36 | -1.27 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 0.54  | 0.34  | 0.28  | 0.25  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -1.23 | -1.65 | -1.88 | -1.99 | -2.04 | -2.04 | -2.00 | -1.93 | -1.83 | -1.71 | -1.57 | -1.42 |
|     | $\sigma^2$ | 0.64  | 0.46  | 0.37  | 0.33  | 0.31  | 0.29  | 0.28  | 0.27  | 0.25  | 0.25  | 0.23  | 0.22  |
| 30  | $\mu$      | -1.29 | -1.76 | -2.06 | -2.26 | -2.39 | -2.47 | -2.52 | -2.55 | -2.54 | -2.52 | -2.48 | -2.41 |
|     | $\sigma^2$ | 0.67  | 0.52  | 0.43  | 0.38  | 0.35  | 0.33  | 0.31  | 0.30  | 0.29  | 0.28  | 0.27  | 0.27  |
| 40  | $\mu$      | -1.31 | -1.82 | -2.16 | -2.40 | -2.58 | -2.71 | -2.81 | -2.88 | -2.92 | -2.96 | -2.97 | -2.96 |
|     | $\sigma^2$ | 0.70  | 0.54  | 0.46  | 0.42  | 0.39  | 0.36  | 0.34  | 0.33  | 0.32  | 0.31  | 0.30  | 0.29  |
| 50  | $\mu$      | -1.33 | -1.86 | -2.21 | -2.48 | -2.69 | -2.85 | -2.98 | -3.09 | -3.17 | -3.23 | -3.28 | -3.31 |
|     | $\sigma^2$ | 0.72  | 0.57  | 0.49  | 0.44  | 0.41  | 0.39  | 0.37  | 0.36  | 0.34  | 0.33  | 0.32  | 0.31  |
| 60  | $\mu$      | -1.34 | -1.88 | -2.25 | -2.53 | -2.76 | -2.95 | -3.10 | -3.23 | -3.34 | -3.42 | -3.50 | -3.55 |
|     | $\sigma^2$ | 0.73  | 0.58  | 0.51  | 0.46  | 0.43  | 0.40  | 0.39  | 0.37  | 0.36  | 0.35  | 0.34  | 0.32  |
| 70  | $\mu$      | -1.34 | -1.90 | -2.28 | -2.57 | -2.82 | -3.02 | -3.19 | -3.34 | -3.46 | -3.56 | -3.65 | -3.73 |
|     | $\sigma^2$ | 0.73  | 0.59  | 0.52  | 0.48  | 0.44  | 0.42  | 0.40  | 0.38  | 0.37  | 0.36  | 0.36  | 0.34  |
| 80  | $\mu$      | -1.35 | -1.91 | -2.30 | -2.61 | -2.86 | -3.07 | -3.26 | -3.41 | -3.55 | -3.67 | -3.77 | -3.86 |
|     | $\sigma^2$ | 0.74  | 0.60  | 0.53  | 0.49  | 0.45  | 0.43  | 0.42  | 0.40  | 0.39  | 0.37  | 0.36  | 0.36  |
| 90  | $\mu$      | -1.36 | -1.91 | -2.31 | -2.63 | -2.90 | -3.11 | -3.31 | -3.47 | -3.62 | -3.76 | -3.87 | -3.97 |
|     | $\sigma^2$ | 0.74  | 0.61  | 0.53  | 0.50  | 0.47  | 0.45  | 0.43  | 0.42  | 0.40  | 0.39  | 0.38  | 0.37  |
| 100 | $\mu$      | -1.36 | -1.92 | -2.33 | -2.65 | -2.92 | -3.15 | -3.35 | -3.53 | -3.69 | -3.82 | -3.95 | -4.06 |
|     | $\sigma^2$ | 0.74  | 0.61  | 0.54  | 0.50  | 0.48  | 0.46  | 0.44  | 0.43  | 0.41  | 0.40  | 0.39  | 0.38  |
| 200 | $\mu$      | -1.38 | -1.96 | -2.39 | -2.74 | -3.04 | -3.30 | -3.54 | -3.76 | -3.95 | -4.14 | -4.31 | -4.46 |
|     | $\sigma^2$ | 0.77  | 0.63  | 0.57  | 0.54  | 0.52  | 0.50  | 0.49  | 0.47  | 0.46  | 0.46  | 0.45  | 0.44  |
| 500 | $\mu$      | -1.39 | -1.98 | -2.43 | -2.79 | -3.11 | -3.40 | -3.66 | -3.90 | -4.13 | -4.33 | -4.54 | -4.72 |
|     | $\sigma^2$ | 0.78  | 0.64  | 0.59  | 0.56  | 0.54  | 0.53  | 0.52  | 0.52  | 0.51  | 0.50  | 0.49  | 0.49  |

TABLE 18

*Finite T correction factors  $\mu_{PG}(t)$  and  $\sigma_{PG}^2(t)$  for the group-mean t-value test statistic of Pedroni ( $PG_t$ ) for case 2*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -1.52 | -1.54 | -1.47 | -1.33 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 0.40  | 0.33  | 0.29  | 0.26  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -1.76 | -1.98 | -2.10 | -2.15 | -2.16 | -2.13 | -2.07 | -1.98 | -1.87 | -1.74 | -1.59 | -1.43 |
|     | $\sigma^2$ | 0.49  | 0.41  | 0.36  | 0.33  | 0.31  | 0.29  | 0.28  | 0.27  | 0.26  | 0.25  | 0.24  | 0.22  |
| 30  | $\mu$      | -1.85 | -2.13 | -2.33 | -2.47 | -2.56 | -2.61 | -2.64 | -2.64 | -2.62 | -2.58 | -2.53 | -2.46 |
|     | $\sigma^2$ | 0.53  | 0.46  | 0.41  | 0.37  | 0.34  | 0.32  | 0.31  | 0.30  | 0.29  | 0.29  | 0.28  | 0.27  |
| 40  | $\mu$      | -1.89 | -2.21 | -2.45 | -2.63 | -2.77 | -2.87 | -2.95 | -3.00 | -3.03 | -3.04 | -3.04 | -3.03 |
|     | $\sigma^2$ | 0.57  | 0.49  | 0.44  | 0.40  | 0.38  | 0.36  | 0.34  | 0.33  | 0.32  | 0.31  | 0.30  | 0.29  |
| 50  | $\mu$      | -1.92 | -2.26 | -2.52 | -2.73 | -2.90 | -3.03 | -3.14 | -3.23 | -3.29 | -3.34 | -3.37 | -3.40 |
|     | $\sigma^2$ | 0.59  | 0.51  | 0.47  | 0.43  | 0.40  | 0.38  | 0.36  | 0.35  | 0.34  | 0.33  | 0.32  | 0.31  |
| 60  | $\mu$      | -1.94 | -2.29 | -2.57 | -2.80 | -2.99 | -3.15 | -3.28 | -3.38 | -3.47 | -3.55 | -3.60 | -3.65 |
|     | $\sigma^2$ | 0.59  | 0.53  | 0.48  | 0.45  | 0.42  | 0.40  | 0.38  | 0.37  | 0.36  | 0.34  | 0.33  | 0.33  |
| 70  | $\mu$      | -1.95 | -2.32 | -2.61 | -2.85 | -3.05 | -3.23 | -3.38 | -3.50 | -3.61 | -3.70 | -3.77 | -3.84 |
|     | $\sigma^2$ | 0.60  | 0.54  | 0.49  | 0.46  | 0.43  | 0.42  | 0.39  | 0.38  | 0.37  | 0.36  | 0.35  | 0.34  |
| 80  | $\mu$      | -1.96 | -2.34 | -2.63 | -2.88 | -3.11 | -3.29 | -3.45 | -3.59 | -3.71 | -3.81 | -3.90 | -3.98 |
|     | $\sigma^2$ | 0.61  | 0.55  | 0.50  | 0.47  | 0.45  | 0.43  | 0.41  | 0.40  | 0.38  | 0.37  | 0.36  | 0.35  |
| 90  | $\mu$      | -1.97 | -2.35 | -2.66 | -2.92 | -3.15 | -3.34 | -3.51 | -3.66 | -3.79 | -3.90 | -4.01 | -4.09 |
|     | $\sigma^2$ | 0.62  | 0.56  | 0.51  | 0.48  | 0.46  | 0.44  | 0.42  | 0.41  | 0.39  | 0.39  | 0.37  | 0.37  |
| 100 | $\mu$      | -1.98 | -2.36 | -2.68 | -2.94 | -3.18 | -3.38 | -3.55 | -3.71 | -3.85 | -3.98 | -4.09 | -4.19 |
|     | $\sigma^2$ | 0.63  | 0.56  | 0.52  | 0.49  | 0.47  | 0.45  | 0.43  | 0.42  | 0.41  | 0.39  | 0.39  | 0.38  |
| 200 | $\mu$      | -2.00 | -2.41 | -2.75 | -3.05 | -3.32 | -3.55 | -3.77 | -3.97 | -4.15 | -4.32 | -4.48 | -4.63 |
|     | $\sigma^2$ | 0.65  | 0.59  | 0.55  | 0.53  | 0.51  | 0.50  | 0.48  | 0.47  | 0.46  | 0.45  | 0.44  | 0.44  |
| 500 | $\mu$      | -2.02 | -2.44 | -2.80 | -3.12 | -3.41 | -3.67 | -3.91 | -4.13 | -4.34 | -4.55 | -4.73 | -4.91 |
|     | $\sigma^2$ | 0.66  | 0.61  | 0.58  | 0.55  | 0.54  | 0.52  | 0.52  | 0.51  | 0.50  | 0.49  | 0.49  | 0.49  |

TABLE 19

*Finite T correction factors  $\mu_{PG}(t)$  and  $\sigma_{PG}^2(t)$  for the group-mean t-value test statistic of Pedroni ( $PG_t$ ) for case 3*

| T/l |            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|-----|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | $\mu$      | -1.67 | -1.54 | -1.37 | -1.15 | -     | -     | -     | -     | -     | -     | -     | -     |
|     | $\sigma^2$ | 0.32  | 0.30  | 0.27  | 0.25  | -     | -     | -     | -     | -     | -     | -     | -     |
| 20  | $\mu$      | -2.09 | -2.16 | -2.20 | -2.19 | -2.15 | -2.08 | -1.99 | -1.88 | -1.74 | -1.60 | -1.44 | -1.27 |
|     | $\sigma^2$ | 0.37  | 0.35  | 0.33  | 0.31  | 0.30  | 0.28  | 0.27  | 0.26  | 0.25  | 0.24  | 0.23  | 0.21  |
| 30  | $\mu$      | -2.23 | -2.39 | -2.51 | -2.59 | -2.63 | -2.65 | -2.65 | -2.63 | -2.58 | -2.53 | -2.47 | -2.38 |
|     | $\sigma^2$ | 0.42  | 0.39  | 0.37  | 0.34  | 0.33  | 0.31  | 0.30  | 0.29  | 0.28  | 0.28  | 0.27  | 0.27  |
| 40  | $\mu$      | -2.31 | -2.51 | -2.67 | -2.79 | -2.89 | -2.96 | -3.01 | -3.04 | -3.05 | -3.05 | -3.04 | -3.00 |
|     | $\sigma^2$ | 0.44  | 0.42  | 0.40  | 0.37  | 0.35  | 0.34  | 0.33  | 0.32  | 0.31  | 0.30  | 0.29  | 0.29  |
| 50  | $\mu$      | -2.35 | -2.58 | -2.77 | -2.92 | -3.05 | -3.16 | -3.24 | -3.30 | -3.35 | -3.38 | -3.40 | -3.41 |
|     | $\sigma^2$ | 0.47  | 0.44  | 0.42  | 0.39  | 0.38  | 0.36  | 0.35  | 0.34  | 0.33  | 0.32  | 0.31  | 0.30  |
| 60  | $\mu$      | -2.38 | -2.63 | -2.84 | -3.01 | -3.16 | -3.29 | -3.40 | -3.48 | -3.55 | -3.61 | -3.66 | -3.69 |
|     | $\sigma^2$ | 0.48  | 0.46  | 0.43  | 0.41  | 0.40  | 0.38  | 0.37  | 0.35  | 0.35  | 0.33  | 0.33  | 0.32  |
| 70  | $\mu$      | -2.40 | -2.66 | -2.88 | -3.07 | -3.24 | -3.38 | -3.51 | -3.61 | -3.70 | -3.78 | -3.85 | -3.89 |
|     | $\sigma^2$ | 0.49  | 0.47  | 0.45  | 0.43  | 0.41  | 0.39  | 0.38  | 0.37  | 0.36  | 0.35  | 0.34  | 0.34  |
| 80  | $\mu$      | -2.42 | -2.68 | -2.92 | -3.12 | -3.30 | -3.46 | -3.60 | -3.71 | -3.82 | -3.91 | -3.99 | -4.06 |
|     | $\sigma^2$ | 0.50  | 0.49  | 0.46  | 0.44  | 0.42  | 0.41  | 0.40  | 0.38  | 0.37  | 0.36  | 0.35  | 0.35  |
| 90  | $\mu$      | -2.44 | -2.71 | -2.95 | -3.17 | -3.35 | -3.52 | -3.67 | -3.79 | -3.91 | -4.01 | -4.10 | -4.18 |
|     | $\sigma^2$ | 0.50  | 0.49  | 0.47  | 0.45  | 0.44  | 0.42  | 0.40  | 0.39  | 0.39  | 0.37  | 0.37  | 0.36  |
| 100 | $\mu$      | -2.44 | -2.72 | -2.97 | -3.19 | -3.39 | -3.56 | -3.72 | -3.86 | -3.99 | -4.09 | -4.19 | -4.28 |
|     | $\sigma^2$ | 0.51  | 0.50  | 0.48  | 0.46  | 0.44  | 0.43  | 0.42  | 0.41  | 0.39  | 0.38  | 0.38  | 0.37  |
| 200 | $\mu$      | -2.49 | -2.80 | -3.08 | -3.34 | -3.57 | -3.78 | -3.98 | -4.16 | -4.33 | -4.49 | -4.63 | -4.77 |
|     | $\sigma^2$ | 0.53  | 0.53  | 0.51  | 0.50  | 0.49  | 0.48  | 0.47  | 0.46  | 0.45  | 0.44  | 0.44  | 0.43  |
| 500 | $\mu$      | -2.52 | -2.85 | -3.15 | -3.42 | -3.68 | -3.92 | -4.14 | -4.35 | -4.55 | -4.73 | -4.91 | -5.08 |
|     | $\sigma^2$ | 0.56  | 0.55  | 0.54  | 0.54  | 0.52  | 0.51  | 0.51  | 0.50  | 0.49  | 0.49  | 0.48  | 0.48  |

Correction factors for Westerlund tests

TABLE 20

Finite  $T$  correction factors  $\mu_{WP}$  and  $\sigma_{WP}^2$  for the pooled variance ratio test statistic of Westerland ( $WP$ ) for case 1

| T/1 | 1          | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10  | $\mu$      | 0.134103 | 0.090341 | 0.059848 | 0.041406 | —        | —        | —        | —        | —        | —        | —        |
|     | $\sigma^2$ | 0.023753 | 0.024009 | 0.017662 | 0.008442 | —        | —        | —        | —        | —        | —        | —        |
| 20  | $\mu$      | 0.130021 | 0.088235 | 0.058862 | 0.039839 | 0.028563 | 0.020743 | 0.015691 | 0.012337 | 0.010055 | 0.008152 | 0.006888 |
|     | $\sigma^2$ | 0.023179 | 0.023379 | 0.016530 | 0.007786 | 0.005509 | 0.001913 | 0.000898 | 0.000473 | 0.000358 | 0.000189 | 0.000116 |
| 30  | $\mu$      | 0.128439 | 0.087240 | 0.057642 | 0.039486 | 0.027710 | 0.020734 | 0.015549 | 0.012131 | 0.009663 | 0.007863 | 0.006583 |
|     | $\sigma^2$ | 0.022383 | 0.022640 | 0.013141 | 0.007704 | 0.003774 | 0.002095 | 0.000995 | 0.000485 | 0.000301 | 0.000164 | 0.000105 |
| 40  | $\mu$      | 0.127737 | 0.087497 | 0.058260 | 0.039498 | 0.027779 | 0.020296 | 0.015351 | 0.011953 | 0.009524 | 0.007786 | 0.006434 |
|     | $\sigma^2$ | 0.022774 | 0.024238 | 0.013711 | 0.007207 | 0.003176 | 0.001875 | 0.000852 | 0.000526 | 0.000259 | 0.000173 | 0.000116 |
| 50  | $\mu$      | 0.127612 | 0.087269 | 0.057848 | 0.039780 | 0.028155 | 0.020321 | 0.015347 | 0.012032 | 0.009611 | 0.007785 | 0.006466 |
|     | $\sigma^2$ | 0.021886 | 0.022561 | 0.014201 | 0.007641 | 0.004064 | 0.001679 | 0.000885 | 0.000480 | 0.000302 | 0.000172 | 0.000109 |
| 60  | $\mu$      | 0.126132 | 0.086633 | 0.058258 | 0.039536 | 0.027969 | 0.020482 | 0.015398 | 0.011930 | 0.009605 | 0.007668 | 0.006420 |
|     | $\sigma^2$ | 0.022576 | 0.021776 | 0.014674 | 0.006875 | 0.003571 | 0.001828 | 0.000855 | 0.000581 | 0.000298 | 0.000149 | 0.000109 |
| 70  | $\mu$      | 0.126749 | 0.087798 | 0.058745 | 0.039786 | 0.027948 | 0.020570 | 0.015422 | 0.011945 | 0.009578 | 0.007713 | 0.006394 |
|     | $\sigma^2$ | 0.022292 | 0.022105 | 0.014588 | 0.008070 | 0.003696 | 0.001921 | 0.000864 | 0.000504 | 0.000310 | 0.000161 | 0.000110 |
| 80  | $\mu$      | 0.127472 | 0.086622 | 0.058596 | 0.039658 | 0.028004 | 0.020706 | 0.015450 | 0.011933 | 0.009544 | 0.007738 | 0.006446 |
|     | $\sigma^2$ | 0.022729 | 0.022478 | 0.015641 | 0.008356 | 0.003693 | 0.002071 | 0.000910 | 0.000438 | 0.000282 | 0.000227 | 0.000120 |
| 90  | $\mu$      | 0.126532 | 0.086117 | 0.058615 | 0.039483 | 0.027891 | 0.020427 | 0.015473 | 0.011944 | 0.009528 | 0.007766 | 0.006423 |
|     | $\sigma^2$ | 0.022875 | 0.022878 | 0.014966 | 0.008170 | 0.003682 | 0.001673 | 0.000929 | 0.000476 | 0.000277 | 0.000176 | 0.000111 |
| 100 | $\mu$      | 0.126276 | 0.087659 | 0.058451 | 0.039214 | 0.027963 | 0.020574 | 0.015349 | 0.011807 | 0.009502 | 0.007778 | 0.006404 |
|     | $\sigma^2$ | 0.021918 | 0.022825 | 0.015706 | 0.006865 | 0.003634 | 0.001831 | 0.000905 | 0.000441 | 0.000285 | 0.000167 | 0.000103 |
| 200 | $\mu$      | 0.125862 | 0.087576 | 0.058500 | 0.039444 | 0.027873 | 0.020609 | 0.015336 | 0.011902 | 0.009598 | 0.007752 | 0.006418 |
|     | $\sigma^2$ | 0.021627 | 0.023599 | 0.014543 | 0.008208 | 0.003388 | 0.002159 | 0.000870 | 0.000459 | 0.000270 | 0.000182 | 0.000117 |
| 500 | $\mu$      | 0.126553 | 0.087158 | 0.058470 | 0.039744 | 0.028117 | 0.020393 | 0.015524 | 0.012077 | 0.009531 | 0.007764 | 0.006423 |
|     | $\sigma^2$ | 0.022755 | 0.022336 | 0.016309 | 0.007948 | 0.003348 | 0.001691 | 0.000949 | 0.000598 | 0.000283 | 0.000178 | 0.000117 |

TABLE 21

Finite  $T$  correction factors  $\mu_{WP}$  and  $\sigma_{WP}^2$  for the pooled variance ratio test statistic of Westerland ( $WP$ ) for case 2

| T/1 | 1                   | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|-----|---------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10  | $\mu$ 0.047254      | 0.032017 | 0.021742 | 0.015676 | —        | —        | —        | —        | —        | —        | —        | —        |
|     | $\sigma^2$ 0.001690 | 0.001583 | 0.001050 | 0.000672 | —        | —        | —        | —        | —        | —        | —        | —        |
| 20  | $\mu$ 0.044920      | 0.029869 | 0.019839 | 0.013613 | 0.009629 | 0.007118 | 0.005514 | 0.004410 | 0.003651 | 0.003092 | 0.002674 | 0.002344 |
|     | $\sigma^2$ 0.001615 | 0.001476 | 0.000988 | 0.000518 | 0.000305 | 0.000108 | 0.000053 | 0.000026 | 0.000016 | 0.000008 | 0.000004 | 0.000003 |
| 30  | $\mu$ 0.044752      | 0.029176 | 0.019242 | 0.013140 | 0.009232 | 0.006730 | 0.005190 | 0.004111 | 0.003309 | 0.002753 | 0.002328 | 0.002002 |
|     | $\sigma^2$ 0.001628 | 0.001439 | 0.000921 | 0.000520 | 0.000213 | 0.000105 | 0.000076 | 0.000034 | 0.000014 | 0.000008 | 0.000004 | 0.000003 |
| 40  | $\mu$ 0.044586      | 0.029360 | 0.019189 | 0.012981 | 0.009127 | 0.006661 | 0.005080 | 0.003963 | 0.003186 | 0.002620 | 0.002187 | 0.001873 |
|     | $\sigma^2$ 0.001638 | 0.001448 | 0.000904 | 0.000517 | 0.000239 | 0.000104 | 0.000057 | 0.000028 | 0.000014 | 0.000007 | 0.000004 | 0.000003 |
| 50  | $\mu$ 0.044771      | 0.028932 | 0.019108 | 0.012929 | 0.009053 | 0.006620 | 0.004985 | 0.003878 | 0.003128 | 0.002558 | 0.002131 | 0.001827 |
|     | $\sigma^2$ 0.001656 | 0.001425 | 0.000925 | 0.000513 | 0.000240 | 0.000122 | 0.000049 | 0.000027 | 0.000014 | 0.000008 | 0.000004 | 0.000003 |
| 60  | $\mu$ 0.044430      | 0.028739 | 0.019168 | 0.012873 | 0.009049 | 0.006574 | 0.004932 | 0.003877 | 0.003106 | 0.002527 | 0.002115 | 0.001792 |
|     | $\sigma^2$ 0.001663 | 0.001406 | 0.000940 | 0.000498 | 0.000262 | 0.000104 | 0.000045 | 0.000025 | 0.000014 | 0.000008 | 0.000004 | 0.000003 |
| 70  | $\mu$ 0.044702      | 0.028932 | 0.018992 | 0.012897 | 0.008975 | 0.006604 | 0.004928 | 0.003864 | 0.003057 | 0.002517 | 0.002095 | 0.001768 |
|     | $\sigma^2$ 0.001672 | 0.001457 | 0.000856 | 0.000480 | 0.000256 | 0.000153 | 0.000051 | 0.000026 | 0.000012 | 0.000009 | 0.000004 | 0.000002 |
| 80  | $\mu$ 0.044662      | 0.028925 | 0.019105 | 0.012893 | 0.008994 | 0.006506 | 0.004935 | 0.003840 | 0.003042 | 0.002492 | 0.002076 | 0.001769 |
|     | $\sigma^2$ 0.001641 | 0.001387 | 0.000924 | 0.000498 | 0.000241 | 0.000107 | 0.000051 | 0.000036 | 0.000013 | 0.000007 | 0.000004 | 0.000003 |
| 90  | $\mu$ 0.044378      | 0.029038 | 0.019095 | 0.012799 | 0.008984 | 0.006511 | 0.004935 | 0.003818 | 0.003045 | 0.002494 | 0.002067 | 0.001753 |
|     | $\sigma^2$ 0.001641 | 0.001402 | 0.000915 | 0.000527 | 0.000241 | 0.000124 | 0.000052 | 0.000024 | 0.000012 | 0.000007 | 0.000004 | 0.000003 |
| 100 | $\mu$ 0.044509      | 0.029144 | 0.019101 | 0.012680 | 0.008978 | 0.006533 | 0.004862 | 0.003814 | 0.003042 | 0.002481 | 0.002066 | 0.001750 |
|     | $\sigma^2$ 0.001681 | 0.001419 | 0.000948 | 0.000427 | 0.000256 | 0.000125 | 0.000044 | 0.000024 | 0.000012 | 0.000007 | 0.000004 | 0.000002 |
| 200 | $\mu$ 0.044339      | 0.029044 | 0.018997 | 0.012613 | 0.008947 | 0.006467 | 0.004939 | 0.003772 | 0.003028 | 0.002464 | 0.002046 | 0.001724 |
|     | $\sigma^2$ 0.001633 | 0.001443 | 0.000960 | 0.000456 | 0.000249 | 0.000110 | 0.000068 | 0.000026 | 0.000011 | 0.000007 | 0.000004 | 0.000003 |
| 500 | $\mu$ 0.044387      | 0.028966 | 0.019123 | 0.012849 | 0.009033 | 0.006463 | 0.004891 | 0.003791 | 0.003028 | 0.002476 | 0.002031 | 0.001718 |
|     | $\sigma^2$ 0.001657 | 0.001398 | 0.000939 | 0.000517 | 0.000276 | 0.000100 | 0.000059 | 0.000025 | 0.000012 | 0.000008 | 0.000004 | 0.000002 |

TABLE 22

Finite  $T$  correction factors  $\mu_{WP}$  and  $\sigma_{WP}^2$  for the pooled variance ratio test statistic of Westerland ( $WP$ ) for case 3

| T/1 | 1          | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10  | $\mu$      | 0.012730 | 0.010672 | 0.008966 | 0.007687 | —        | —        | —        | —        | —        | —        | —        |
|     | $\sigma^2$ | 0.000048 | 0.000043 | 0.000035 | 0.000029 | —        | —        | —        | —        | —        | —        | —        |
| 20  | $\mu$      | 0.011001 | 0.008786 | 0.007101 | 0.005734 | 0.004736 | 0.003982 | 0.003372 | 0.002916 | 0.002553 | 0.002266 | 0.001855 |
|     | $\sigma^2$ | 0.000047 | 0.000039 | 0.000032 | 0.000022 | 0.000015 | 0.000011 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 30  | $\mu$      | 0.010624 | 0.008457 | 0.006722 | 0.005418 | 0.004382 | 0.003598 | 0.003018 | 0.002557 | 0.002194 | 0.001918 | 0.001692 |
|     | $\sigma^2$ | 0.000046 | 0.000040 | 0.000029 | 0.000022 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 40  | $\mu$      | 0.010505 | 0.008336 | 0.006592 | 0.005268 | 0.004257 | 0.003500 | 0.002886 | 0.002433 | 0.002087 | 0.001794 | 0.001564 |
|     | $\sigma^2$ | 0.000047 | 0.000039 | 0.000031 | 0.000022 | 0.000014 | 0.000010 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 50  | $\mu$      | 0.010484 | 0.008274 | 0.006546 | 0.005196 | 0.004194 | 0.003424 | 0.002833 | 0.002374 | 0.002018 | 0.001737 | 0.001509 |
|     | $\sigma^2$ | 0.000046 | 0.000039 | 0.000031 | 0.000021 | 0.000014 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 60  | $\mu$      | 0.010435 | 0.008215 | 0.006538 | 0.005186 | 0.004178 | 0.003380 | 0.002807 | 0.002346 | 0.001987 | 0.001703 | 0.001477 |
|     | $\sigma^2$ | 0.000045 | 0.000038 | 0.000030 | 0.000021 | 0.000015 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 70  | $\mu$      | 0.010402 | 0.008205 | 0.006503 | 0.005139 | 0.004152 | 0.003379 | 0.002784 | 0.002326 | 0.001968 | 0.001686 | 0.001455 |
|     | $\sigma^2$ | 0.000045 | 0.000039 | 0.000030 | 0.000021 | 0.000015 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 80  | $\mu$      | 0.010388 | 0.008174 | 0.006471 | 0.005153 | 0.004144 | 0.003373 | 0.002782 | 0.002316 | 0.001958 | 0.001678 | 0.001446 |
|     | $\sigma^2$ | 0.000045 | 0.000039 | 0.000030 | 0.000022 | 0.000015 | 0.000009 | 0.000006 | 0.000004 | 0.000002 | 0.000001 | 0.000001 |
| 90  | $\mu$      | 0.010419 | 0.008218 | 0.006489 | 0.005150 | 0.004121 | 0.003355 | 0.002767 | 0.002299 | 0.001955 | 0.001666 | 0.001436 |
|     | $\sigma^2$ | 0.000045 | 0.000040 | 0.000030 | 0.000022 | 0.000015 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 100 | $\mu$      | 0.010409 | 0.008227 | 0.006459 | 0.005150 | 0.004122 | 0.003371 | 0.002761 | 0.002308 | 0.001946 | 0.001663 | 0.001427 |
|     | $\sigma^2$ | 0.000046 | 0.000039 | 0.000030 | 0.000021 | 0.000014 | 0.000010 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 200 | $\mu$      | 0.010385 | 0.008174 | 0.006468 | 0.005101 | 0.004106 | 0.003333 | 0.002727 | 0.002277 | 0.001924 | 0.001641 | 0.001233 |
|     | $\sigma^2$ | 0.000046 | 0.000039 | 0.000029 | 0.000021 | 0.000014 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |
| 500 | $\mu$      | 0.010391 | 0.008218 | 0.006429 | 0.005109 | 0.004097 | 0.003330 | 0.002740 | 0.002287 | 0.001915 | 0.001637 | 0.001225 |
|     | $\sigma^2$ | 0.000046 | 0.000039 | 0.000029 | 0.000021 | 0.000014 | 0.000009 | 0.000006 | 0.000004 | 0.000003 | 0.000002 | 0.000001 |

TABLE 23

Finite  $T$  correction factors  $\mu_{WG}$  and  $\sigma_{WG}^2$  for the group-mean variance ratio test statistic of Westerland ( $WG$ ) for case 1

| T/1 | 1          | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10  | $\mu$      | 0.097280 | 0.062168 | 0.041707 | 0.029765 | —        | —        | —        | —        | —        | —        | —        |
|     | $\sigma^2$ | 0.006535 | 0.003618 | 0.001855 | 0.000996 | —        | —        | —        | —        | —        | —        | —        |
| 20  | $\mu$      | 0.094159 | 0.060167 | 0.040204 | 0.027977 | 0.020670 | 0.015628 | 0.012248 | 0.009874 | 0.008156 | 0.006809 | 0.005869 |
|     | $\sigma^2$ | 0.006367 | 0.003594 | 0.001898 | 0.000994 | 0.000563 | 0.000319 | 0.000188 | 0.000118 | 0.000077 | 0.000050 | 0.000035 |
| 30  | $\mu$      | 0.093253 | 0.059739 | 0.039924 | 0.027793 | 0.020274 | 0.015393 | 0.011995 | 0.009622 | 0.007835 | 0.006528 | 0.005556 |
|     | $\sigma^2$ | 0.006264 | 0.003569 | 0.001886 | 0.001007 | 0.000557 | 0.000322 | 0.000192 | 0.000120 | 0.000078 | 0.000052 | 0.000036 |
| 40  | $\mu$      | 0.093045 | 0.059747 | 0.040104 | 0.027893 | 0.020345 | 0.015247 | 0.011901 | 0.009470 | 0.007748 | 0.006447 | 0.005409 |
|     | $\sigma^2$ | 0.006313 | 0.003604 | 0.001901 | 0.001011 | 0.000566 | 0.000317 | 0.000194 | 0.000121 | 0.000077 | 0.000051 | 0.000036 |
| 50  | $\mu$      | 0.093674 | 0.059925 | 0.039915 | 0.027897 | 0.020311 | 0.015320 | 0.011843 | 0.009528 | 0.007751 | 0.006403 | 0.005425 |
|     | $\sigma^2$ | 0.006336 | 0.003620 | 0.001897 | 0.001031 | 0.000563 | 0.000324 | 0.000193 | 0.000123 | 0.000080 | 0.000053 | 0.000037 |
| 60  | $\mu$      | 0.092438 | 0.059770 | 0.039945 | 0.027881 | 0.020247 | 0.015314 | 0.011856 | 0.009403 | 0.007746 | 0.006349 | 0.005359 |
|     | $\sigma^2$ | 0.006223 | 0.003588 | 0.001909 | 0.001023 | 0.000565 | 0.000327 | 0.000194 | 0.000121 | 0.000080 | 0.000052 | 0.000037 |
| 70  | $\mu$      | 0.092707 | 0.060086 | 0.040101 | 0.028007 | 0.020303 | 0.015356 | 0.011872 | 0.009470 | 0.007740 | 0.006353 | 0.005353 |
|     | $\sigma^2$ | 0.006266 | 0.003637 | 0.001915 | 0.001039 | 0.000567 | 0.000330 | 0.000192 | 0.000122 | 0.000080 | 0.000053 | 0.000036 |
| 80  | $\mu$      | 0.092990 | 0.059679 | 0.040178 | 0.027801 | 0.020363 | 0.015383 | 0.011912 | 0.009467 | 0.007690 | 0.006343 | 0.005377 |
|     | $\sigma^2$ | 0.006286 | 0.003613 | 0.001960 | 0.001014 | 0.000564 | 0.000332 | 0.000196 | 0.000121 | 0.000080 | 0.000052 | 0.000038 |
| 90  | $\mu$      | 0.092902 | 0.059475 | 0.040038 | 0.027731 | 0.020213 | 0.015297 | 0.011904 | 0.009457 | 0.007682 | 0.006399 | 0.005351 |
|     | $\sigma^2$ | 0.006301 | 0.003555 | 0.001934 | 0.001013 | 0.000565 | 0.000326 | 0.000199 | 0.000123 | 0.000079 | 0.000054 | 0.000037 |
| 100 | $\mu$      | 0.092761 | 0.060109 | 0.039912 | 0.027805 | 0.020303 | 0.015355 | 0.011817 | 0.009372 | 0.007663 | 0.006391 | 0.005367 |
|     | $\sigma^2$ | 0.006279 | 0.003681 | 0.001899 | 0.001007 | 0.000567 | 0.000331 | 0.000196 | 0.000120 | 0.000079 | 0.000054 | 0.000037 |
| 200 | $\mu$      | 0.092586 | 0.059996 | 0.040131 | 0.027805 | 0.020329 | 0.015301 | 0.011848 | 0.009421 | 0.007752 | 0.006360 | 0.005359 |
|     | $\sigma^2$ | 0.006262 | 0.003658 | 0.001926 | 0.001021 | 0.000570 | 0.000332 | 0.000193 | 0.000121 | 0.000082 | 0.000054 | 0.000037 |
| 500 | $\mu$      | 0.092833 | 0.059939 | 0.039880 | 0.027927 | 0.020456 | 0.015321 | 0.011953 | 0.009471 | 0.007720 | 0.006383 | 0.005357 |
|     | $\sigma^2$ | 0.006280 | 0.003640 | 0.001903 | 0.001040 | 0.000576 | 0.000328 | 0.000201 | 0.000125 | 0.000080 | 0.000054 | 0.000037 |

TABLE 24

Finite  $T$  correction factors  $\mu_{WG}$  and  $\sigma_{WG}^2$  for the group-mean variance ratio test statistic of Westerland ( $WG$ ) for case 2

| T/1 | 1          | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10  | $\mu$      | 0.033845 | 0.022279 | 0.015499 | 0.011554 | —        | —        | —        | —        | —        | —        | —        |
|     | $\sigma^2$ | 0.000534 | 0.000286 | 0.000136 | 0.000067 | —        | —        | —        | —        | —        | —        | —        |
| 20  | $\mu$      | 0.031467 | 0.020144 | 0.013503 | 0.009566 | 0.007055 | 0.004388 | 0.003619 | 0.003070 | 0.002656 | 0.002337 | 0.002086 |
|     | $\sigma^2$ | 0.000519 | 0.000278 | 0.000135 | 0.000065 | 0.000031 | 0.000016 | 0.000008 | 0.000003 | 0.000002 | 0.000001 | 0.000001 |
| 30  | $\mu$      | 0.031119 | 0.019598 | 0.013061 | 0.009119 | 0.006714 | 0.005094 | 0.004026 | 0.003279 | 0.002725 | 0.002313 | 0.001994 |
|     | $\sigma^2$ | 0.000520 | 0.000273 | 0.000131 | 0.000063 | 0.000031 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 40  | $\mu$      | 0.030974 | 0.019634 | 0.012962 | 0.009010 | 0.006567 | 0.005006 | 0.003926 | 0.003155 | 0.002601 | 0.002183 | 0.001862 |
|     | $\sigma^2$ | 0.000517 | 0.000276 | 0.000133 | 0.000063 | 0.000032 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 50  | $\mu$      | 0.031012 | 0.019428 | 0.012872 | 0.008931 | 0.006514 | 0.004938 | 0.003848 | 0.003084 | 0.002547 | 0.002125 | 0.001801 |
|     | $\sigma^2$ | 0.000521 | 0.000272 | 0.000132 | 0.000063 | 0.000031 | 0.000016 | 0.000008 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 60  | $\mu$      | 0.030854 | 0.019335 | 0.012873 | 0.008909 | 0.006475 | 0.004905 | 0.003817 | 0.003065 | 0.002517 | 0.002094 | 0.001782 |
|     | $\sigma^2$ | 0.000515 | 0.000267 | 0.000133 | 0.000063 | 0.000031 | 0.000016 | 0.000008 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 70  | $\mu$      | 0.030937 | 0.019337 | 0.012838 | 0.008912 | 0.006459 | 0.004882 | 0.003804 | 0.003050 | 0.002484 | 0.002081 | 0.001761 |
|     | $\sigma^2$ | 0.000520 | 0.000269 | 0.000130 | 0.000063 | 0.000031 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 80  | $\mu$      | 0.030938 | 0.019390 | 0.012803 | 0.008898 | 0.006470 | 0.004861 | 0.003800 | 0.003032 | 0.002469 | 0.002064 | 0.001745 |
|     | $\sigma^2$ | 0.000522 | 0.000274 | 0.000133 | 0.000064 | 0.000031 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 90  | $\mu$      | 0.030816 | 0.019383 | 0.012828 | 0.008885 | 0.006442 | 0.004844 | 0.003781 | 0.003020 | 0.002467 | 0.002059 | 0.001735 |
|     | $\sigma^2$ | 0.000516 | 0.000273 | 0.000132 | 0.000063 | 0.000031 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 100 | $\mu$      | 0.030807 | 0.019439 | 0.012813 | 0.008851 | 0.006436 | 0.004853 | 0.003763 | 0.003023 | 0.002466 | 0.002049 | 0.001732 |
|     | $\sigma^2$ | 0.000518 | 0.000275 | 0.000133 | 0.000062 | 0.000031 | 0.000016 | 0.000008 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 200 | $\mu$      | 0.030743 | 0.019368 | 0.012756 | 0.008795 | 0.006430 | 0.004821 | 0.003773 | 0.002985 | 0.002456 | 0.002033 | 0.001715 |
|     | $\sigma^2$ | 0.000518 | 0.000273 | 0.000131 | 0.000061 | 0.000031 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |
| 500 | $\mu$      | 0.030787 | 0.019406 | 0.012816 | 0.008845 | 0.006434 | 0.004814 | 0.003747 | 0.002990 | 0.002443 | 0.002032 | 0.001705 |
|     | $\sigma^2$ | 0.000517 | 0.000273 | 0.000133 | 0.000062 | 0.000032 | 0.000016 | 0.000009 | 0.000005 | 0.000003 | 0.000002 | 0.000001 |

TABLE 25

Finite  $T$  correction factors  $\mu_{WG}$  and  $\sigma_{WG}^2$  for the group-mean variance ratio test statistic of Westerland ( $WG$ ) for case 3

| T/l | 1          | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11        | 12        |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|-----------|
| 10  | $\mu$      | 0.010960 | 0.009182 | 0.007771 | 0.006731 | -        | -        | -        | -        | -        | -         | -         |
|     | $\sigma^2$ | 0.000022 | 0.000016 | 0.000011 | 0.000008 | -        | -        | -        | -        | -        | -         | -         |
| 20  | $\mu$      | 0.009083 | 0.007217 | 0.005833 | 0.004756 | 0.003381 | 0.002909 | 0.002553 | 0.002270 | 0.002038 | 0.001853  | 0.001701  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000011 | 0.000007 | 0.000005 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |
| 30  | $\mu$      | 0.008691 | 0.006856 | 0.005458 | 0.004424 | 0.003612 | 0.002557 | 0.002195 | 0.001911 | 0.001685 | 0.001503  | 0.001349  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000005 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |
| 40  | $\mu$      | 0.008563 | 0.006733 | 0.005320 | 0.004267 | 0.003489 | 0.002899 | 0.002429 | 0.002071 | 0.001794 | 0.001565  | 0.001377  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000005 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000004 | 0.0000003 |
| 50  | $\mu$      | 0.008540 | 0.006673 | 0.005272 | 0.004220 | 0.003431 | 0.002834 | 0.002370 | 0.002010 | 0.001732 | 0.001508  | 0.001322  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000004 | 0.0000003 |
| 60  | $\mu$      | 0.008497 | 0.006632 | 0.005259 | 0.004197 | 0.003409 | 0.002795 | 0.002345 | 0.001987 | 0.001703 | 0.001471  | 0.001290  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000005 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |
| 70  | $\mu$      | 0.008475 | 0.006618 | 0.005230 | 0.004160 | 0.003384 | 0.002786 | 0.002319 | 0.001964 | 0.001680 | 0.001456  | 0.001270  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000004 | 0.0000003 |
| 80  | $\mu$      | 0.008455 | 0.006582 | 0.005201 | 0.004154 | 0.003366 | 0.002780 | 0.002318 | 0.001953 | 0.001674 | 0.001445  | 0.001260  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |
| 90  | $\mu$      | 0.008481 | 0.006599 | 0.005211 | 0.004154 | 0.003359 | 0.002771 | 0.002305 | 0.001941 | 0.001666 | 0.001435  | 0.001251  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |
| 100 | $\mu$      | 0.008453 | 0.006606 | 0.005184 | 0.004156 | 0.003358 | 0.002768 | 0.002300 | 0.001944 | 0.001659 | 0.001432  | 0.001243  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000004 | 0.0000003 |
| 200 | $\mu$      | 0.008441 | 0.006568 | 0.005193 | 0.004116 | 0.003347 | 0.002743 | 0.002273 | 0.001919 | 0.001638 | 0.001409  | 0.001226  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000004 | 0.0000003 |
| 500 | $\mu$      | 0.008437 | 0.006594 | 0.005166 | 0.004120 | 0.003336 | 0.002739 | 0.002277 | 0.001921 | 0.001631 | 0.001408  | 0.001223  |
|     | $\sigma^2$ | 0.000021 | 0.000015 | 0.000010 | 0.000007 | 0.000004 | 0.000003 | 0.000002 | 0.000001 | 0.000001 | 0.0000005 | 0.0000003 |



Correction factors for system tests

TABLE 26

*Finite T correction factors  $\mu_{LR}$  and  $\sigma_{LR}^2$  for case 1*

| T/m-k |            | 1    | 2     | 3     | 4     | 5     | 6      | 7      | 8      | 9      | 10     | 11     | 12     |
|-------|------------|------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| 10    | $\mu$      | 1.00 | 4.97  | 11.40 | 19.78 | –     | –      | –      | –      | –      | –      | –      | –      |
|       | $\sigma^2$ | 1.75 | 8.32  | 19.39 | 34.71 | –     | –      | –      | –      | –      | –      | –      | –      |
| 20    | $\mu$      | 1.06 | 5.45  | 12.90 | 23.03 | 35.61 | 50.24  | 66.71  | 84.95  | 104.78 | 125.83 | 148.15 | 171.74 |
|       | $\sigma^2$ | 1.92 | 9.05  | 21.03 | 37.62 | 58.62 | 84.27  | 113.31 | 148.14 | 184.39 | 226.09 | 271.19 | 320.41 |
| 30    | $\mu$      | 1.09 | 5.66  | 13.54 | 24.47 | 38.20 | 54.51  | 73.08  | 93.92  | 116.69 | 141.48 | 167.88 | 195.98 |
|       | $\sigma^2$ | 2.01 | 9.48  | 21.75 | 39.14 | 61.75 | 88.48  | 117.93 | 152.72 | 193.06 | 237.08 | 284.01 | 336.21 |
| 40    | $\mu$      | 1.10 | 5.74  | 13.88 | 25.30 | 39.71 | 56.88  | 76.86  | 99.28  | 123.98 | 150.95 | 180.01 | 211.09 |
|       | $\sigma^2$ | 2.06 | 9.73  | 22.29 | 40.10 | 63.23 | 89.36  | 121.90 | 158.13 | 198.57 | 244.92 | 293.25 | 346.30 |
| 50    | $\mu$      | 1.10 | 5.82  | 14.09 | 25.79 | 40.62 | 58.49  | 79.28  | 102.71 | 128.85 | 157.32 | 188.24 | 221.35 |
|       | $\sigma^2$ | 2.05 | 9.90  | 22.95 | 41.23 | 63.97 | 91.55  | 124.31 | 161.67 | 202.75 | 247.66 | 298.42 | 357.47 |
| 60    | $\mu$      | 1.12 | 5.85  | 14.28 | 26.11 | 41.31 | 59.67  | 80.99  | 105.26 | 132.33 | 161.95 | 194.07 | 228.72 |
|       | $\sigma^2$ | 2.15 | 9.83  | 23.44 | 41.90 | 65.21 | 92.74  | 127.47 | 164.55 | 207.47 | 254.13 | 304.52 | 358.61 |
| 70    | $\mu$      | 1.12 | 5.91  | 14.37 | 26.39 | 41.81 | 60.47  | 82.26  | 107.12 | 134.89 | 165.41 | 198.64 | 234.42 |
|       | $\sigma^2$ | 2.14 | 10.08 | 23.44 | 42.19 | 66.12 | 94.90  | 128.93 | 166.05 | 208.39 | 257.97 | 307.79 | 369.28 |
| 80    | $\mu$      | 1.12 | 5.94  | 14.45 | 26.56 | 42.16 | 61.08  | 83.33  | 108.53 | 136.97 | 168.09 | 202.06 | 238.98 |
|       | $\sigma^2$ | 2.12 | 10.09 | 23.68 | 42.41 | 66.40 | 95.85  | 129.08 | 169.05 | 211.98 | 262.02 | 313.65 | 371.03 |
| 90    | $\mu$      | 1.12 | 5.96  | 14.53 | 26.77 | 42.42 | 61.60  | 84.06  | 109.80 | 138.45 | 170.22 | 204.96 | 242.70 |
|       | $\sigma^2$ | 2.13 | 10.22 | 23.77 | 43.50 | 66.64 | 96.19  | 130.49 | 170.71 | 213.41 | 260.29 | 317.19 | 375.56 |
| 100   | $\mu$      | 1.13 | 5.96  | 14.57 | 26.84 | 42.69 | 62.04  | 84.73  | 110.63 | 139.85 | 171.96 | 207.29 | 245.47 |
|       | $\sigma^2$ | 2.16 | 10.27 | 23.97 | 43.18 | 67.53 | 96.97  | 132.94 | 170.64 | 215.28 | 266.49 | 320.99 | 377.86 |
| 200   | $\mu$      | 1.13 | 6.04  | 14.83 | 27.44 | 43.88 | 63.90  | 87.75  | 115.12 | 146.11 | 180.60 | 218.52 | 259.80 |
|       | $\sigma^2$ | 2.19 | 10.37 | 24.48 | 44.17 | 69.79 | 100.86 | 136.66 | 178.12 | 224.39 | 279.63 | 336.48 | 394.70 |
| 500   | $\mu$      | 1.14 | 6.09  | 14.96 | 27.82 | 44.56 | 65.17  | 89.67  | 118.03 | 150.13 | 186.00 | 225.74 | 269.25 |
|       | $\sigma^2$ | 2.22 | 10.60 | 24.81 | 45.29 | 71.04 | 103.68 | 141.29 | 183.52 | 233.34 | 289.01 | 344.85 | 413.23 |





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